A universal density profile from hierarchical clustering

Simon White
Max Planck Institute for Astrophysics

CIRM Workshop, Marseille, July 2023

Cosmic structure formation

$$a\frac{\partial f}{\partial t} + \mathbf{v}\,.\frac{\partial f}{\partial \mathbf{x}} - \nabla\phi\,.\frac{\partial f}{\partial \mathbf{v}} = 0, \quad \nabla^2\phi = 4\pi G a^{-1}\int (f - \bar{f})\; d^3v,$$

with $(\mathbf{x}, \mathbf{v})$ comoving with the cosmic expansion, $a(t)$.

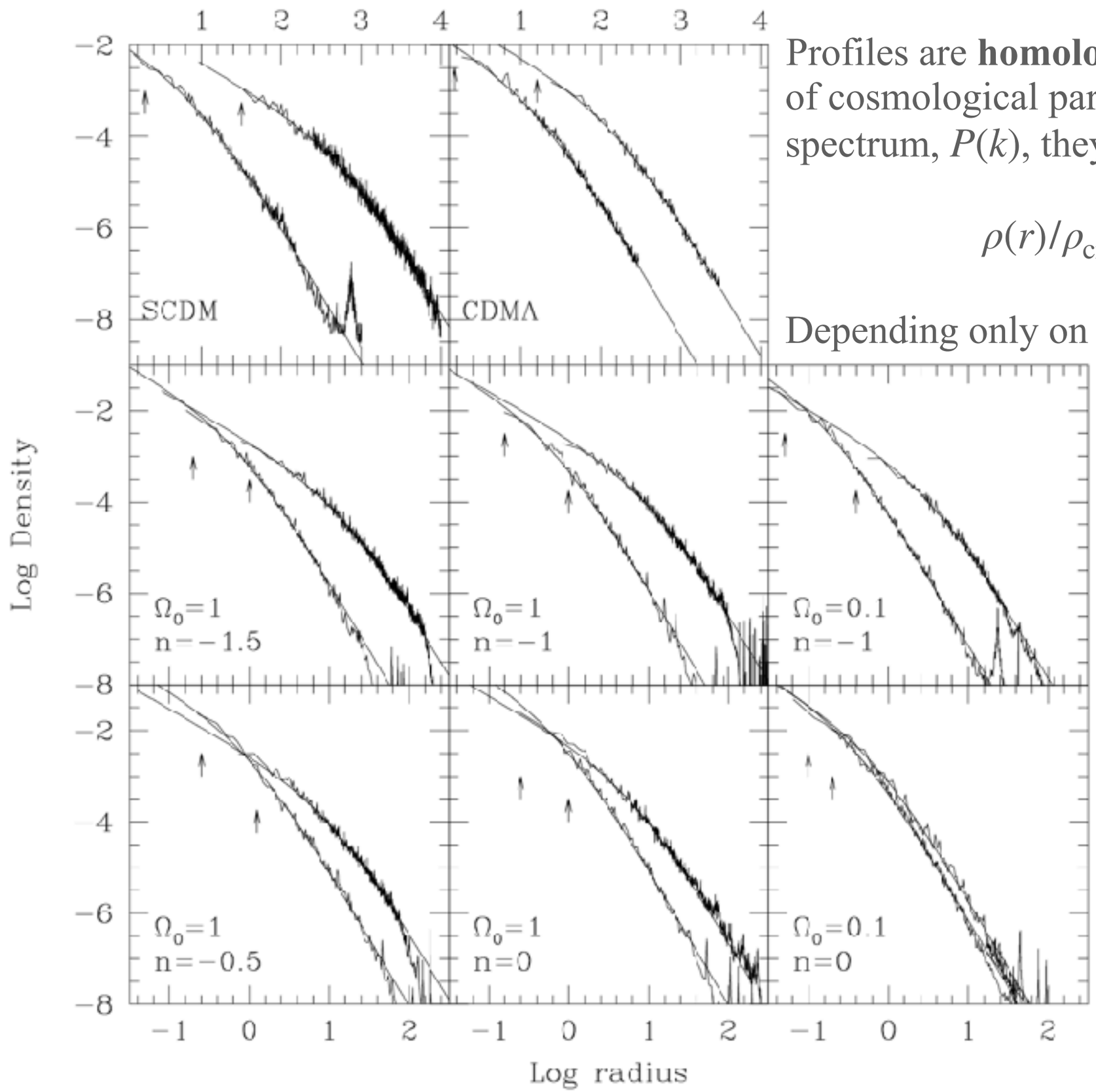
Initial conditions at some early time t_i

$$f(\boldsymbol{x}, \boldsymbol{v}, t_i) = \rho(t_i)\,[1 + \delta(\boldsymbol{x})]\,N\,[\{\boldsymbol{v} - \boldsymbol{V}(\boldsymbol{x})\}/\sigma]$$

where $\rho(t_i)$ is the mean mass density of CDM,
$\delta(\boldsymbol{x})$ is a Gaussian random field with finite variance $\ll 1$,
$\boldsymbol{V}(\boldsymbol{x}) = \nabla\psi(\boldsymbol{x})$ where $\nabla^2\psi(\boldsymbol{x}) \propto \delta(\boldsymbol{x})$
and N is standard normal with $\sigma^2 << \langle|\mathbf{V}|^2\rangle$

$P(k)$ is the (isotropic) power spectrum of the initial density field

NFW claims about spherically averaged halo density profiles



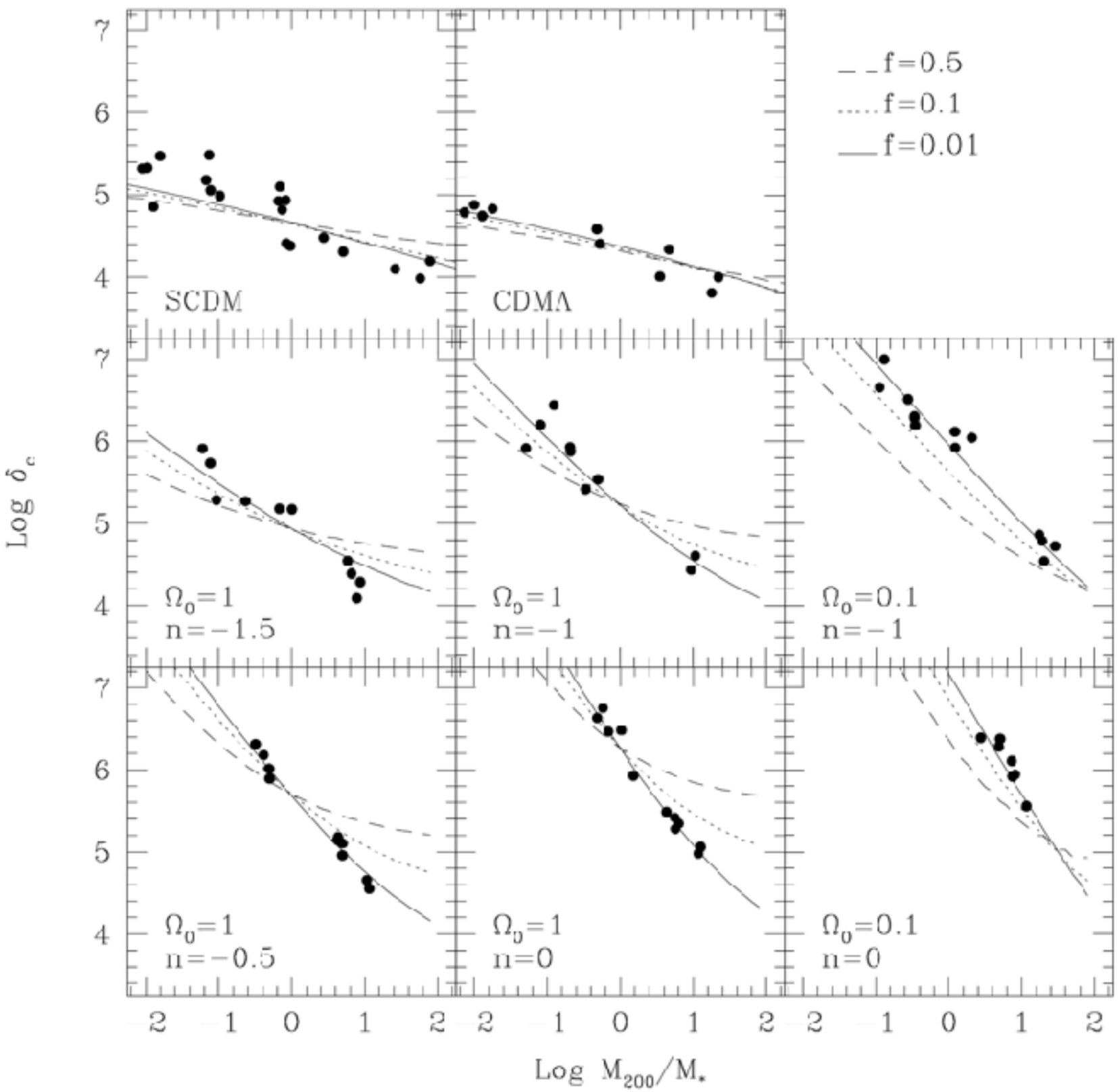
Profiles are **homologous**. Independent of halo mass, of cosmological parameters and of initial linear power spectrum, $P(k)$, they are well fit by the simple formula,

$$\rho(r)/\rho_{\rm crit} = \delta_s r_s^3 / r(r+r_s)^2$$

Depending only on the two scale parameters, δ_s and r_s

Navarro, Frenk & White 1997

NFW claims about spherically averaged halo density profiles



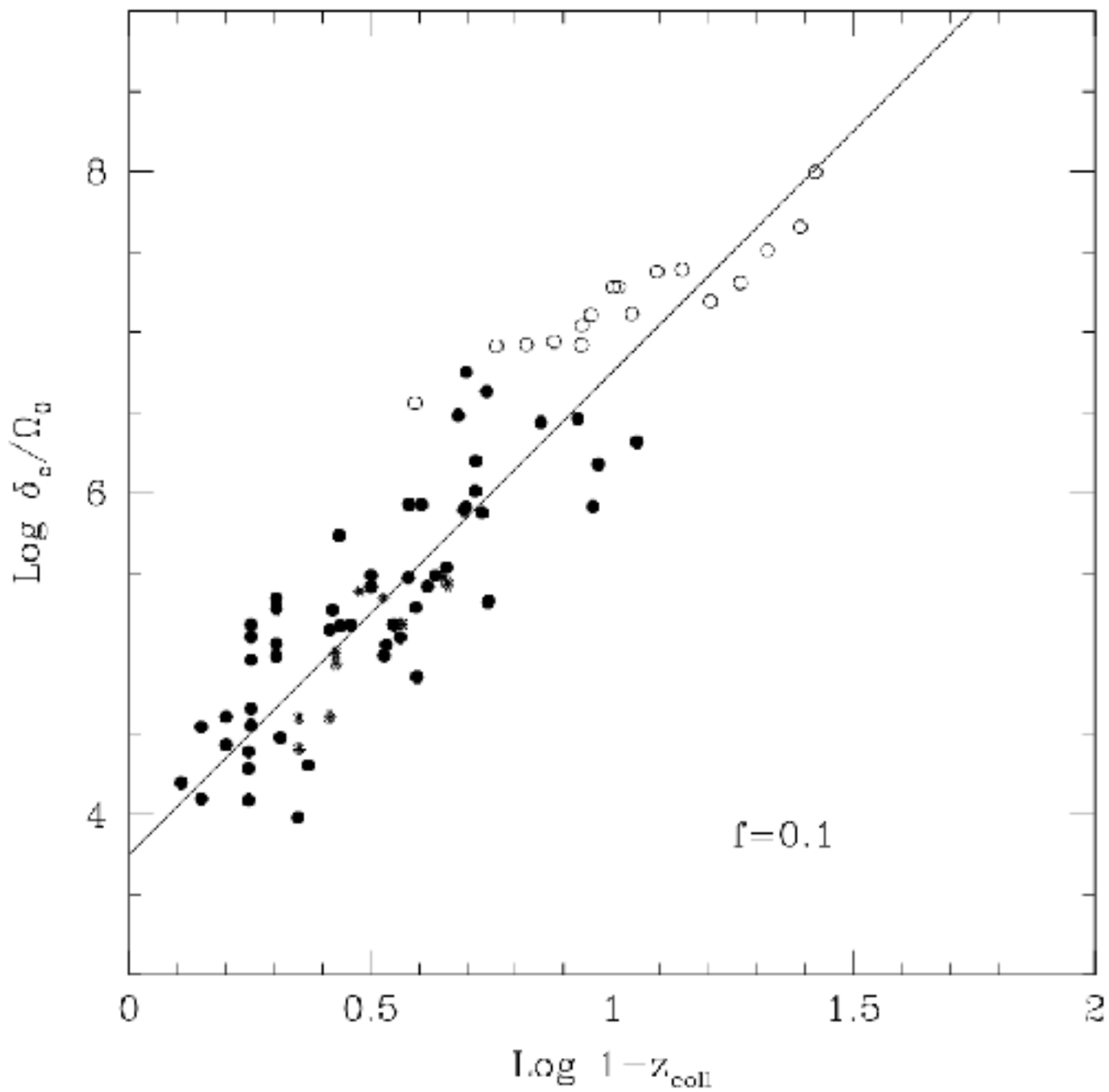
The characteristic density of a halo depends on its mass

Lower mass halos are denser

The halo mass-density relation depends strongly on cosmological parameters and on $P(k)$

Navarro, Frenk & White 1997

NFW claims about spherically averaged halo density profiles



The characteristic density of halos of all masses in all cosmologies and for all $P(k)$ is proportional to the density of the universe at the time $z_{\rm coll}$ when half of the total halo mass was first in significant nonlinear lumps (e.g. $> 0.1 M_{\rm halo}$)

$$\delta_c = A\Omega_0(1 + z_{\rm coll})^3$$

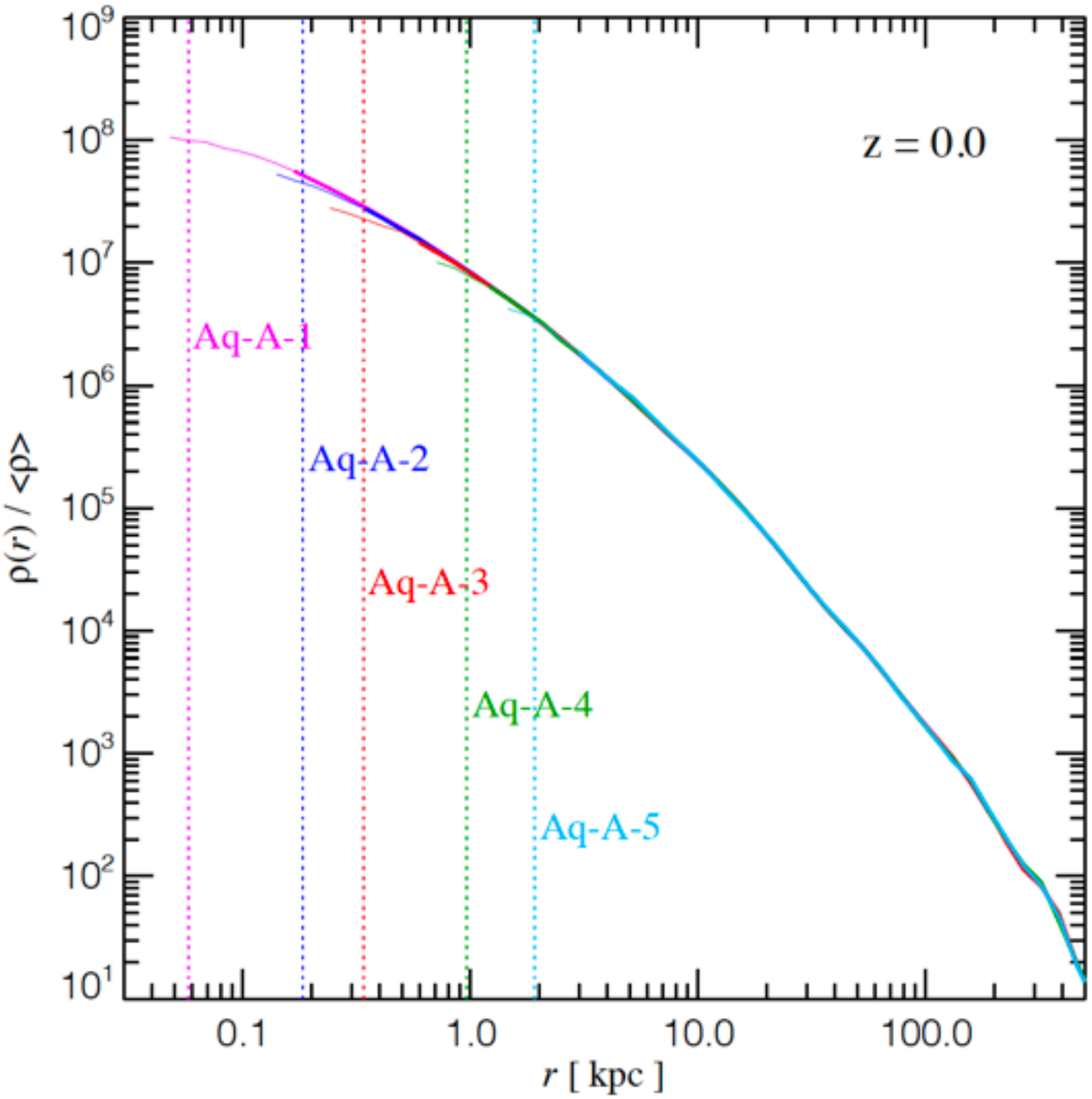
for a universal constant A

The characteristic density of halos thus reflects their assembly history

Navarro, Frenk & White 1997

Halos converge to NFW outside $r_{\mathrm{Power}}(t_f)$

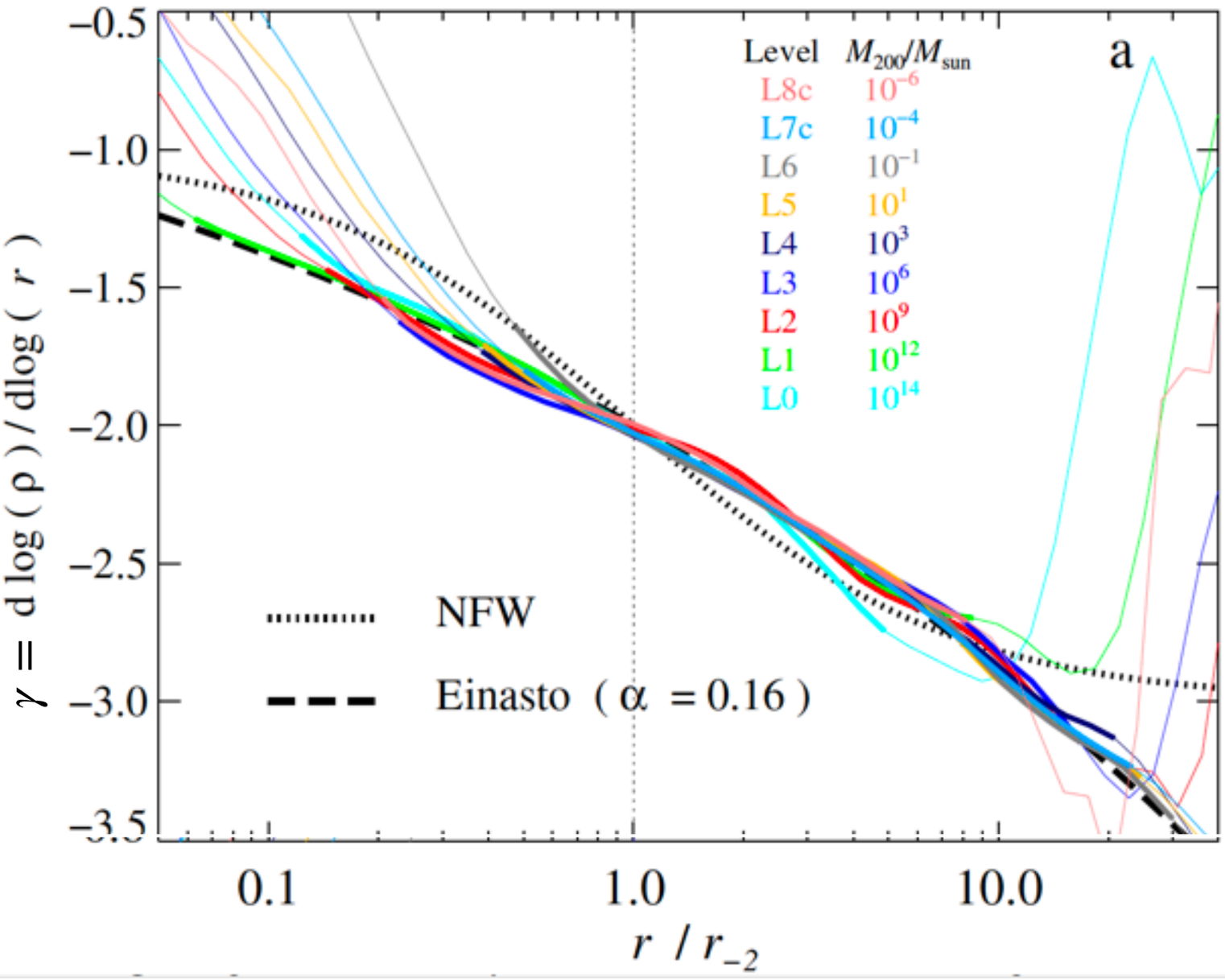
Springel et al 2008



The NFW shape is not a consequence of 2-body relaxation/discreteness

In ΛCDM halos γ declines with radius

Wang, Bose et al 2020

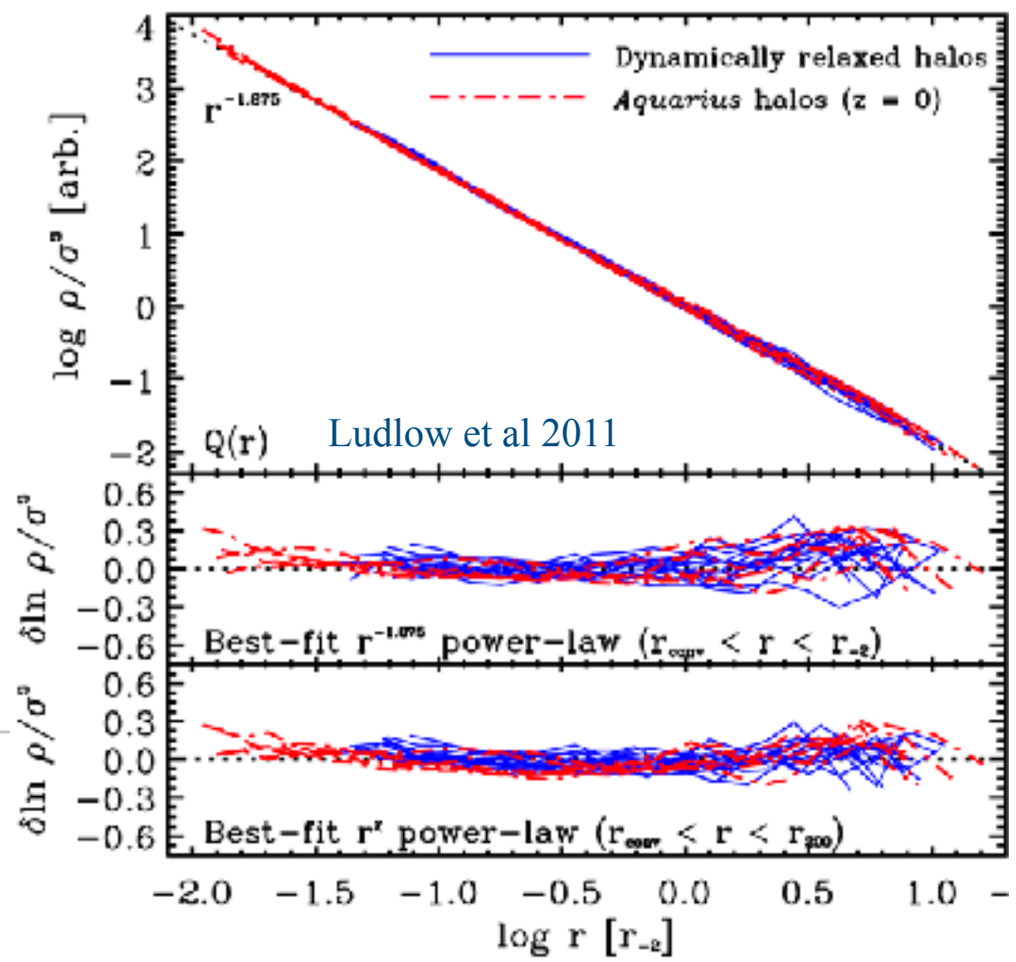


The shape of ΛCDM density profiles is independent of mass, e.g. relative to M_*

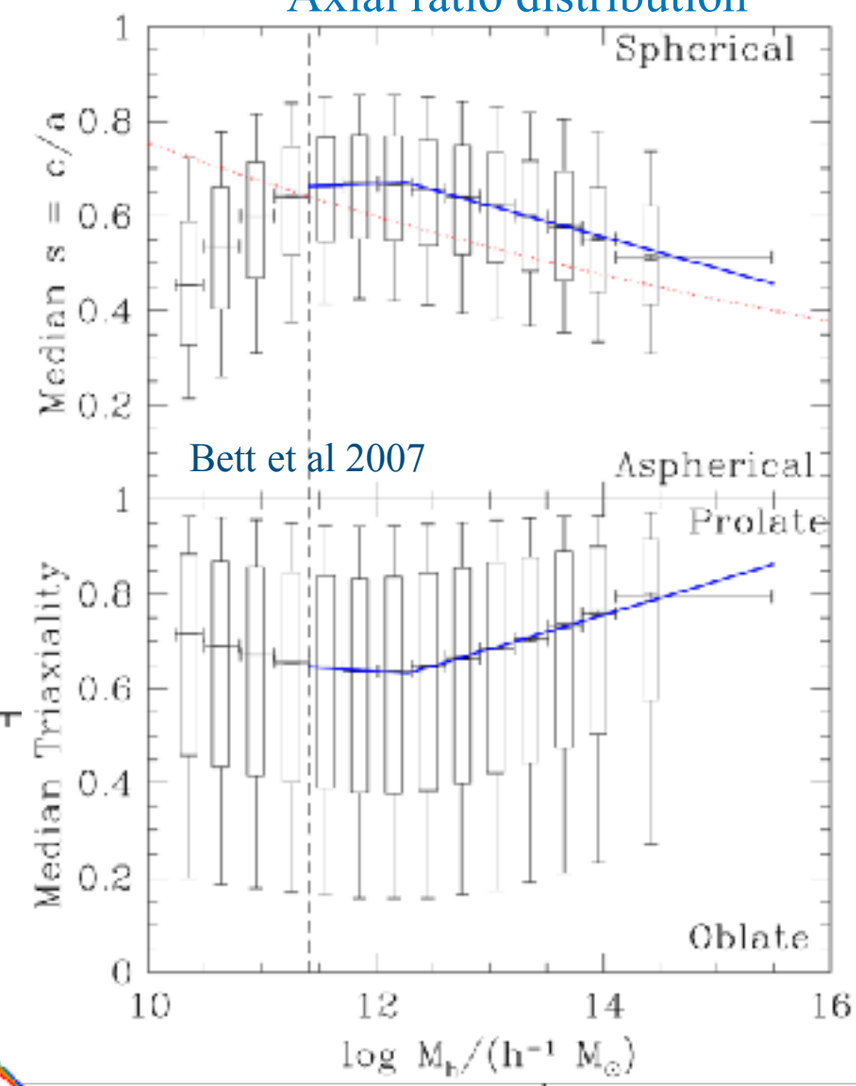
No dependence on linear power spectrum slope, see also halos in $P \propto k^n$ cosmologies

Other "universalities" of ΛCDM halos

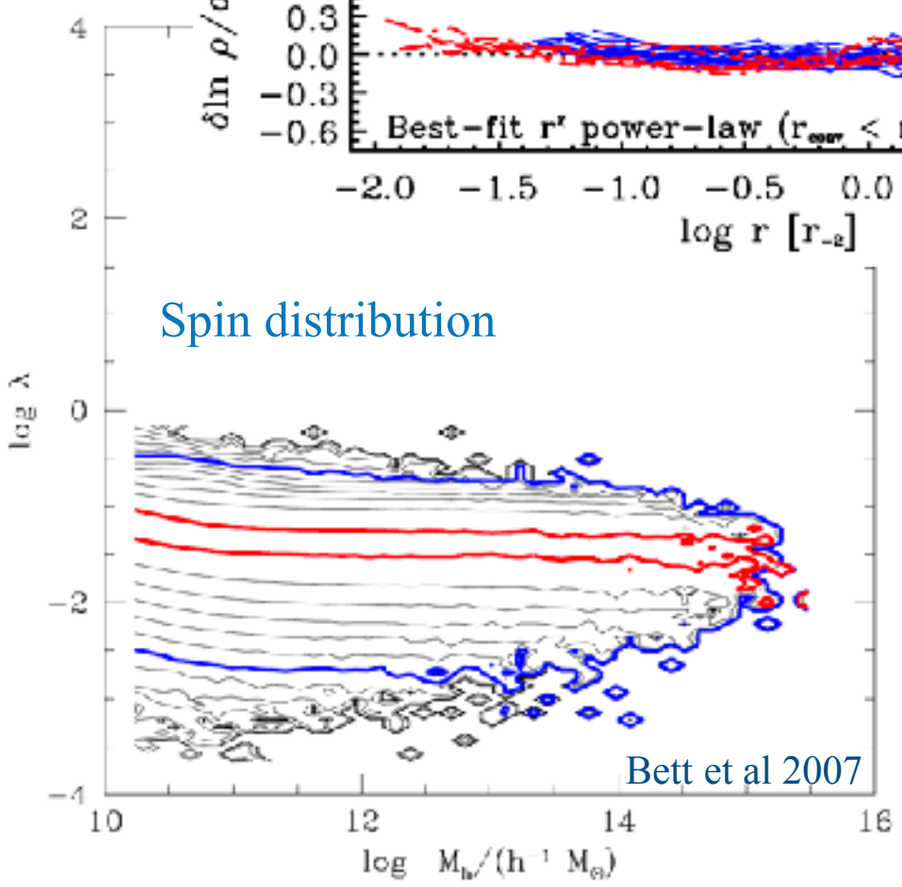
Pseudo-phase-space density: $Q \equiv \rho/\sigma^3$

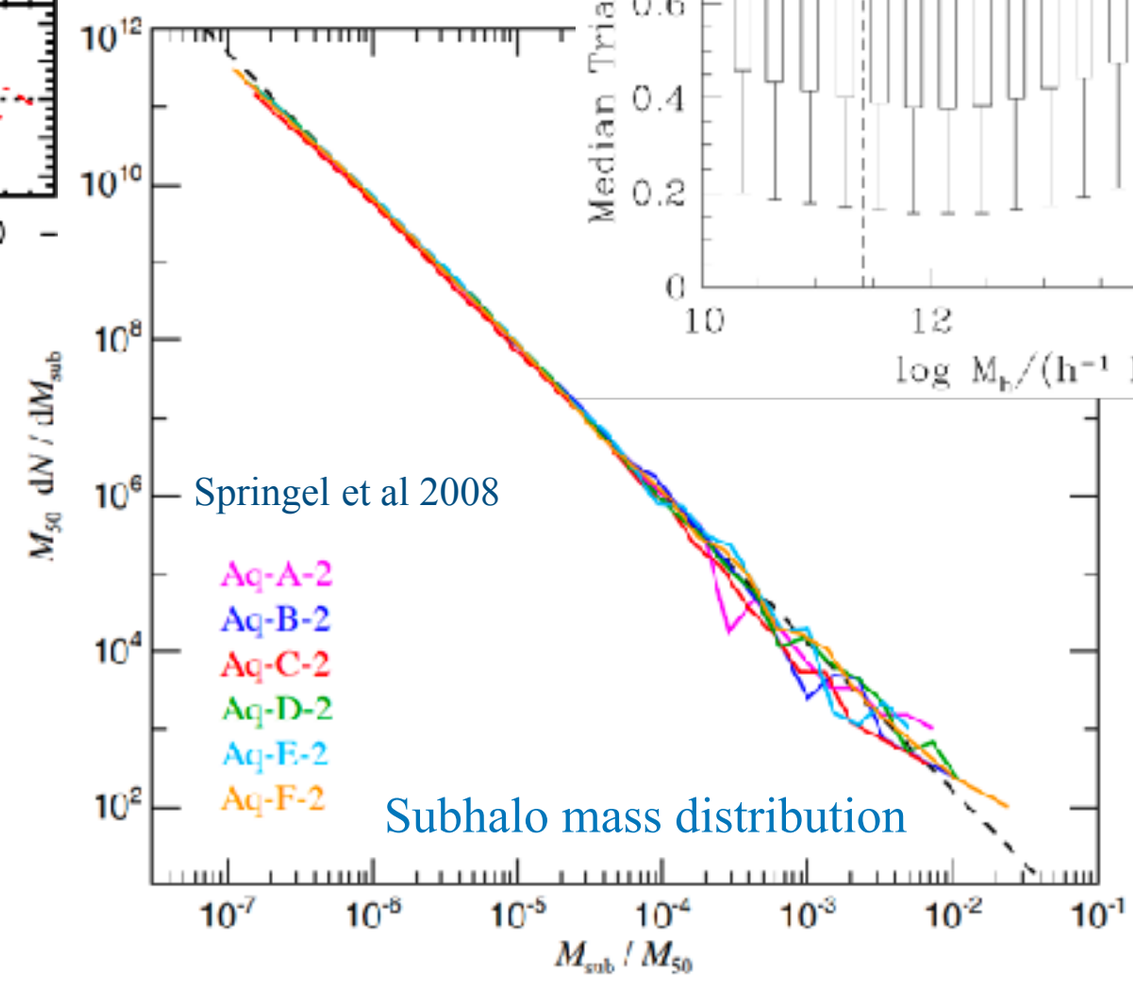


Axial ratio distribution



Spin distribution

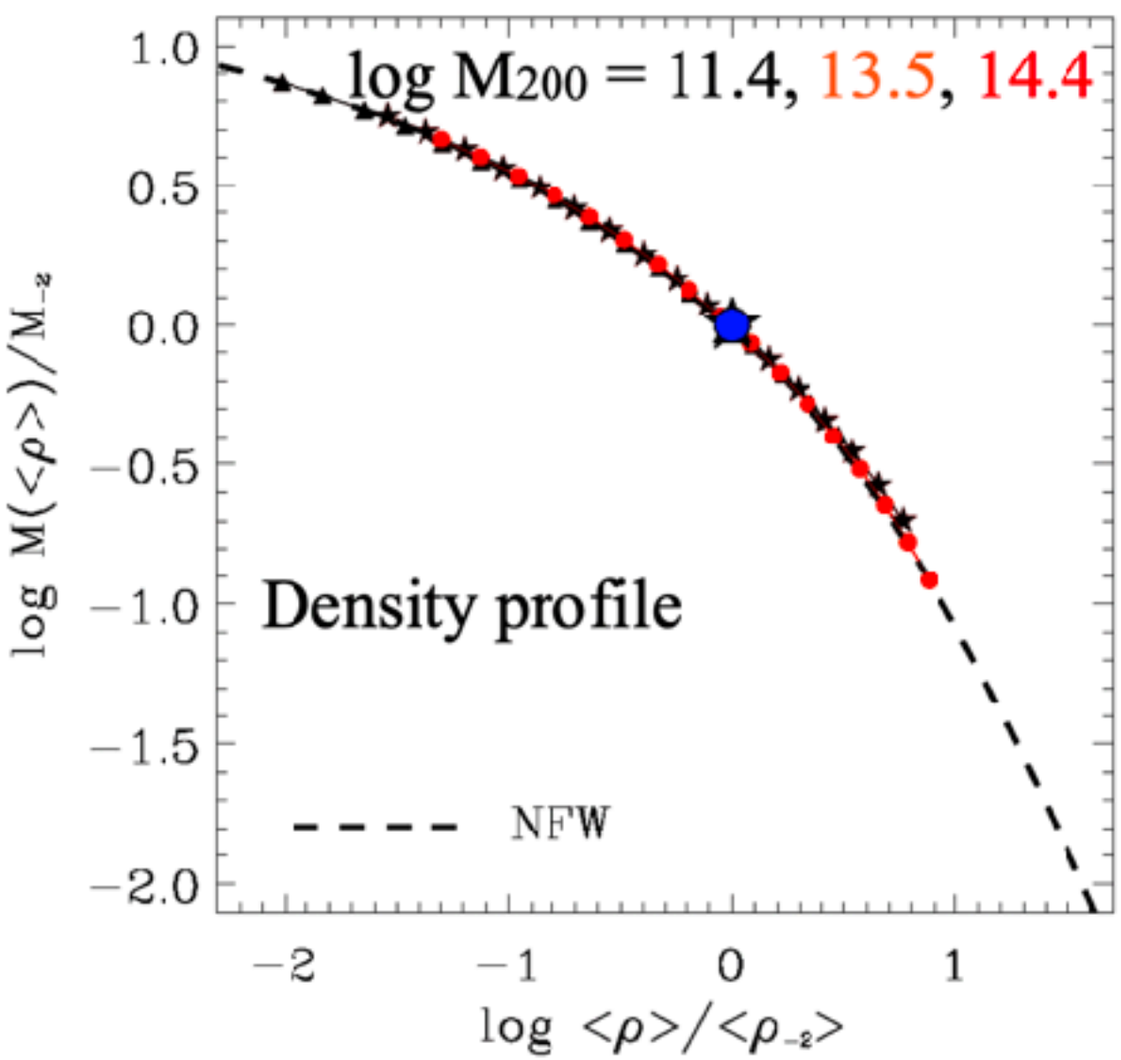


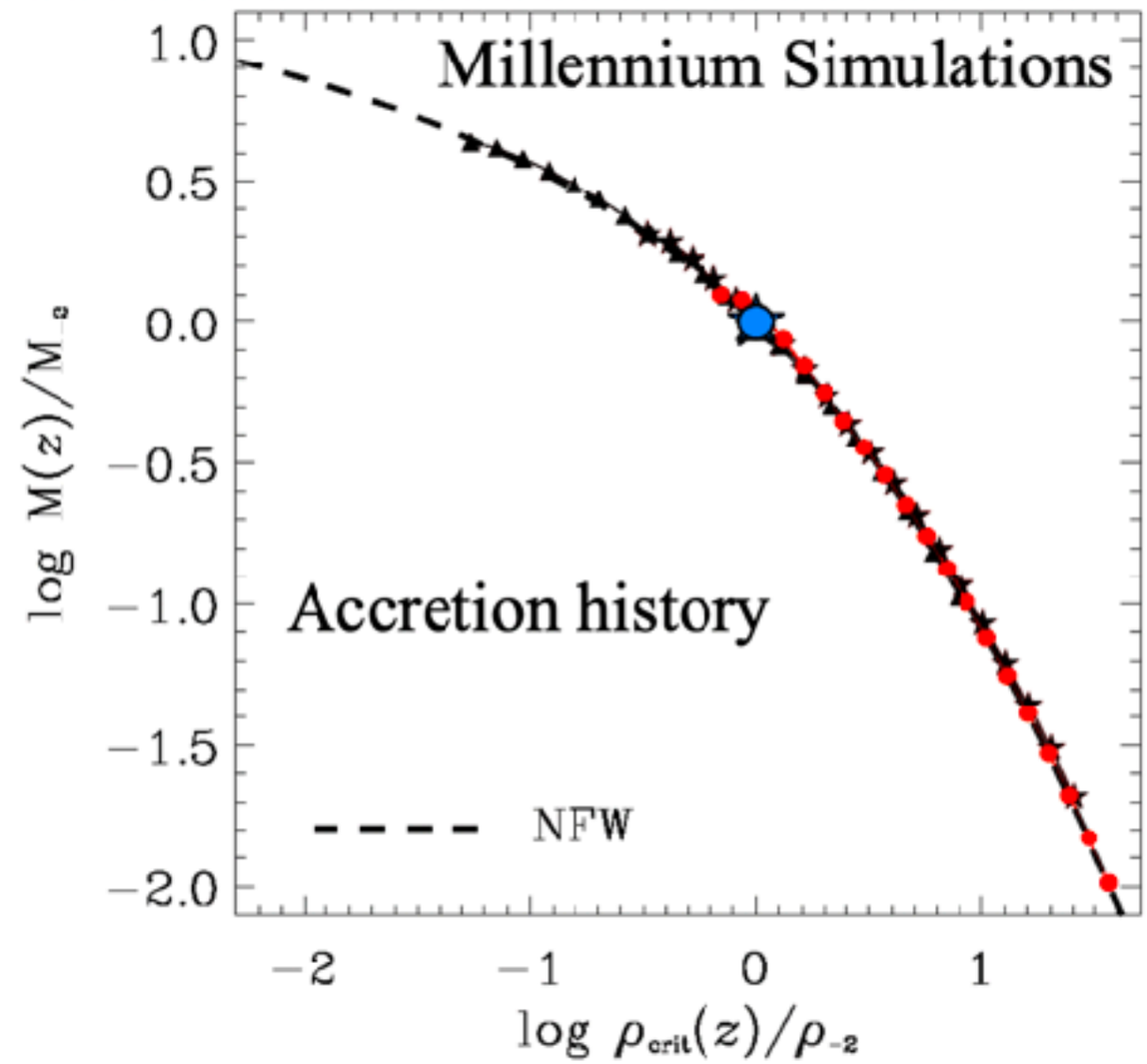


Subhalo mass distribution

The connection to halo assembly

Ludlow et al 2014





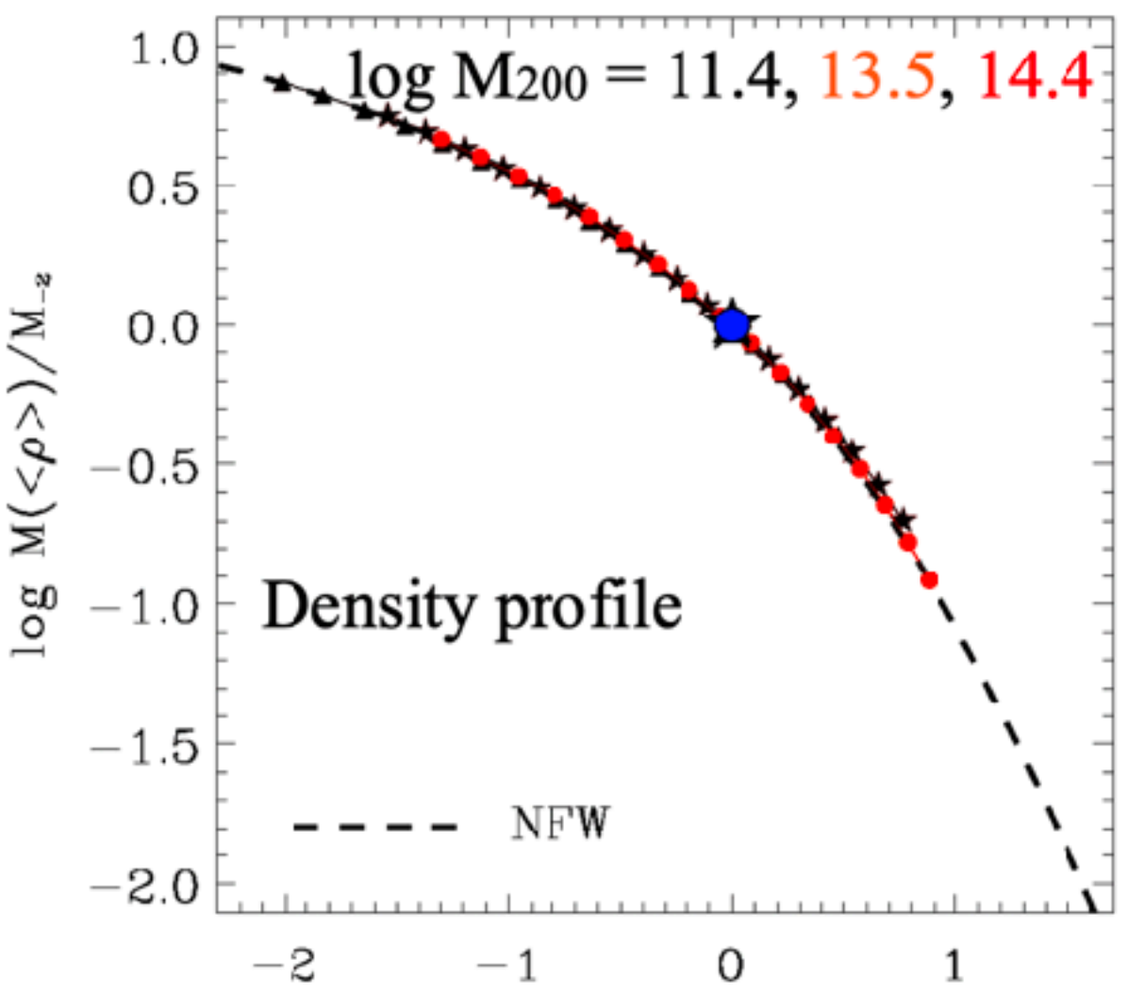
The mean profiles of ΛCDM halos *are* tightly linked to their mean growth histories

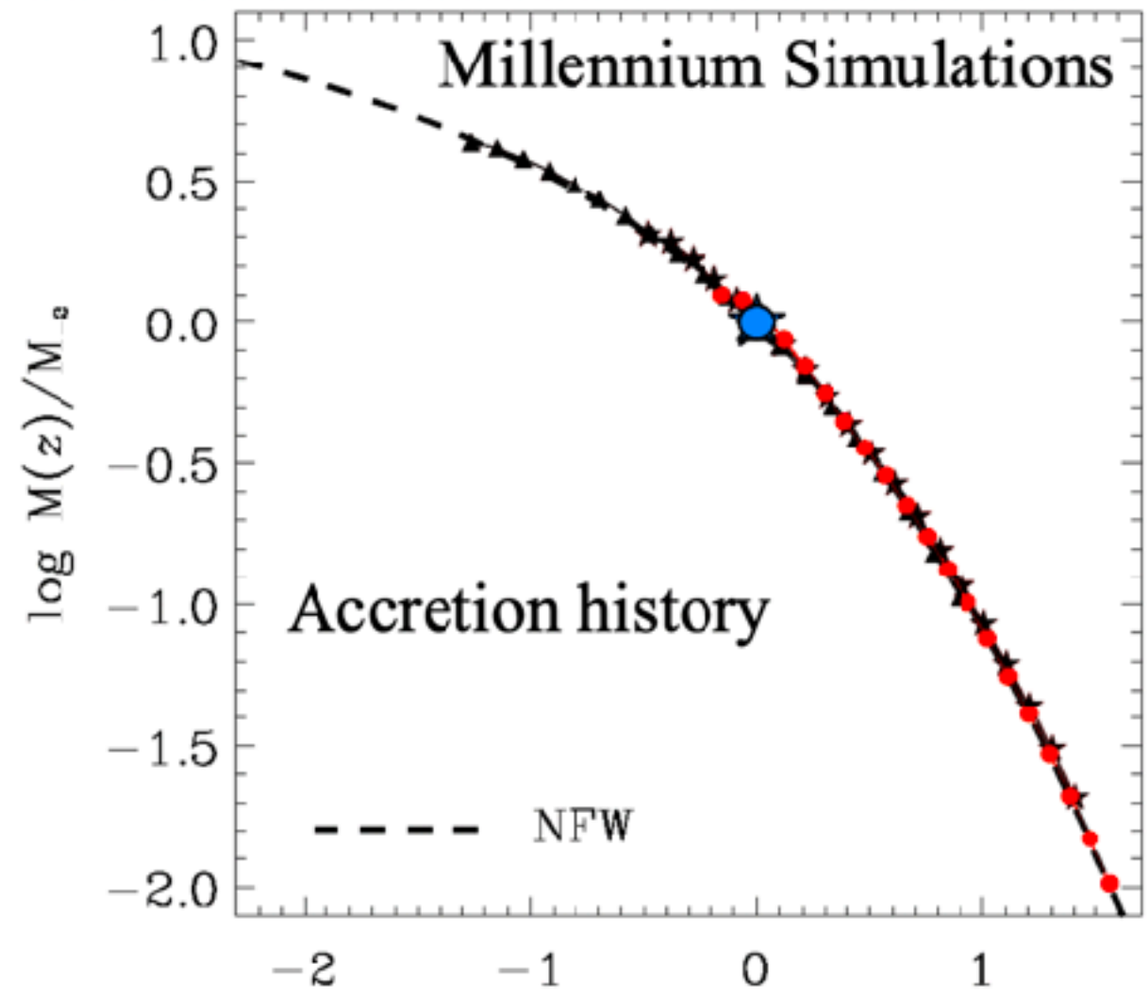
→ Violent relaxation is weak

A "universal" growth history shape

The connection to halo assembly

Ludlow et al 2014





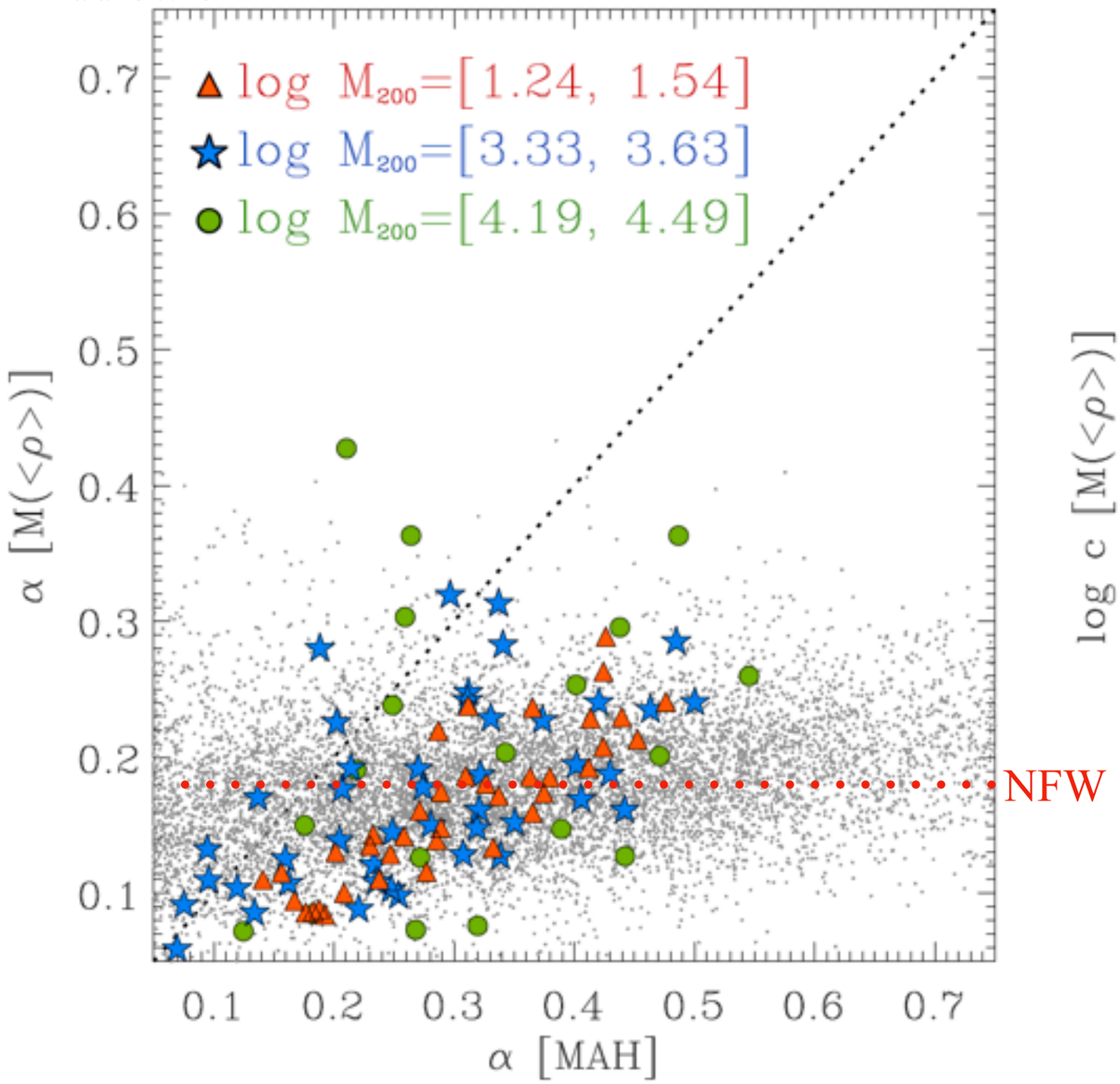
The mean profiles of ΛCDM halos *are* tightly linked to their mean growth histories

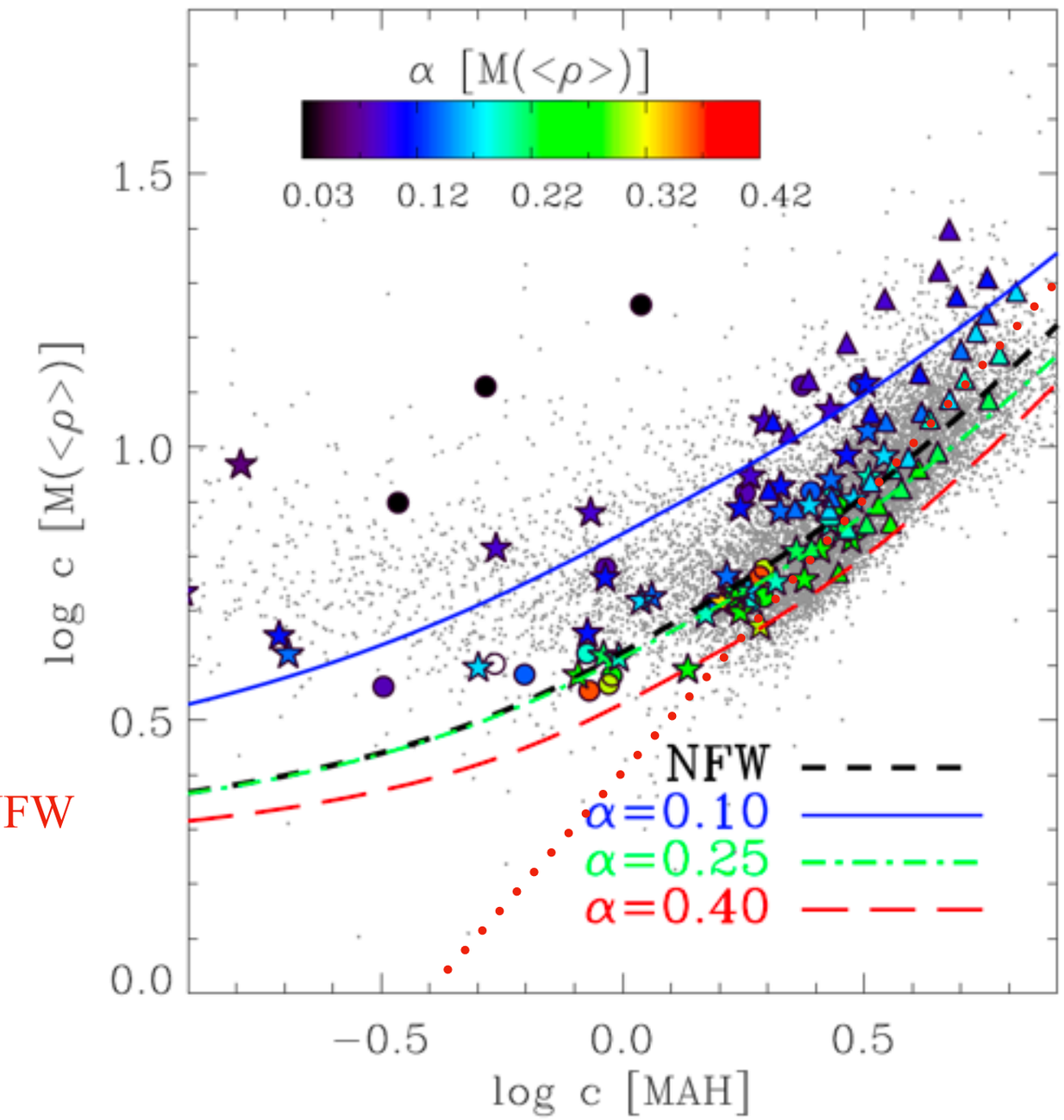
→ Violent relaxation is weak

A "universal" growth history shape

Convergent evolution?

Ludlow et al 2014





Profile *c* reflects MAH *c* nearly linearly, but profiles are closer to NFW than MAH's: convergence driven by weak violent relaxation

Self-similar halo growth

Consider a power-law ellipsoidal linear density perturbation within an otherwise uniform EdS universe:

$$\delta(\mathbf{x}, t) = (t/t_0)^{2/3}(\mathbf{x}\,.\,A\,.\,\mathbf{x})^{-\alpha/2}, \qquad |A| = 1$$
$$= (t/t_0)^{2/3} M(\mathbf{x})^{-\alpha/3}$$

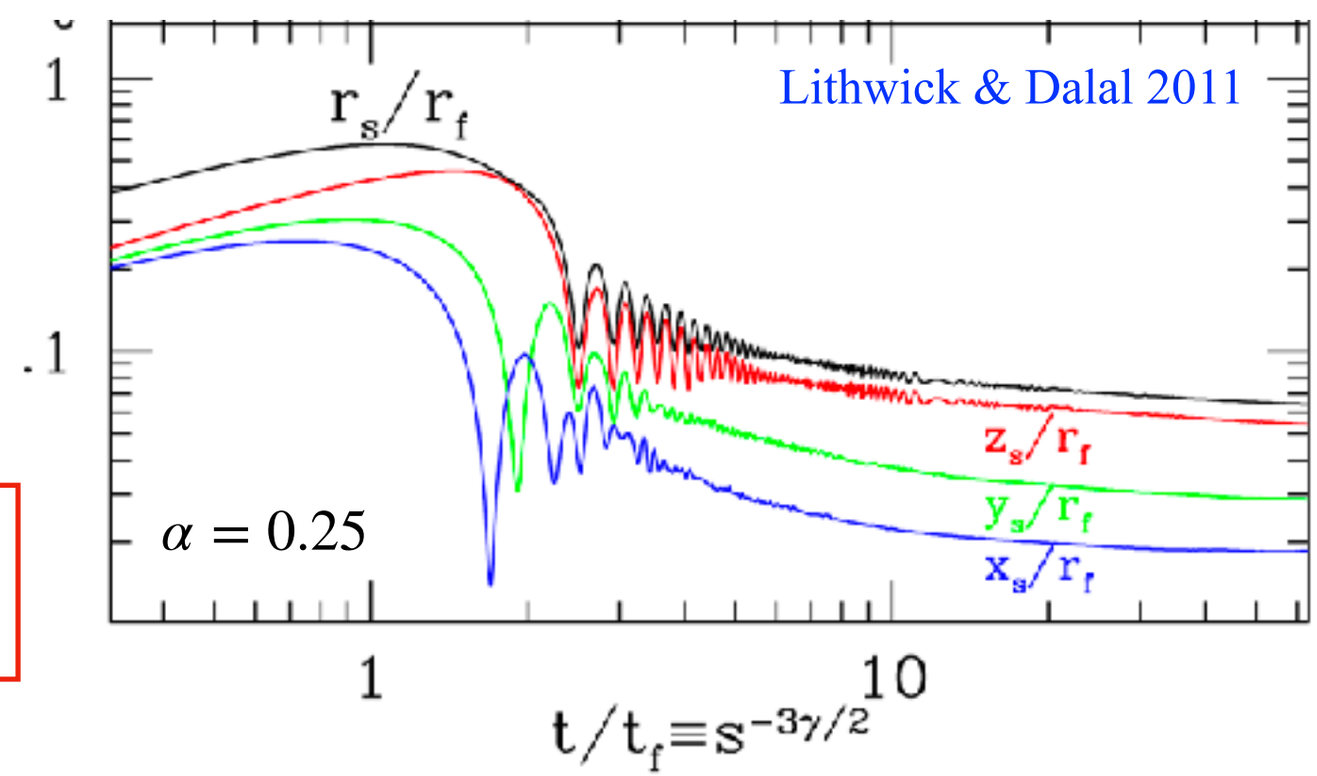
The halo mass thus increases as: $M_{\rm halo}(t) \propto t^{2/\alpha}$

Within the halo: $\rho \propto r^{-\gamma}$ → $t_{\rm orb} \propto r^{\gamma/2}$, $M \propto r^{3-\gamma}$ → $M \propto t_{\rm orb}^{(6-2\gamma)/\gamma}$

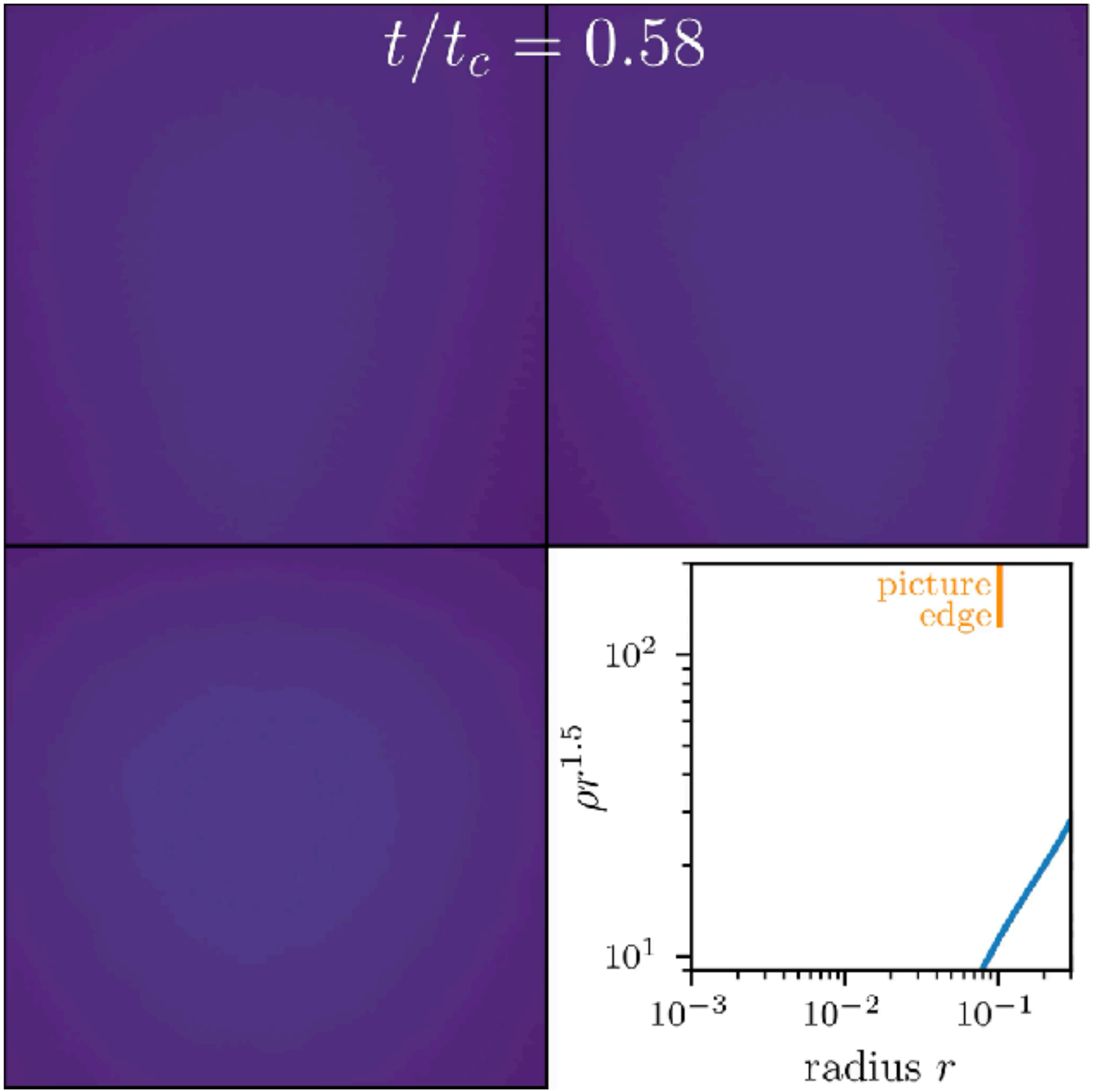
If $M(t_{\rm orb}) \propto M_{\rm halo}(t = t_{\rm orb})$,
then $2/\alpha = (6 - 2\gamma)/\gamma$

→ $\gamma = 3\alpha/(1 + \alpha)$

This is *not* NFW-like, but rather a power law with γ depending on α



Prompt cusp formation in a ΛCDM density peak

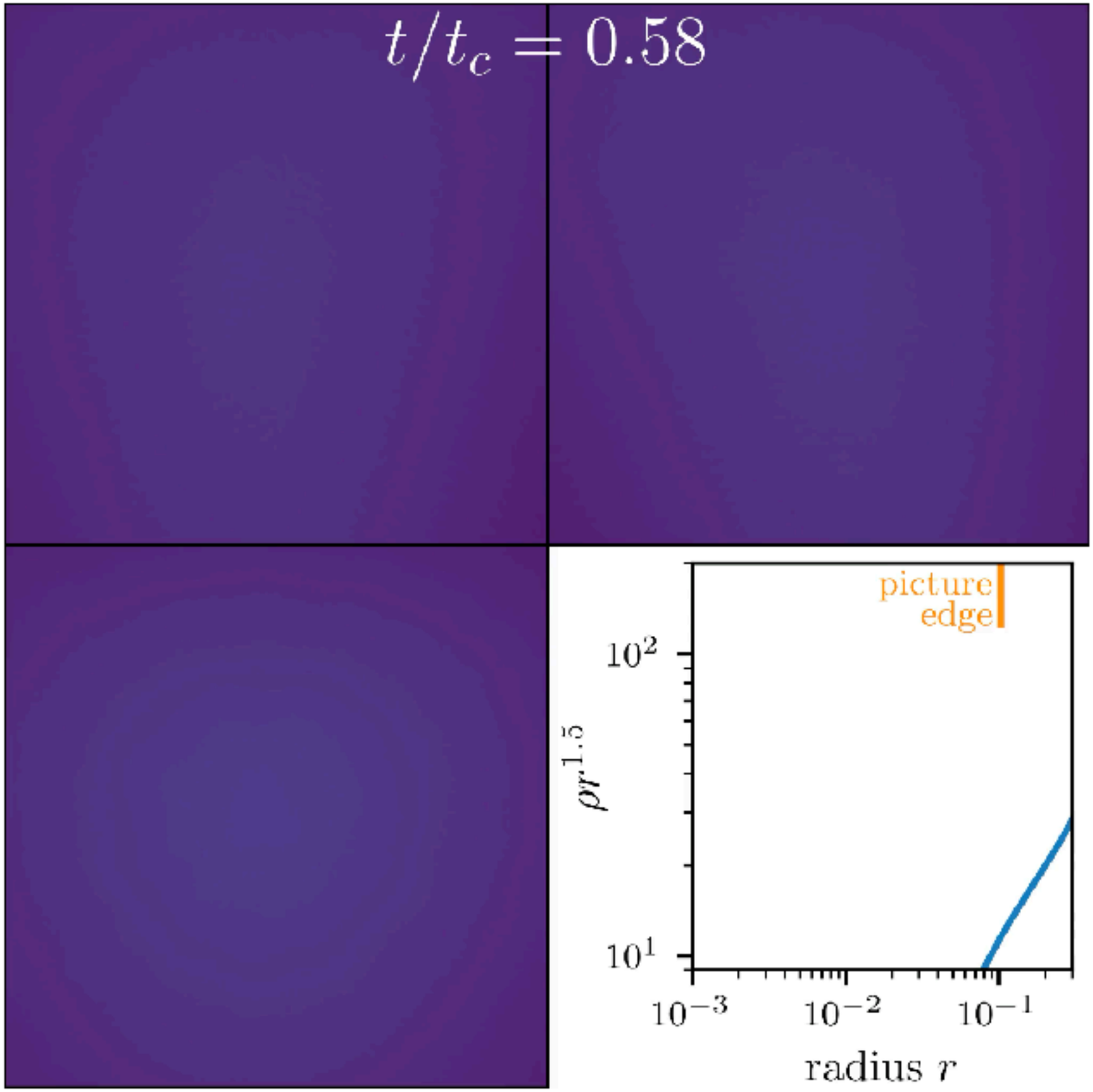


$t_c \longrightarrow z = 87$

$M_{pk} \sim 10^{-6} M_{\rm sun}$

Delos & White 2023

Prompt cusp formation in a ΛCDM density peak



$t_c \longrightarrow z = 87$

$M_{pk} \sim 10^{-6} M_{\rm sun}$

Prompt cusp formation differs qualitatively from "normal" halo formation

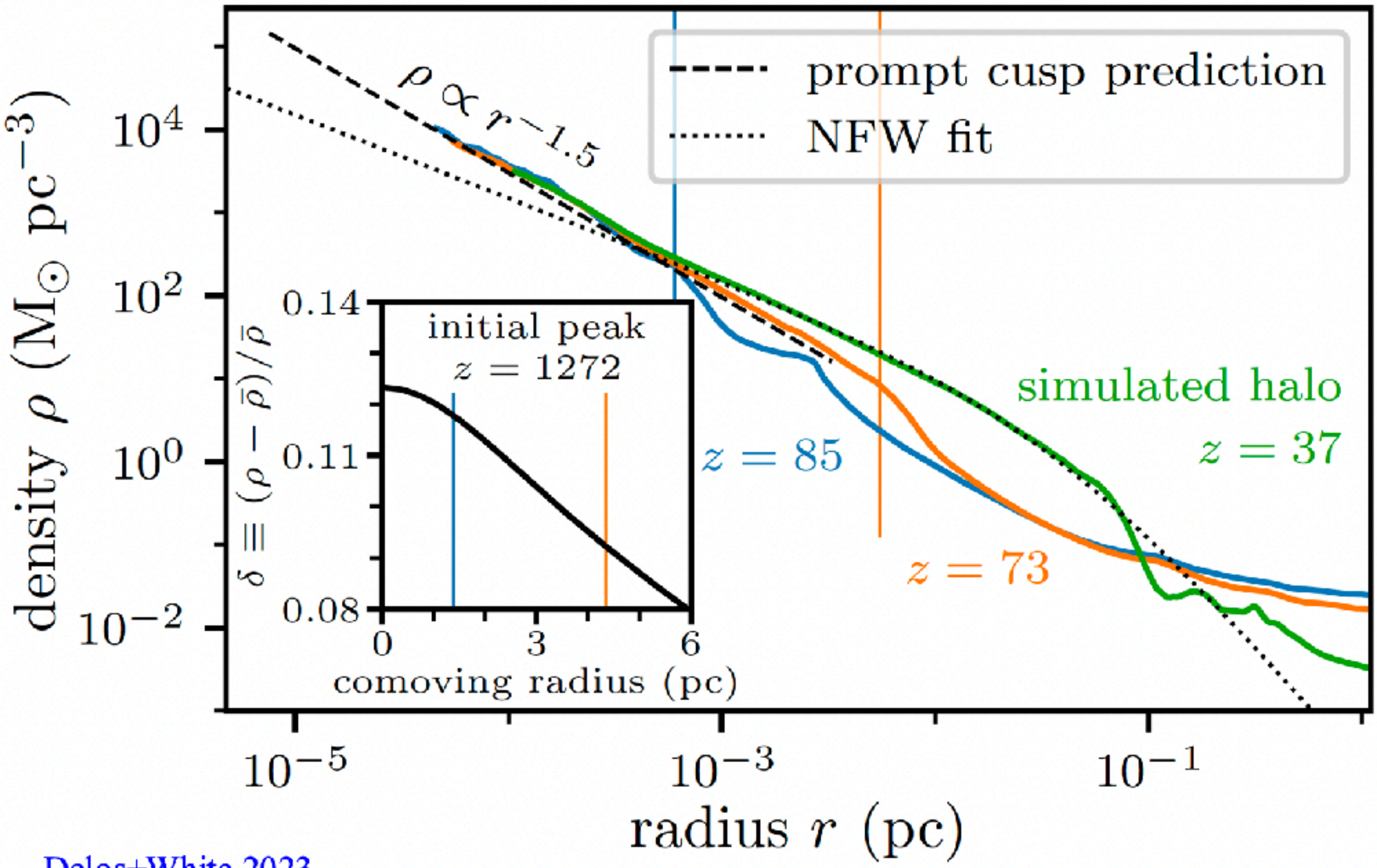
Violent relaxation is important

No close link of profile to cusp growth history

A "universal" profile *different* from NFW

Delos & White 2023

Prompt cusp and subsequent halo growth for a peak with $z_{coll} = 87$



Delos+White 2023

- Near-universal NFW-like profiles form by hierarchical clustering from:
  — **gaussian** initial linear density fluctuations with
  — **broad** $P(k)$ without strong features.

- The profile **shape** is only very weakly dependent on
  — halo mass M_h/M_*
  — the mean slope of $P(k)$
  — the background cosmological parameters

- The characteristic **density** of halos depends on all three of these factors

- The profile shape and the characteristic density of CDM halos is tightly linked to **assembly history**, with violent relaxation reducing deviations from NFW shape.

- Non-hierarchical formation from non-gaussian initial conditions can produce non-NFW halo density profiles which may or may not be linked to halo assembly history

Thus the "universality" of NFW-like density profiles appears to be a consequence of convergent evolution from near-universal halo assembly histories for Vlasov-Poisson evolution from gaussian linear density fluctuations with a broad $P(k)$

Excursion set calculation of halo mass growth

Let $p(M_1, z_1 \,|\, M_0, z_0)dM_1$ be the distribution of progenitor halo mass M_1 at z_1 for individual mass elements which are part of a halo of mass M_0 at z_0. Then

$$dN = \frac{M_0}{M_1} p(M_1, z_1 \,|\, M_0, z_o)dM_1$$

is the number distribution of progenitors by mass. One can estimate a typical mass for the most massive progenitor at redshift z_1 by solving for $M_{\text{m.m.}}$ in

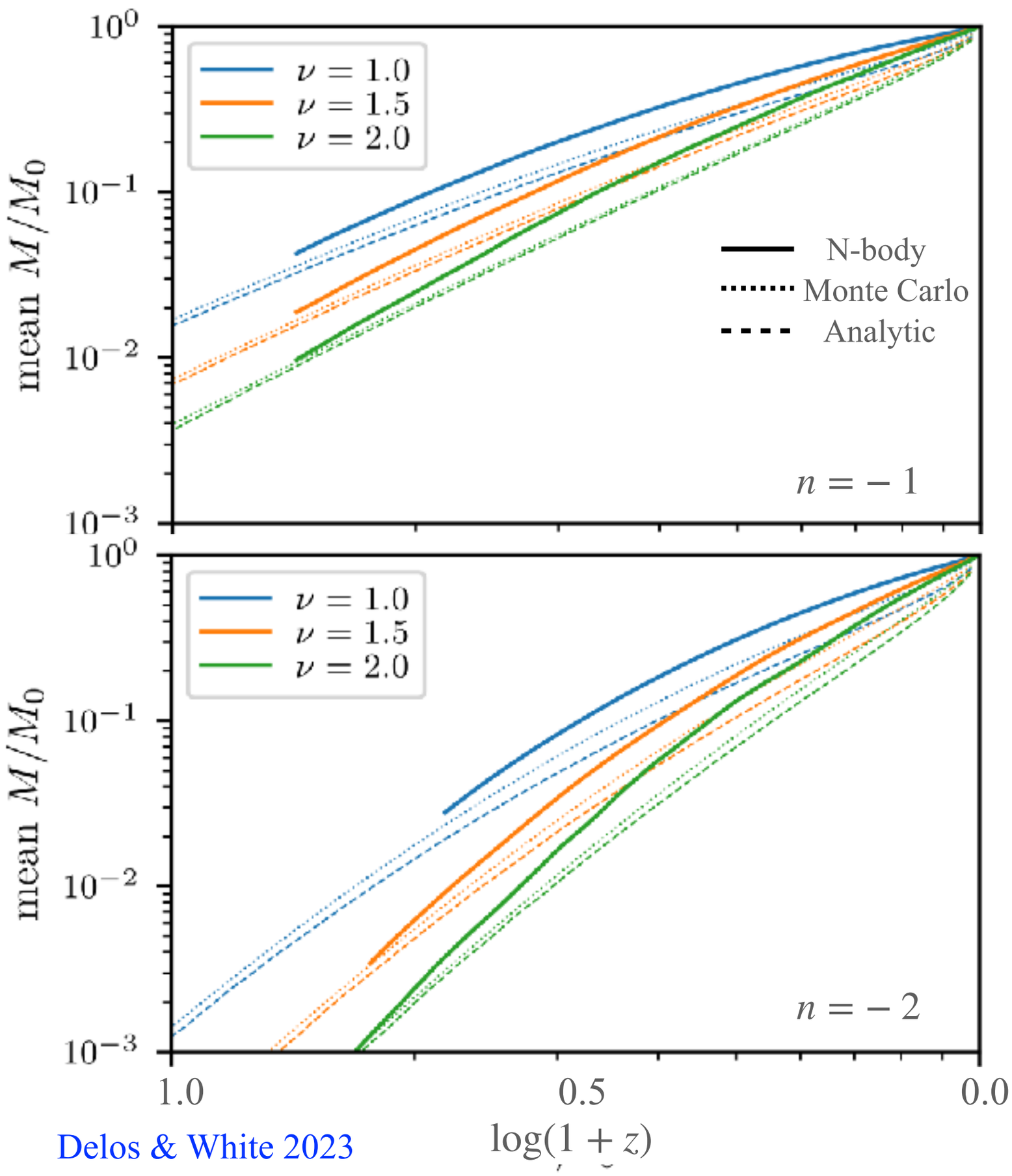
$$M_{\text{m.m.}}(z_1 \,|\, M_0, z_0) = \int_{M_{\text{m.m.}}}^{M_0} \frac{dN}{dM_1} \, M_1 \, dM_1 \, .$$

For an EdS universe with $P(k) \propto k^n$, $\sigma^2(M) \propto M^{-(3+n)/3}$, w.l.o.g. $z_0 = 0$, and

$$M_{\text{m.m.}}/M_0 = \text{erfc } A\left((M_{\text{m.m.}}/M_0)^{-(3+n)/3} - 1\right)^{-1/2}$$

for a sharp-k filter, where $A = \frac{1}{\sqrt{2}}\left(\frac{M_0}{M_*}\right)^{(3+n)/6} z_1$, $\sigma(M_*) = \delta_c = 1.686$,

so $M_{\text{m.m}}/M_0 = f(A, n)$ — Is this approximately "universal"?



Delos & White 2023

Analytic estimates of halo assembly histories agree qualitatively with both MC realisations of excursion set trajectories and N-body reconstructions

The universality of halo density profiles may be just a consequence of gaussian statistics