The origin of NFW

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Inverted NFW, the search for an explanation?

Self-similar halo growth

Consider a power-law ellipsoidal linear density perturbation within an otherwise uniform EdS universe:

$$\delta(\mathbf{x}, t) = (t/t_0)^{2/3}(\mathbf{x} \cdot A \cdot \mathbf{x})^{-\alpha/2}, \qquad |A| = 1$$
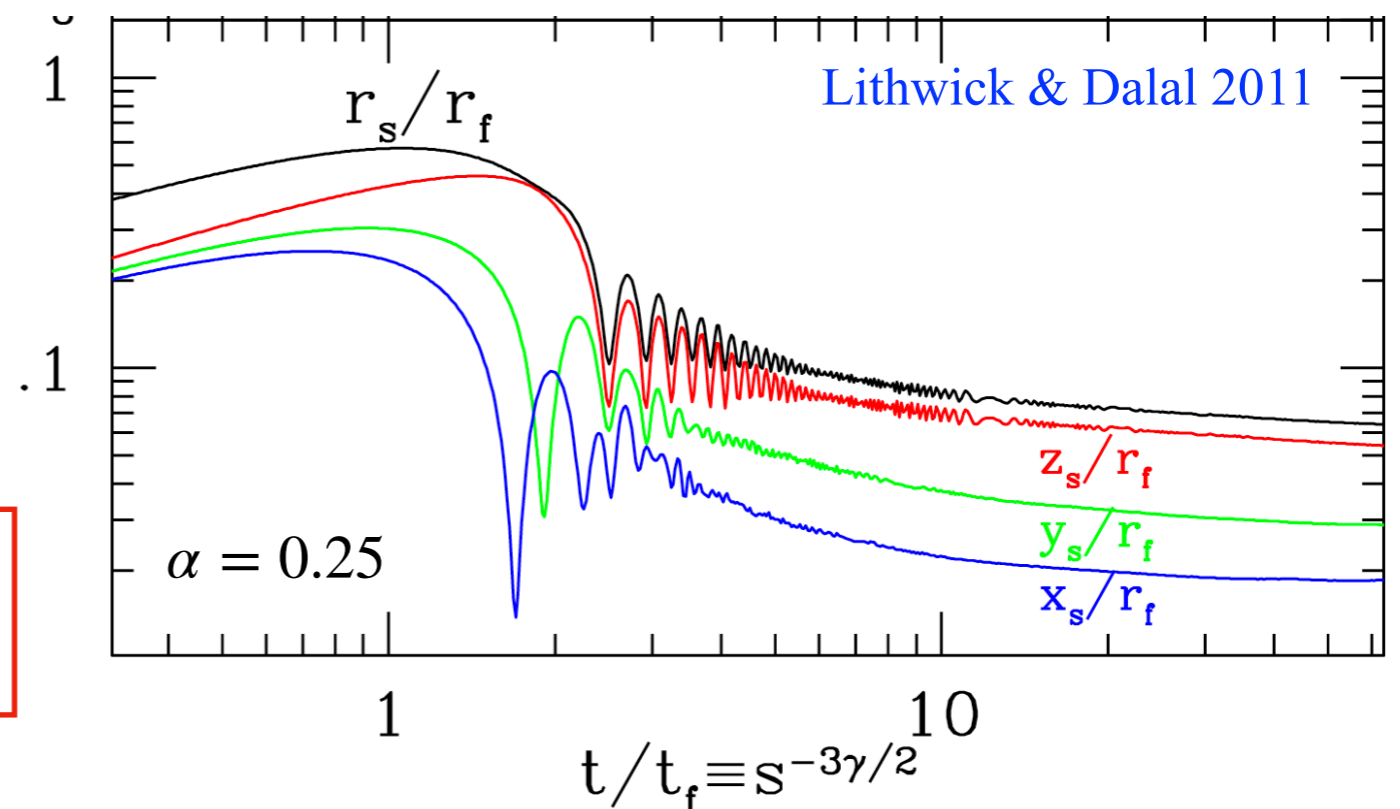
$$= (t/t_0)^{2/3}M(\mathbf{x})^{-\alpha/3}$$

The halo mass thus increases as: $\qquad M_{\text{halo}}(t) \propto t^{2/\alpha}$

Within the halo: $\quad \rho \propto r^{-\gamma} \quad \longrightarrow \quad t_{\text{orb}} \propto r^{\gamma/2}, M \propto r^{3-\gamma} \quad \longrightarrow \quad M \propto t_{\text{orb}}^{(6-2\gamma)/\gamma}$

If $M(t_{\text{orb}}) \propto M_{\text{halo}}(t = t_{\text{orb}})$, then $\quad 2/\alpha = (6 - 2\gamma)/\gamma$
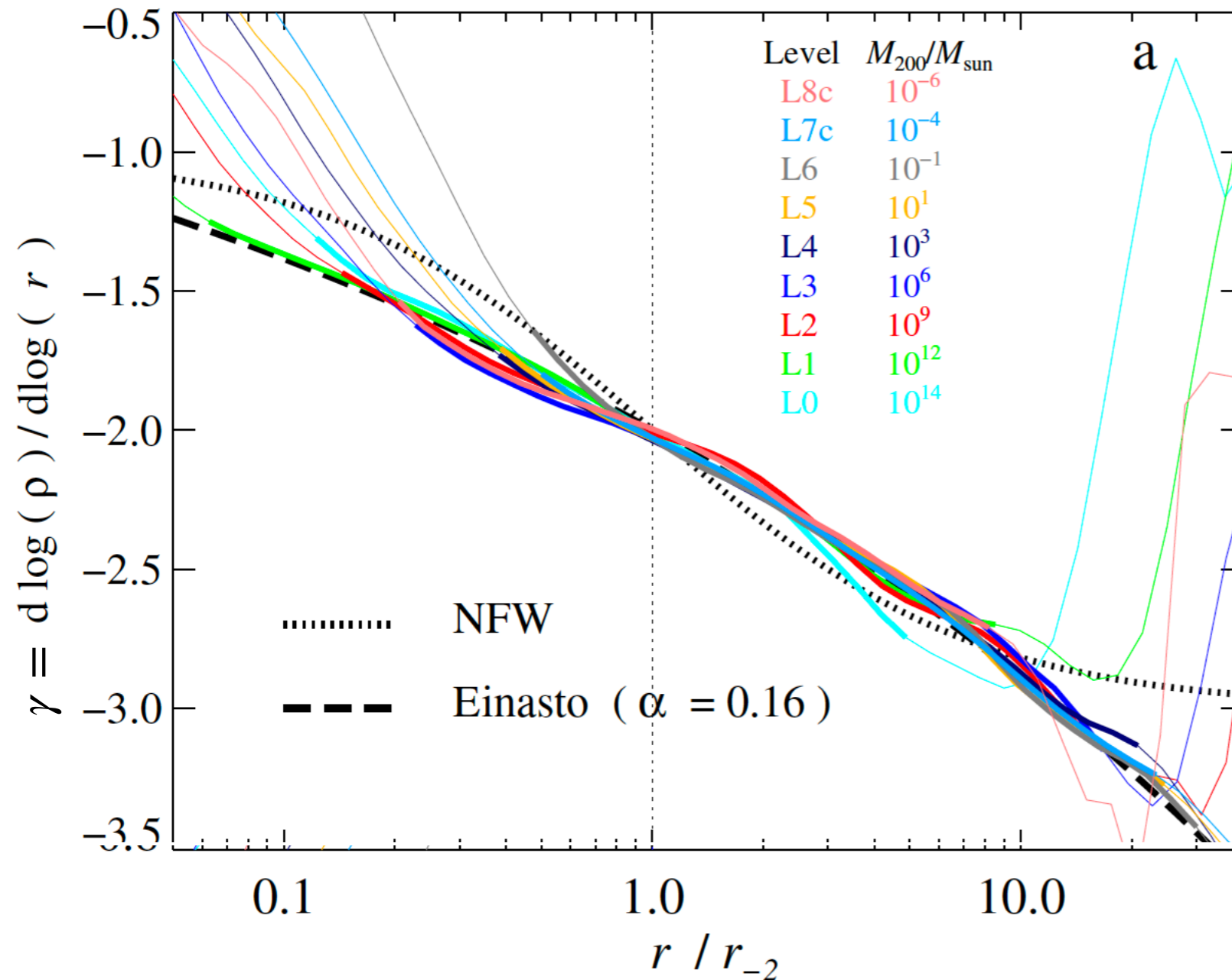
$$\longrightarrow \quad \gamma = 3\alpha/(1 + \alpha)$$

This is *not* NFW-like, but rather a power law with γ depending on α

Lithwick & Dalal 2011

r_s/r_f

z_s/r_f

y_s/r_f

x_s/r_f

$\alpha = 0.25$

$t/t_f \equiv s^{-3\gamma/2}$
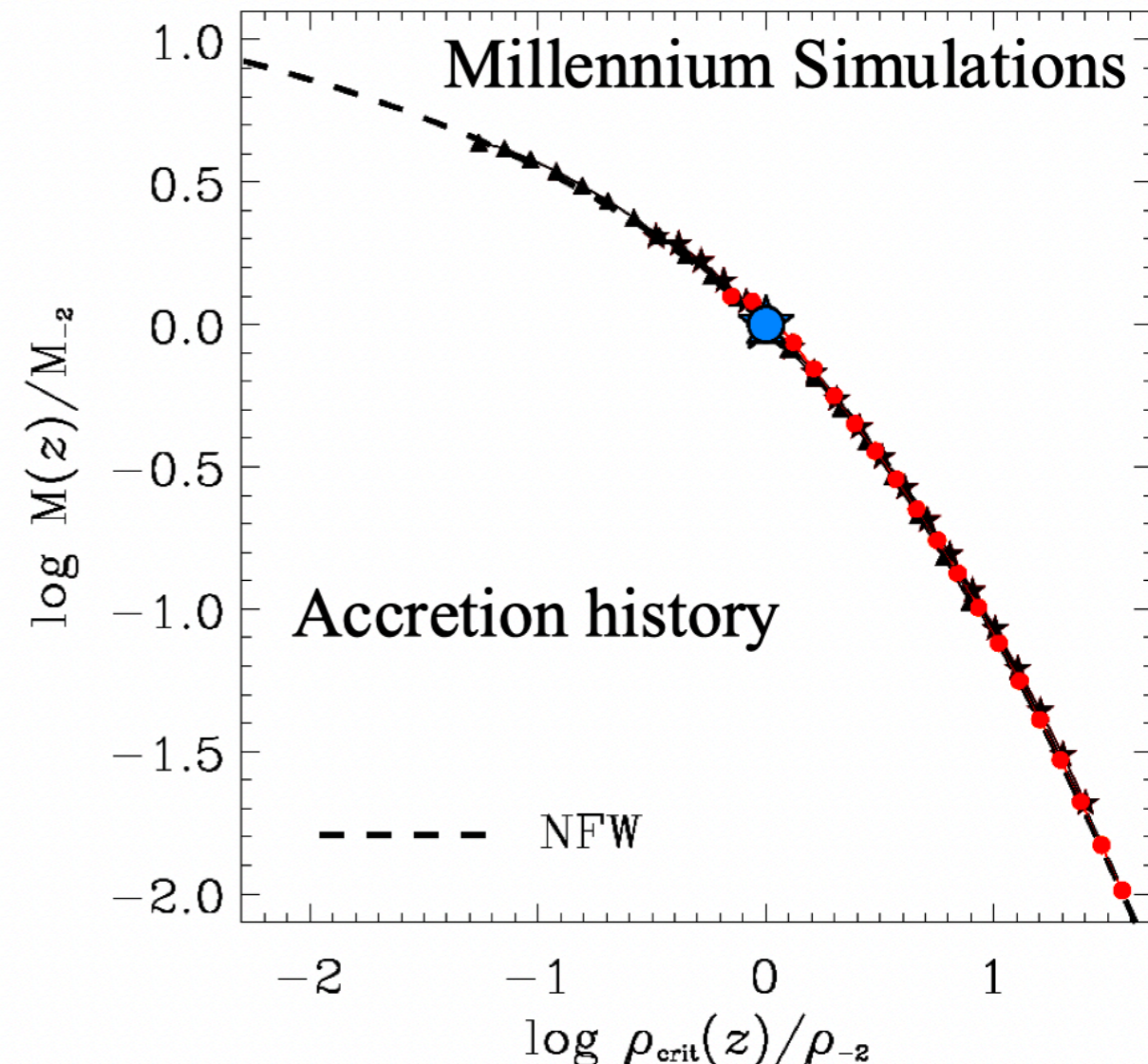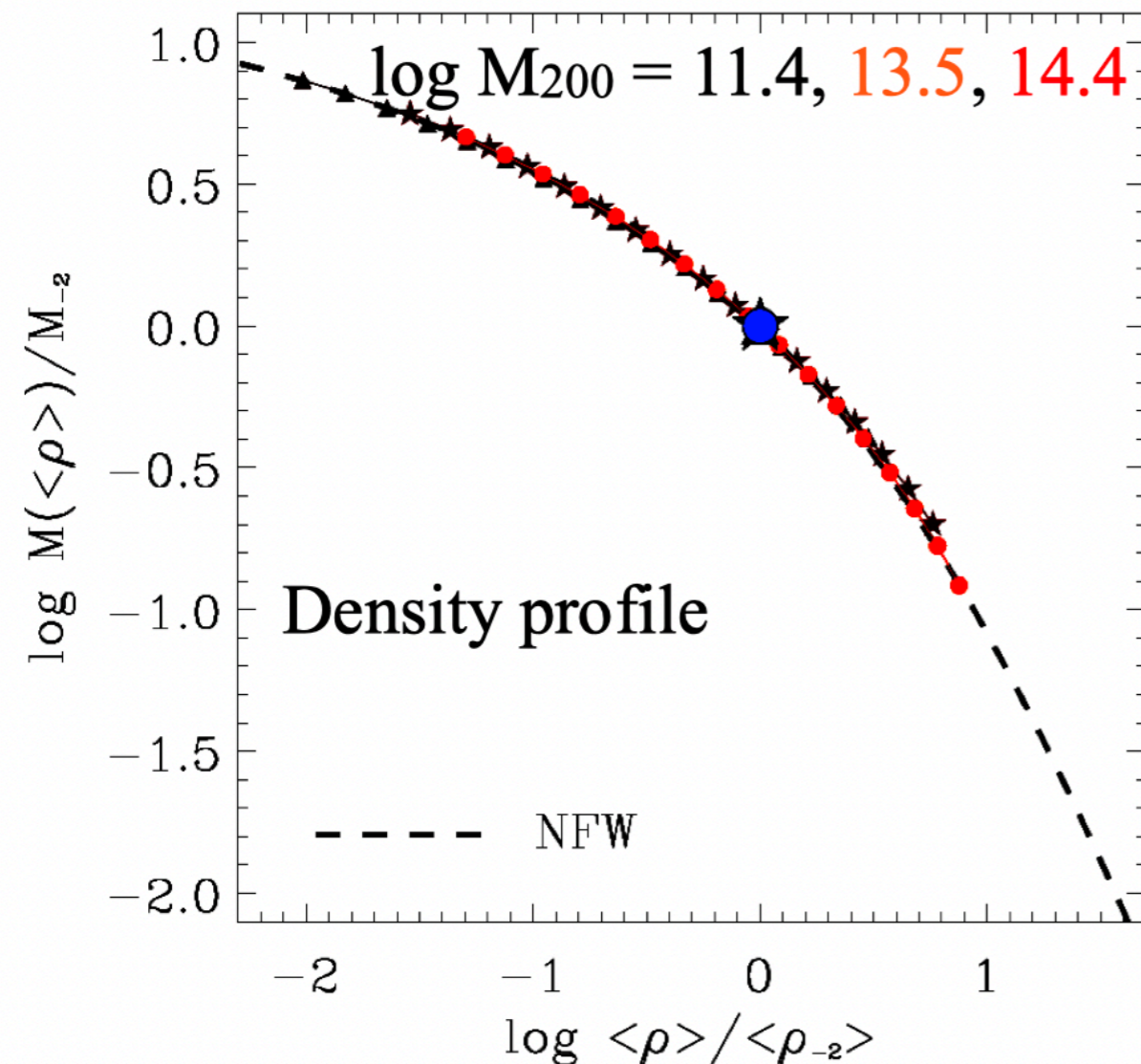
In ΛCDM halos γ declines with radius

The shape of ΛCDM density profiles is independent of mass, e.g. relative to M_*

No dependence on linear power spectrum slope, see also halos in $P \propto k^n$ cosmologies

The connection to halo assembly

Ludlow et al 2014

The mean profiles of ΛCDM halos *are* tightly linked to their mean growth histories
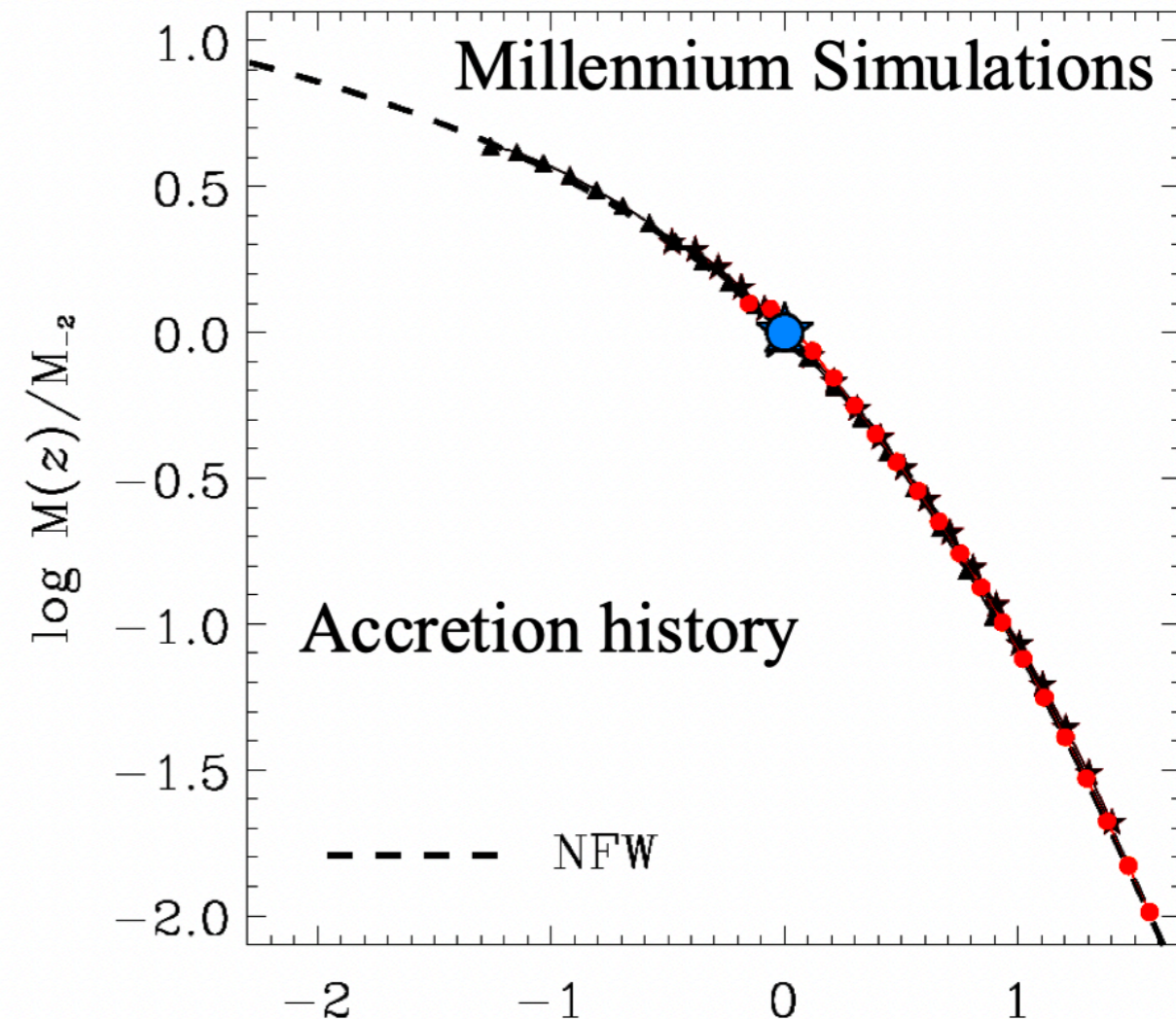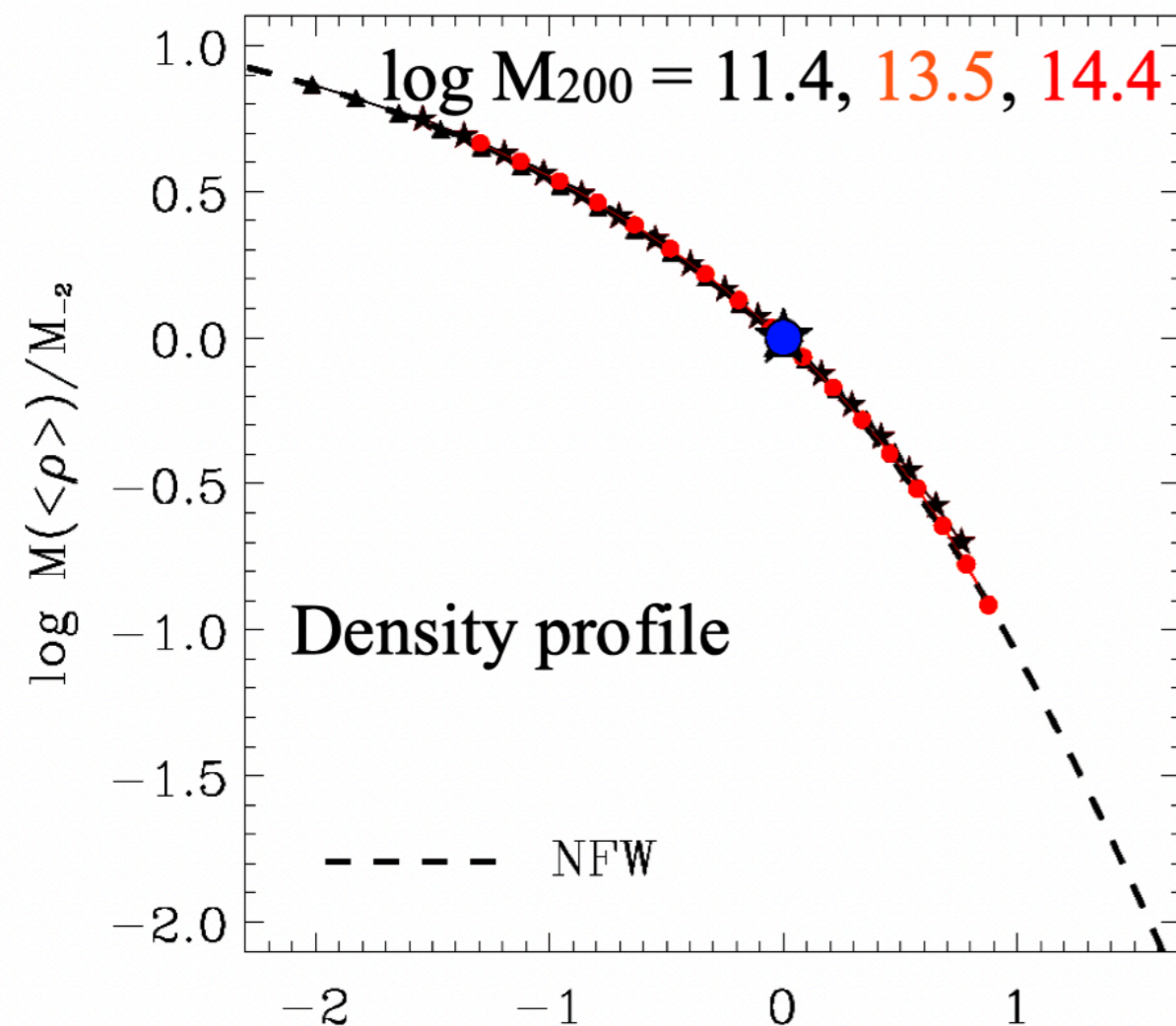
Violent relaxation is weak

A "universal" growth history shape

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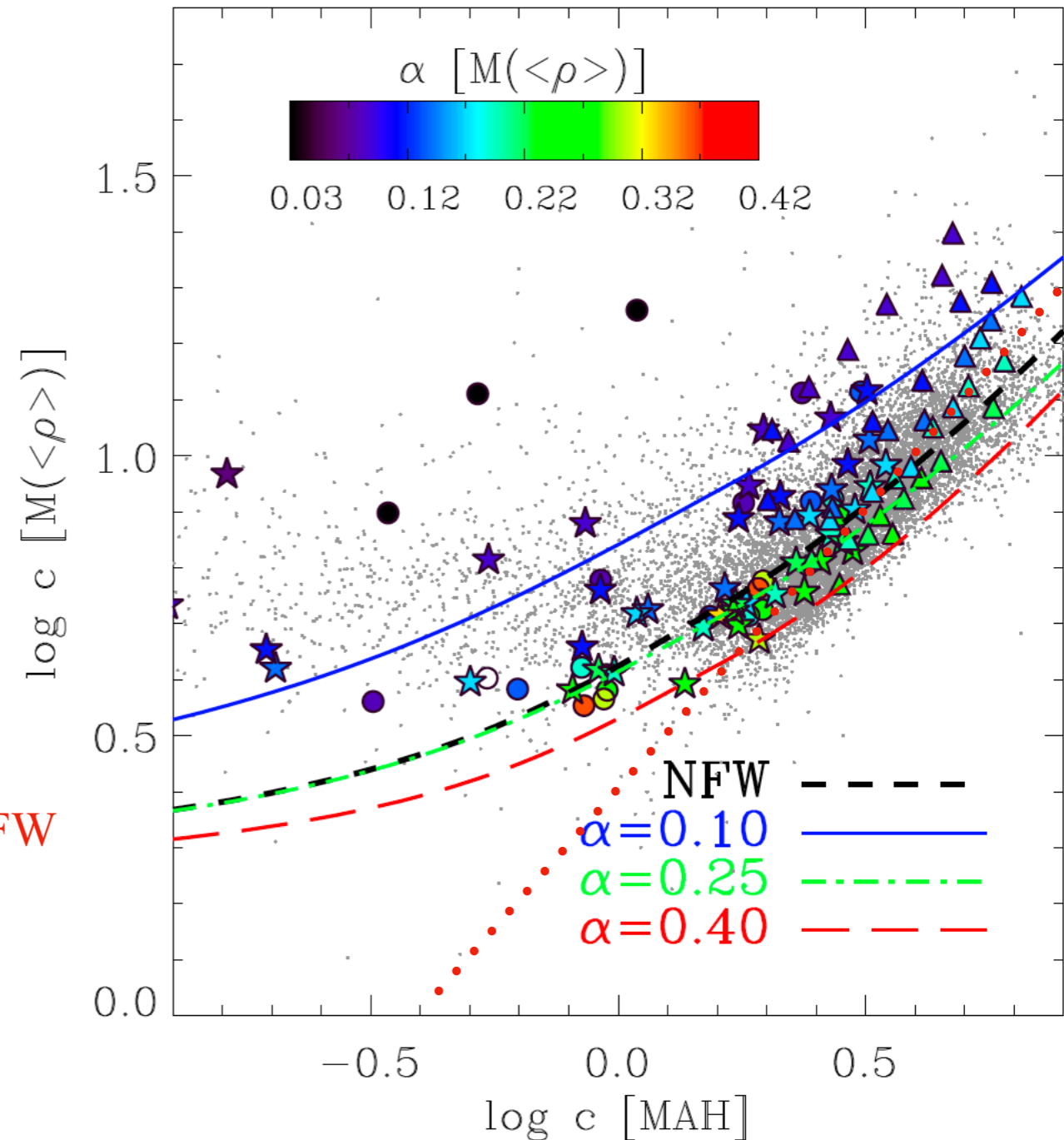
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Convergent evolution?

Ludlow et al 2014
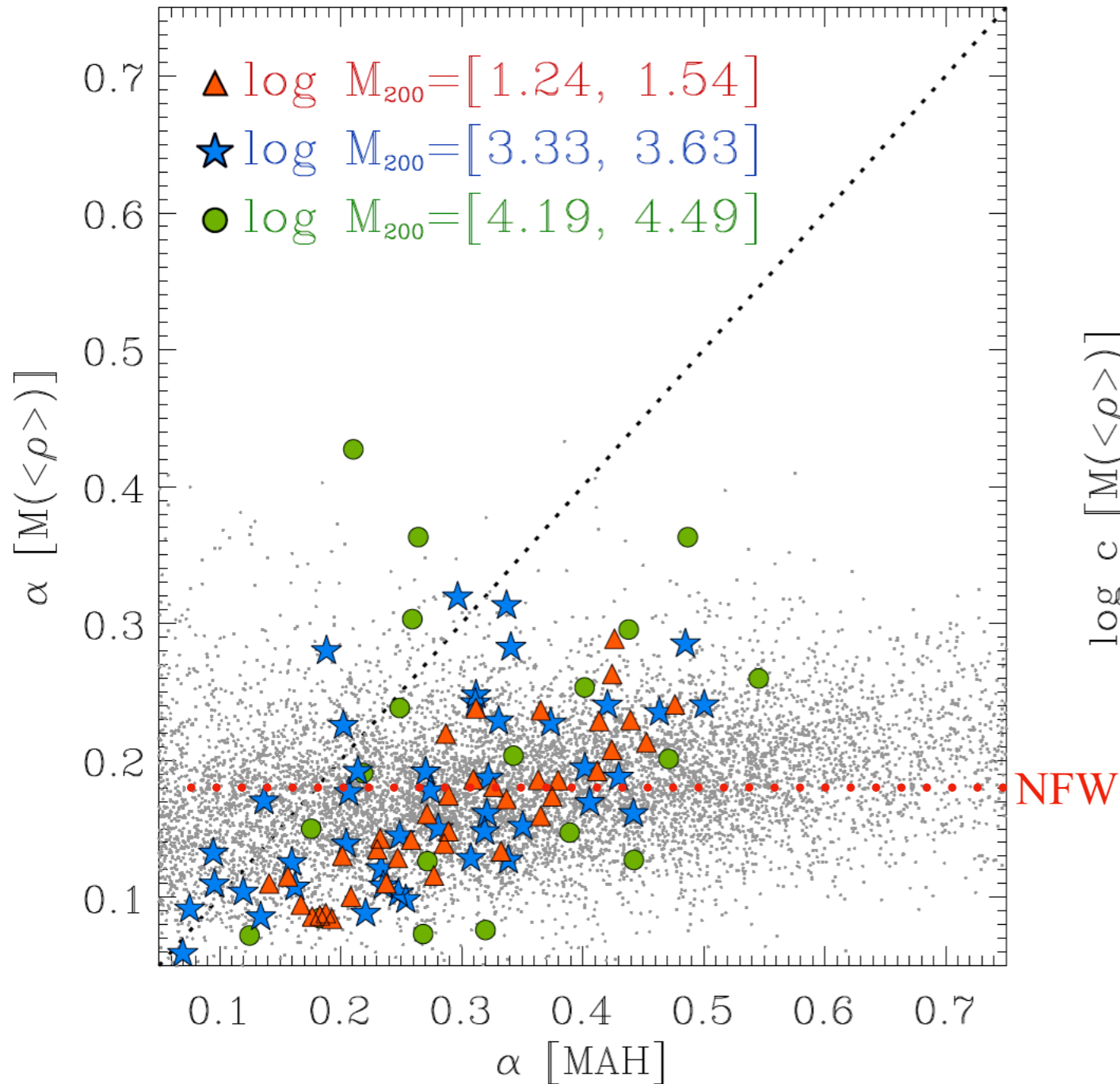
Profile *c* reflects MAH *c* nearly linearly, but profiles are closer to NFW than MAH's: convergence driven by weak violent relaxation

Halos converge to NFW outside $r_{\text{Power}}(t_f)$

Springel et al 2008

z = 0.0

Aq-A-1

Aq-A-2

Aq-A-3

Aq-A-4

Aq-A-5

$\rho(r) / \langle \rho \rangle$

r [kpc]
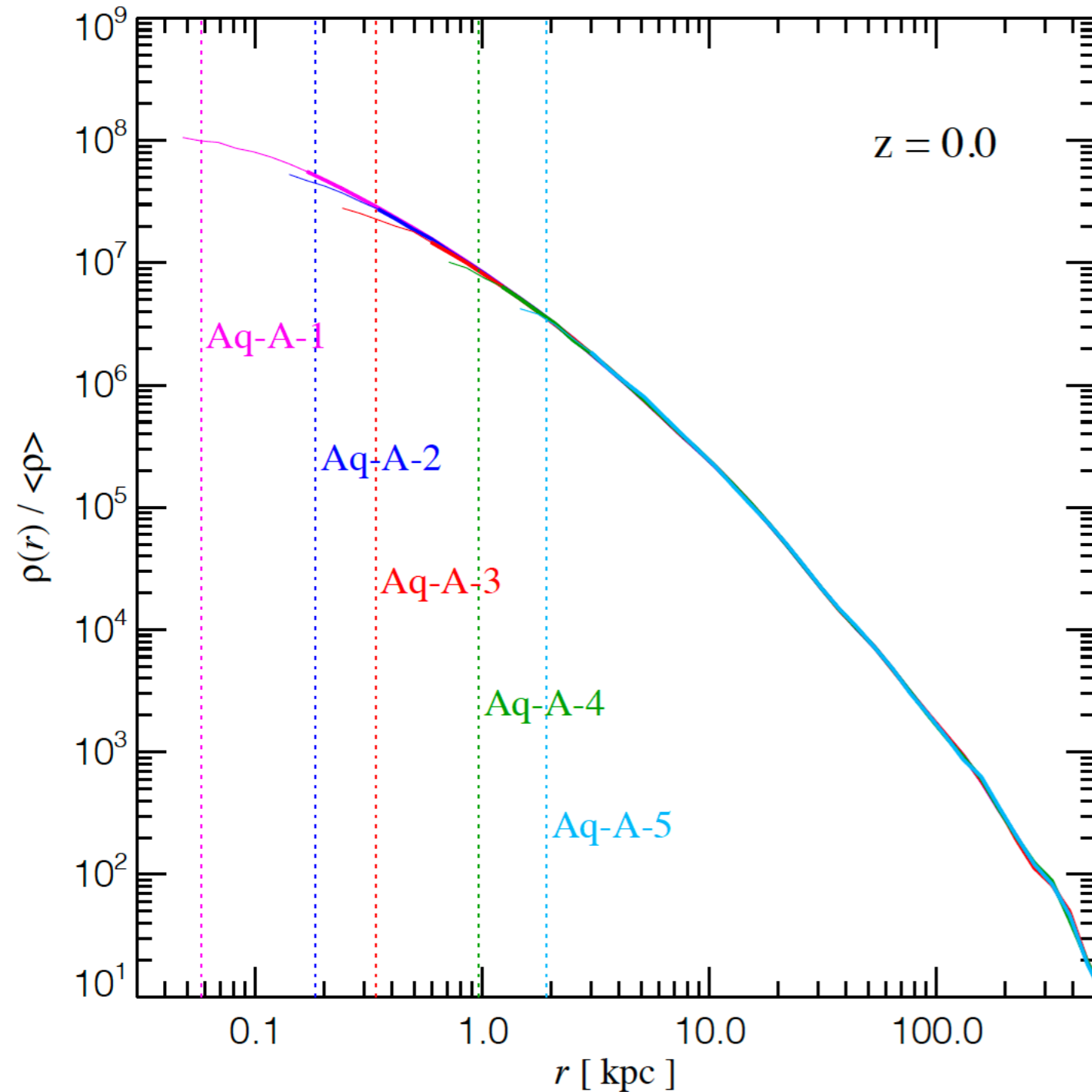
The NFW shape is not a consequence of 2-body relaxation/discreteness

Prompt cusp formation in a ΛCDM density peak

$t/t_c = 0.58$

$t_c \longrightarrow z = 87$

$M_{pk} \sim 10^{-6} M_{\mathrm{sun}}$

$\rho r^{1.5}$

picture edge

radius r

Delos & White 2023
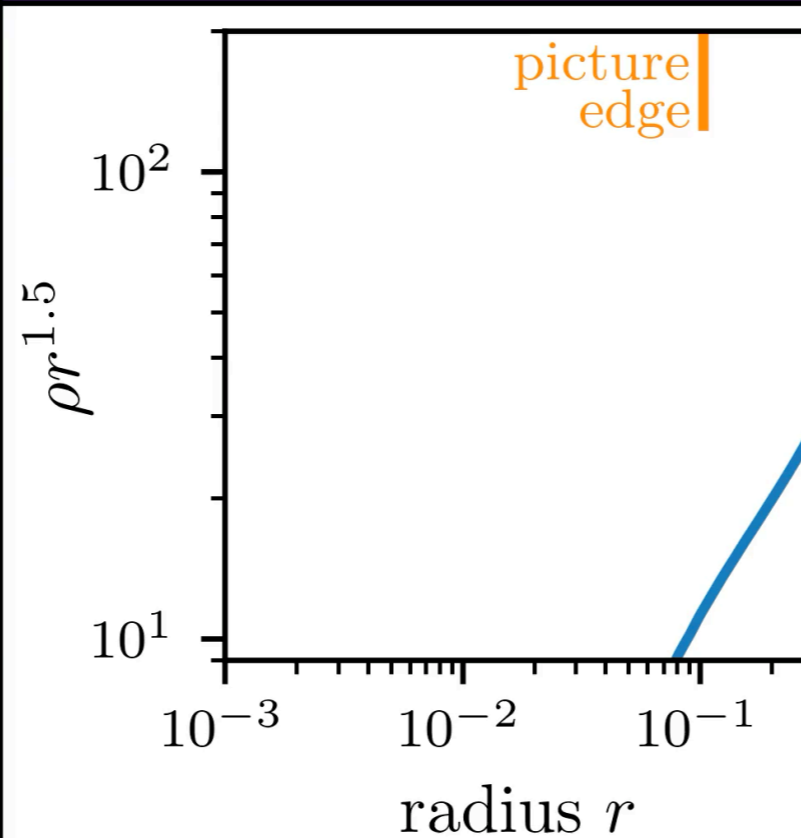
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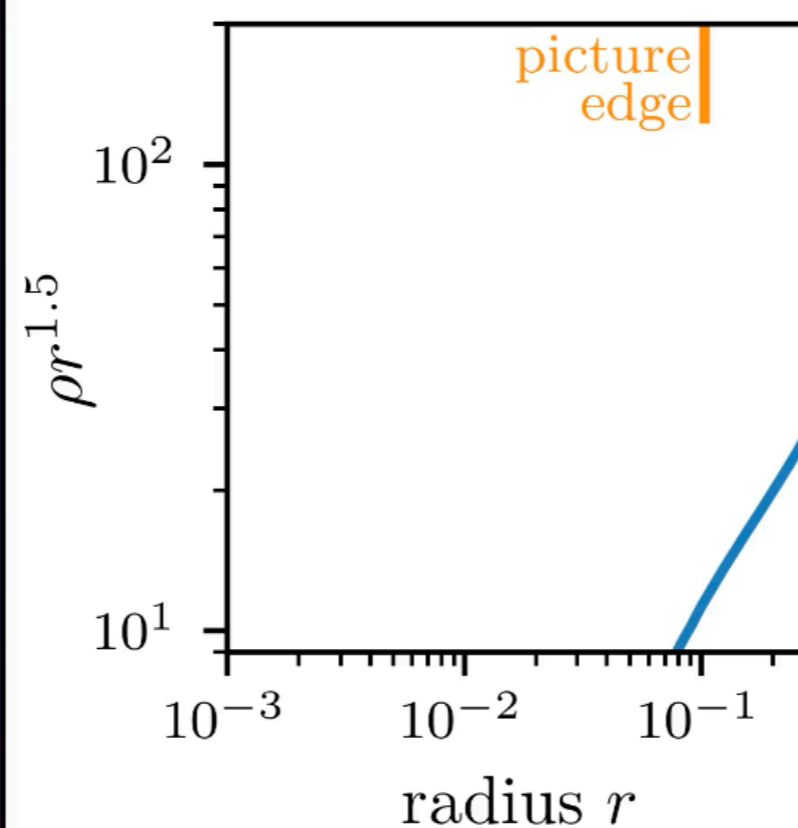
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Prompt cusp formation differs qualitatively from "normal" halo formation

Violent relaxation is important

No close link of profile to cusp growth history

A "universal" profile *different* from NFW

picture edge

$\rho r^{1.5}$

radius r

Prompt cusp and subsequent halo growth for a peak with $z_{coll} = 87$
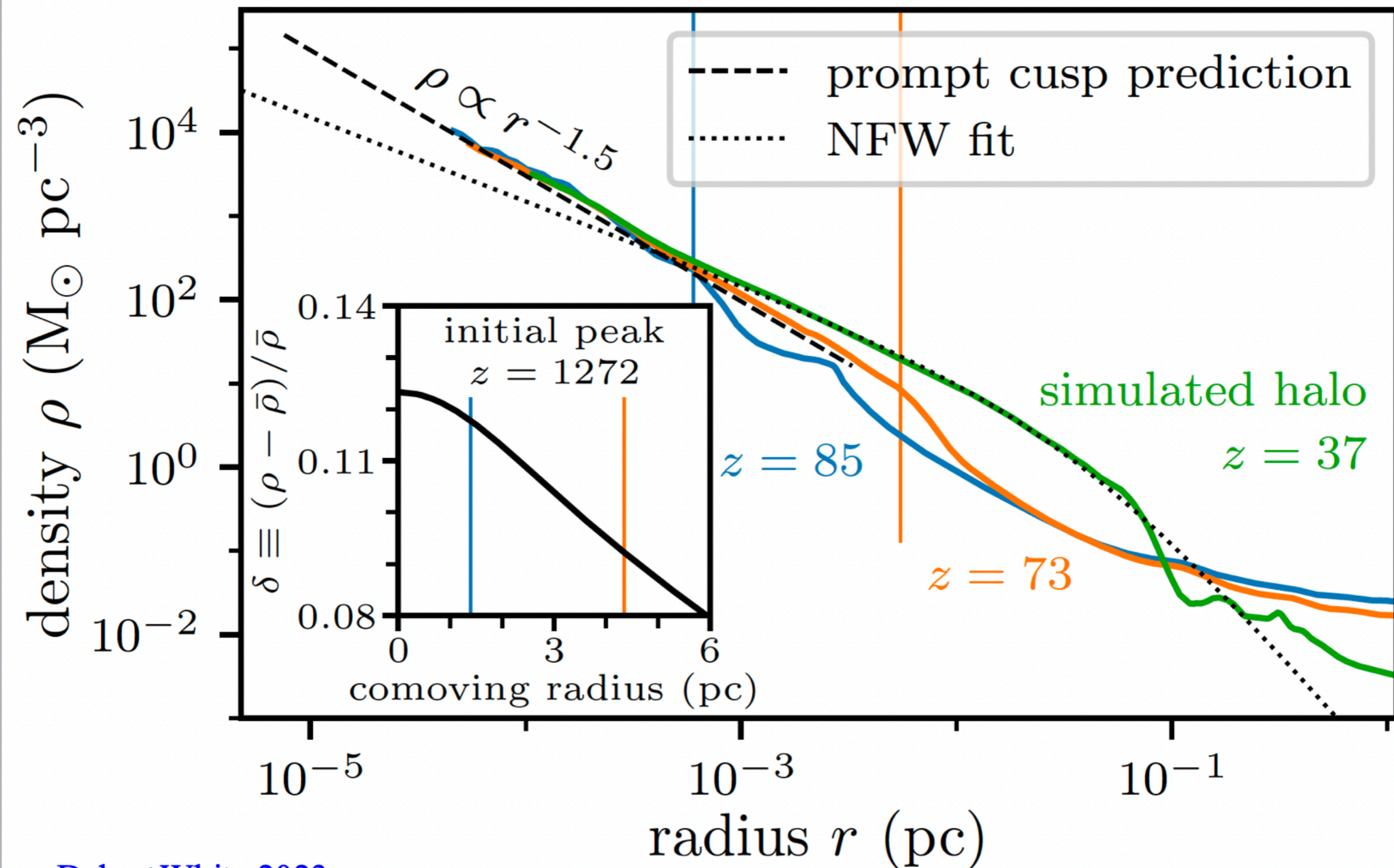
prompt cusp prediction

NFW fit

$\rho \propto r^{-1.5}$

density ρ (M$_\odot$ pc^{-3})

radius r (pc)

simulated halo

$z = 37$

$z = 85$

$z = 73$

initial peak
$z = 1272$

$\delta \equiv (\rho - \bar{\rho})/\bar{\rho}$

comoving radius (pc)

Delos+White 2023

Excursion set calculation of halo mass growth

Let $p(M_1, z_1 | M_0, z_0)dM_1$ be the distribution of progenitor halo mass M_1 at z_1 for individual mass elements which are part of a halo of mass M_0 at z_0. Then

$$dN = \frac{M_0}{M_1} p(M_1, z_1 | M_0, z_o)dM_1$$

is the number distribution of progenitors by mass. For Poisson sampling from this distribution, the mean mass of the most massive progenitor would be given by

$$\langle M_{\mathrm{halo}}\rangle(z_1 | M_0, z_0) = \int_{M_1=0}^{M_0} dN \, M_1 \, \exp\left(-\int_{M_1}^{M_0} dN \right).$$

For an EdS universe with $P(k) \propto k^n$, $\quad \sigma^2(M) \propto M^{-(3+n)/3}$, w.l.o.g. $z_0 = 0$, and

$$\langle M_{\mathrm{halo}}\rangle/M_0 = \sqrt{\frac{2}{\pi}} \int_0^\infty dZ \, \exp\left(-Z^2/2 - \sqrt{\frac{2}{\pi}} \int_Z^\infty dZ'\left(\frac{A^2 + Z'^2}{Z'^2}\right)^{3/(3+n)} \exp\left(-Z'^2/2\right) \right)$$

for a sharp-k filter, where $\quad A = \left(\frac{M_0}{M_*}\right)^{(3+n)/6} z_1, \qquad \sigma(M_*) = \delta_c = 1.686$

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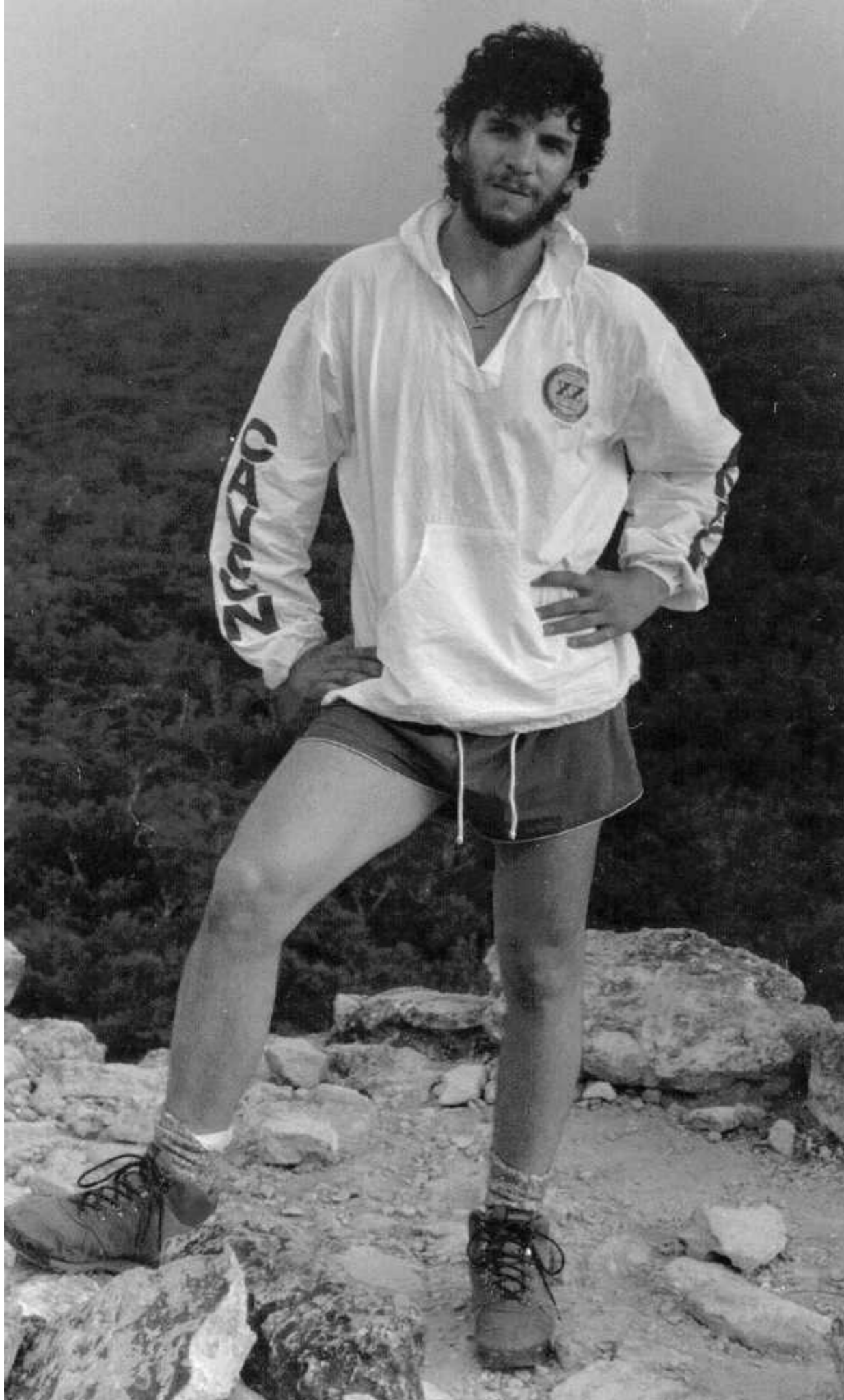- Violent relaxation smooths profiles but does not mix radially

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The universal NFW shape is a consequence of convergent evolution + near-universal hierarchical growth histories from gaussian I.C.'s

Thanks for
the ride,
Don Julio