PiTP 2009: "Computational Astrophysics"
Lectures on Collisionless Dynamics and SPH
Exercise Set 2

Volker Springel

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1 Creating galaxy clusters models and merging them

Here we would like to study the merger of two toy galaxy clusters, with non-radiative gas physics. Go through the following steps:

- Create initial conditions for a collisionless Hernquist halo
- Add a gas component in approximate hydrostatic equilibrium to the halo
- Check the stability of the created isolated initial conditions
- Put two copies of the halo onto a collision course, and simulate their merger
- Look at the time-evolution of the temperature and X-ray emission during the merger

1.1 Creating a Hernquist halo

A collisionless Hernquist halo has a density profile

$$\rho(r) = \frac{M}{2\pi} \frac{a}{r(r+a)^3} \tag{1}$$

and gravitational potential

$$\Phi(r) = -\frac{GM}{r+a} \tag{2}$$

Its distribution function is only a function of specific energy, and can be derived analytically. It is given by

$$f(E) = \frac{1}{\sqrt{2}(2\pi)^3(GMa)^{3/2}} \frac{\sqrt{e}}{(1-e)^2} \left[(1-2e)(8e^2 - 8e - 3) + \frac{3\sin^{-1}(\sqrt{e})}{\sqrt{e(1-e)}} \right], \quad (3)$$

where $e = -aE/(GM)$, and $E = \mathbf{v}^2/2 + \Phi$.

For definiteness, we pick $M = 10^{15}\,h^{-1}\mathrm{M}_\odot$ and $a = 500\,h^{-1}\mathrm{kpc}$. (As we will use the Gadget2 code to experiment with the initial conditions, it is recommended to work in internal Gadget2 units throughout, with $[L] = h^{-1}\mathrm{kpc}$, $[L/T] = \mathrm{km/sec}$, and $[M] = 10^{10}\,h^{-1}\mathrm{Mpc}$. In these units, the gravitational constant has the numerical value $G = 43007.1$)

Write code that creates an N-body realization (e.g. with $N = 50000$, or with $N = 10000$ for testing) of the above solution of the collisionless Boltzmann equation, and write this as a Gadget2 initial conditions file. You can do this in any language, but IDL is probably simplest if your familiar with it. To help you doing this, I have prepared a dummy script that contains the appropriate write statement (*SetupICs_dummy.pro* on the web-site) – but you still have to fill in the correct coordinates, velocities and masses!

Some hints for this task: Create a realization of the density structure first. To this end, draw uniform random numbers between $[0, 1[$, and map each number q to a radius r via $q = M(< r)/M$, where $M(< r)$ is the enclosed mass inside r. Next, draw random spherical angular coordinates that give you a uniform distribution on the unit sphere and convert to cartesian coordinates. The velocity distribution is a bit more tricky. For each particle you can calculate the potential energy at its radius. Draw uniformly distributed random numbers between the minimum and maximum allowed velocity magnitude at this radius, and use the rejection technique to randomly accept velocities in proportion to the velocity distribution function $p(v)$ at that radius. Observe that we have $p(v)\,\mathrm{d}v \propto f(E)v^2\mathrm{d}v$. For each velocity magnitude that is kept, you can then initialize a randomly oriented velocity vector.

1.2 Adding gas

Now we want to turn 20% of the mass of the halo into gas, which we want to set-up in approximate hydrostatic equilibrium. For simplicity, we simply use the same density profile as for the dark matter, and use for the internal energy per unit mass of the gas

$$u(r) = \frac{3}{2}\sigma_{1\mathrm{d}}^2(r), \quad (4)$$

where $\sigma_{1\mathrm{d}}$ is the one-dimensional velocity dispersion of the dark matter particles in the Hernquist halo. Why does this set the gas in approximate hydrostatic equilibrium? And why is this way of setting up the gas not really correct? (We shall ignore this incorrectness here and accept that the system will go through a short transient initially, before it really settles into equilibrium.)

Make a particle realization of both dark matter and gas (with $N_{\rm dm} = 10000$ and $N_{\rm gas} = 10000$). Write everything as a Gadget2 initial conditions file.

1.3 Check the stability of your isolated models

Use Gadget2 to evolve both the pure DM and the DM+gas halos for several Gyrs. (Hint: Base your Makefile and parameterfile on the files *galaxy.Makefile* and *galaxy.param* examples contained in the *parameterfiles* directory, and modify them as needed.) The halos should remain reasonable stable end evolve very little, after a short initial transient. Monitor the evolution of the kinetic, thermal and potential energies to see whether there is any significant secular evolution. You could also look at the density profile at different times.

1.4 Collide the clusters

We now set-up a collision of clusters on a head-on encounter. Create new initial conditions where you put two copies of your cluster model a distance $2\,h^{-1}$Mpc apart initially. Give them an encounter velocity corresponding to a zero-energy orbit. (Treat the clusters as point masses for that purpose, and ignore the fact that the halos will already overlap slightly at the initial time.)

Then run a simulation of the merger, until a time of $3\,h^{-1}$Gyr, when the merger has completed and the remnant is reasonably relaxed. Use Gadget2 in pure tree-mode. Make a couple of images at different times to visually check what is going on.

1.5 The X-ray emission during the merger

Measure the quantity

$$L_X = \int \rho_{\rm gas}(\mathbf{r})^2 \,{\rm d}V \tag{5}$$

as a function of time, which is a rough proxy for the expected X-ray emission (ignoring the $\sqrt{T}$ temperature dependence of bremsstrahlung emission and complications such as metal lines for the moment). Devise a method to efficiently extract $L_X(t)$ from the SPH snapshots, and plot L_X as a function of time. Also look at the evolution of the mean mass-weighted temperature as a function of time.