

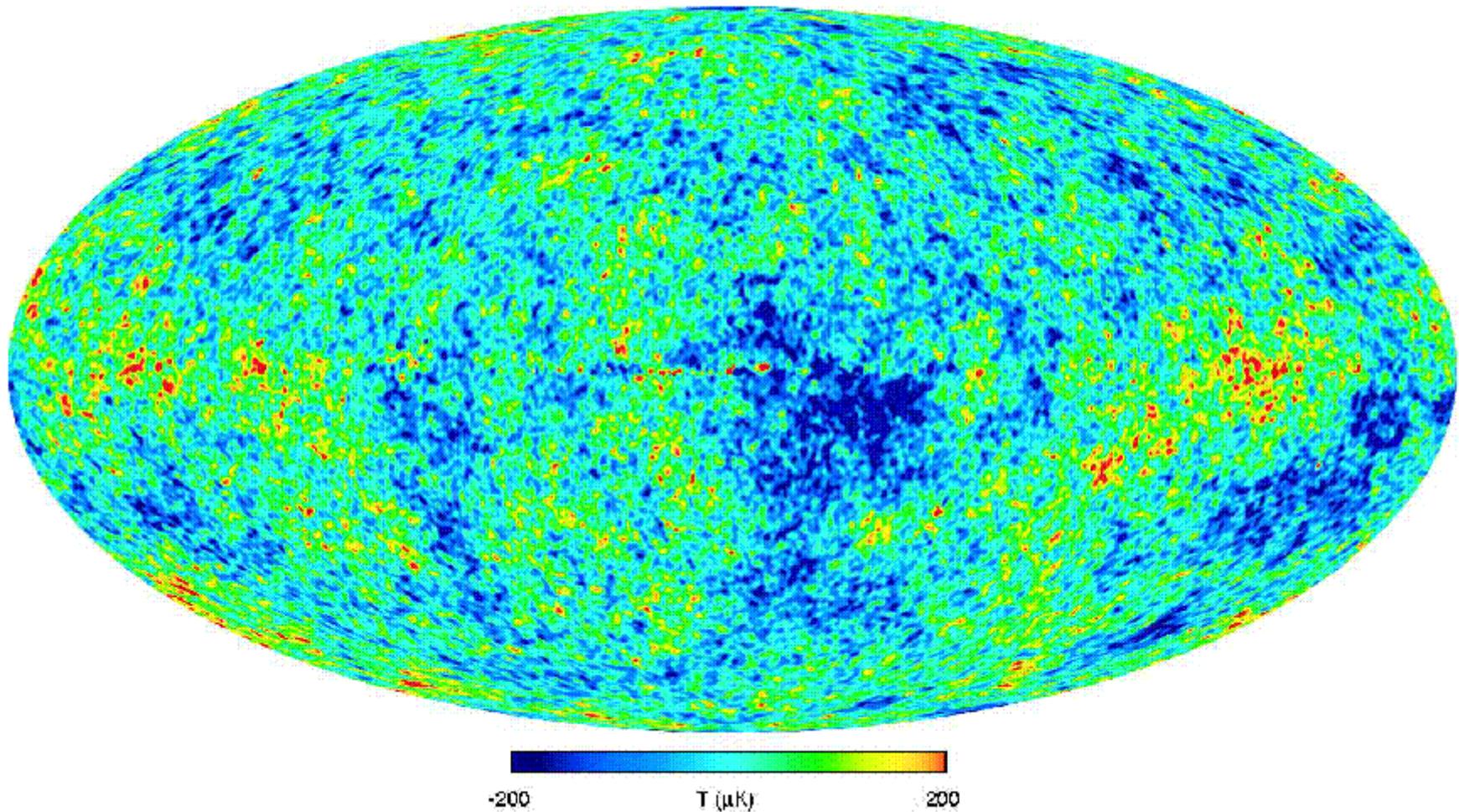
*Putting Gravity to
Work, Cambridge
July 2008*

The (very) small-scale structure of dark matter halos

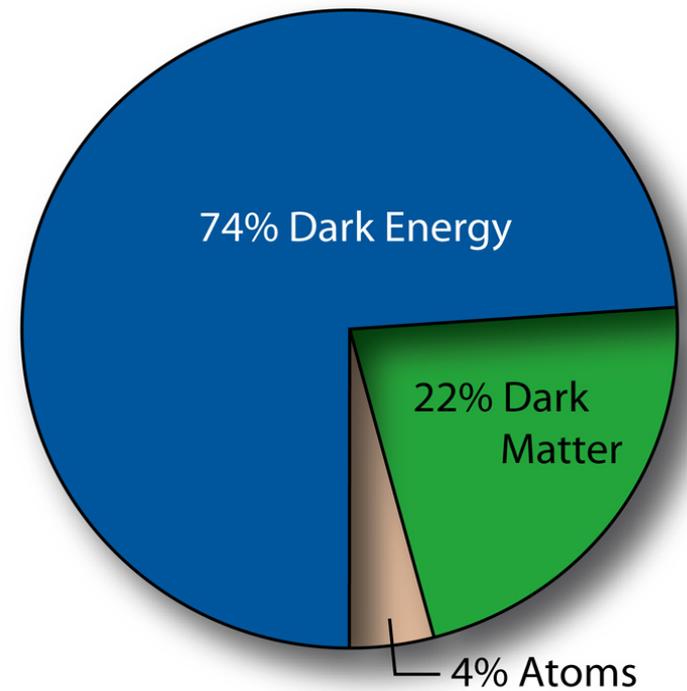
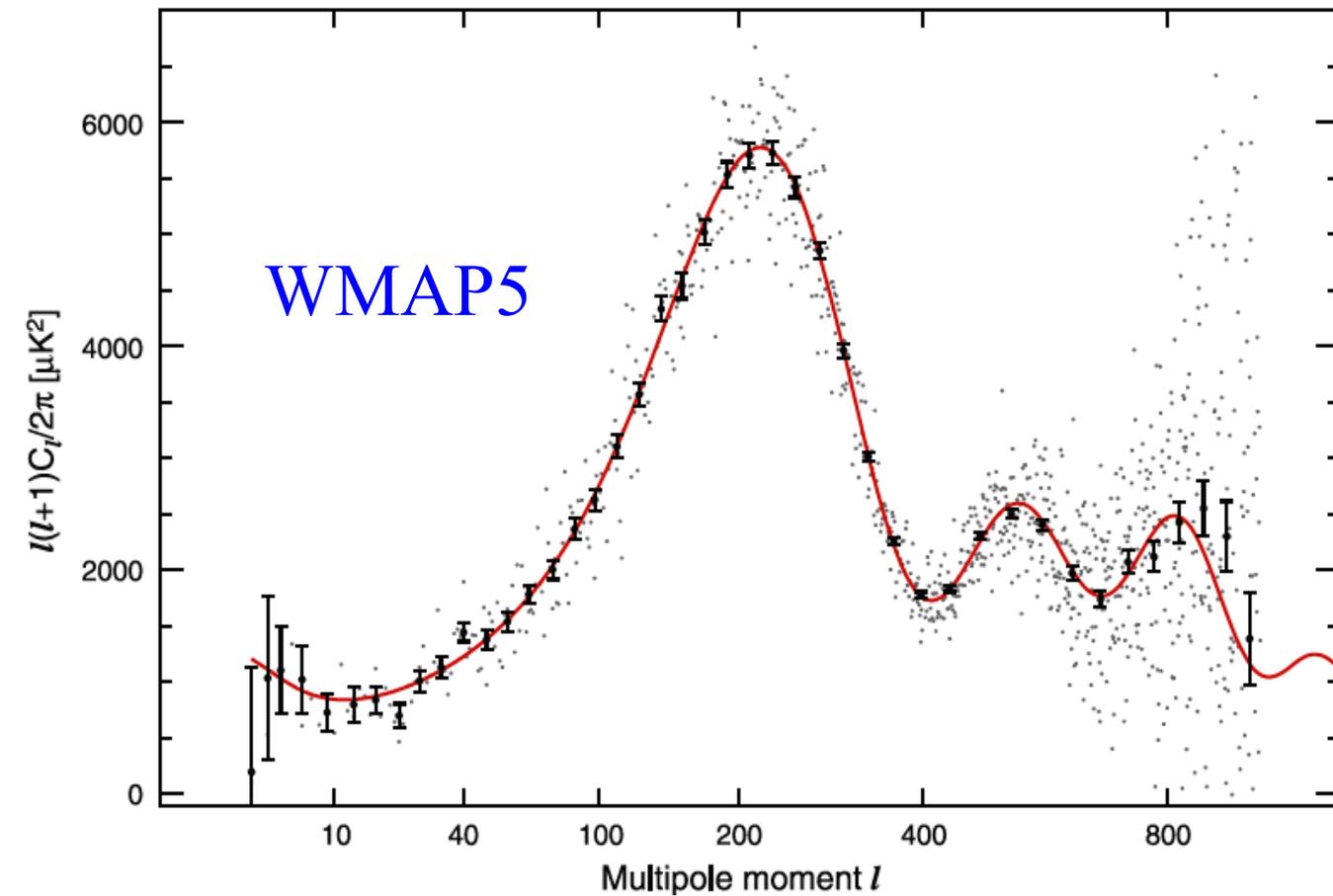
Simon White

Max Planck Institute for Astrophysics

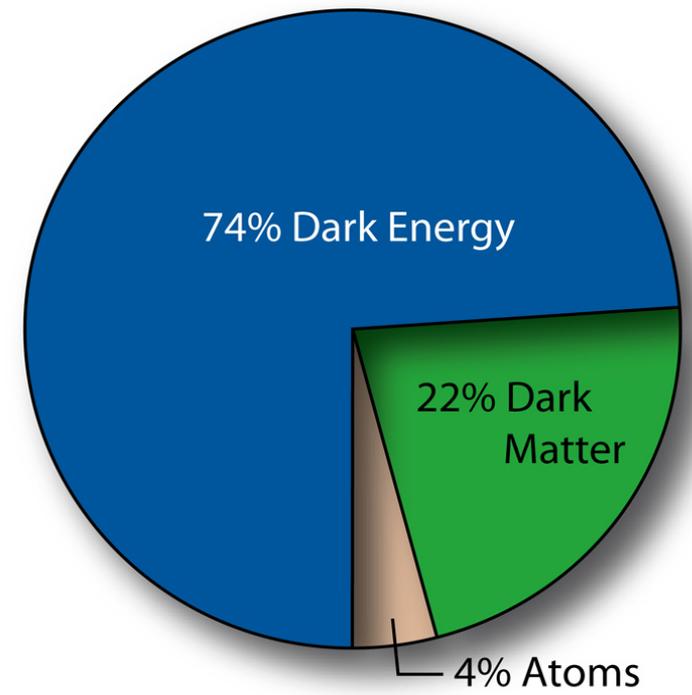
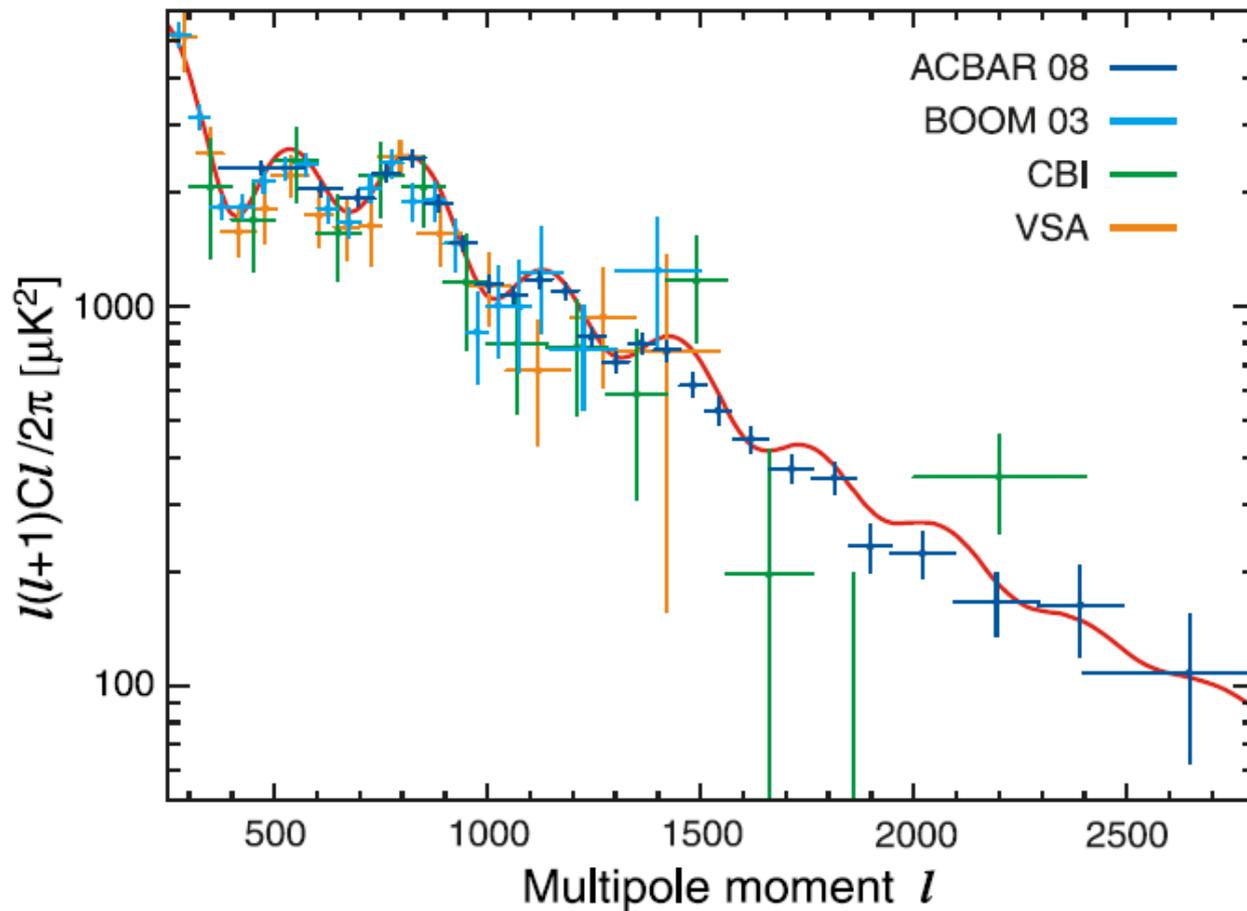
The *WMAP* of the whole CMB sky



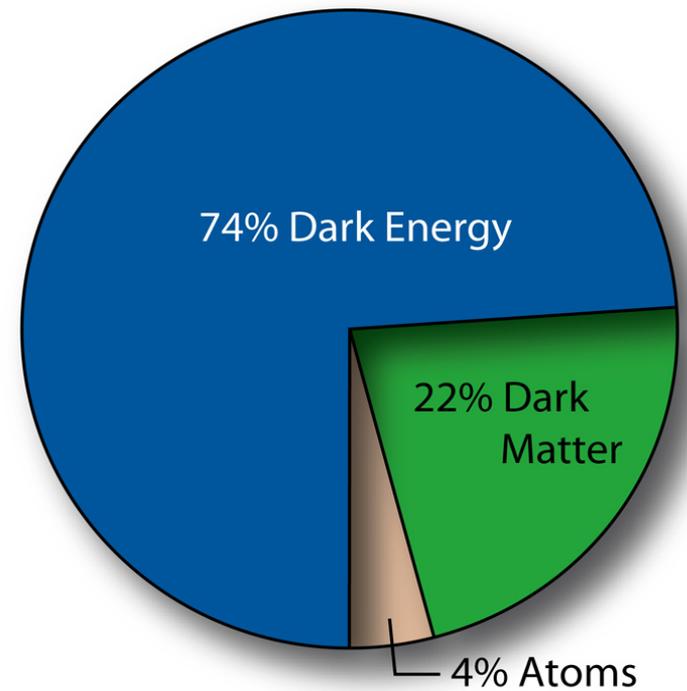
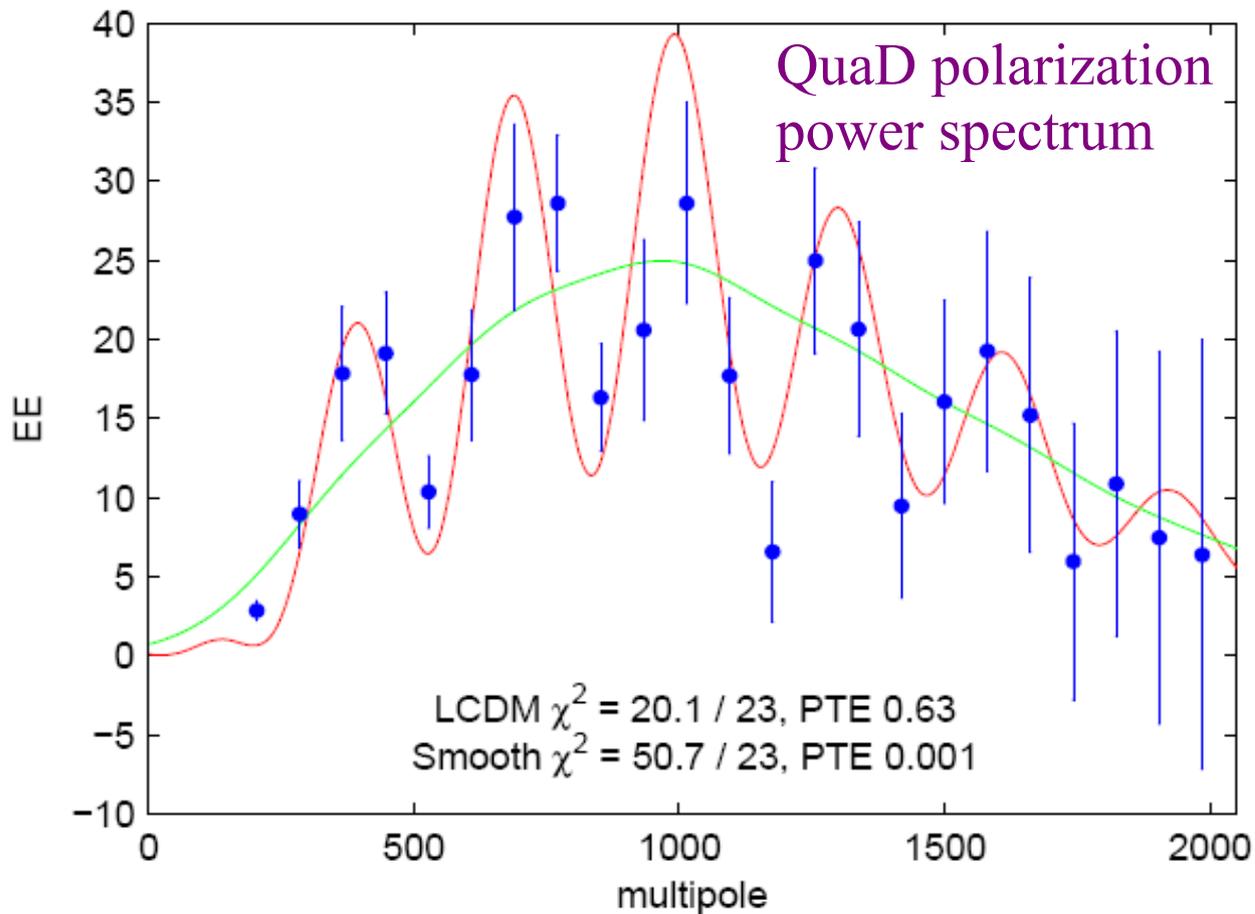
Bennett et al 2003



parameter	symbol	WMAP-5		comment
		alone	+ BAO + SNe	
CMB temperature	T_{CMB}	2.728 ± 0.004 K	–	from (Fixsen <i>et al.</i> 1996)
total matter density	Ω_{tot}	$1.099^{+0.100}_{-0.085}$	1.0052 ± 0.0064	assuming spatial flatness here and below
matter density	$\Omega_{\text{m}0}$	0.258 ± 0.03	0.279 ± 0.015	
baryon density	$\Omega_{\text{b}0}$	0.0441 ± 0.0030	0.0462 ± 0.0015	
cosmological constant	$\Omega_{\Lambda 0}$	0.742 ± 0.03	0.721 ± 0.015	
Hubble constant	h	$0.719^{+0.026}_{-0.027}$	0.701 ± 0.013	
power-spectrum normalisation	σ_8	0.796 ± 0.036	0.817 ± 0.026	
age of the Universe in Gyr	t_0	13.69 ± 0.13	13.73 ± 0.12	
decoupling redshift	z_{dec}	1087.9 ± 1.2	1088.2 ± 1.1	
reionisation optical depth	τ	0.087 ± 0.017	0.084 ± 0.016	
spectral index	n_s	$0.963^{+0.014}_{-0.015}$	$0.960^{+0.014}_{-0.013}$	



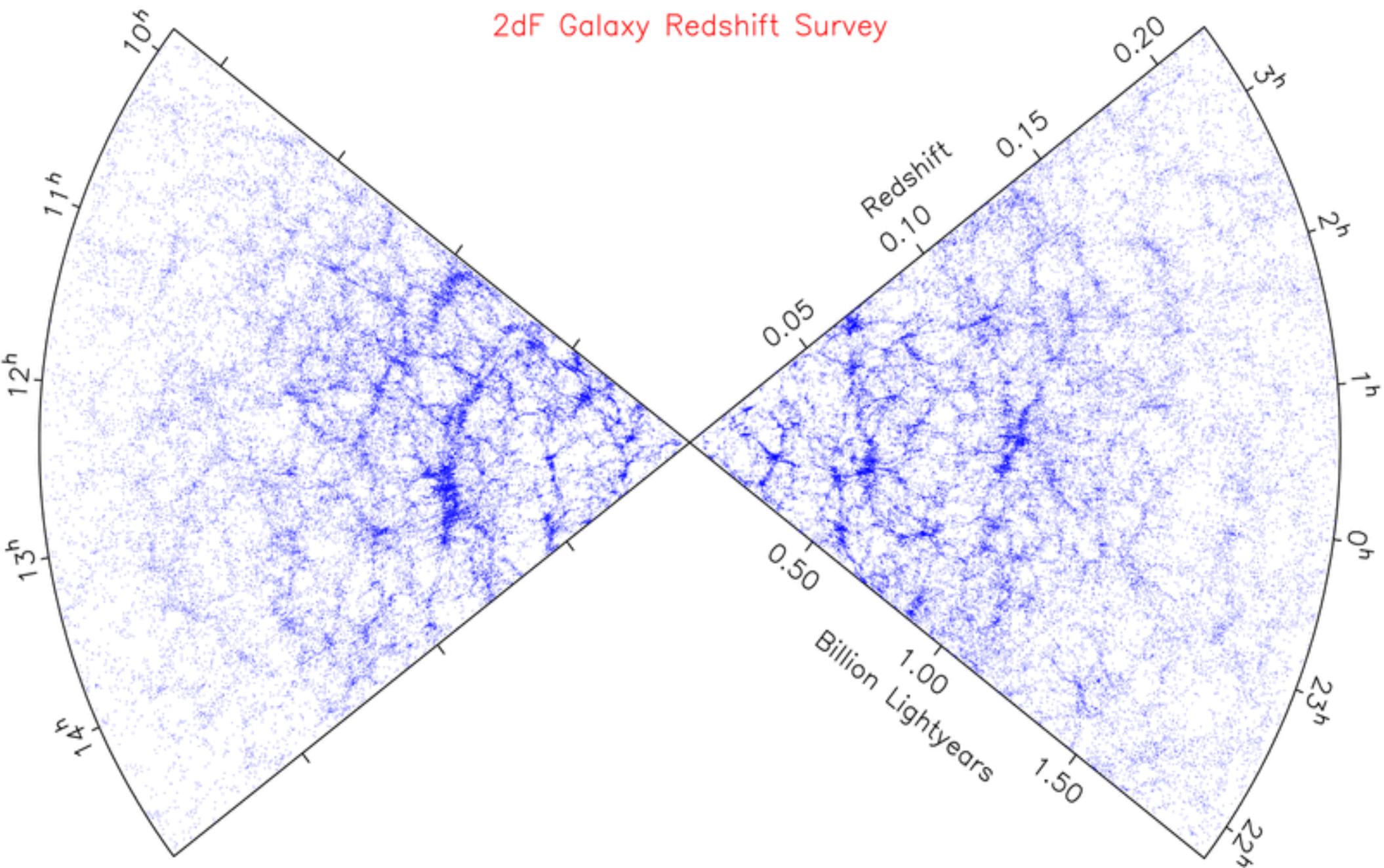
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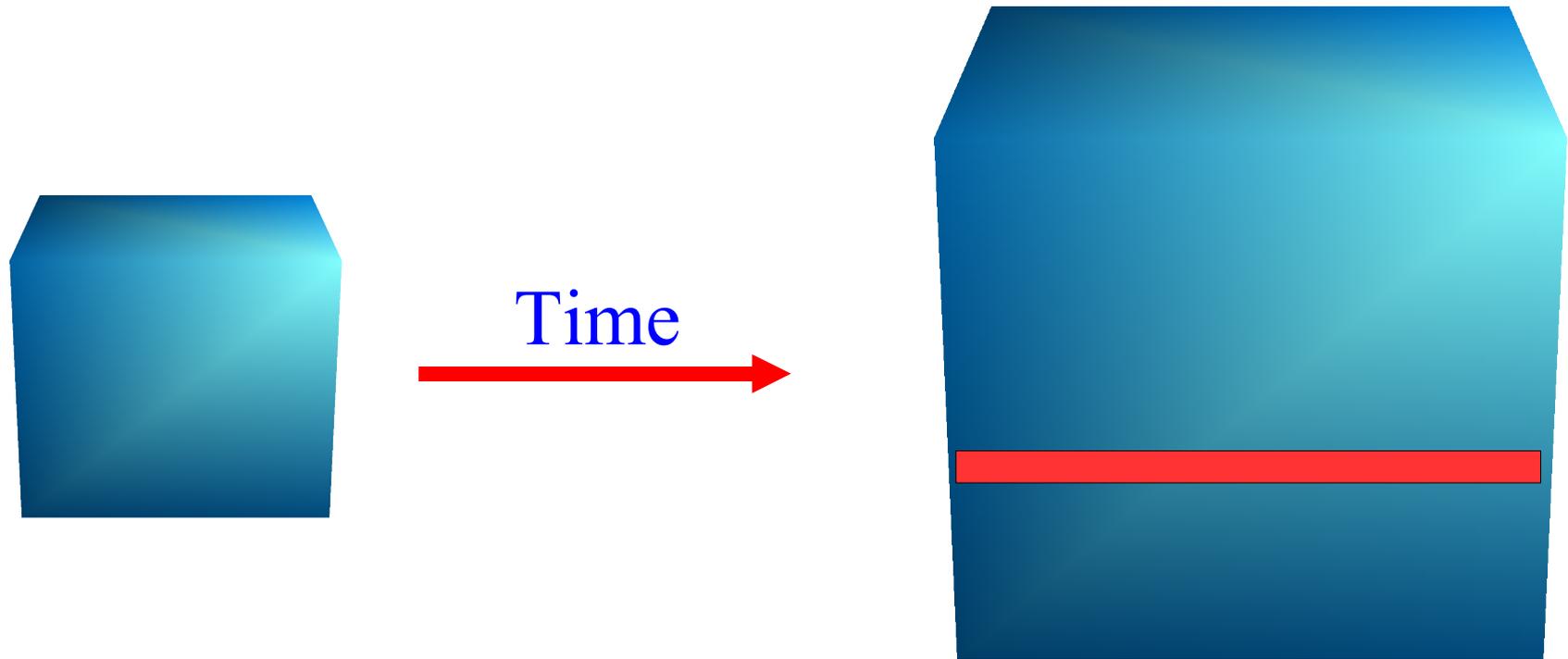
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Nearby large-scale structure

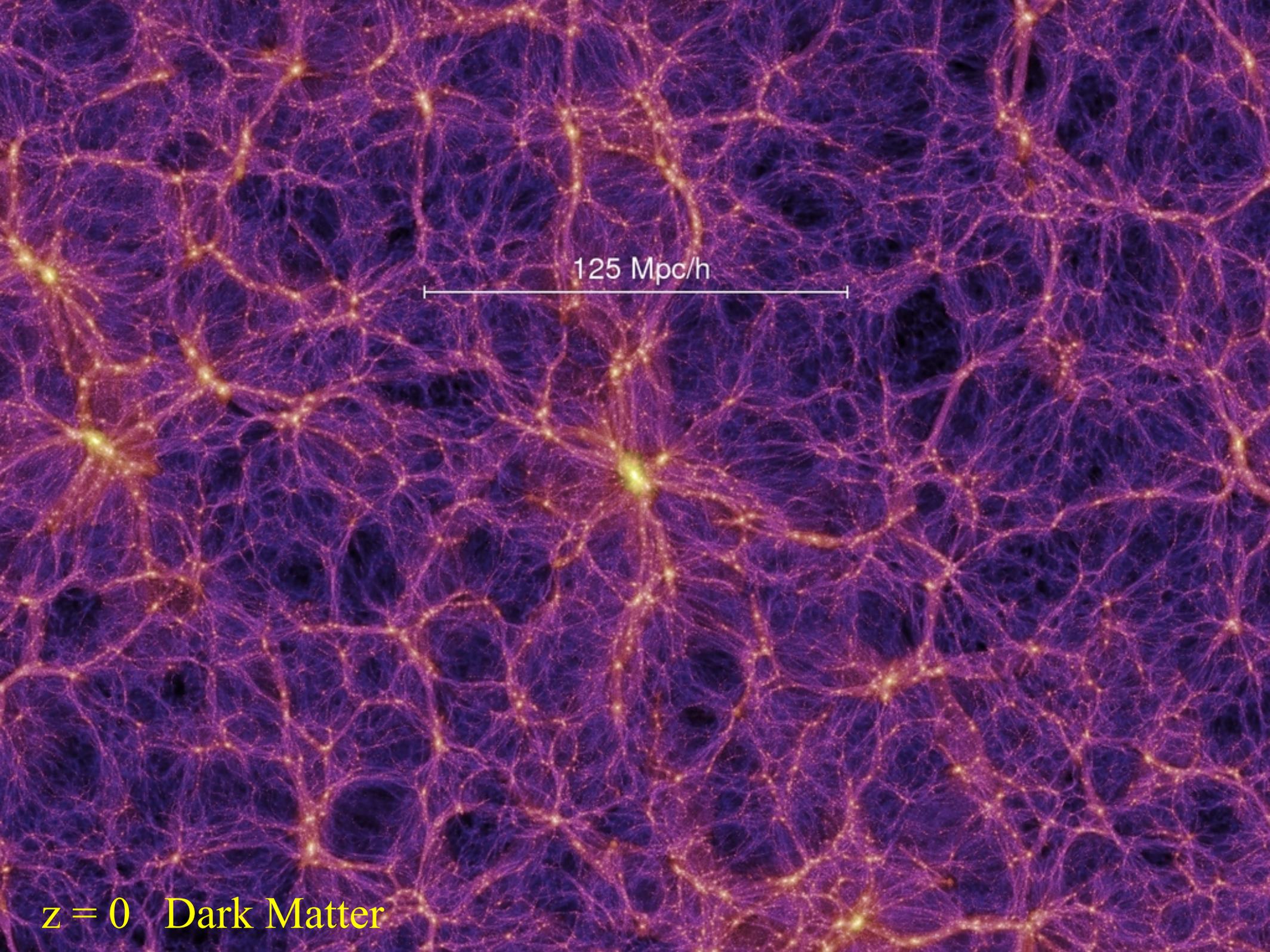
2dF Galaxy Redshift Survey



Evolving the Universe in a computer

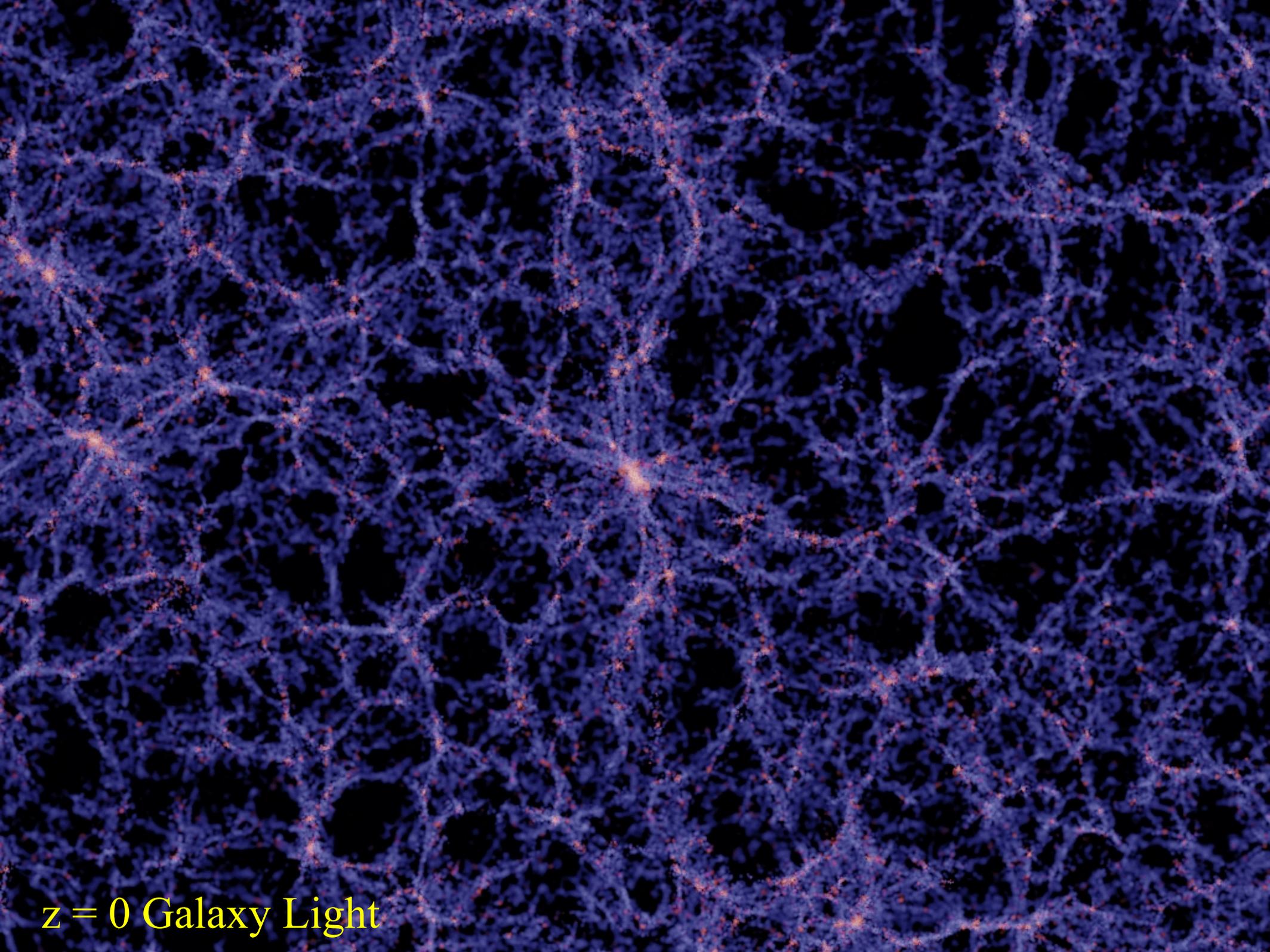


- Follow the matter in an expanding cubic region
- Start 400,000 years after the Big Bang
- Match initial conditions to the observed Microwave Background
- Calculate evolution forward to the present day

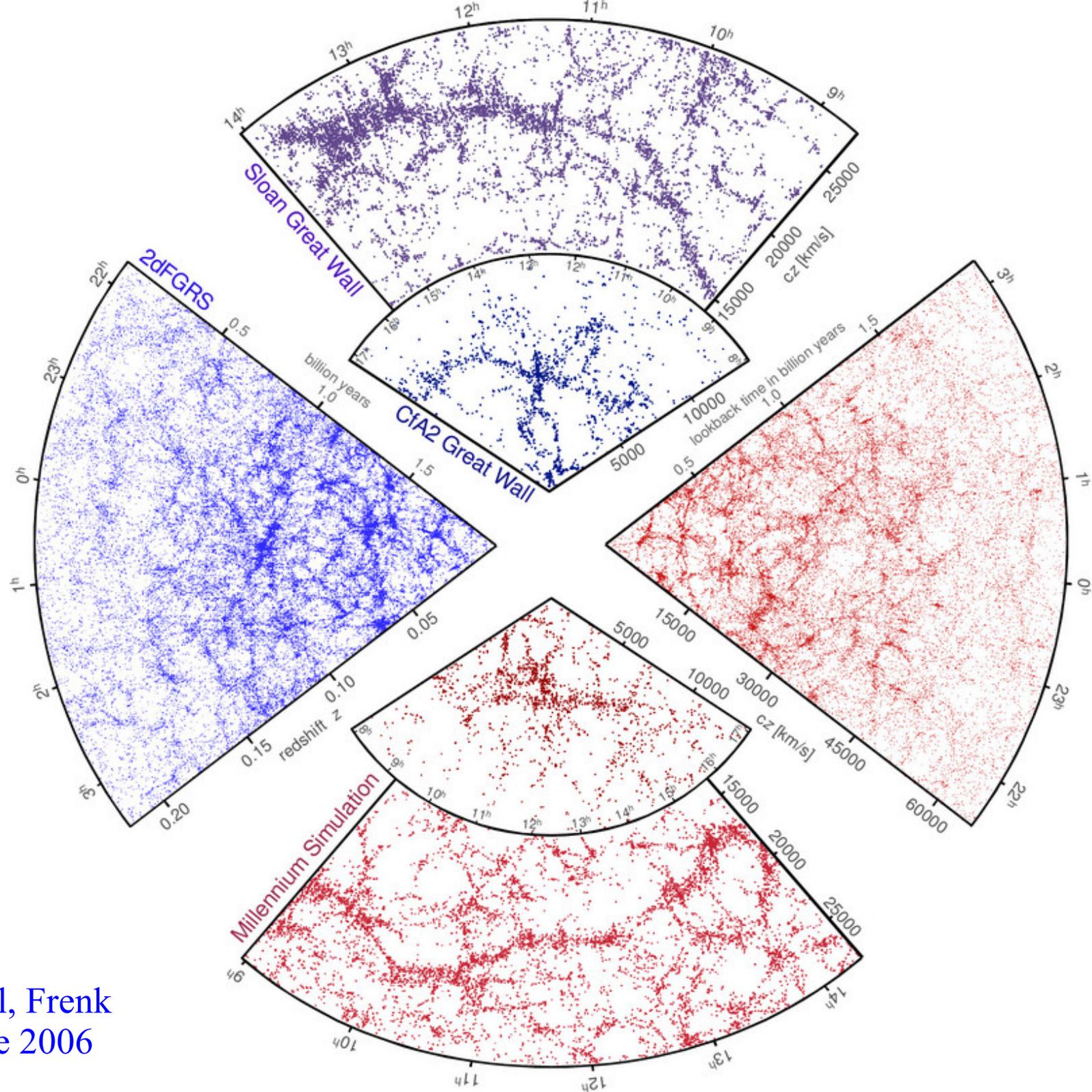


125 Mpc/h

$z = 0$ Dark Matter



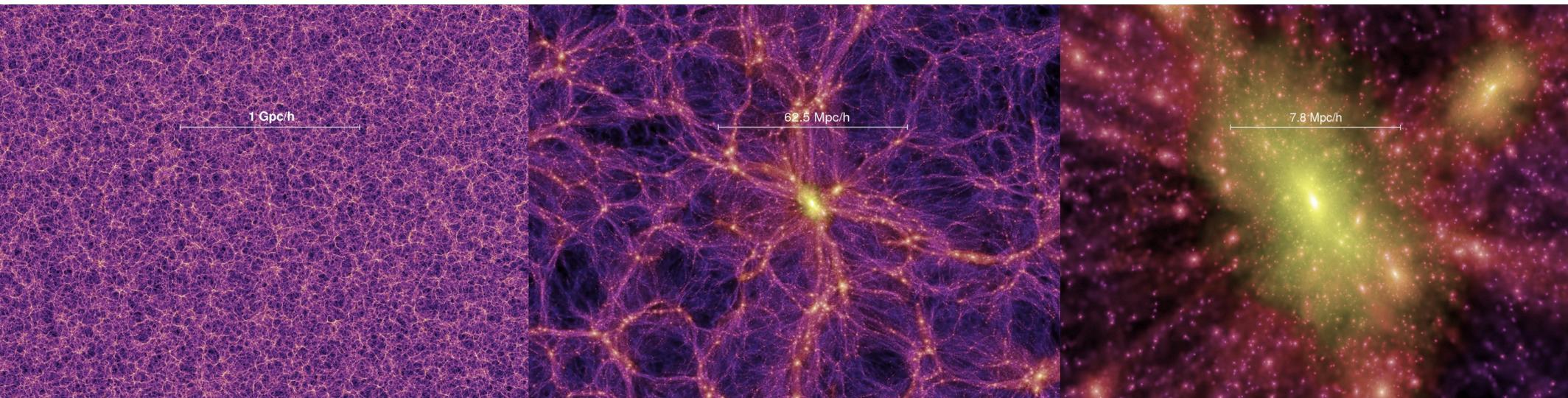
$z = 0$ Galaxy Light



Springel, Frenk
& White 2006

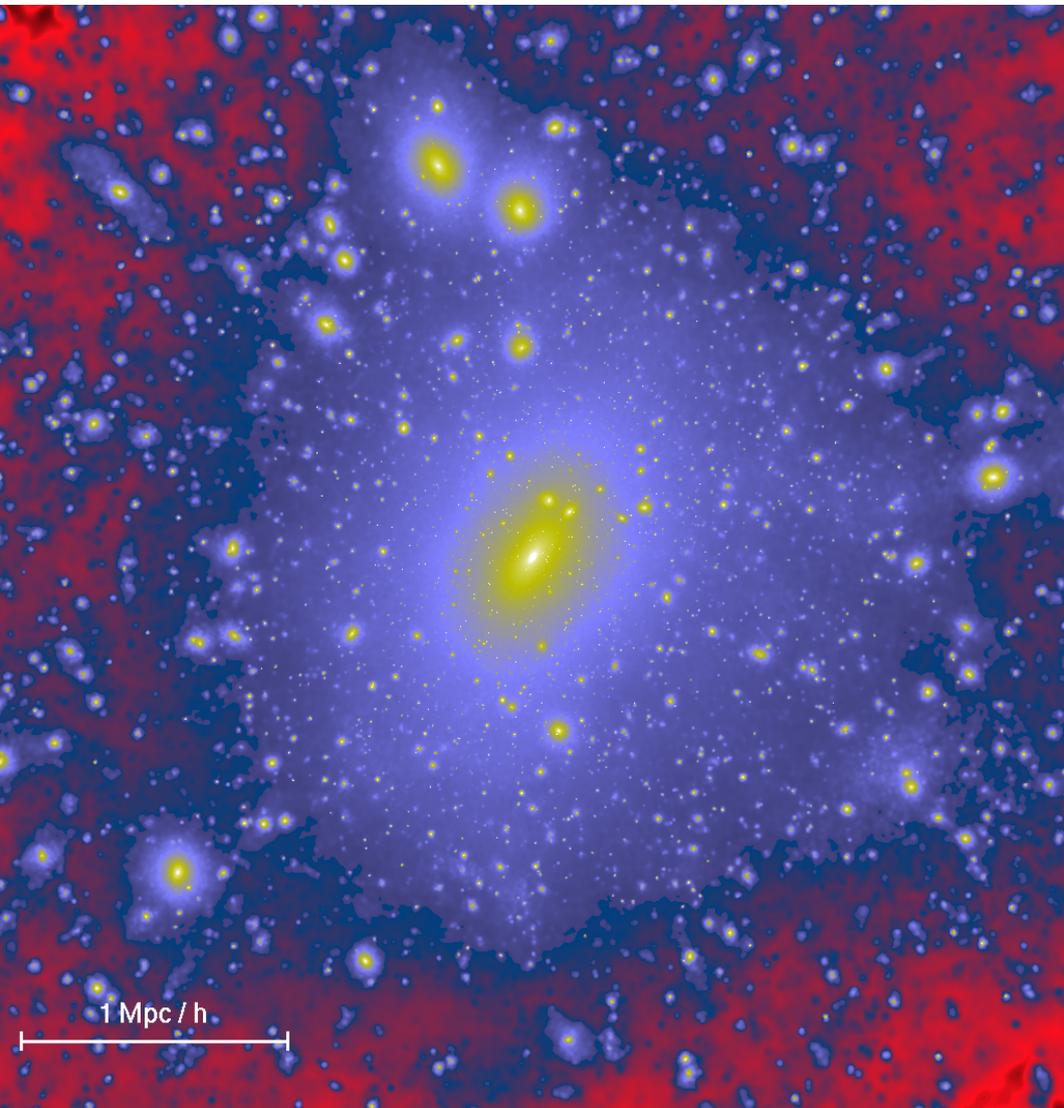
Visualizing Darkness

- The smooth becomes rough with the passing of time
- Uniformity, filamentarity, hierarchy – it all depends on scale

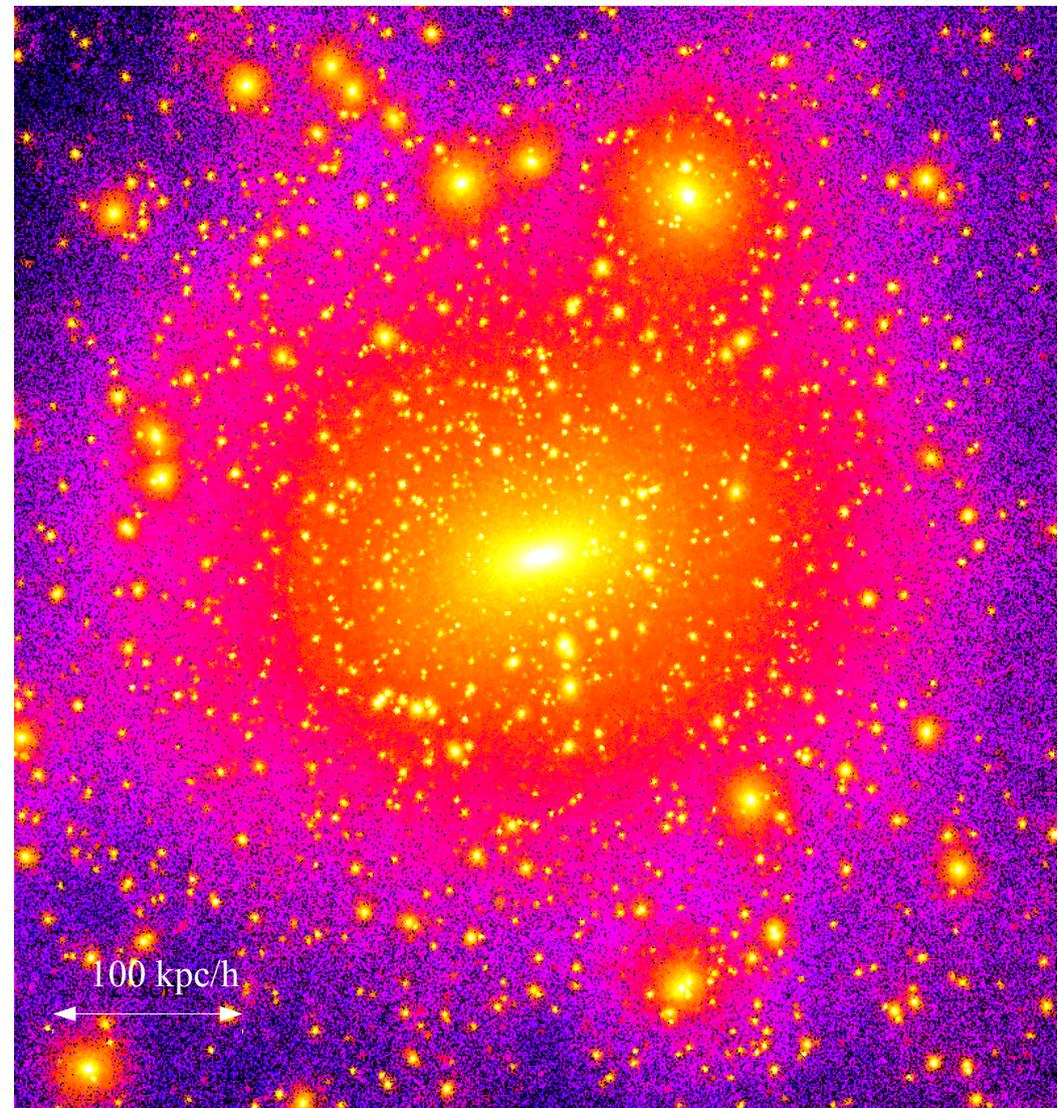


The dark matter structure of Λ CDM halos

A rich galaxy cluster halo
Springel et al 2001



A 'Milky Way' halo
Power et al 2002

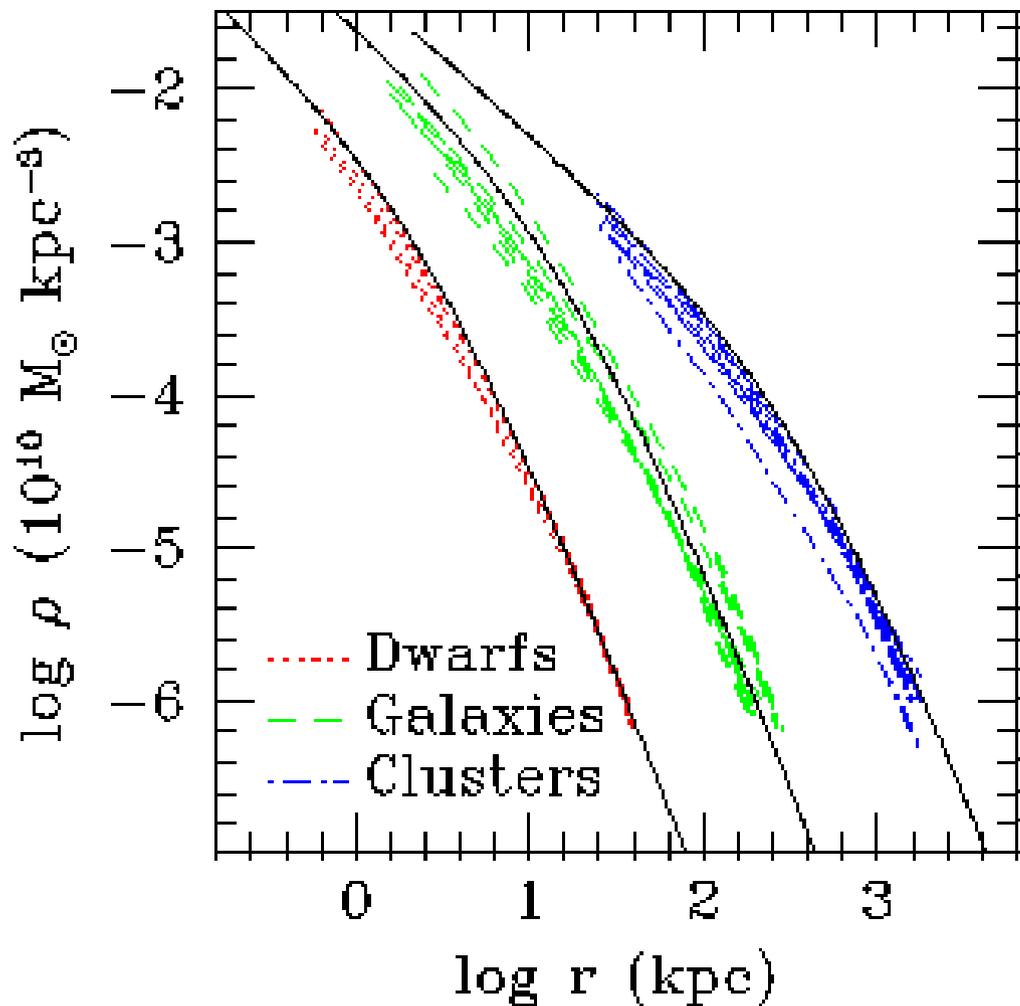


Λ CDM galaxy halos (without galaxies!)

- Halos extend to ~ 10 times the 'visible' radius of galaxies and contain ~ 10 times the mass in the visible regions
 - Halos are not spherical but approximate triaxial ellipsoids
 - more prolate than oblate
 - axial ratios greater than two are common
 - "Cuspy" density profiles with outwardly increasing slopes
 - $d \ln \rho / d \ln r = \gamma$ with $\gamma < -2.5$ at large r
 - $\gamma > -1.2$ at small r
 - Substantial numbers of self-bound subhalos contain $\sim 10\%$ of the halo's mass and have $dN/dM \sim M^{-1.8}$
-  Most substructure mass is in most massive subhalos

Density profiles of dark matter halos

Navarro, Frenk & White 1996



The average dark matter density of a dark halo depends on distance from halo centre in a very similar way in halos of all masses at all times
-- a universal profile shape --

$$\rho(r)/\rho_{crit} \approx \delta r_s / r(1 + r/r_s)^2$$

More massive halos and halos that form earlier have higher densities (bigger δ)

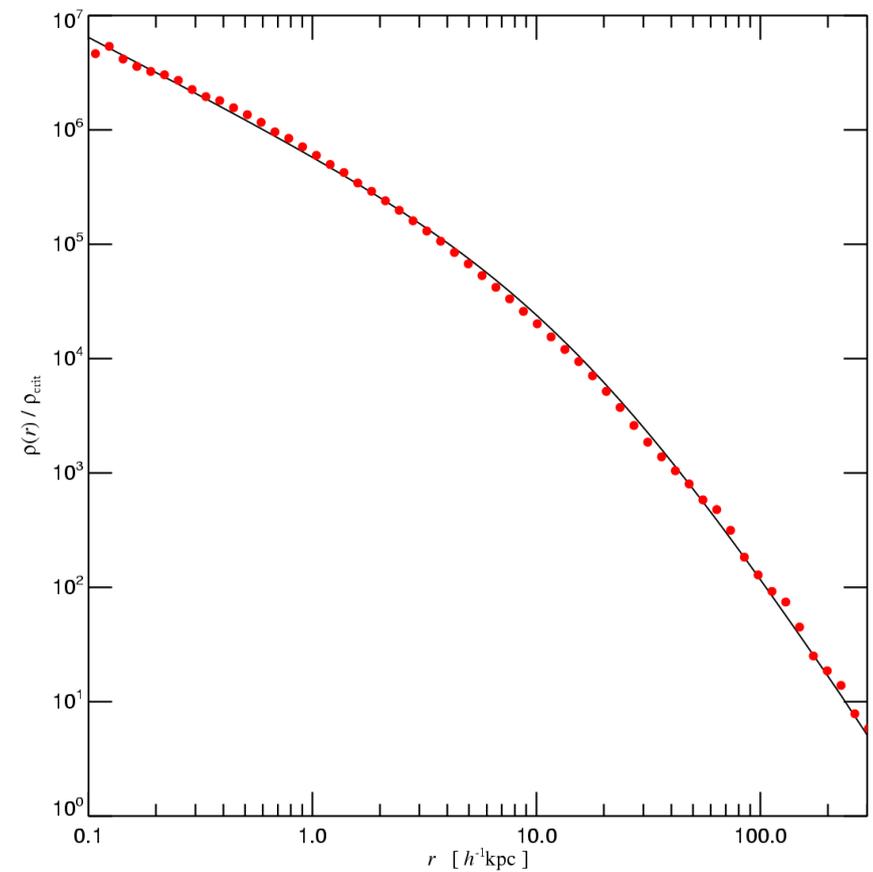
A high-resolution Milky Way halo

Navarro et al 2006

$$N_{200} \sim 3 \times 10^7$$



600 kpc



“Milky Way” halo

$z = 1.5$

$N_{200} = 3 \times 10^6$



“Milky Way” halo

$$z = 1.5$$

$$N_{200} = 94 \times 10^6$$



“Milky Way” halo

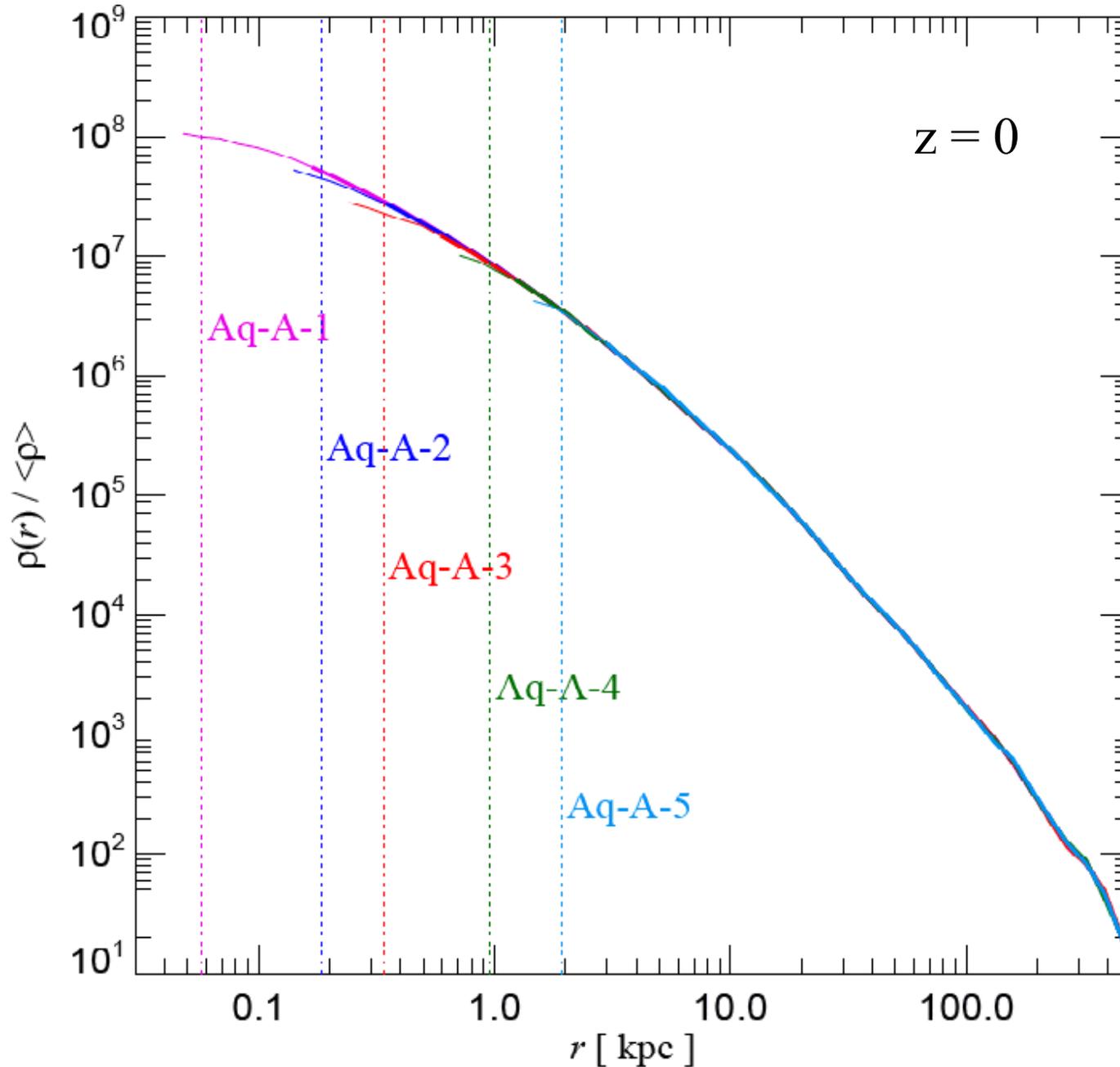
$$z = 1.5$$

$$N_{200} = 750 \times 10^6$$



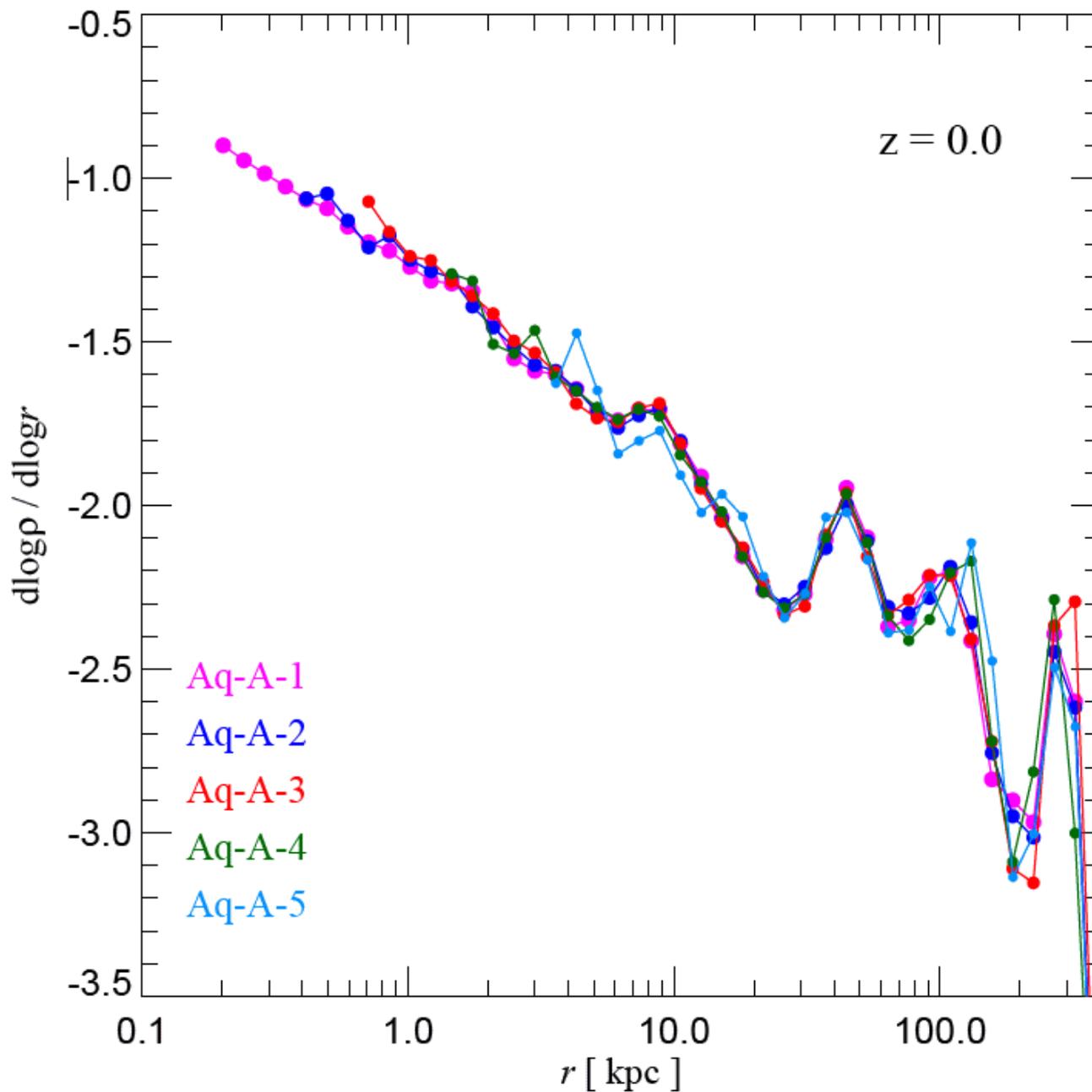
How well do density profiles converge?

Aquarius Project: Springel et al 2008



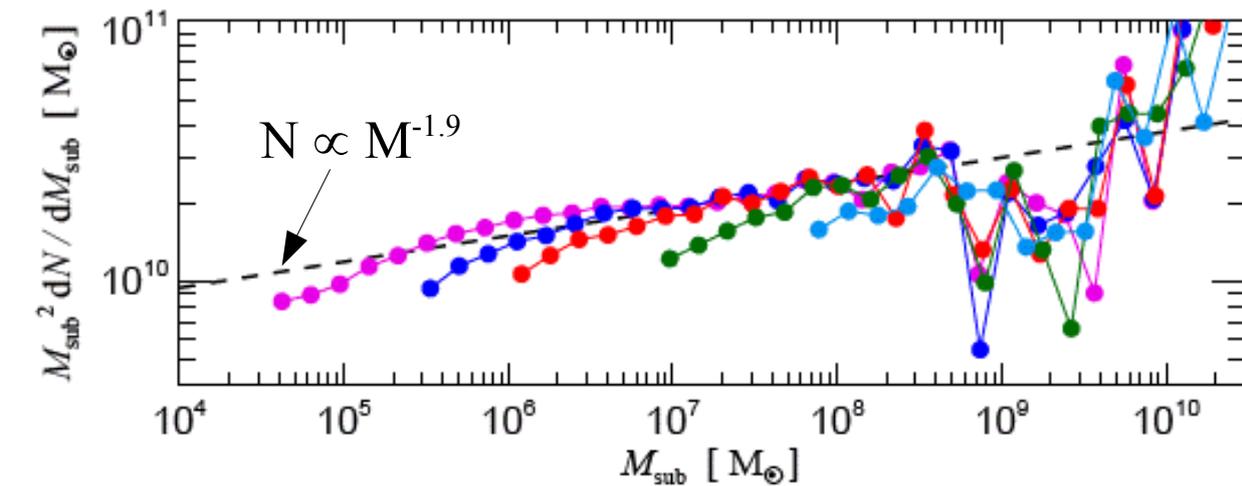
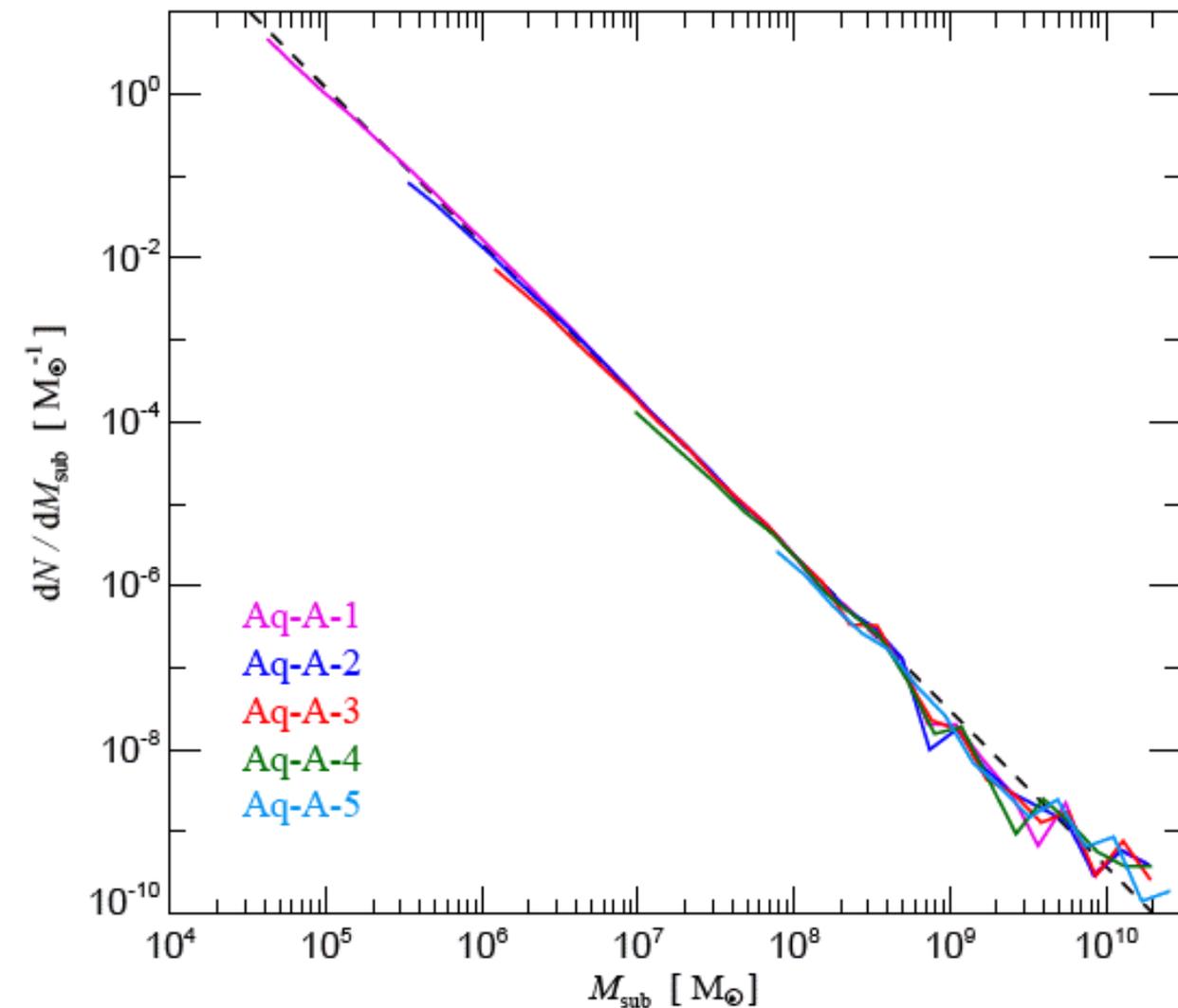
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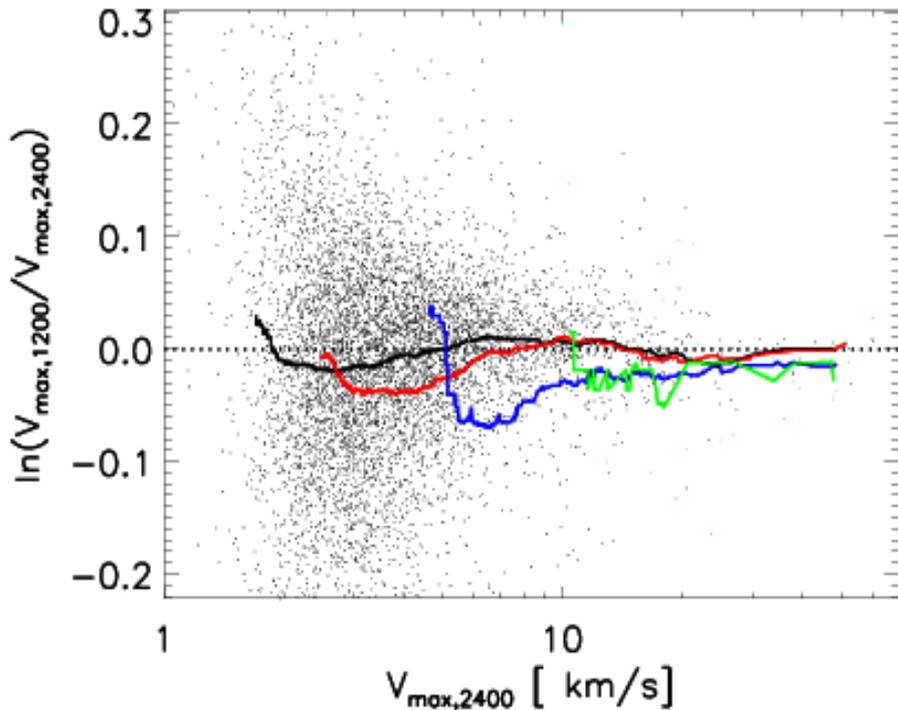
How well does substructure converge?

Aquarius Project: Springel et al 2008

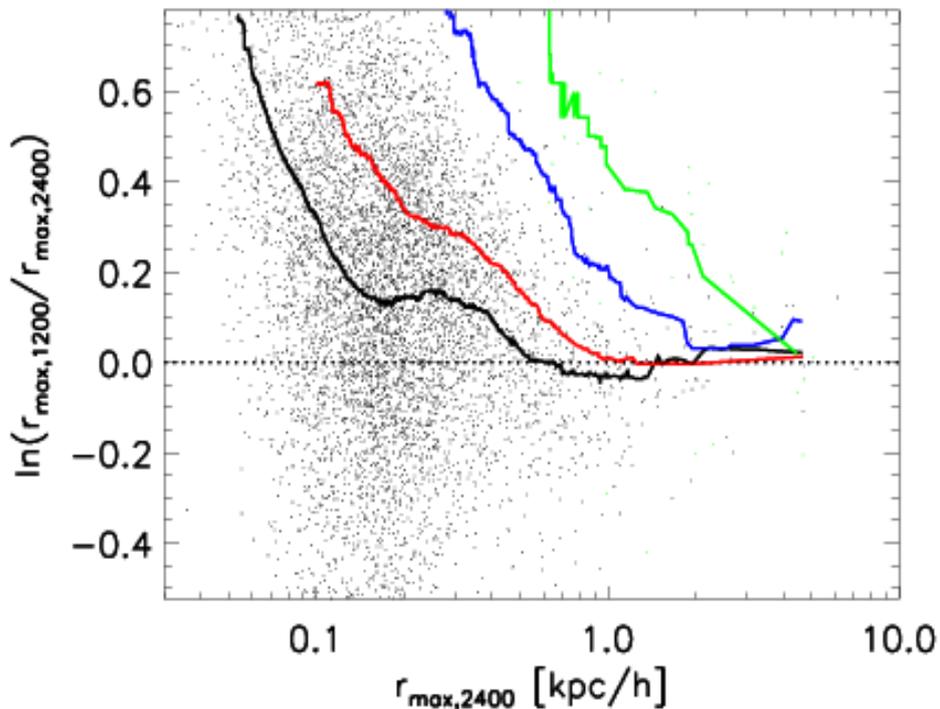


How well does substructure converge?

Aquarius Project: Springel et al 2008



Convergence in the size and maximum circular velocity for individual subhalos cross-matched between simulation pairs.

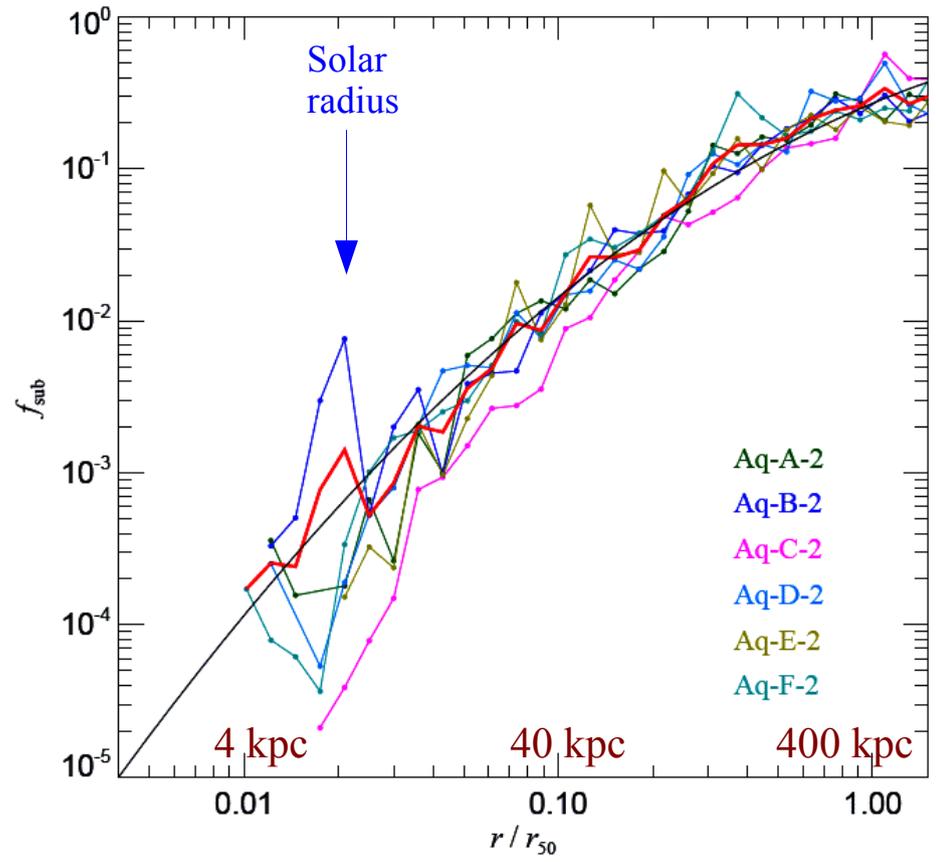
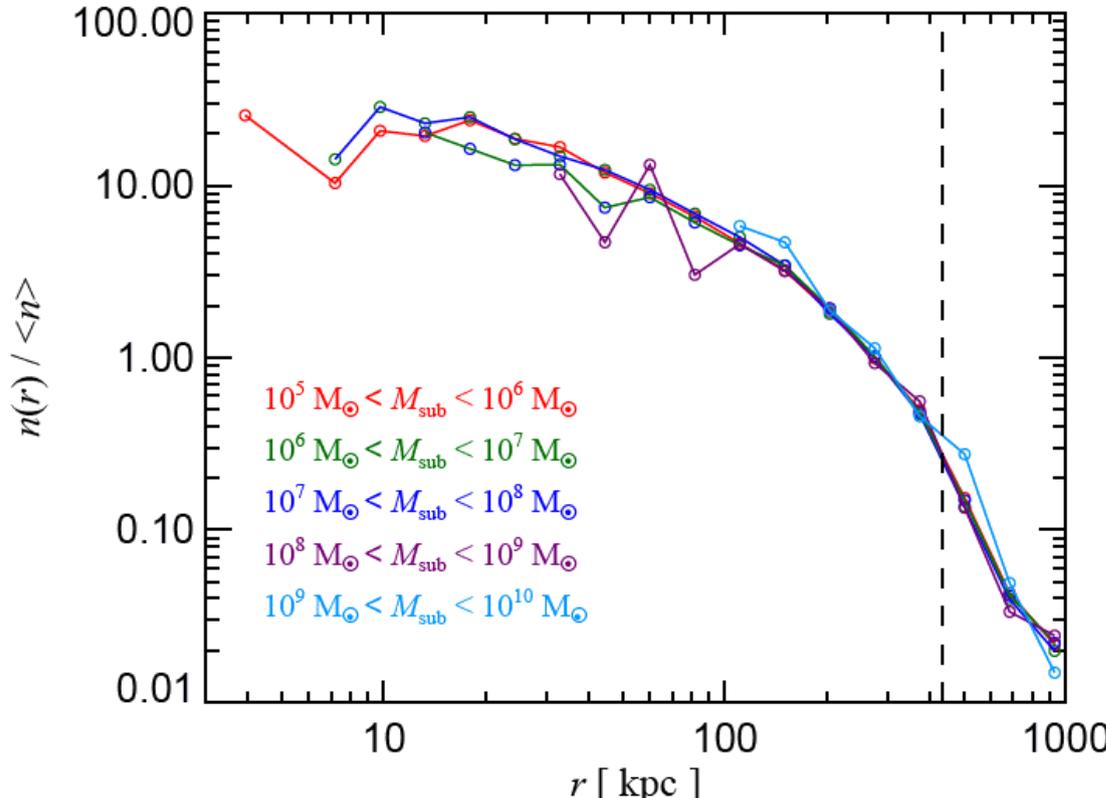


Biggest simulation gives convergent results for

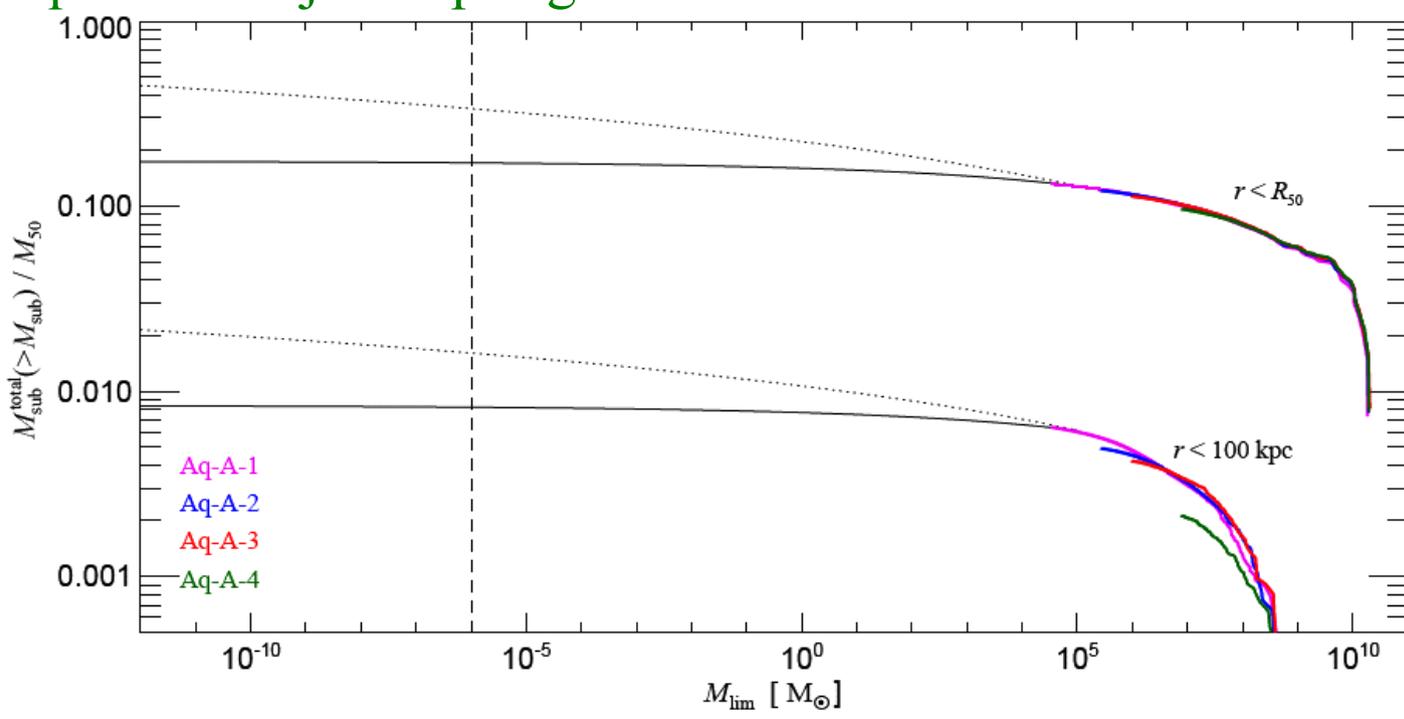
$$V_{\max} > 1.5 \text{ km/s}$$

$$r_{\max} > 165 \text{ pc}$$

Much smaller than the halos inferred for even the faintest dwarf galaxies



Aquarius Project: Springel et al 2008



- All mass subhalos are similarly distributed
- A small fraction of the inner mass in subhalos
- $\ll 1\%$ of the mass near the Sun is in subhalos

Small-scale structure of the CDM distribution

- Direct detection involves bolometers/cavities of meter scale which are sensitive to particle momentum
 - what is the density structure between m and kpc scales?
 - how many streams intersect the detector at any time?
- Intensity of annihilation radiation depends on
$$\int \rho^2(\mathbf{x}) \langle \sigma v \rangle dV$$
 - what is the density distribution around individual CDM particles on the annihilation interaction scale?

Predictions for detection experiments depend on the CDM distribution on scales far below those accessible to simulation

→ We require a good theoretical understanding of mixing and small-scale structure

Small-scale structure affects DM detection?

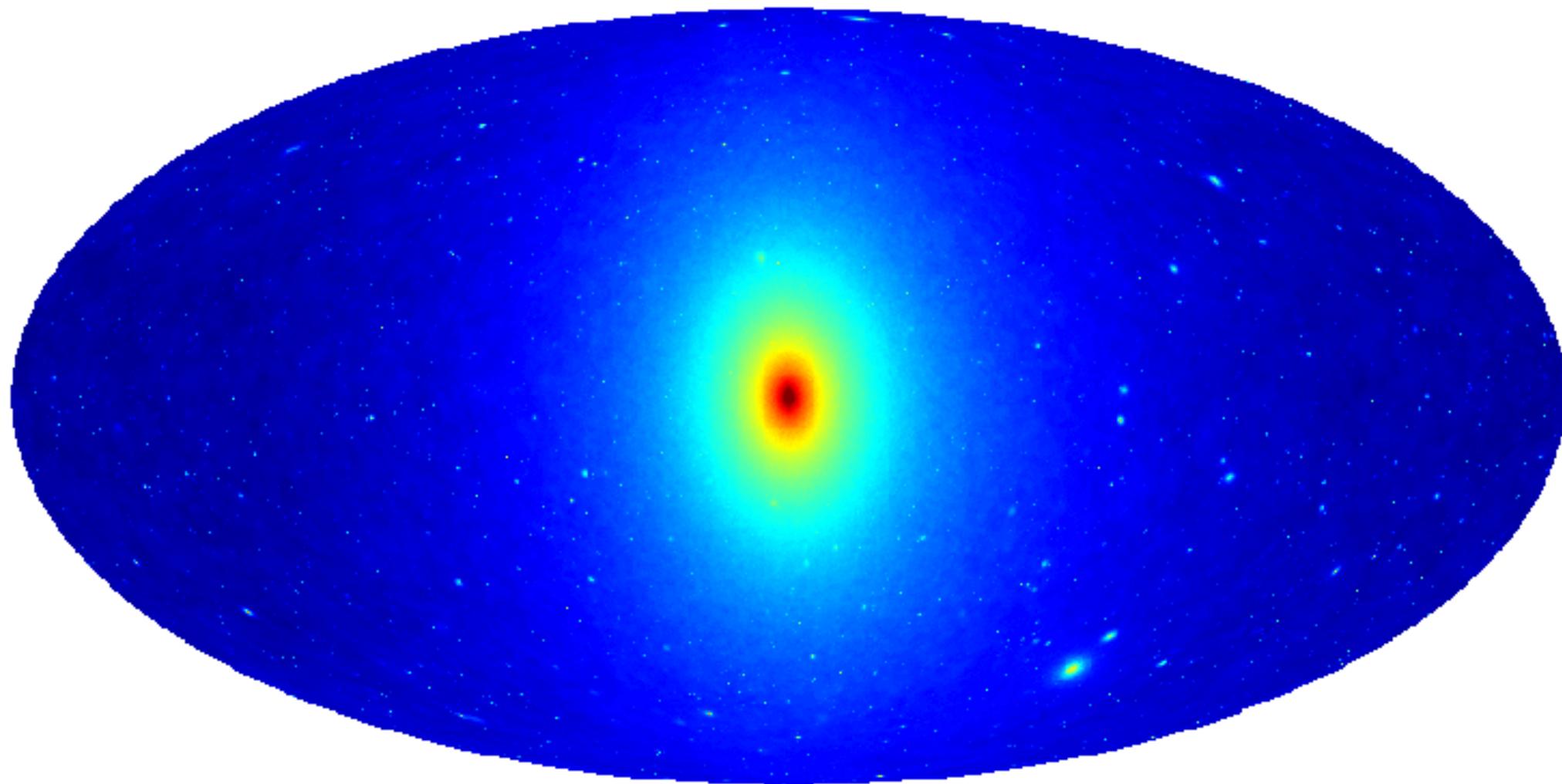
- Halo DM is mostly in small (e.g. Earth mass?) clumps?
 - ▶ direct detectors typically live in low density regions?
- DM streams —▶ non-Maxwellian, “clumpy” $f(\mathbf{v})$?
 - ▶ direct detectors will see an irregular energy distribution?
- Small (Earth-mass?) clumps dominate observable annihilation signal?
- Dwarf Spheroidals/subhalos are best targets for detecting annihilation?
(and are boosted by sub-substructure)
- Smooth halo annihilation emission is dominated by caustics?

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Milky Way halo seen in DM annihilation radiation

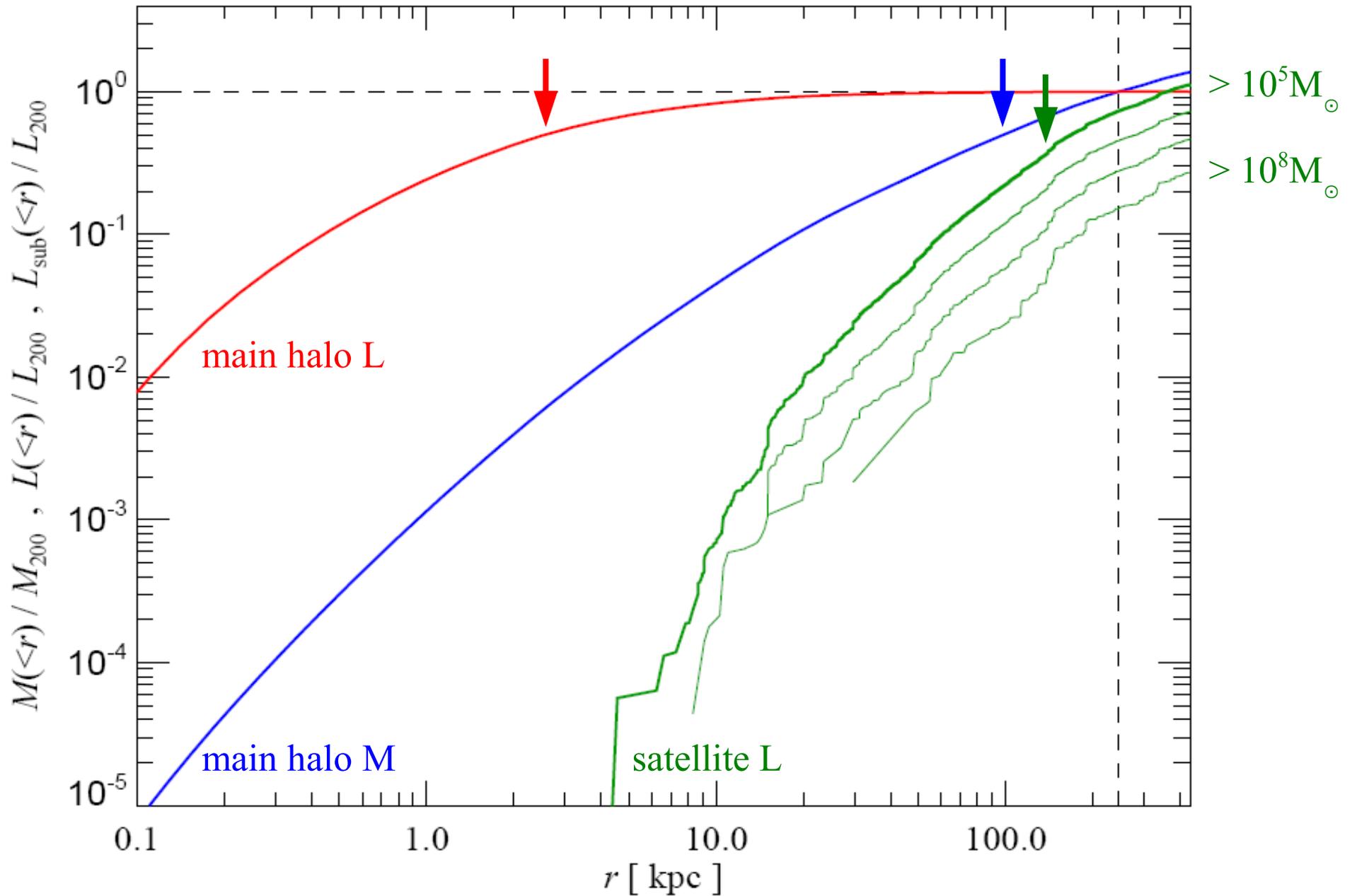
Aquarius simulation: $N_{200} = 1.1 \times 10^9$



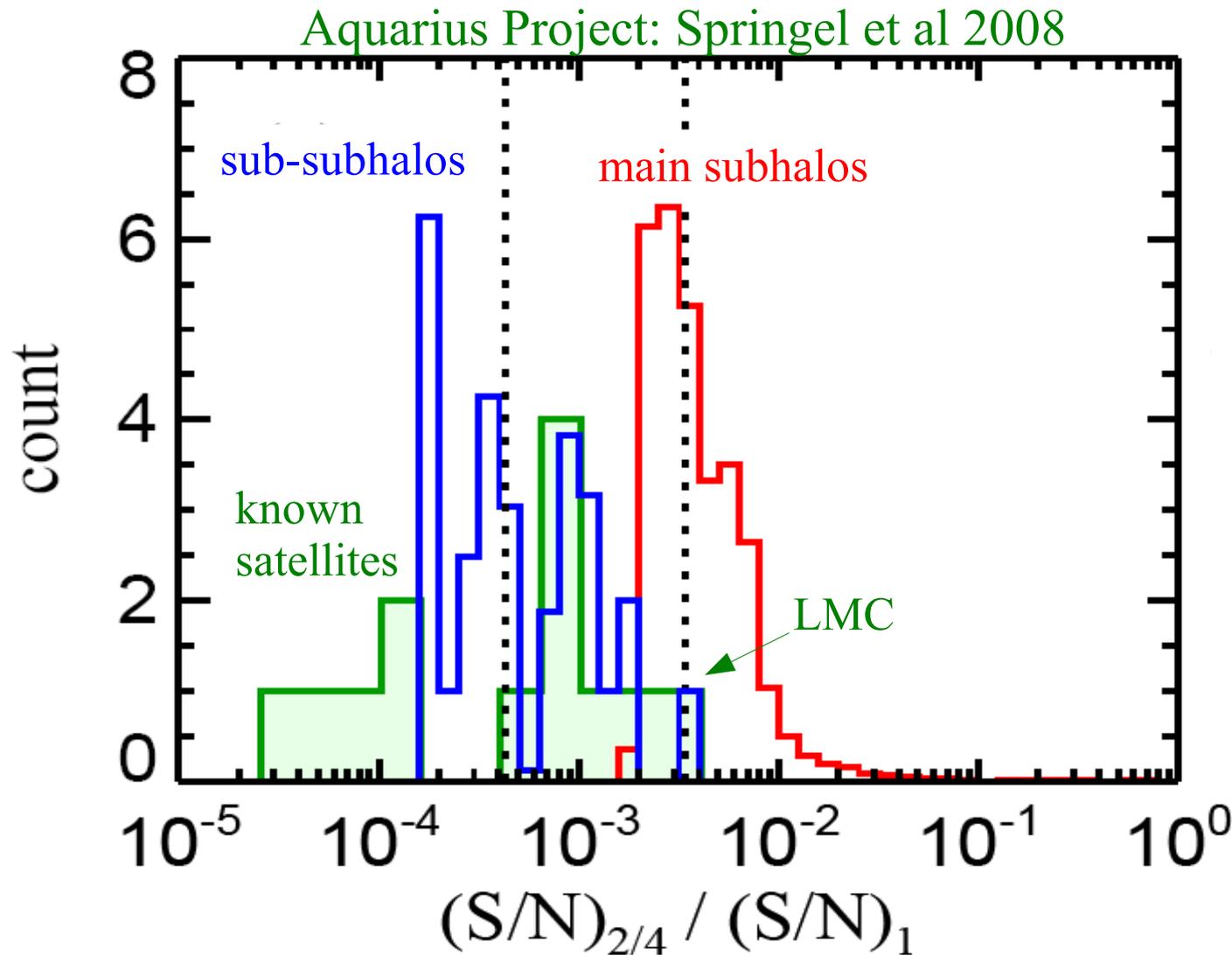
14.  18. $\text{Log} (M_{\text{sun}}^2 \text{ kpc}^{-5} \text{ sr}^{-1})$

Mass and annihilation radiation profiles of a MW halo

Aquarius Project: Springel et al 2008



S/N for detecting subhalos in units of that for detecting the main halo
30 highest S/N objects, assuming use of optimal filters



- Highest S/N subhalos have 1% of S/N of main halo
- Highest S/N subhalos have 10 times S/N of known satellites
- Substructure of subhalos has no influence on detectability

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Cold Dark Matter at high redshift (e.g. $z \sim 10^5$)

Well *after* CDM particles become nonrelativistic, but *before* they dominate the cosmic density, their distribution function is

$$f(\mathbf{x}, \mathbf{v}, t) = \rho(t) [1 + \delta(\mathbf{x})] N[\{\mathbf{v} - \mathbf{V}(\mathbf{x})\}/\sigma]$$

where $\rho(t)$ is the mean mass density of CDM,

$\delta(\mathbf{x})$ is a Gaussian random field with finite variance $\ll 1$,

$\mathbf{V}(\mathbf{x}) = \nabla \psi(\mathbf{x})$ where $\nabla^2 \psi(\mathbf{x}) \propto \delta(\mathbf{x})$

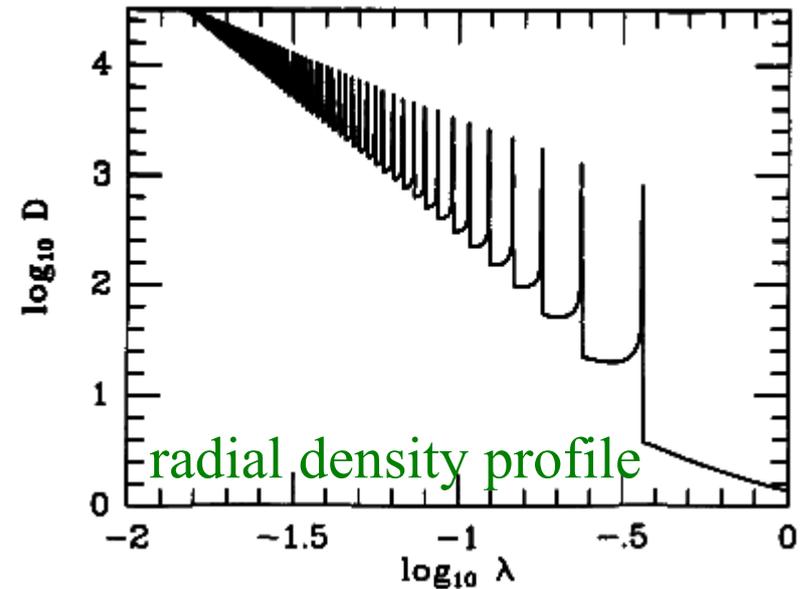
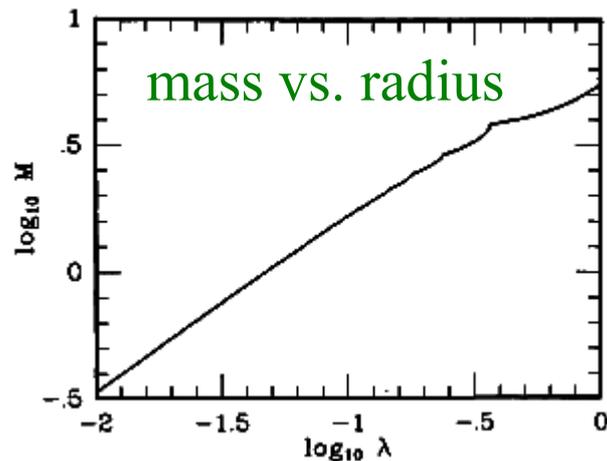
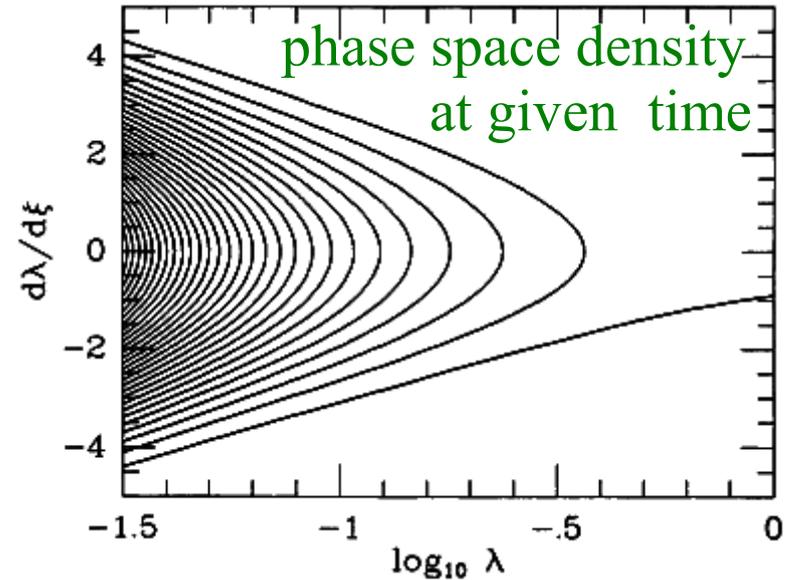
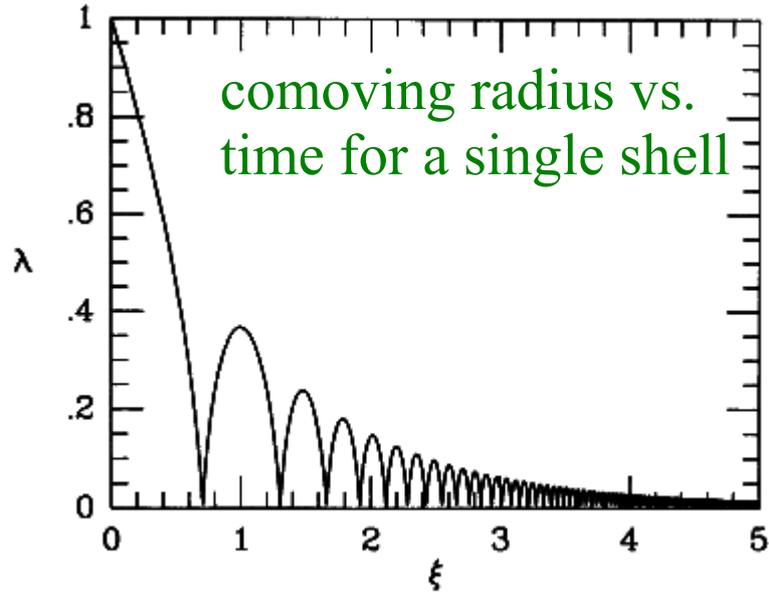
and N is standard normal with $\sigma^2 \ll \langle |\mathbf{V}|^2 \rangle$

CDM occupies a thin 3-D 'sheet' within the full 6-D phase-space and its projection onto \mathbf{x} -space is near-uniform.

$Df/Dt = 0$ \longrightarrow only a 3-D subspace is occupied at later times.
Nonlinear evolution leads to a complex, multi-stream structure.

Similarity solution for spherical collapse in CDM

Bertschinger 1985



Evolution of CDM structure

Consequences of $Df / Dt = 0$

- The 3-D phase sheet can be stretched and folded but not torn
- At least 1 sheet must pass through every point \mathbf{x}
- In nonlinear objects there are typically many sheets at each \mathbf{x}
- Stretching which reduces a sheet's density must also reduce its velocity dispersions to maintain $f = \text{const.}$
- At a caustic, at least one velocity dispersion must $\longrightarrow \infty$
- All these processes can be followed in fully general simulations by tracking the phase-sheet local to each simulation particle

The geodesic deviation equation

Particle equation of motion: $\dot{X} = \begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{v}} \end{bmatrix} = \begin{bmatrix} \mathbf{v} \\ -\nabla\phi \end{bmatrix}$

Offset to a neighbor: $\delta\dot{X} = \begin{bmatrix} \delta\mathbf{v} \\ \mathbf{T} \cdot \delta\mathbf{x} \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{I} \\ \mathbf{T} & 0 \end{bmatrix} \cdot \delta X$; $\mathbf{T} = -\nabla(\nabla\phi)$

Write $\delta X(t) = D(X_0, t) \cdot \delta X_0$, then differentiating w.r.t. time gives,

$$\dot{D} = \begin{bmatrix} 0 & \mathbf{I} \\ \mathbf{T} & 0 \end{bmatrix} \cdot D \quad \text{with } D_0 = I$$

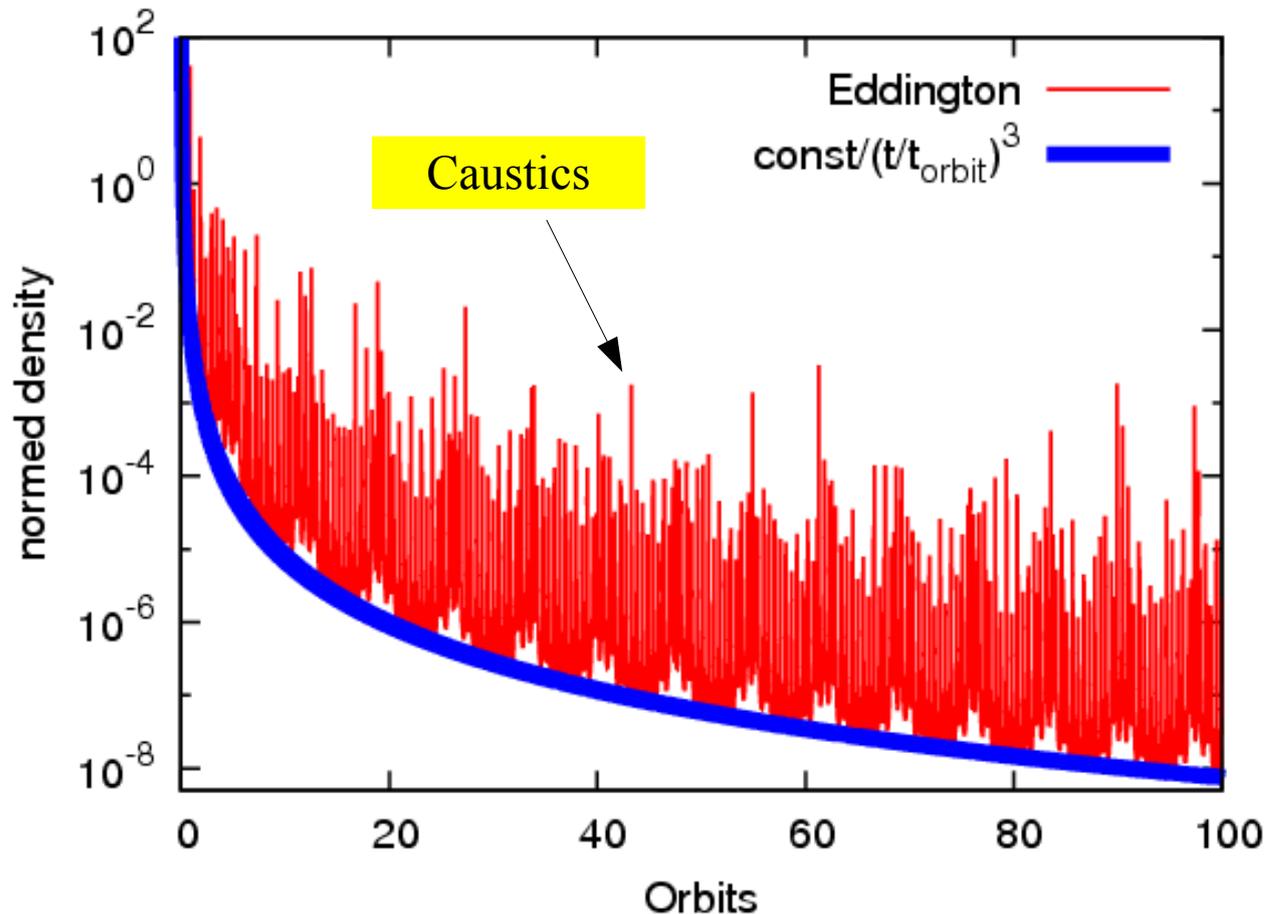
- Integrating this equation together with each particle's trajectory gives the evolution of its local phase-space distribution
- No symmetry or stationarity assumptions are required
- $\det(D) = 1$ at all times by Liouville's theorem
- For CDM, $1/|\det(D_{\mathbf{xx}})|$ gives the decrease in local 3D space density of each particle's phase sheet. Switches sign and is infinite at caustics.

Static highly symmetric potentials

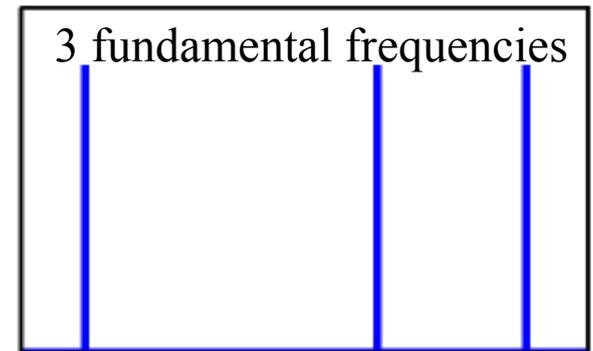
Mark Vogelsberger, Amina Helmi, Volker Springel

Axisymmetric Eddington potential

$$\Phi(r, \theta) = v_h^2 \log(r^2 + d^2) + \frac{\beta^2 \cos^2 \theta}{r^2}$$



Spectral analysis of orbit:

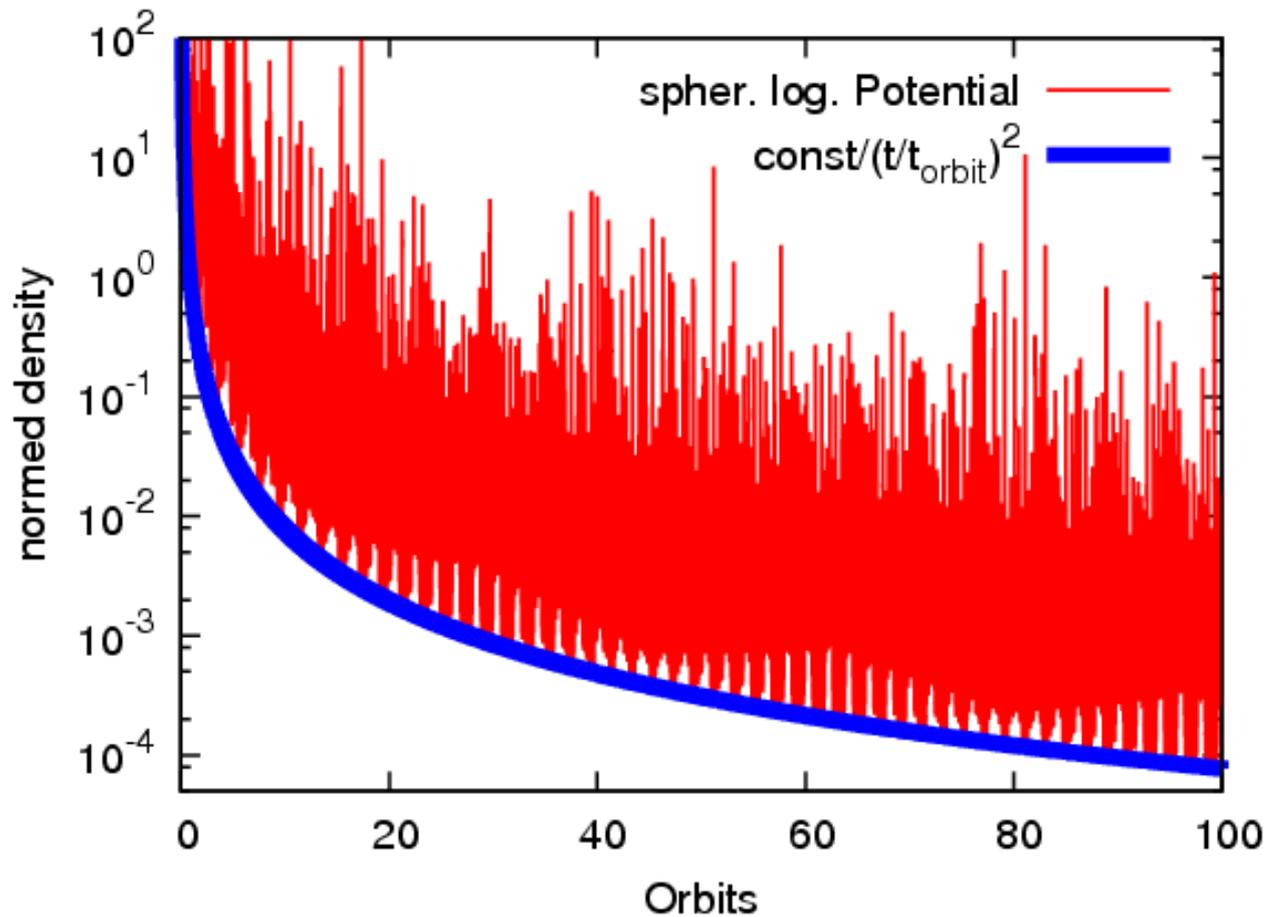


density decreases like $1/t^3$

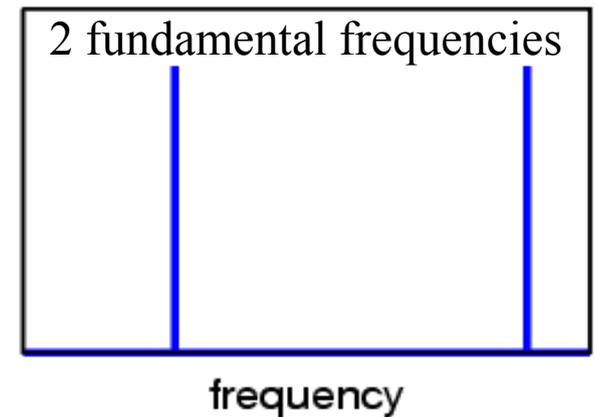
Changing the number of frequencies

Spherical logarithmic potential

$$\Phi(r, \theta) = v_h^2 \log(r^2 + d^2)$$



Spectral analysis of orbit:



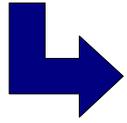
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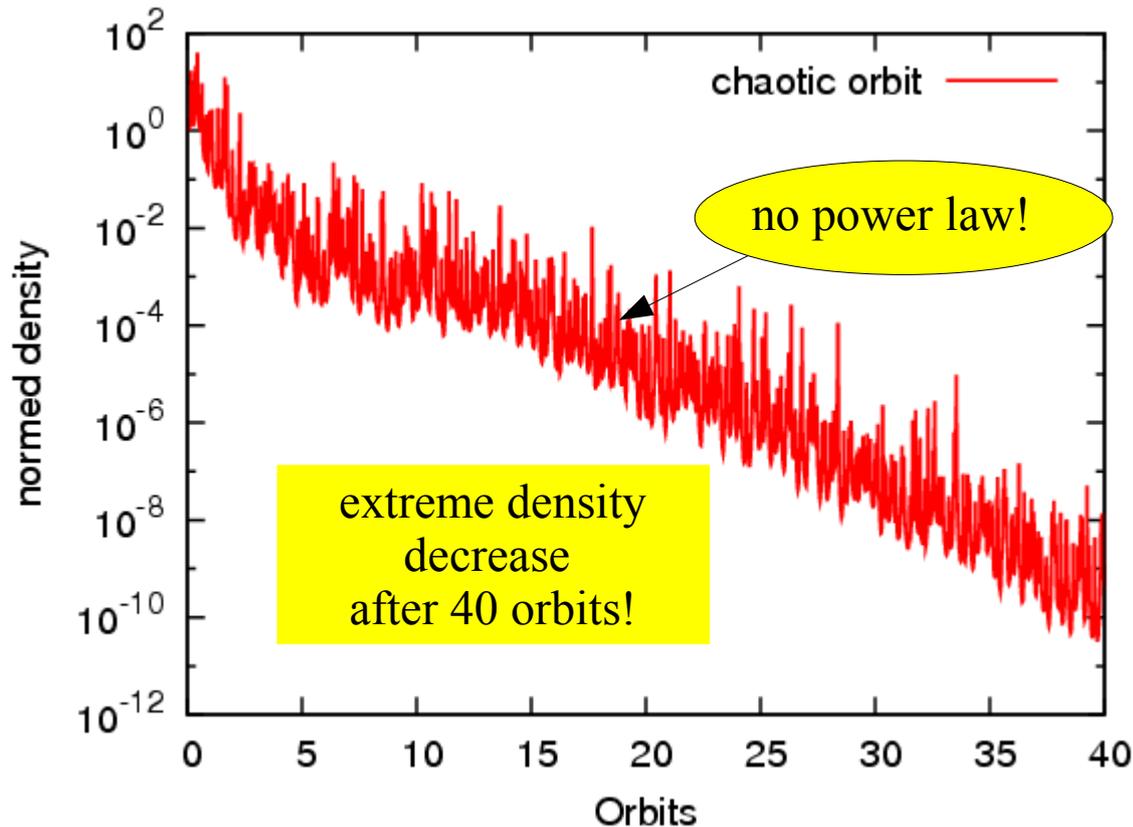
Number of fundamental frequencies dictates the density decrease of the stream

Chaotic mixing

chaotic motion implies a **rapid stream density decrease** \longrightarrow **rapid mixing**



density decrease is **not** like a power law anymore



how to find chaotic regions in phase space?

Common method:

- Lyapunov exponents
- frequency analysis (NAFF)
- ...

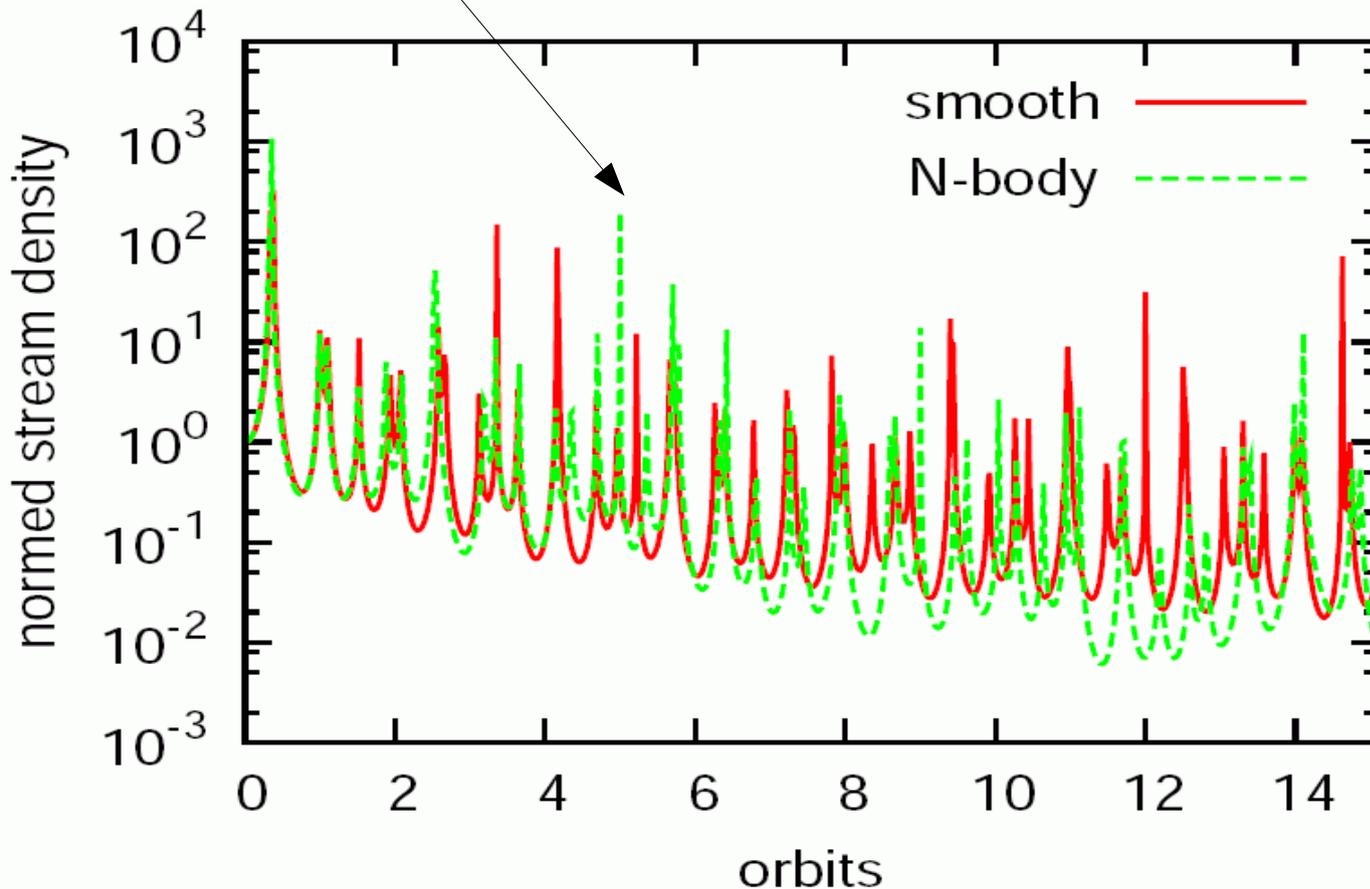
Compare frequency analysis results with geodesic deviation equation results

A particle orbit in a live Halo

spherical Hernquist
density profile

caustics resolved in N-body
halo!

$$\rho(r) = \frac{M}{2\pi} \frac{a}{r} \frac{1}{(r+a)^3}$$

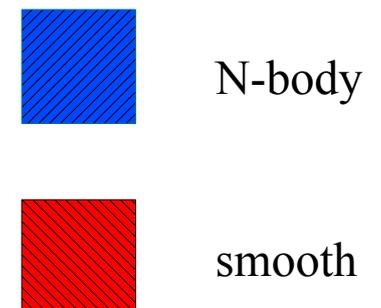
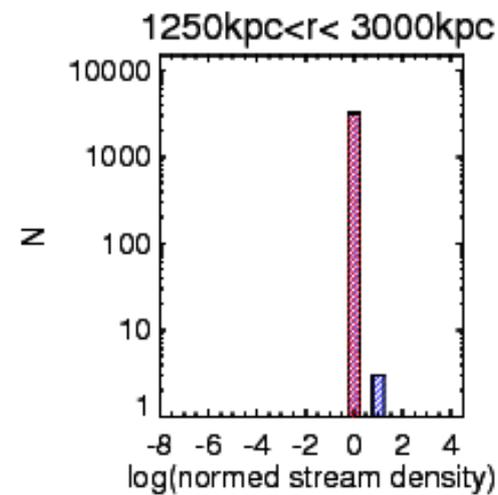
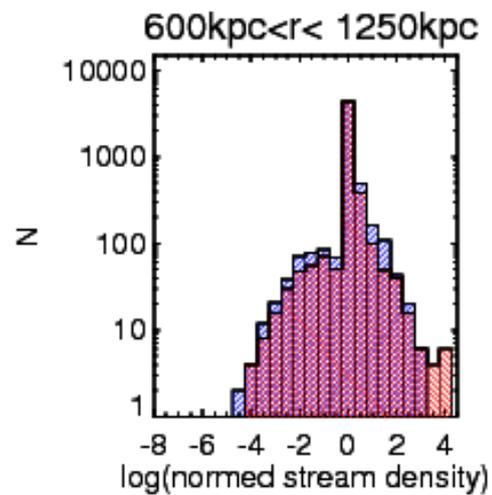
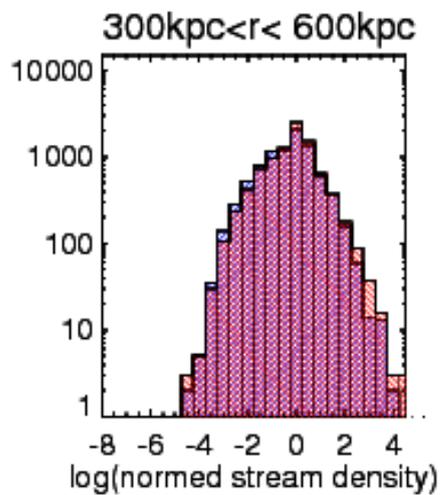
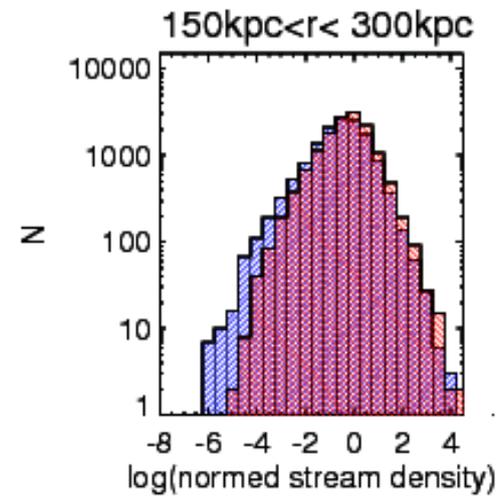
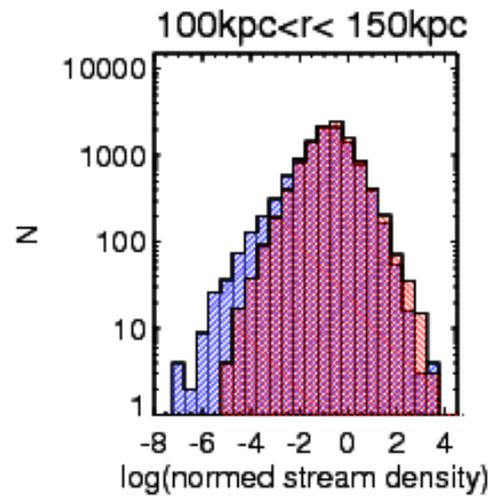
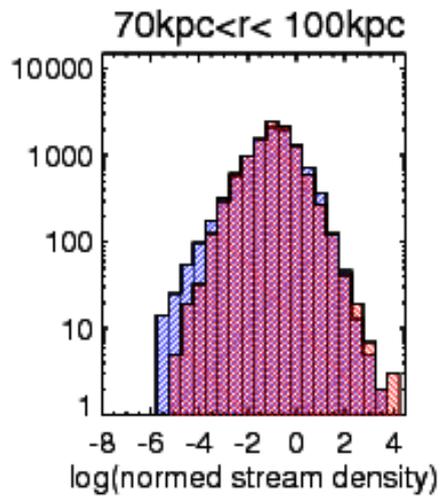
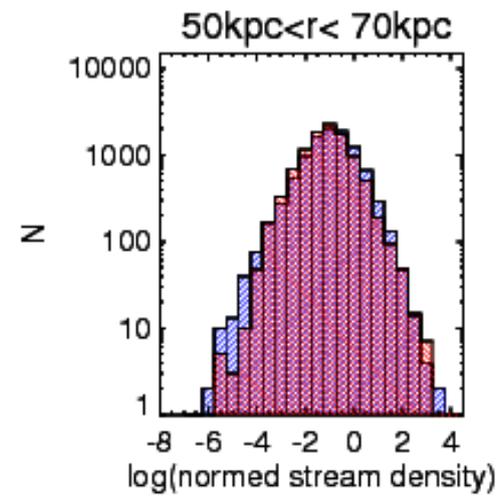
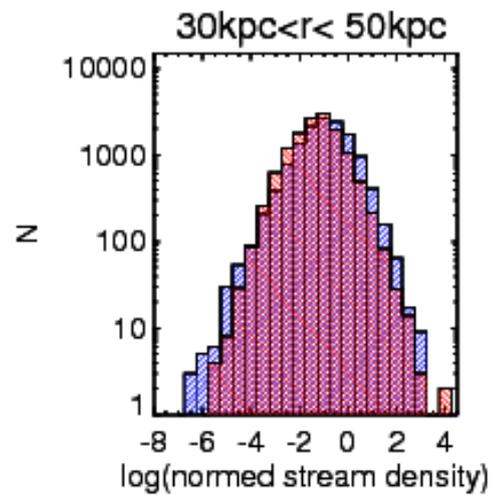
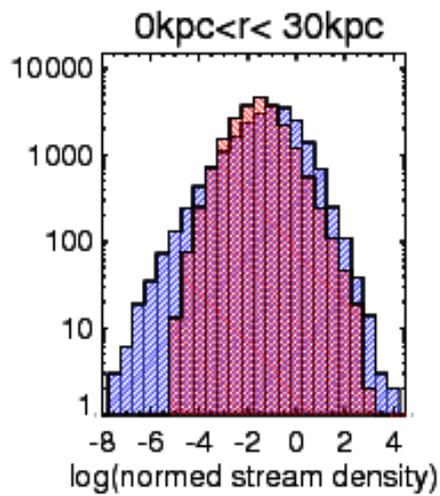


general shape and
caustic spacing/number
very similar!

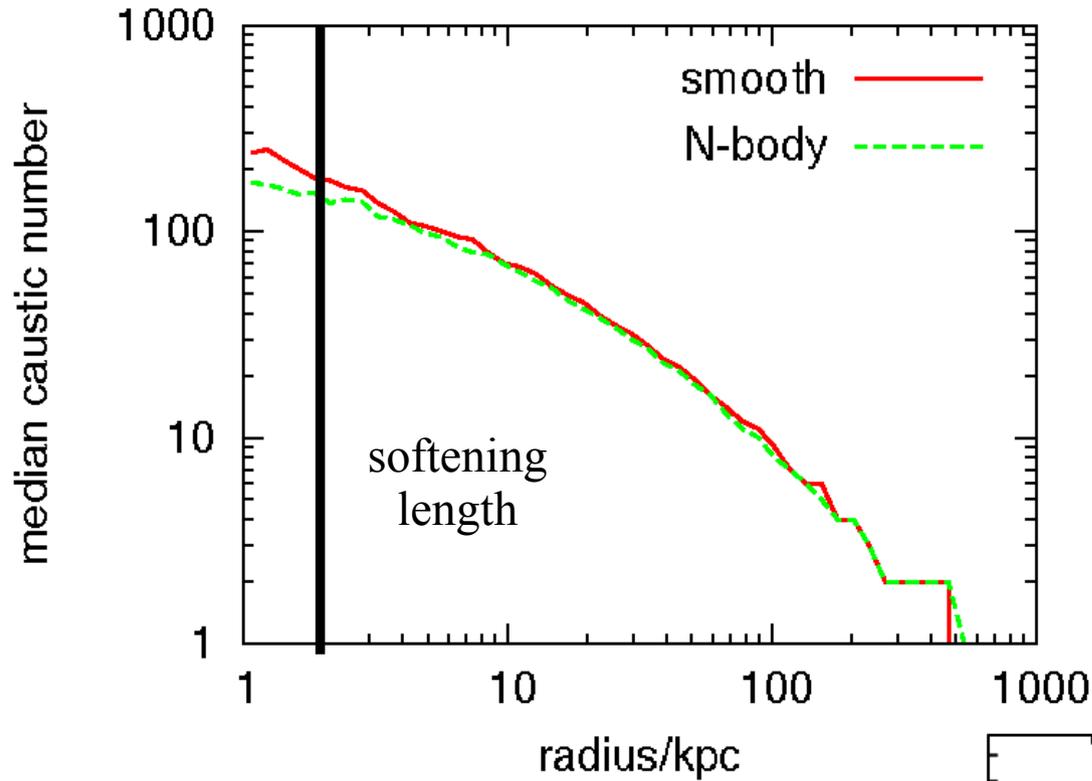
phase-space density
conservation: 10^{-8}

All DM particles

discreteness:
some overly
low densities



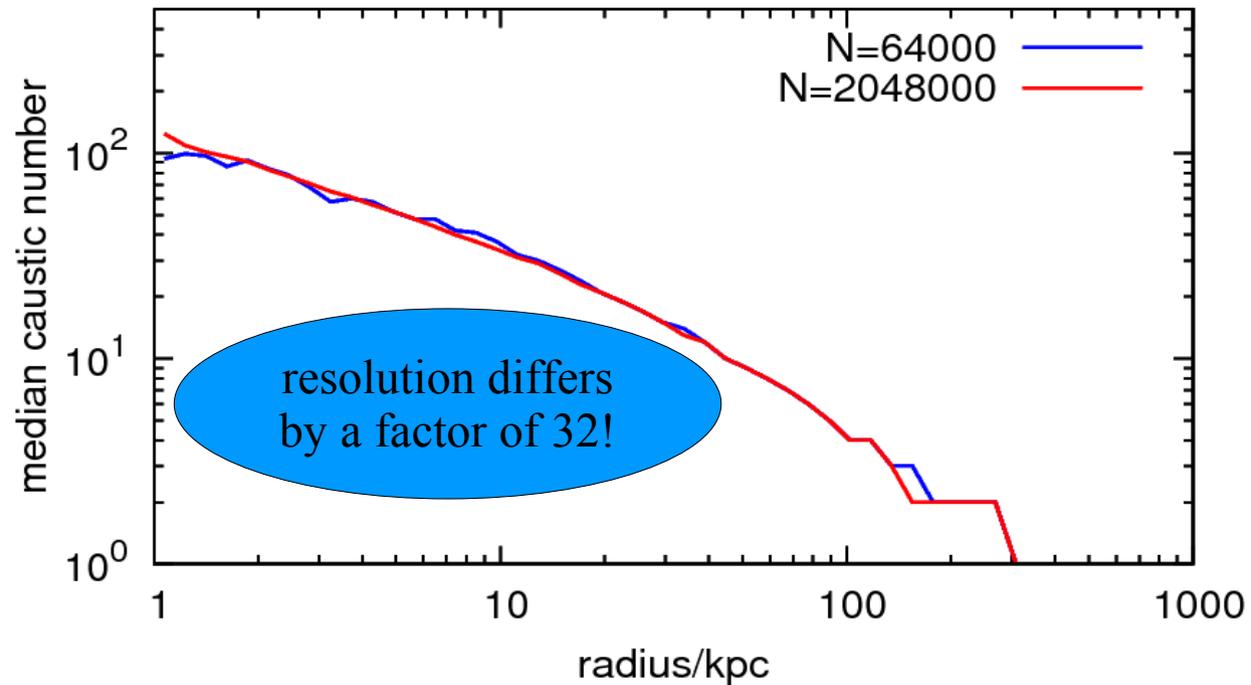
Number of Caustic Passages

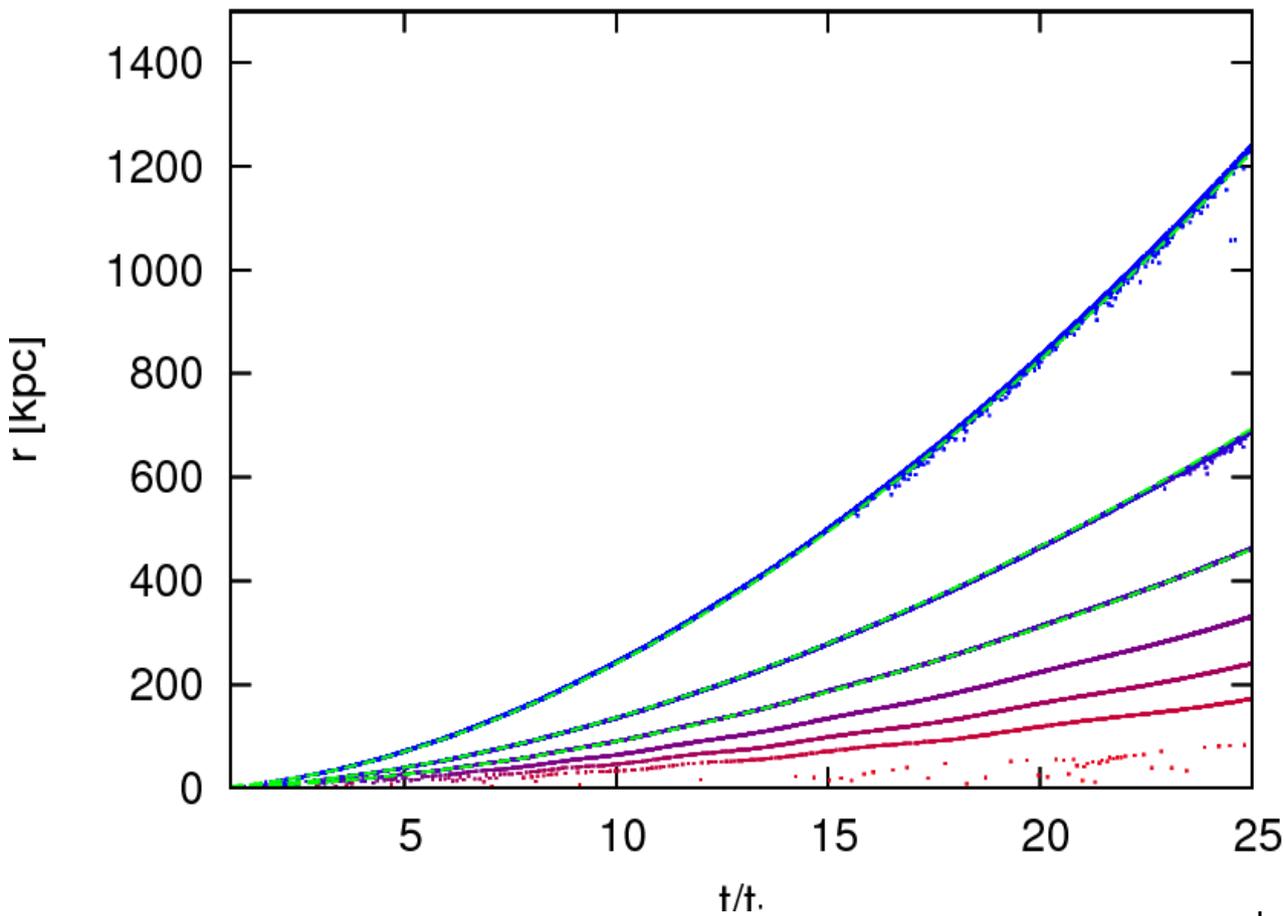


analytic and N-body
results nearly the same!

**Annihilation boost
factor estimates
due to caustics
should be very robust!**

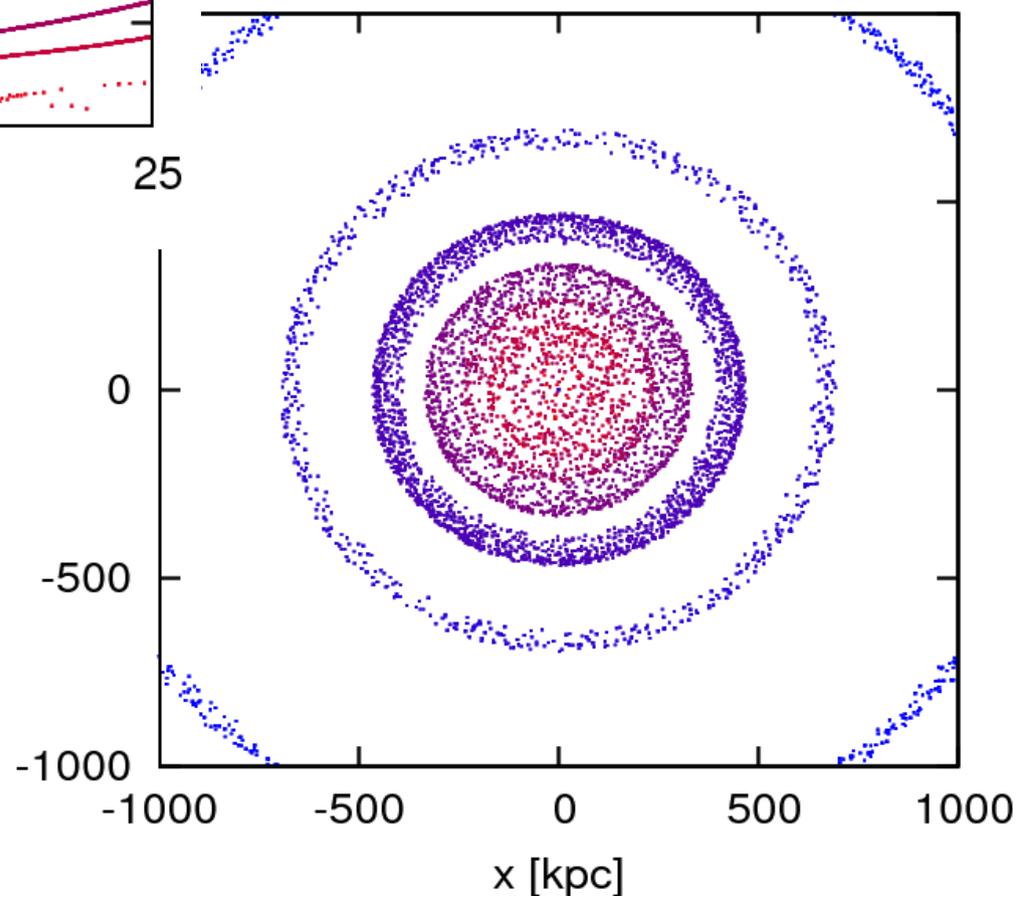
**Very stable against
particle number
and softening length!**

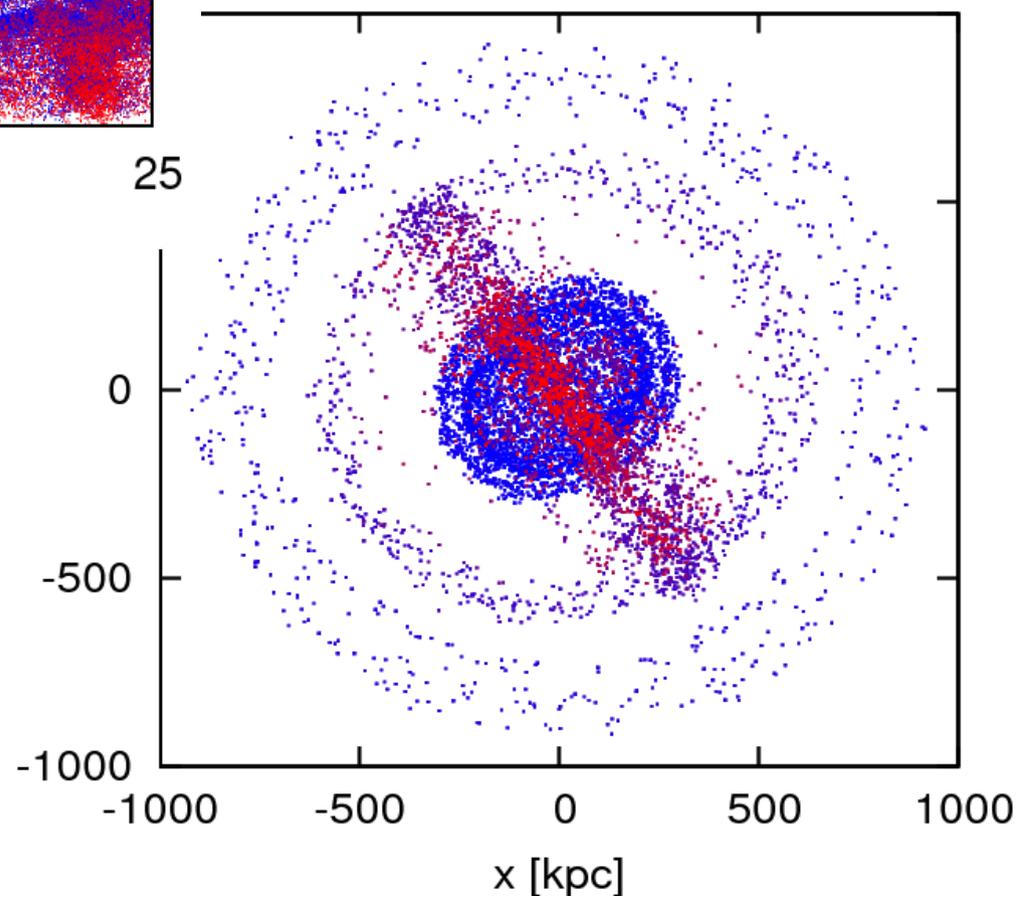
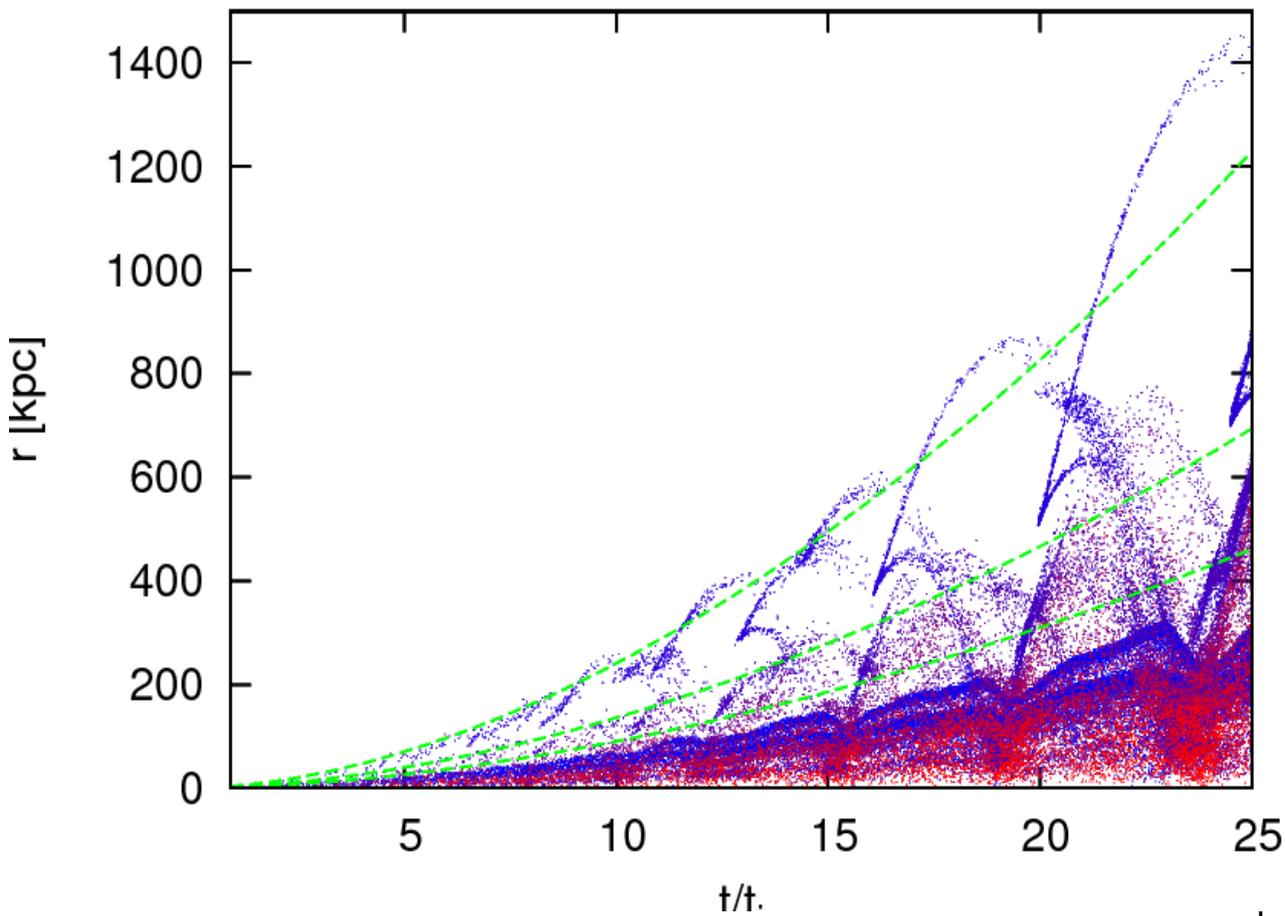




Caustic structure and evolution for growth from spherical self-similar IC's

1-D (spherical) gravity





Caustic structure and evolution for growth from spherical self-similar IC's

Fully 3-D gravity

Small-scale structure affects DM detection?

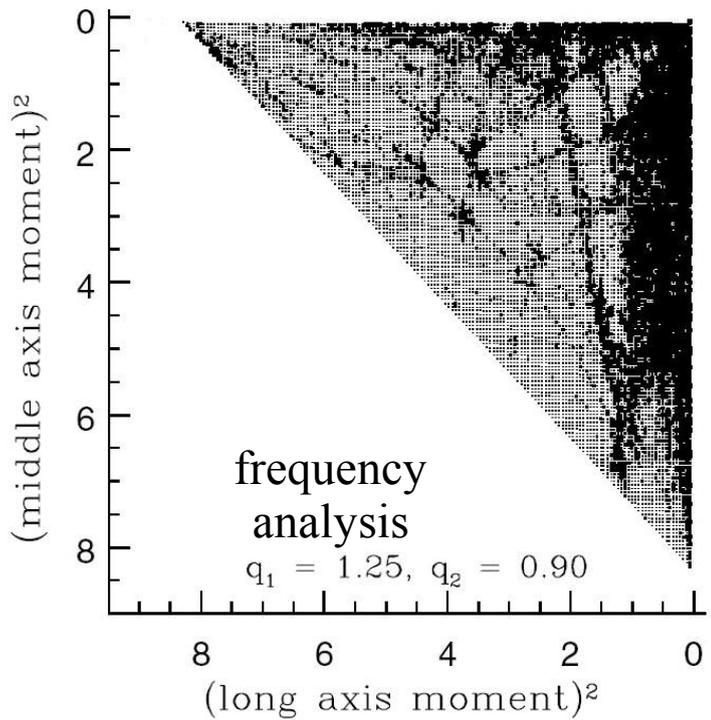
- ~~• Halo DM is mostly in small (e.g. Earth mass?) clumps?
→ direct detectors typically live in low density regions?~~
- DM streams → non-Maxwellian, “clumpy” $f(\mathbf{v})$?
→ direct detectors will see an irregular energy distribution?
- ~~• Small (Earth-mass?) clumps dominate observable annihilation signal?~~
- ~~• Dwarf Spheroidals/subhalos are best targets for detecting annihilation?
(and are boosted by sub-substructure)~~
- Smooth halo annihilation emission is dominated by caustics?

Small-scale structure affects DM detection?

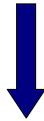
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- Small (Earth-mass?) clumps dominate observable annihilation signal?
- Dwarf Spheroidals/subhalos are best targets for detecting annihilation?
(and are boosted by sub-substructure)
- Smooth halo annihilation emission is dominated by caustics?

Conclusions

- GDE robustly identifies caustic passages and gives fair stream density estimates for particles in fully 3-D CDM simulations
- Many streams are present at each point well inside a CDM halo (at least 100,000 at the Sun's position)
 - quasi-Gaussian signal in direct detection experiments
- Caustic structure is more complex in realistic 3-D situations than in matched 1-D models but the caustics are weaker
 - negligible boosting of annihilation signal due to caustics
- Boost due to small substructures has *no* effect on γ -detectability

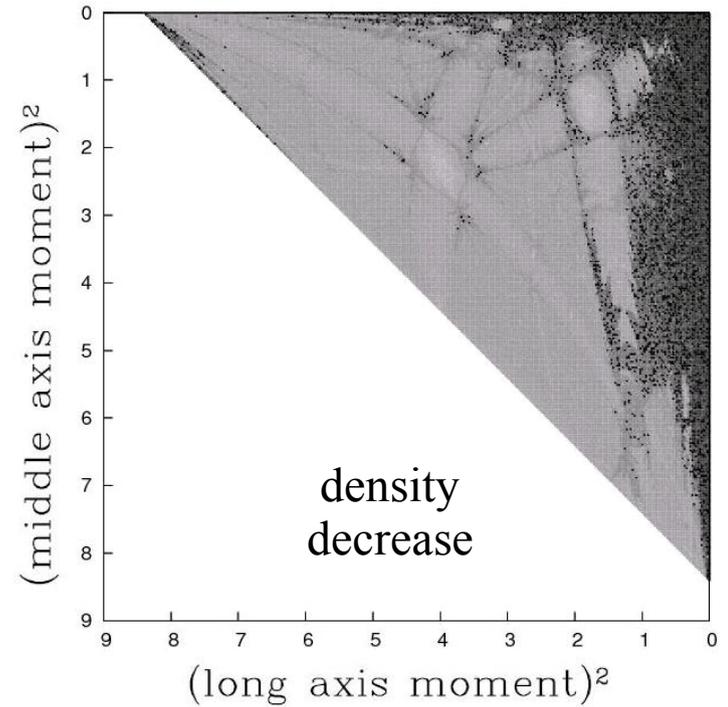


moderate triaxiality

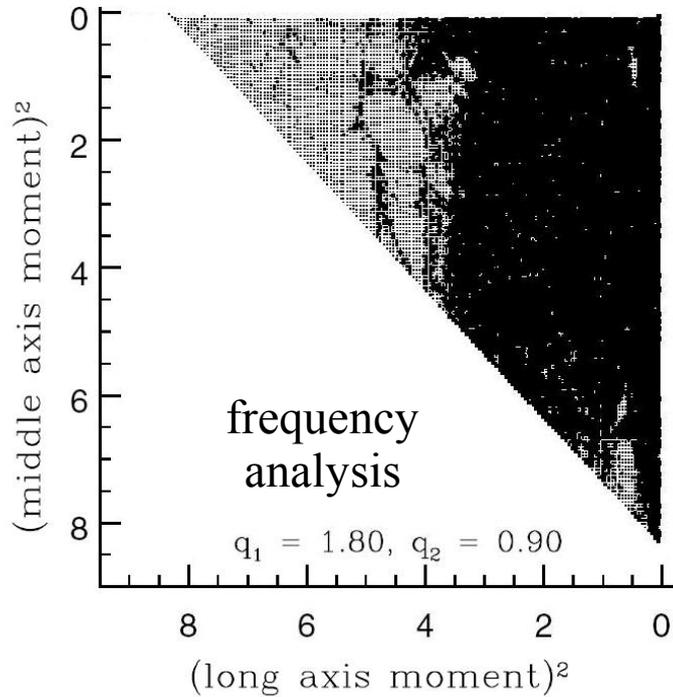


small fraction of chaotic orbits

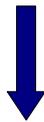
stream density mostly decaying like a power law



Papaphilippou & Laskar 1998



high triaxiality



large fraction of chaotic orbits

stream density mostly decaying much faster than a power law

integrate 10^5 different orbits

