COSMOLOGICAL FUZZY DARK MATTER SIMULATIONS

Differences to cold dark matter

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Outline

Introduction

- Motivation and theoretical background
- The fuzzy dark matter equations
- Impact of fuzzy dark matter on structure formation
  - #1: Initial conditions
  - #2: Dynamics

Numerical methods and challenges

Simulations and results

- Dark matter power spectrum
- Halo mass function
- Dark matter halo profiles

Summary

What is “fuzzy dark matter”?

- F(C)DM, BECDM, ULDM, ELBDM, ψDM, quantum-wave DM, (ultra-light) axion(-like) DM (ULA, ALP)…
- New **extremely light scalar** particle ($m \approx 10^{-22}$ eV!)
- Non-thermal production mechanism (thus not ultra-hot)
- Aggregations of bosons can form a **Bose–Einstein condensate**
- Quantum effects counteract gravity at **small scales** (uncertainty principle), erase structure
- Tiny mass
  $\Rightarrow$ large de Broglie wavelength ($\lambda \sim 1/m$)
  $\Rightarrow$ **macroscopic quantum effects** on kpc scales

Motivation for fuzzy dark matter

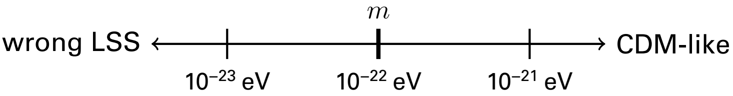
Particle physics perspective:

- Original concept – strong CP problem:
  Why doesn't QCD violate CP symmetry?
- Solved by Peccei–Quinn $\mathrm{U}(1)$ symmetry
  and (pseudo-)scalar field (*axion*!)
  Peccei and Quinn (1977)!
- Fuzzy dark matter is **not** the QCD axion, but axion-like particles are a common feature of early-universe theories

Astrophysics perspective:

- Small-scale challenges (cusp–core, missing satellites, ...)

→ Ultra-light scalars: WIMP alternative, could improve this



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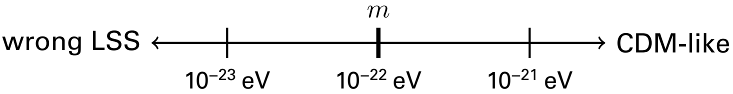

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The fuzzy dark matter equations

Schrödinger–Poisson system

- Add a scalar field to the Einstein–Hilbert action of general relativity

$$S = \frac{1}{\hbar c^2} \int \mathrm{d}^4 x \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - \frac{1}{2} \frac{m^2 c^2}{\hbar^2} \phi^2 - \frac{\lambda}{\hbar^2 c^2} \phi^4 \right)$$

- Non-relativistic limit yields the Schrödinger equation

$$i\hbar \left(\partial_t \psi + \frac{3}{2} H \psi \right) = -\frac{\hbar^2}{2m} \nabla^2 \psi + m \Phi \psi$$

- Mean field approximation: interpretation as **single macroscopic wave function of BE condensate** with density $\rho = m|\psi|^2$
- **“FDM equations”: non-linear Schrödinger–Poisson system**

$$i\hbar \partial_t \psi_\mathrm{c} = -\frac{\hbar^2}{2ma^2} \nabla_\mathrm{c}^2 \psi_\mathrm{c} + \frac{m}{a} \Phi_\mathrm{c} \psi_\mathrm{c}$$
$$\nabla_\mathrm{c}^2 \Phi_\mathrm{c} = 4\pi G m (|\psi_\mathrm{c}|^2 - \langle |\psi_\mathrm{c}|^2 \rangle)$$

Only a single scale, determined by $\frac{\hbar}{m}$ ($\rightarrow$ wavelength)

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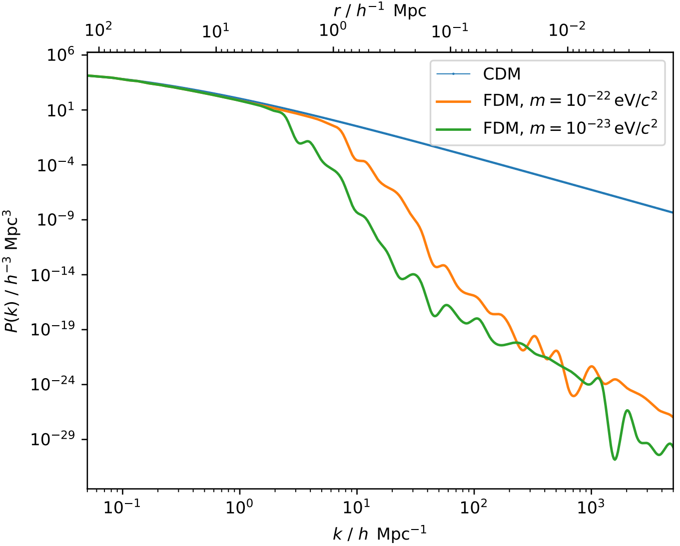
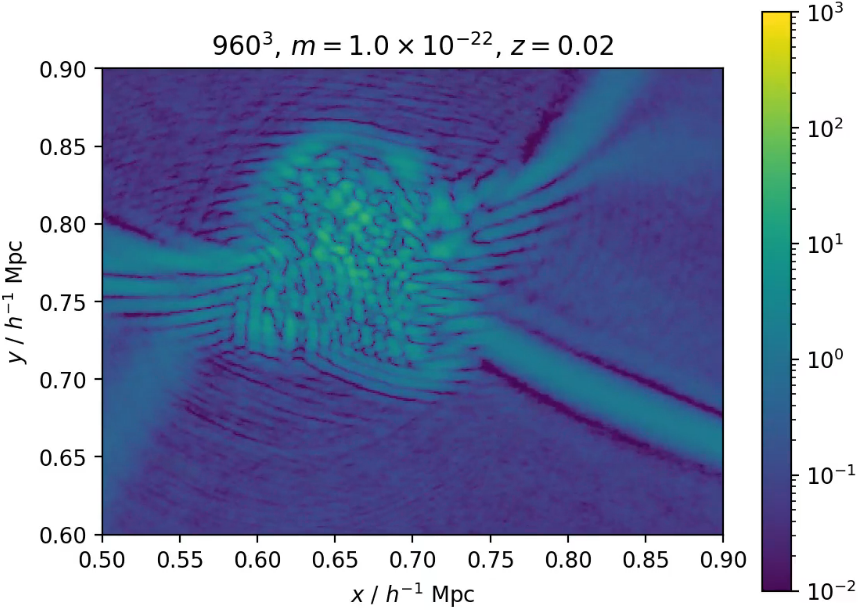
Impact of fuzzy dark matter – #1: Initial conditions



(Only CDM initial conditions for now → comparability)

Seeds of structure suppressed below quantum wavelength

Impact of fuzzy dark matter – #2: Dynamics



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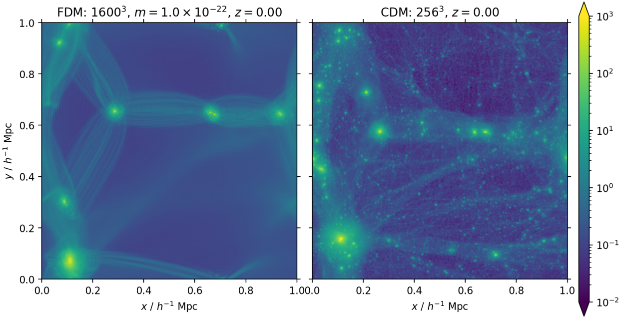

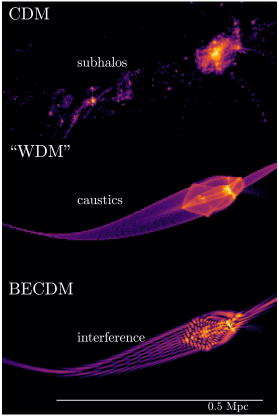

- Quantum fluctuations, interference patterns
- Small-scale (sub-)structure suppressed

Mocz, Fialkov, et al. (2020)

Using pseudo-spectral methods to simulate fuzzy dark matter

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2ma^2}\nabla^2\psi + \frac{m}{a}\Phi\psi$$
$$\nabla^2\Phi = 4\pi Gm(|\psi|^2 - \langle|\psi|^2\rangle)$$

- Symmetrized split-step Fourier method ("kick–drift–kick")
- Algorithm:

$\psi \leftarrow e^{-i\frac{m}{\hbar}\frac{\Delta t}{2}\Phi}\psi$	kick
$\psi \leftarrow \mathrm{IFFT}\left(e^{-i\frac{\hbar}{m}\frac{\Delta t}{2}k^2}\,\mathrm{FFT}(\psi)\right)$	drift
$\Phi \leftarrow \mathrm{IFFT}\left(-\frac{1}{k^2}\,\mathrm{FFT}\left(4\pi Gm(\lvert\psi\rvert^2 - \langle\lvert\psi\rvert^2\rangle)\right)\right)$	update potential
$\psi \leftarrow e^{-i\frac{m}{\hbar}\frac{\Delta t}{2}\Phi}\psi$	kick

Choice of time step: $\Delta t < \min\left(\frac{4}{9\pi}\frac{m}{\hbar}a^2\Delta x^2 \;\;,\; 2\pi\frac{\hbar}{m}a\frac{1}{|\Phi_{\max}|}\right)$

- "Exact" solution
- Automatic conservation of mass
- Can adapt existing particle–mesh (PM) code
- Simple implementation

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Correspondence of CDM and FDM initial conditions

- Cold dark matter (CDM): Collisionless fluid, described by a phase space distribution function $f(\vec{x}, \vec{v})$
- Can construct a wave function ψ from a distribution function f:

$$\psi(\vec{x}) \sim \sum_{\vec{v}} \sqrt{f(\vec{x}, \vec{v})} e^{im/\hbar\, \vec{x} \cdot \vec{v} + R_{\vec{v}}}$$

For "cold"/single-stream distribution function:

$$\psi = \sqrt{\frac{\rho}{m}} e^{i\alpha}$$

$$\vec{v} = \frac{\hbar}{m} \nabla \alpha$$

- Grid discretization implies a maximum velocity which can be represented

Mocz, Lancaster, et al. (2018), Mocz, Fialkov, et al. (2020)

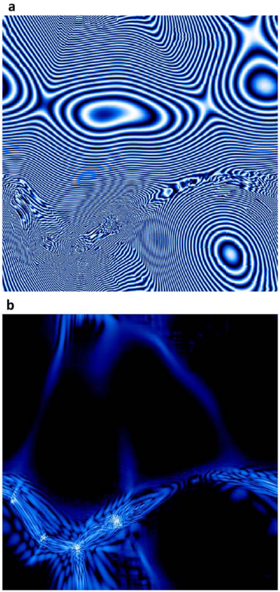
$$v < \frac{\hbar}{m} \frac{\pi}{\Delta x}$$

Why is it hard to simulate fuzzy dark matter?

Computational challenges

- **Both** large scales and small (kpc-scale) de Broglie wavelength must be resolved for correct evolution
- High velocities require high resolution even in low-density regions
  Velocity criterion: $v < \frac{\hbar}{m} \frac{\pi}{\Delta x}$
- **Time step criterion:** $\Delta t \sim \Delta x^2$
  (seems to be approach-independent)

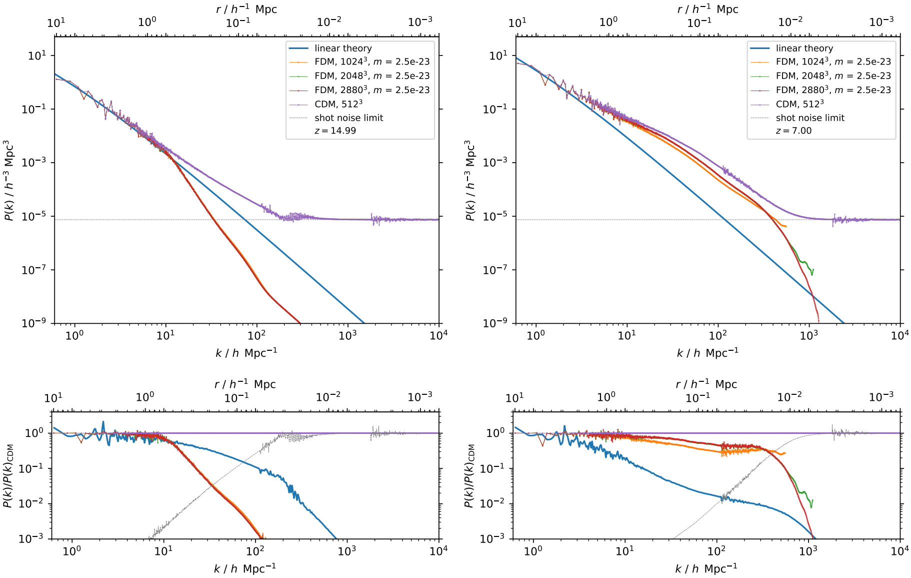
- (Tooling: Hydrodynamics codes are designed for N-body simulations)



Schive, Chiueh, and Broadhurst (2014)

First results: Dark matter power spectrum

10 Mpc/h box simulations



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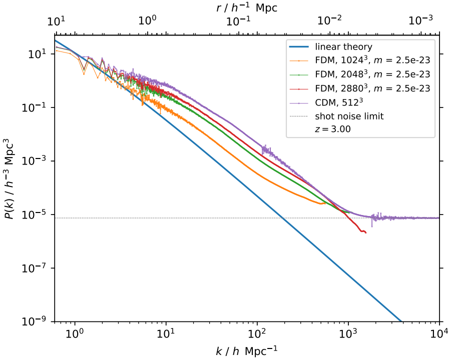
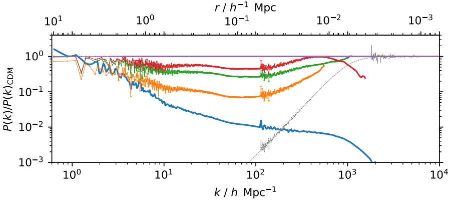
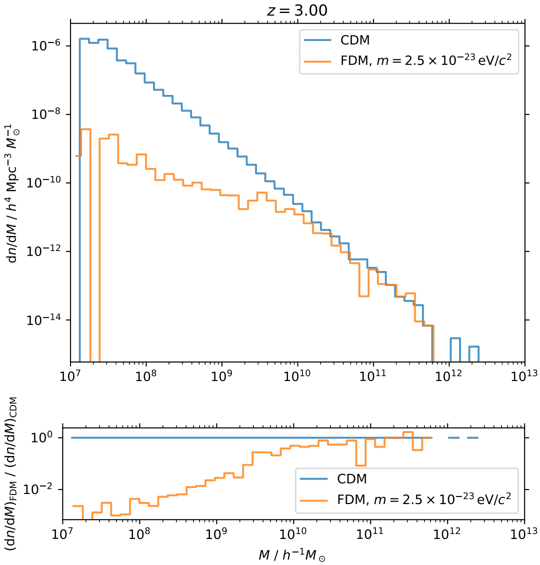




- Agreement on large scales
- Delayed structure formation
- Suppression on small scales

First results: Halo mass function

10 Mpc/h box simulations

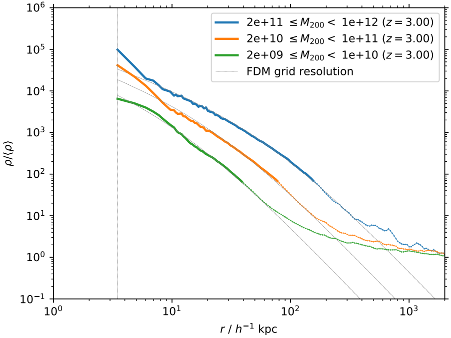


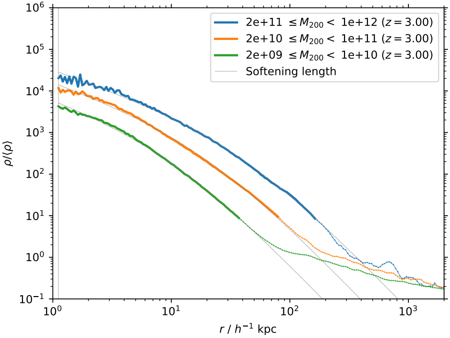
- Fewer low-mass objects

First results: Dark matter halo profiles (stacked)

10 $^{\text{Mpc}}/_{h}$ box simulations

FDM ($m = 2.5 \times 10^{-23}\ ^{\text{eV}}/_{c^2}$) CDM





- Fuzzy dark matter halos form central cores with flat density profiles
- Adaptive resolution better suited to show inner halos

Summary

Main problems for fuzzy dark matter simulations:

1. **Time integration** $\Delta t \sim \Delta x^2$
2. **Rapid oscillations** even in low-density regions
3. **Large dynamic range**: “large”-scale structure simulations must still resolve de Broglie wavelength, limited to $\lesssim 10\,\mathrm{Mpc}/h$ box
4. “New” field without decades of experience or refined codes/methods as for CDM

Features of FDM dynamics compared to CDM:

1. (Modified initial power spectrum, small scales suppressed)
2. **Suppression** of structure below de Broglie wavelength ($\approx$ kpc) $\rightarrow$ Heisenberg uncertainty principle
3. Formation of **halo cores**
4. Fluctuating $\approx$ kpc quantum **interference patterns**

References

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