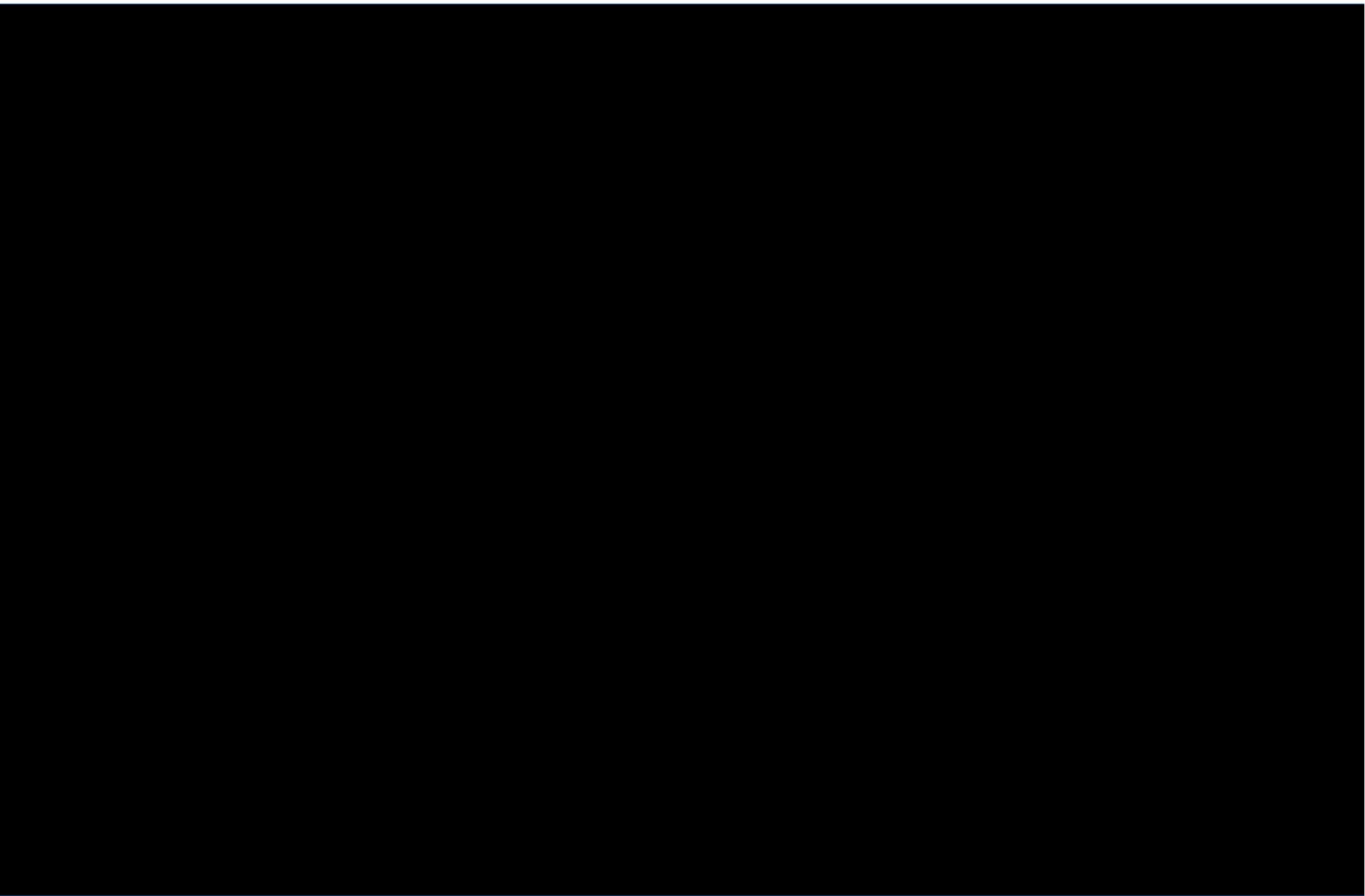
A visualization of the cosmic web, showing a complex network of filaments and nodes. The filaments are thin, blue, and interconnected, forming a web-like structure. The nodes are larger, denser regions of yellow and orange, representing galaxy clusters and superclusters. The background is a deep blue, suggesting the vastness of space.

*Dark Side of the Universe
Buenos Aires, July 2019*

Nonlinear structure in the DM distribution

*Simon White
MPI for Astrophysics*



Newtonian “experiment” with 100 million bodies – forming a dark matter halo

The four elements of Λ CDM halos

I Smooth background halo

- NFW-like cusped density profile
- near-ellipsoidal equidensity contours

II Bound subhalos

- most massive typically 1% of main halo mass
- total mass of all subhalos $\lesssim 10\%$
- less centrally concentrated than the smooth component

III Tidal streams

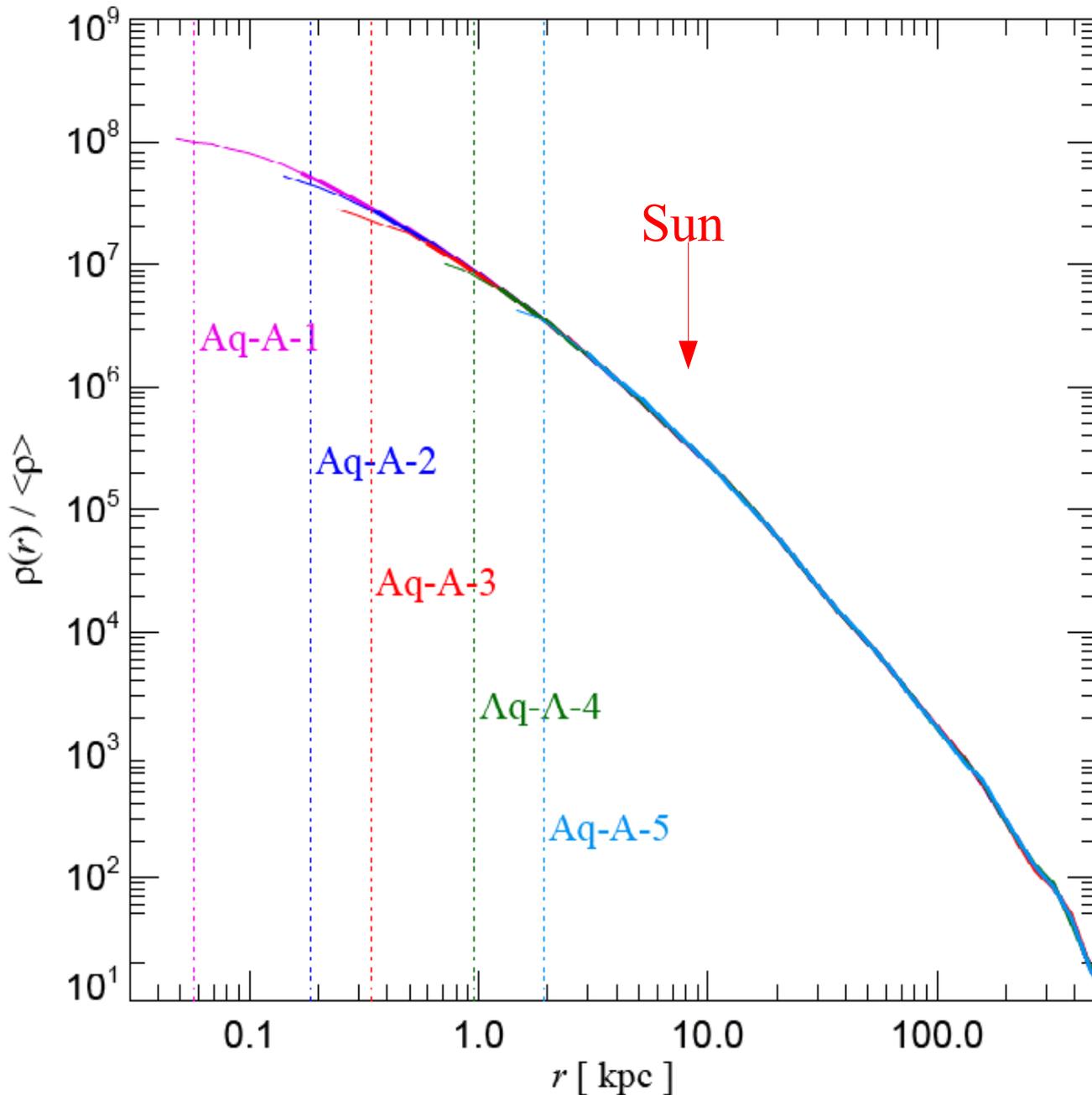
- remnants of tidally disrupted subhalos

IV Fundamental streams

- consequence of smooth and cold initial conditions
- very low internal velocity dispersions
- produce density caustics at projective catastrophes

I. Smooth background halo

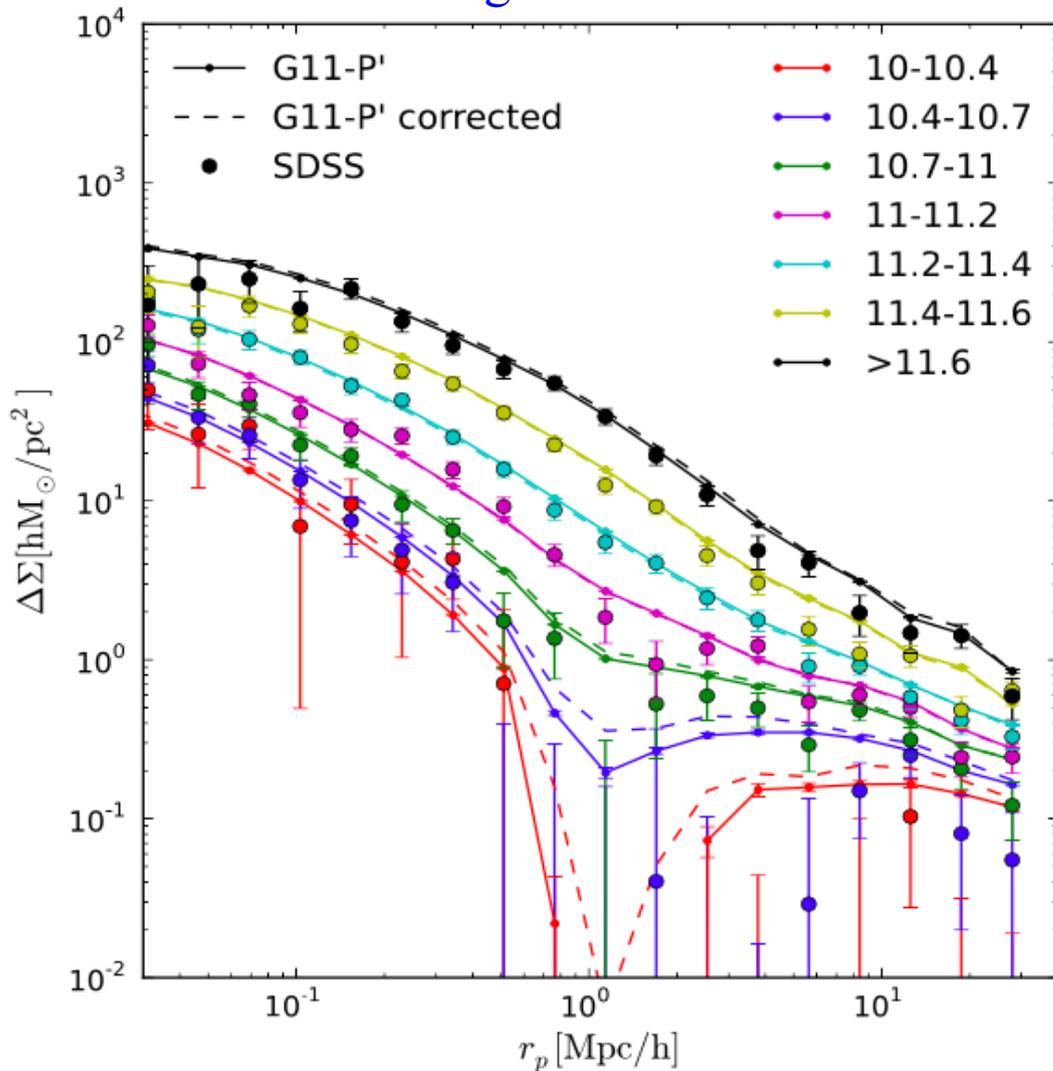
Aquarius Project: Springel et al 2008



Density profiles of simulated DM-only Λ CDM halos are now very well determined -- to radii well inside the Sun's position

Λ CDM halo profiles vs lensing observations

Wang et al 2016



Weak lensing profiles around stacks of isolated SDSS galaxies as a function of their stellar mass.

Predictions from a SDSS “mock” catalogue made from a SAM in the Planck cosmology with parameters adjusted to fit galaxy abundances.

No further parameter adjustment to fit lensing/clustering observations.

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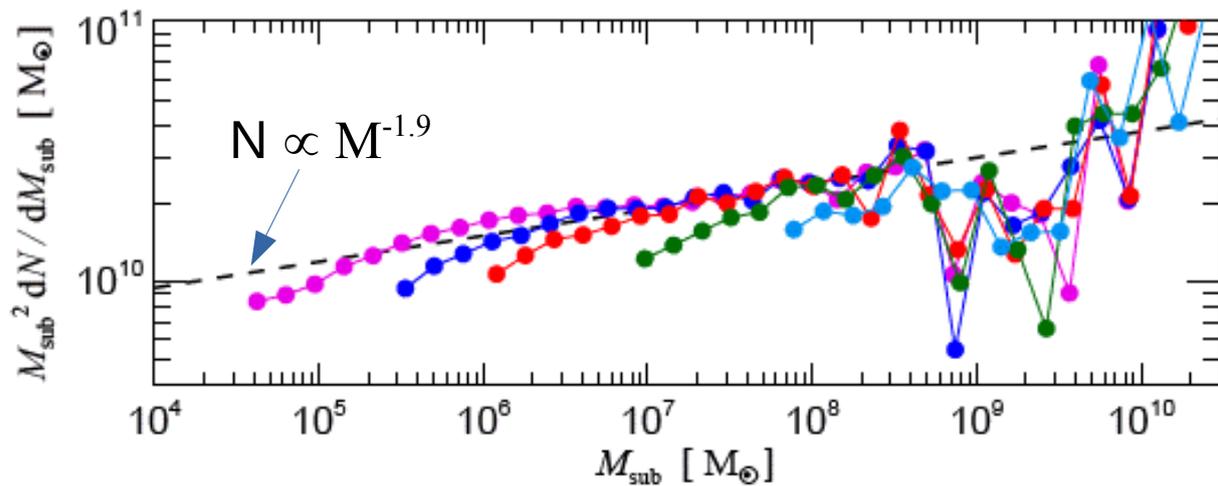
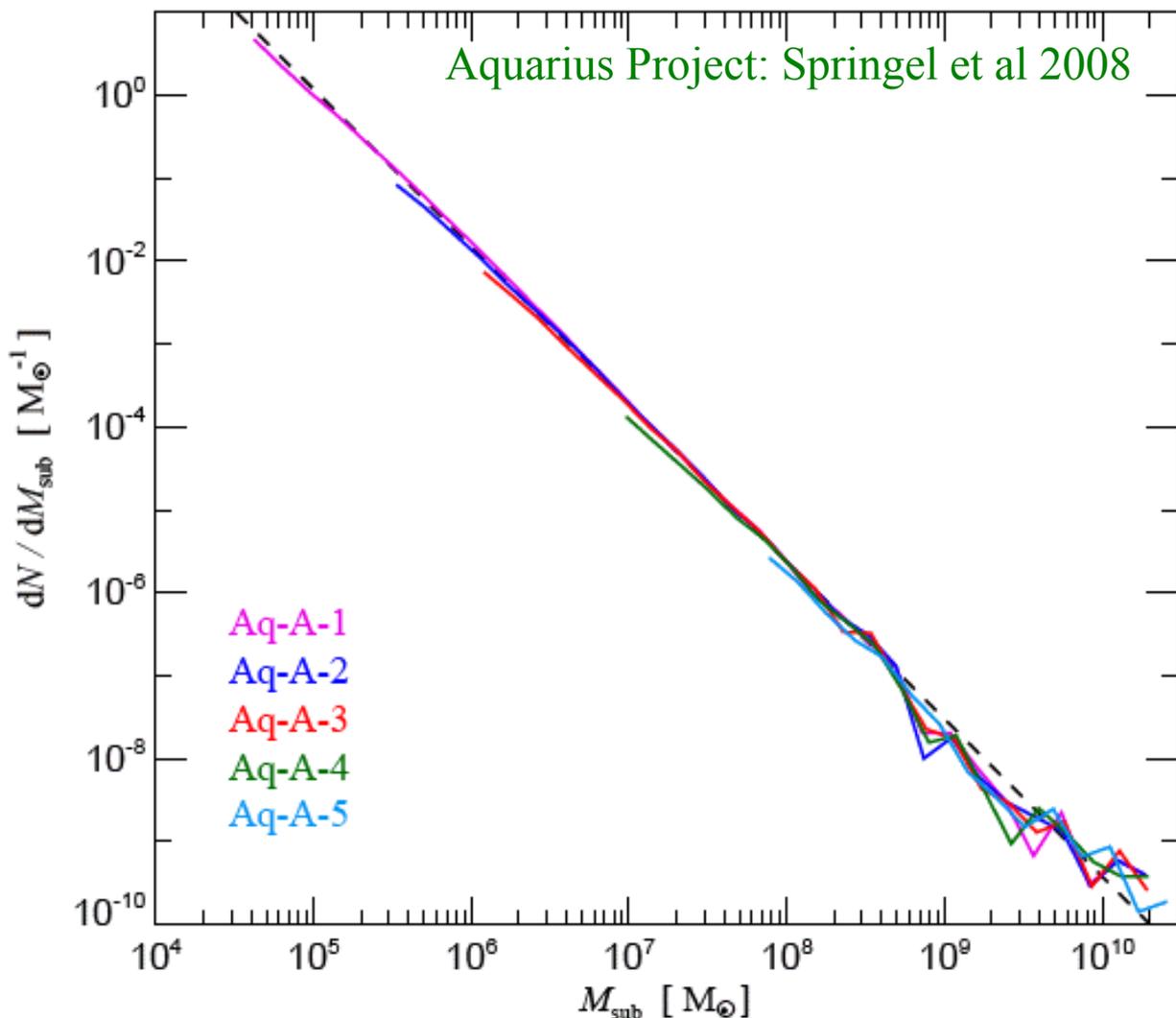
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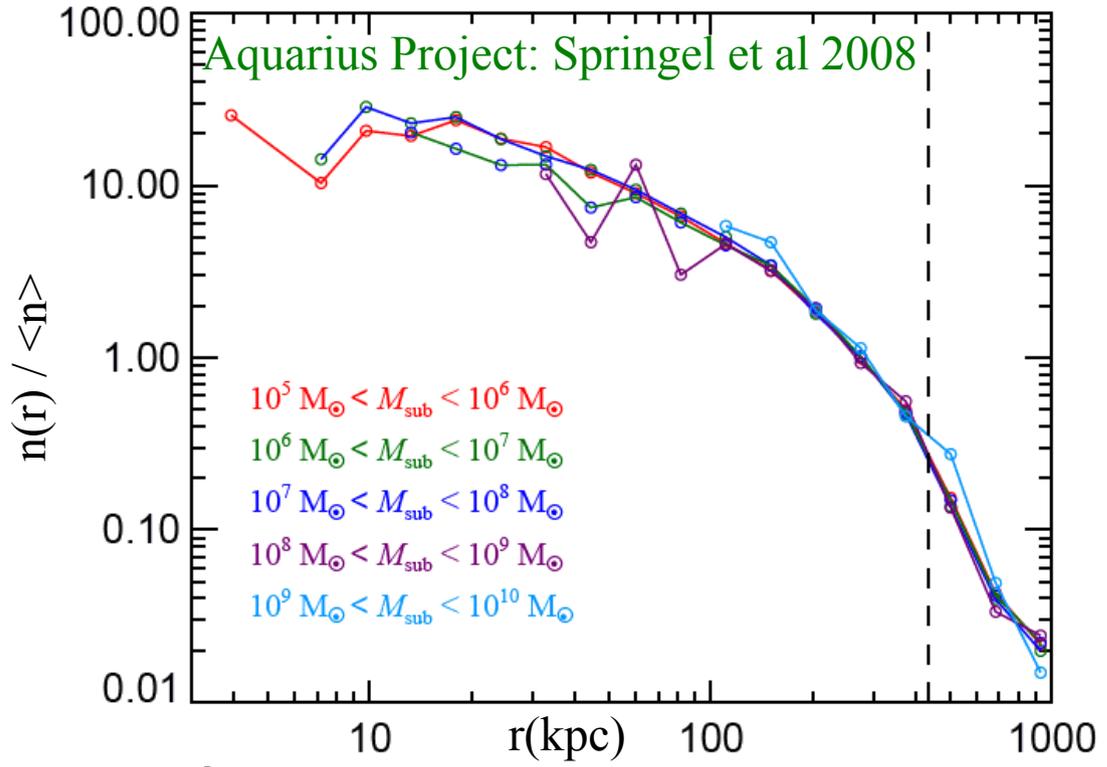
II. Bound subhalos

Abundance of self-bound subhalos is measured to below $10^{-7} M_{\text{halo}}$

Most subhalo mass is in the biggest objects (just)



Aquarius Project: Springel et al 2008



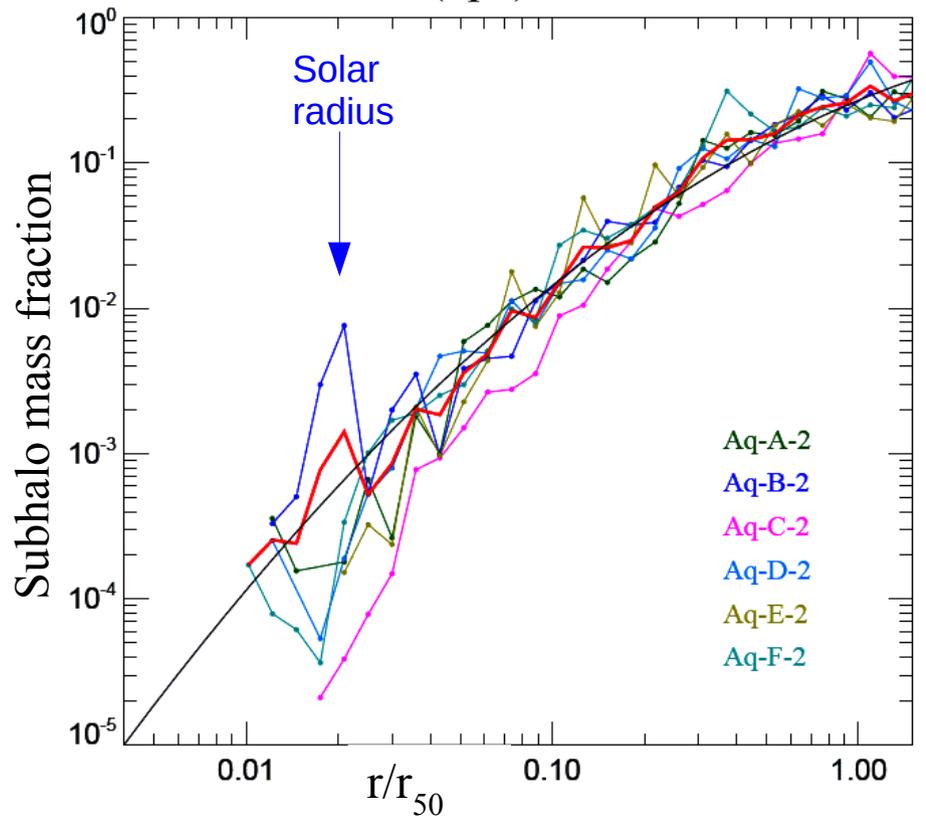
II. Bound subhalos

Abundance of self-bound subhalos is measured to below $10^{-7} M_{\text{halo}}$

Most subhalo mass is in the biggest objects (just)

The radial distribution of subhaloes is shallow and independent of their mass

Only a very small fraction of the mass near the Sun is in subhaloes



Bound subhalos: conclusions

Substructure is primarily in the outermost parts of halos

The radial distribution of subhalos is almost mass-independent

The total mass in subhalos converges (weakly) at small m

Subhalos contain a very small mass fraction in the inner halo ($\sim 0.1\%$ near the Sun) and so will *not* be relevant for direct detection experiments

(Small) subhalos *dominate* the total annihilation luminosity at large radius

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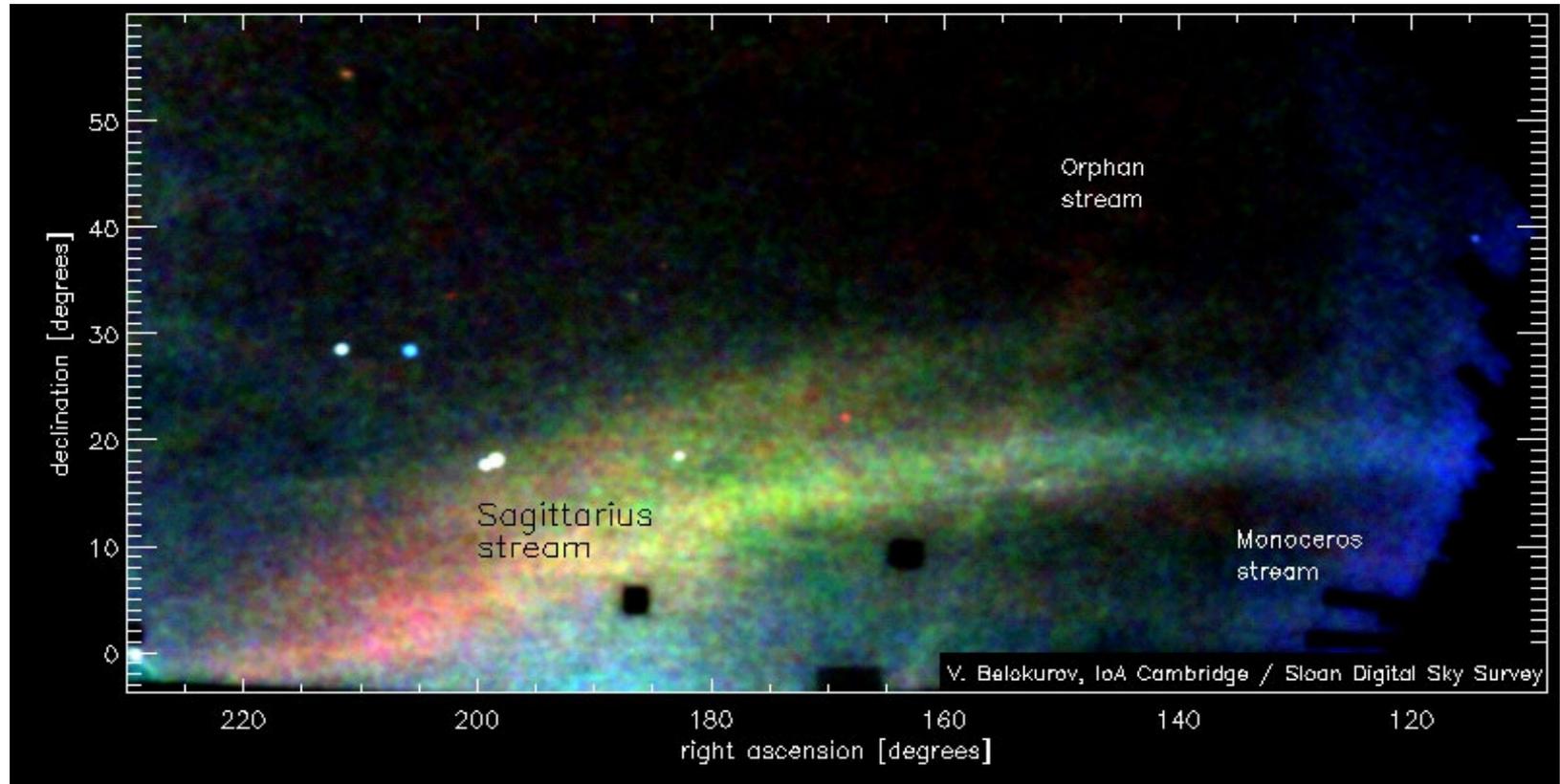
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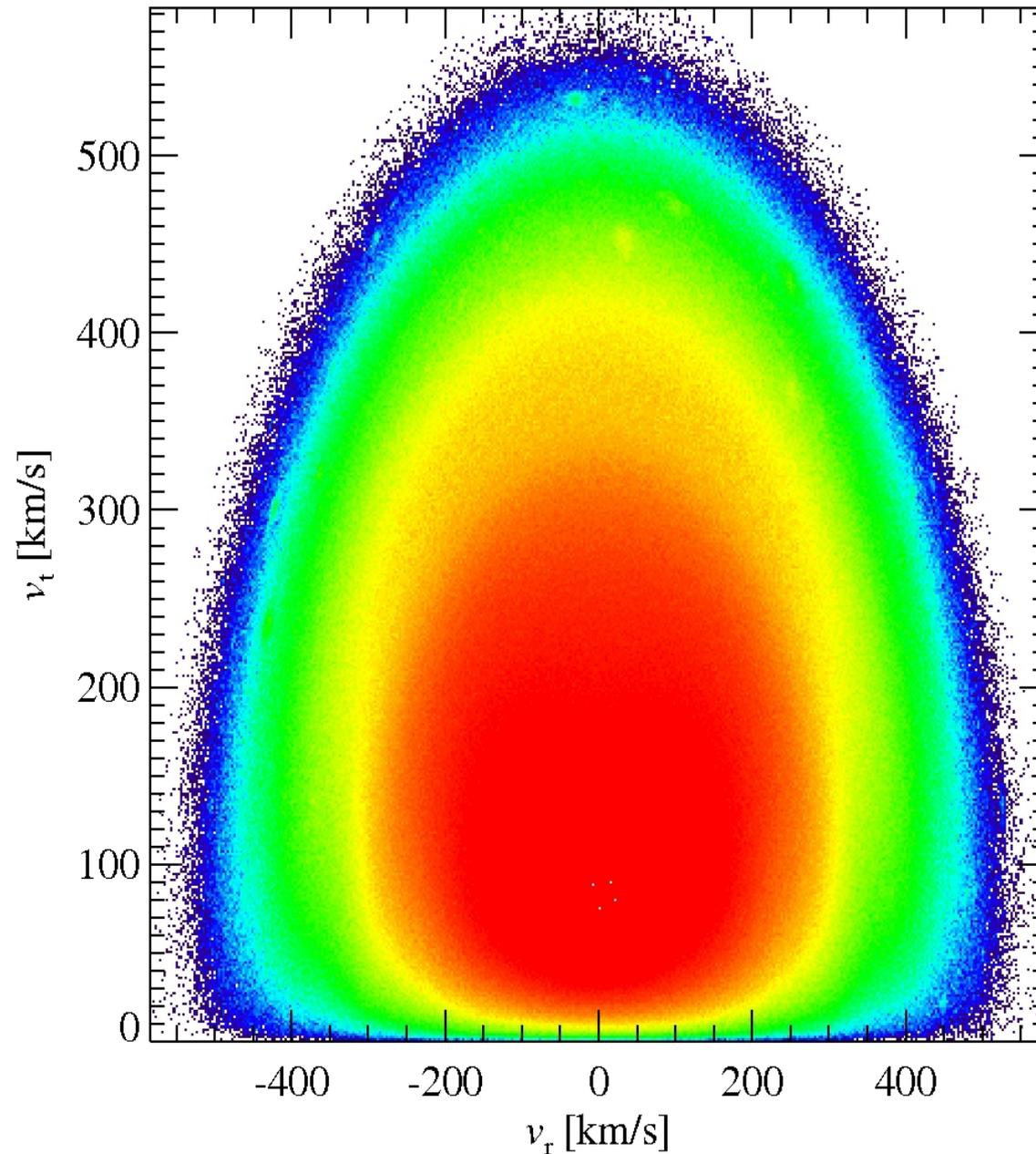
III. Tidal Streams



- Produced by partial or total tidal disruption of subhalos
- Analogous to observed stellar streams in the Galactic halo
- Distributed along/around orbit of subhalo (c.f. meteor streams)
- Localised in almost 1-D region of 6-D phase-space (\underline{x} , \underline{v})

Dark matter phase-space structure in the inner MW

M. Maciejewski



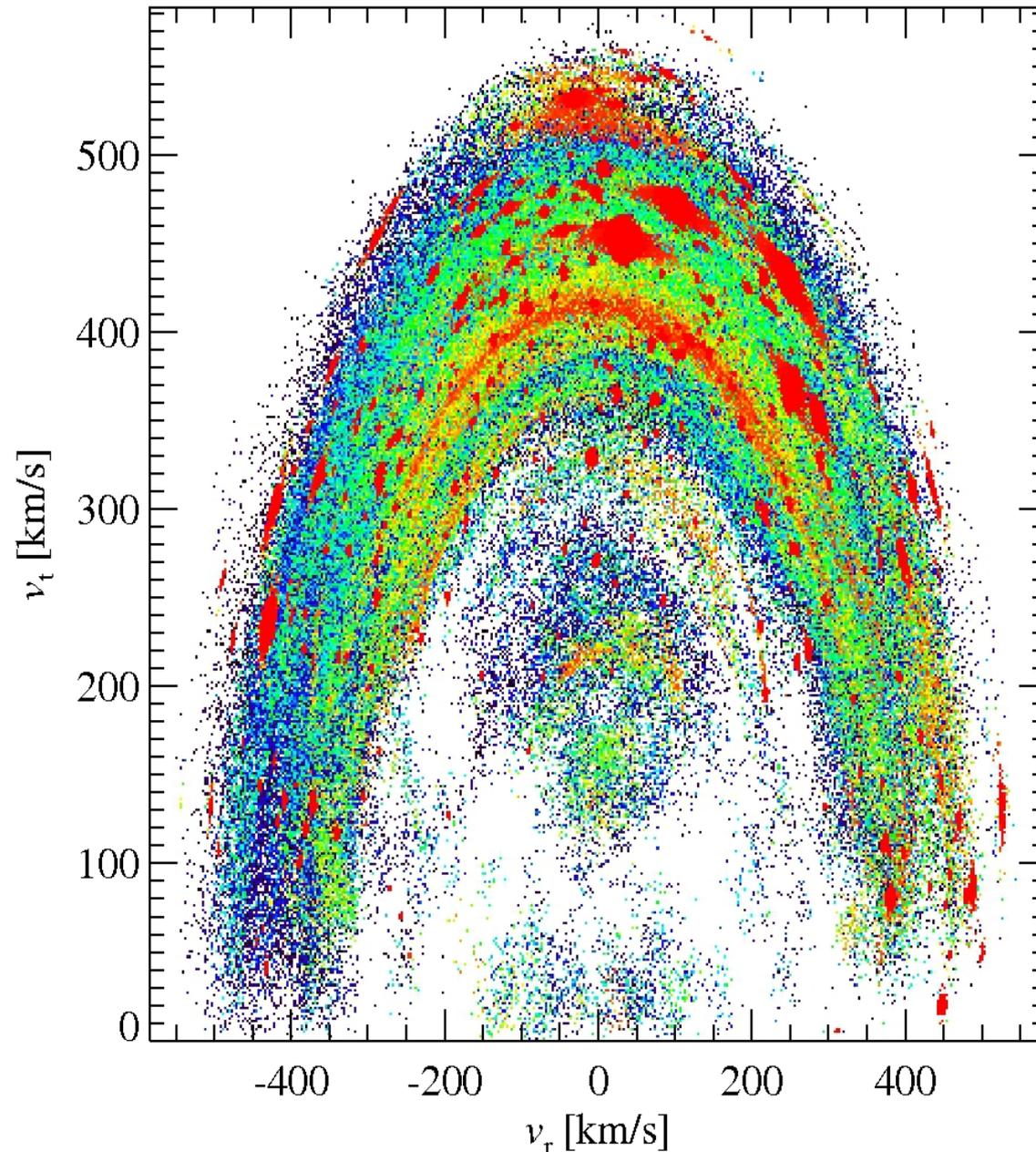
$6 \text{ kpc} < r < 12 \text{ kpc}$

All particles

$N = 3.8 \times 10^7$

Dark matter phase-space structure in the inner MW

M. Maciejewski



$6 \text{ kpc} < r < 12 \text{ kpc}$

Particles in detected
phase-space structure

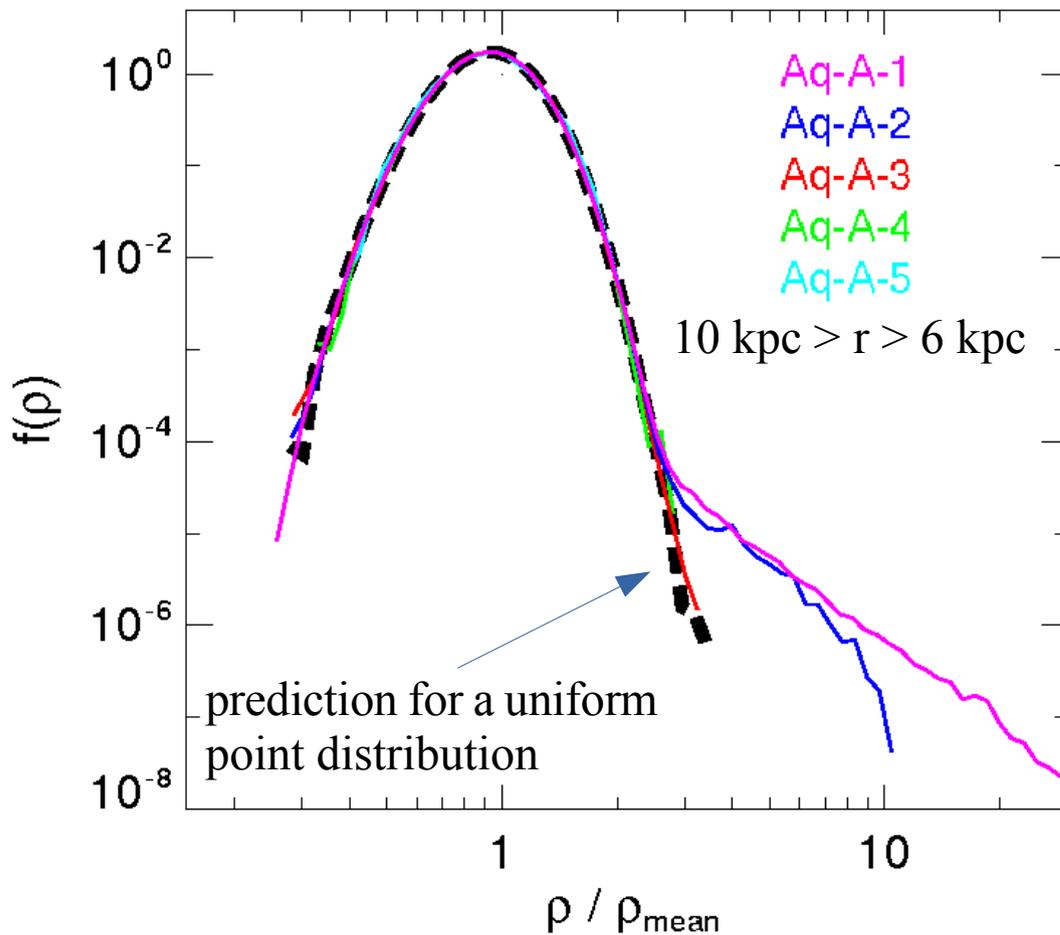
$N = 2.6 \times 10^5$
in tidal streams

$N = 3.9 \times 10^4$
in subhalos

→ only $\sim 1\%$ of the
DM signal is in strong
tidal streams

Local density in the inner halo compared to a smooth ellipsoidal model

Vogelsberger et al 2008



Estimate a density ρ at each point by adaptively smoothing using the 64 nearest particles

Fit to a smooth density profile stratified on similar ellipsoids

The chance of a random point lying in a substructure is $< 10^{-4}$

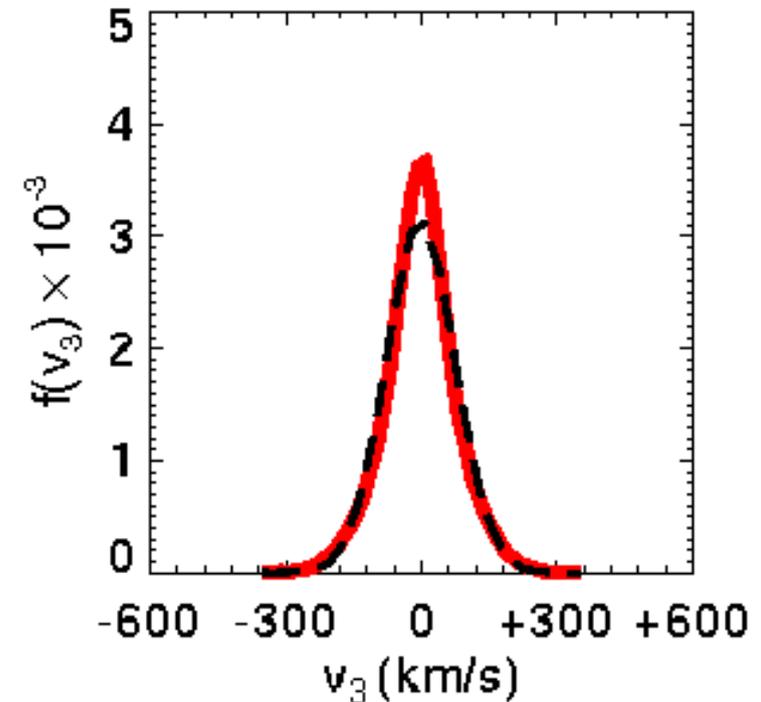
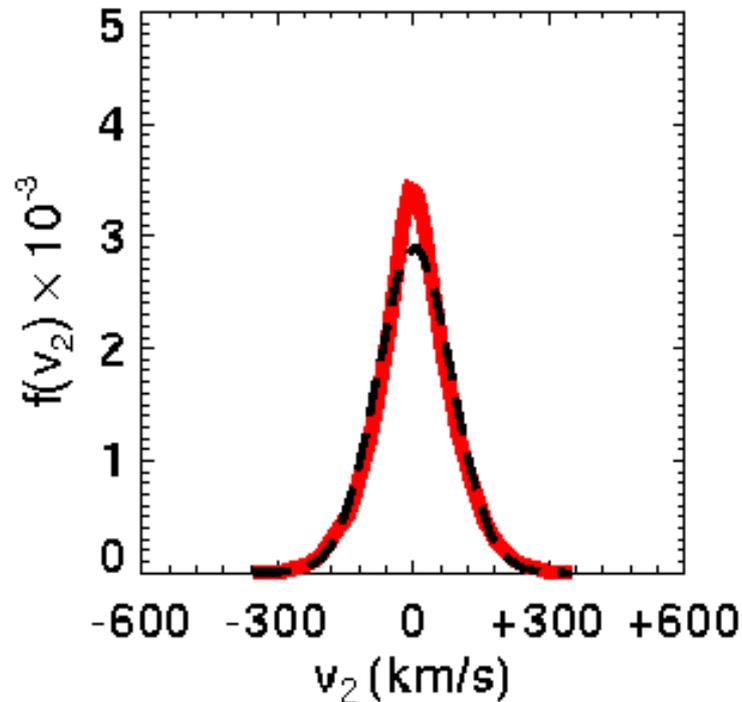
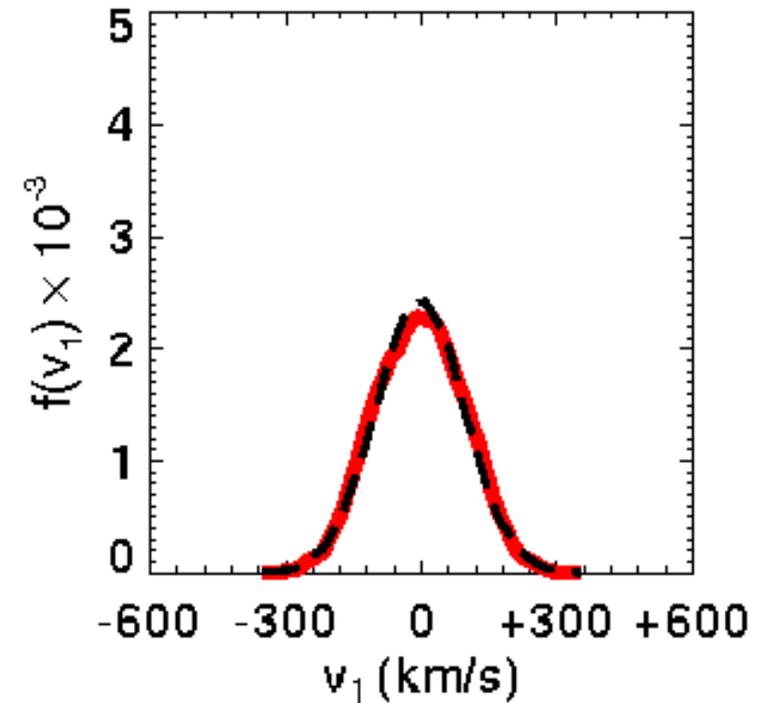
The *rms* scatter about the smooth model for the remaining points is only about 4%

Local velocity distribution

Velocity histograms for particles in a typical $(2\text{kpc})^3$ box at $R = 8$ kpc

Distributions are smooth, near-Gaussian and different in different directions

No individual streams are visible



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IV. Fundamental streams

After CDM particles become nonrelativistic, but *before* nonlinear objects form (e.g. $z > 100$) their distribution function is

$$f(\mathbf{x}, \mathbf{v}, t) = \rho(t) [1 + \delta(\mathbf{x}, t)] N [\{\mathbf{v} - \mathbf{V}(\mathbf{x}, t)\} / \sigma]$$

where $\rho(t)$ is the mean mass density of CDM,

$\delta(\mathbf{x}, t)$ is a Gaussian random field with finite variance $\ll 1$,

$\mathbf{V}(\mathbf{x}, t) = \nabla \psi(\mathbf{x}, t)$ where $\nabla^2 \psi \propto \delta$,

and N is normal with $\sigma^2 \ll \langle |\mathbf{V}|^2 \rangle$ (today $\sigma \sim 0.1$ cm/s)

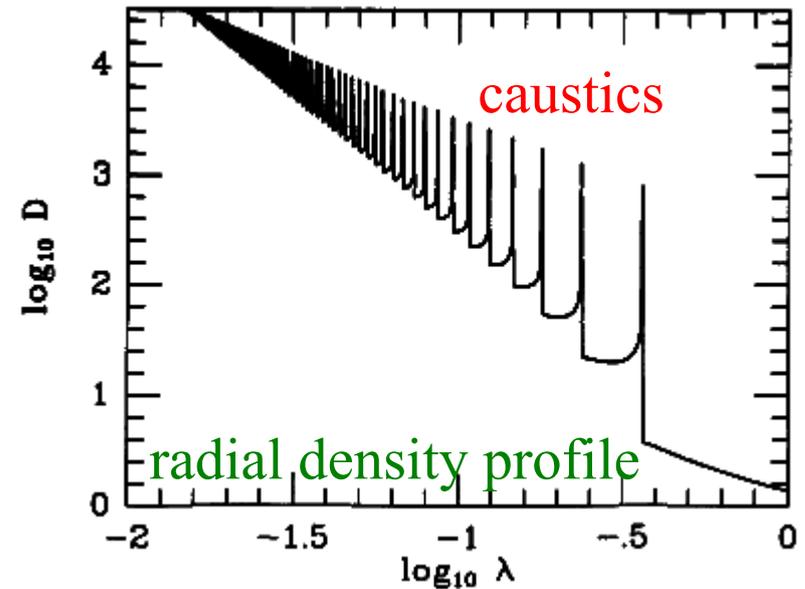
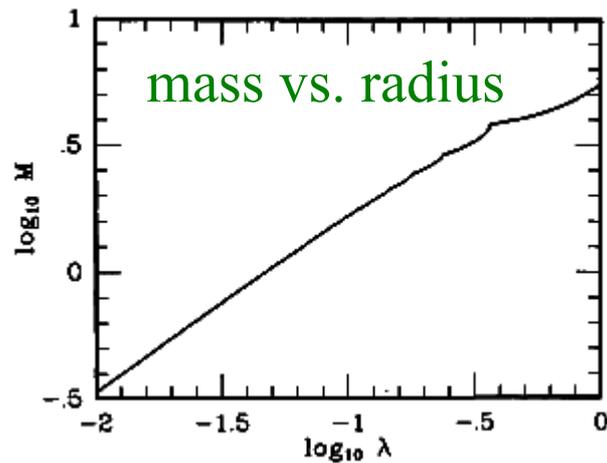
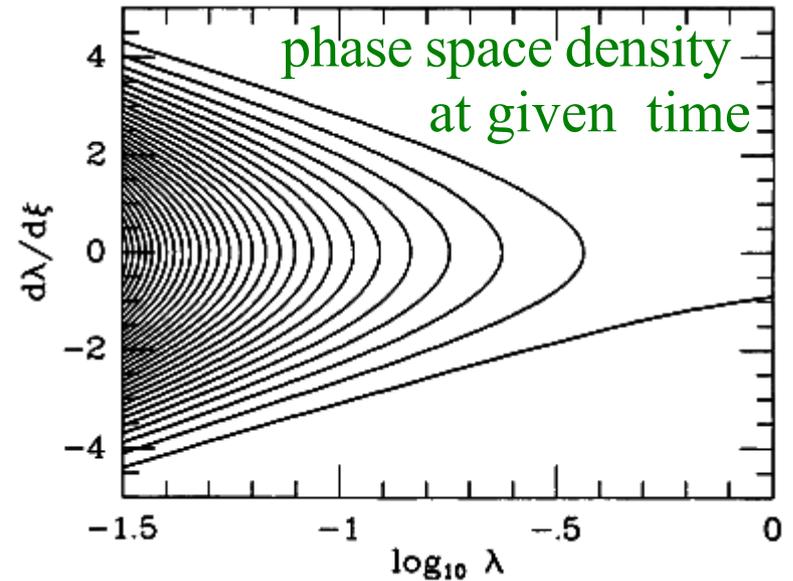
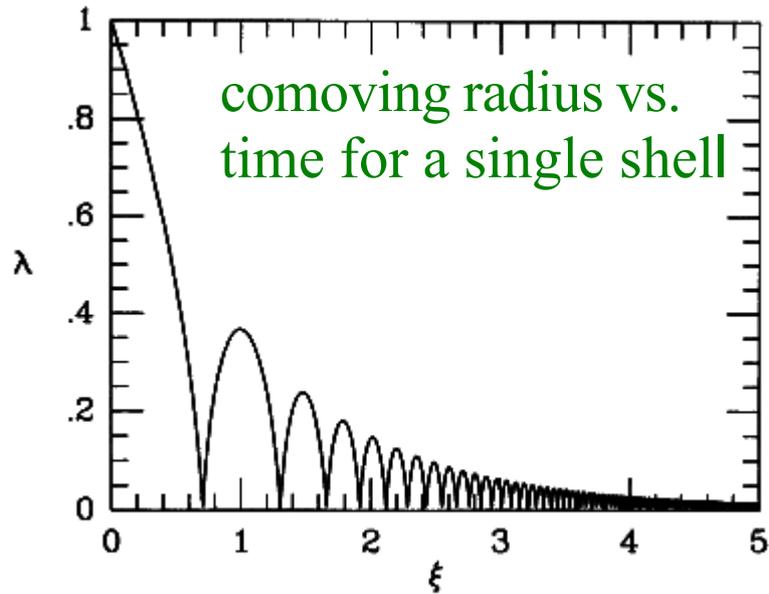
CDM occupies a thin 3-D 'sheet' within the full 6-D phase-space and its projection onto \mathbf{x} -space is near-uniform.

$Df / Dt = 0$  only a 3-D subspace is occupied at *all* times.

Nonlinear evolution leads to multi-stream structure and caustics

Similarity solution for spherical collapse in CDM

Bertschinger 1985



IV. Fundamental streams

Consequences of $Df/Dt = 0$

The 3-D phase sheet can be stretched and folded but not torn

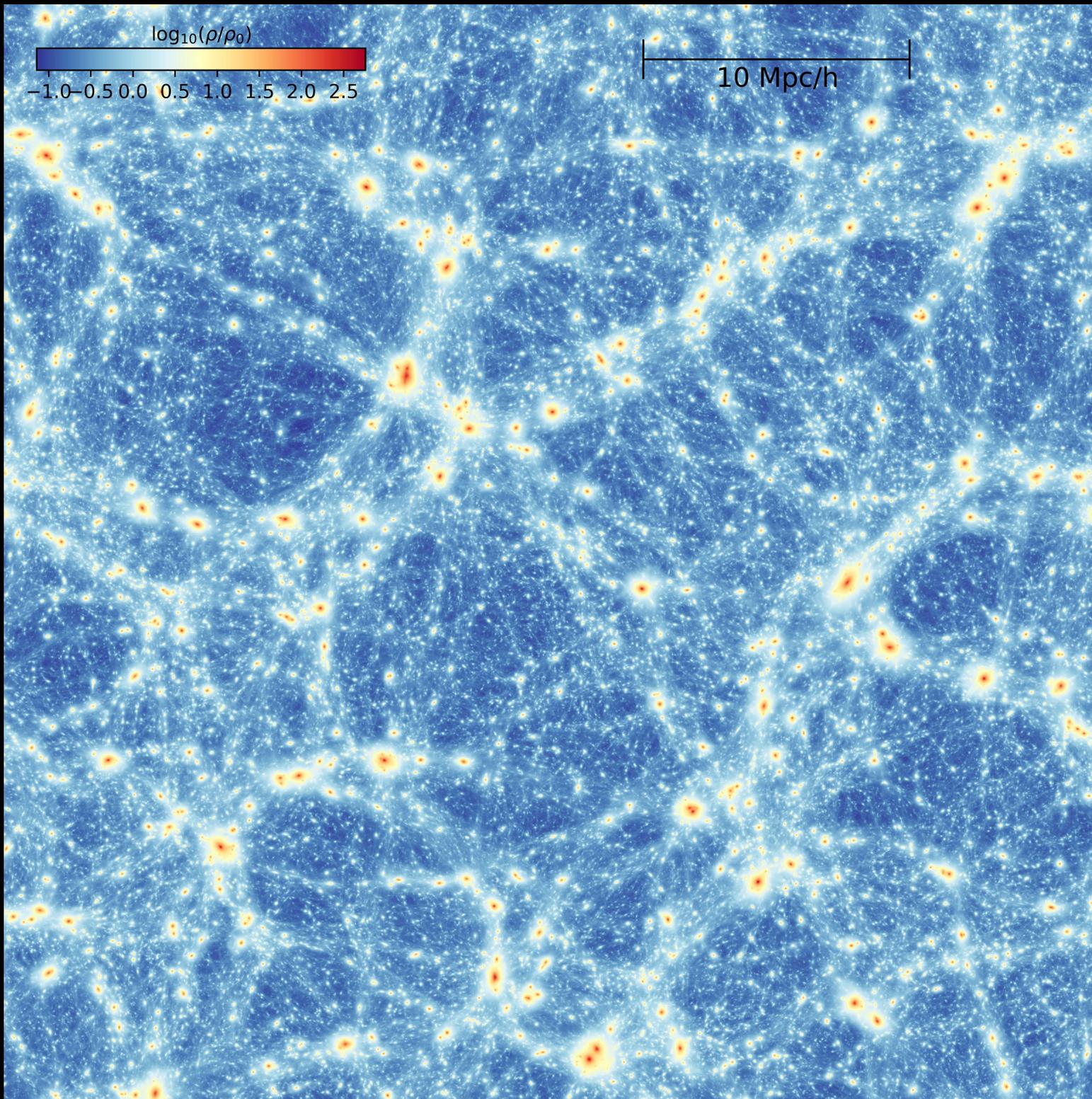
At least one sheet must pass through every point \mathbf{x}

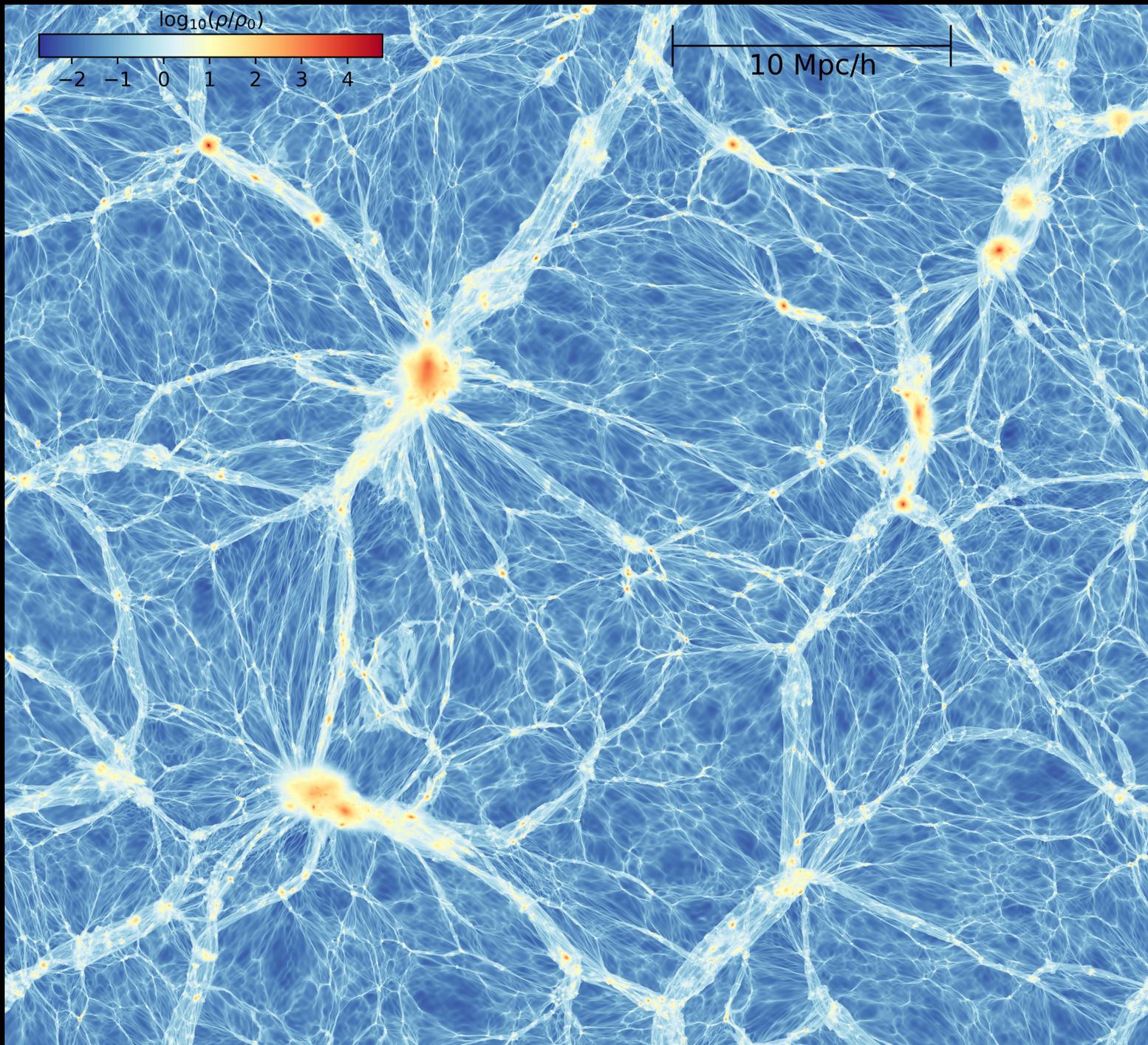
In nonlinear objects there are typically many sheets at each \mathbf{x}

Stretching which reduces a sheet's density must also reduce its velocity dispersions to maintain $f = \text{const.}$ $\longrightarrow \sigma \sim \rho^{1/3}$

At a caustic, at least one velocity dispersion must $\longrightarrow \infty$ and the integrated annihilation cross-section also diverges

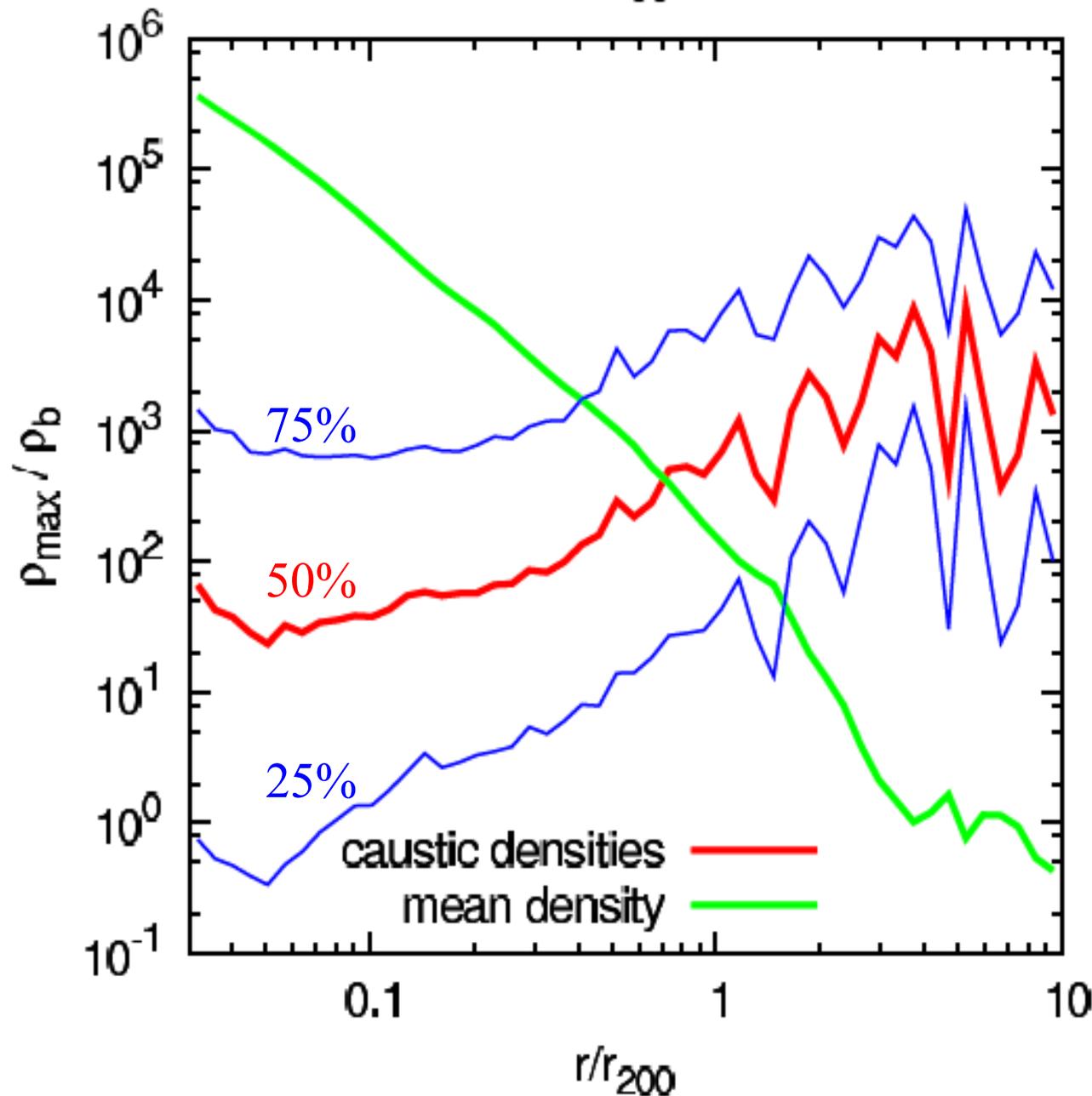
All these processes can be followed in **fully general** simulations by tracking the phase-sheet local to each simulation particle





Radial distribution of peak density at caustics

Vogelsberger & White 2011

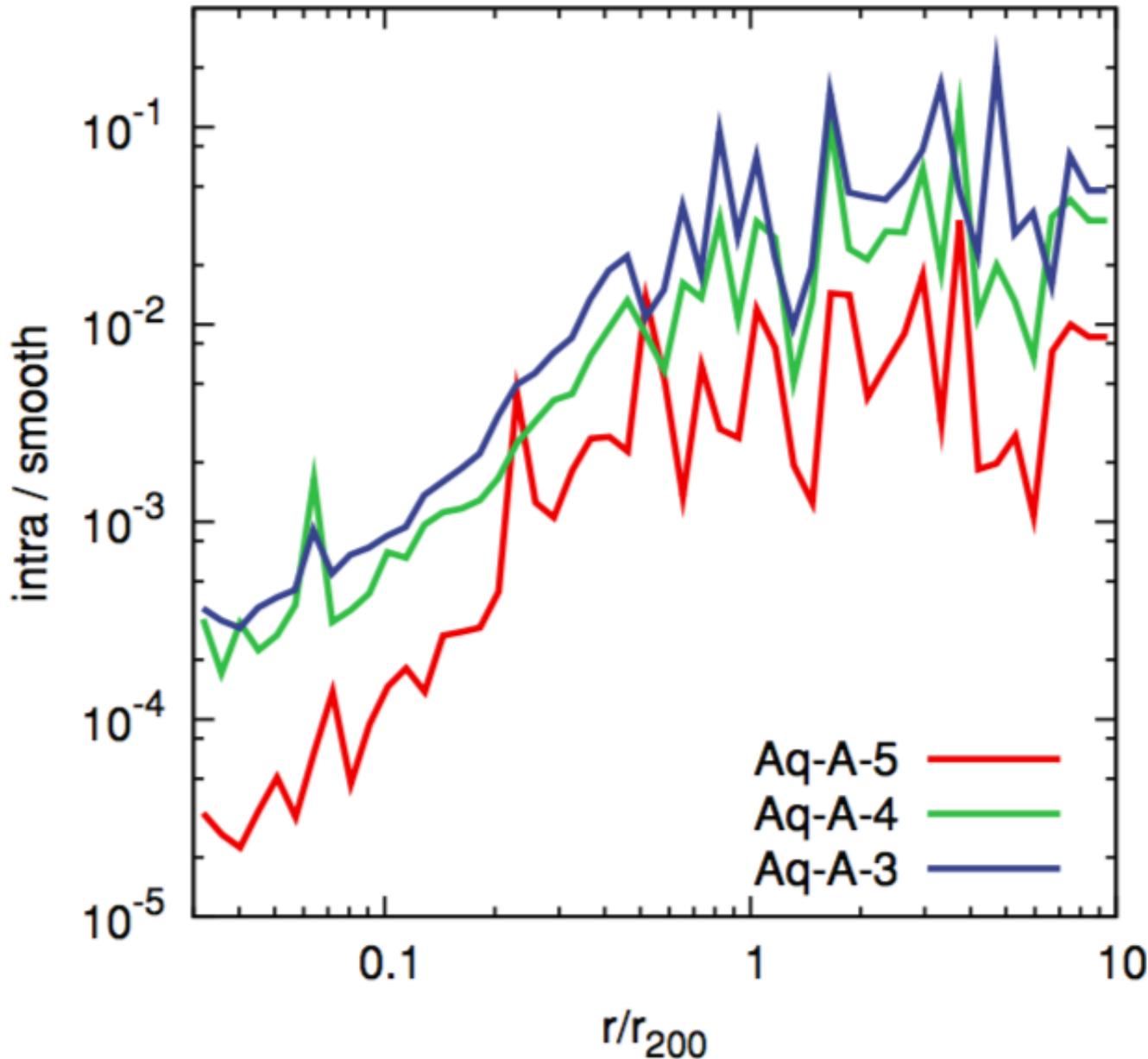


Milky Way mass halo

Initial velocity dispersion
assumes a standard
WIMP with
 $m = 100 \text{ GeV}/c^2$

Fraction of annihilation luminosity from caustics

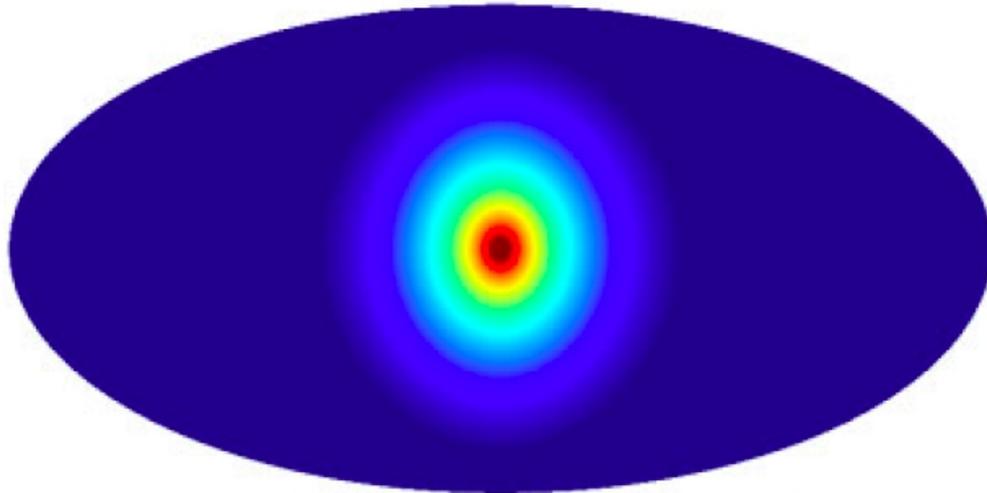
Vogelsberger & White 2011



Initial velocity dispersion assumes a standard WIMP with $m = 100 \text{ GeV}/c^2$

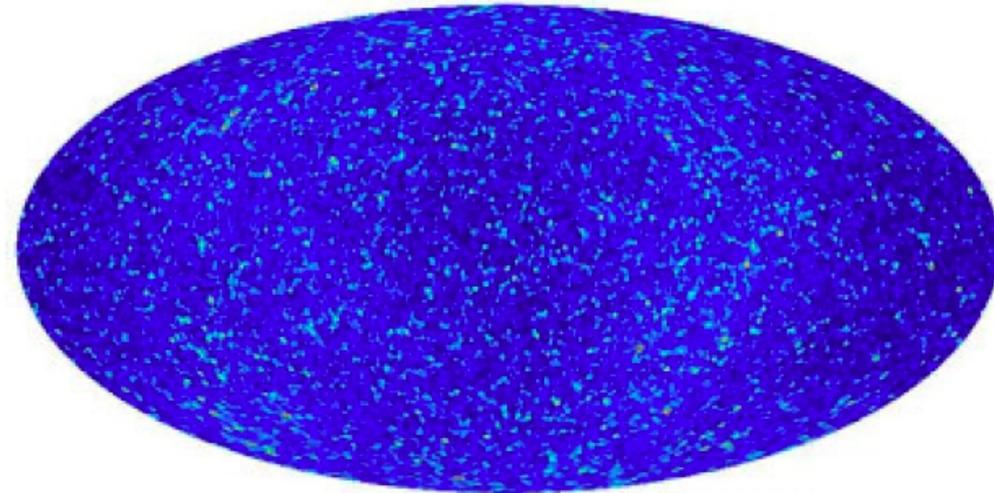
Note: caustic emission is compared to that from the smooth DM component here, but the dominant emission at large radius is from small subhaloes

smooth main halo emission (MainSm)



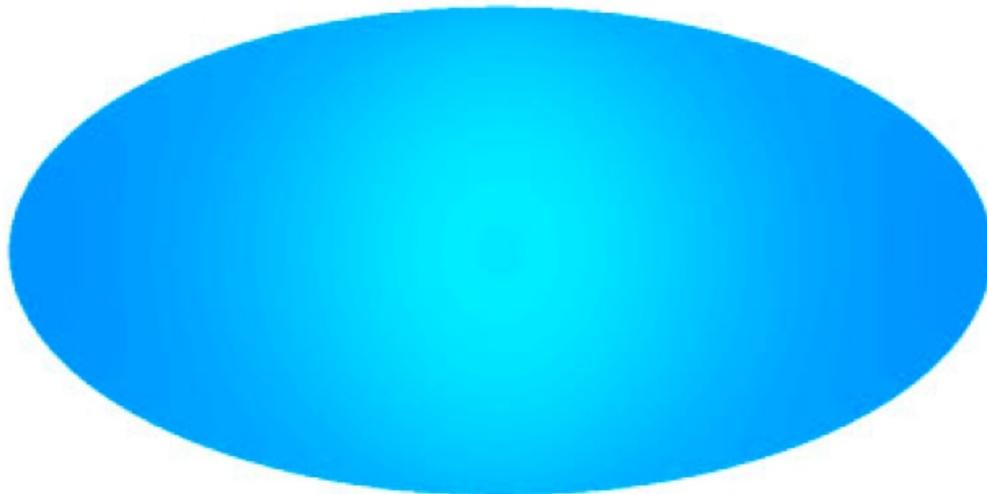
-0.50  2.0 Log(Intensity)

emission from resolved subhalos (SubSm+SubSub)



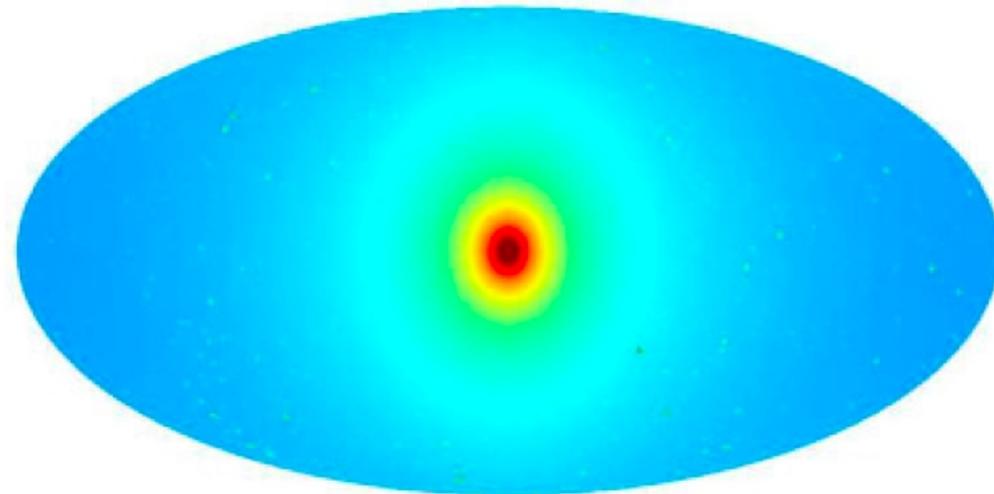
-3.0  2.0 Log(Intensity)

unresolved subhalo emission (MainUn)



-0.50  2.0 Log(Intensity)

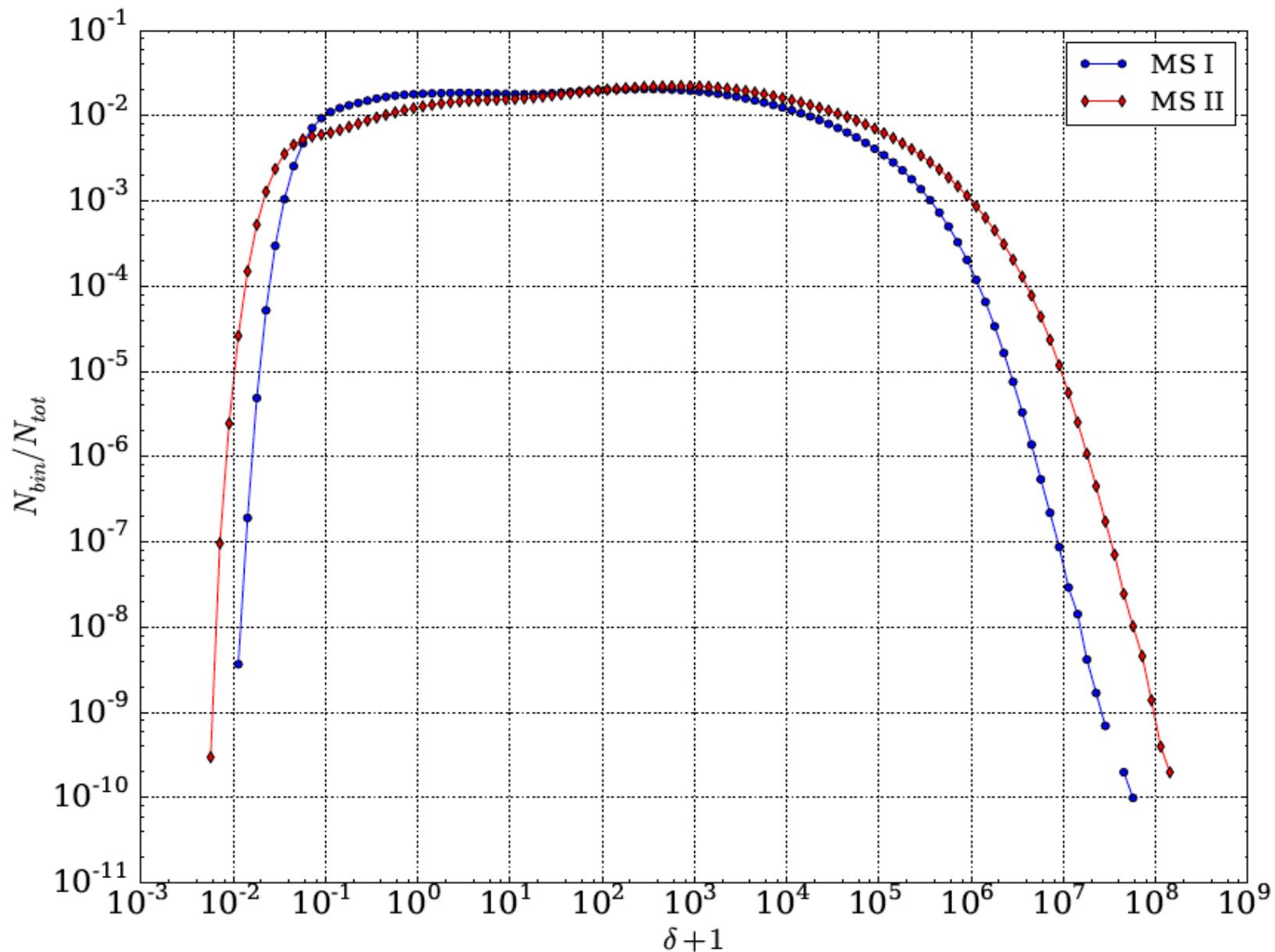
total emission



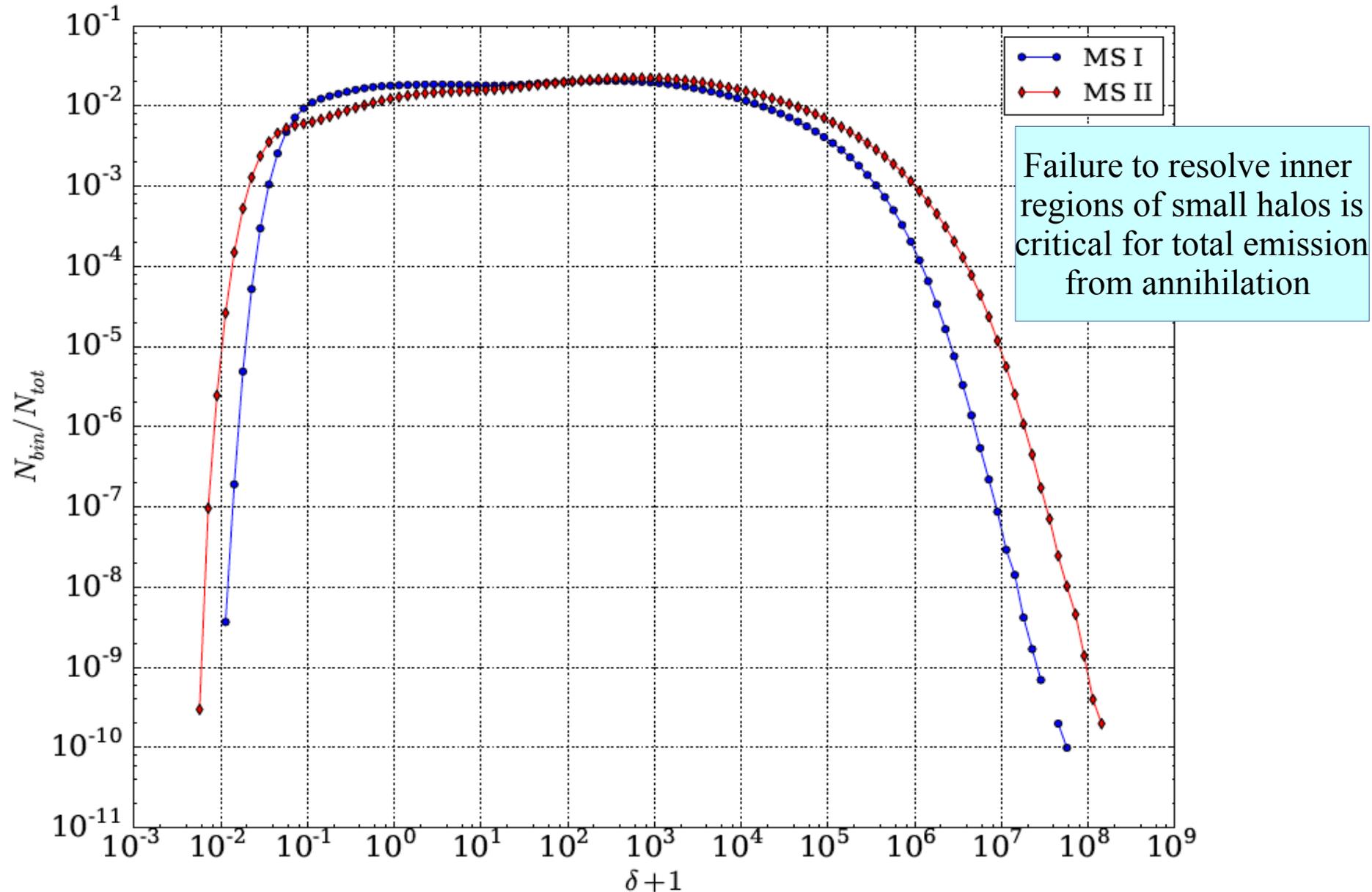
-0.50  2.0 Log(Intensity)

- Halo annihilation flux dominated by that from unresolved small halos but this is nearly uniform over the sky
- Flux from the Galactic centre dominates that from resolved subhalos by a large factor, but relative detectability depends critically on noise sources

Voronoi-estimated DM densities at the particle positions in the two Millennium Simulations, estimated as: $\rho_i \propto 1 / V_{\text{Vor},i}$

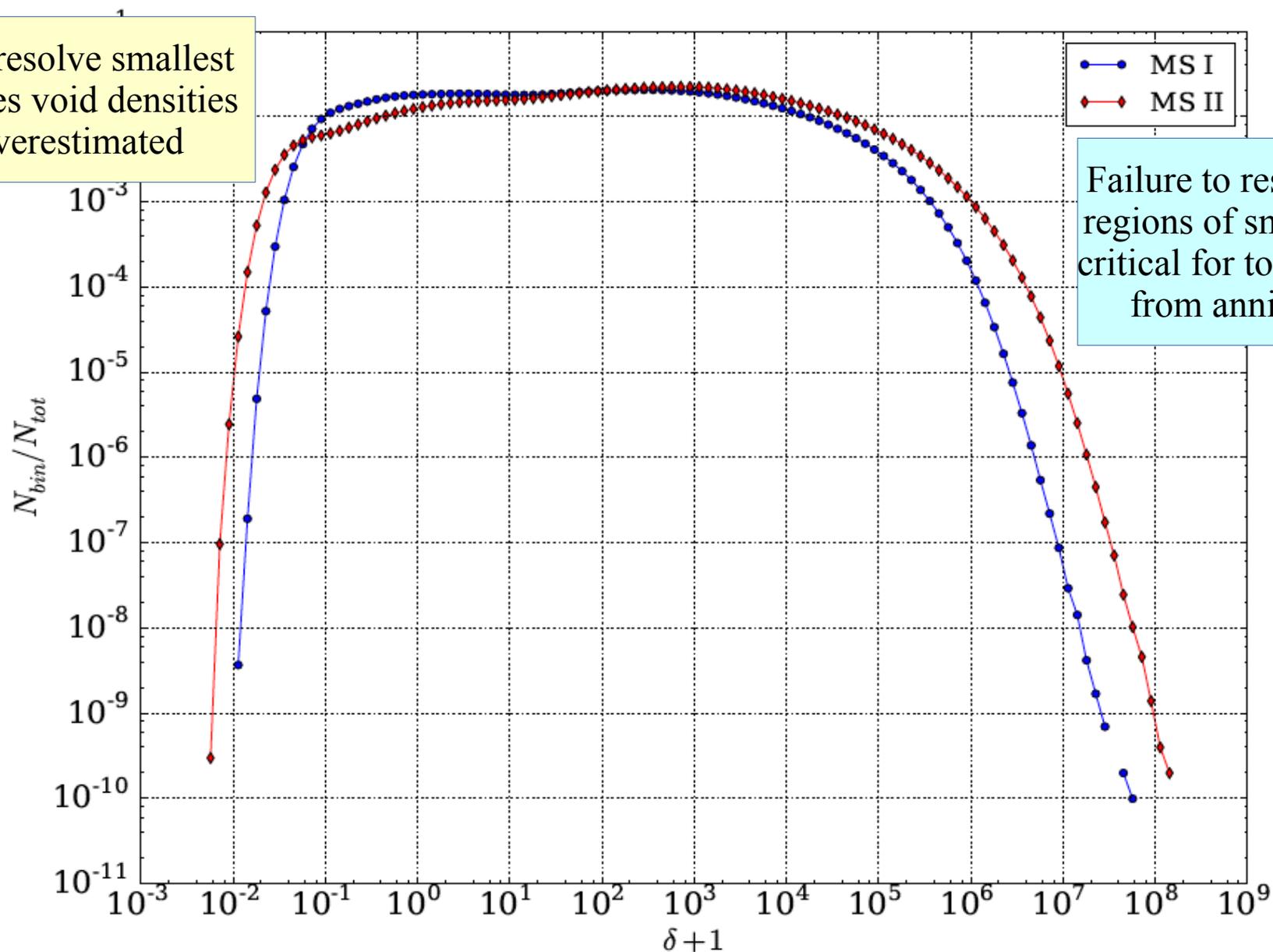


Voronoi-estimated DM densities at the particle positions in the two Millennium Simulations, estimated as: $\rho_i \propto 1 / V_{\text{Vor},i}$



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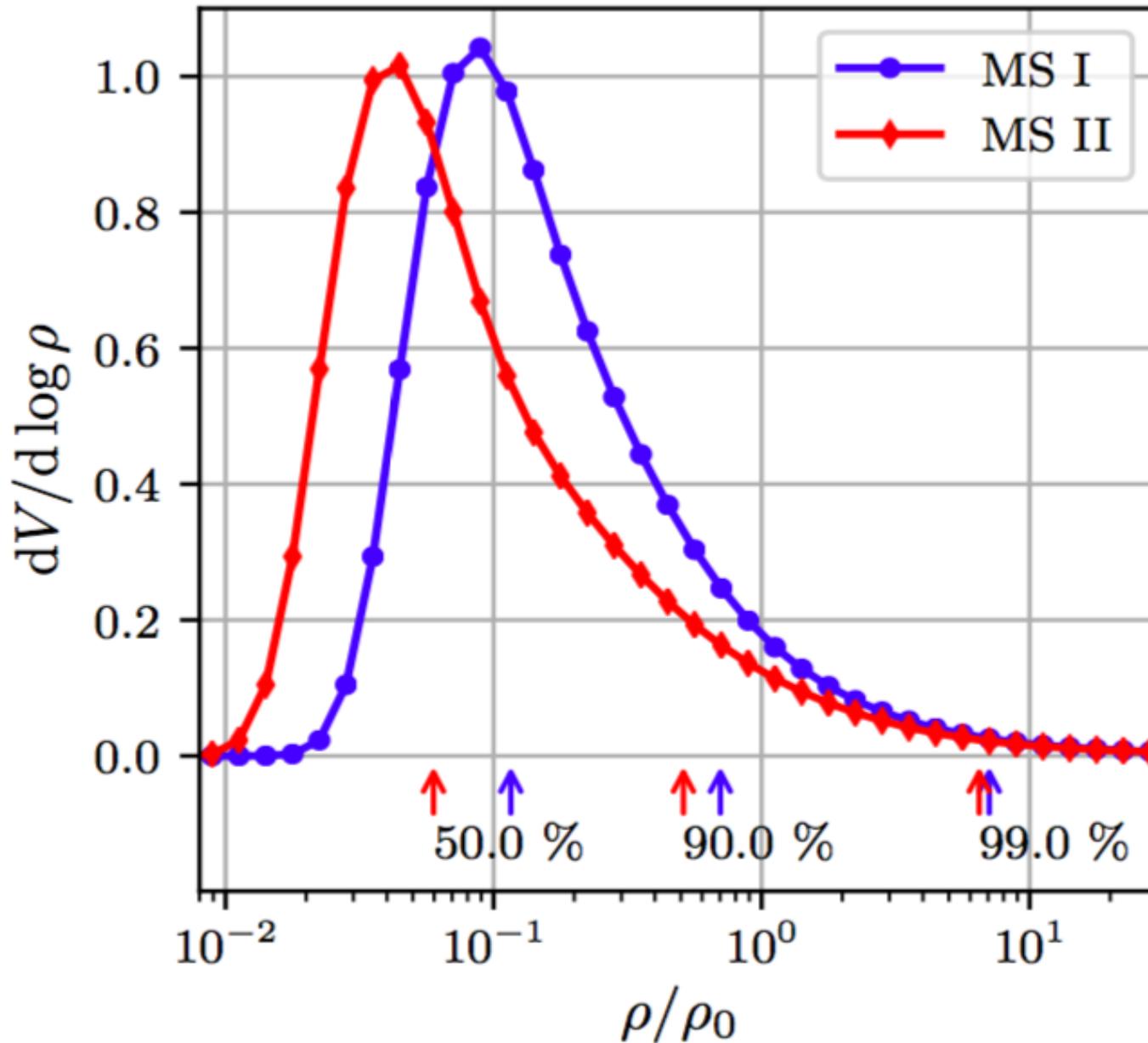
Failure to resolve smallest halos causes void densities to be overestimated



Failure to resolve inner regions of small halos is critical for total emission from annihilation

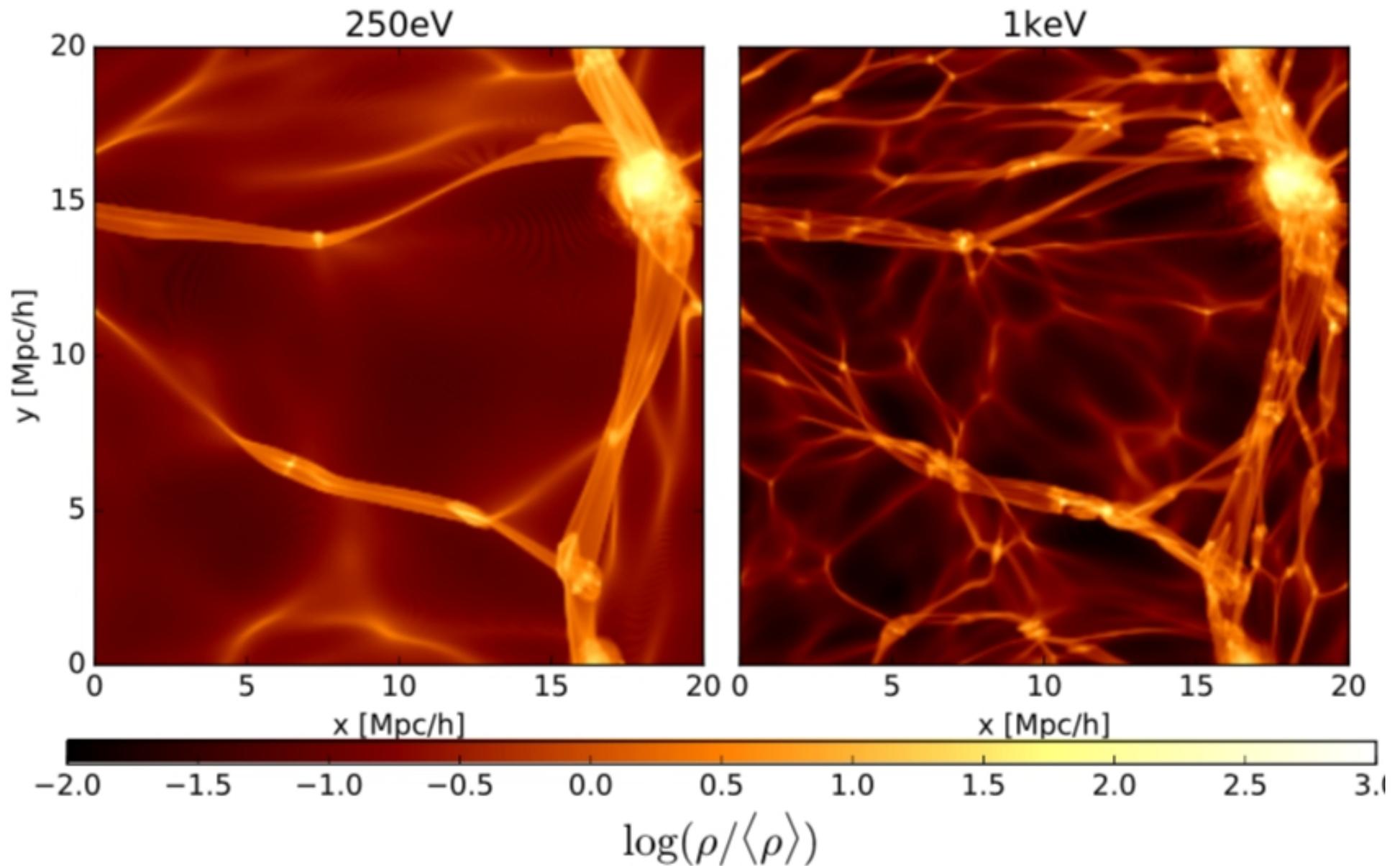
Volume-weighted density distributions in the two MS.

Stuecker et al 2018

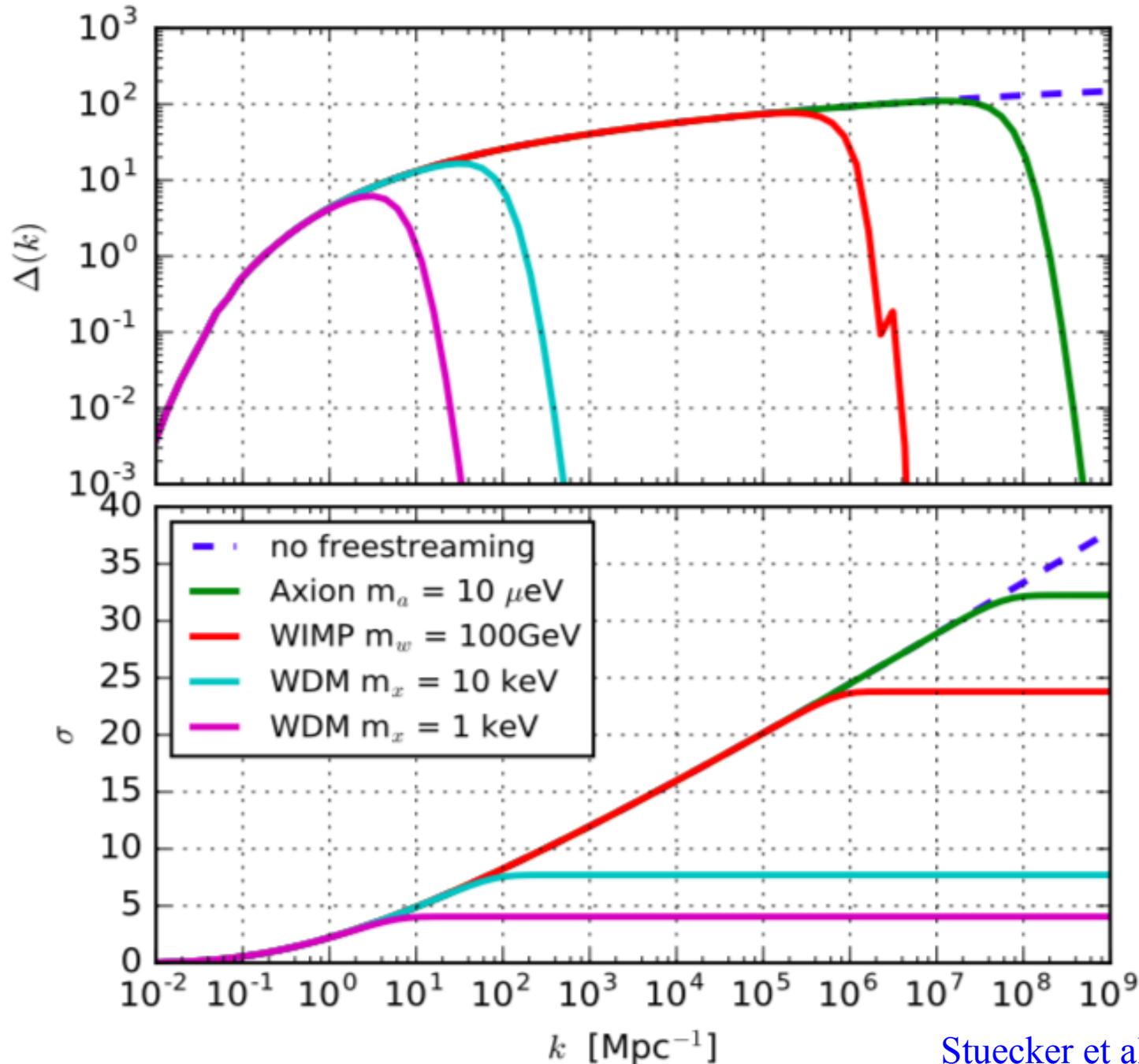


What is the median density of the Universe?

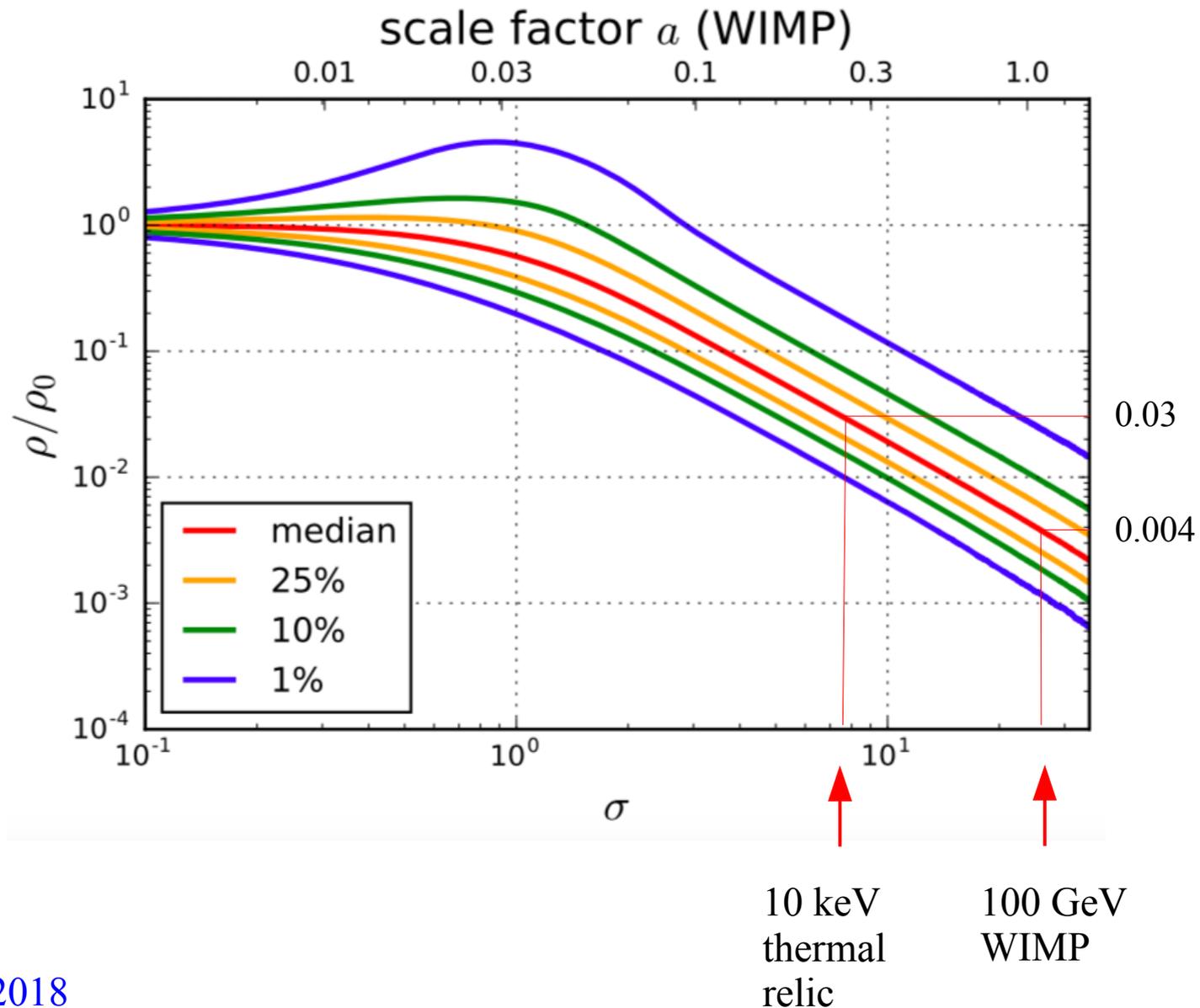
The median density is sensitive to the amount of small-scale structure: voids are emptier with more small-scale structure.



The amount of small-scale structure depends on the nature of the dark matter.



In an excursion set model, the density distribution in single stream regions depends only on σ , hence on the nature of DM



The VVV simulation

Planck cosmology

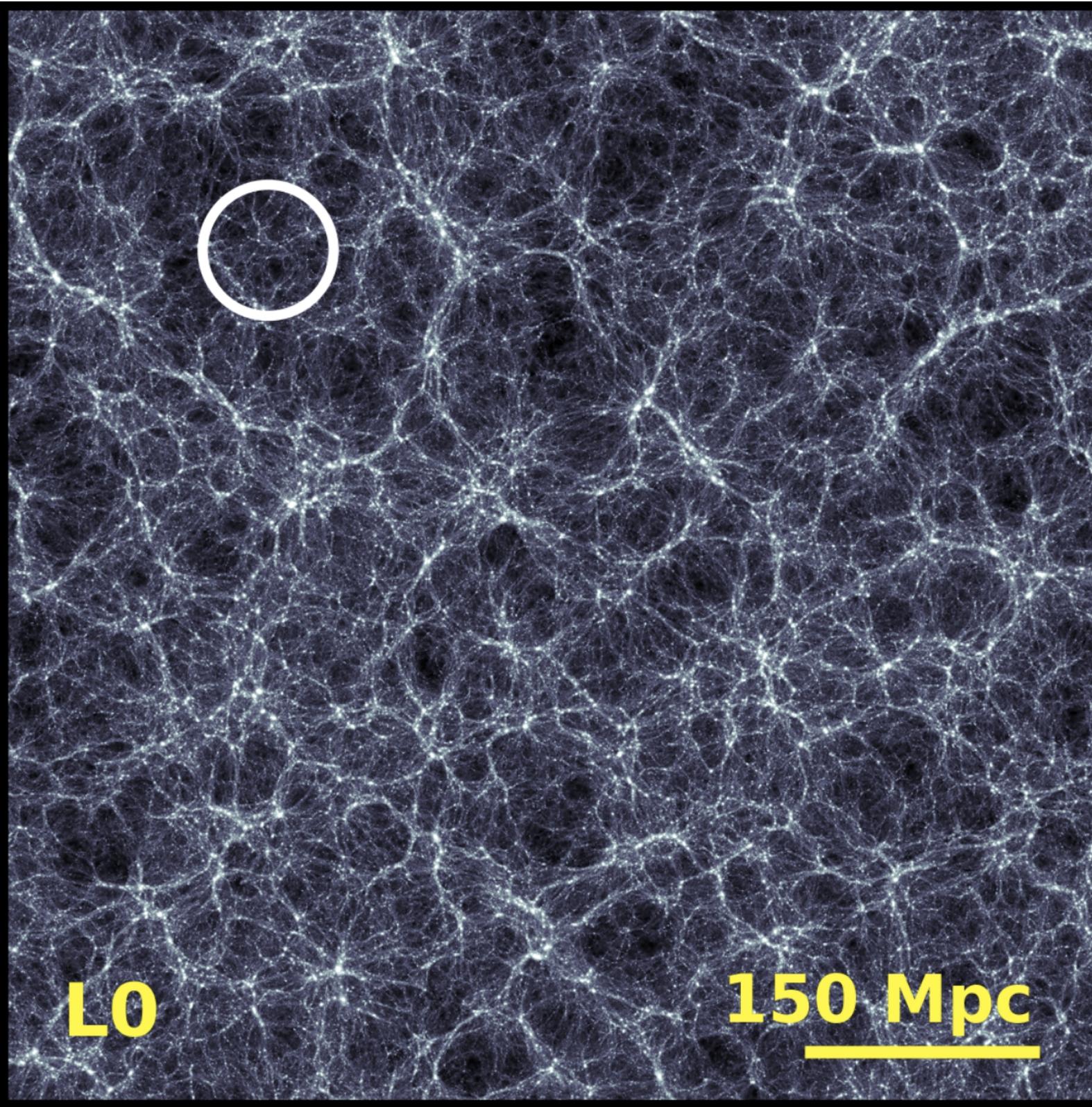
Dark matter only

Dynamic range of
30 orders of
magnitude in mass

Base Level

L0

150 Mpc



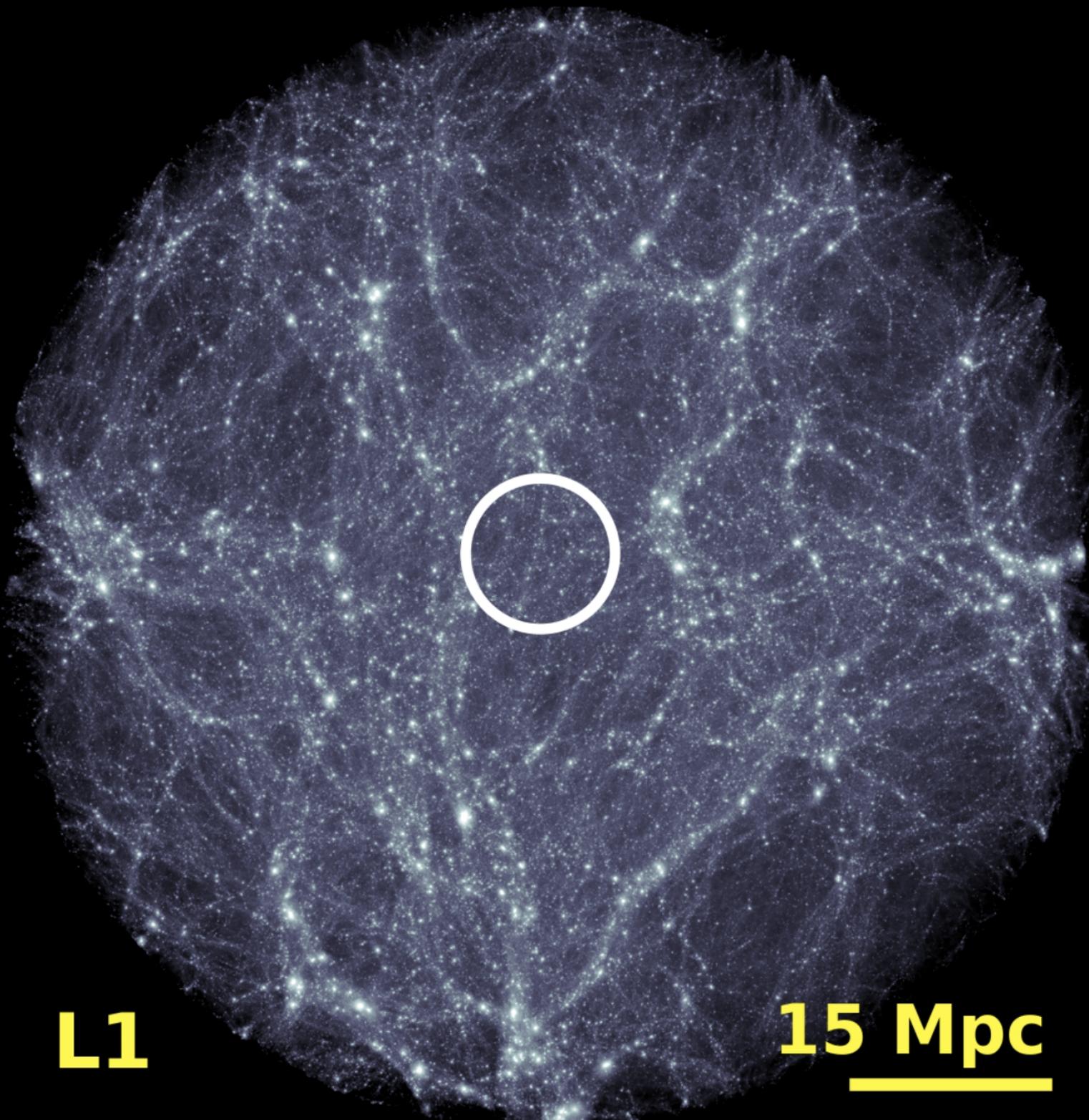
The VVV simulation

Planck cosmology

Dark matter only

Dynamic range of
30 orders of
magnitude in mass

Zoom Level 1



L1

15 Mpc

The VVV simulation

Planck cosmology

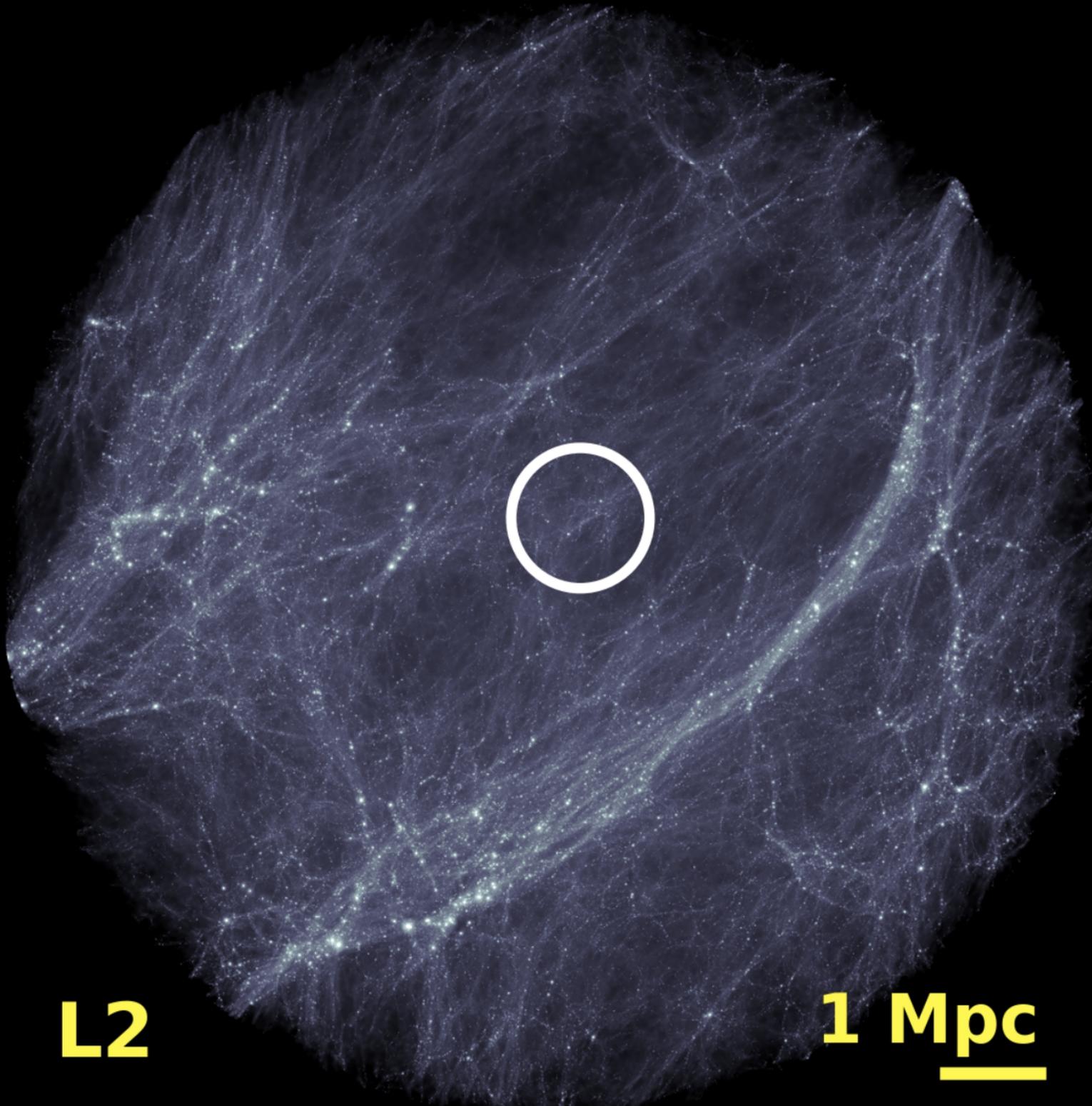
Dark matter only

Dynamic range of
30 orders of
magnitude in mass

Zoom Level 2

L2

1 Mpc



The VVV simulation

Planck cosmology

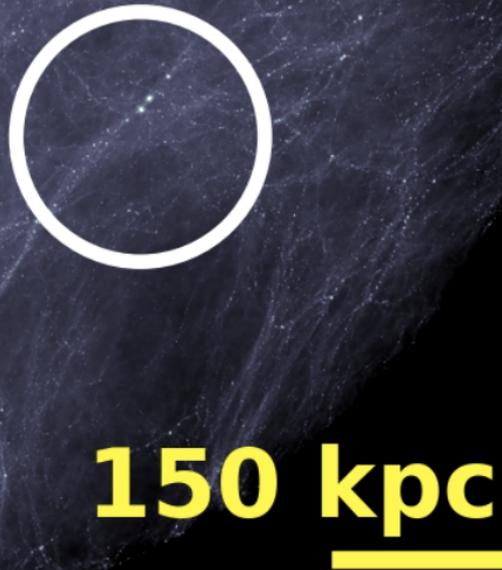
Dark matter only

Dynamic range of
30 orders of
magnitude in mass

Zoom Level 3

L3

150 kpc



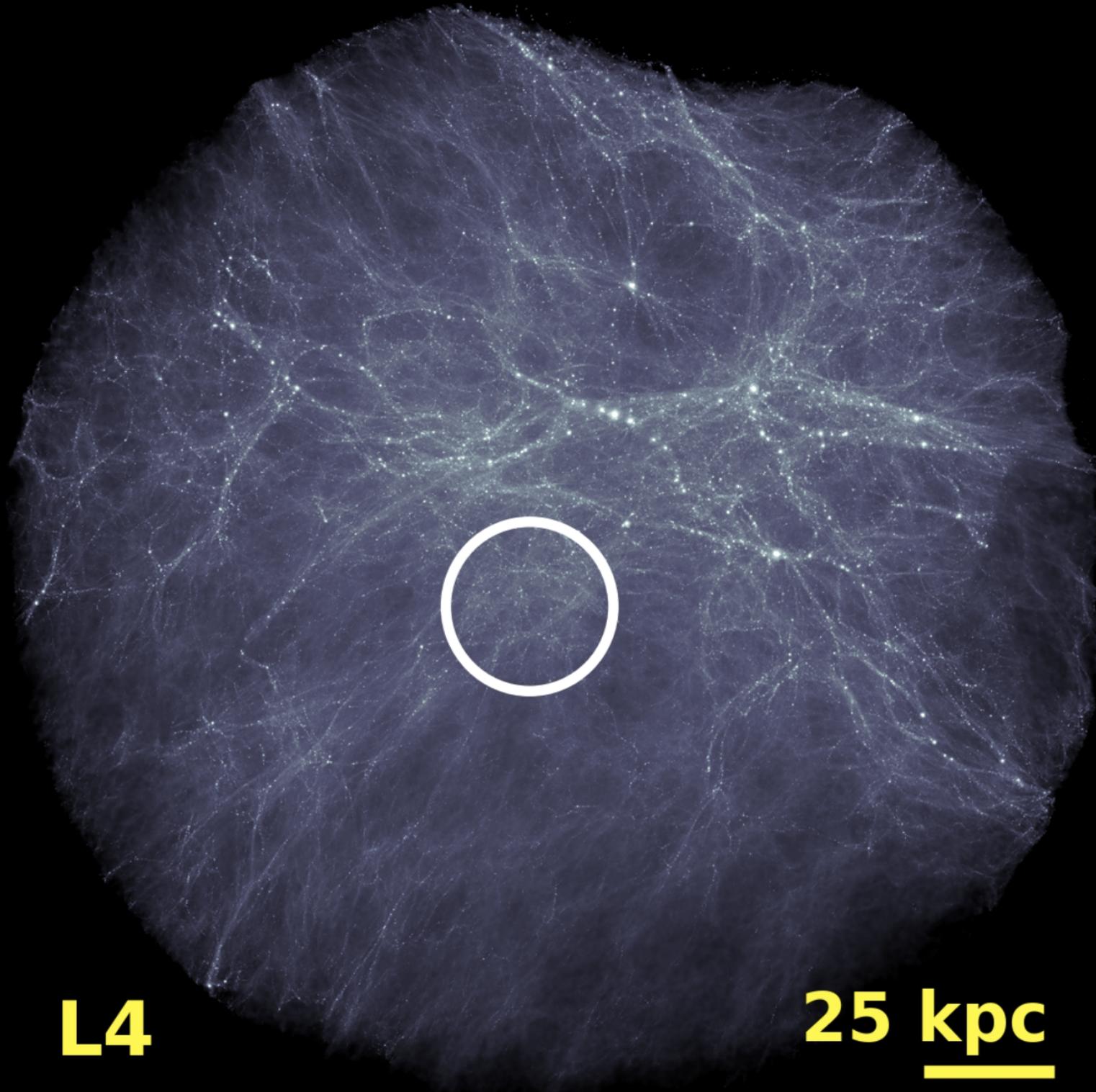
The VVV simulation

Planck cosmology

Dark matter only

Dynamic range of
30 orders of
magnitude in mass

Zoom Level 4



L4

25 kpc

The VVV simulation

Planck cosmology

Dark matter only

Dynamic range of
30 orders of
magnitude in mass

Zoom Level 5

L5

5 kpc



The VVV simulation

Planck cosmology

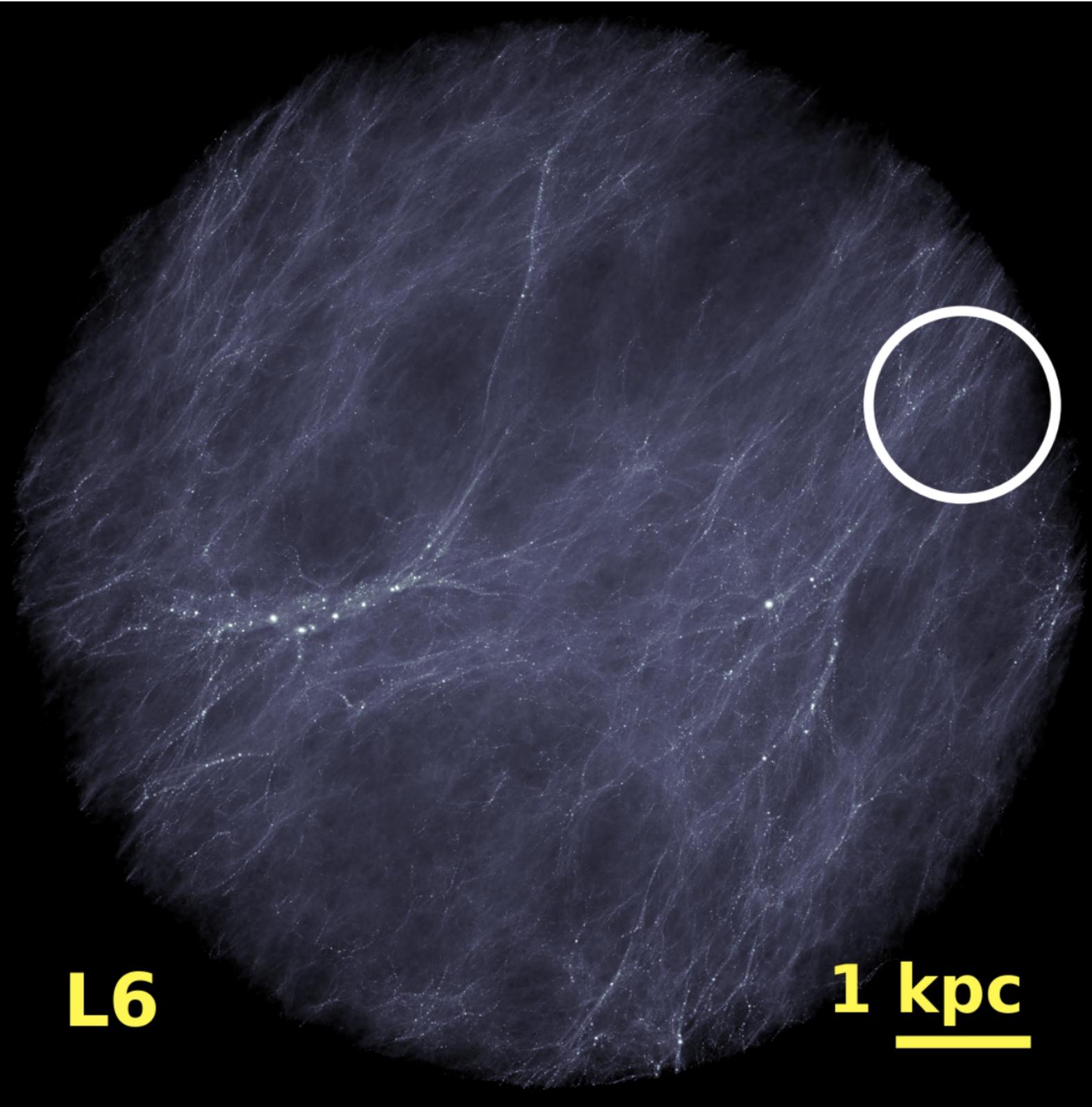
Dark matter only

Dynamic range of
30 orders of
magnitude in mass

Zoom Level 6

L6

1 kpc



The VVV simulation

Planck cosmology

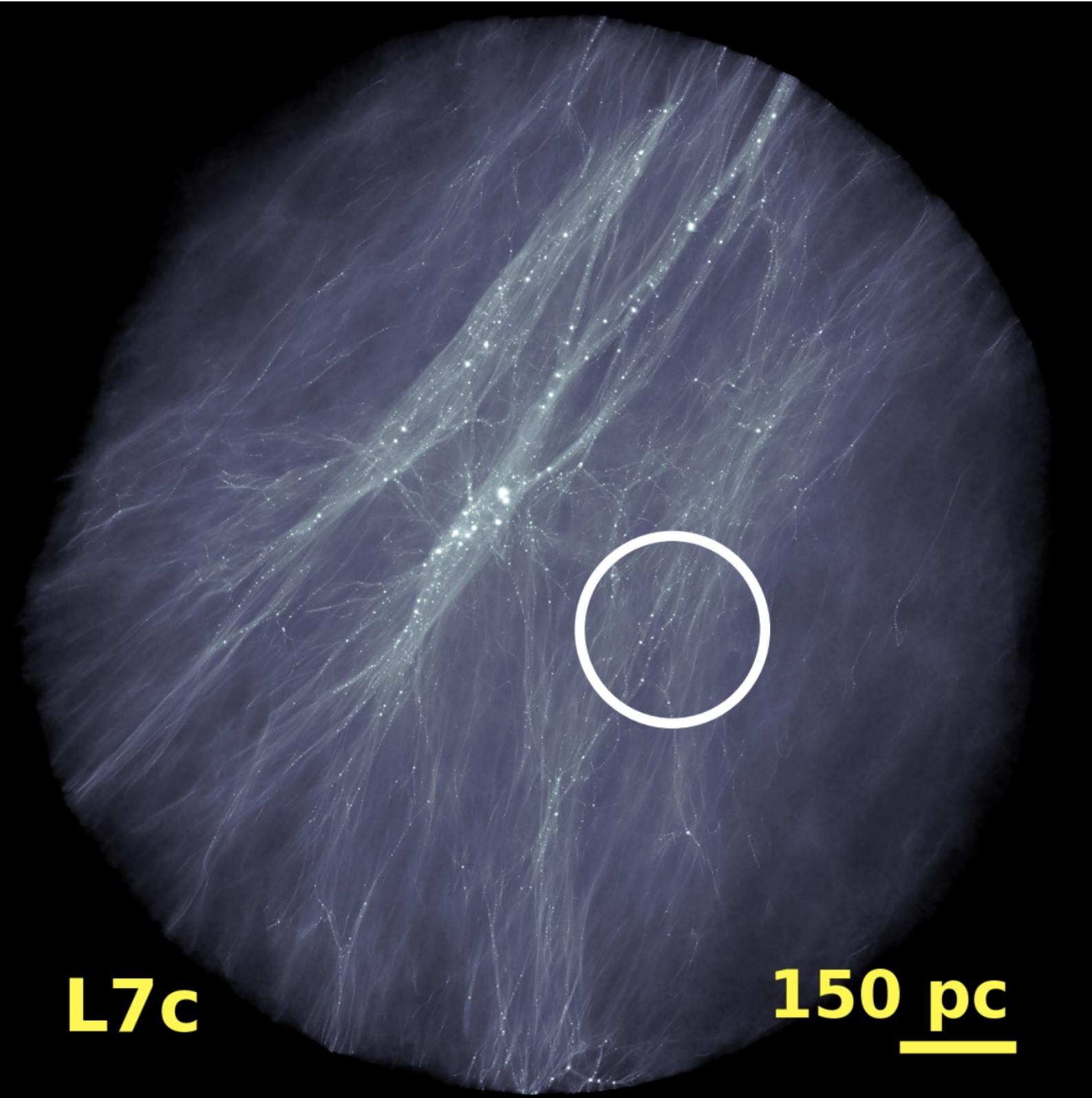
Dark matter only

Dynamic range of
30 orders of
magnitude in mass

Zoom Level 7

L7c

150 pc

The VVV simulation

Planck cosmology

Dark matter only

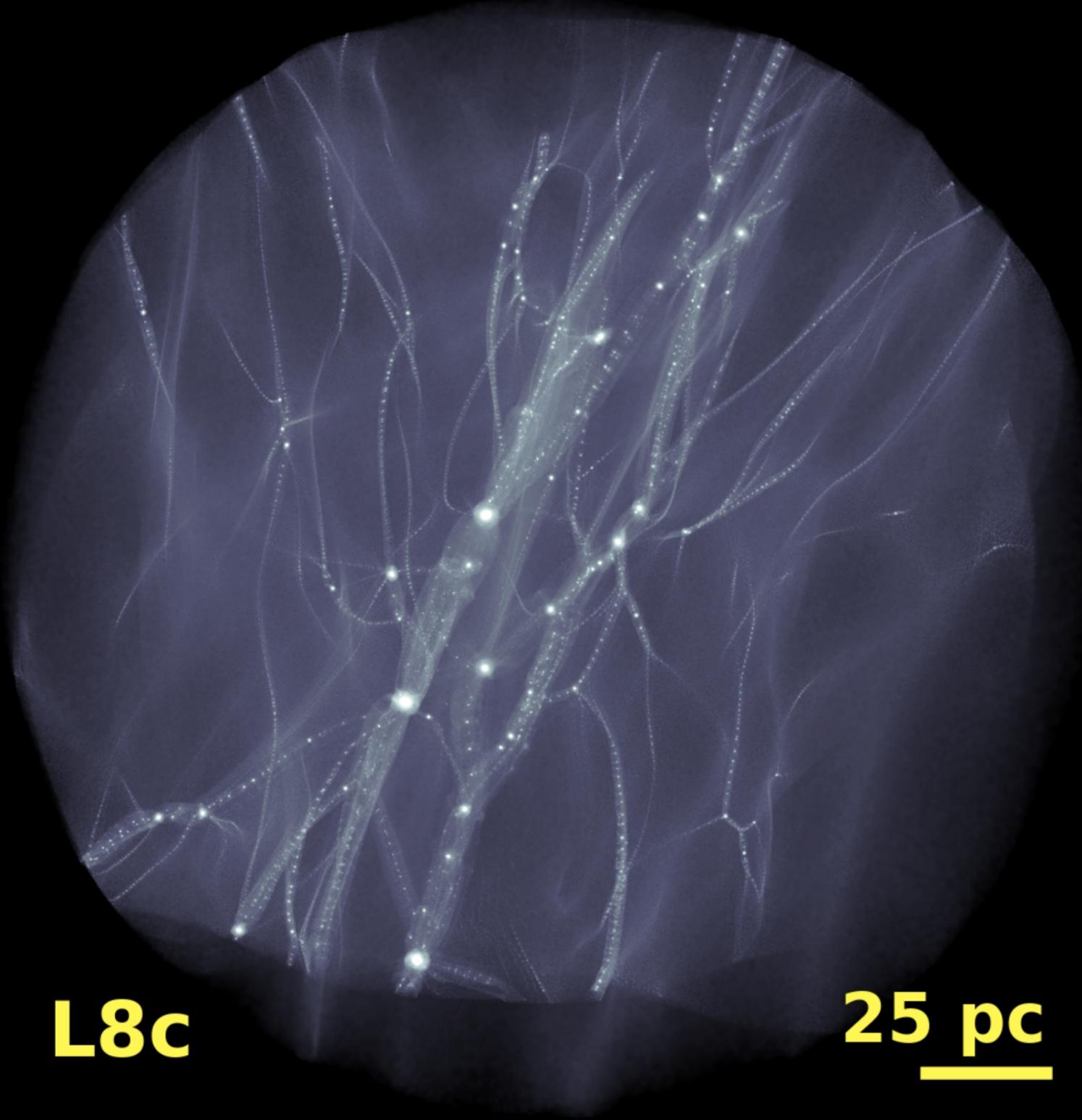
Dynamic range of
30 orders of
magnitude in mass

Zoom Level 8

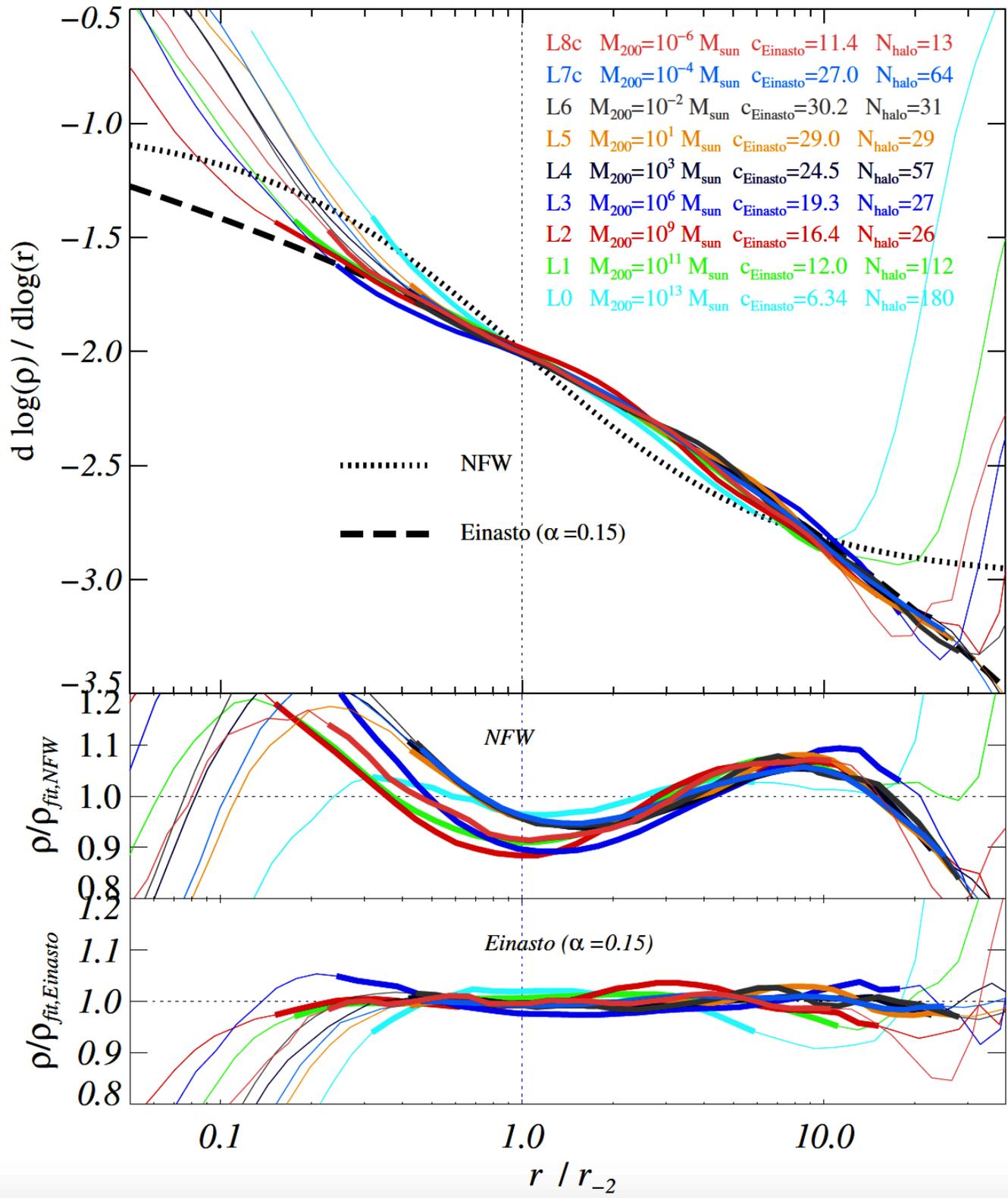
The density of
this region is
only 0.4% of the
cosmic mean

L8c

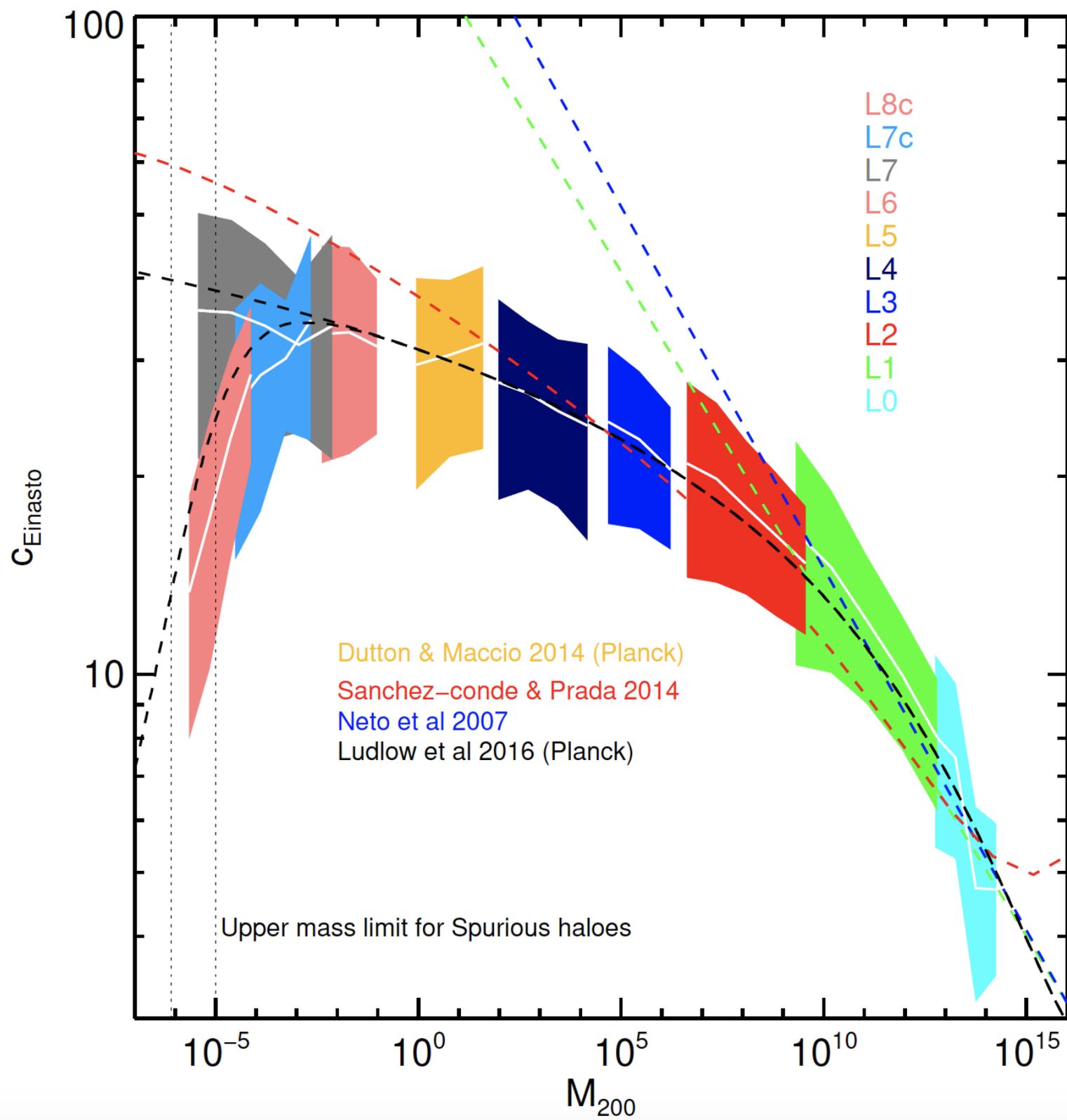
25 pc



Density profile shapes



Over 19 orders of magnitude in halo mass and 4 orders of magnitude in halo density, the mean density profiles of halos are fit by NFW to within 20% and by Einasto with $\alpha = 0.15$ to within 7%



Concentration-mass relation

Over the full 20 orders of magnitude probed, the relation of Ludlow et al (2016) is followed quite closely.

There is a turndown at 1000 Earth masses due to the free-streaming limit.

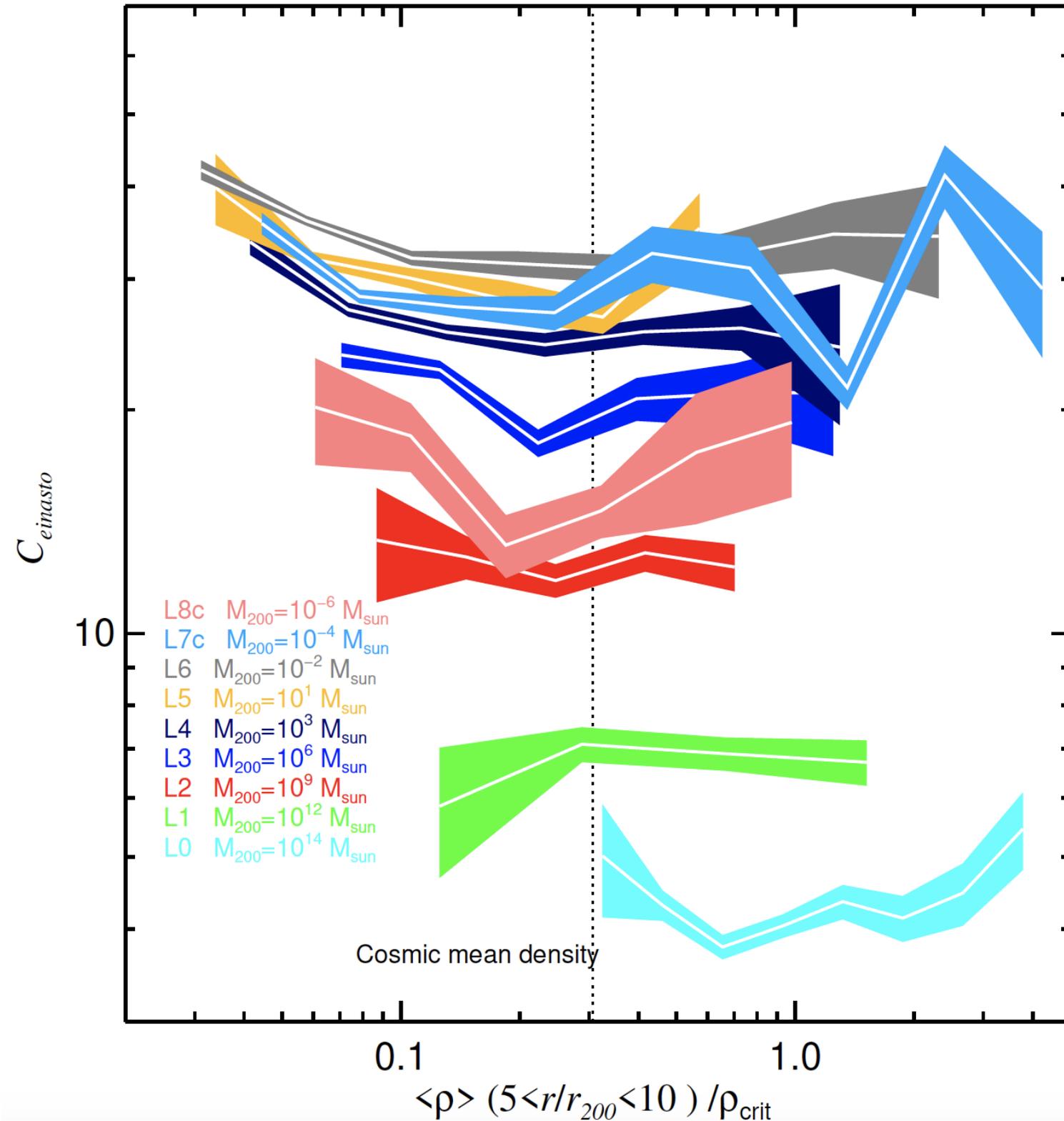
The scatter does not depend strongly on halo mass.

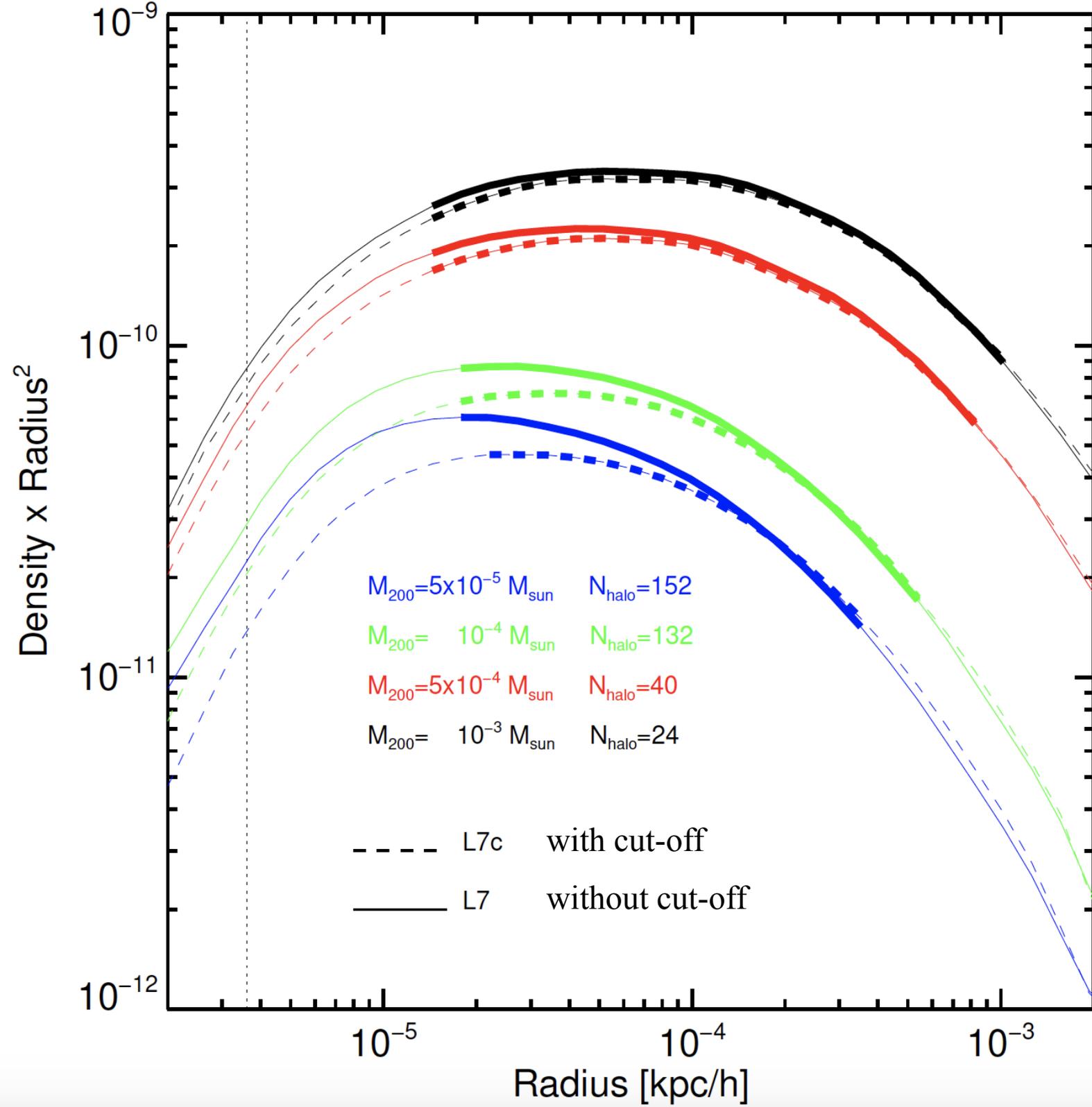
Wang, Bose et al 2019

Concentration-density relation

At given halo mass, concentration does not depend on *local* environment density.

The *range* of local environment density does not depend strongly on halo mass





Free-streaming effects on halo density profiles

The concentration of halos near the cut-off mass is reduced by free-streaming

To conclude...

- Trivariate gaussian models for the DM velocity distribution should predict the signal accurately in direct DM-detection experiments
- DM substructures/streams have negligible impact on such experiments
- Caustics have at most minor effects on halo images when these are viewed in emission due to DM annihilation
- Such images are dominated by emission from small subhalos at large radii, giving a smooth and almost flat profile in projection
- The *typical* DM density in the Universe (also that in the environment of low-mass halos) is much less than the mean, $\sim 0.004 \bar{\rho}$ for a WIMP
- Halos of all masses have NFW-like profiles at $z = 0$ with a mass-concentration relation much shallower than most of those published