

Beijing, July 2010

# The KIAA Lectures

**Formation of the Milky Way and other  
galaxies in the  $\Lambda$ CDM cosmogony**

*Simon White*

*Max Planck Institute for Astrophysics*

# Lecture Plan

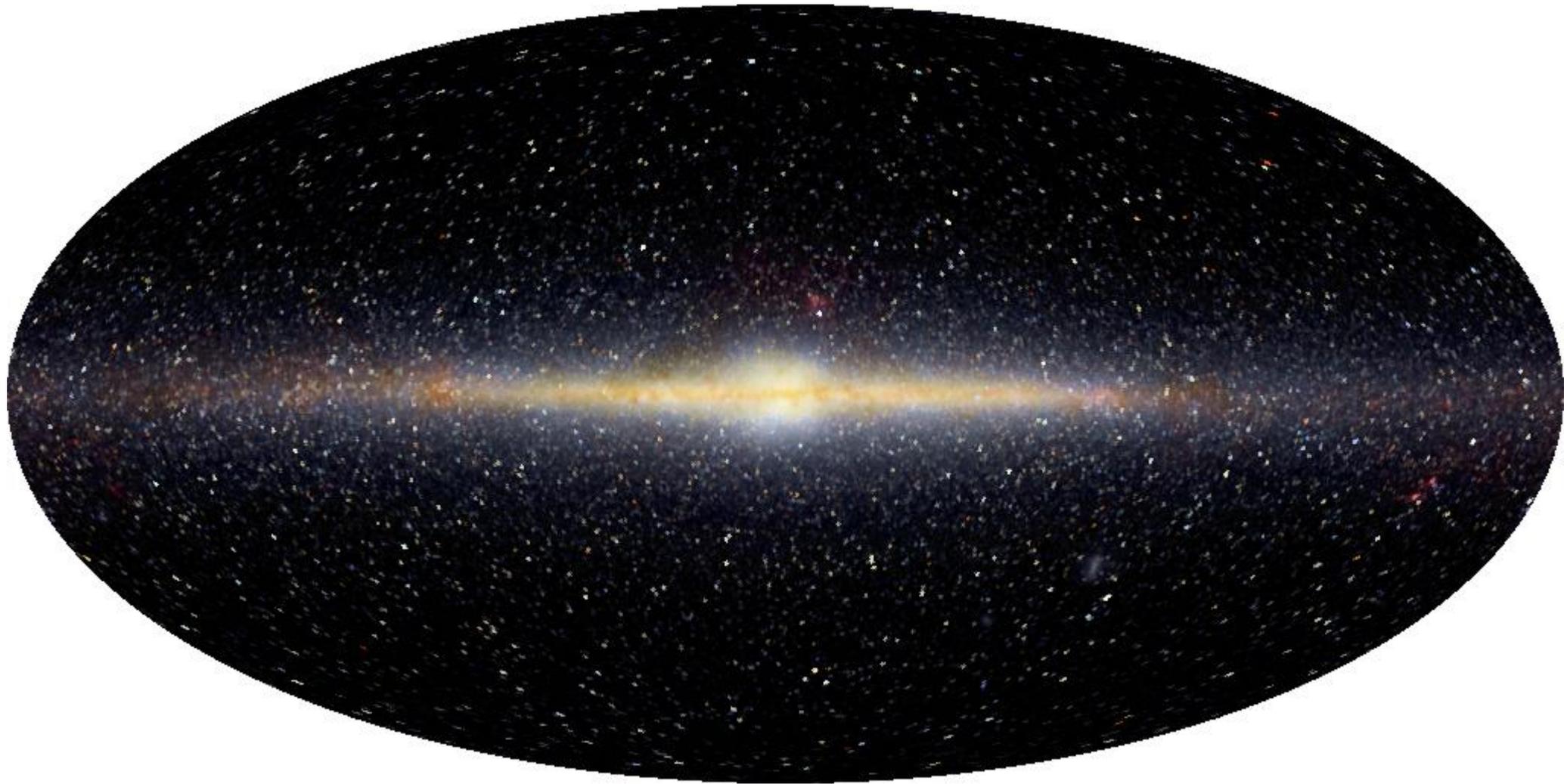
- I** Introduction to the standard model of cosmogony  
--- ingredients, observational support, possible problems
- II** Halos – structure growth, PS and EPS theory, applications
- III** Halos – assembly histories, profiles, core problems
- IV** Halos – shapes, substructure, satellites, DM detection
- V** Abundance matching, MW halo mass + formation efficiency
- VI** Gas cooling and condensation (hot and cold), disk formation
- VII** Simulations – techniques, what can and can't be done

# The pillars of the standard cosmogony

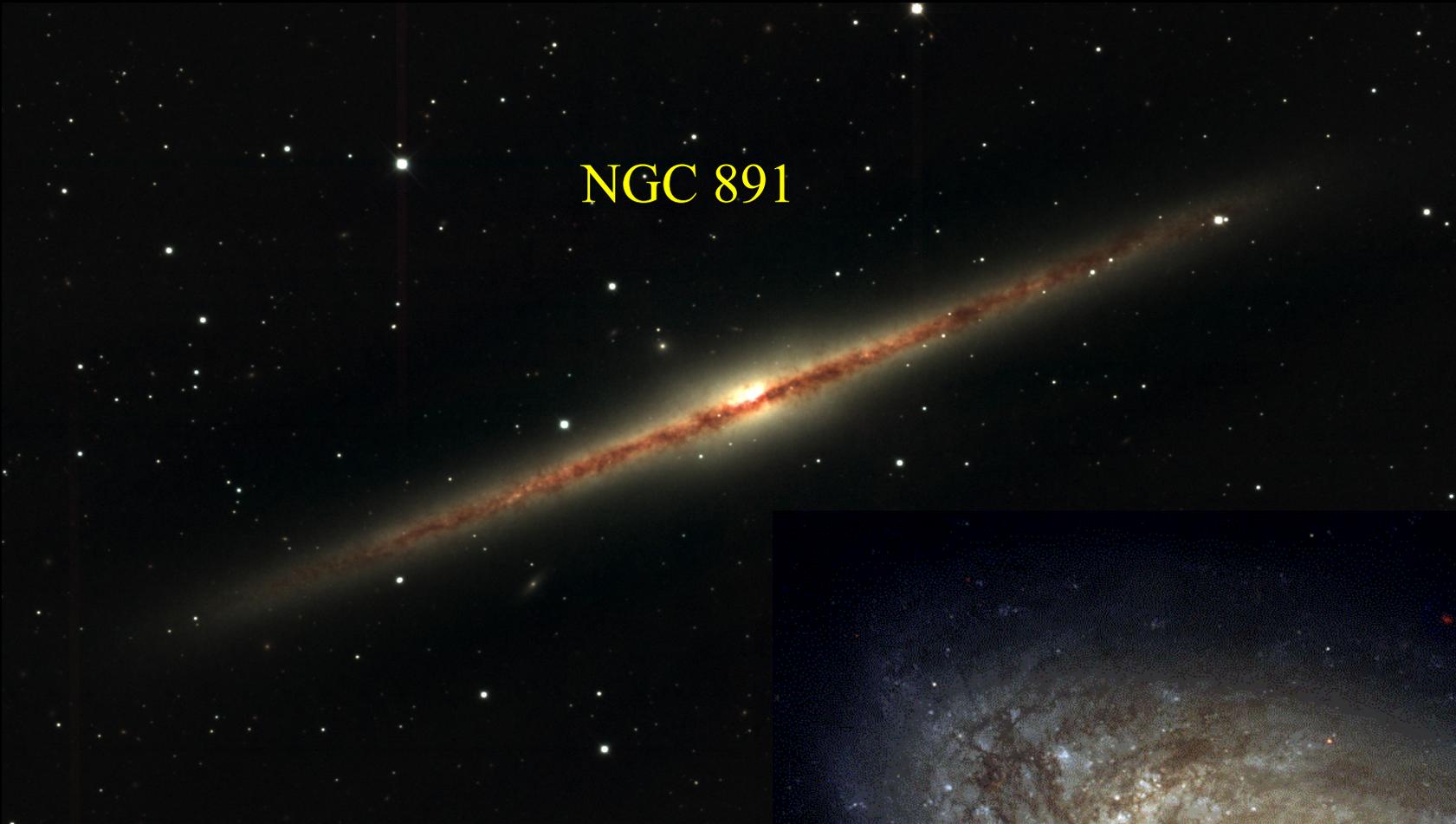
- I** The expansion of the Universe
  - non-static, indeed, accelerating! —→ Dark Energy?
- II** The uniform abundance of light elements
  - a Hot Big Bang with low baryon density
- III** The Cosmic Microwave Background (CMB)
  - uniformity —→ large-scale isotropy/homogeneity
  - Planckian spectrum —→ a hot, dense early Universe
  - anisotropies —→ initial conditions for structure formation
- IV** Large-scale structure – low-redshift quasilinear structure
- V** Gravitational lensing – properties of present-day DM halos
- VI** Ly  $\alpha$  forest – small-scale structure at  $z=3$



# COBE's near-infrared map of the whole sky



NGC 891



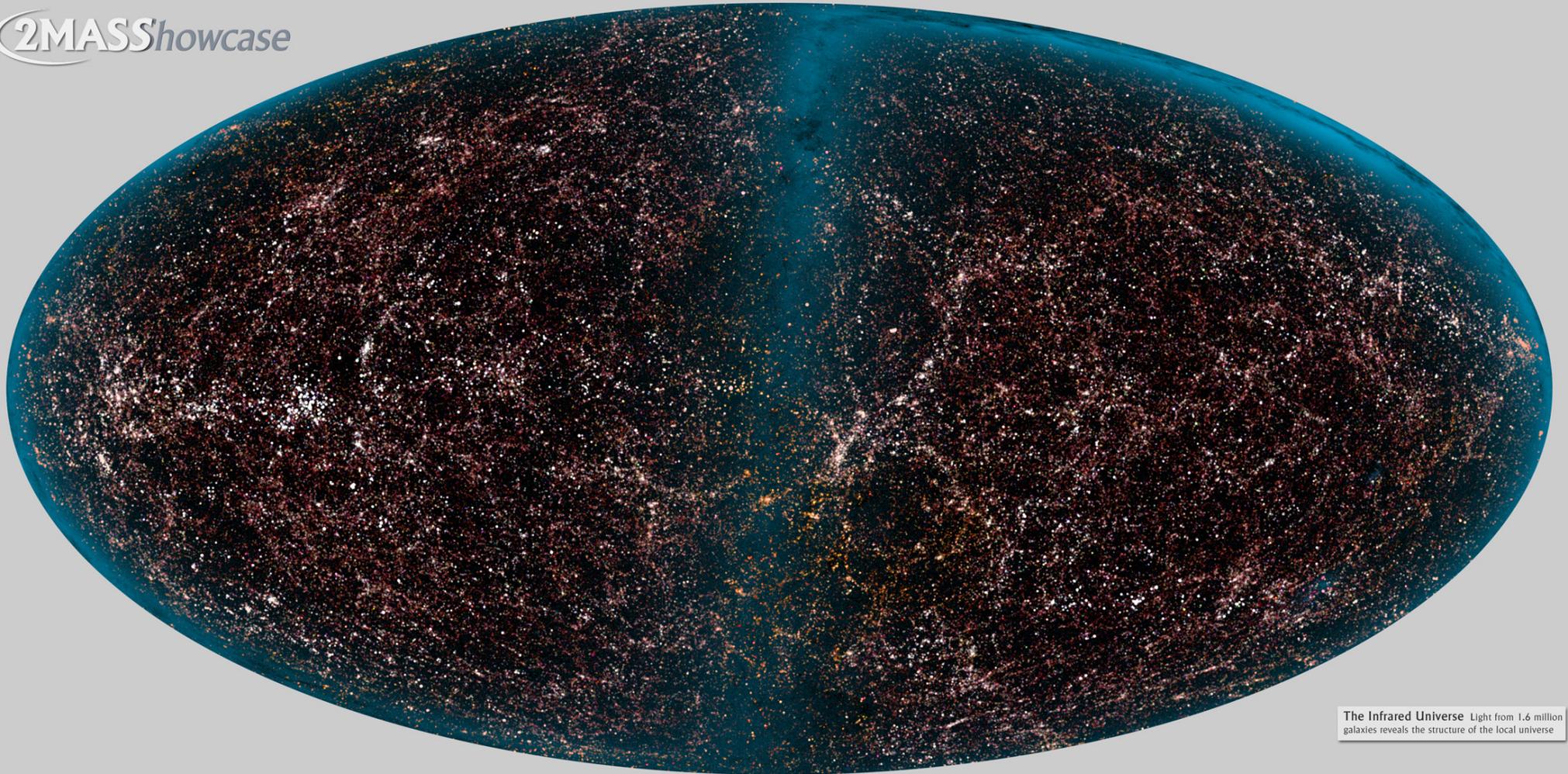
**Spiral galaxies  
like our own**



NGC 4414

# Galaxy map of the whole sky

2MASS Showcase

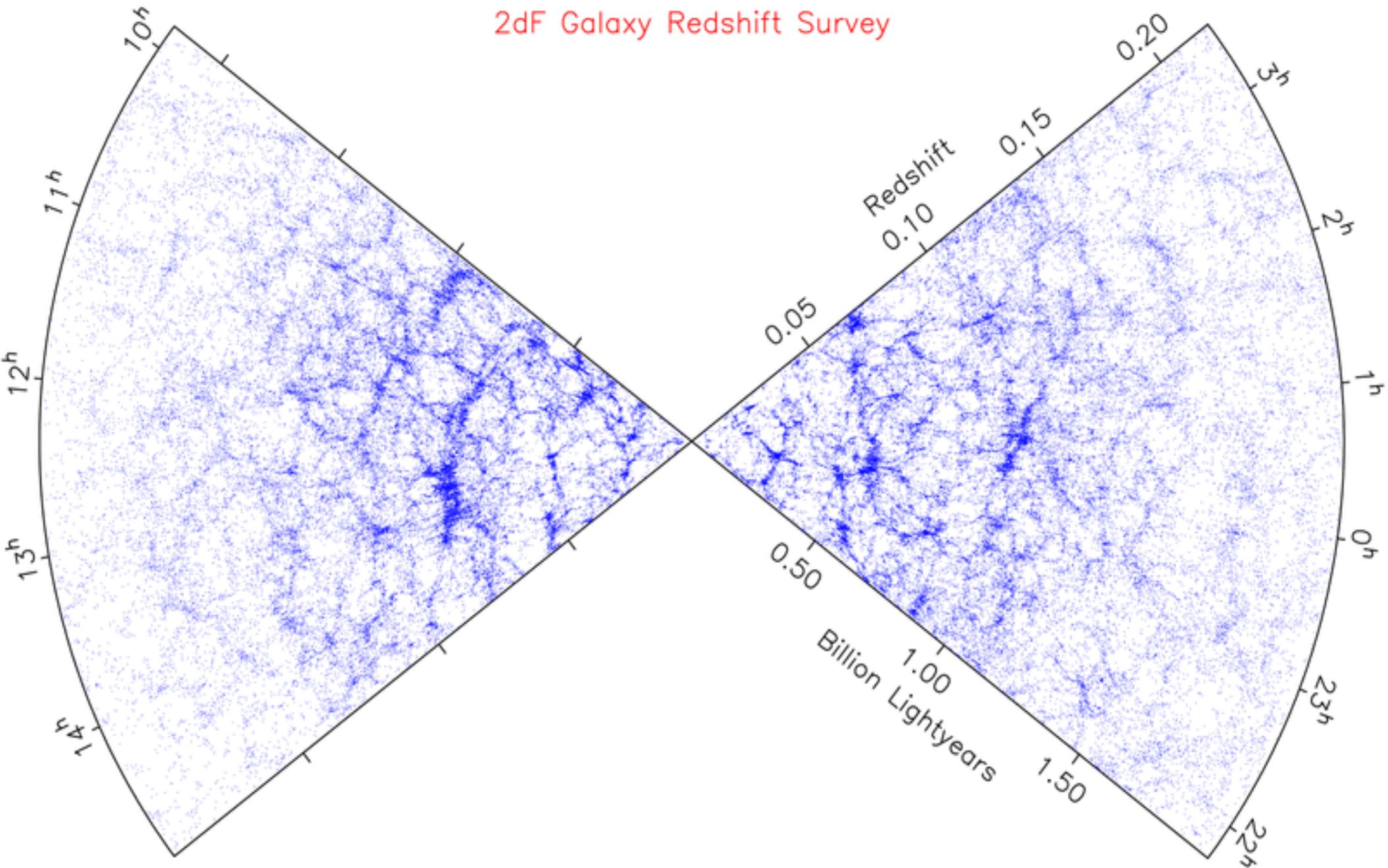


**The Infrared Universe** Light from 1.6 million galaxies reveals the structure of the local universe

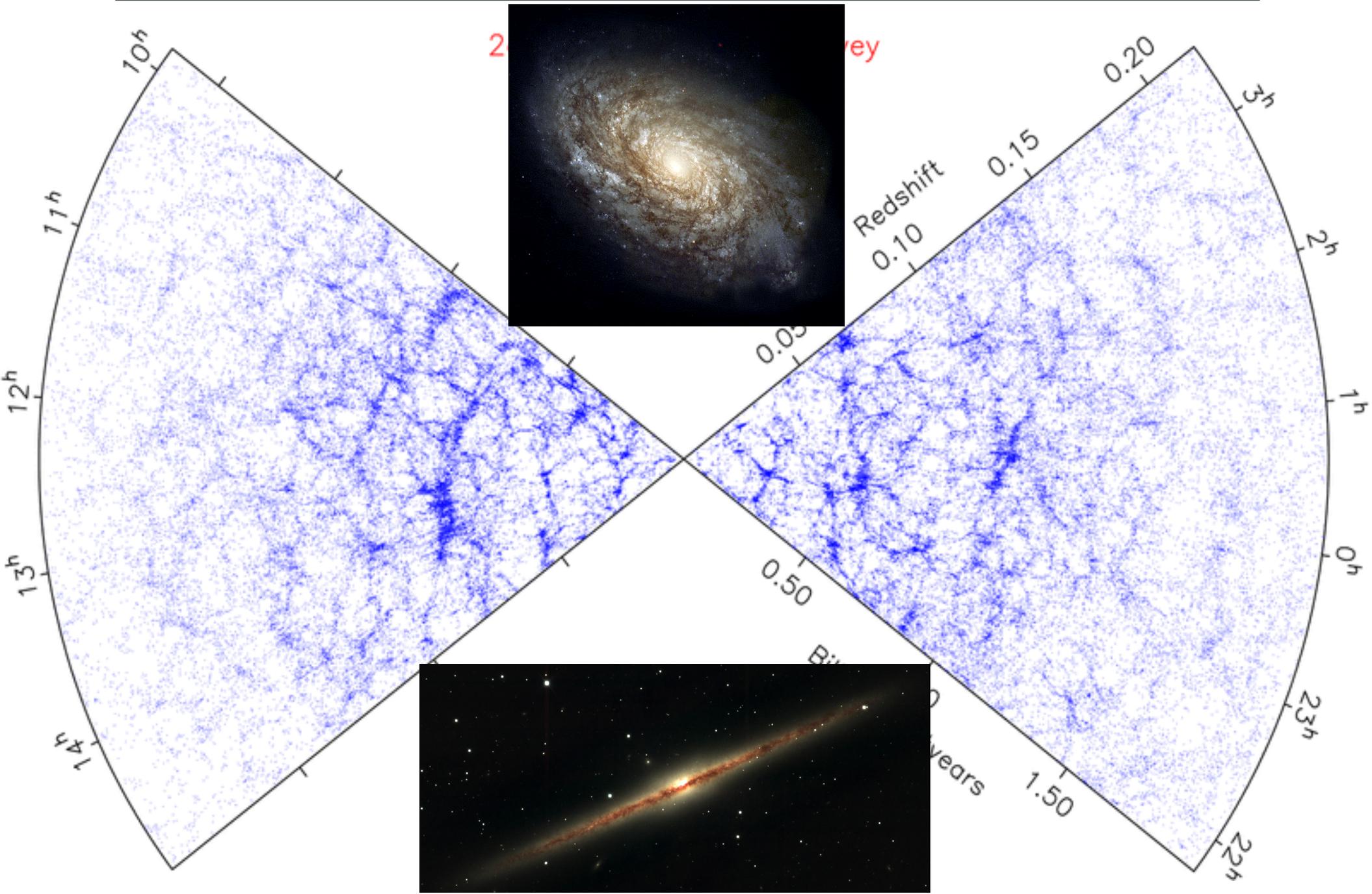
Two Micron All Sky Survey Image Mosaic: Infrared Processing and Analysis Center/Caltech & University of Massachusetts

# Large-scale structure in today's Universe

2dF Galaxy Redshift Survey



# Large-scale structure in today's Universe



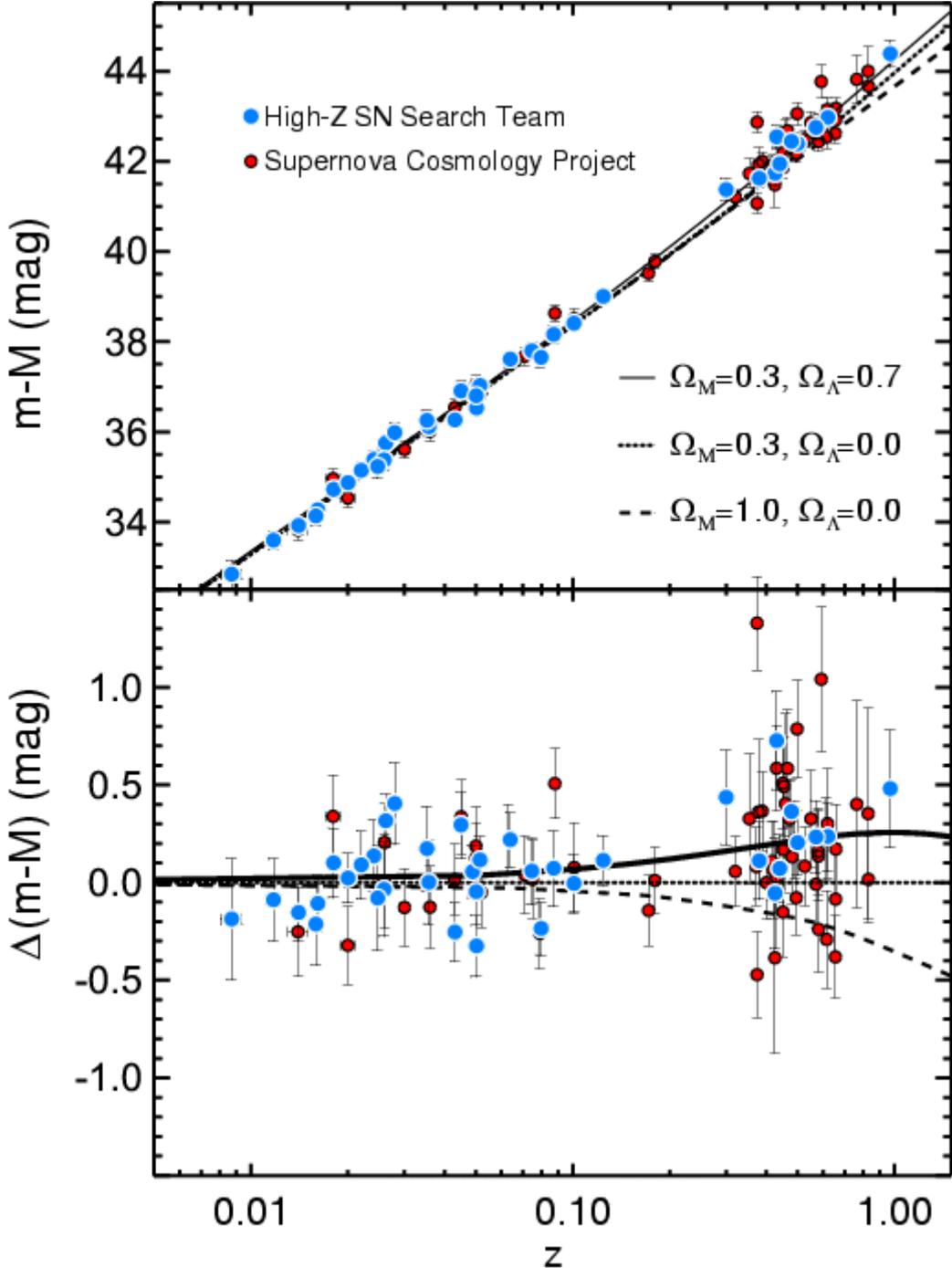


The Hubble  
ultradeep field

A 300 hour  
exposure with  
the HST

The assembly  
history of  
galaxies like  
our own can be  
observed in  
such images

# An accelerating Universe



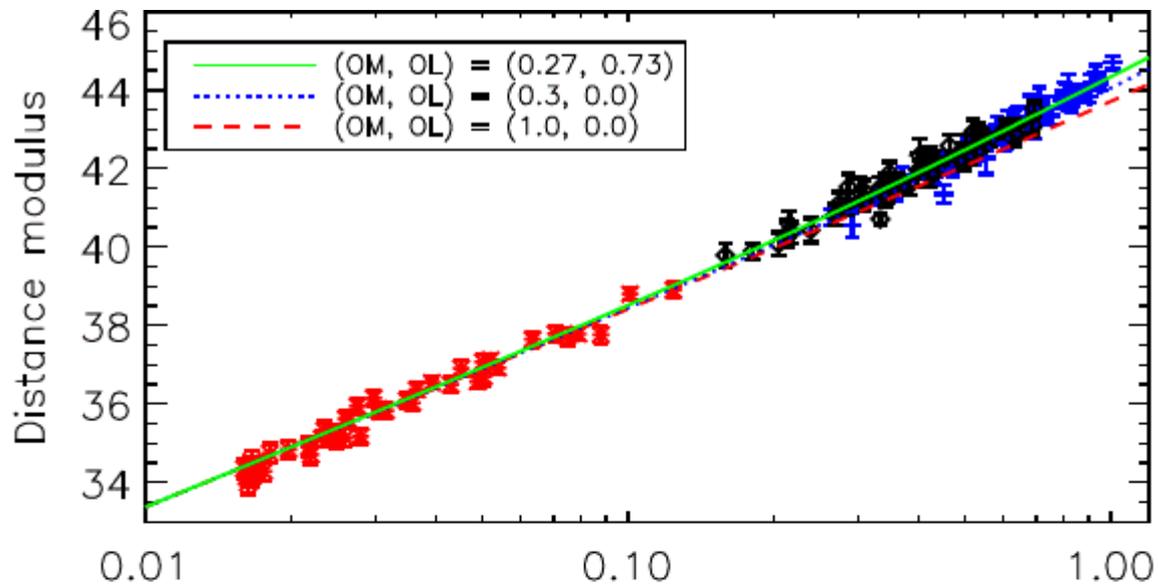
State of the observations 2003

- Distant supernovae appear less bright than expected
- Today the cosmic expansion is accelerating *not* slowing down
- The dominant contribution to the cosmic mass/energy budget must have *negative* pressure

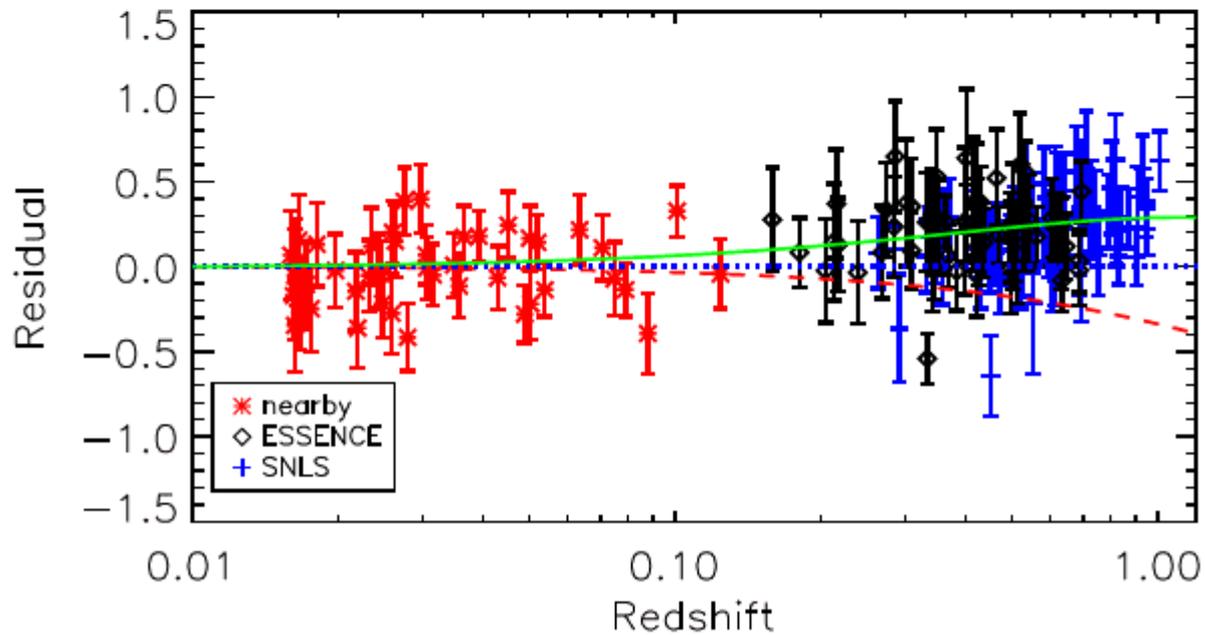


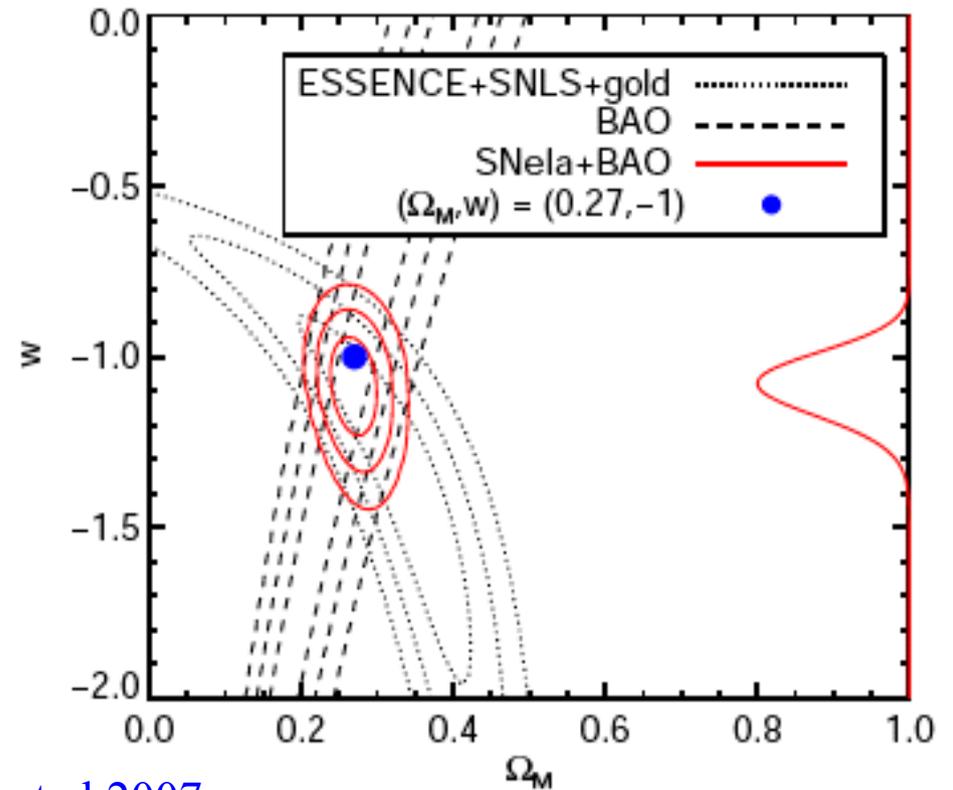
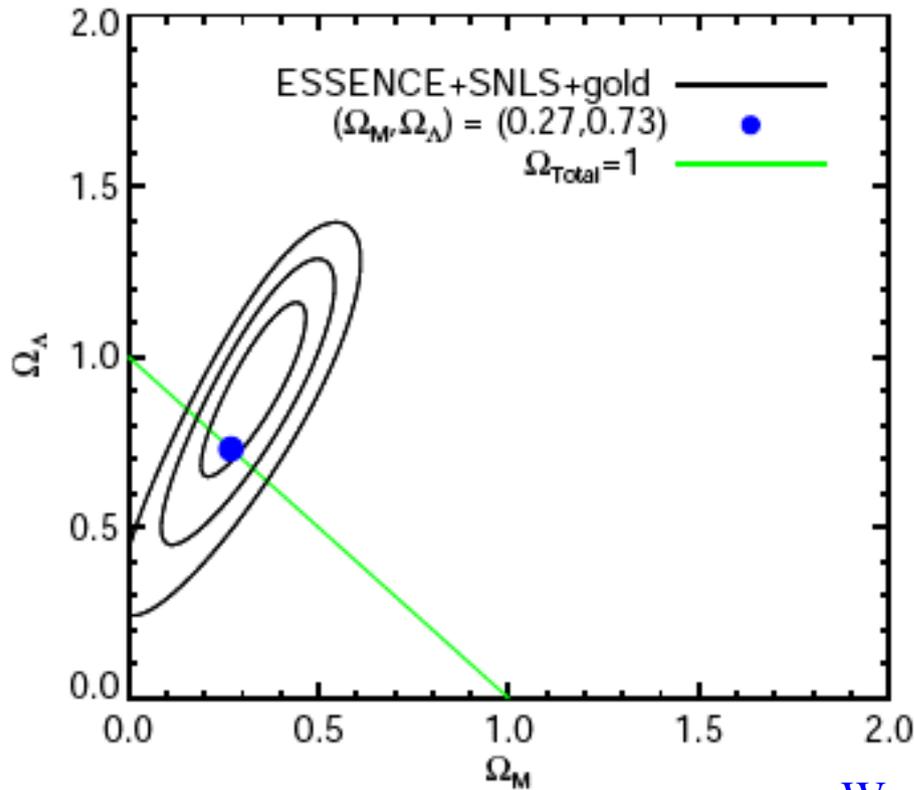
$$\ddot{R} / R = -4\pi/3 G (\rho + 3 p)$$

Dark Energy, Quintessence, a Cosmological Constant?



The ESSENCE Survey  
Wood-Vasey et al 2007





Wood-Vasey et al 2007

- The SN data require an accelerated expansion today
- With large-scale structure data, they imply a flat Universe with DE
- The DE appears to behave “like” a cosmological constant,  $w \approx -1$
- The implied parameters agree with those obtained independently from the cosmic microwave background

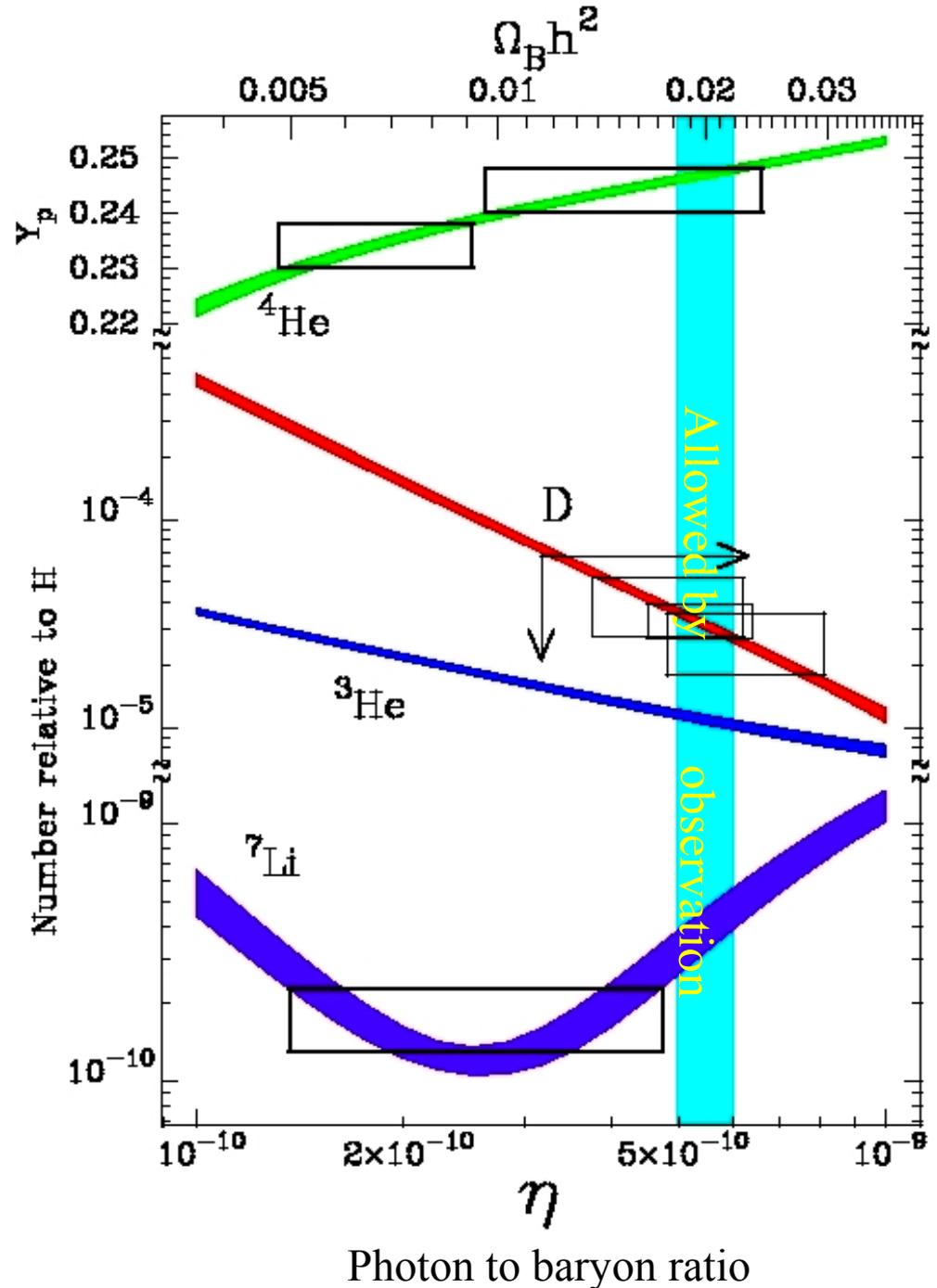
# “Explanations” for Dark Energy

- A cosmological constant (i.e. another constant of gravity)
- Dynamical Dark Energy, e.g. quintessence
- A result of “leakage” from higher dimensions
- A reflection of the need to extend/modify General Relativity
- A consequence of the nonlinear behaviour of GR
- The result of systematics in the SN data

# Element formation in the Early Universe

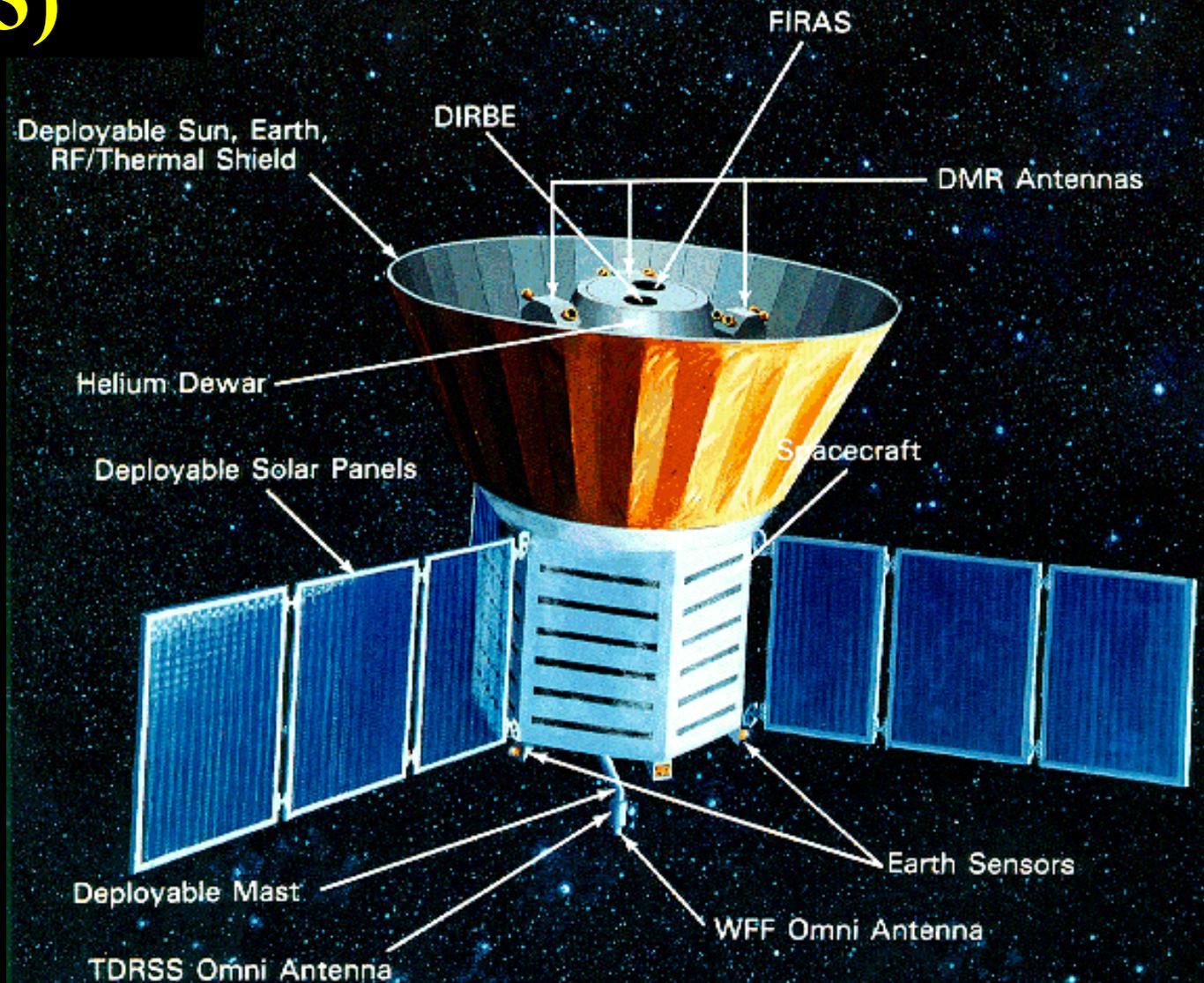
- During the very first 3 minutes the Universe "cools" to  $10^9$  C
- The nuclei of a few light elements are formed
  - Hydrogen ( $^1\text{H}$ ,  $^2\text{H}$ )
  - Helium ( $^3\text{He}$ ,  $^4\text{He}$ )
  - Lithium ( $^7\text{Li}$ )
- All the other elements formed later through nuclear reactions in stars and stellar explosions

The atoms in our bodies were made in stars



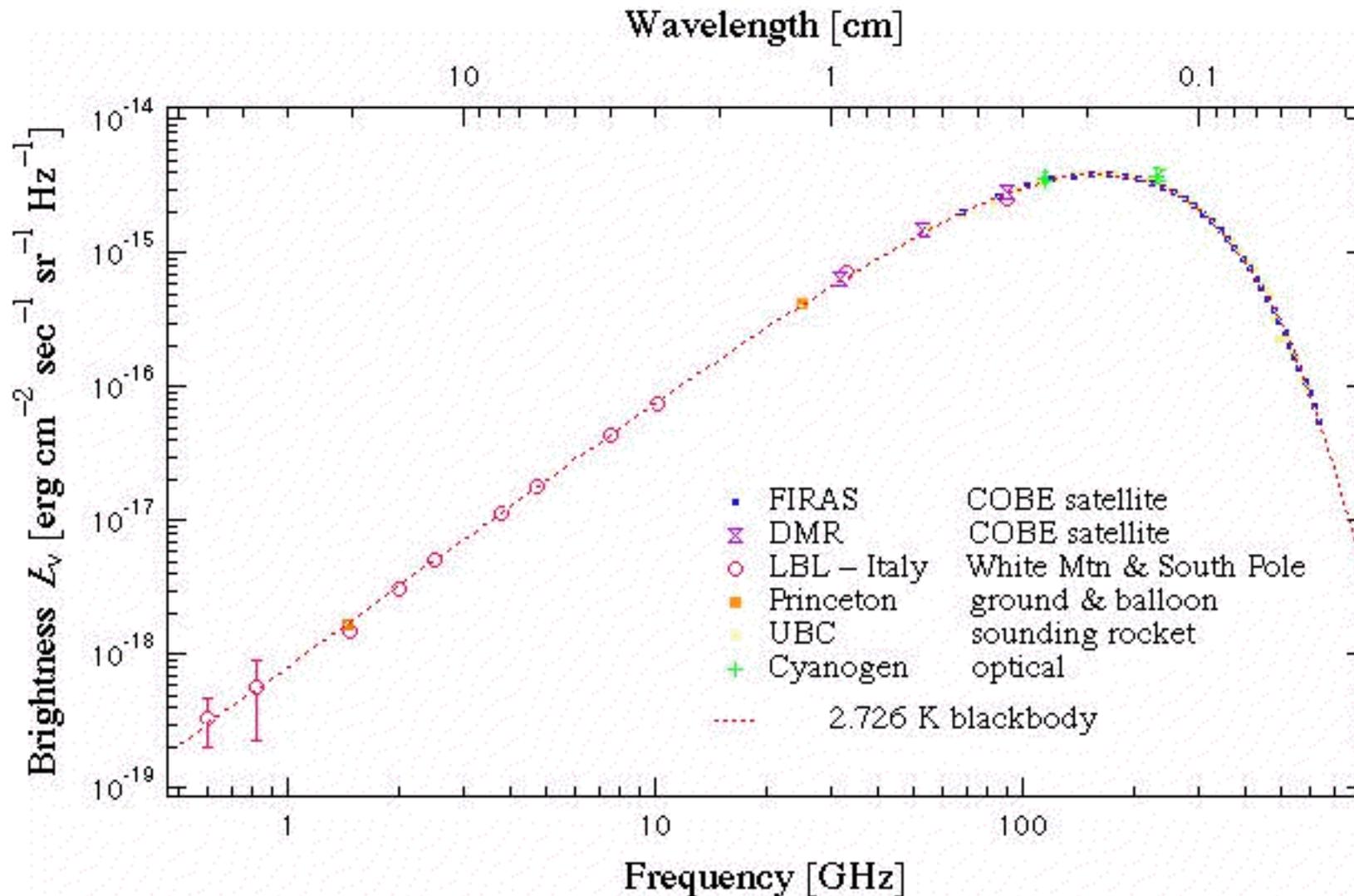
# The COBE satellite (1989 - 1993)

- Two instruments made maps of the whole sky in microwaves and in infrared radiation
- One instrument took a precise spectrum of the sky in microwaves



2006 Physics Nobel Prize

# Spectrum of the microwave background



- Spectrum matches a Planckian black-body to better than 1 in  $10^{-4}$
- The early universe was hot, smooth and in thermal equilibrium
- No significant energy input later than  $\sim 1$  month after the Big Bang

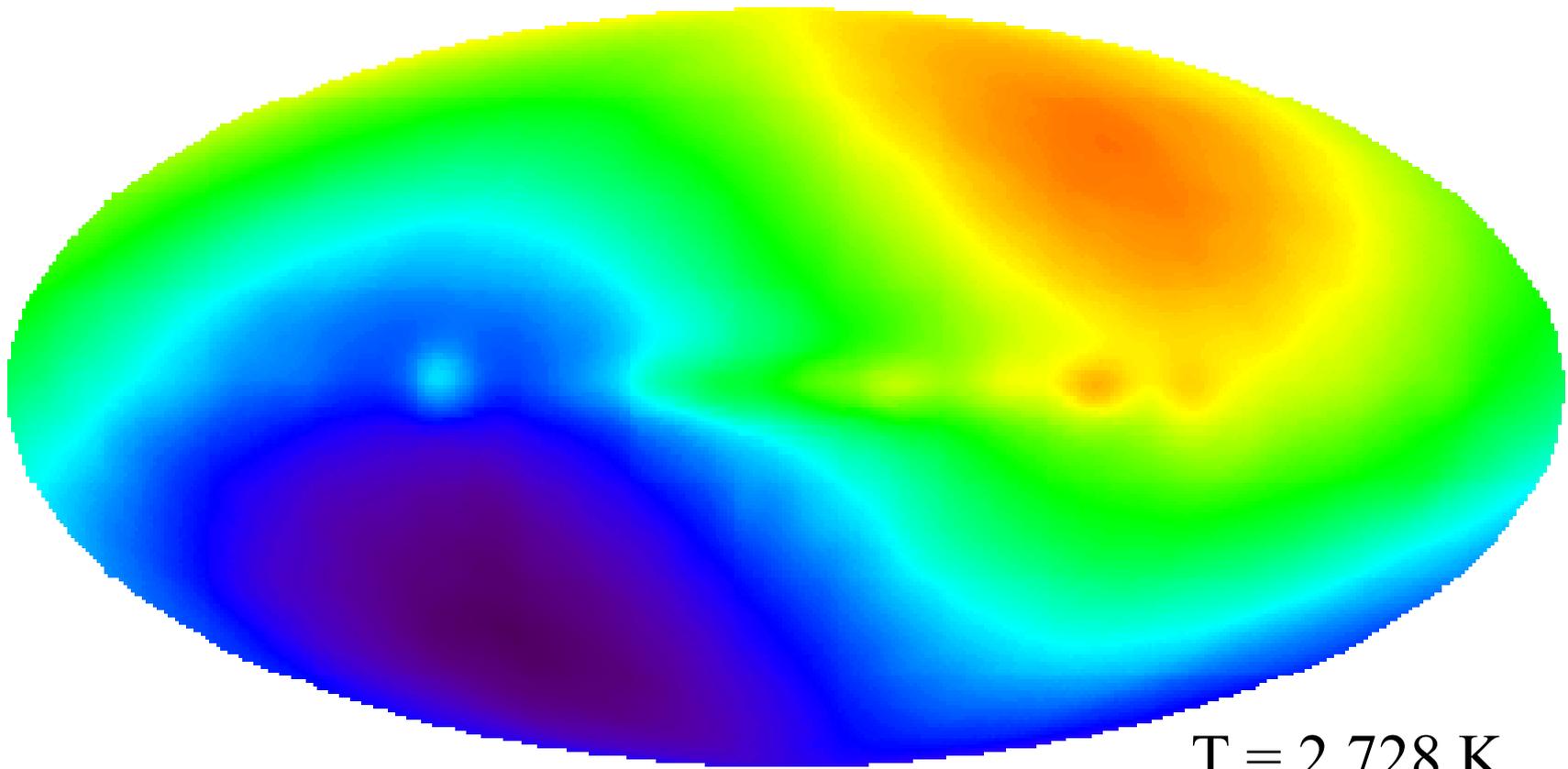
# COBE's temperature map of the entire sky



$T = 2.728 \text{ K}$   
 $\Delta T = 0.1 \text{ K}$

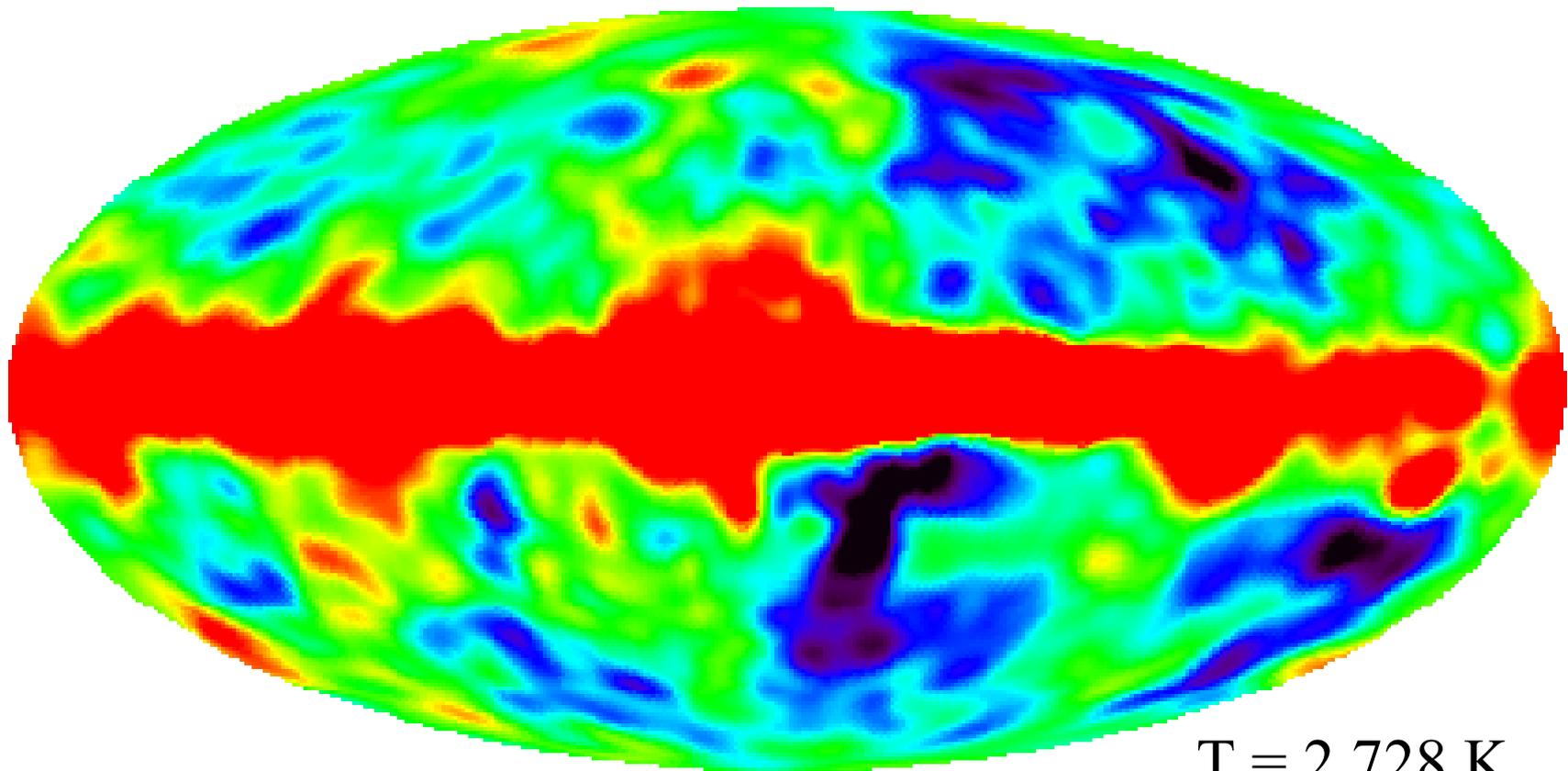
The large-scale Universe is isotropic!

# COBE's temperature map of the entire sky



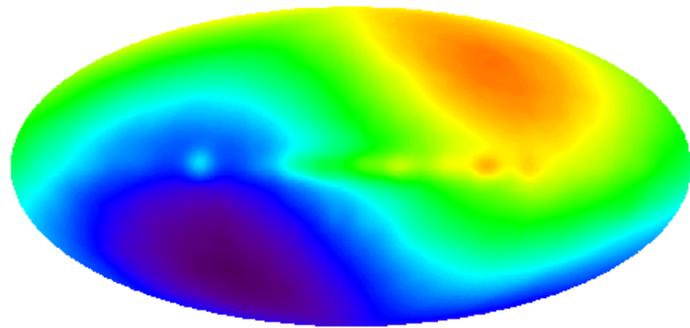
$T = 2.728 \text{ K}$   
 $\Delta T = 0.0034 \text{ K}$

# COBE's temperature map of the entire sky



$T = 2.728 \text{ K}$   
 $\Delta T = 0.00002 \text{ K}$

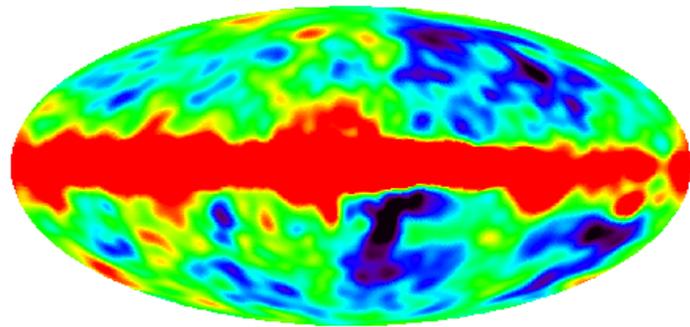
# Structure in the COBE map



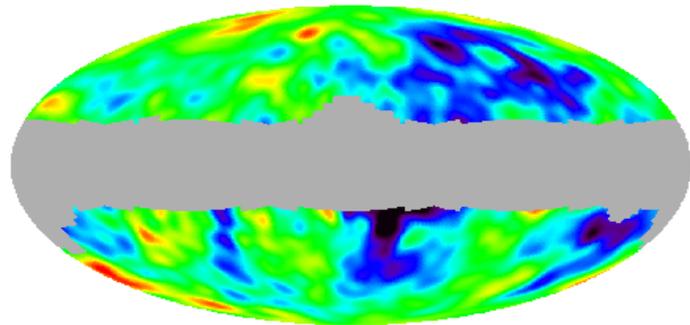
- One side of the sky is 'hot', the other is 'cold'

→ the Earth's motion through the Cosmos

$$V_{\text{Milky Way}} = 600 \text{ km/s}$$



- Radiation from hot gas and dust in our own Milky Way



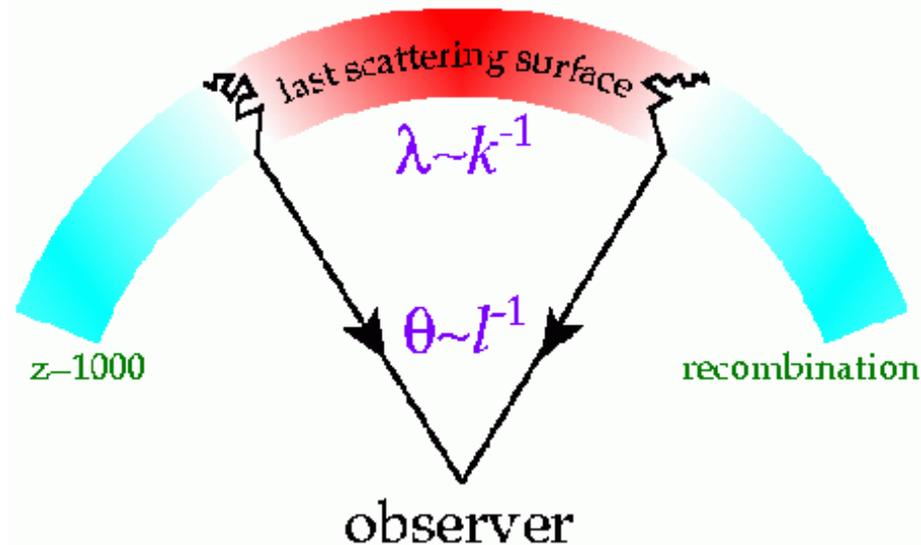
- Structure in the Microwave Background itself

# Structure in the Microwave Background

- The structure lies in cosmic 'clouds',  $\sim 4 \times 10^{10}$  1-yrs away
- It reflects weak “sound” waves,  $A \sim 10^{-4}$ , in the clouds
- At the time the Universe was only 400,000 years old, and was 1,000 times smaller and 1,000 times hotter than today

The *pattern* of structure reflects

- A: The global geometry and topology of the Universe
- B: The constituents and thermal evolution of the Universe
- C: The process which generated the structure

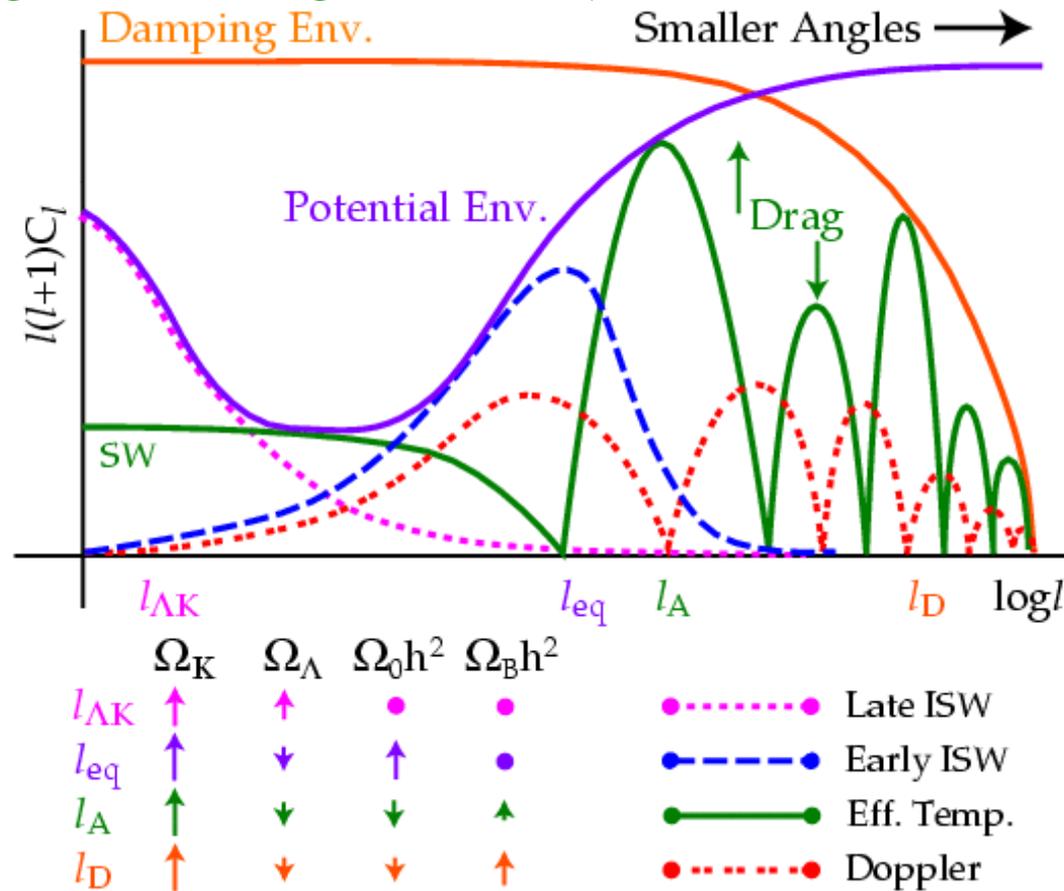
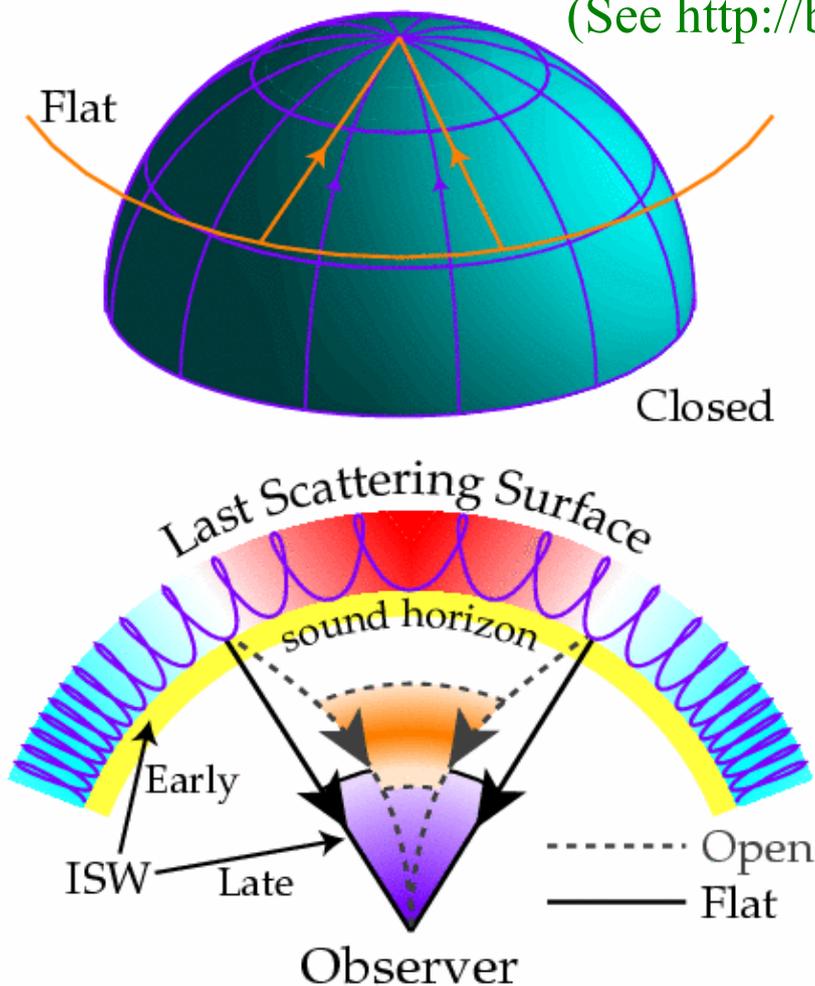


Observed fluctuations are gravitationally modified sound waves of amplitude  $\sim 10^{-5}$  propagating in the photon-baryon-DM fluid in the last scattering surface at  $z \sim 1000$   $t \sim 370,000$  yr

This (linear) sound wave field can be characterised by its Power Spectrum, the mean square amplitude of waves as a function of comoving spatial wavenumber  $k \sim 2\pi / \lambda$

The corresponding CMB temperature fluctuation field can be characterised by its power spectrum as a function of  $l \sim 2\pi / \theta$

(See <http://background.uchicago.edu/~whu>)



## Physical effects on the CMB power spectrum

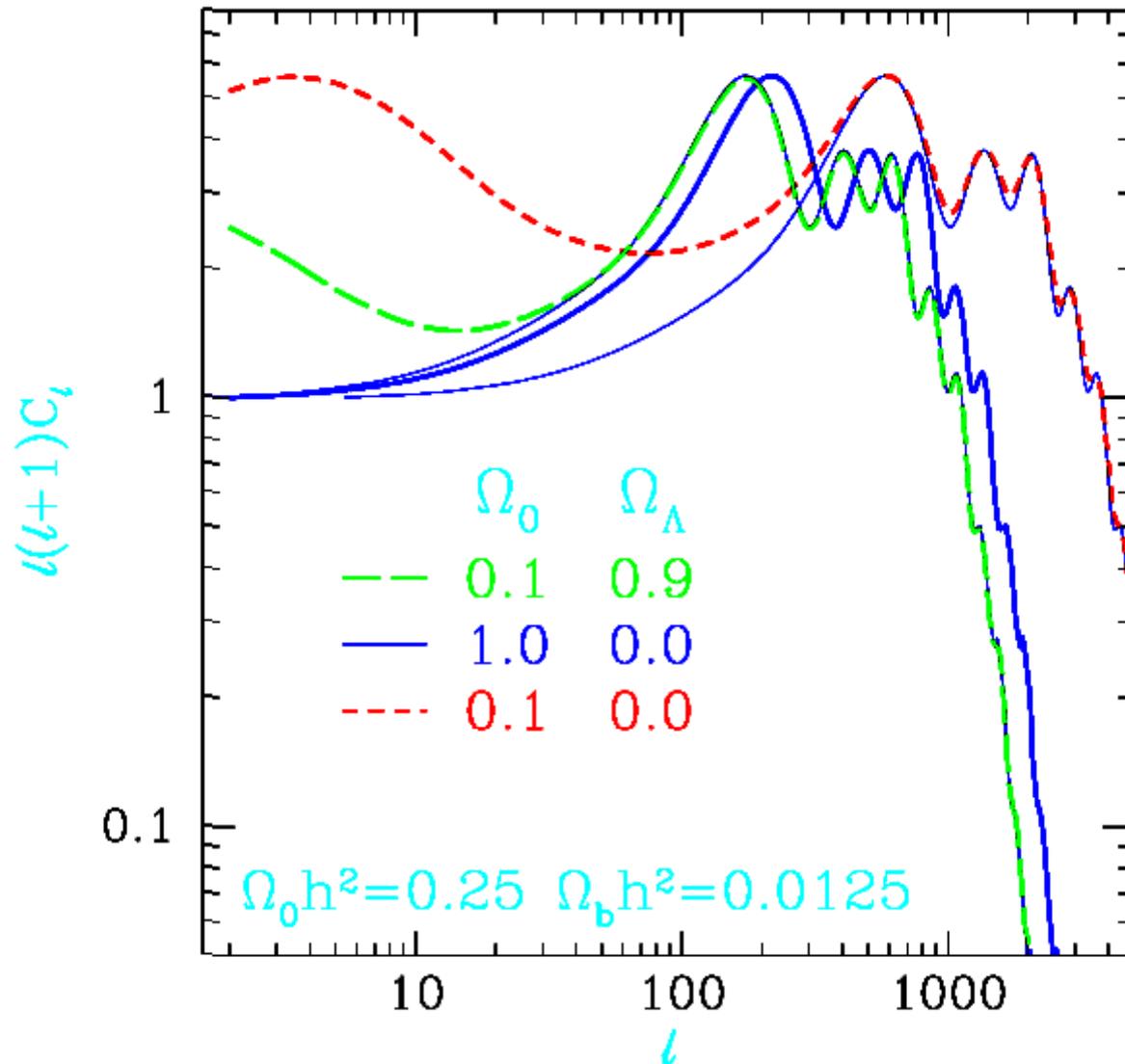
**Geometry:** The global geometry links physical lengths at  $z=1000$  to angular scales on the sky  $\longrightarrow$  the  $l$ -positions of the peaks

**Temperature fluctuations** in the gas at  $z=1000$  due to compression/rarefaction in sound waves

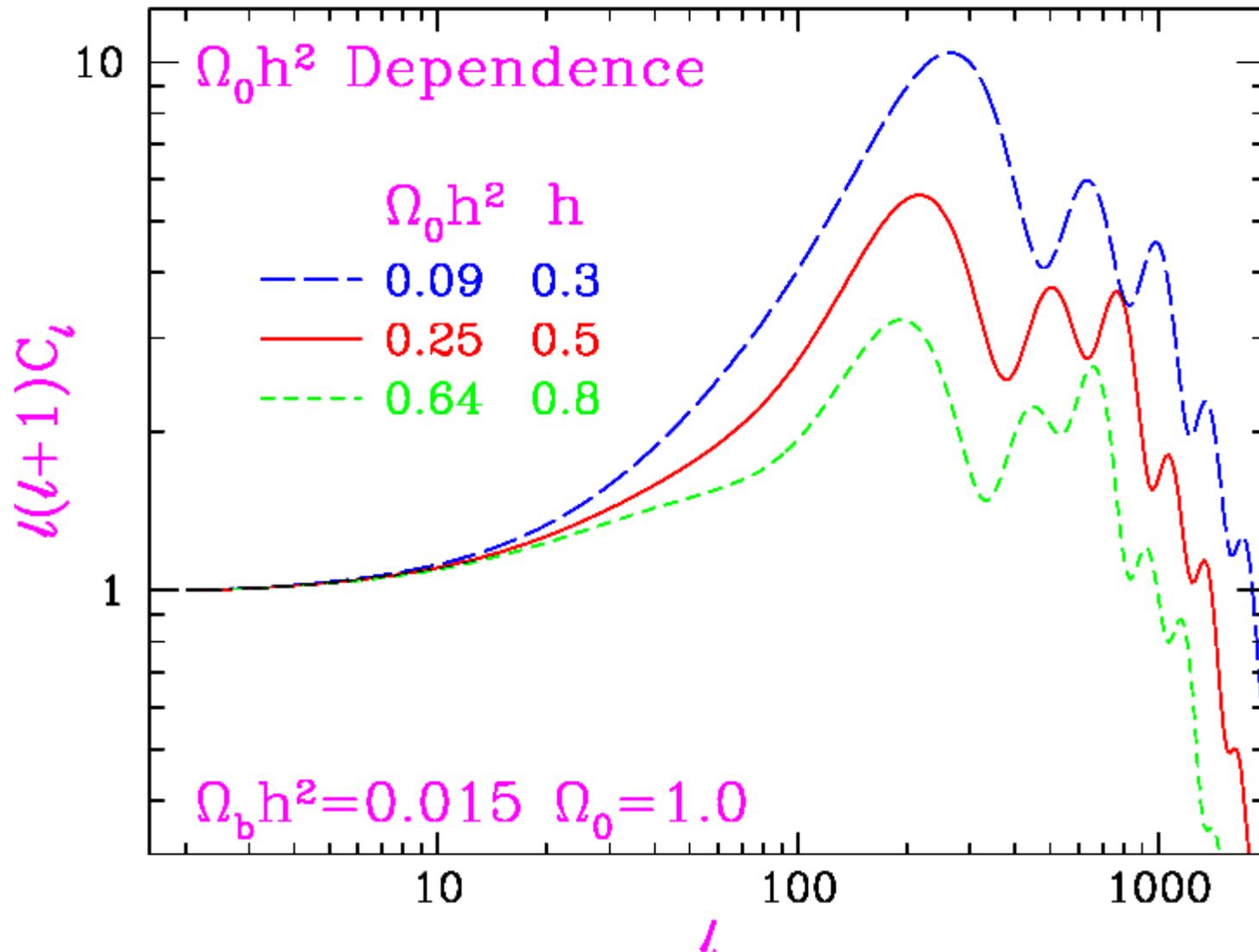
**Motions** in the gas at  $z=1000$  due to the sound waves

**Gravitational redshifts** at  $z \sim 1000$  and at low  $z$  (Sachs-Wolfe effect)

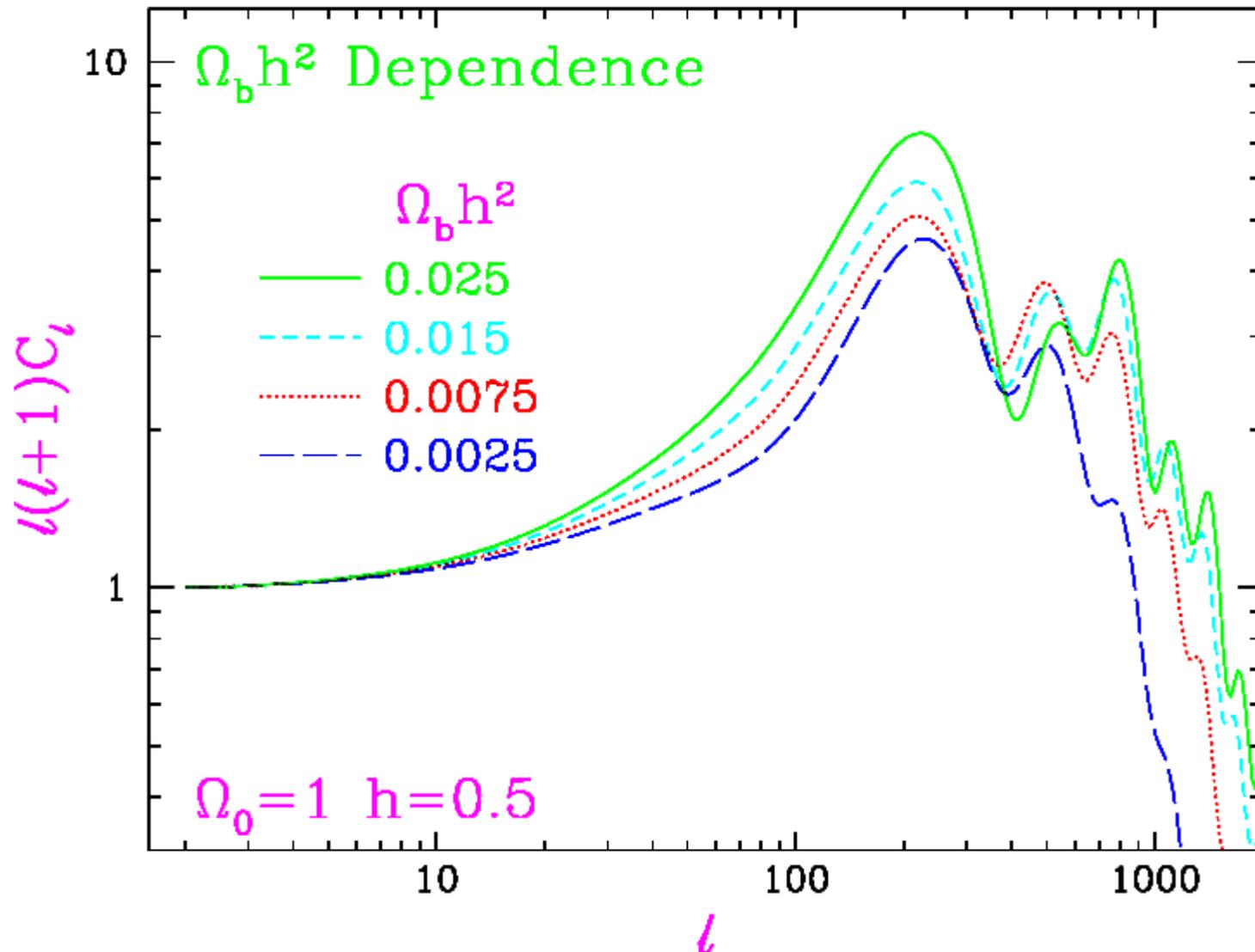
Variation of power spectrum with changing angular size distance to the last scattering surface. The change at low  $l$  is due to the low redshift integrated Sachs-Wolfe effect as the universe becomes curvature or  $\Lambda$  dominated.



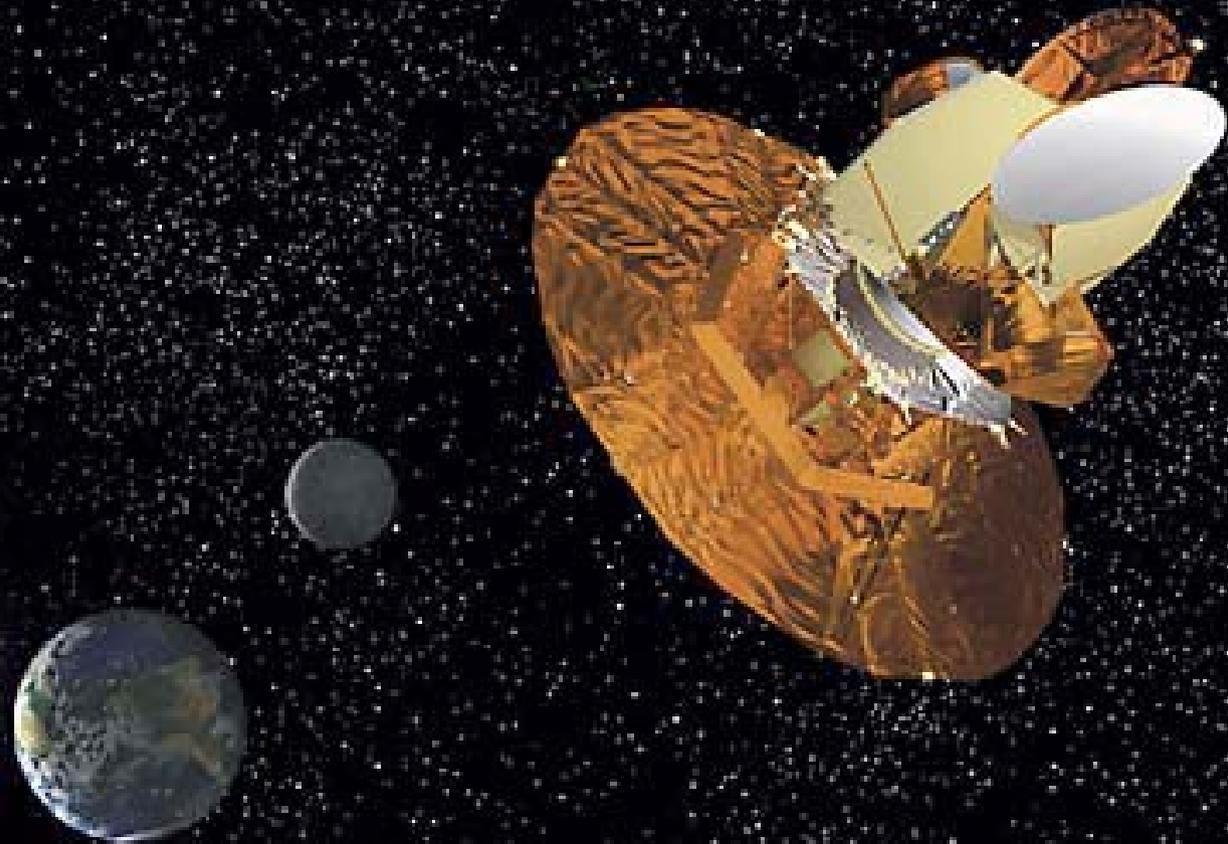
Variation of power spectrum with mass density for fixed baryon and photon density and a flat universe without  $\Lambda$



Variation of the power spectrum with the baryon density. The effect at high  $l$  is due to diffusive damping of the short-wavelength sound waves (Silk damping).

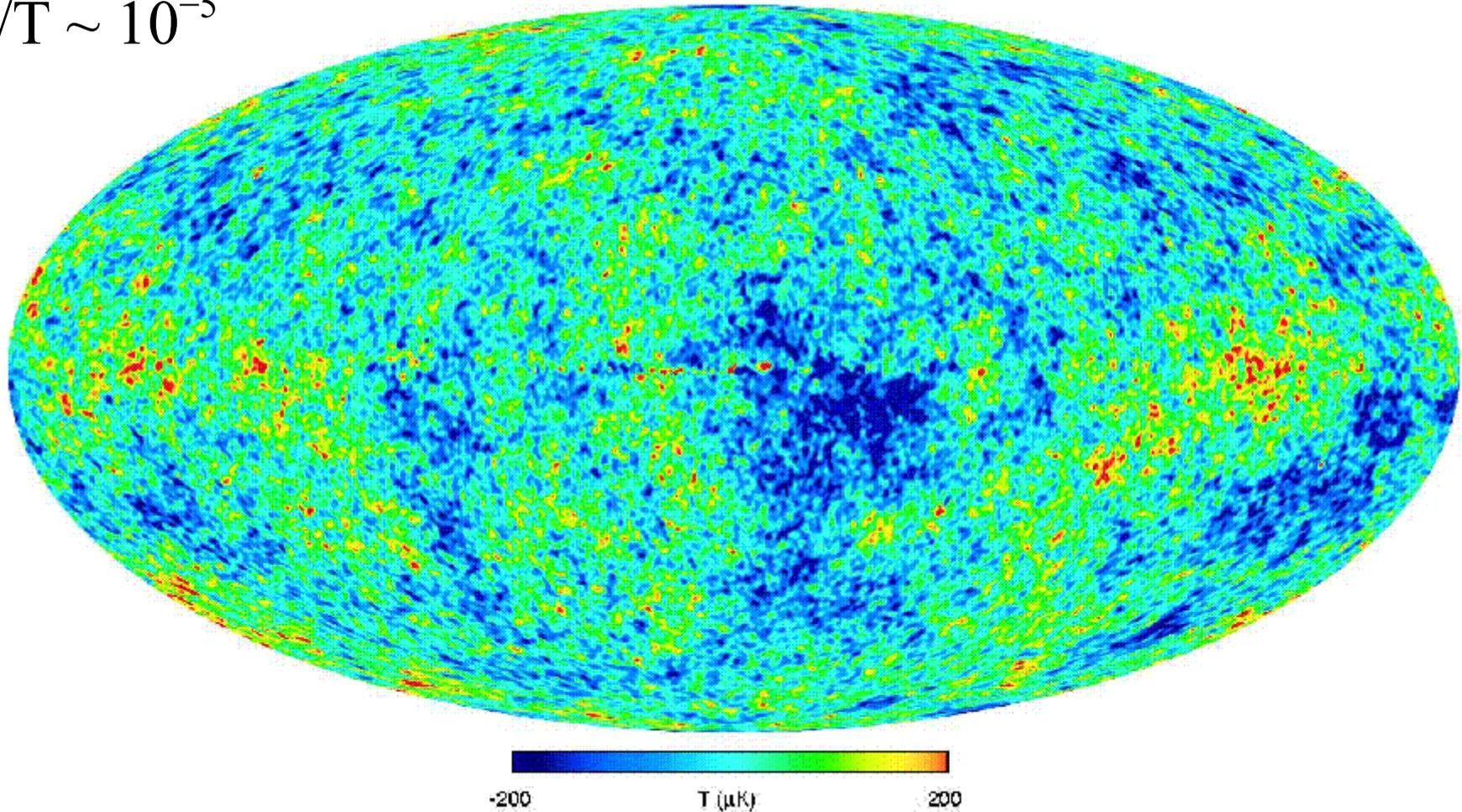


# The *WMAP* Satellite at Lagrange-Point L2



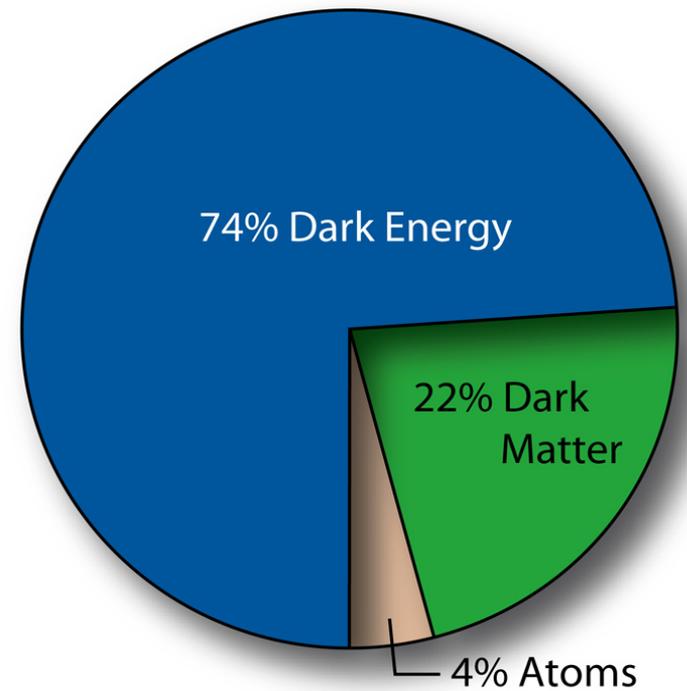
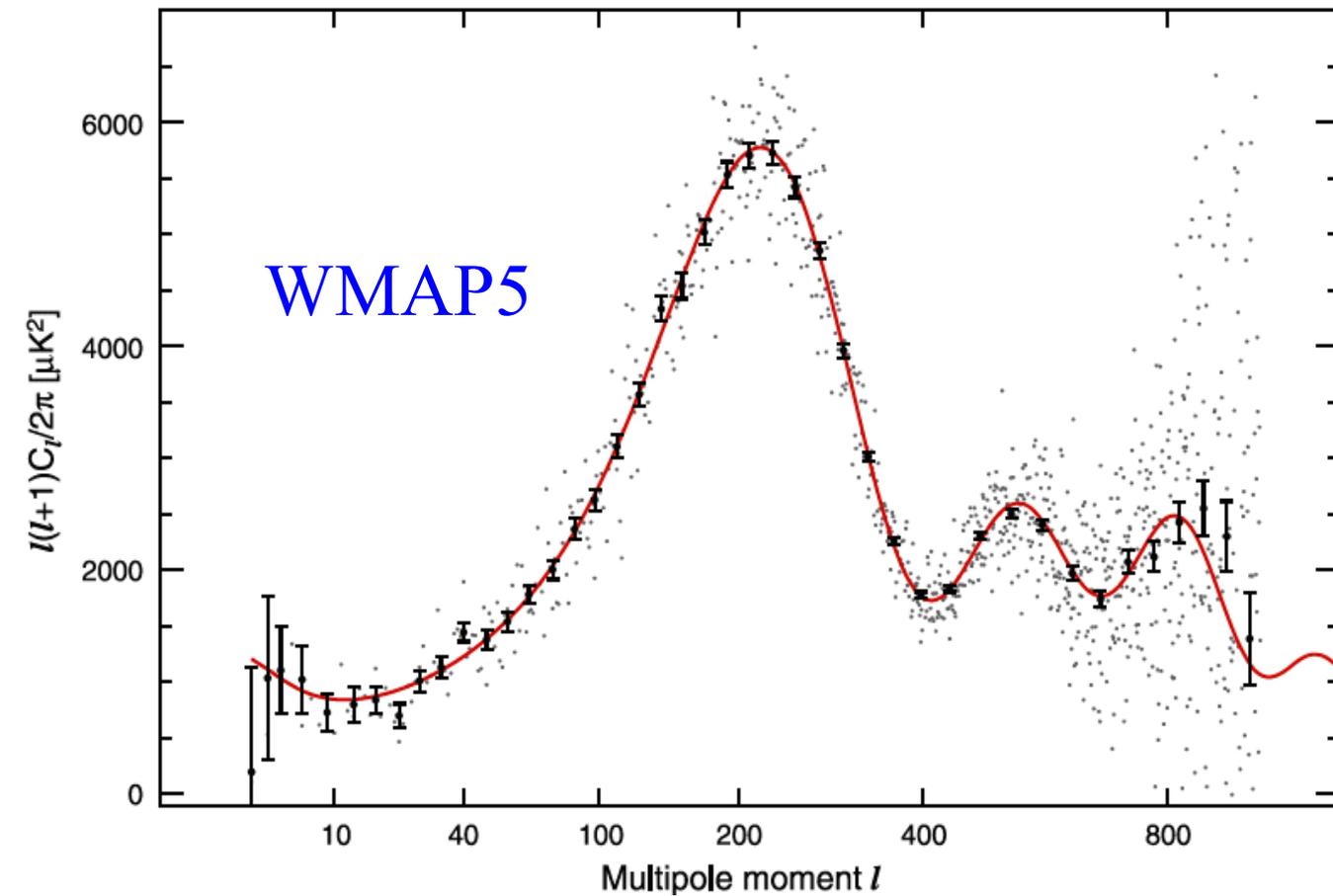
# The *WMAP* of the whole CMB sky

$$\Delta T/T \sim 10^{-5}$$

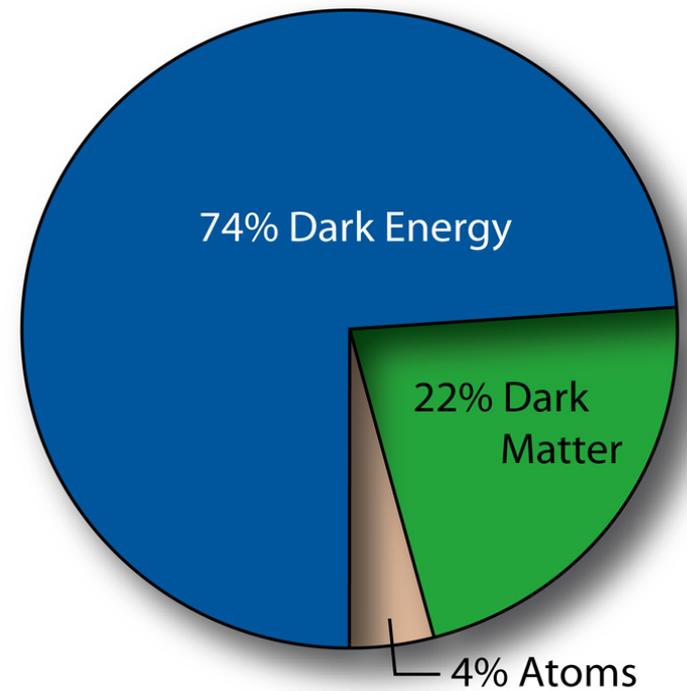
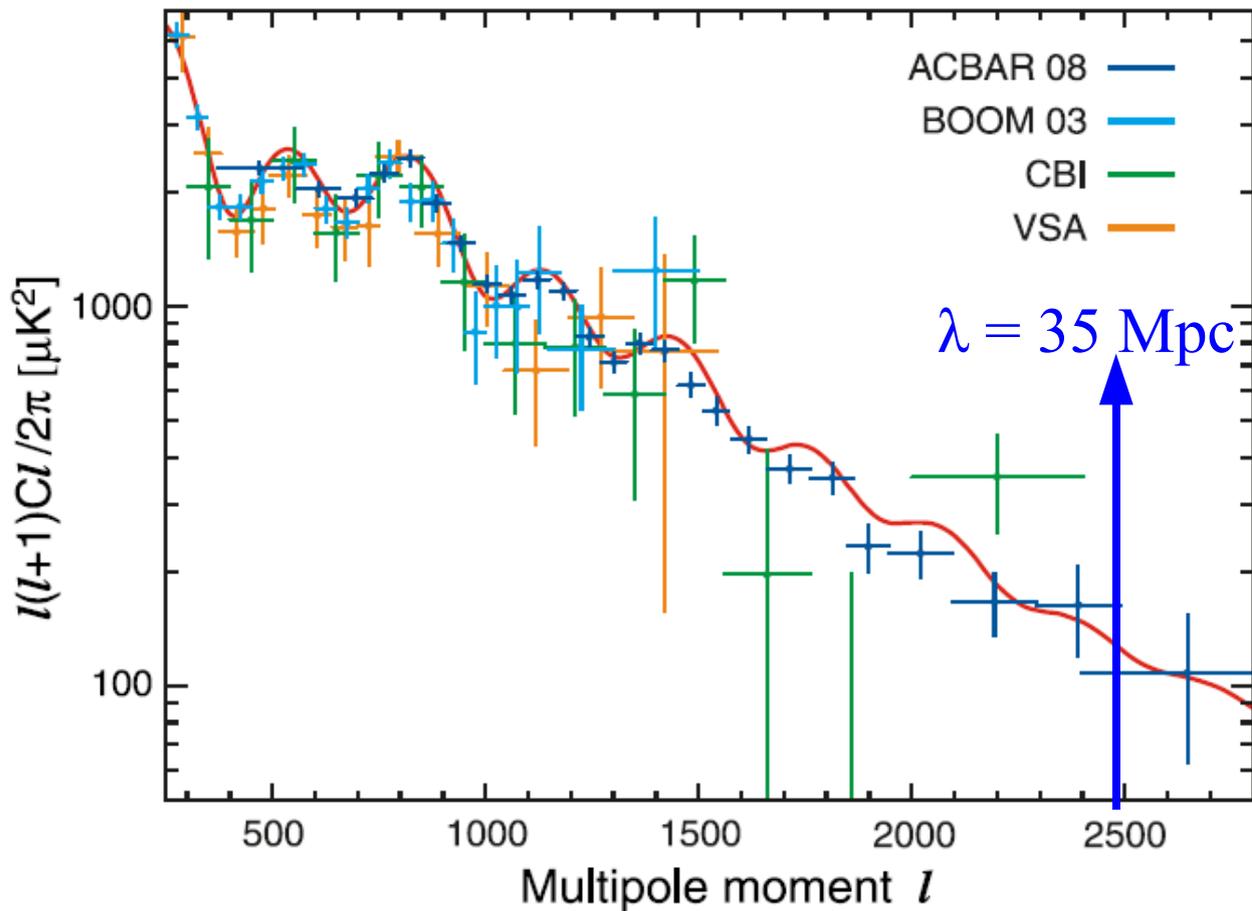


The initial conditions for  
all structure formation

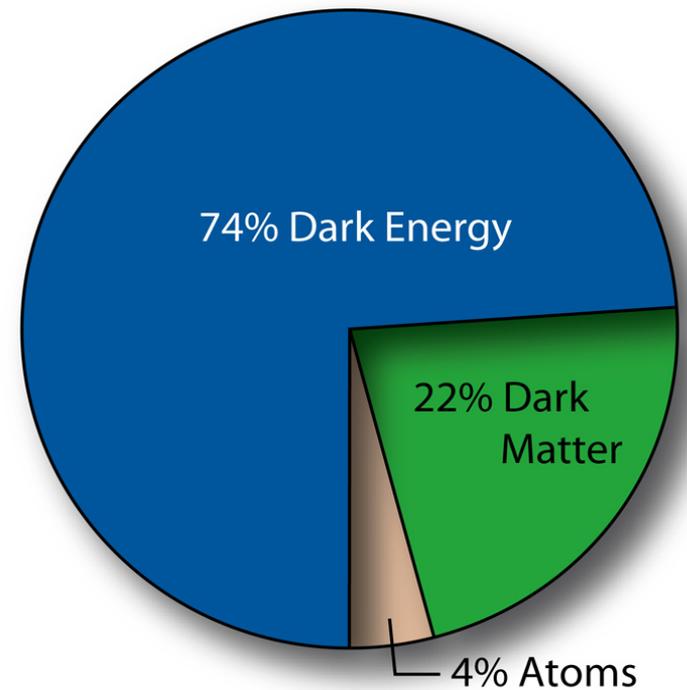
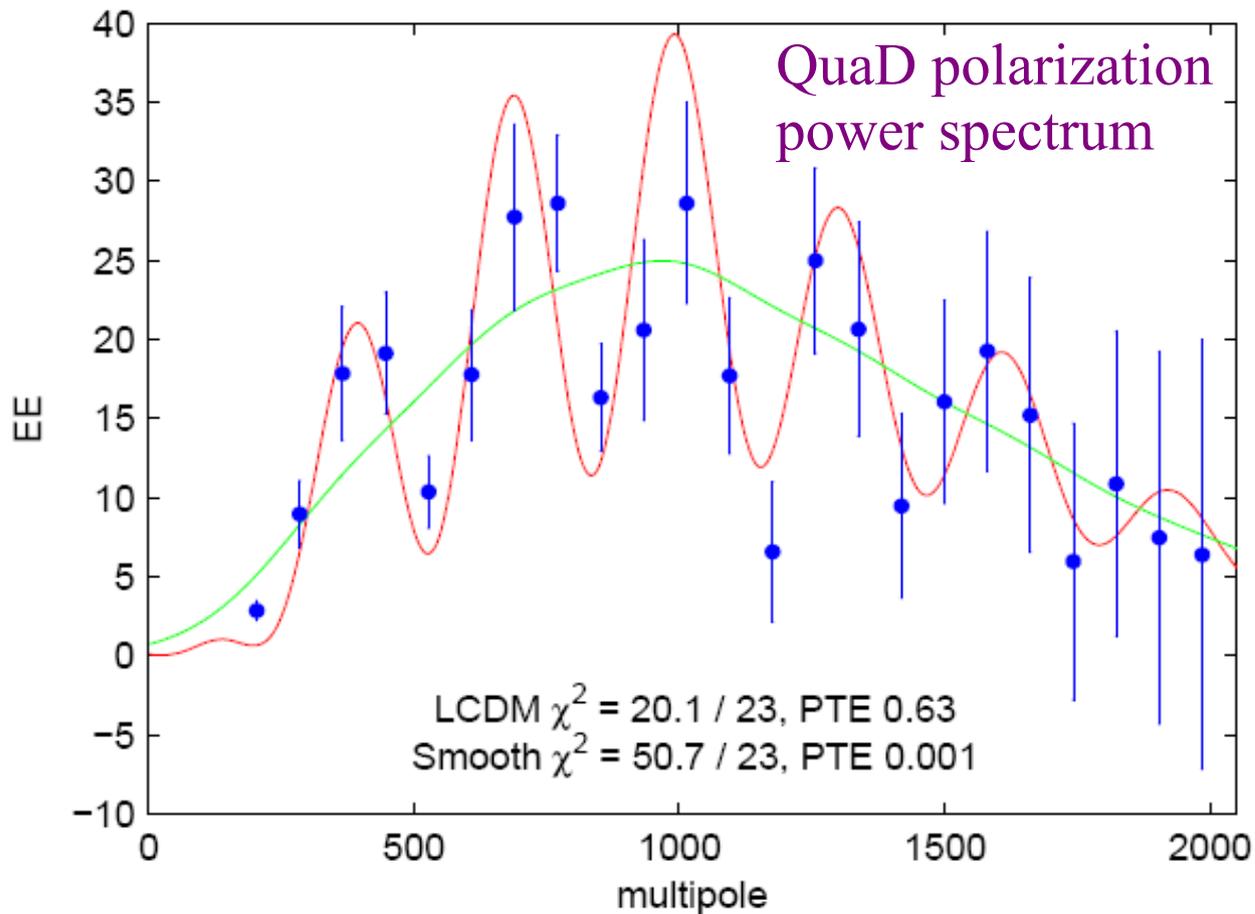
Bennett et al 2003



parameter	symbol	WMAP-5		comment
		alone	+ BAO + SNe	
CMB temperature	$T_{\text{CMB}}$	$2.728 \pm 0.004 \text{ K}$	–	from (Fixsen <i>et al.</i> 1996)
total matter density	$\Omega_{\text{tot}}$	$1.099^{+0.100}_{-0.085}$	$1.0052 \pm 0.0064$	assuming spatial flatness here and below
matter density	$\Omega_{\text{m}0}$	$0.258 \pm 0.03$	$0.279 \pm 0.015$	
baryon density	$\Omega_{\text{b}0}$	$0.0441 \pm 0.0030$	$0.0462 \pm 0.0015$	
cosmological constant	$\Omega_{\Lambda 0}$	$0.742 \pm 0.03$	$0.721 \pm 0.015$	
Hubble constant	$h$	$0.719^{+0.026}_{-0.027}$	$0.701 \pm 0.013$	
power-spectrum normalisation	$\sigma_8$	$0.796 \pm 0.036$	$0.817 \pm 0.026$	
age of the Universe in Gyr	$t_0$	$13.69 \pm 0.13$	$13.73 \pm 0.12$	
decoupling redshift	$z_{\text{dec}}$	$1087.9 \pm 1.2$	$1088.2 \pm 1.1$	
reionisation optical depth	$\tau$	$0.087 \pm 0.017$	$0.084 \pm 0.016$	
spectral index	$n_s$	$0.963^{+0.014}_{-0.015}$	$0.960^{+0.014}_{-0.013}$	



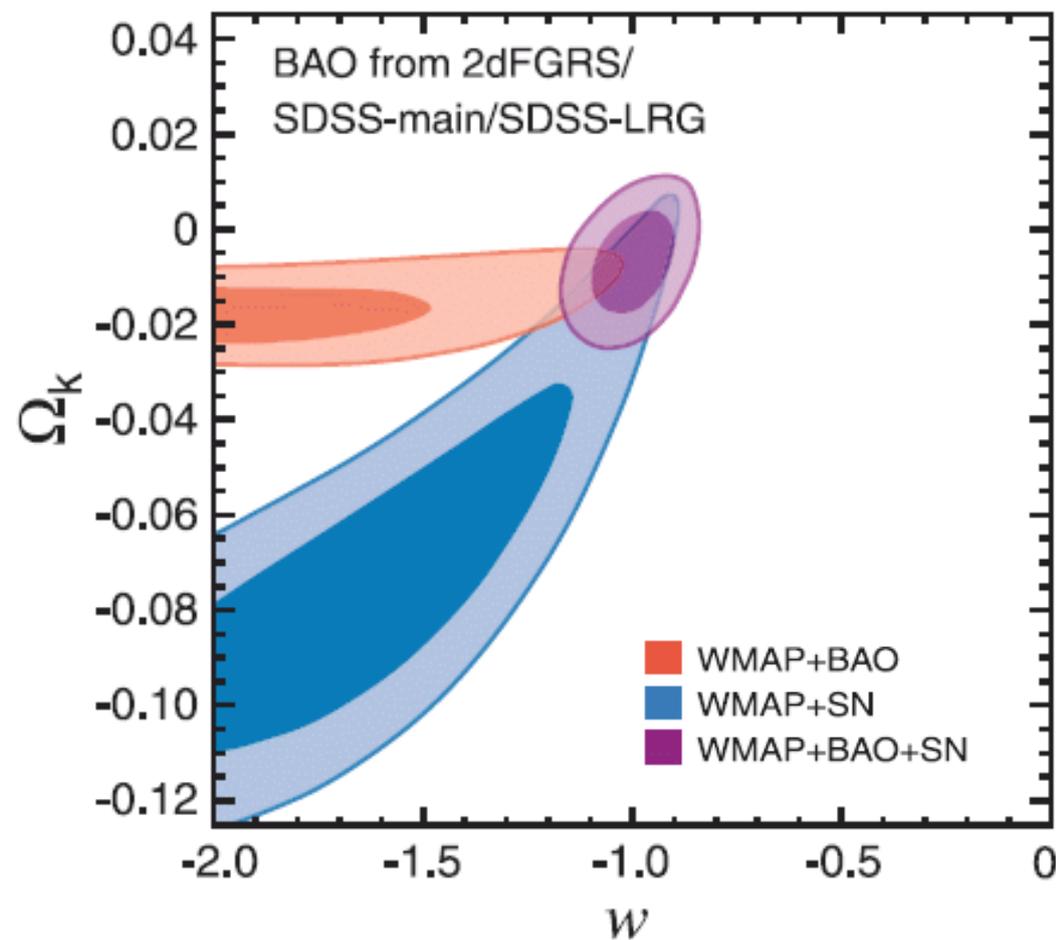
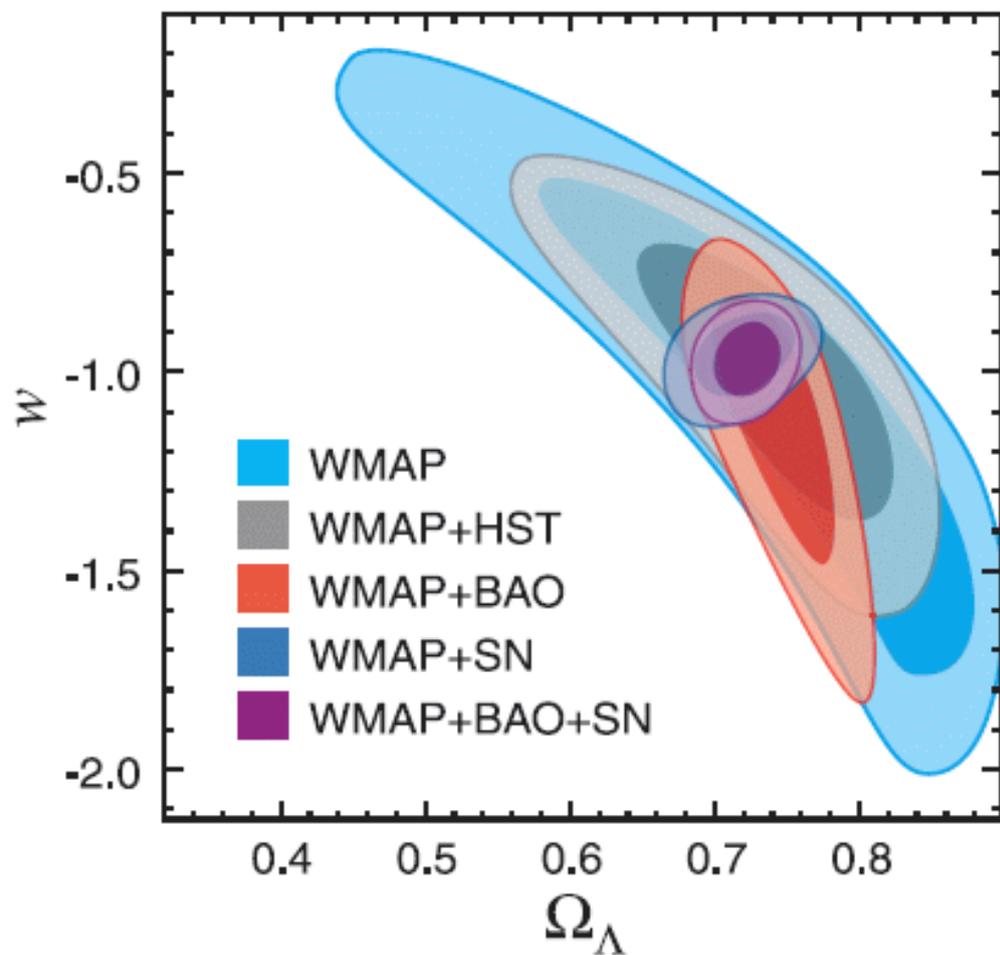
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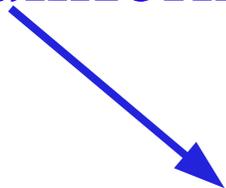
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# Putting it all together: consistency and complementarity

Komatsu et al 2008



Stays uniform



74% Dark Energy

22% Dark Matter



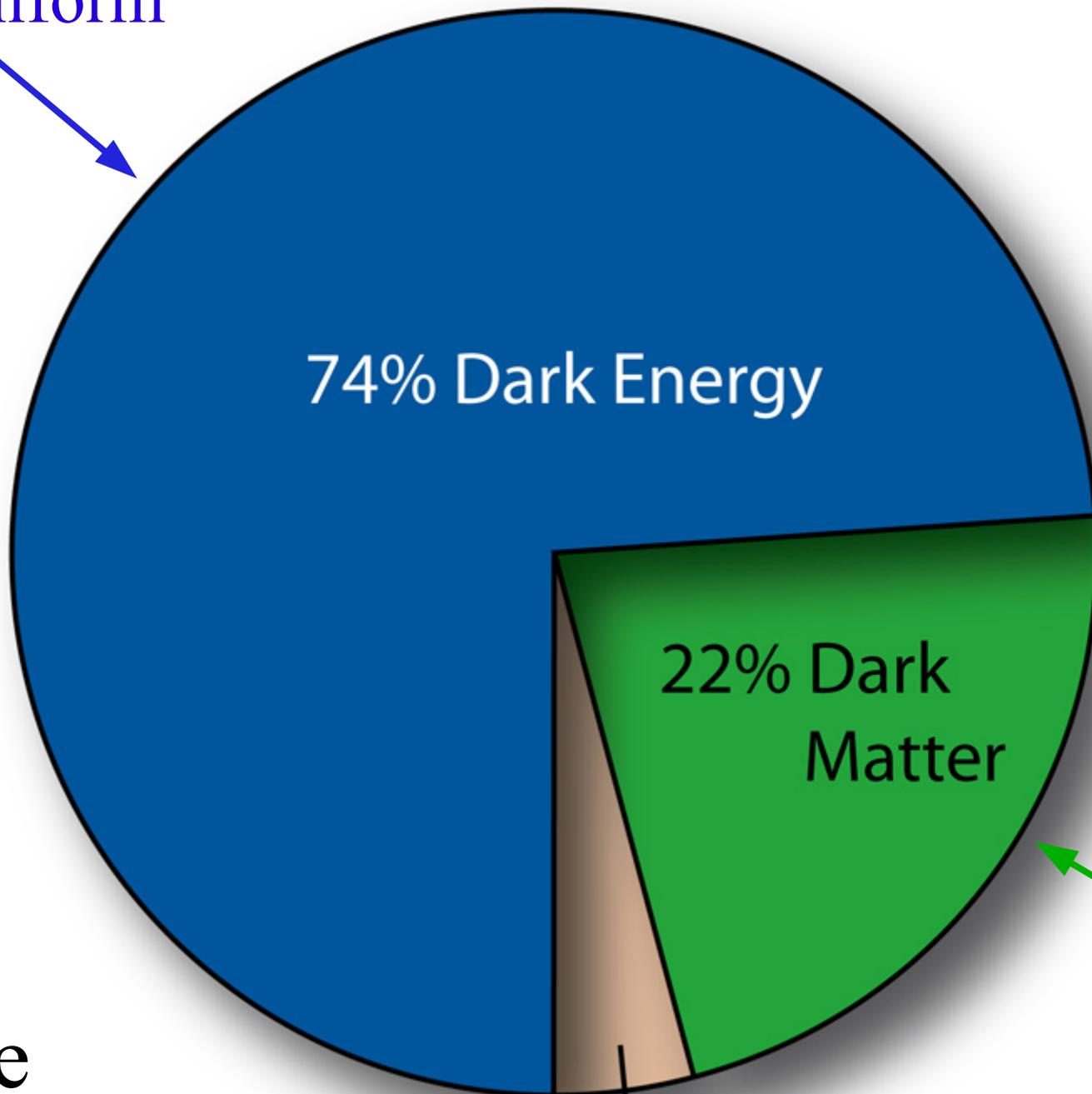
Clumps with time

4% Atoms



Visible!

Today's Universe according to WMAP



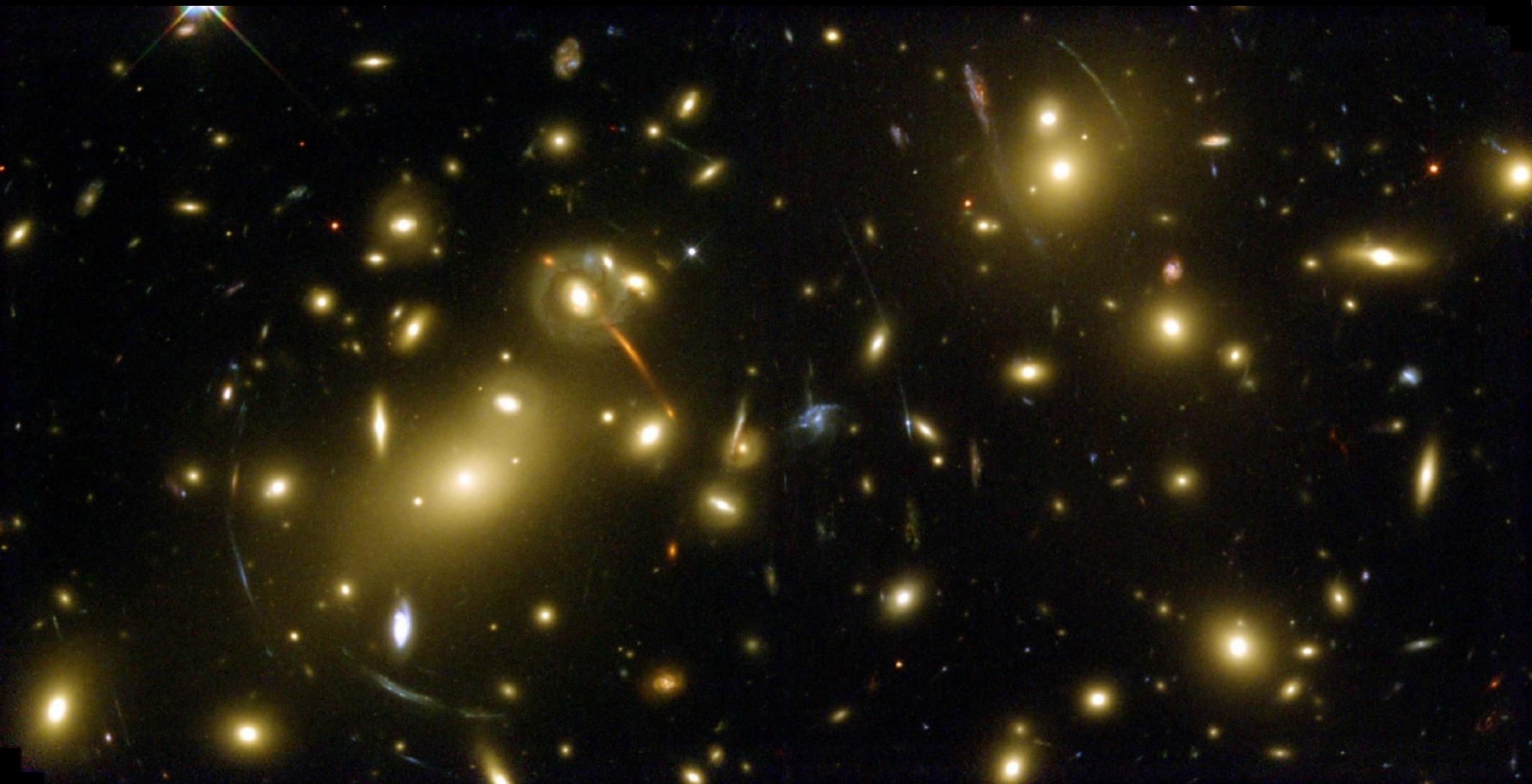
# What have we learned from WMAP?

- Our Universe is flat -- its geometry is that imagined by Euclid
- Only a small fraction of it is made of ordinary matter -- about 4.5%  
    → there is a lot of dark, nonbaryonic matter (about 23%)  
    (which can be “seen” through gravitational lensing)
- Most of it must be a new kind of dark energy (perhaps a cosmological constant) as also inferred from the apparently accelerating expansion
- All structure in the Universe originated as quantum zero-point fluctuations of the *vacuum*, perhaps  $10^{-30}$  s after the Big Bang!

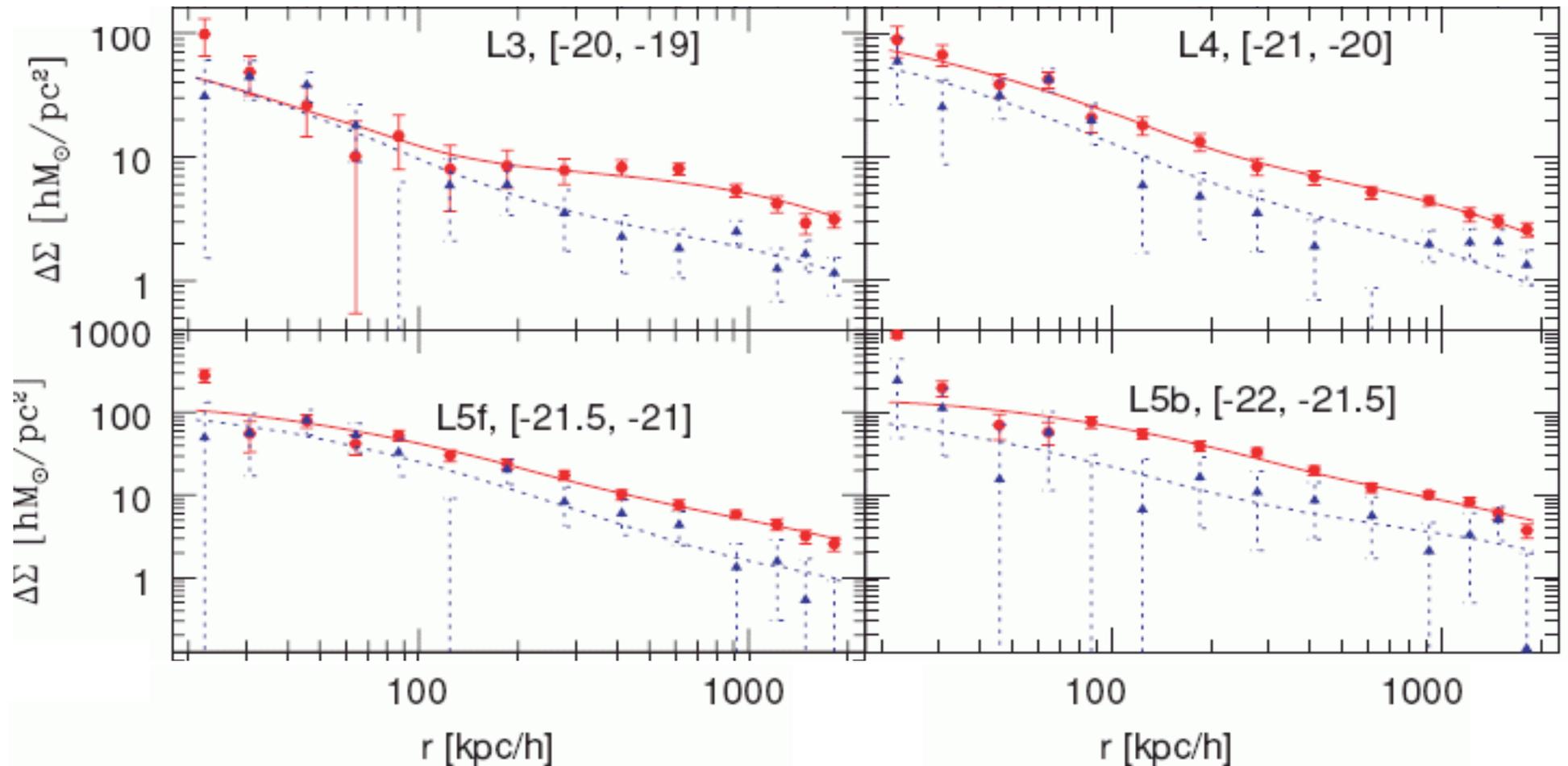
Everything has formed from nothing

# Gravitational lensing by a galaxy cluster

Abell 2218  $z=0.17$

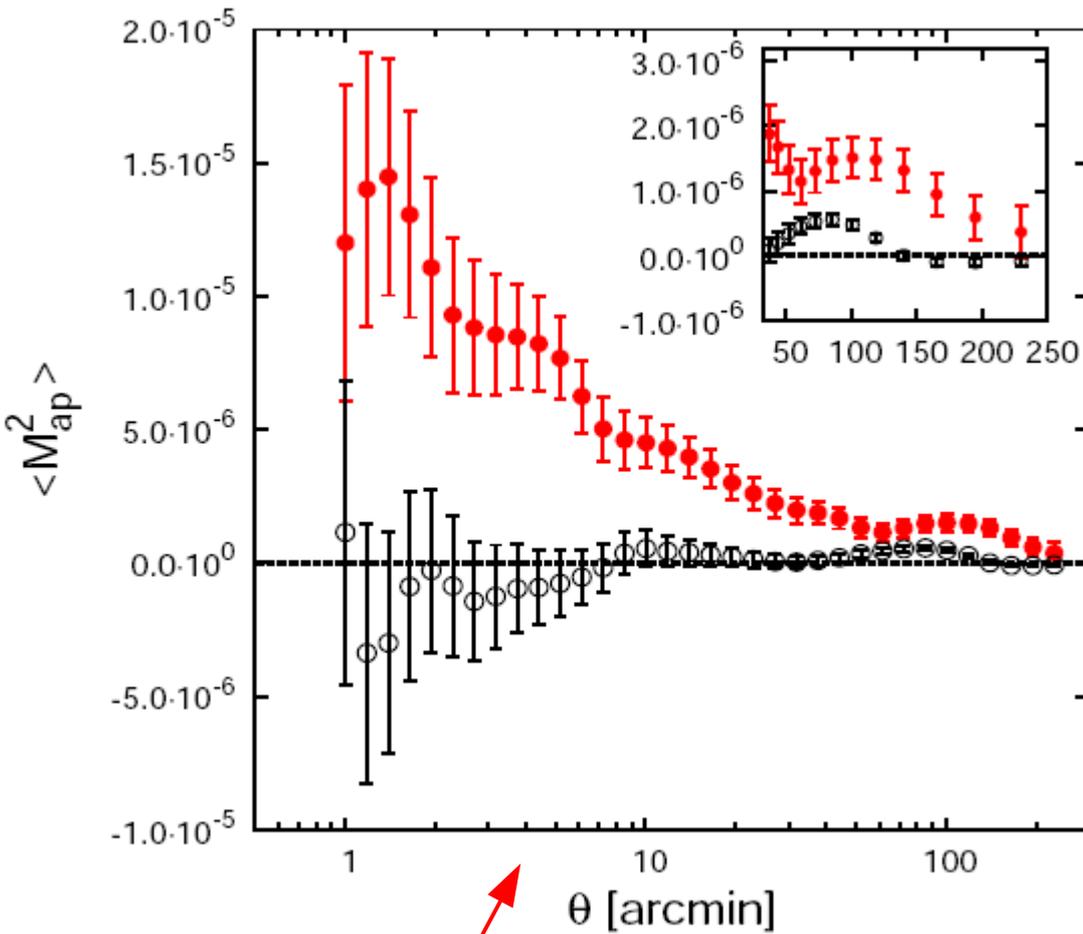


# Mean galaxy halo density profiles from SDSS lensing



Points are mean projected excess surface densities of halos of **red** and **blue** SDSS galaxies in different bins of absolute magnitude. Lines are a standard  $\Lambda\text{CDM}$  model

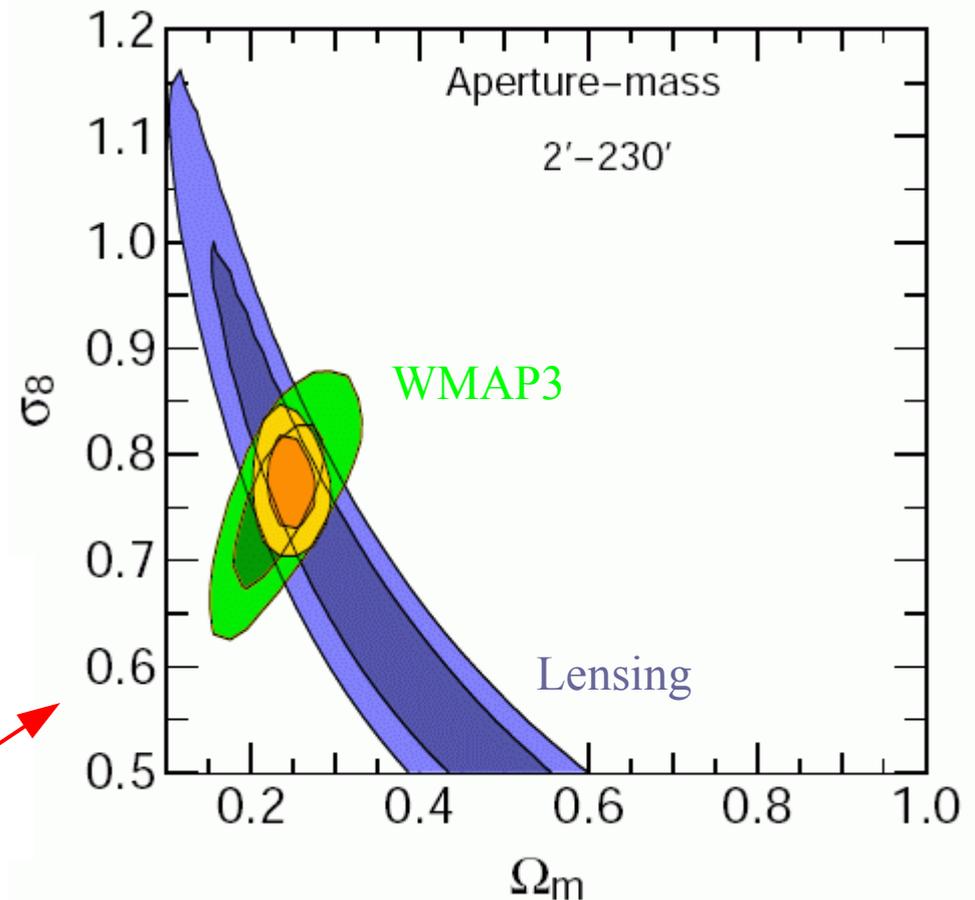
# Large-scale structure from weak lensing



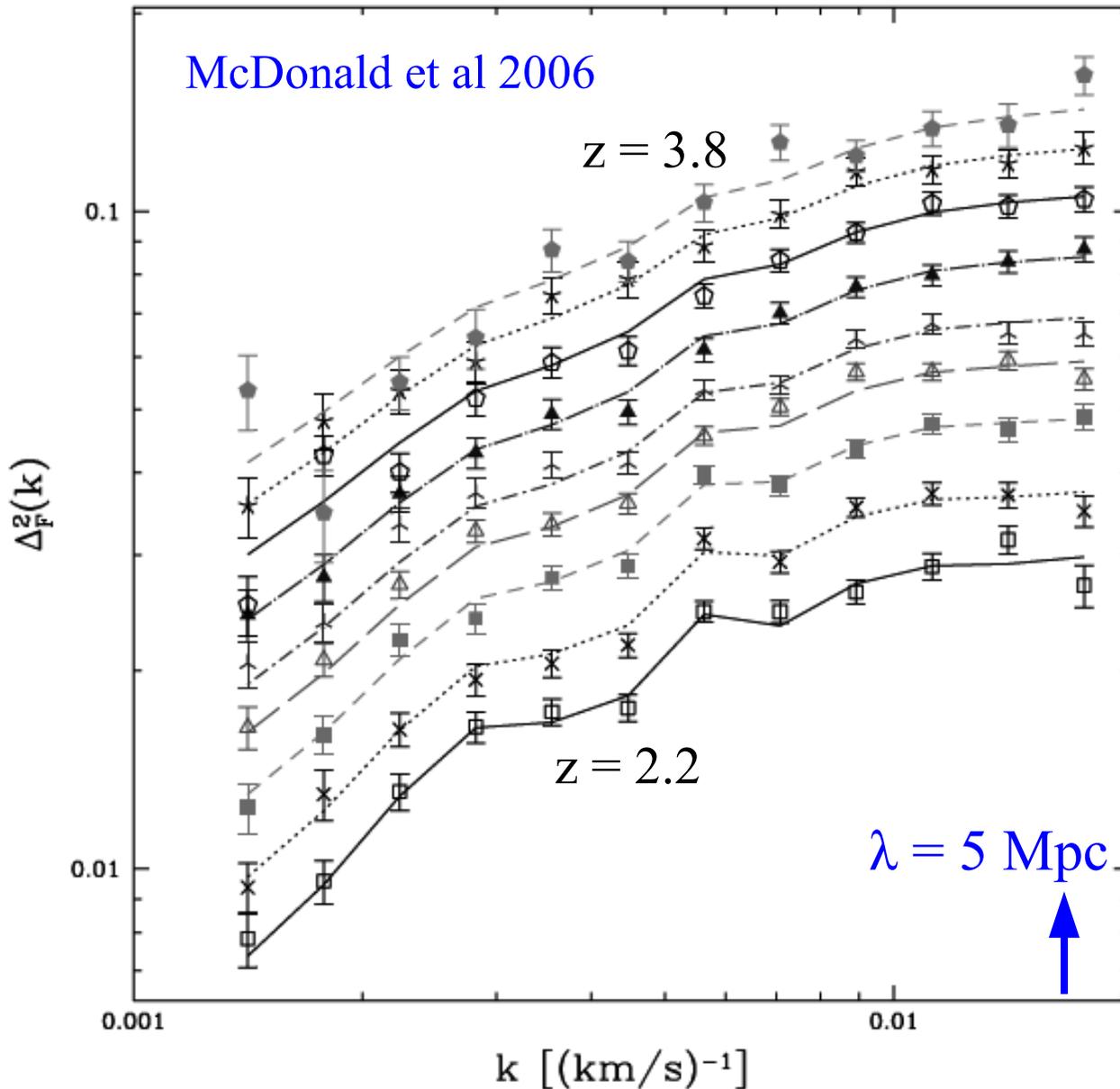
*rms* mass fluctuation in a compensated circular aperture  
implied parameter constraints

Fu et al 2008

Signal on 1' to 10' scales is dominated by the *nonlinear* contribution from dark halos



# Power spectrum of the Ly $\alpha$ forest in QSO spectra



Confirms  $\Lambda$ CDM model  
down to scales which  
make *dwarf* galaxies

Points are the SDSS measures, lines are a standard  $\Lambda$ CDM model

# Baryon wiggles in the galaxy distribution

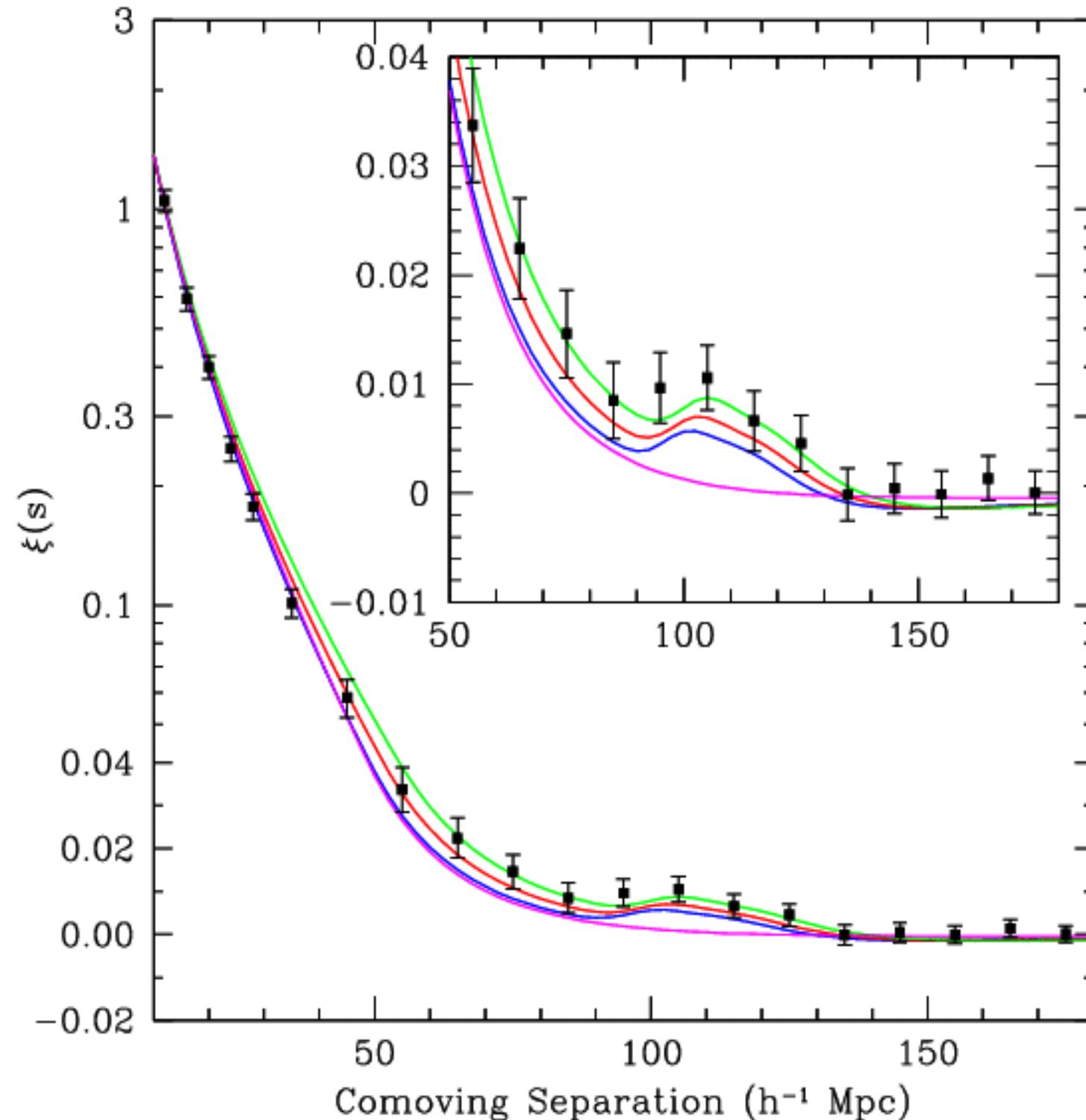
Eisenstein et al 2005

Galaxy correlation function from the SDSS luminous red galaxy survey.

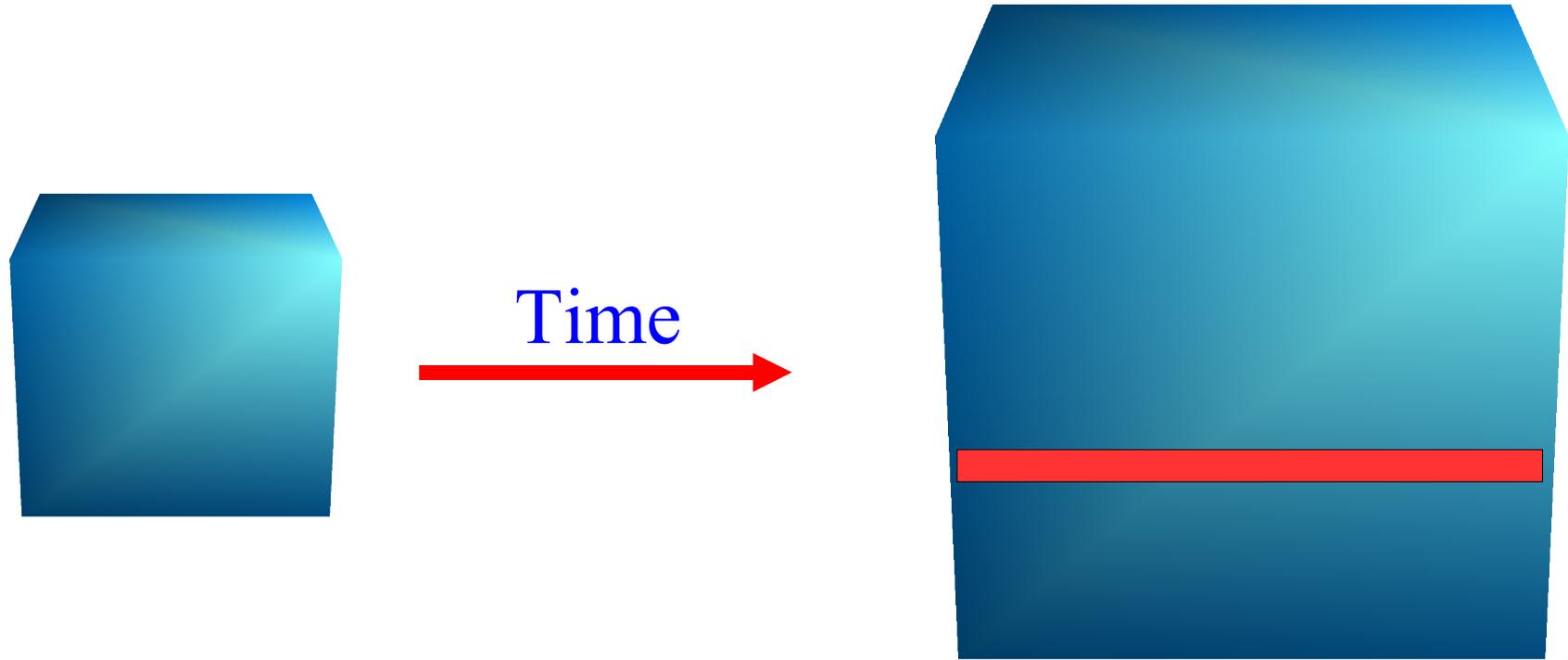
$$\langle z \rangle \sim 0.3$$

Confirms feature seen in the CMB

$$\rightarrow \frac{d_A(z=0.3)}{d_A(z=1000)}$$



# Evolving the Universe in a computer



- Follow the matter in an expanding cubic region
- Start 400,000 years after the Big Bang
- Match initial conditions to the observed Microwave Background
- Calculate evolution forward to the present day

# Views of the dark matter in a Virtual Universe

- The growth of dark matter structures in a thin slice
- A zoom from the whole visible Universe into a galaxy cluster
- A flight through the dark matter distribution
- The assembly of the Milky Way's halo

# Milky Way formation and the $\Lambda$ CDM cosmogony

- The structure predicted by the standard model is confirmed...
    - ...down to MW scale in the initial conditions (CMB)
    - ...down to MW halo scale at  $z=0$  (LSS + lensing)
    - ...down to dwarf galaxy scale at  $z=2 - 4$  (Ly  $\alpha$  forest)
  - The evolution of structure in the dominant DM component can be simulated with few uncertainties
  - The population of MW-like galaxies can be *observed* over the period corresponding to assembly and to formation of stars
- Old ELS and SZ pictures are obsolete and unhelpful!
- (unrelated to the formation of the main MW components [disk+bulge] and with no significant explanatory power for the stellar halo)

# Outstanding issues for the $\Lambda$ CDM cosmogony

- The nature of Dark Energy
- The nature of Dark Matter
- The low DM content of galaxy cores (cusp problem)
- The low abundance of dwarf galaxies (luminosity function and satellite problems)

# Lecture Plan

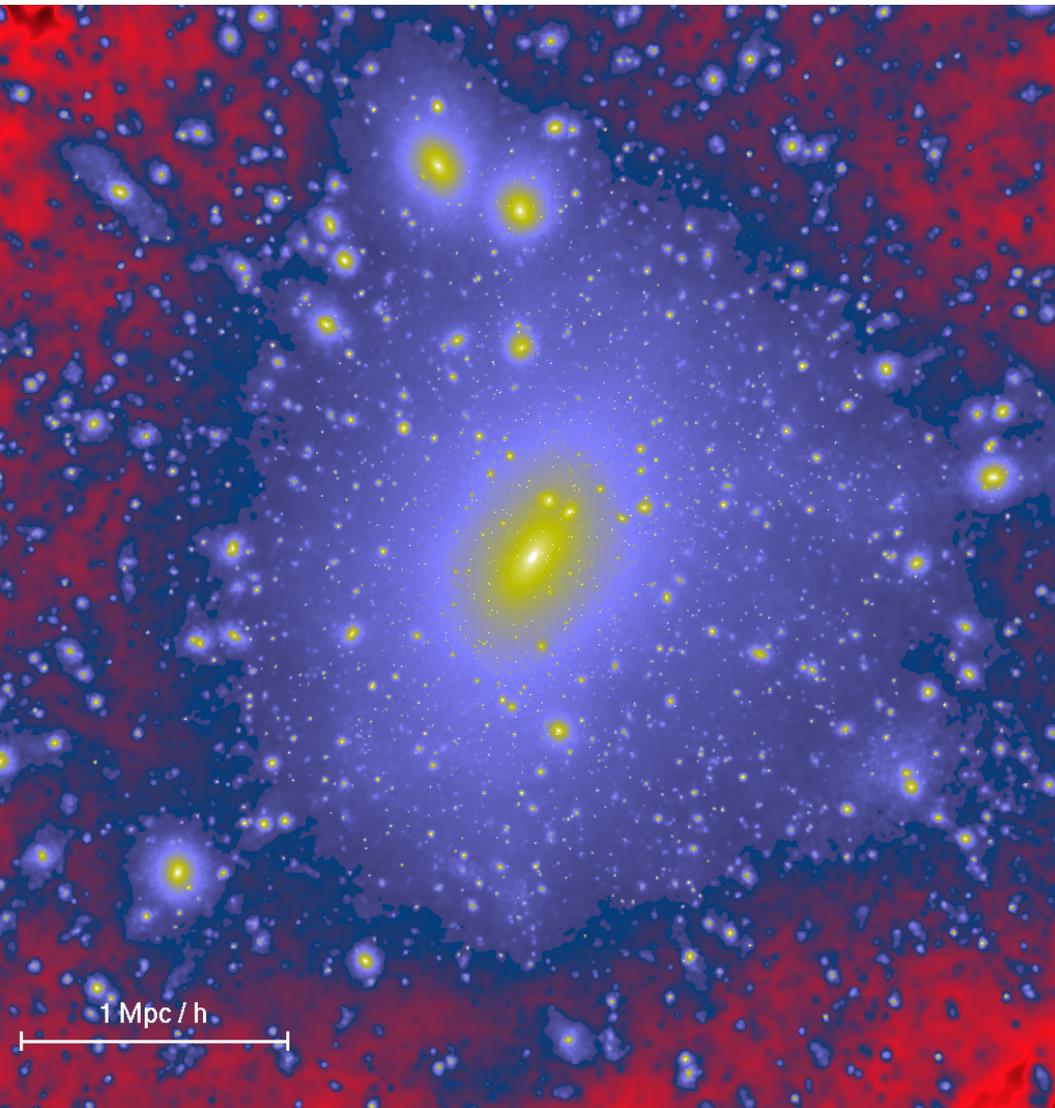
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# Dark matter halos are the basic units of nonlinear structure

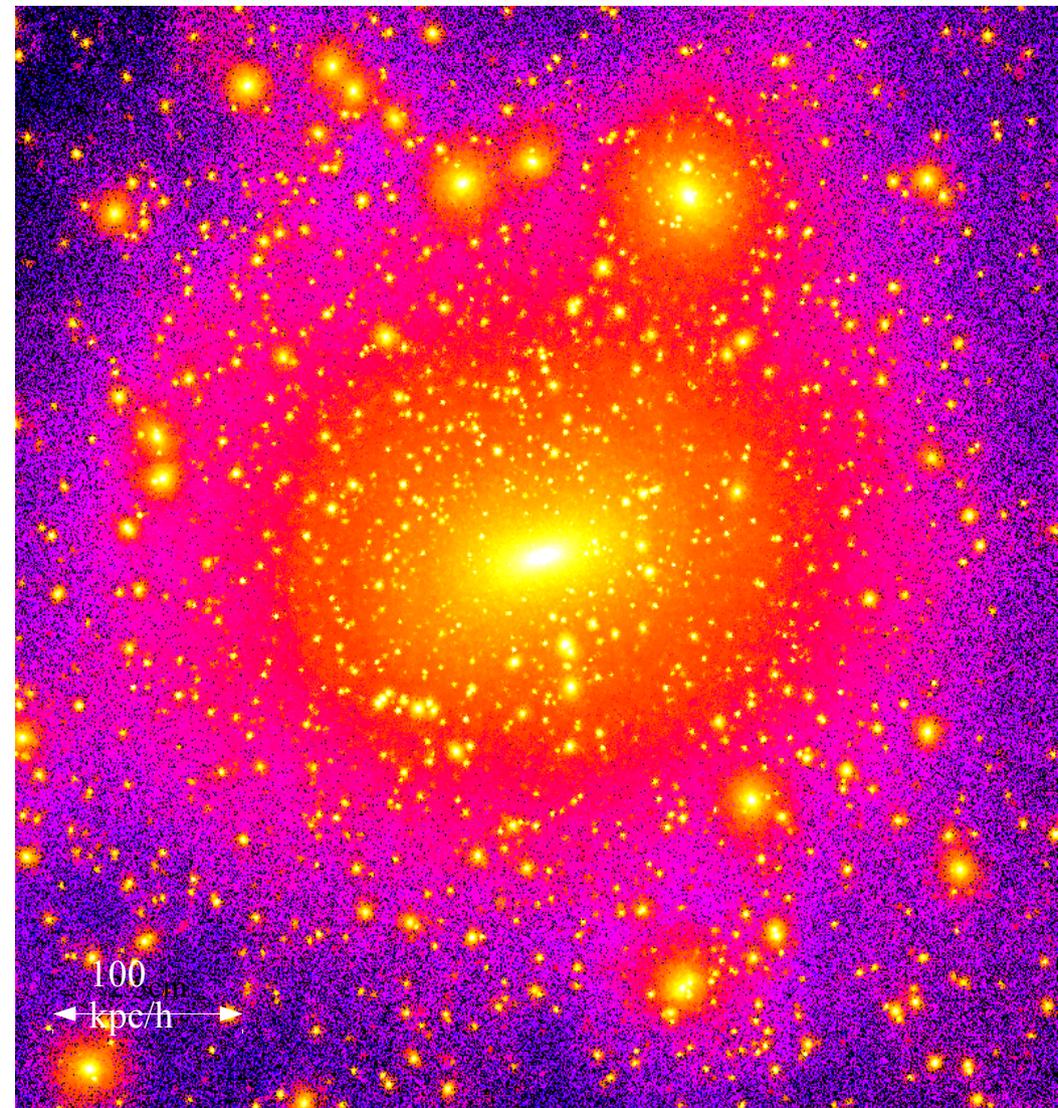
- Is all dark matter part of some halo?
- Was this always the case?
- How do halos grow? accretion? merging?
- How are they distributed?
- What is their internal structure?
  - density profile
  - shape
  - subhalo population – mass/radial distributions, evolution
  - caustics
- How do these properties affect DM detection experiments?
- How do they affect/are they affected by the baryonic matter
- What is the relation between galaxies and their halos?

# $\Lambda$ CDM halos

A rich galaxy cluster halo  
Springel et al 2001



A 'Milky Way' halo  
Power et al 2002



# A simple model for structure formation

In linear theory in a dust universe

$$\delta(\mathbf{x}, z) = D(z) \delta_o(\mathbf{x}) = (2\pi)^{-3/2} D(z) \int d^3k \delta_k \exp(-i \mathbf{k} \cdot \mathbf{x})$$

where we define  $D(0) = 1$

Consider the smoothed density field

$$\delta_s(\mathbf{x}, z; k_c) = (2\pi)^{-3/2} D(z) \int_{|\mathbf{k}| < k_c} d^3k \delta_k \exp(-i \mathbf{k} \cdot \mathbf{x})$$

and define  $\langle \delta_s(\mathbf{x}, z; k_c)^2 \rangle_x = D^2(z) \sigma_o^2(k_c)$ ,  $M_s = 6\pi^2 \rho_o k_c^{-3}$

As  $k_c$  grows from 0 to  $\infty$ , the smoothing mass decreases from  $\infty$  to 0, and  $\delta_s(\mathbf{x}, z; k_c)$  executes a random walk

For a gaussian linear overdensity field

$$\Delta \delta_s = \delta_s(\mathbf{x}, z; k_c + \Delta k_c) - \delta_s(\mathbf{x}, z; k_c)$$

is independent of  $\delta_s$  and has variance  $D^2 \Delta \sigma_o^2$

---- A Markov random walk ----

# The “Press & Schechter” Ansatz

A uniform spherical “top hat” perturbation virialises when its extrapolated linear overdensity is  $\delta_c \approx 1.69$

Assume that at redshift  $z$ , the mass element initially at  $\mathbf{x}$  is part of a virialised object with the largest mass  $M$  for which

$$\delta_s(\mathbf{x}, z; k_c(M)) \geq \delta_c$$

This is the Markov walk's first upcrossing of the barrier  $\delta_s = \delta_c$

The fraction of all points with first upcrossing below  $k_c$  is then the fraction of cosmic mass in objects with mass above  $M_s(k_c)$

$$\rightarrow n(M, z) dM = \frac{\rho_0}{\sqrt{(2\pi)M^2}} \frac{\delta_c}{D\sigma_0} \frac{d \ln \sigma_0^2}{d \ln M} \exp \left[ -\frac{1}{2} \left( \frac{\delta_c}{D\sigma_0} \right)^2 \right]$$

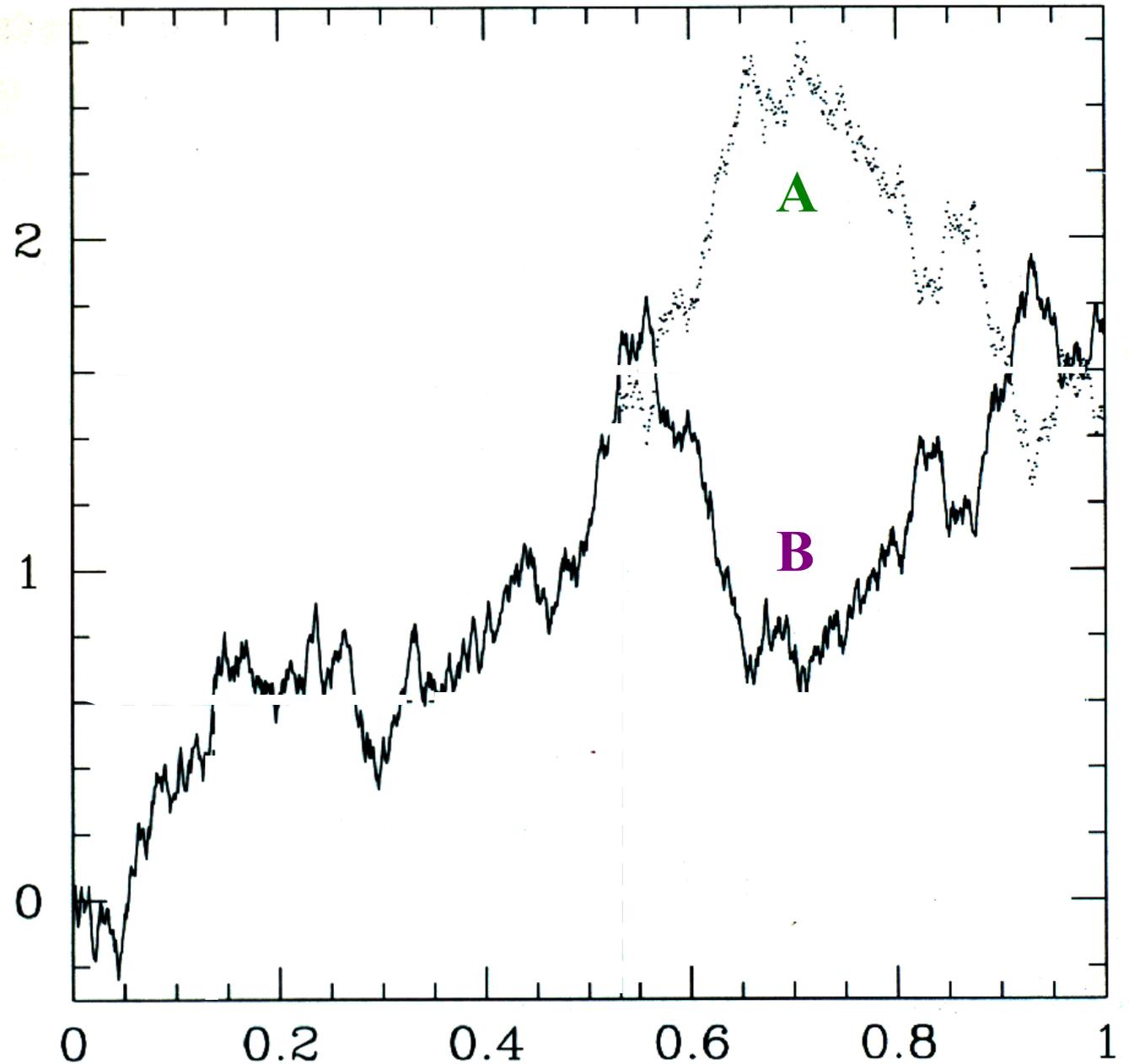
# Overdensity vs smoothing at a given position

If the density field is smoothed using a sharp filter in  $k$ -space, then each step in the random walk is independent of all earlier steps

A Markov process

The walks shown at positions **A** and **B** are equally probable

initial overdensity  $\delta_s/D(\tau)$



variance  $\sigma_0^2(k_c)$  of smoothed field

← mass

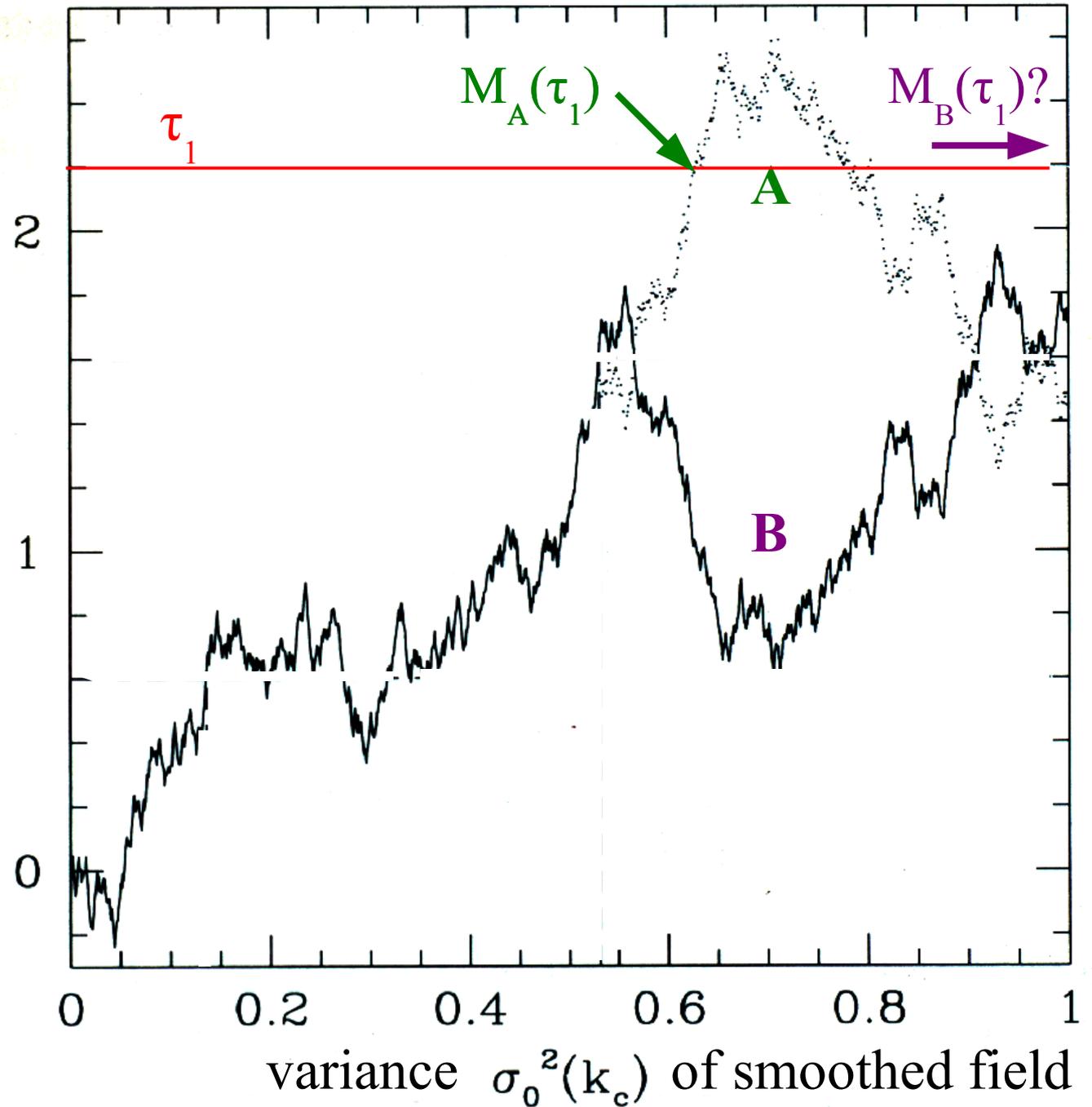
← spatial scale

# Overdensity vs smoothing at a given position

At an early time  $\tau_1$   
**A** is part of a quite massive halo

**B** is part of a very low mass halo or no halo at all

initial overdensity  $\delta_s/D(\tau)$



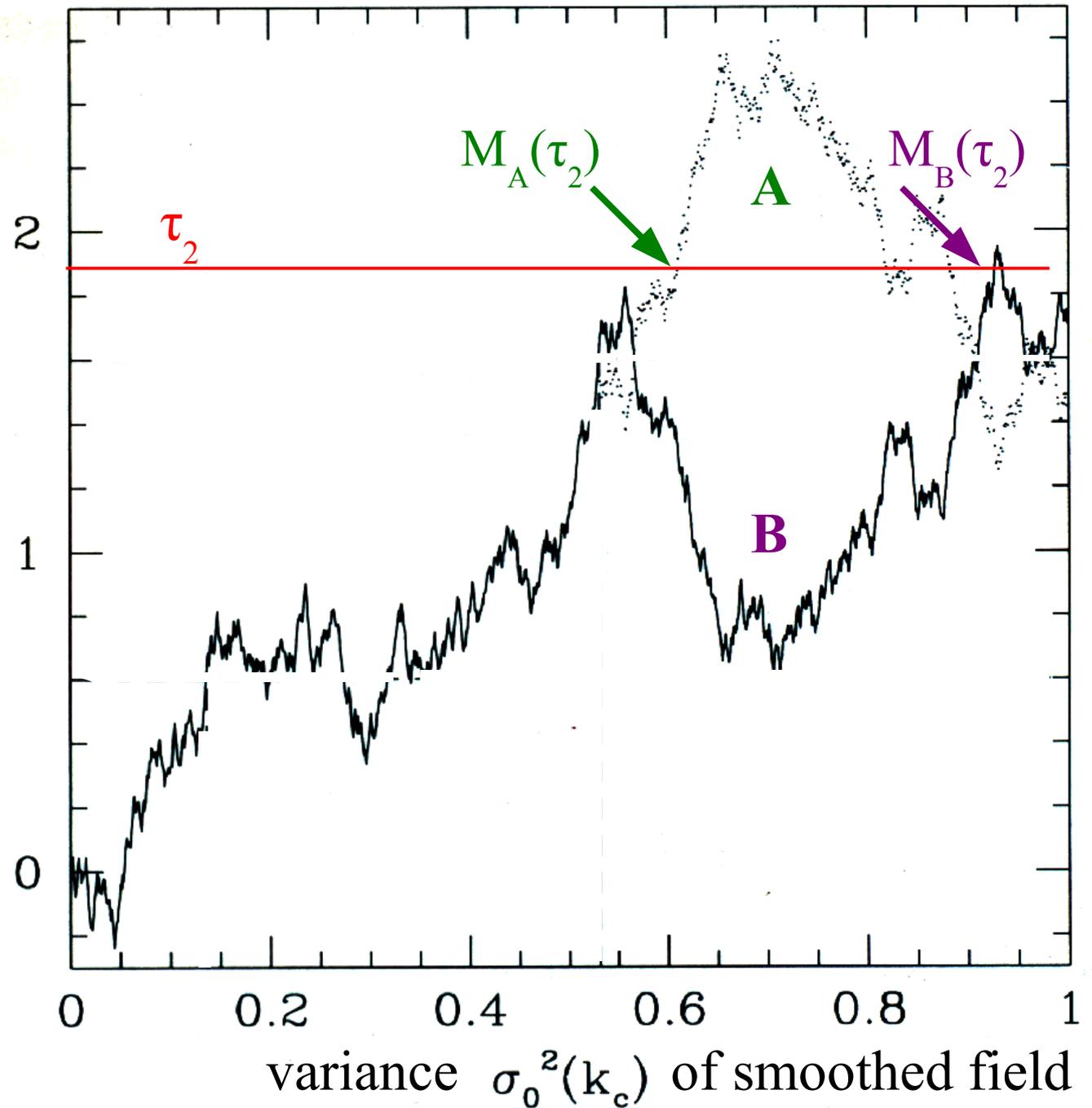
← mass

← spatial scale

# Overdensity vs smoothing at a given position

Later, at time  $\tau_2$   
**A**'s halo has grown slightly by accretion  
**B** is now part of a moderately massive halo

initial overdensity  $\delta_s/D(\tau)$



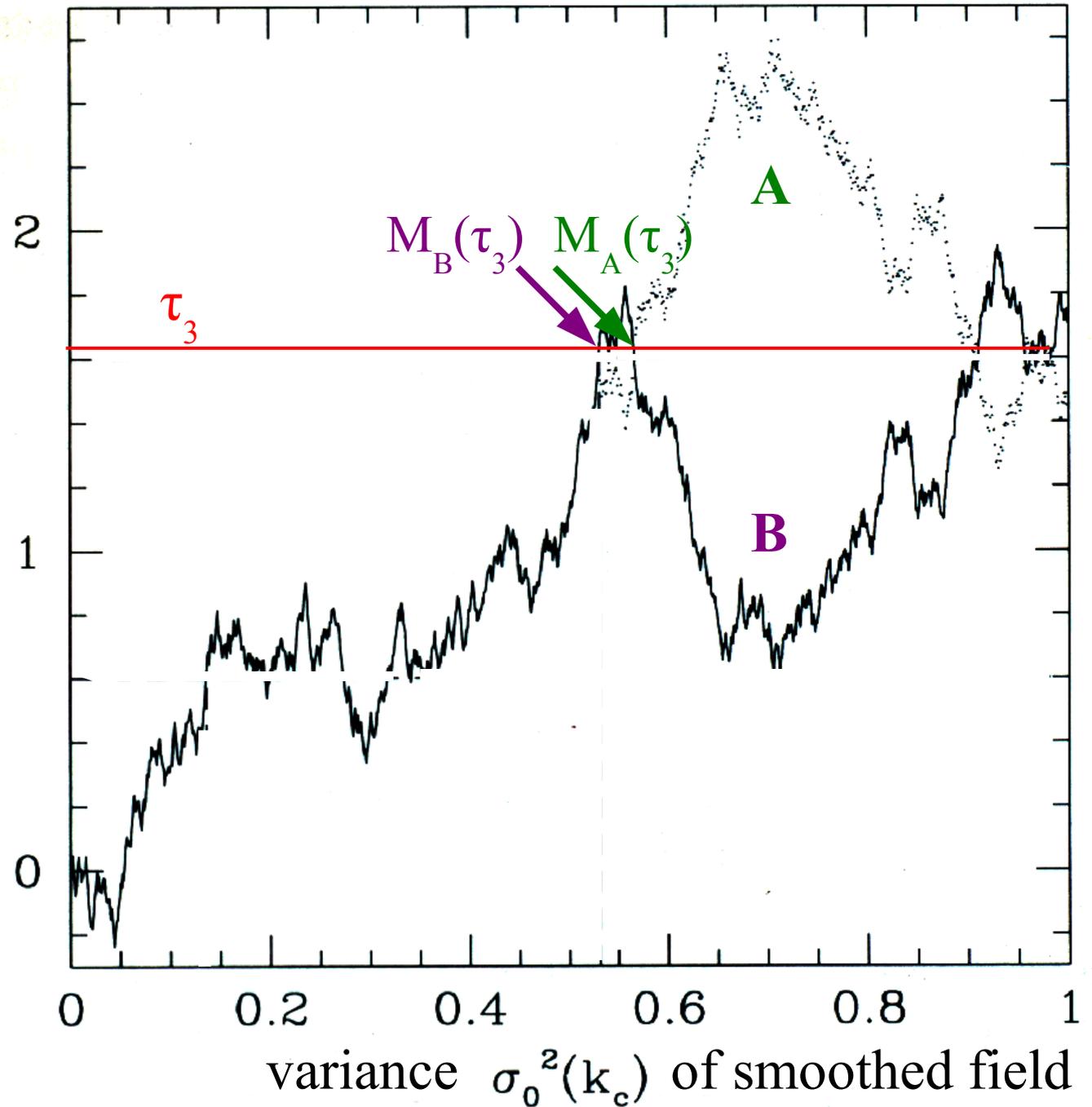
← mass  
← spatial scale

# Overdensity vs smoothing at a given position

A bit later, time  $\tau_3$   
**A**'s halo has grown further by accretion

**B**'s halo has merged again and is now more massive than **A**'s halo

initial overdensity  $\delta_s/D(\tau)$



← mass

← spatial scale

# Overdensity vs smoothing at a given position

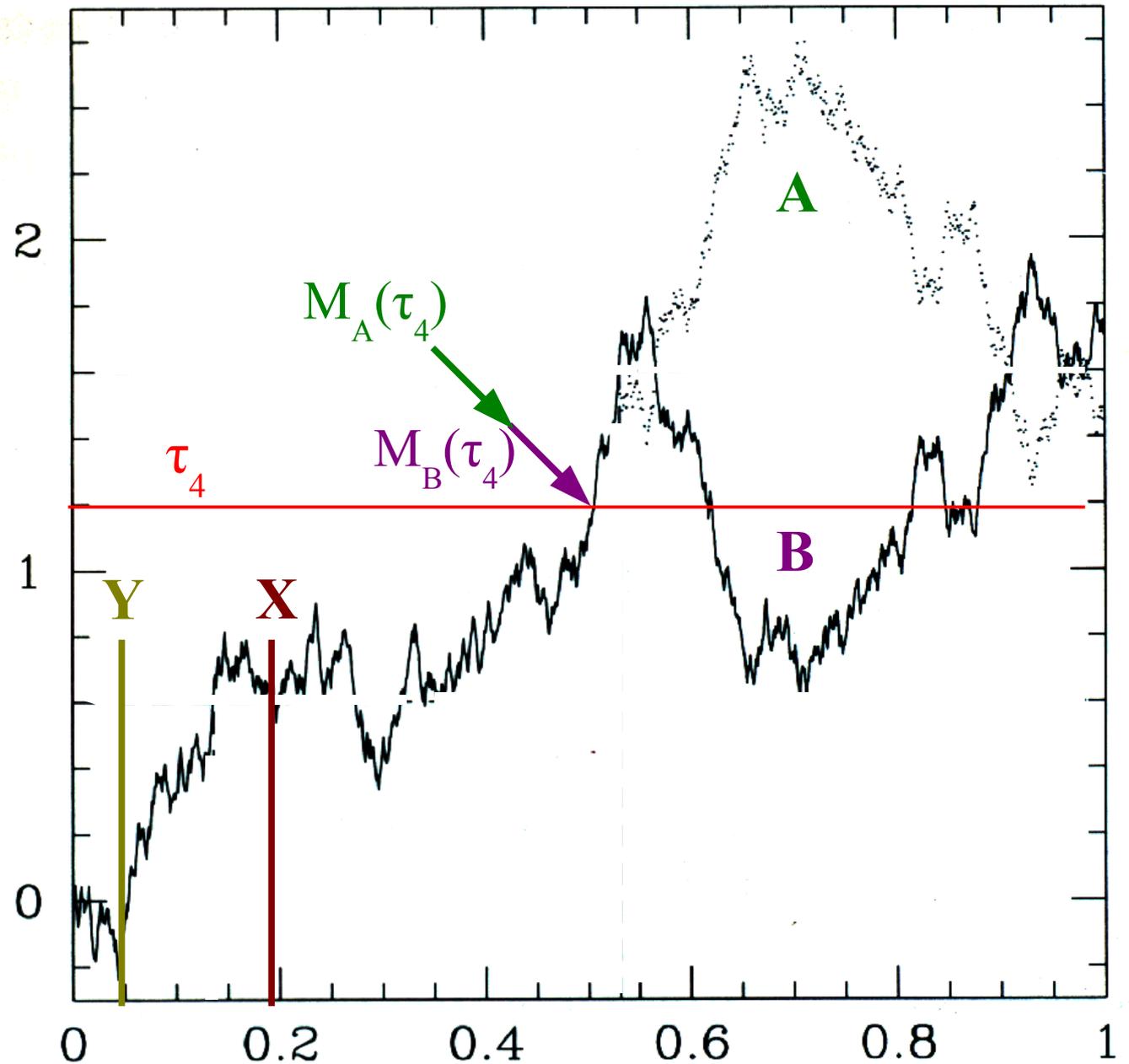
Still later, e.g.  $\tau_4$

**A** and **B** are part of halos which follow identical merging/accretion histories

On scale **X** they are embedded in a high density region.

On larger scale **Y** in a low density region

initial overdensity  $\delta_s/D(\tau)$



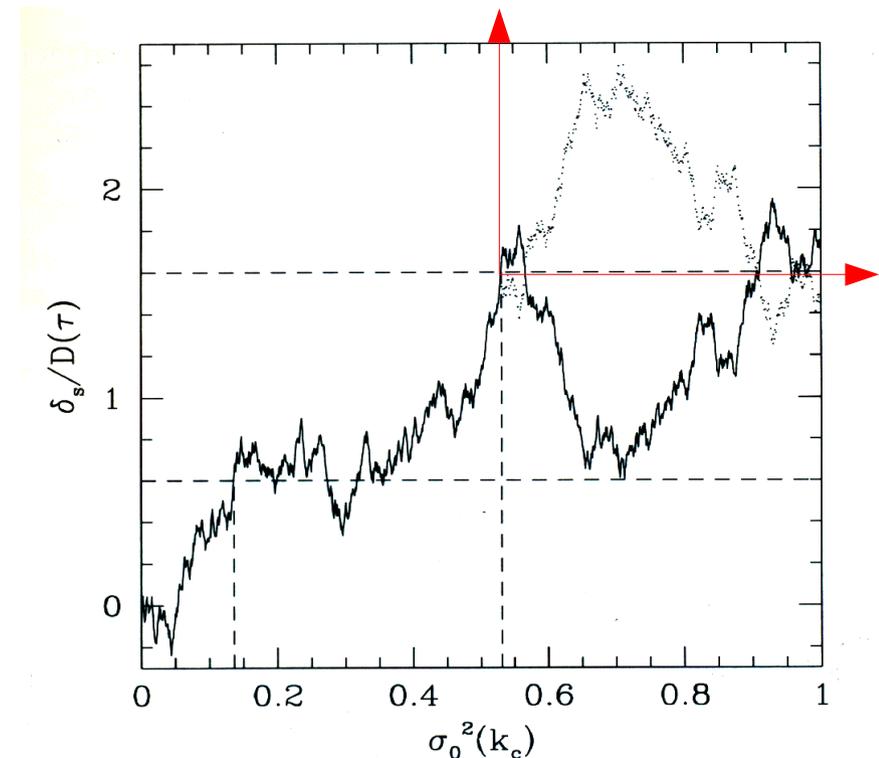
variance  $\sigma_0^2(k_c)$  of smoothed field

← mass

← spatial scale

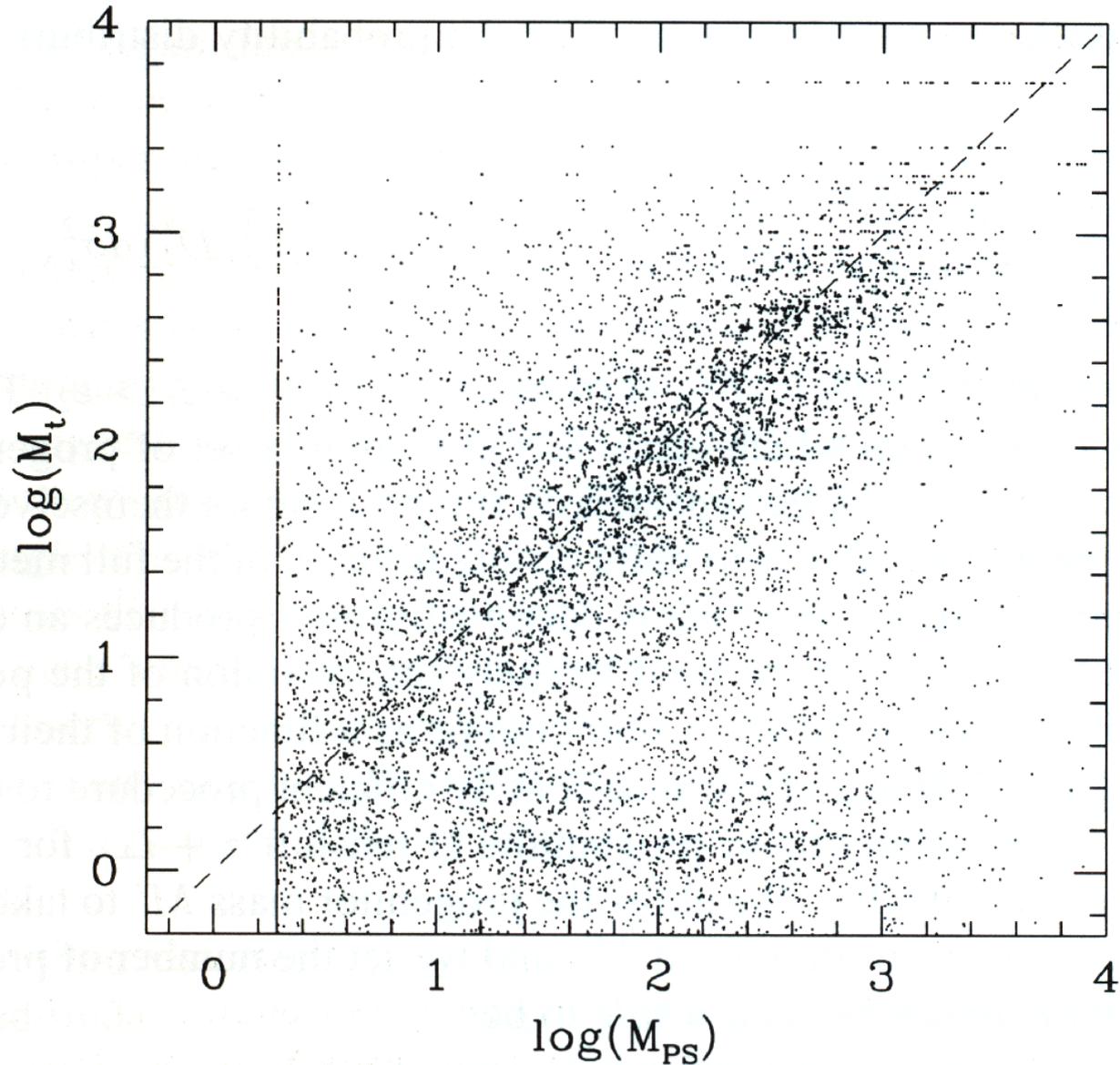
# Consequences of the Markov nature of EPS walks

- The assembly history of a halo is independent of its future
- The assembly history of a halo is independent of its environment
- The internal structure of a halo is independent of its environment
- The mass distribution of progenitors of a halo of given  $M$  and  $z$  is obtained simply by changing the origin to  $\sigma_0^2(M)$  and  $\delta_c/D(z)$
- The resulting formulae can be used to obtain descendant distributions and merger rates
- A similar argument gives formulae for the clustering bias of halos



# Does it work point by point?

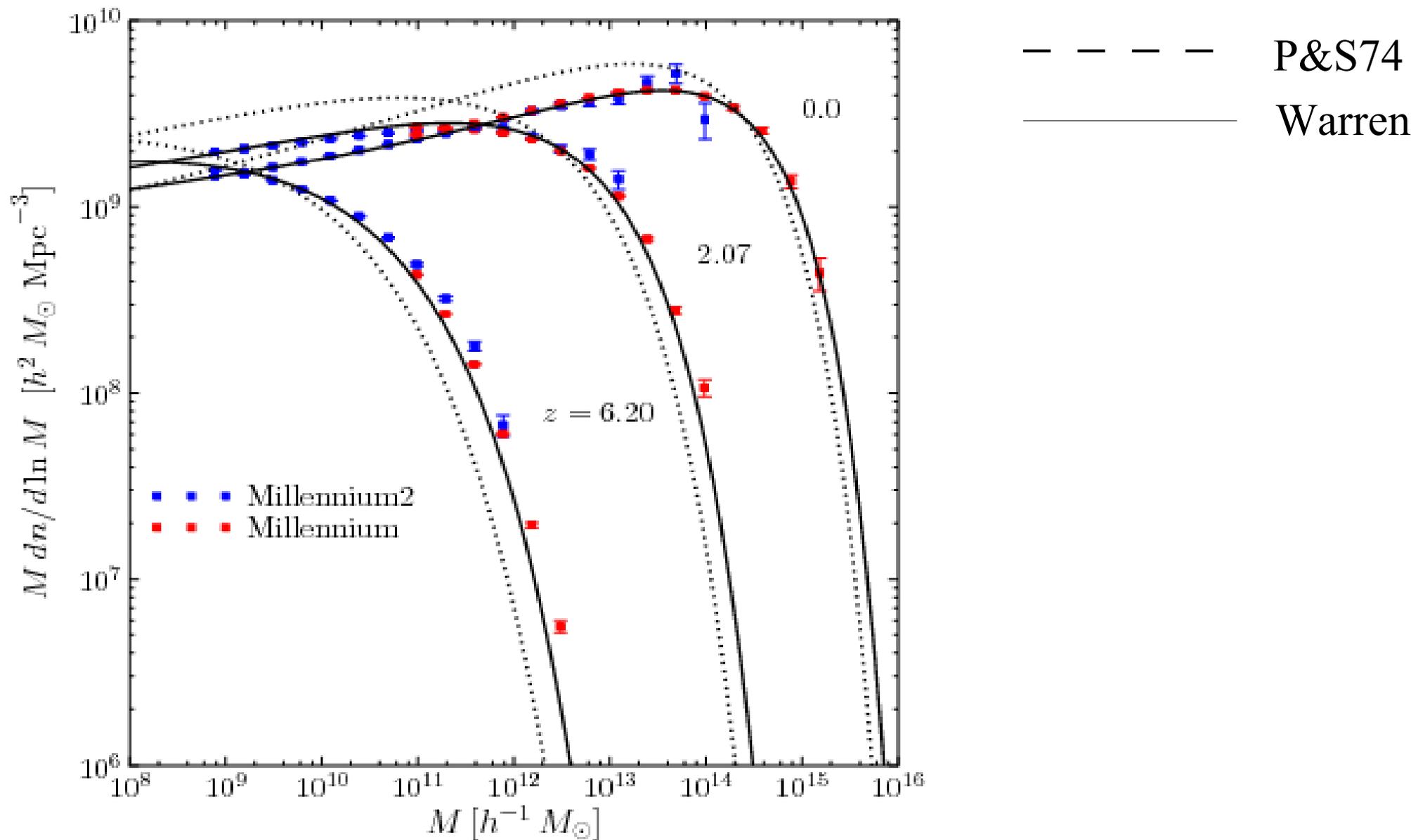
Mass of the halo in which the particle is actually found



Halo mass predicted for each particle by its own sharp k-space random walk

# Does it work statistically?

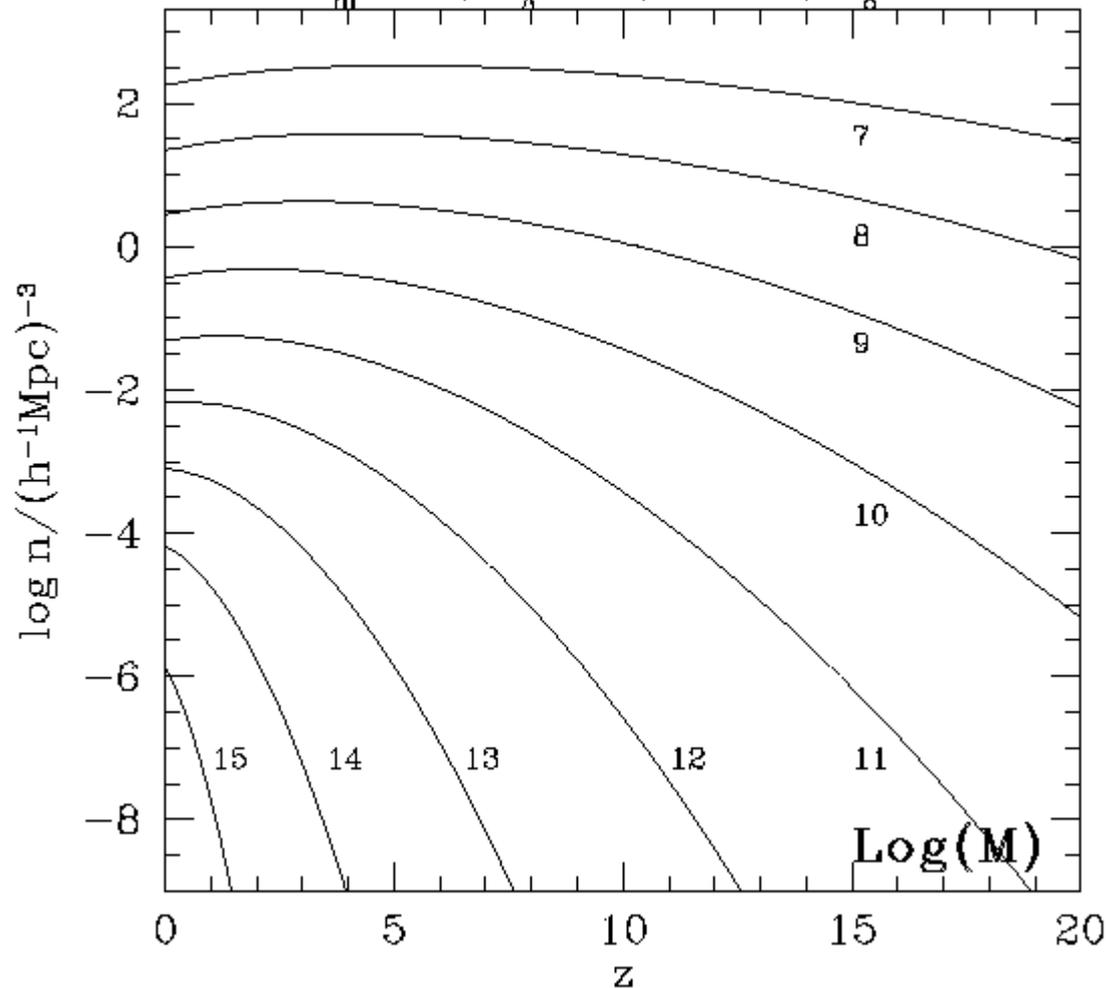
Boylan-Kolchin et al 2009



# Evolution of halo abundance in $\Lambda$ CDM

Mo & White 2002

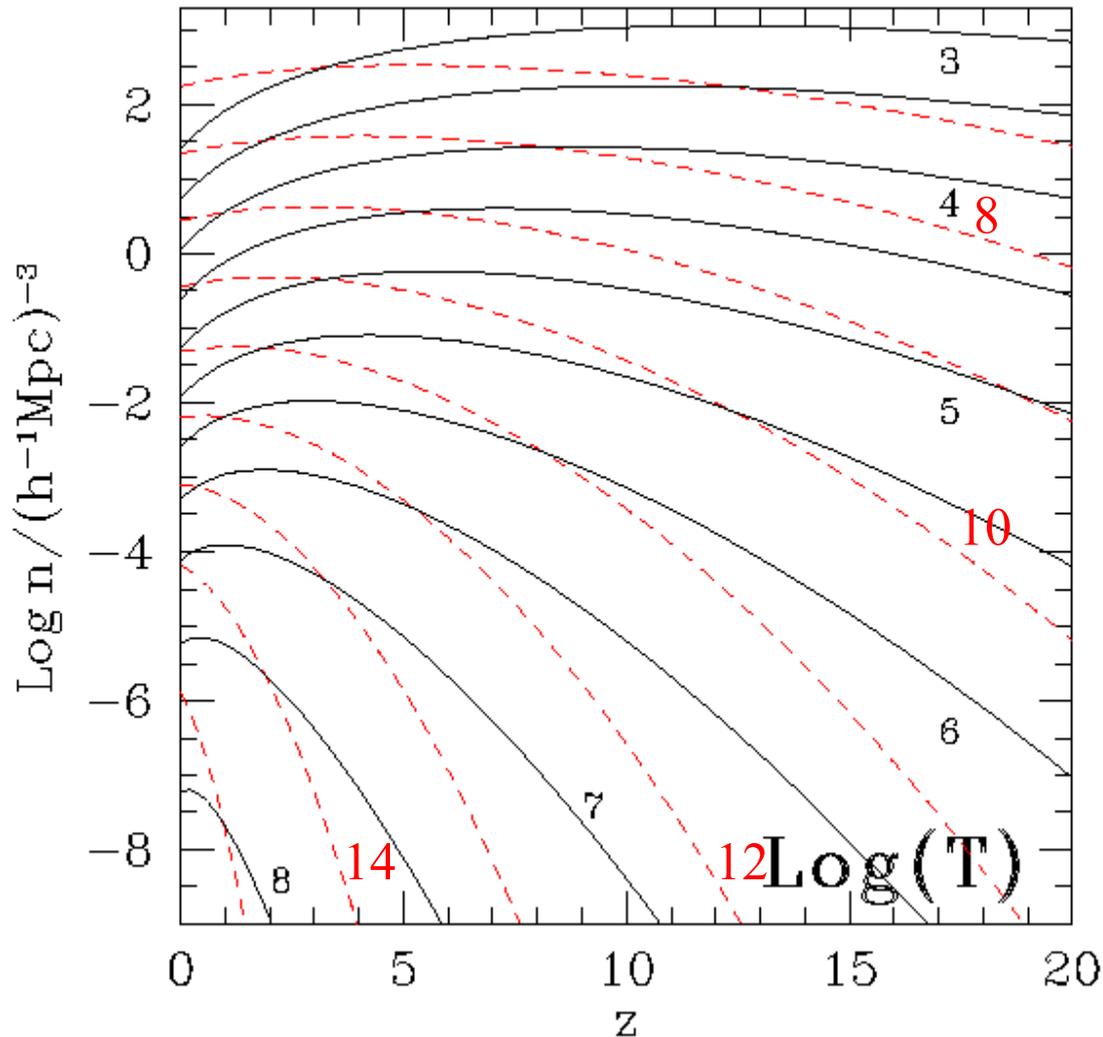
$\Omega_m=0.3, \Omega_\Lambda=0.7, h=0.7, \sigma_8=0.9$



- Abundance of rich cluster halos drops rapidly with  $z$
- Abundance of Milky Way mass halos drops by less than a factor of 10 to  $z=5$
- $10^9 M_\odot$  halos are almost as common at  $z=10$  as at  $z=0$

# Evolution of halo abundance in $\Lambda$ CDM

Mo & White 2002

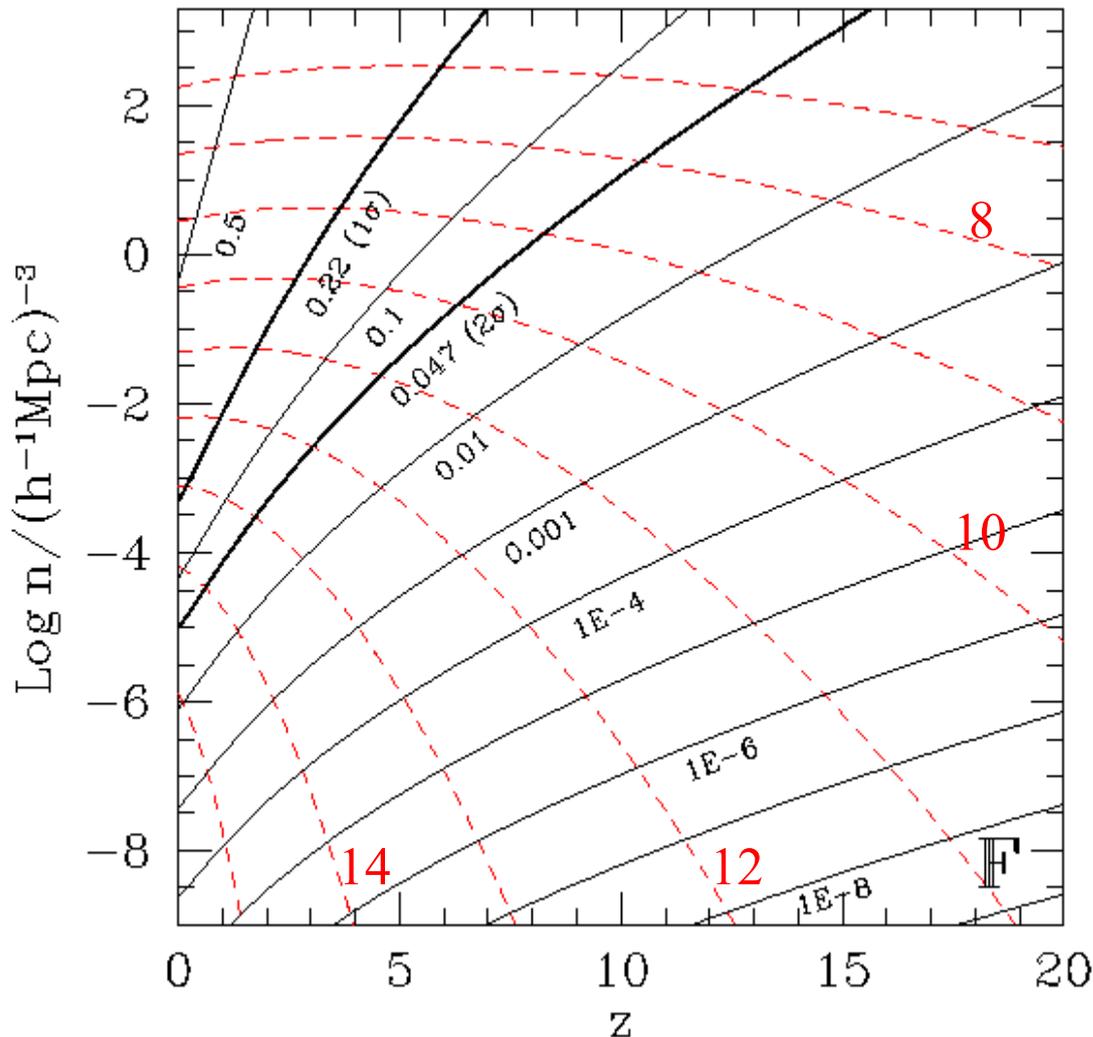


- Temperature increases with both mass and redshift  

$$T \propto M^{2/3} (1+z)$$
- Halos with virial temperature  $T = 10^7$  K are as abundant at  $z = 2$  as at  $z=0$
- Halos with virial temperature  $T = 10^6$  K are as abundant at  $z = 8$  as at  $z=0$
- Halos of mass  $> 10^{7.5} M_{\odot}$  have  $T > 10^4$  K at  $z=20$  and so can cool by H line emission

# Evolution of halo abundance in $\Lambda$ CDM

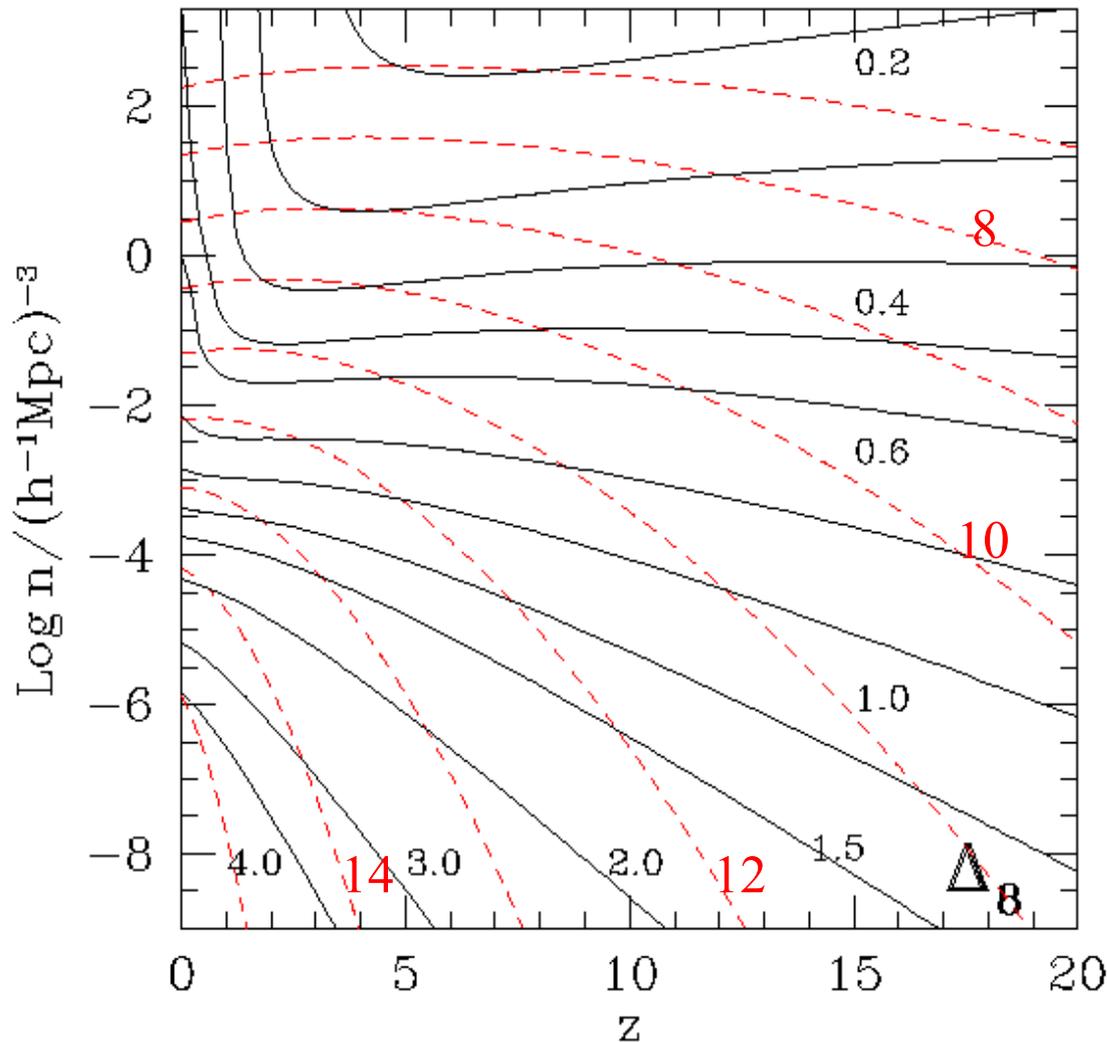
Mo & White 2002



- Half of all mass is in halos more massive than  $10^{10} M_{\odot}$  at  $z=0$ , but only 10% at  $z=5$ , 1% at  $z=9$  and  $10^{-6}$  at  $z=20$
- 1% of all mass is in halos more massive than  $10^{15} M_{\odot}$  at  $z=0$
- 40% of all mass at  $z=0$  is in halos which cannot confine photoionised gas
- 1% of all mass at  $z=15$  is in halos hot enough to cool by H line emission

# Evolution of halo abundance in $\Lambda$ CDM

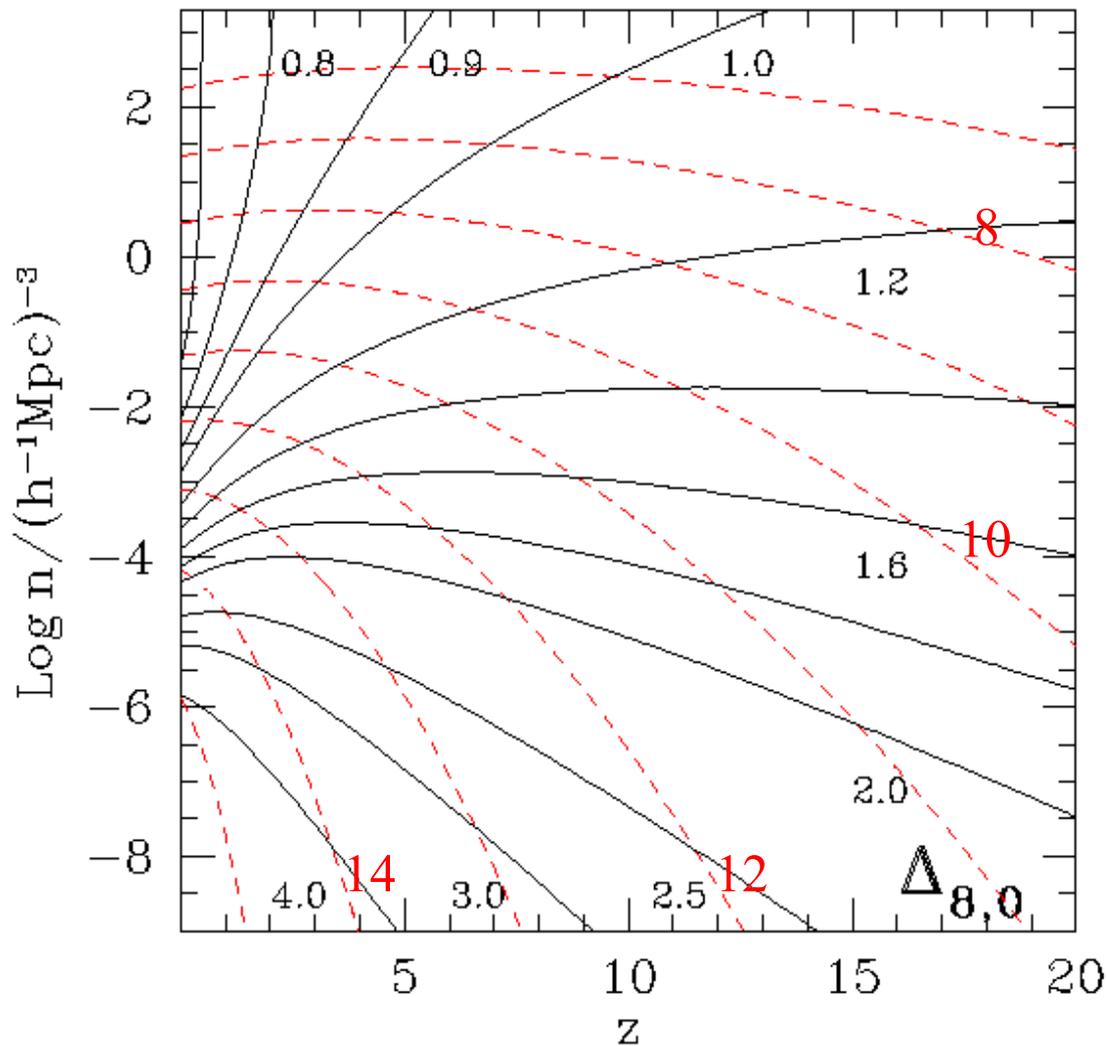
Mo & White 2002



- Halos with the abundance of  $L_*$  galaxies at  $z=0$  are equally strongly clustered at all  $z < 20$
- Halos of given mass or virial temperature are more clustered at *higher*  $z$

# Evolution of halo abundance in $\Lambda$ CDM

Mo & White 2002



- The remnants (stars and heavy elements) from all star-forming systems at  $z > 6$  are today more clustered than  $L_*$  galaxies
- The remnants of objects which at any  $z > 2$  had an abundance similar to that of present-day  $L_*$  galaxies are today more clustered than  $L_*$  galaxies

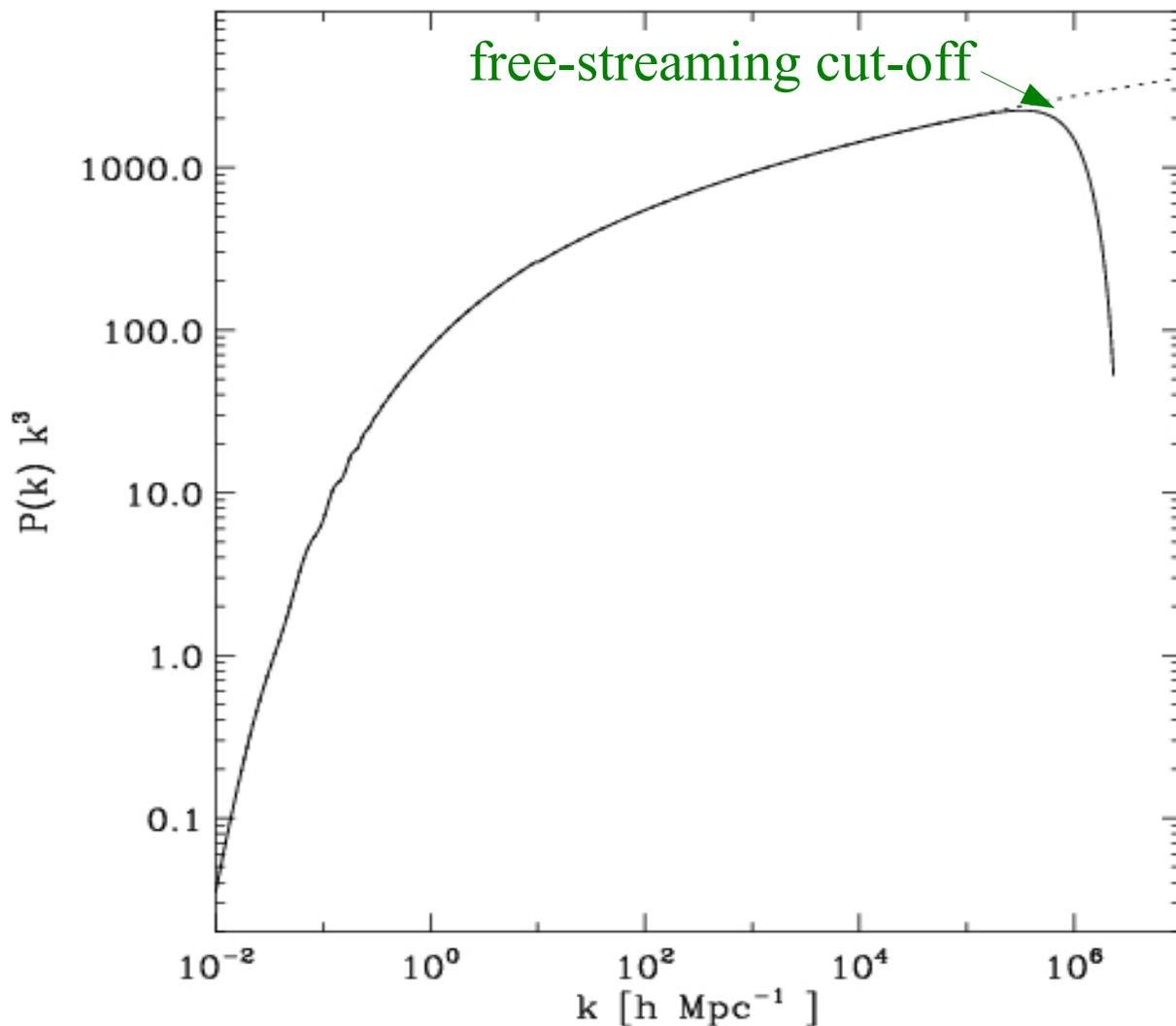
# Lecture Plan

- I** Introduction to the standard model of cosmogony  
--- ingredients, observational support, possible problems
- II** Halos – structure growth, PS and EPS theory, applications
- III** Halos – assembly histories, profiles, core problems
- IV** Halos – shapes, substructure, satellites, DM detection
- V** Abundance matching, MW halo mass + formation efficiency
- VI** Gas cooling and condensation (hot and cold), disk formation
- VII** Simulations – techniques, what can and can't be done

# EPS statistics for the standard $\Lambda$ CDM cosmology

Millennium Simulation cosmology:  $\Omega_m = 0.25$ ,  $\Omega_\Lambda = 0.75$ ,  $n=1$ ,  $\sigma_8 = 0.9$

Angulo & White 2010



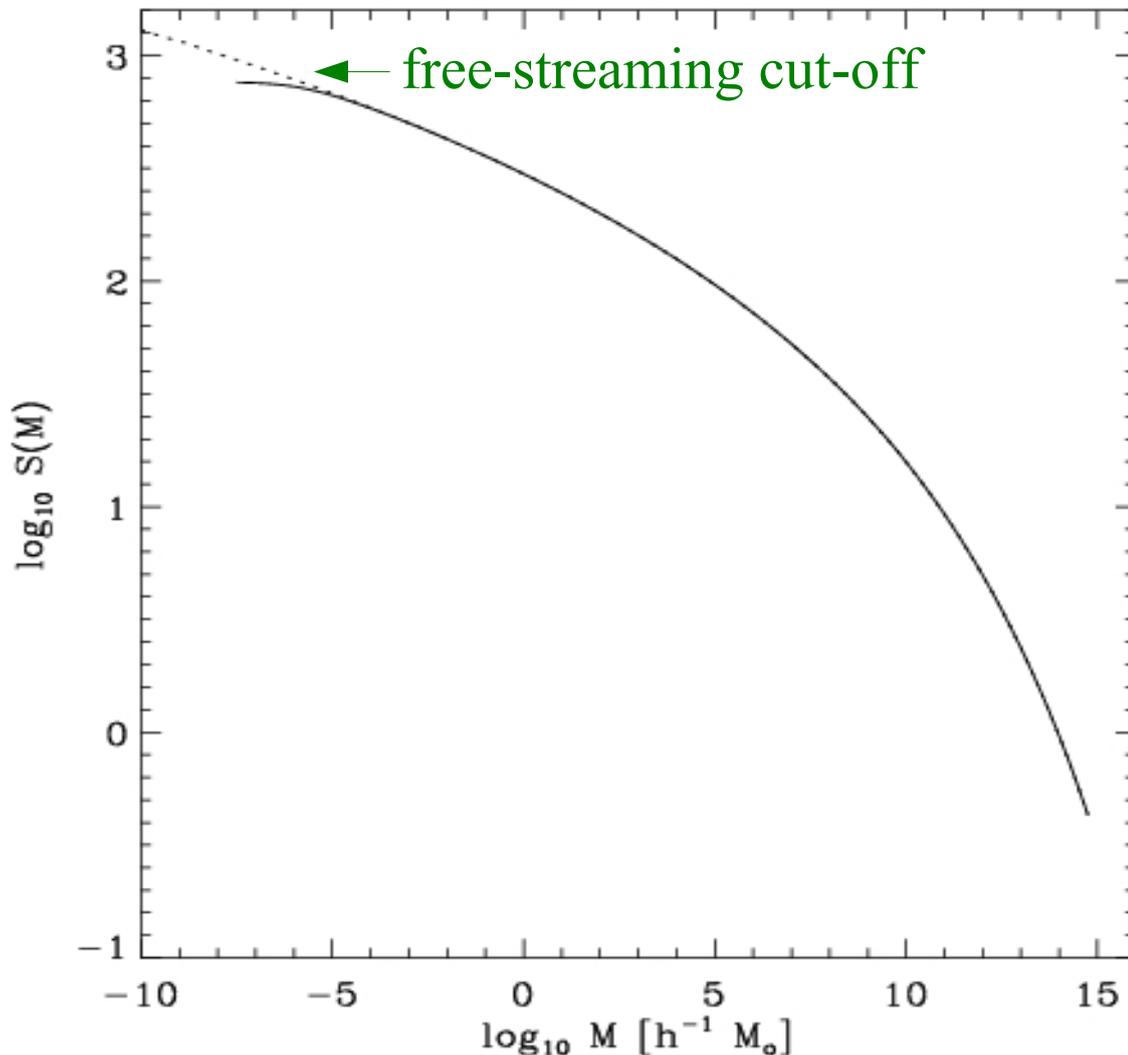
The linear power spectrum in “power per octave” form

Assumes a 100GeV wimp following Green et al (2004)

# EPS statistics for the standard $\Lambda$ CDM cosmology

Millennium Simulation cosmology:  $\Omega_m = 0.25$ ,  $\Omega_\Lambda = 0.75$ ,  $n=1$ ,  $\sigma_8 = 0.9$

Angulo & White 2010

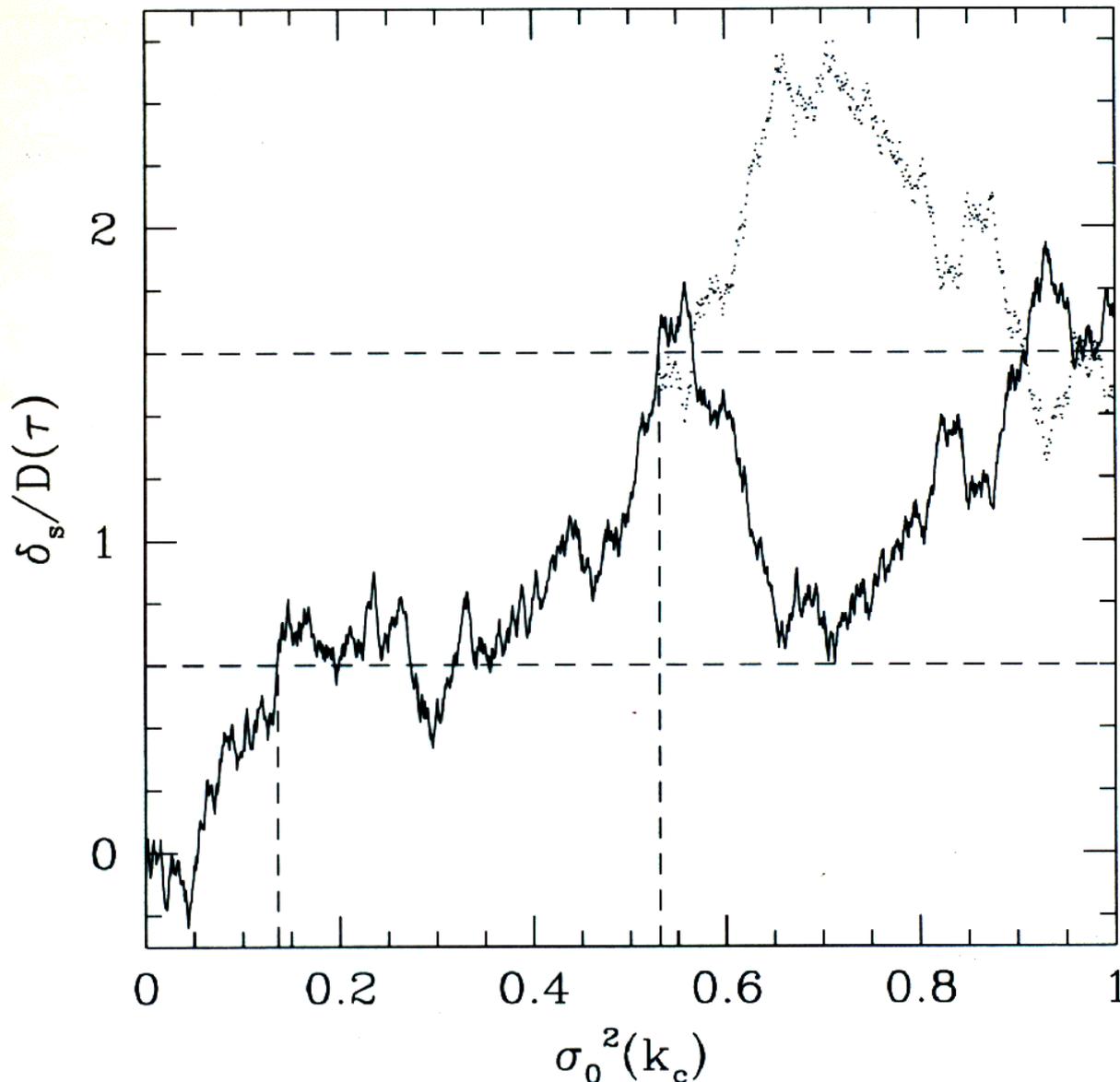


Variance of linear density fluctuation within spheres containing mass  $M$ , extrapolated to  $z = 0$

As  $M \rightarrow 0$ ,  $S(M) \rightarrow 720$

# EPS statistics for the standard $\Lambda$ CDM cosmology

Millennium Simulation cosmology:  $\Omega_m = 0.25$ ,  $\Omega_\Lambda = 0.75$ ,  $n=1$ ,  $\sigma_8 = 0.9$



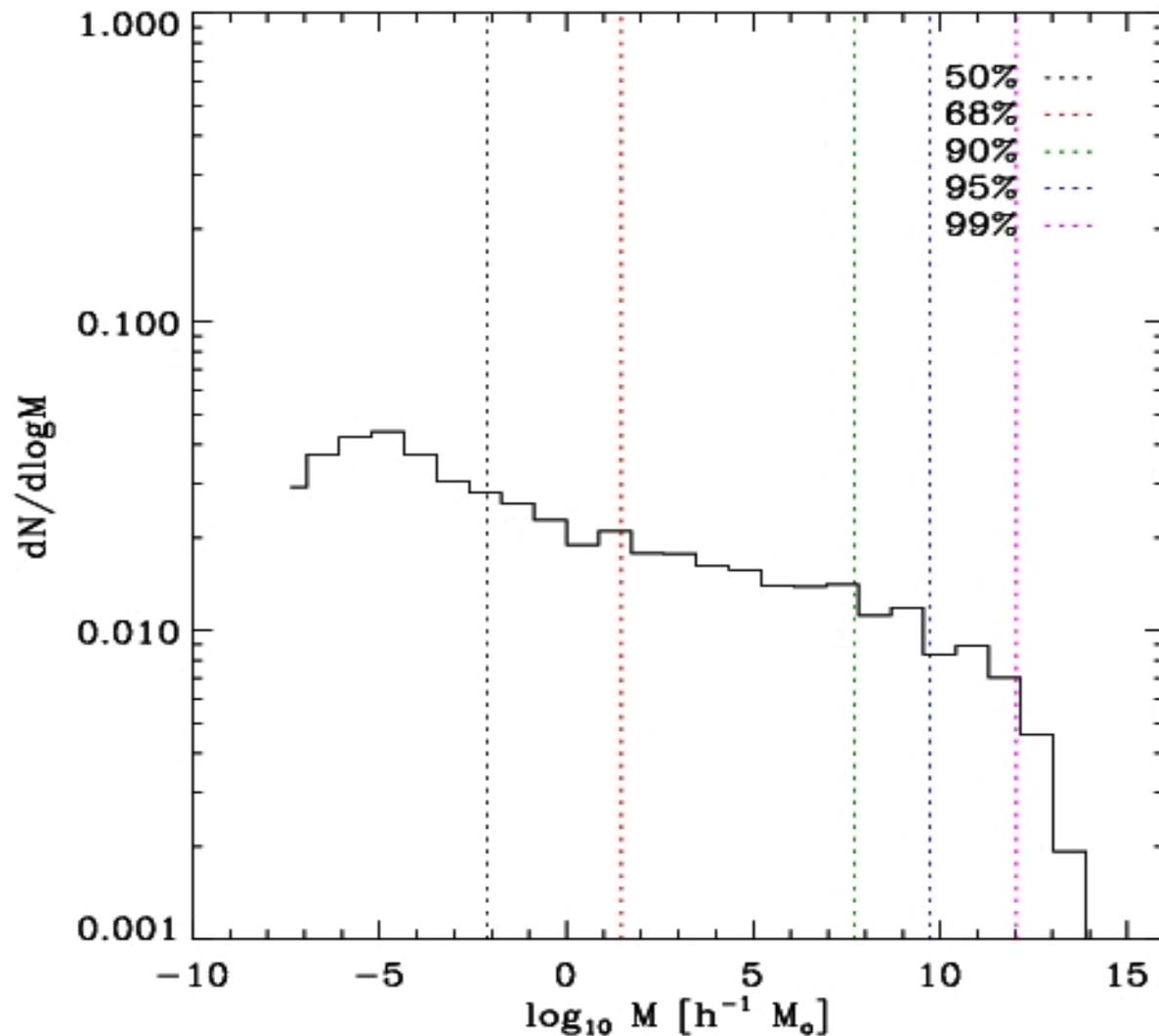
If these Markov random walks are scaled so the maximum variance is 720 and the vertical axis is multiplied by  $\sqrt{720}$ , then they represent complete halo assembly histories for random CDM particles.

An ensemble of walks thus represents the probability distribution of assembly histories

# EPS statistics for the standard $\Lambda$ CDM cosmology

Millennium Simulation cosmology:  $\Omega_m = 0.25$ ,  $\Omega_\Lambda = 0.75$ ,  $n=1$ ,  $\sigma_8 = 0.9$

Angulo & White 2010



Distribution of the masses of the first halos for a random set of dark matter particles

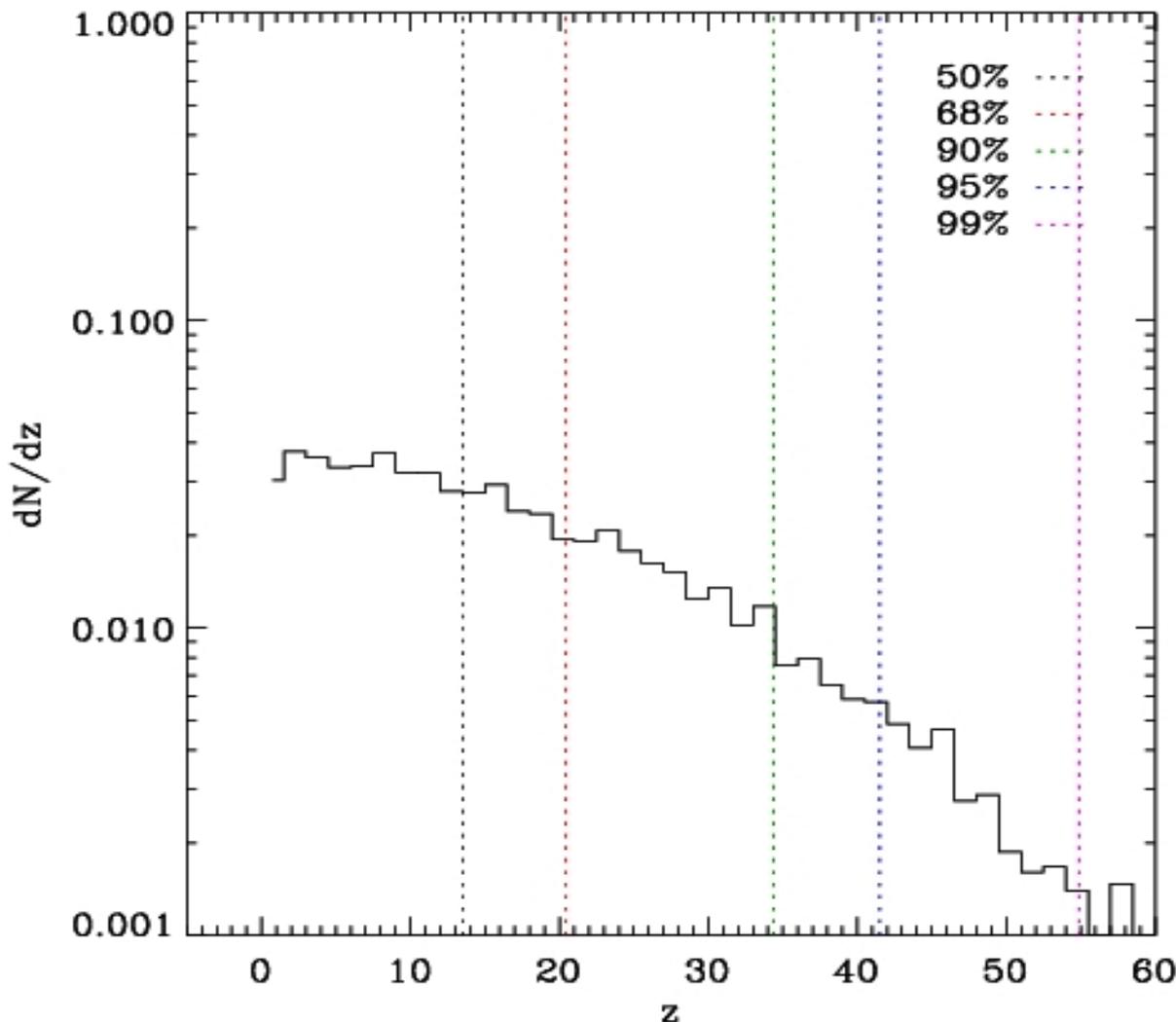
The median is  $10^{-2} M_\odot$

For 10% of the mass the first halo has  $M > 10^7 M_\odot$

# EPS statistics for the standard $\Lambda$ CDM cosmology

Millennium Simulation cosmology:  $\Omega_m = 0.25$ ,  $\Omega_\Lambda = 0.75$ ,  $n=1$ ,  $\sigma_8 = 0.9$

Angulo & White 2010



Distribution of the collapse redshifts of the first halos for a random set of dark matter particles

The median is  $z = 13$

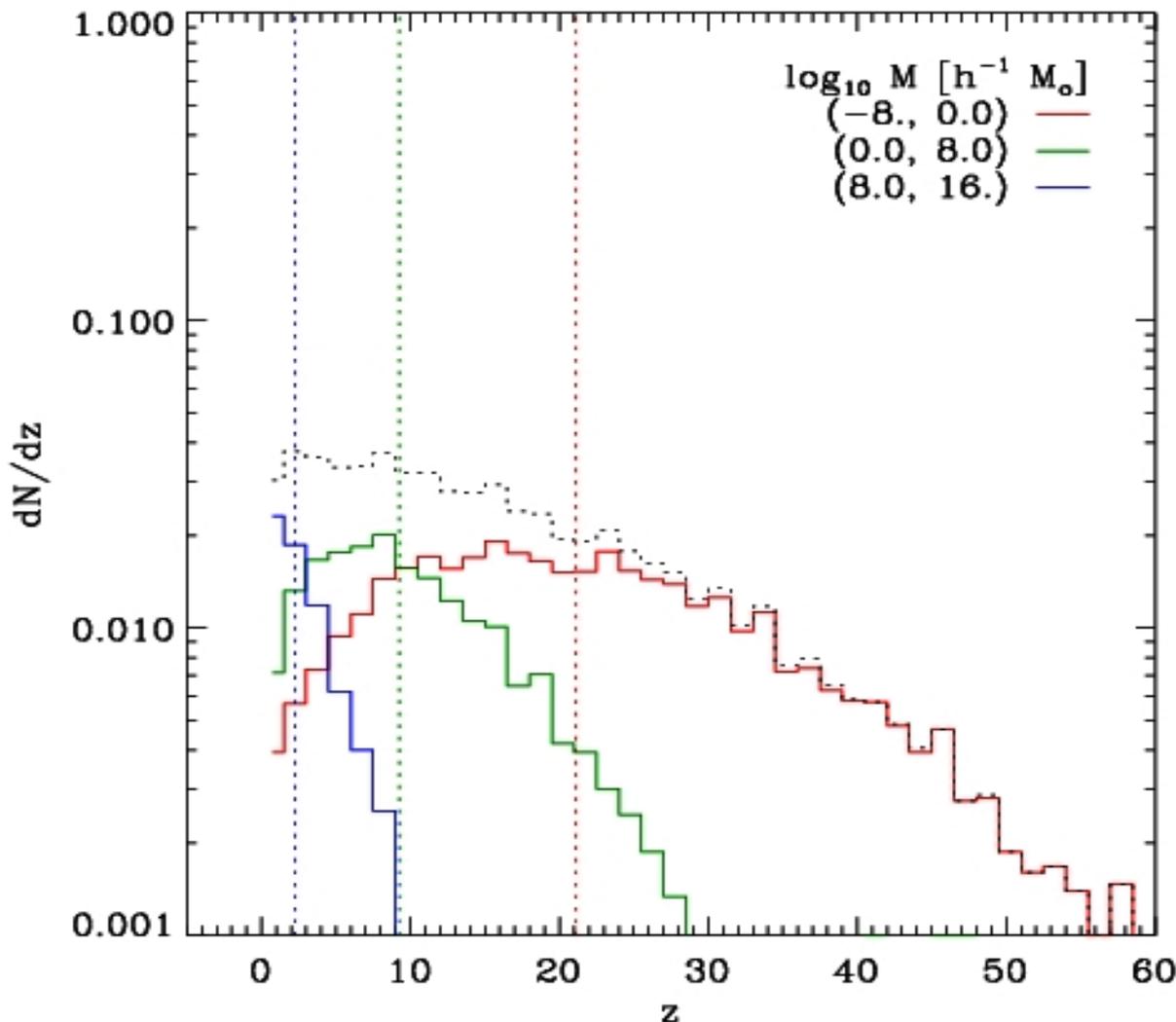
For 10% of the mass the first halo collapses at  $z > 34$

For 1% at  $z > 55$

# EPS statistics for the standard $\Lambda$ CDM cosmology

Millennium Simulation cosmology:  $\Omega_m = 0.25$ ,  $\Omega_\Lambda = 0.75$ ,  $n=1$ ,  $\sigma_8 = 0.9$

Angulo & White 2010



Distribution of the collapse redshifts of the first halos for dark matter particles split by the mass of the first object

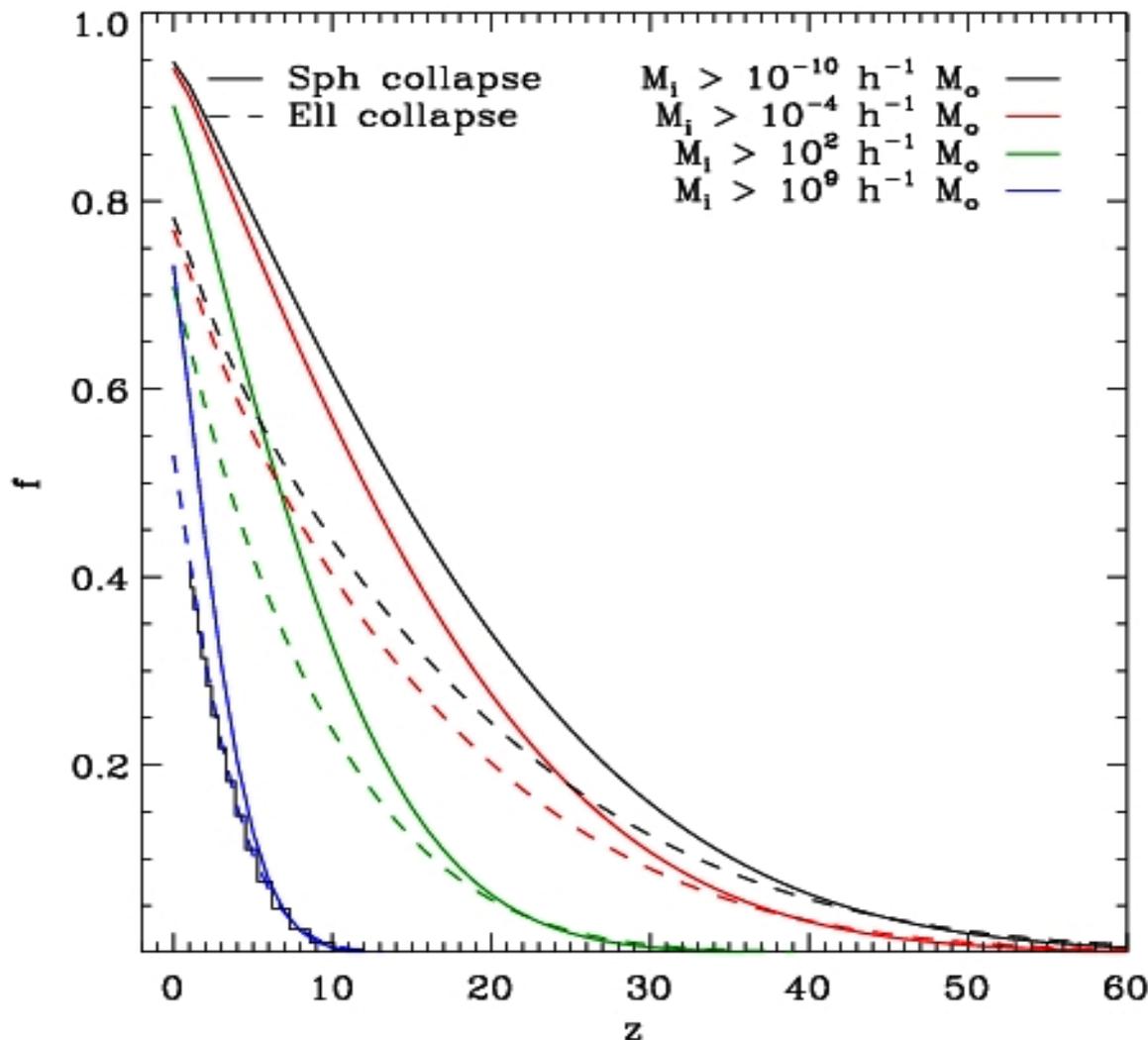
The high redshift tail is entirely due to matter in small first halos

For first halo masses below a solar mass, the median collapse redshift is  $z = 21$

# EPS statistics for the standard $\Lambda$ CDM cosmology

Millennium Simulation cosmology:  $\Omega_m = 0.25$ ,  $\Omega_\Lambda = 0.75$ ,  $n=1$ ,  $\sigma_8 = 0.9$

Angulo & White 2010



Total mass fraction in halos

At  $z = 0$  about 5% (Sph) or 20% (Ell) of the mass is still diffuse

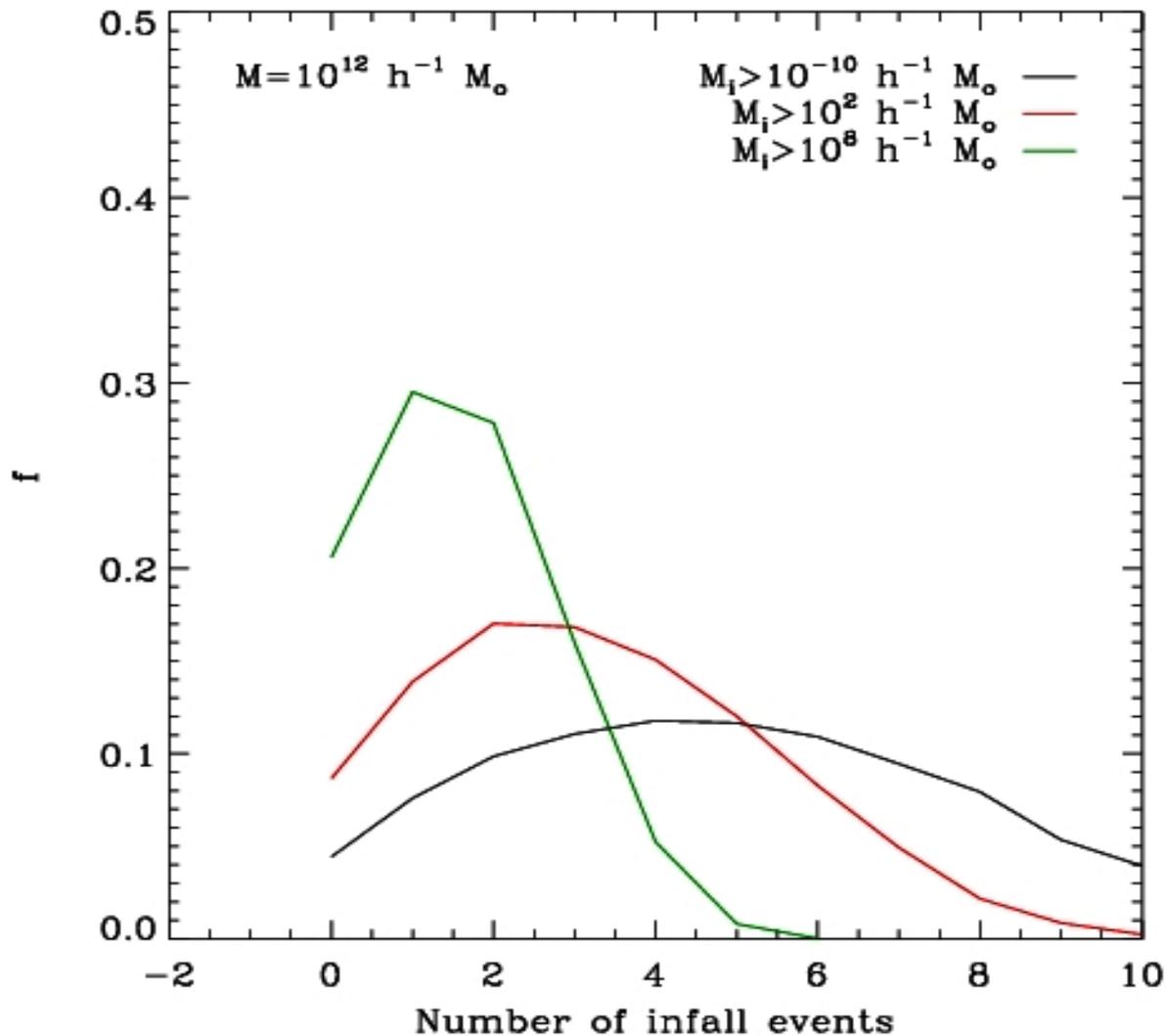
Beyond  $z = 50$  almost all the mass is diffuse

Only at  $z < 2$  (Sph) or  $z < 0.5$  (Ell) is most mass in halos with  $M > 10^8 M_\odot$ . The “Ell” curve agrees with simulations

# EPS statistics for the standard $\Lambda$ CDM cosmology

Millennium Simulation cosmology:  $\Omega_m = 0.25$ ,  $\Omega_\Lambda = 0.75$ ,  $n=1$ ,  $\sigma_8 = 0.9$

Angulo & White 2010



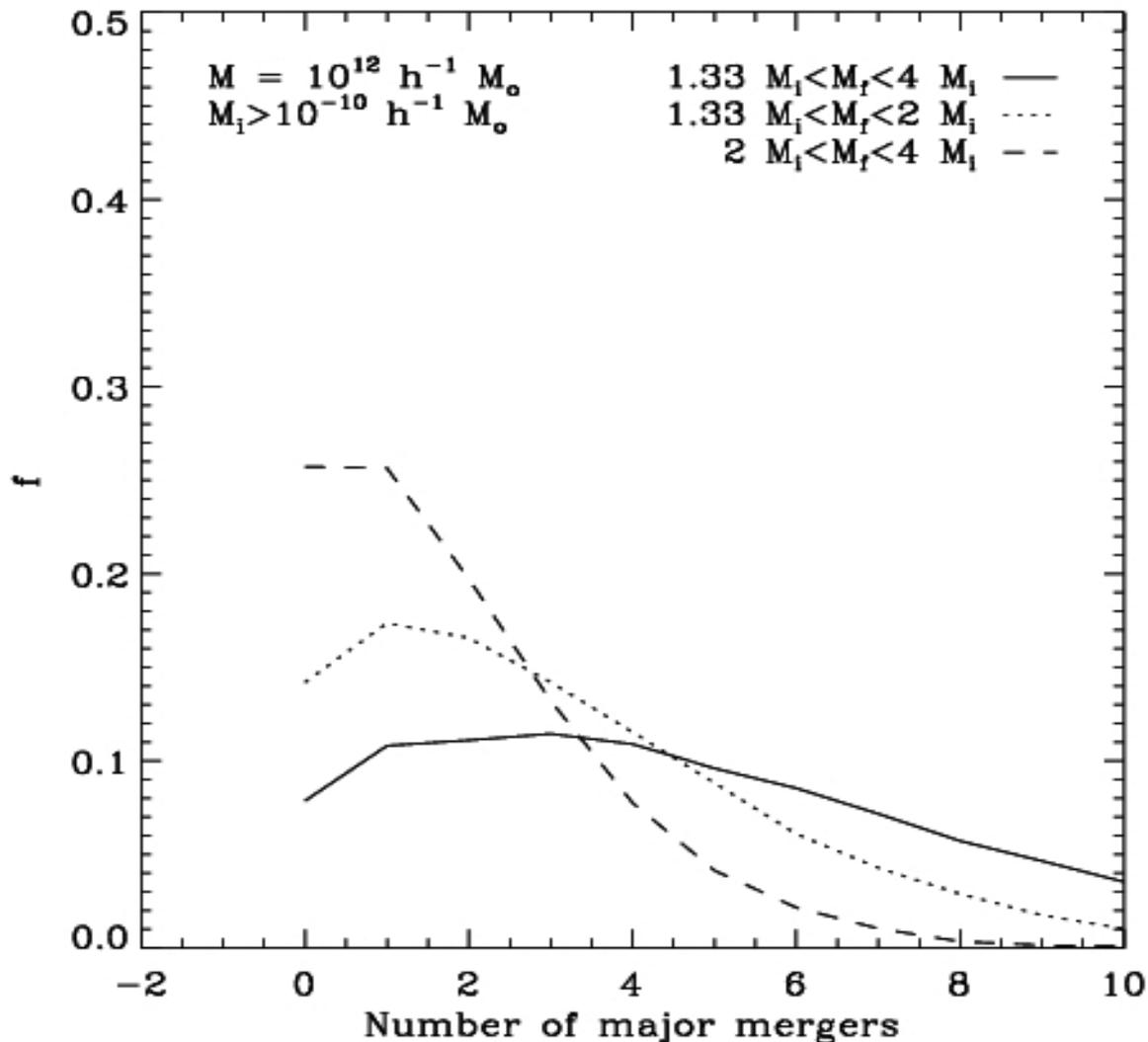
The typical mass element in a “Milky Way” halo goes through  $\sim 5$  “infall events” where its halo falls into a halo bigger than itself.

Typically only one of these is as part of a halo with  $M > 10^8 M_\odot$ .

# EPS statistics for the standard $\Lambda$ CDM cosmology

Millennium Simulation cosmology:  $\Omega_m = 0.25$ ,  $\Omega_\Lambda = 0.75$ ,  $n=1$ ,  $\sigma_8 = 0.9$

Angulo & White 2010



The typical mass element in a “Milky Way” halo goes through  $\sim 3$  “major mergers” where the two halos are within a factor of 3 in mass

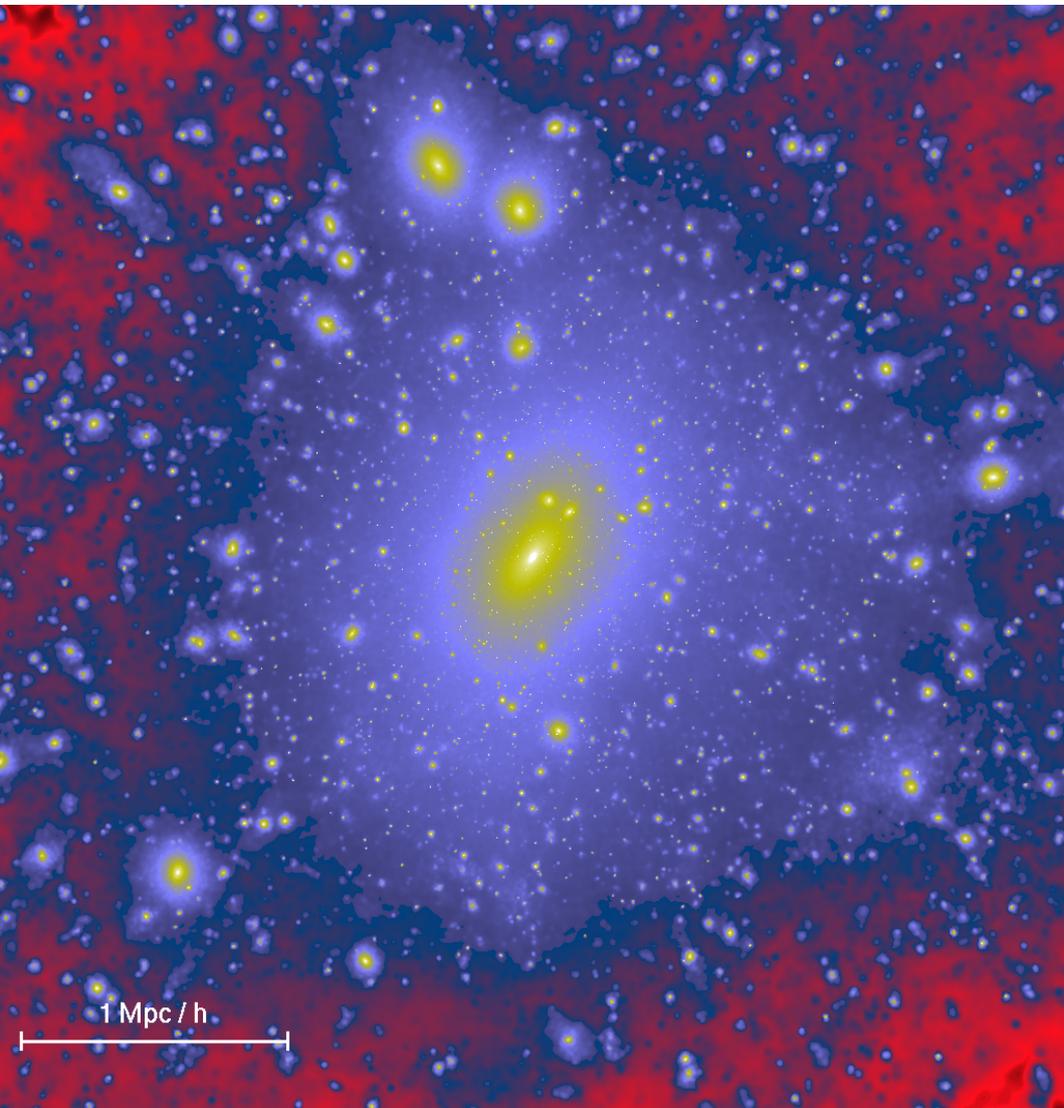
The majority of these occur when the element is part of the larger halo

# EPS halo assembly: conclusions

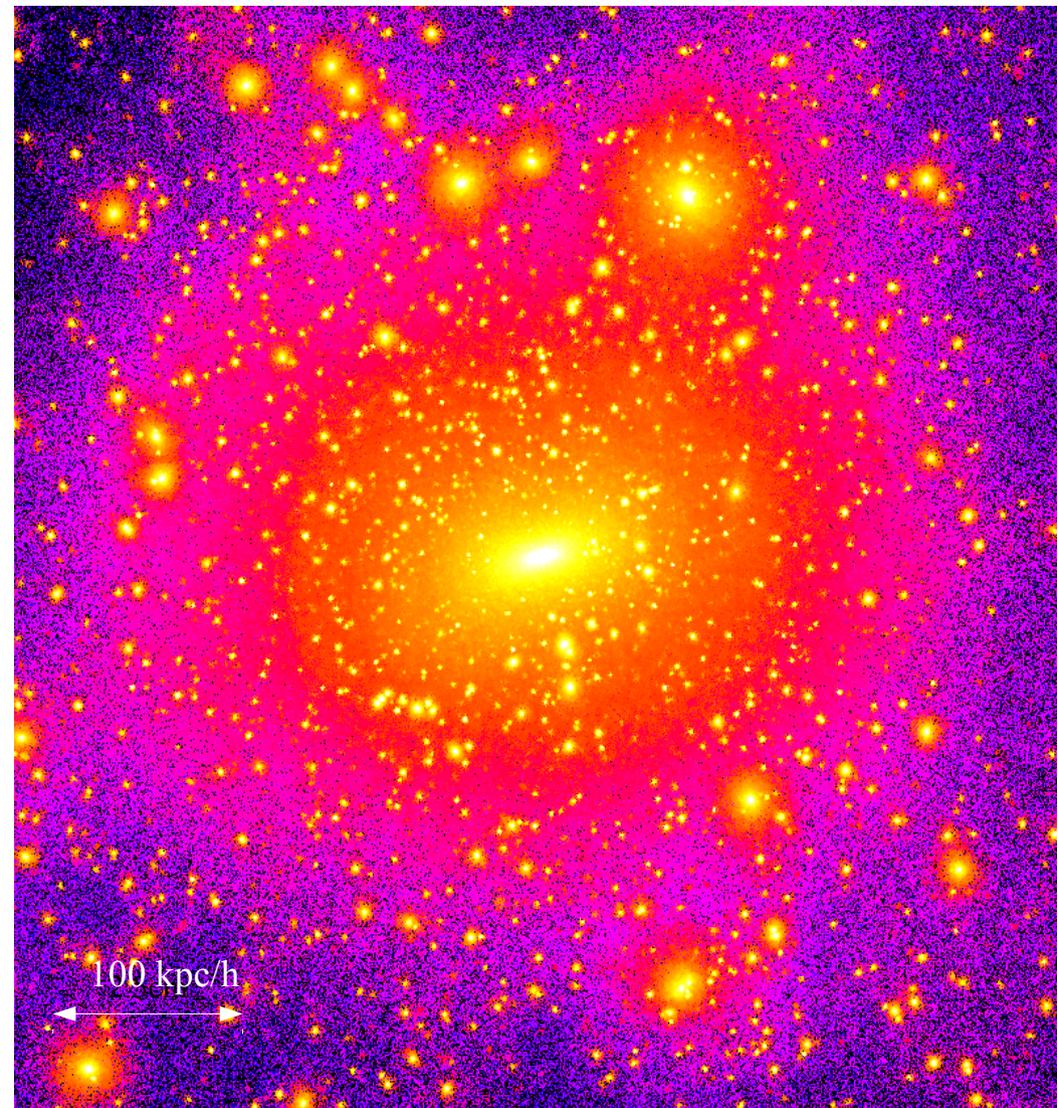
- The typical first halo is much more massive than the free streaming mass
- First halos typically collapse quite late  $z \sim 13$
- Halo growth occurs mainly by accretion of much smaller halos
- There are rather few “generations” of accretion/merger events
- Major mergers are not a major part of the growth of many halos

# The dark matter structure of $\Lambda$ CDM halos

A rich galaxy cluster halo  
Springel et al 2001



A 'Milky Way' halo  
Power et al 2002

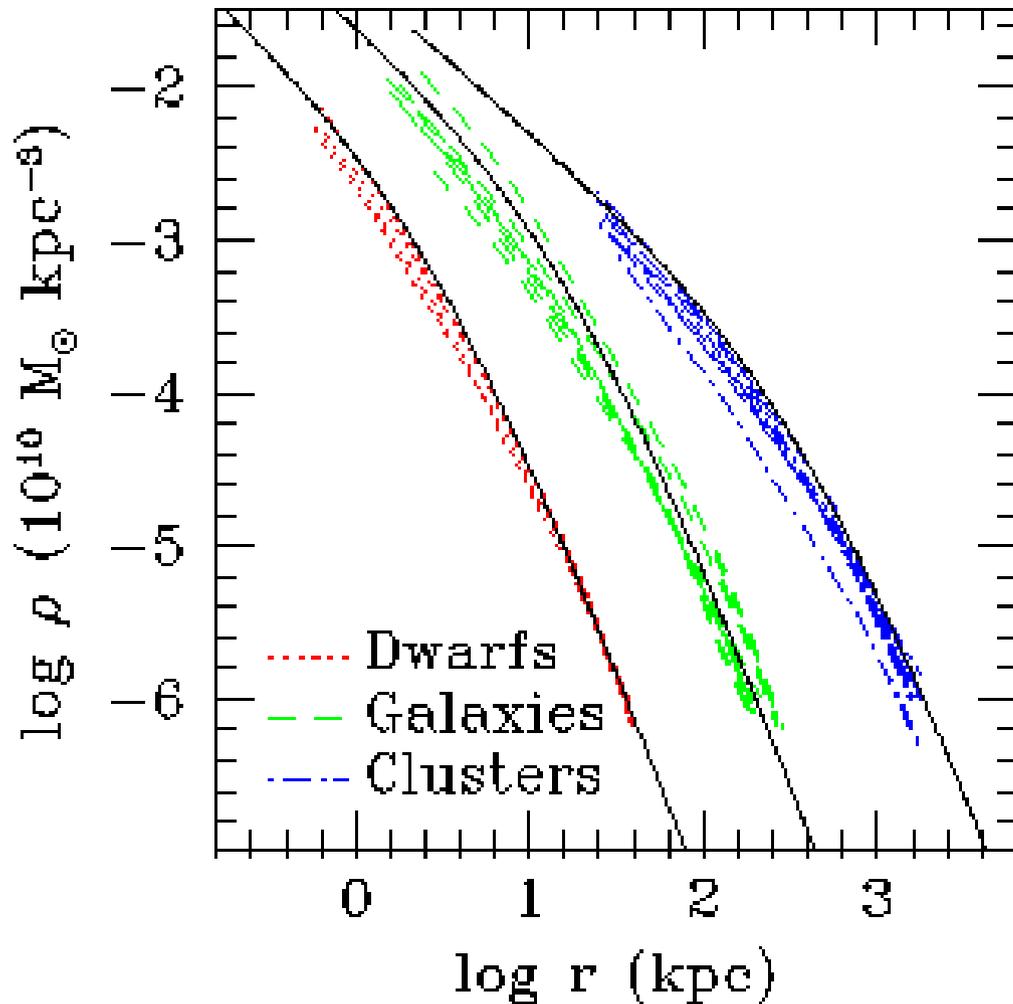


# $\Lambda$ CDM galaxy halos (without galaxies!)

- Halos extend to  $\sim 10$  times the 'visible' radius of galaxies and contain  $\sim 10$  times the mass in the visible regions
  - Halos are not spherical but approximate triaxial ellipsoids
    - more prolate than oblate
    - axial ratios greater than two are common
  - "Cuspy" density profiles with outwardly increasing slopes
    - $d \ln \rho / d \ln r = \gamma$  with  $\gamma < -2.5$  at large  $r$
    - $\gamma > -1.2$  at small  $r$
  - Substantial numbers of self-bound subhalos contain  $\sim 10\%$  of the halo's mass and have  $dN/dM \sim M^{-1.8}$
-  Most substructure mass is in most massive subhalos

# Density profiles of dark matter halos

Navarro, Frenk & White 1996



The average dark matter density of a dark halo depends on distance from halo centre in a very similar way in halos of all masses at all times

-- a universal profile shape --

$$\rho(r)/\rho_{crit} \approx \delta r_s / r(1 + r/r_s)^2$$

Less massive halos and halos that form earlier have higher densities (bigger  $\delta$ )

Concentration  $c = r_{200} / r_s$  is an alternative density measure  
Beware variety of definitions!

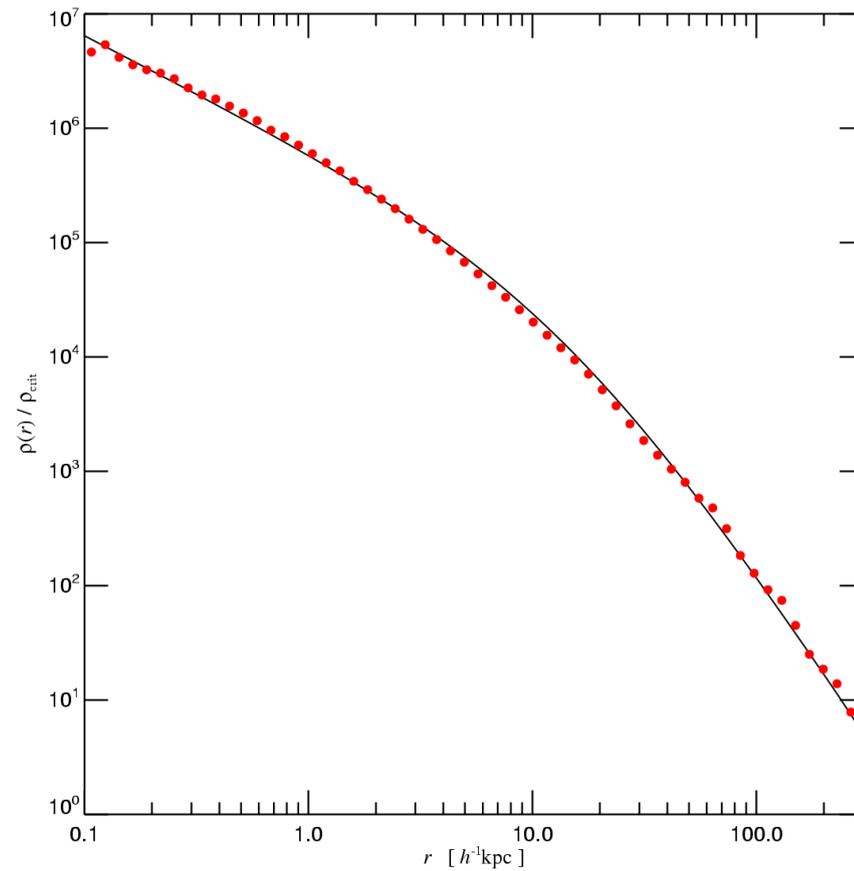
**NFW profiles may not be pretty....**



...but they work surprisingly well



600 kpc



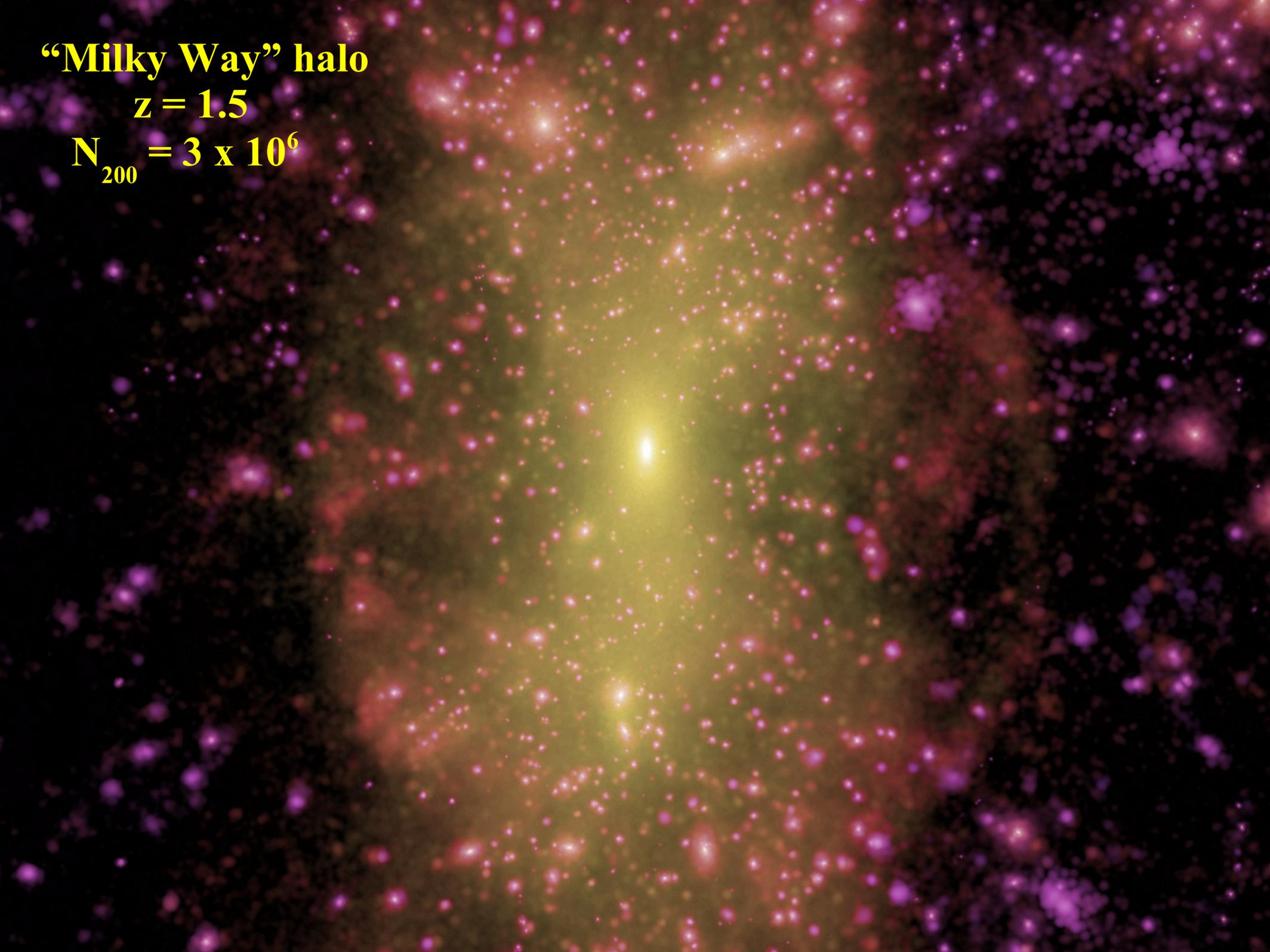
$$N_{200} \sim 3 \times 10^7$$

Navarro et al 2006

**“Milky Way” halo**

**$z = 1.5$**

**$N_{200} = 3 \times 10^6$**



**“Milky Way” halo**

**$z = 1.5$**

**$N_{200} = 94 \times 10^6$**



**“Milky Way” halo**

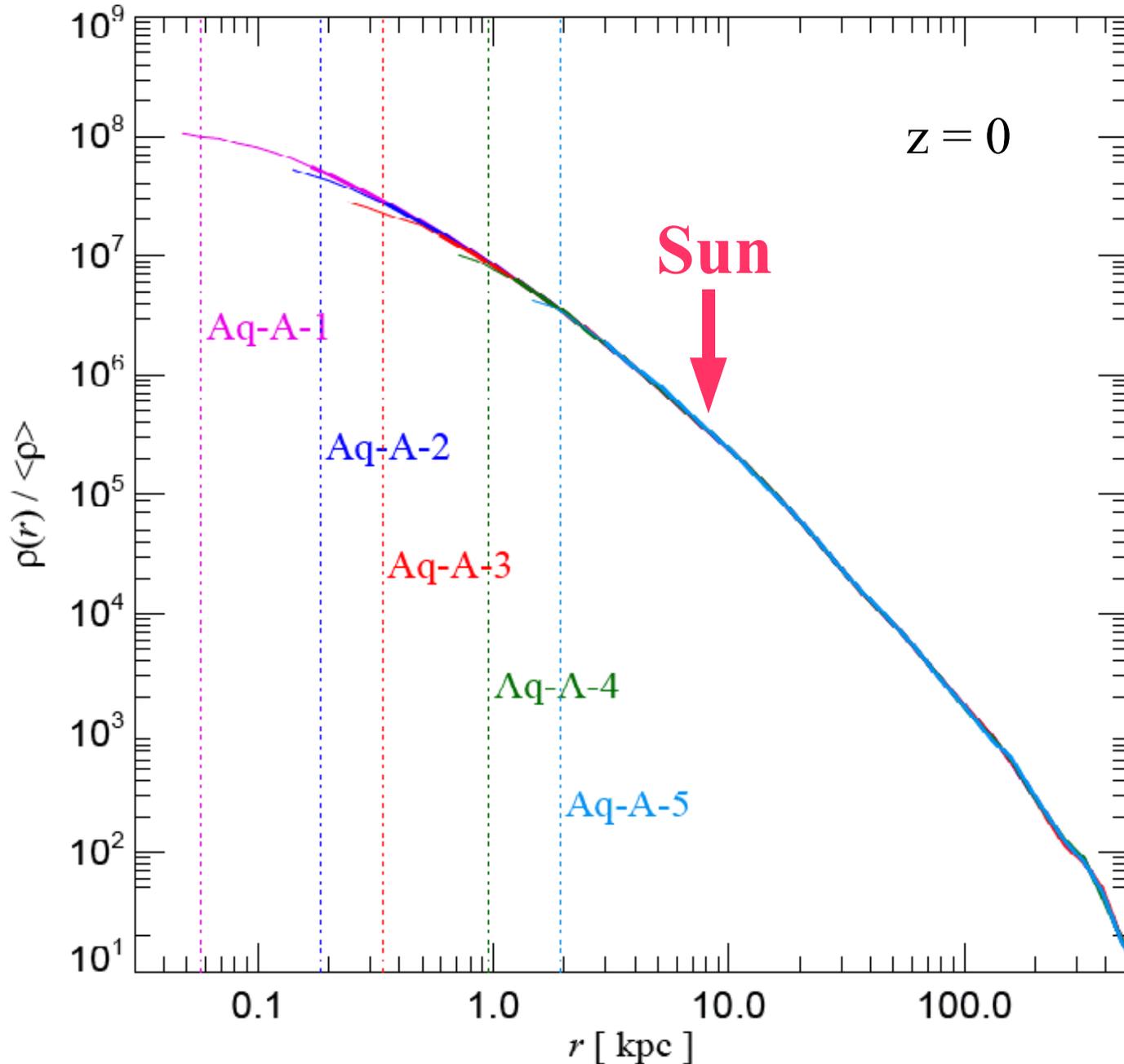
$$z = 1.5$$

$$N_{200} = 750 \times 10^6$$



# How well do density profiles converge?

Aquarius Project: Springel et al 2008



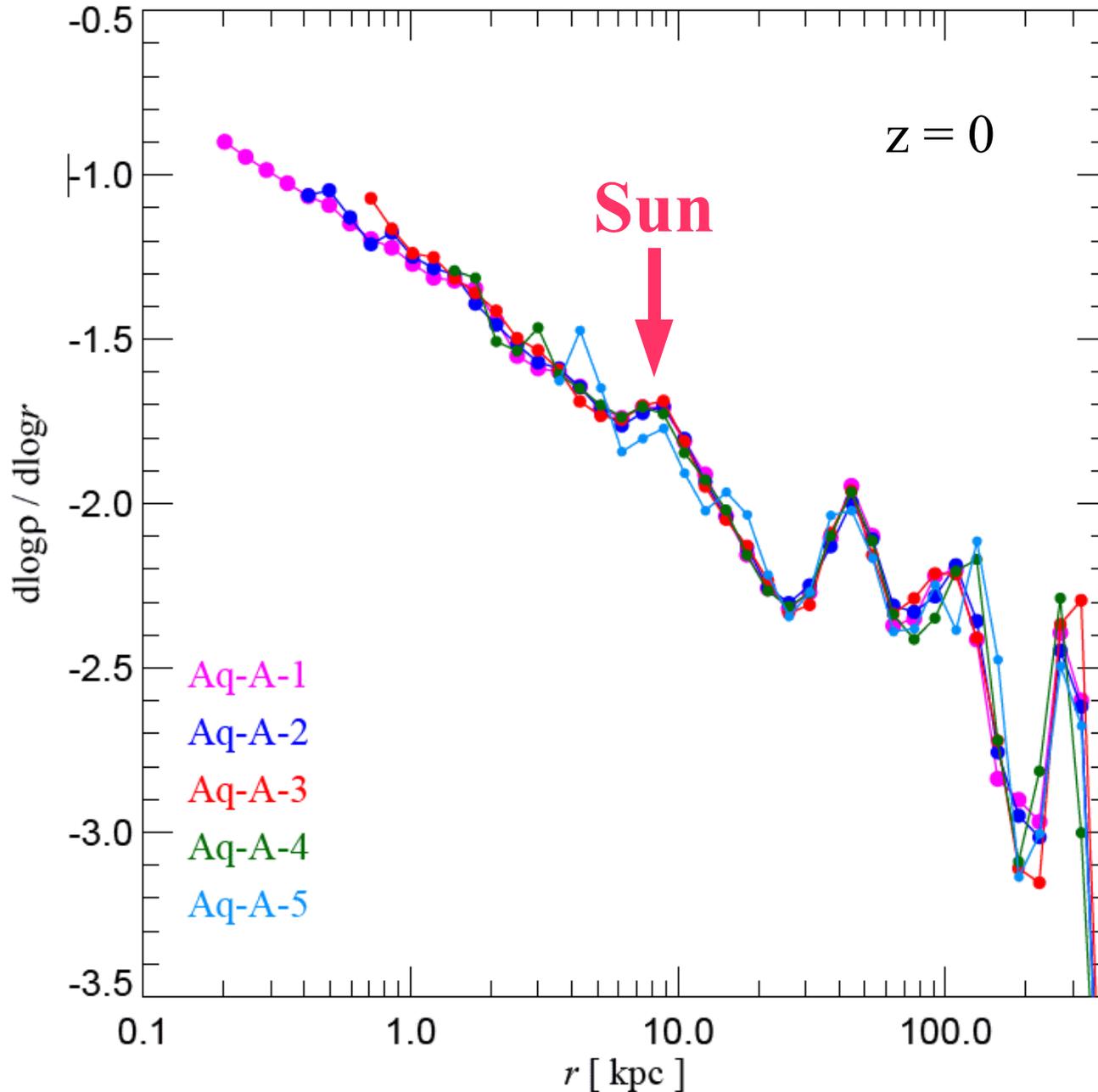
Mass and length resolution vary by factors of 2000 and 35

Convergence excellent to radii well below that of the Sun's orbit

Real DM profile will be modified by growth of the visible Galaxy

# How well do density profiles converge?

Aquarius Project: Springel et al 2008



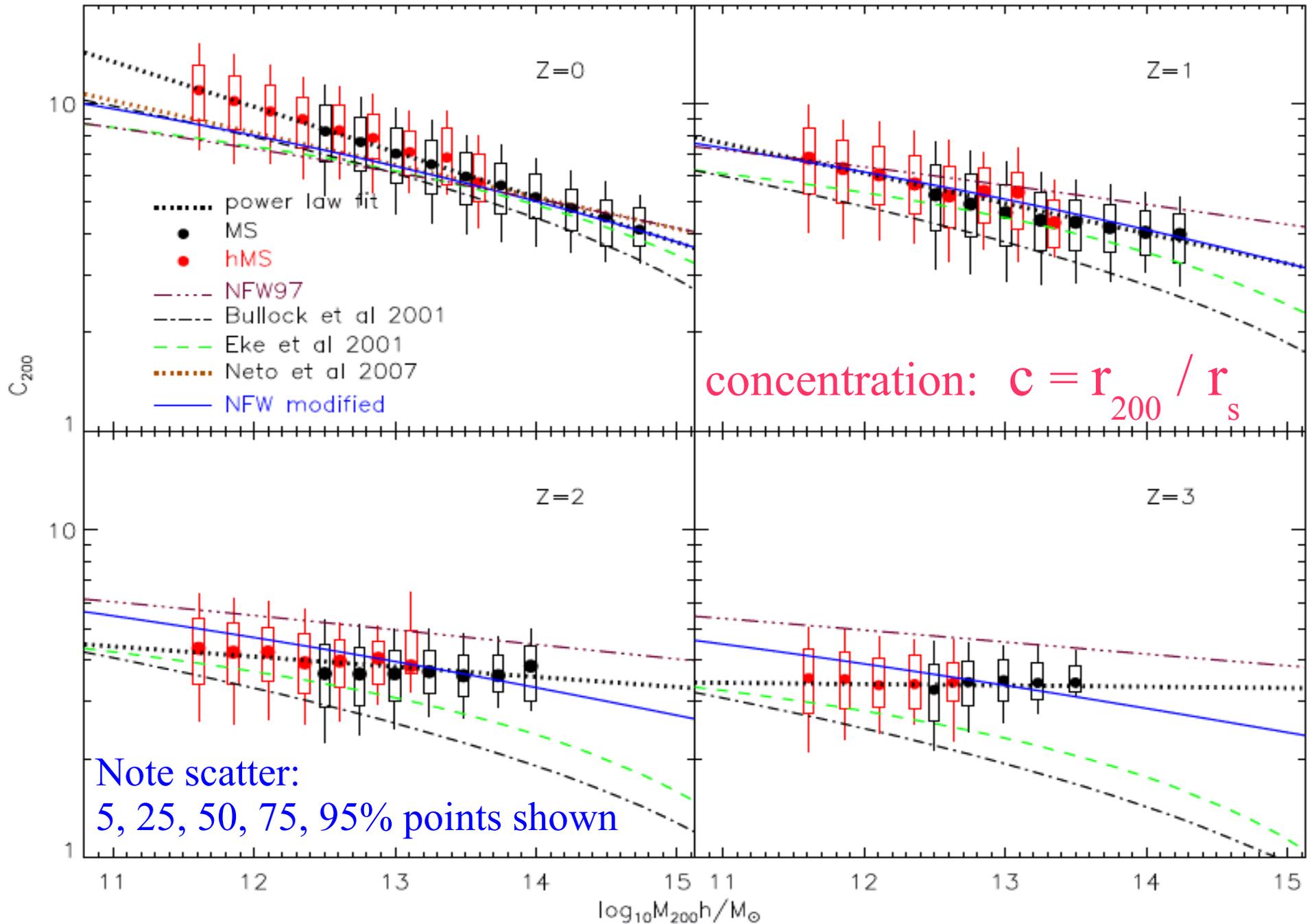
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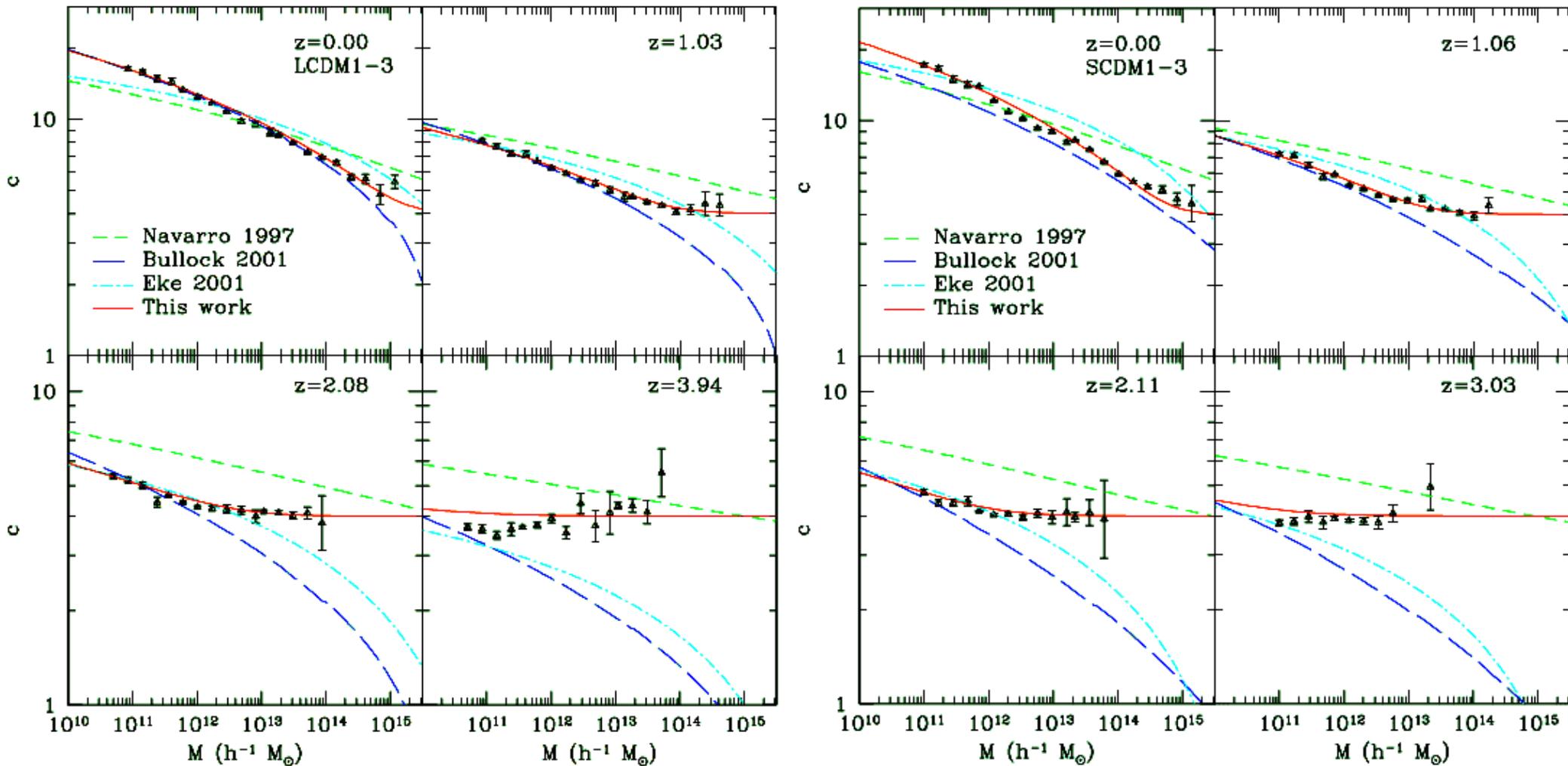
# Concentration scatter and trend with M and z

Gao et al 2008



# Concentration trends with $M$ , $z$ and cosmology

Zhao et al 2008

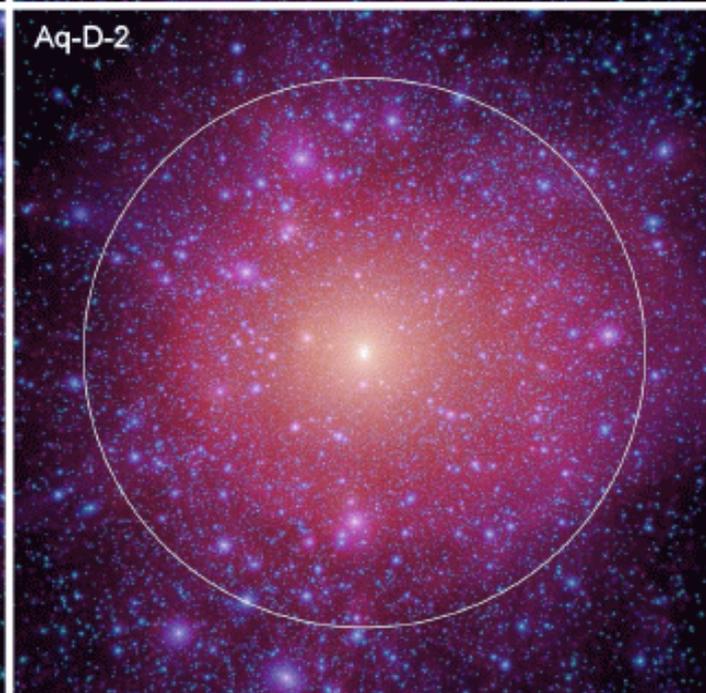
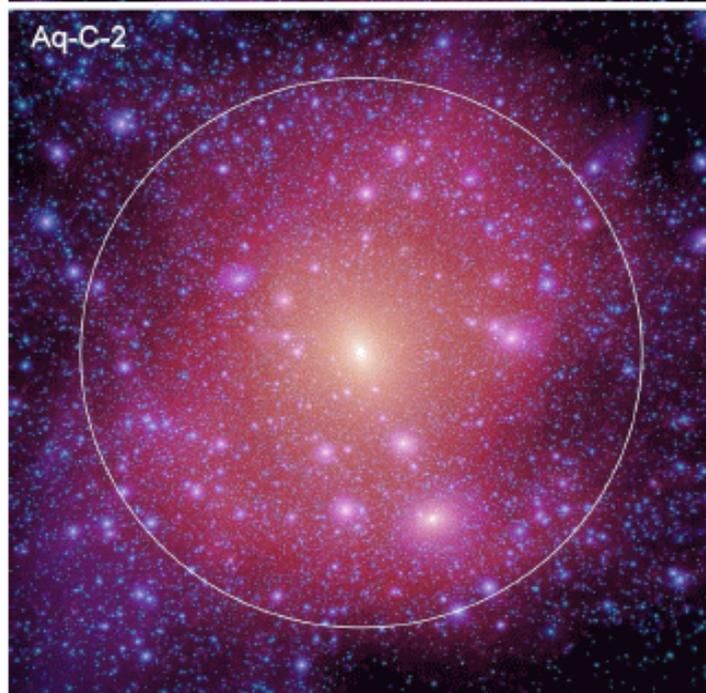
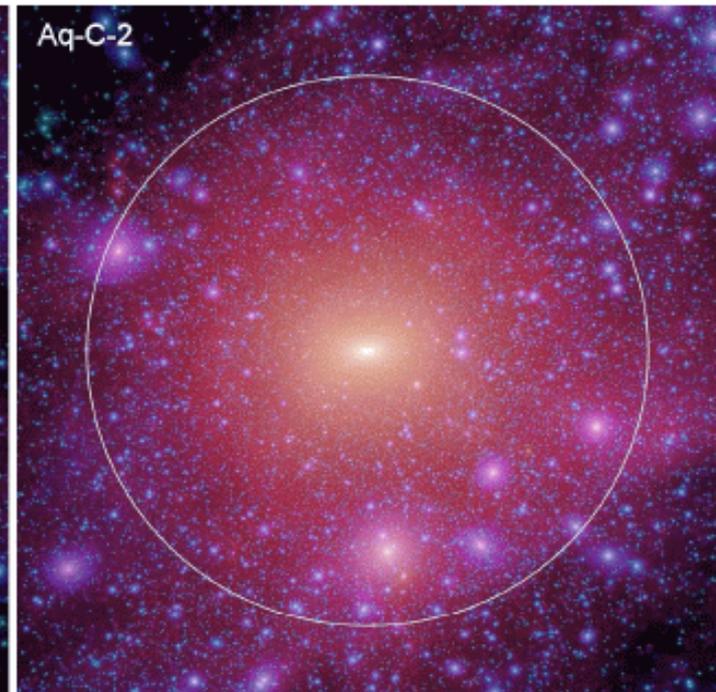
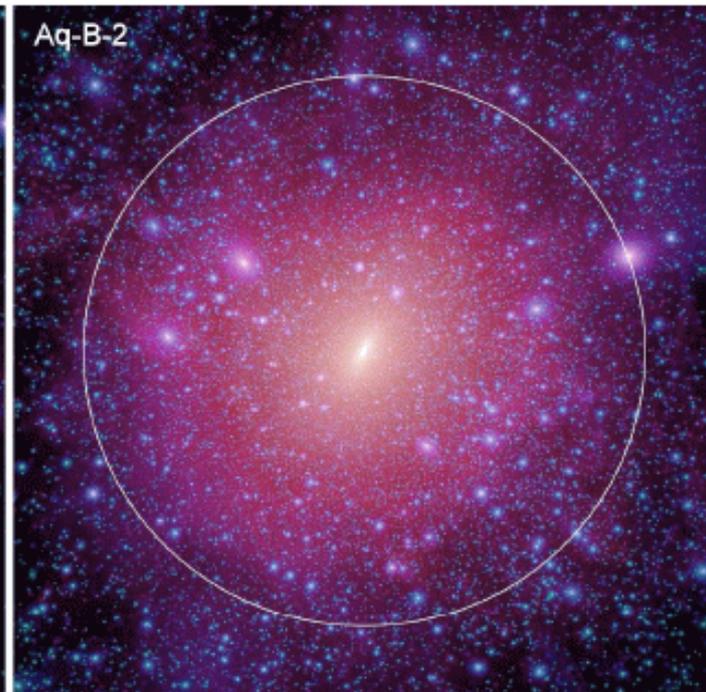
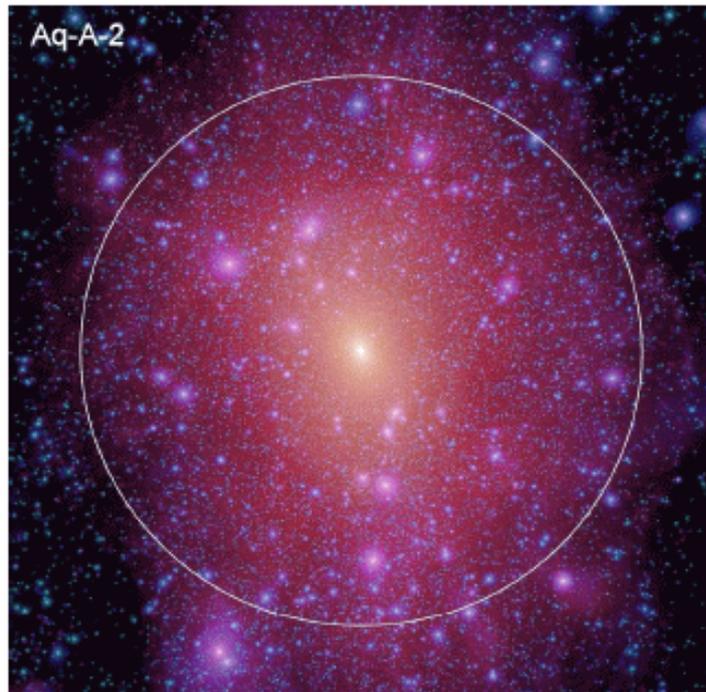


The concentration of halos depends on mass,  $\Omega_m$  and redshift

Related to cosmic expansion between formation and observation times

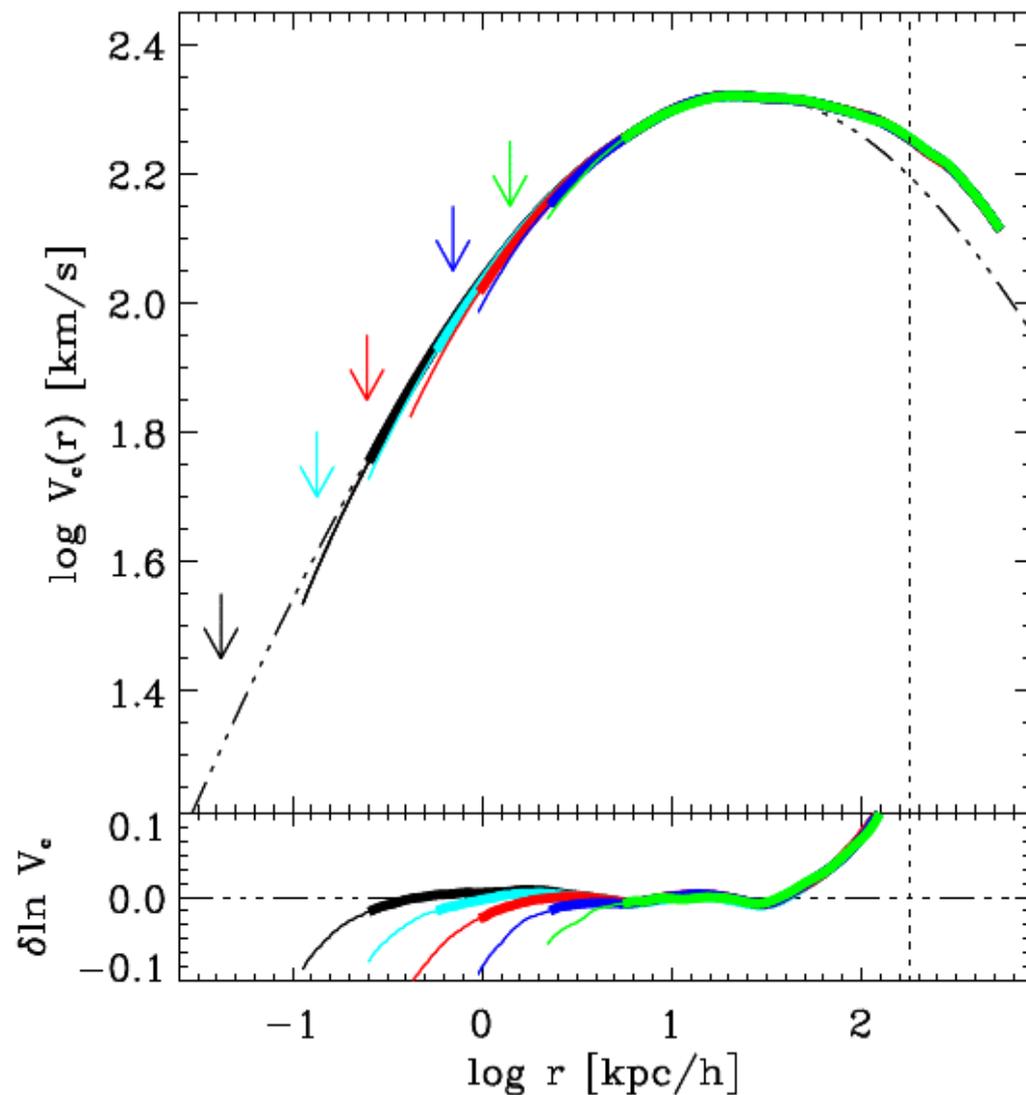
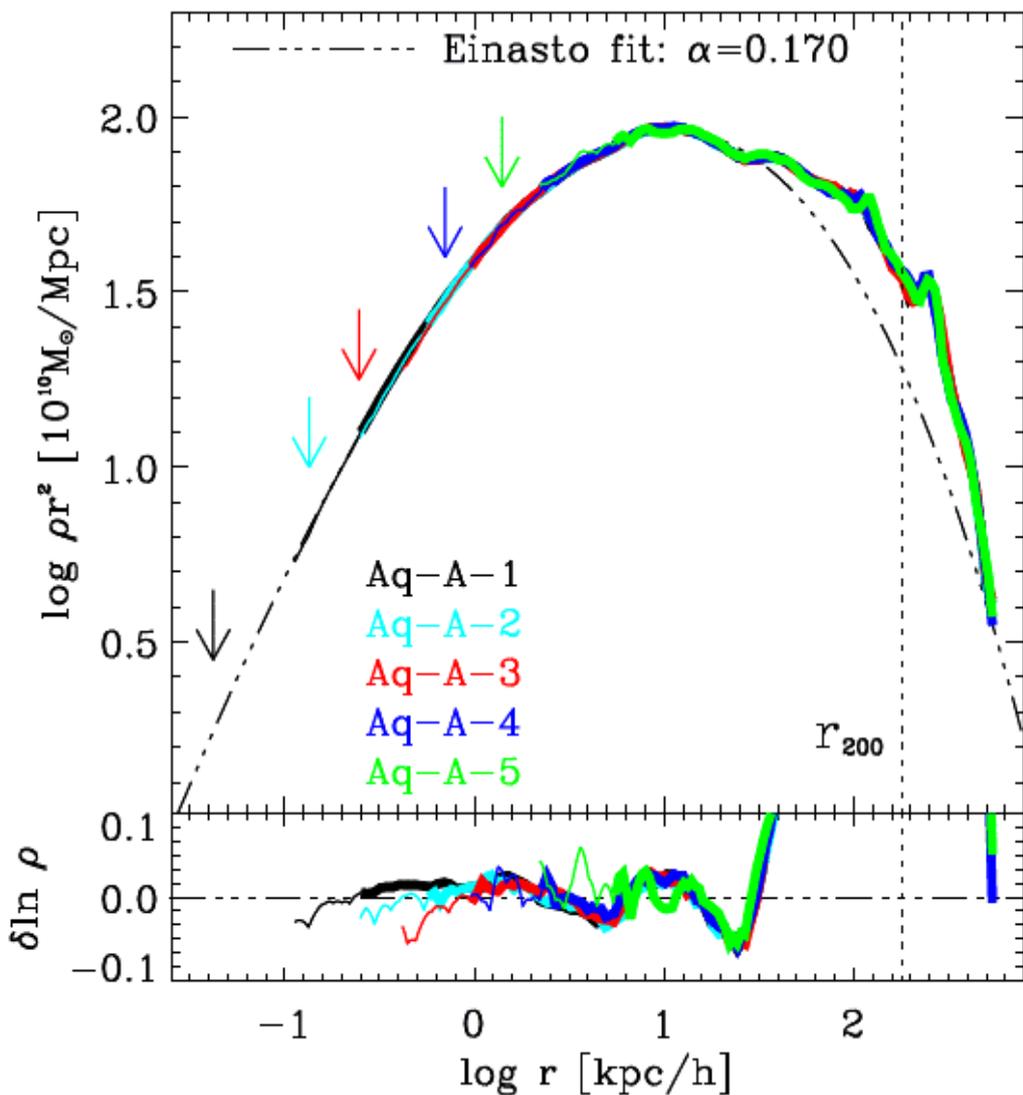
# The Aquarius halos

Springel et al 2008



# The Einasto profile fits the inner cusps

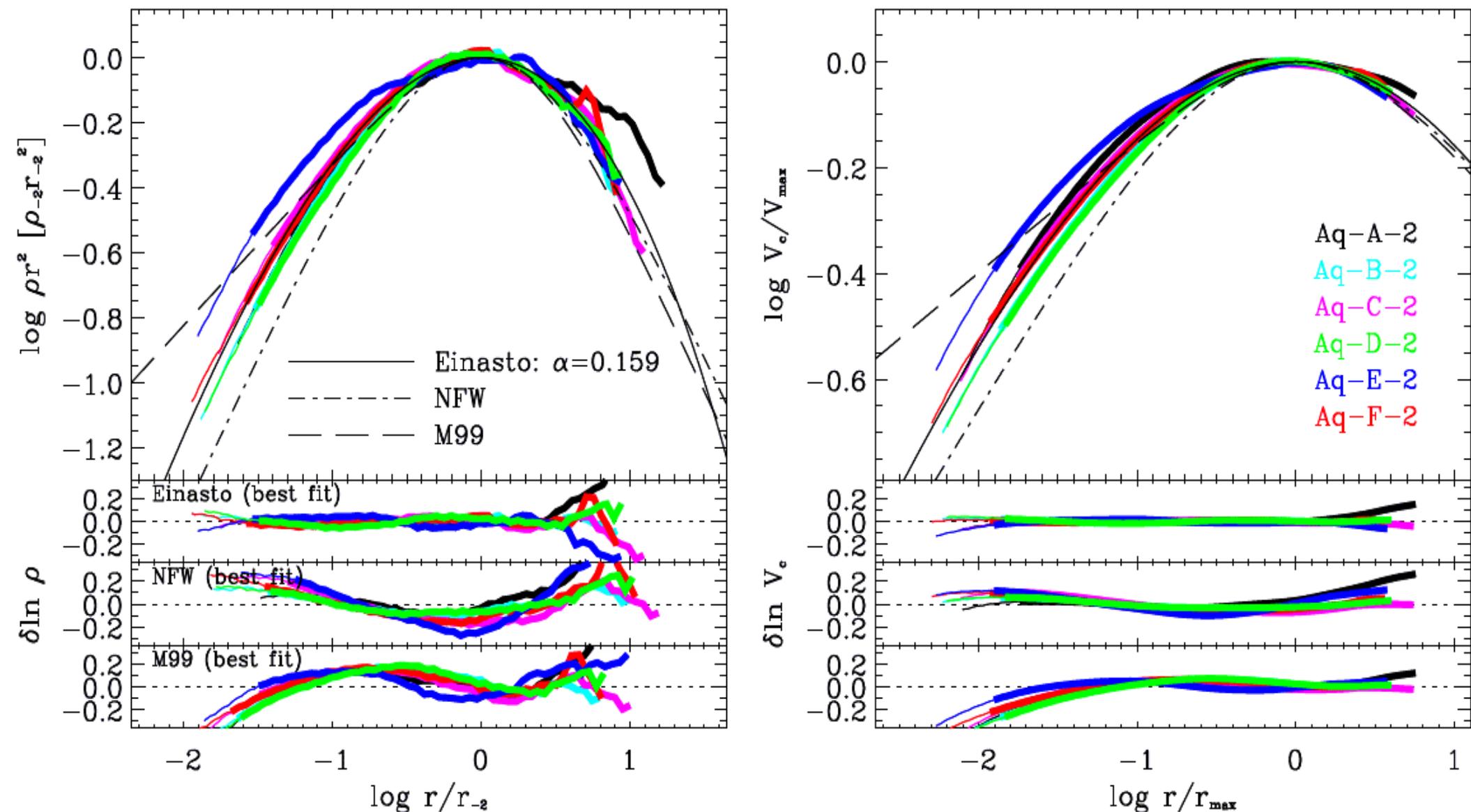
Navarro et al 2009



Einasto's (1965) profile:  $\ln \rho(r) / \rho_{-2} = -2 / \alpha [(r / r_{-2})^\alpha - 1]$

# The Einasto profile fits the inner cusps

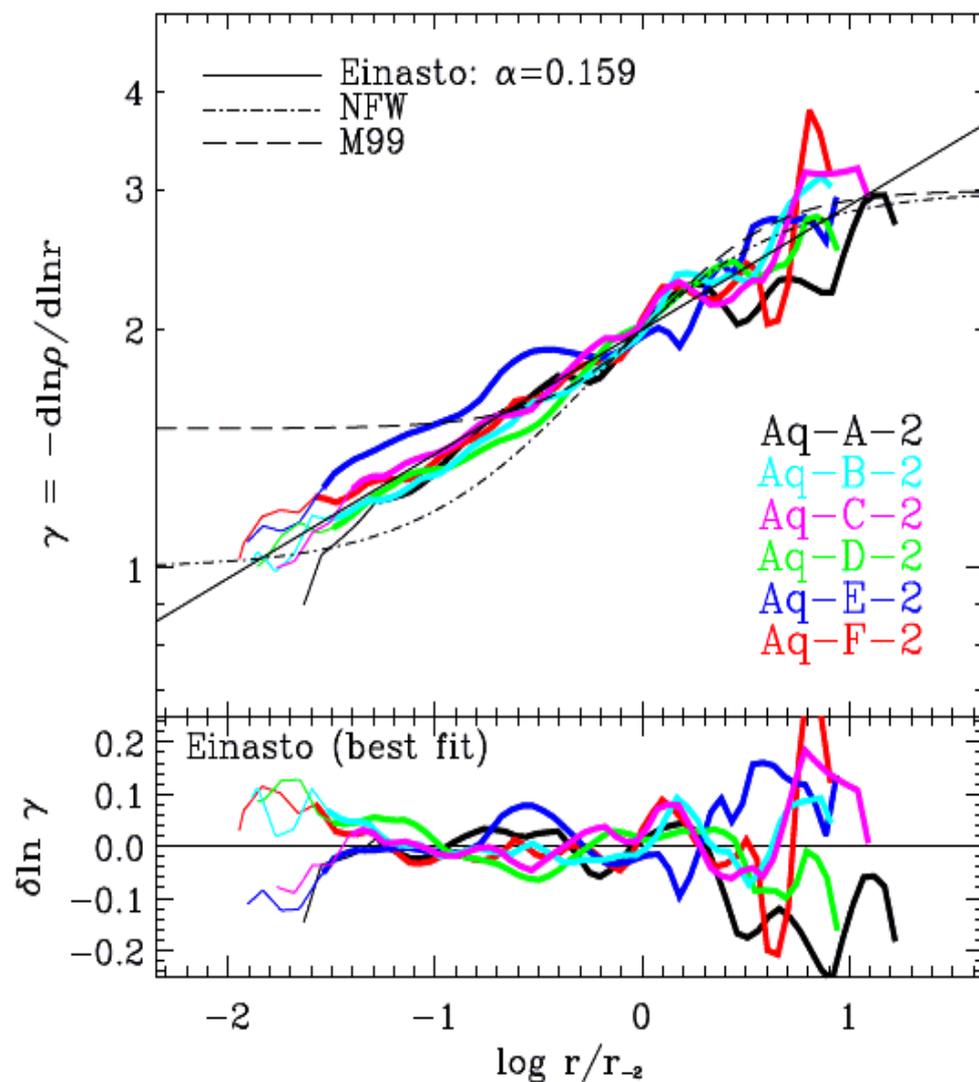
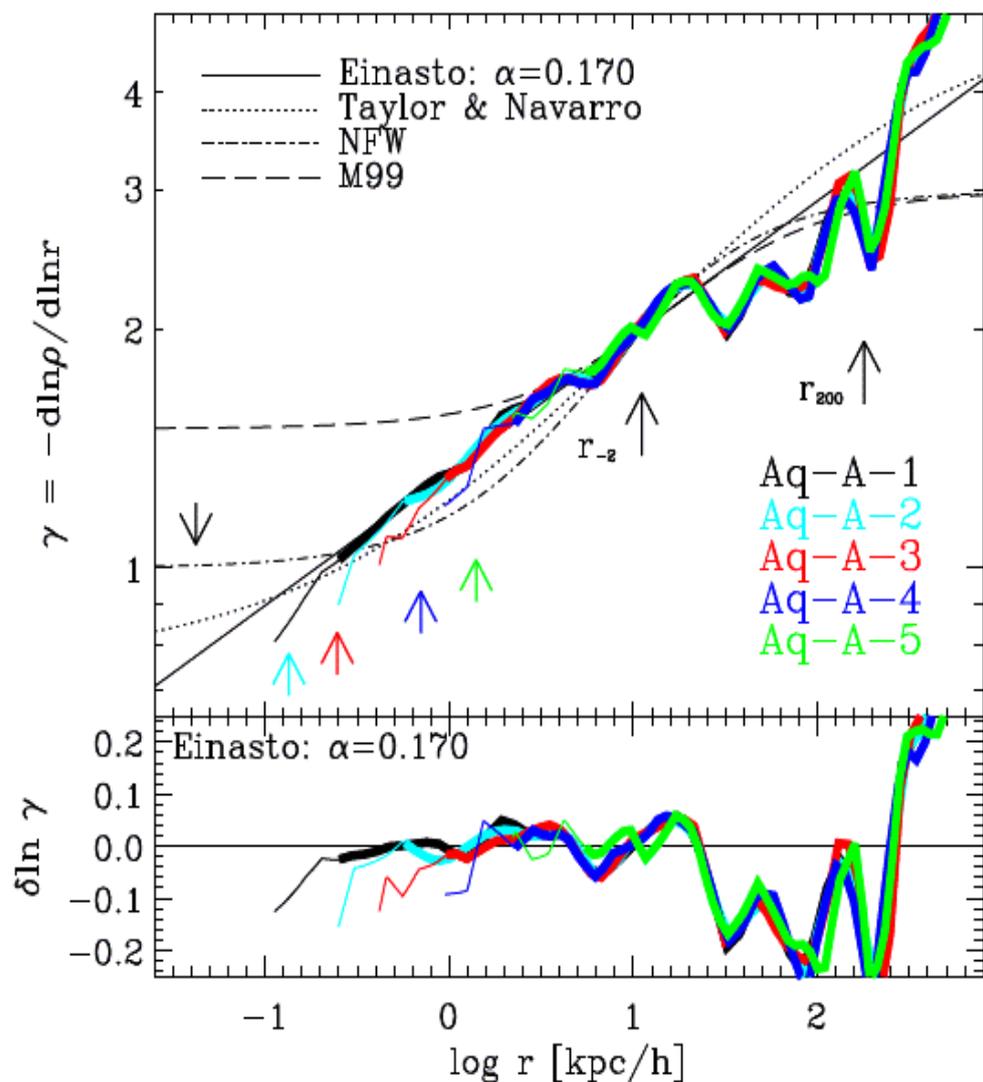
Navarro et al 2009



Einasto's (1965) profile:  $\ln \rho(r) / \rho_{-2} = -2 / \alpha \left[ (r / r_{-2})^\alpha - 1 \right]$

# The Einasto profile fits the inner cusps

Navarro et al 2009

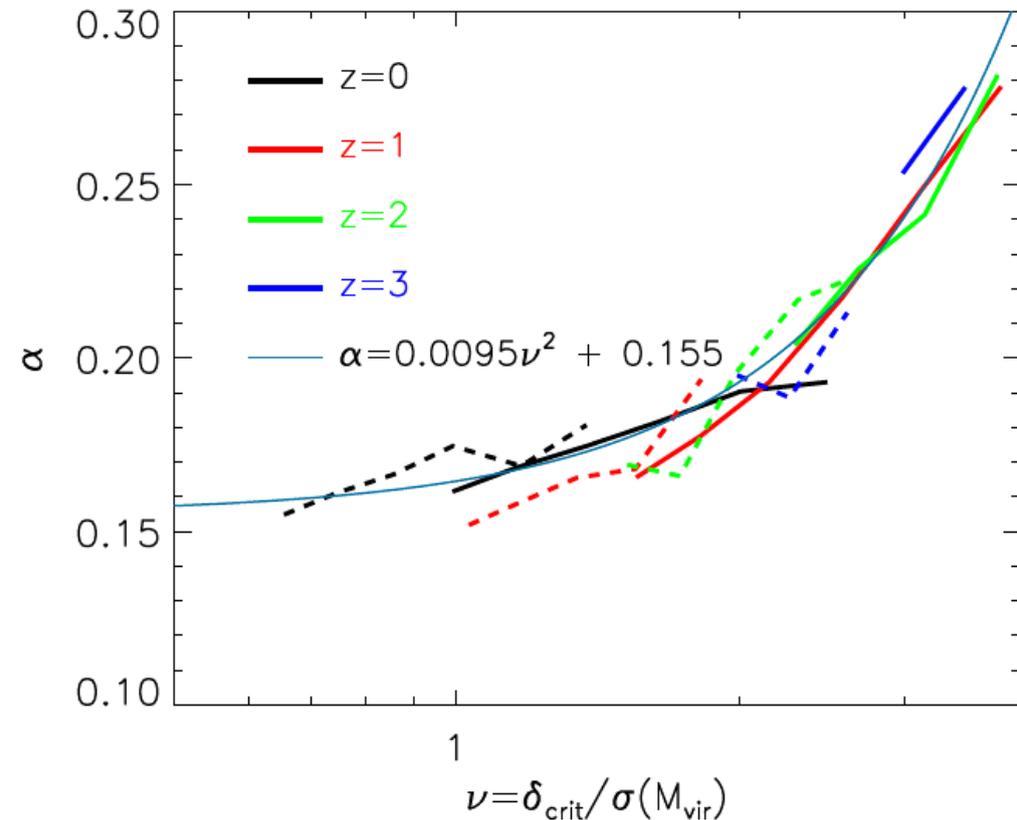
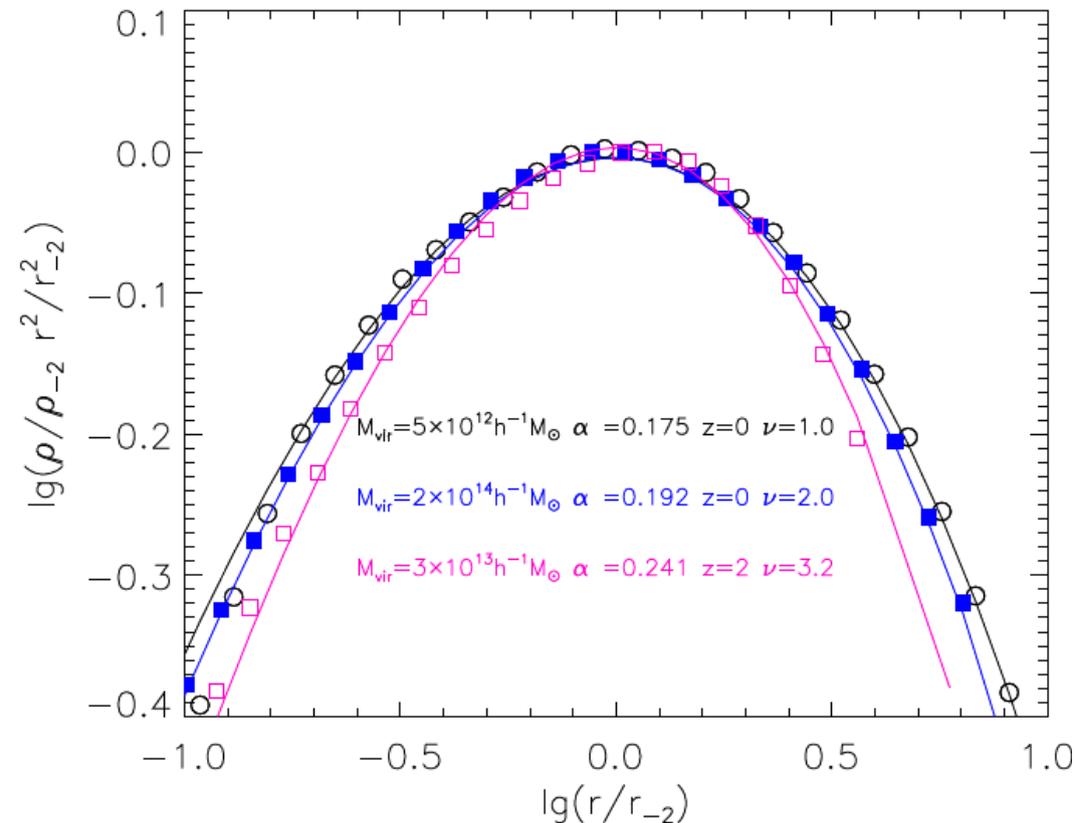


Einasto's (1965) profile:  $\ln \rho(r) / \rho_{-2} = -2 / \alpha [(r / r_{-2})^\alpha - 1]$

# The Einasto $\alpha$ varies with mass

Gao et al 2008

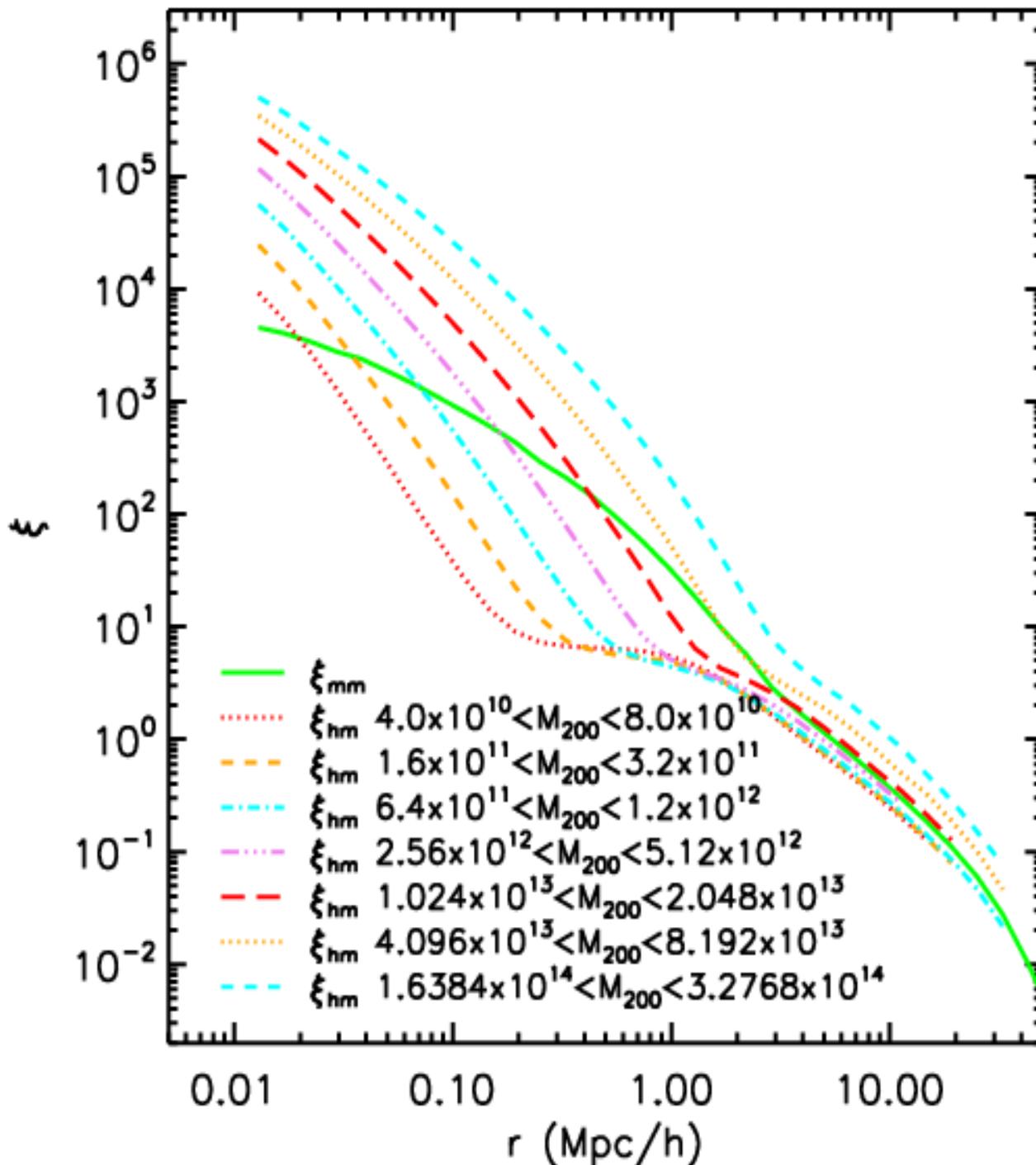
Results for stacked halos in the Millennium Simulation



Einasto's (1965) profile:  $\ln \rho(r) / \rho_{-2} = -2 / \alpha [(r / r_{-2})^\alpha - 1]$

# Mean profiles to much larger radii

Hayashi & White 2008



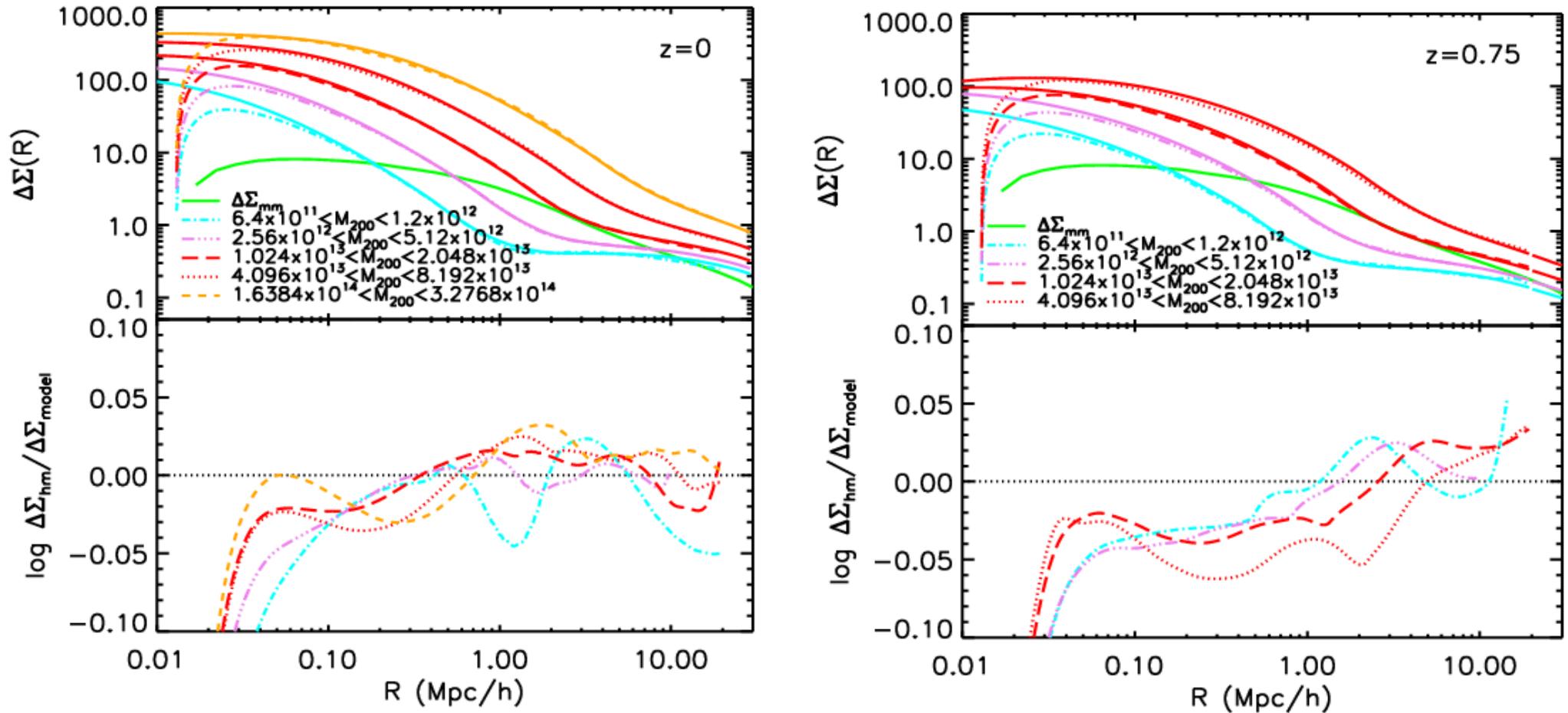
At large radii, the mean density profile  $\bar{\rho}(r) \propto \xi_{lin}(r)$ , the *linear* mass correlation function

To a good approximation

$$\bar{\rho}(r) = \max[ \rho_{Ein}(r), b \xi_{lin}(r) ]$$

# A lensing test of the DM paradigm?

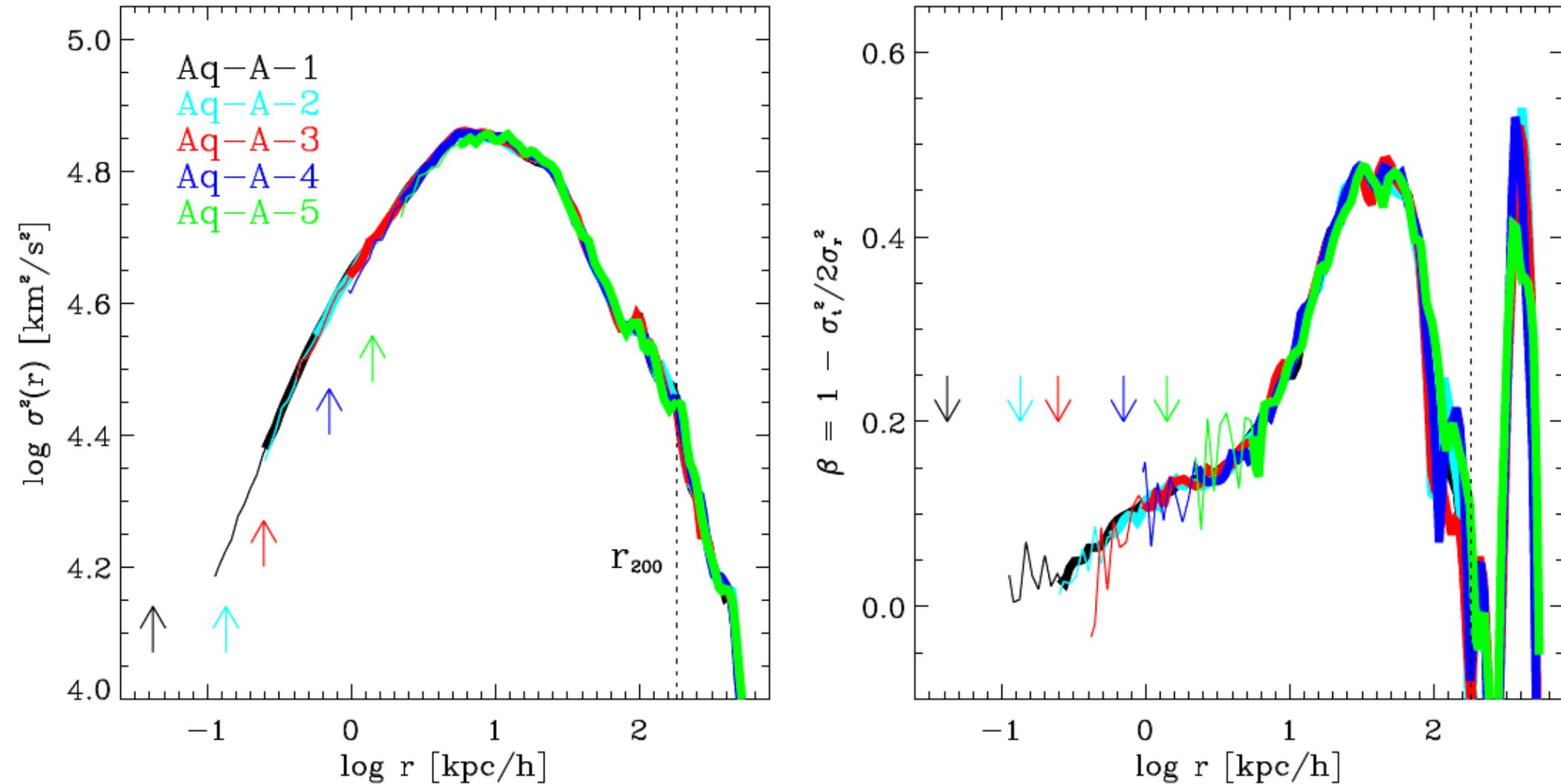
Hayashi & White 2008



Gravitational lensing allows *measurement* of these mean profiles  
The characteristic shape is a direct test of the DM paradigm

# Velocity dispersion profiles

Navarro et al 2009

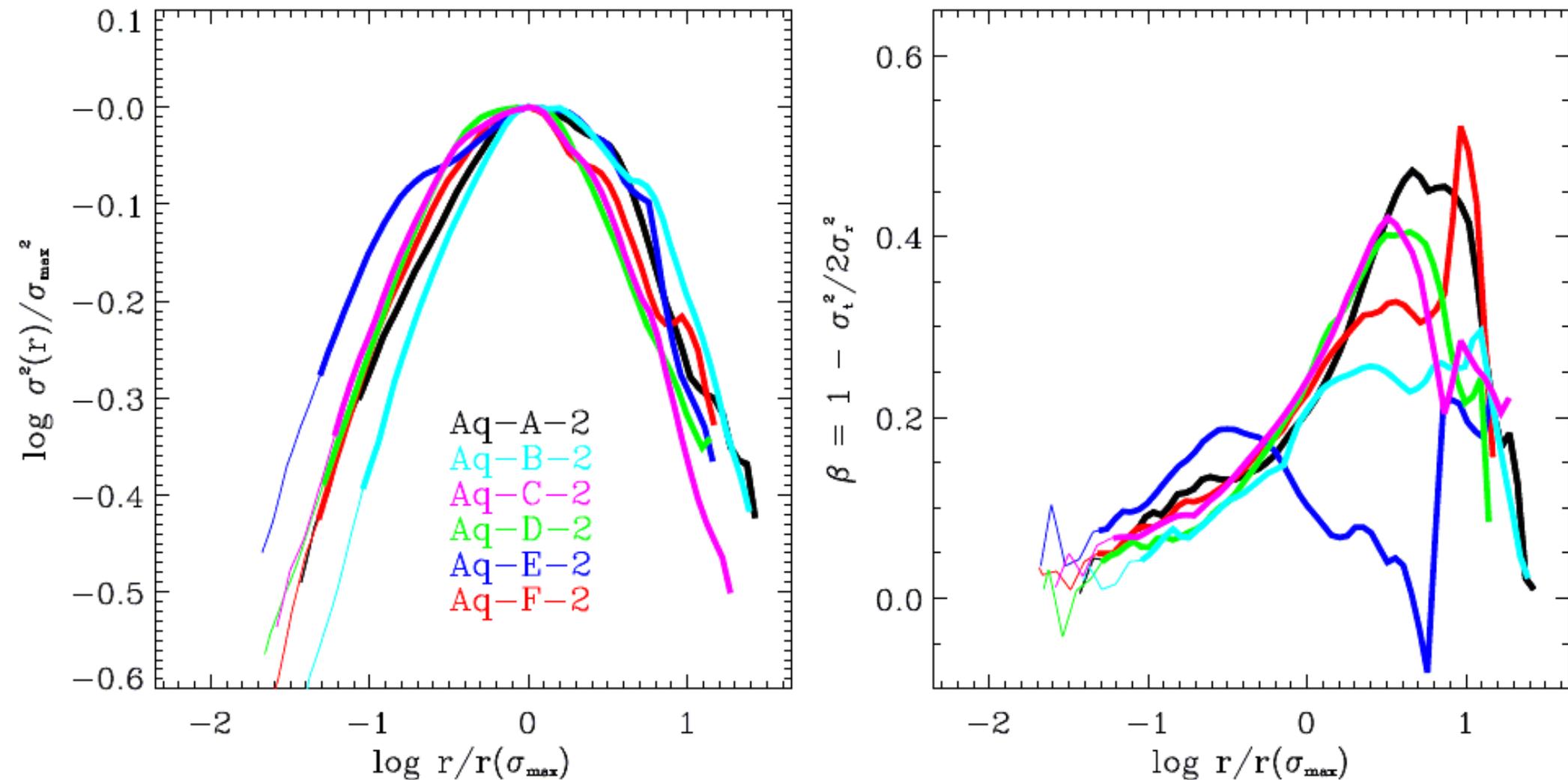


Results are well converged

Velocity dispersion and anisotropy peak at intermediate radii

# Velocity dispersion profiles

Navarro et al 2009



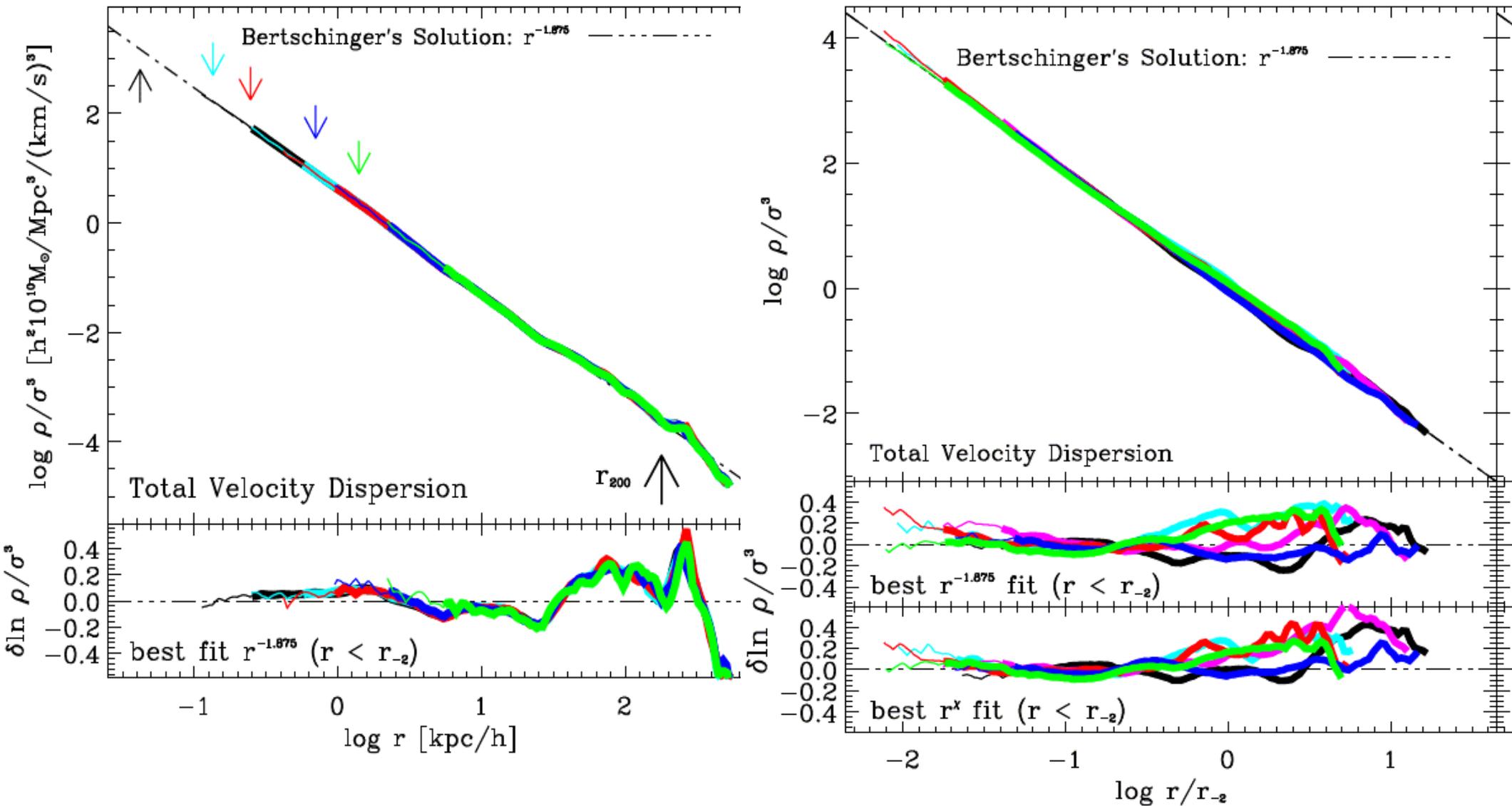
Results are well converged

Velocity dispersion and anisotropy peak at intermediate radii

Profiles vary significantly between halos

# Pseudo-phase-space density profiles

Navarro et al 2009



Shape variations in the density and velocity dispersion profiles compensate to make  $\rho(r) / \sigma(r)^3$  an almost universal power law

# Halo profiles: conclusions

- The NFW formula fits spherically averaged profiles of most objects to within 10% out to at least  $2 r_s$
- The characteristic density (or concentration) varies with mass, redshift and cosmology
- The Einasto formula fits better – its additional shape parameter varies systematically with mass
- There is no indication of *any* “asymptotic inner power law”
- The scatter among halos is larger than the Einasto-NFW difference
- Mean profiles change shape dramatically at large radius ( $\delta < 10$ )
- Velocity dispersion profiles show considerable variation
- Variations in  $\rho(r)$  and  $\sigma(r)$  compensate to give power law  $\rho/\sigma^3$

# Halo profiles: observational issues

$\Lambda$ CDM halos agree well with the observed structure of

**Galaxy halos** (10 to 200 kpc: lensing, satellite motions)

**Cluster halos** (30 to 2000 kpc: lensing, X-ray data on gas)

They appear to disagree with data in the inner parts of galaxies

**Milky Way**  $M_{200} = 1(2) \times 10^{12} M_{\odot} \rightarrow M(R_{\odot}) = 3.3(4.5) \times 10^{10} M_{\odot}$

to compare with  $M_*(R_{\odot}) = 5 \times 10^{10} M_{\odot}$

**LSB/dwarf galaxies** Rotation curves rise slower than expected

The “Cusp Problem”

Baryonic physics complicates this comparison

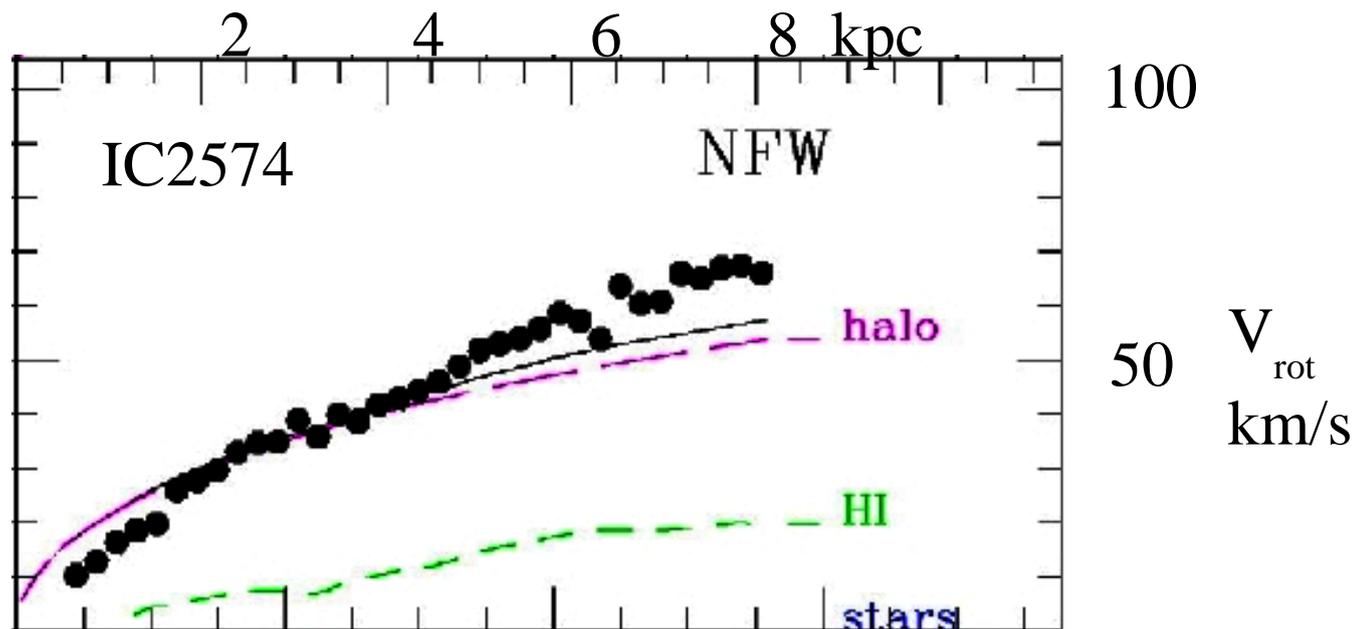
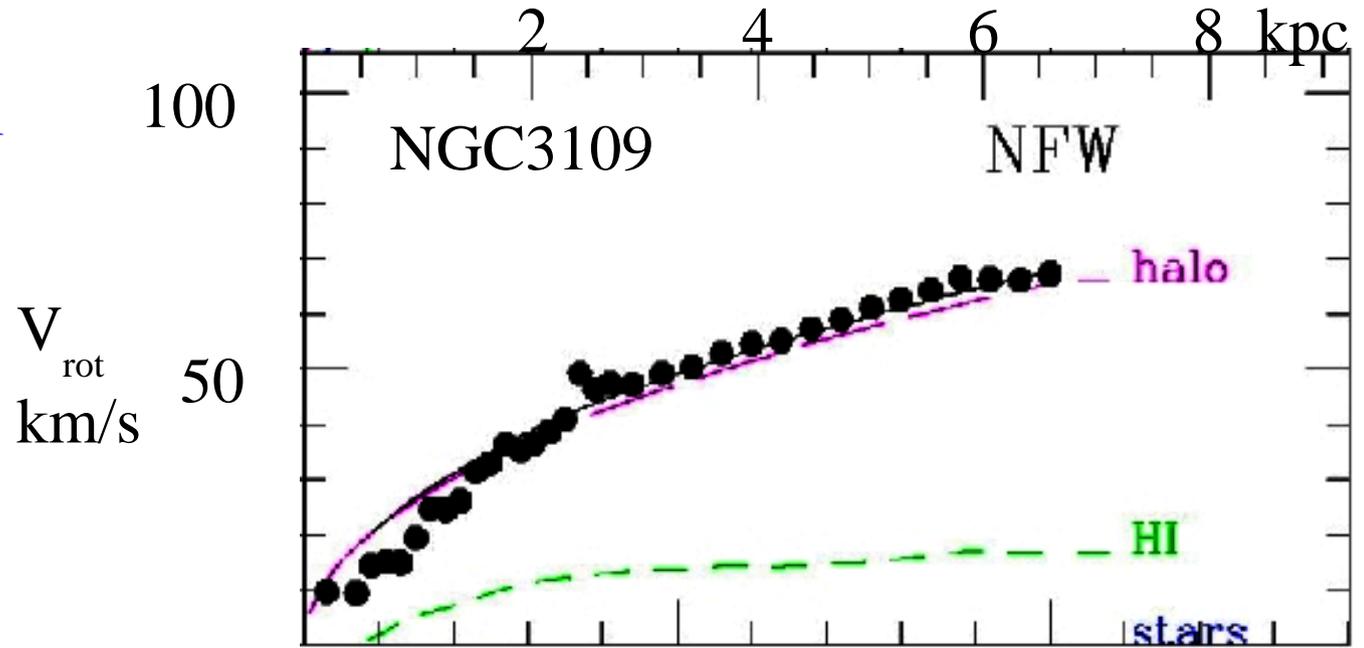
**Adiabatic compression** increase may central DM densities

**Sudden expulsion of gas** may decrease central DM densities

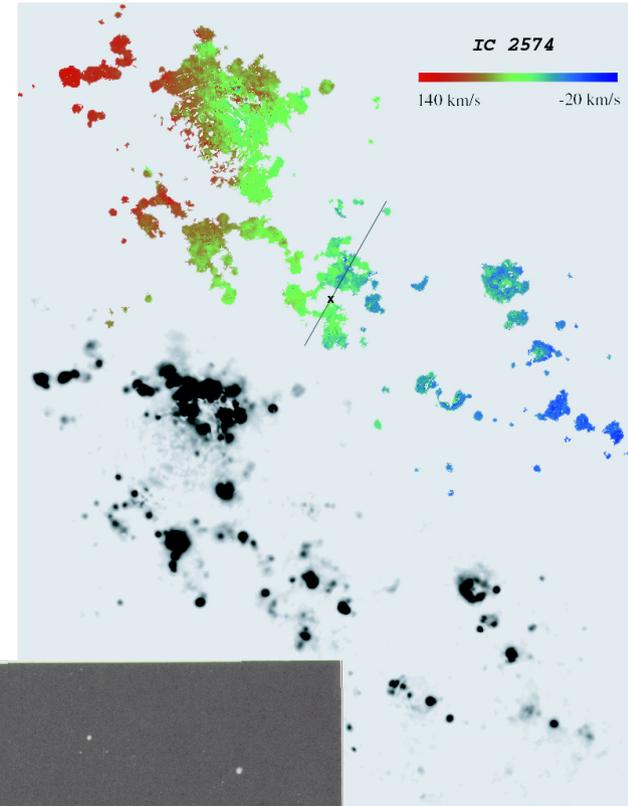
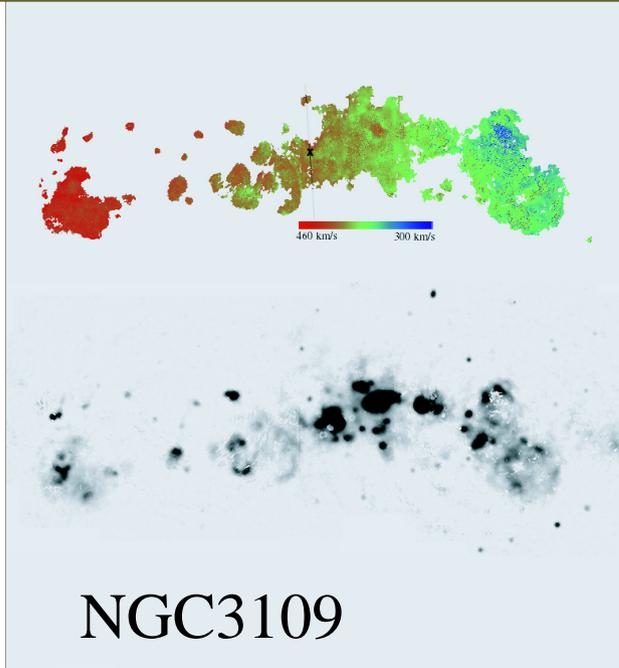
both are quite uncertain

# High quality rotation curves for nearby dwarfs: I

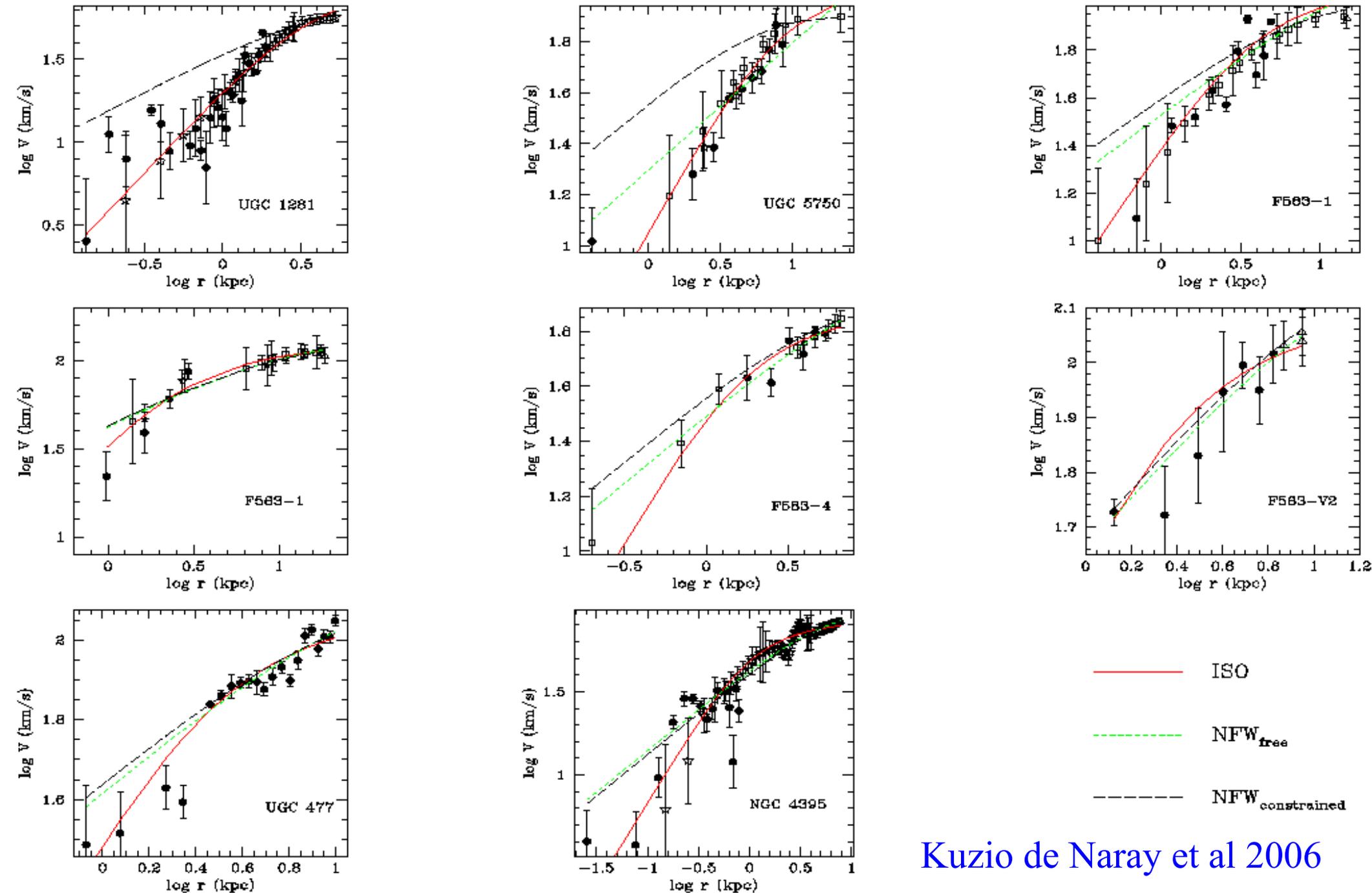
Blais-Ouellette, Amram  
& Carignan 2001



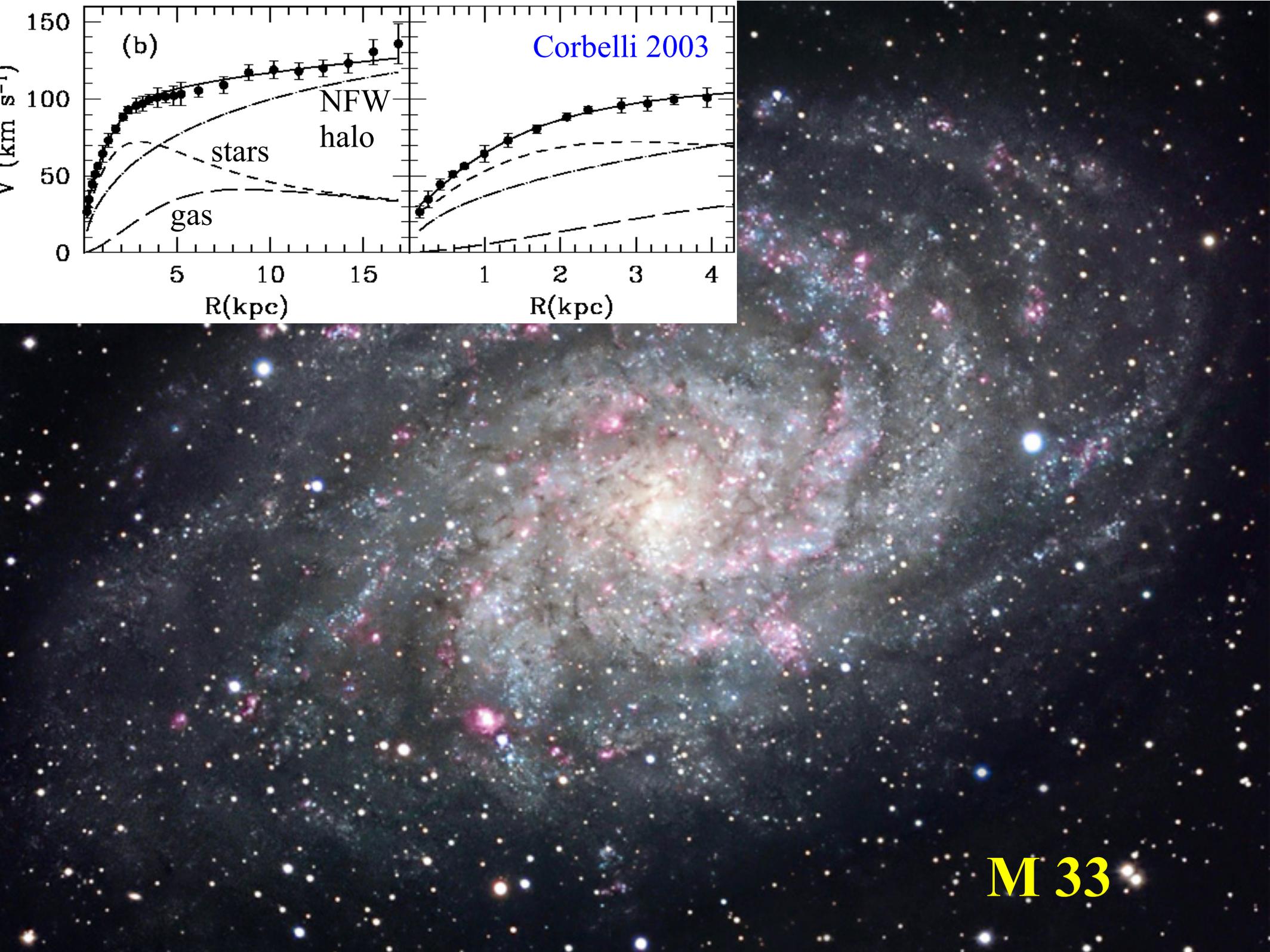
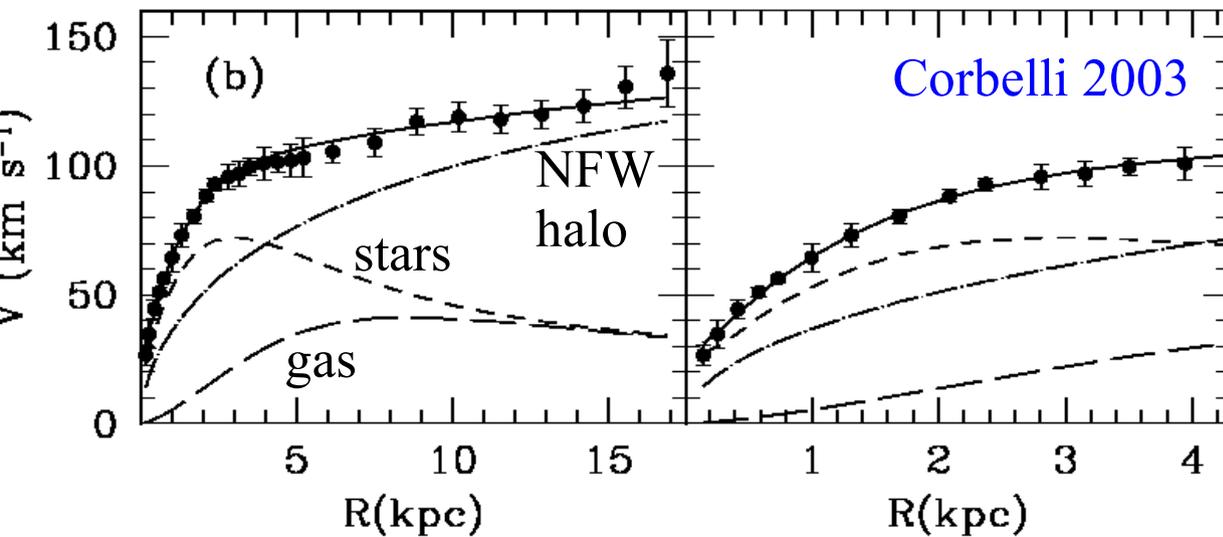
# High quality rotation curves for nearby dwarfs: II



# Inner rotation curves of low SB galaxies



Kuzio de Naray et al 2006



**M 33**

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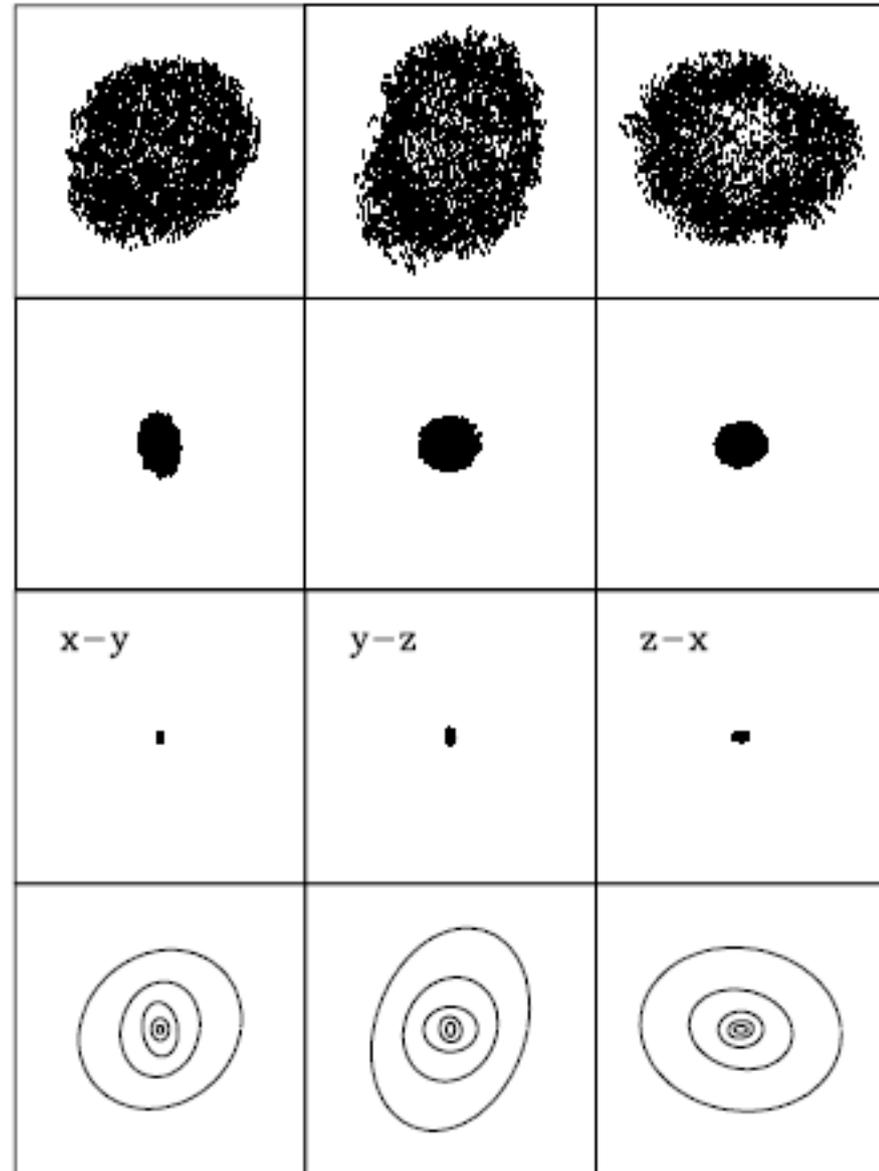
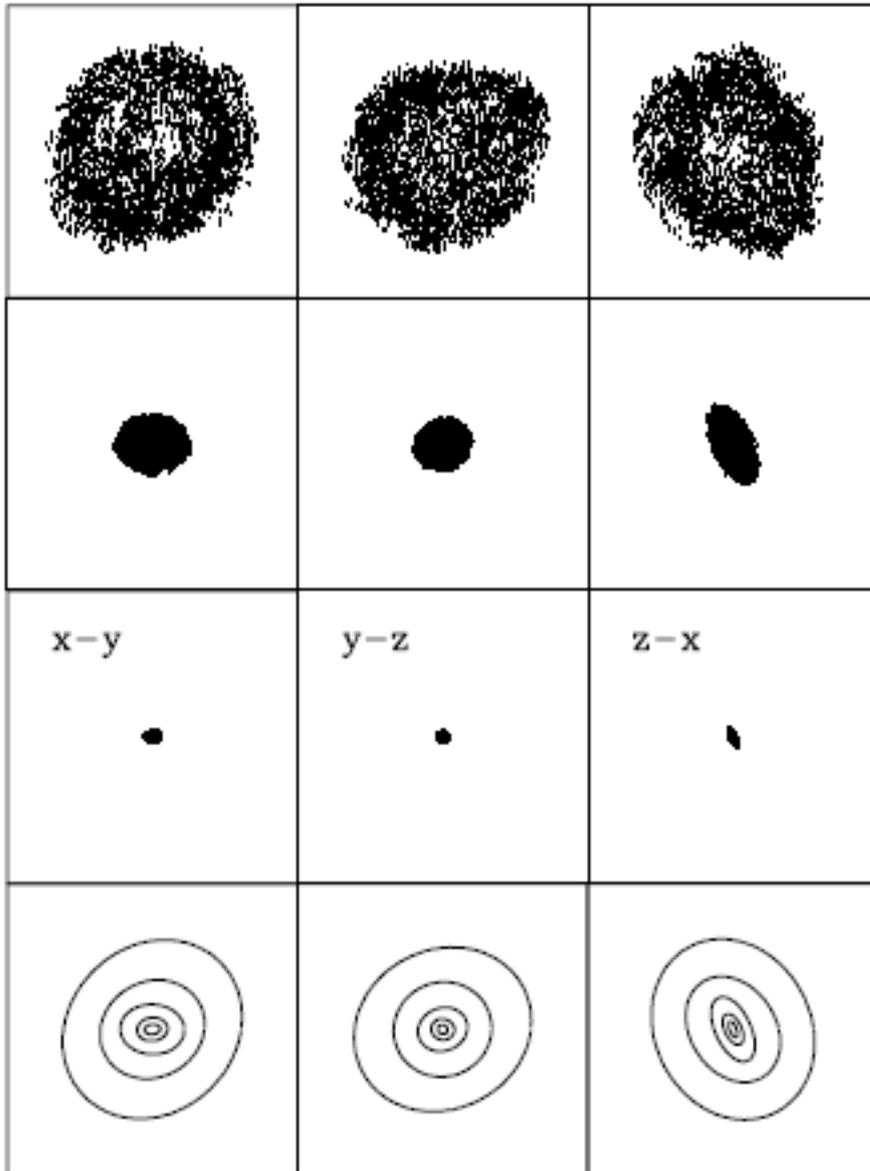
# Shapes of halo equidensity surfaces

Group

Jing & Suto 2002

Galaxy

$\delta$

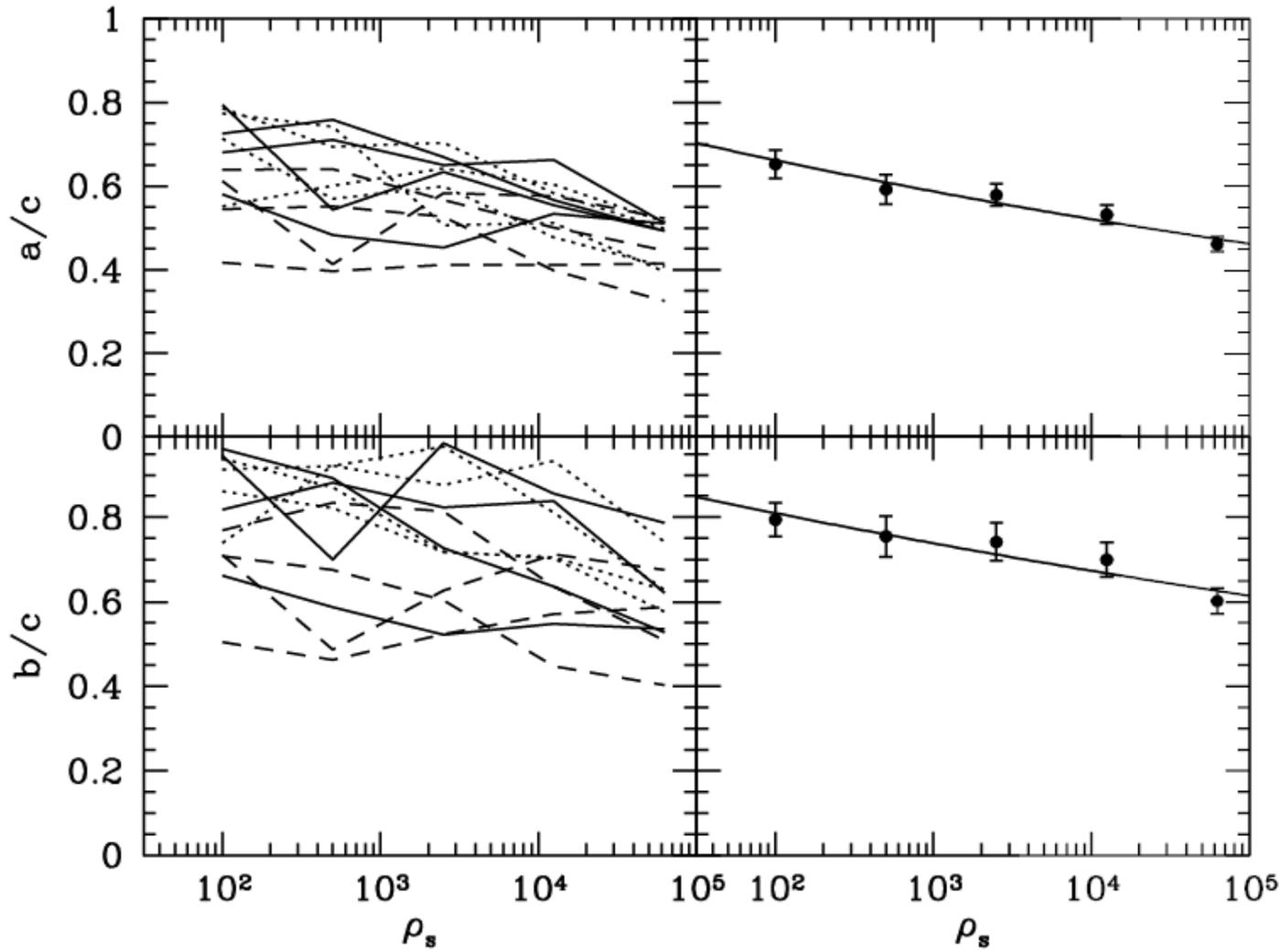


100

2500

6250

# Shapes of halo equidensity surfaces



Jing & Suto 2002

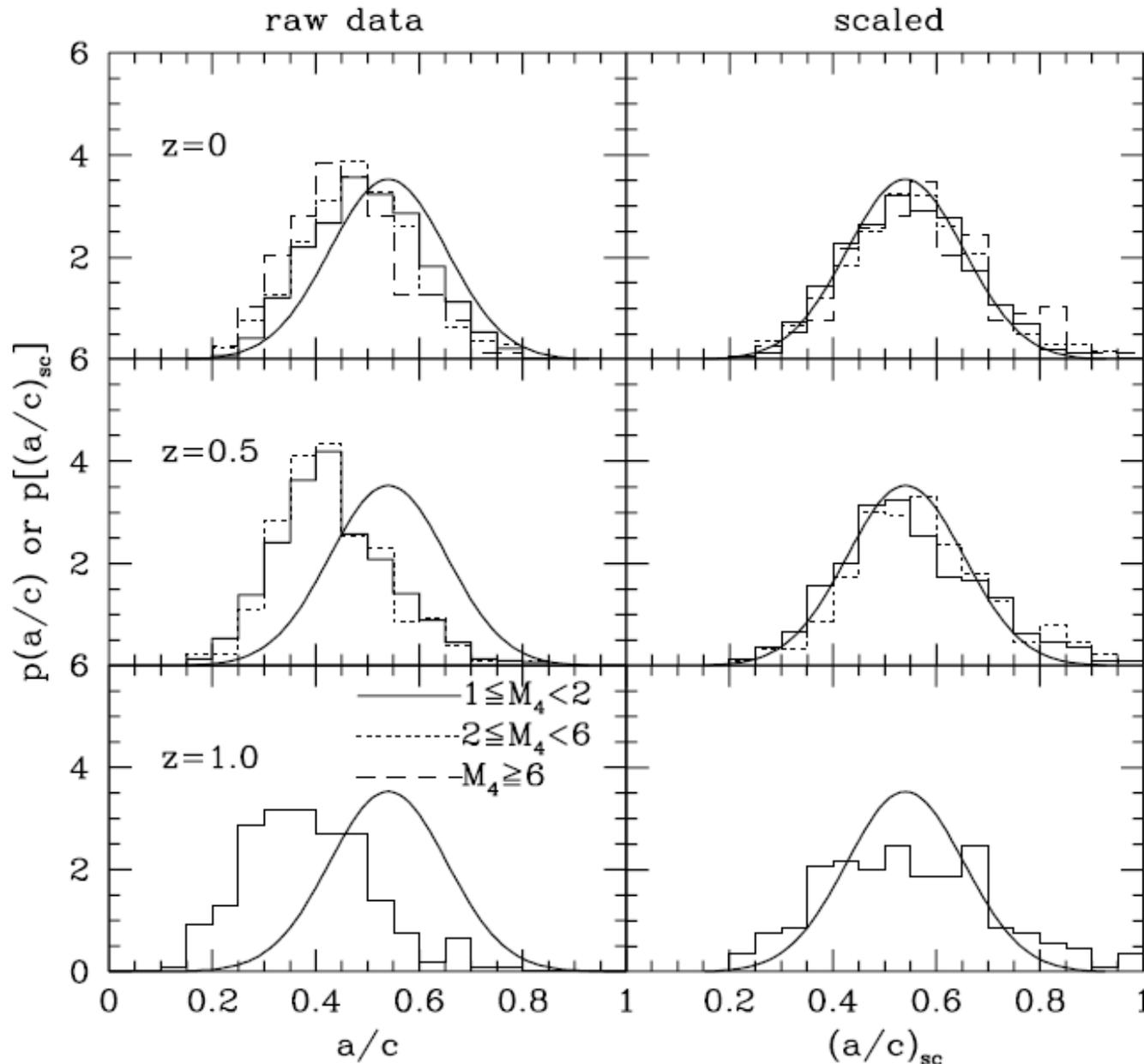
Shapes become systematically less spherical with decreasing radius

There is a lot of scatter between halos

Principal axes may change direction

# Shapes of halo equidensity surfaces

$$\Omega_0=1.0, \Lambda_0=0.0, \sigma_8=0.55$$



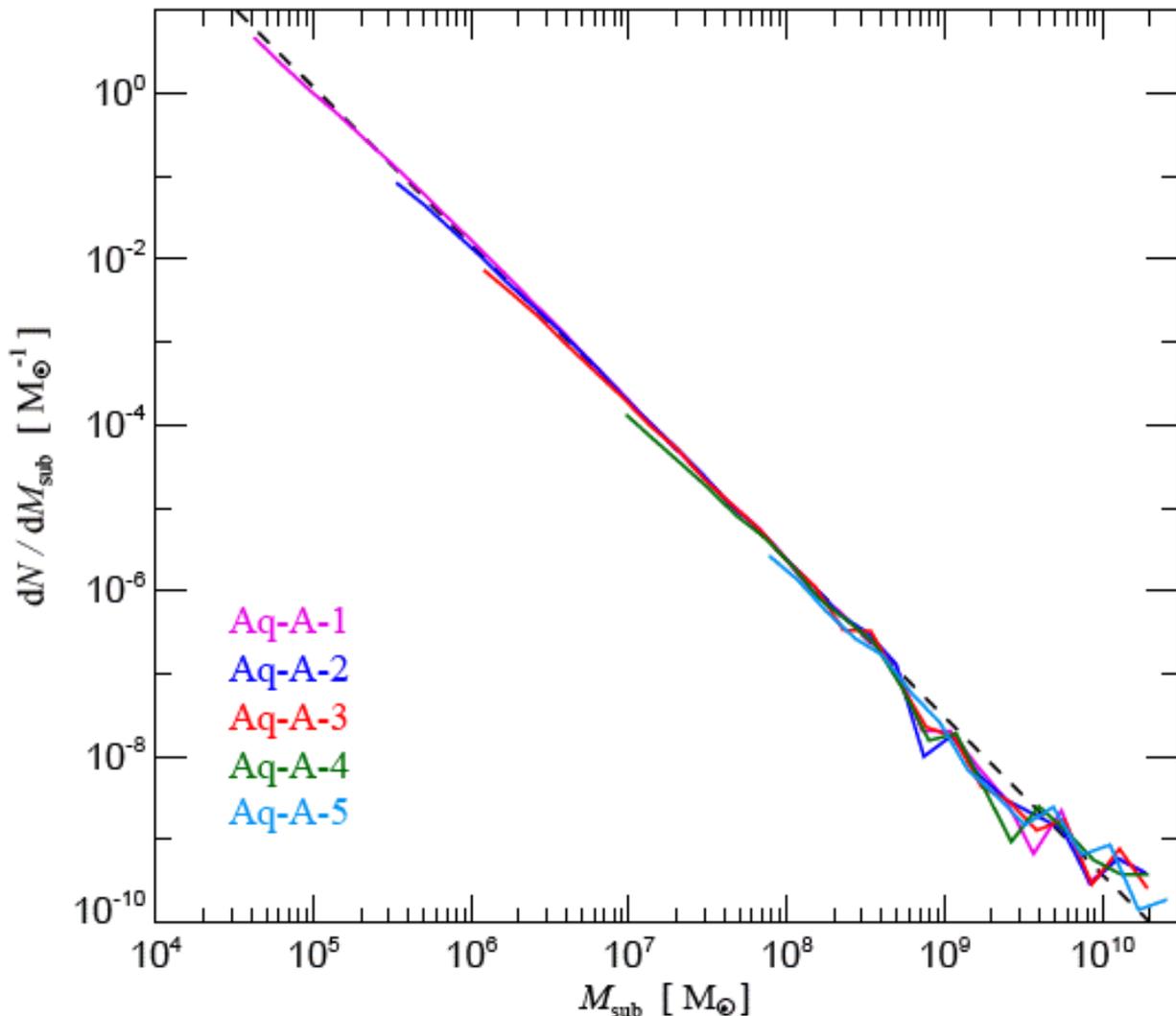
Jing & Suto 2002

Shapes become systematically less spherical with increasing mass

A simple scaling leaves a “universal” result for the axis ratio distributions

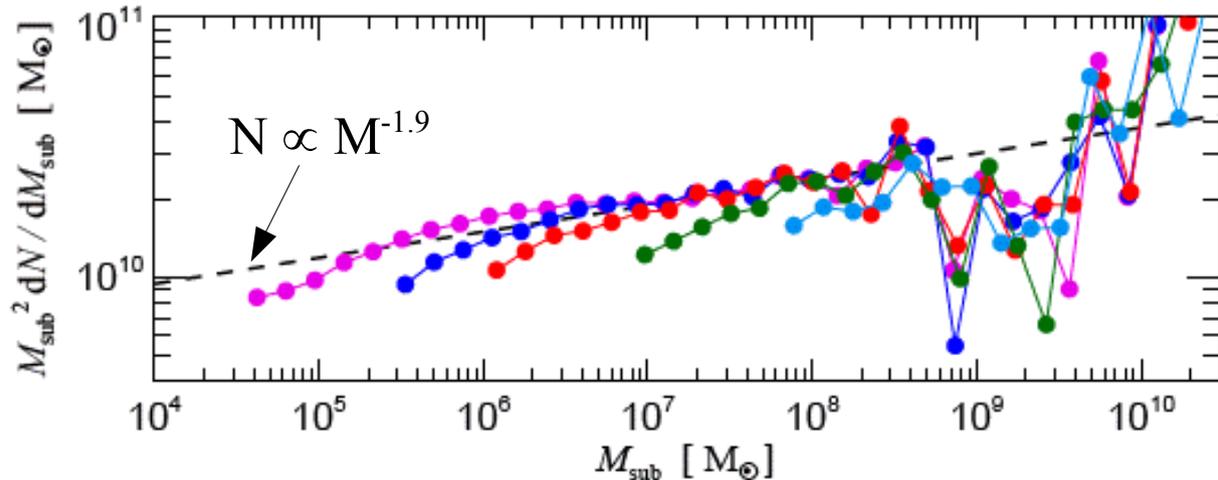
# How well does substructure converge?

Springel et al 2008



Differential number count of subhalos with  $r < 400$  kpc

Convergence is excellent



Slope is slightly shallower than  $-2$   $\rightarrow$  most subhalo mass is in the biggest objects

# How well does substructure converge?

Aquarius Project: Springel et al 2008

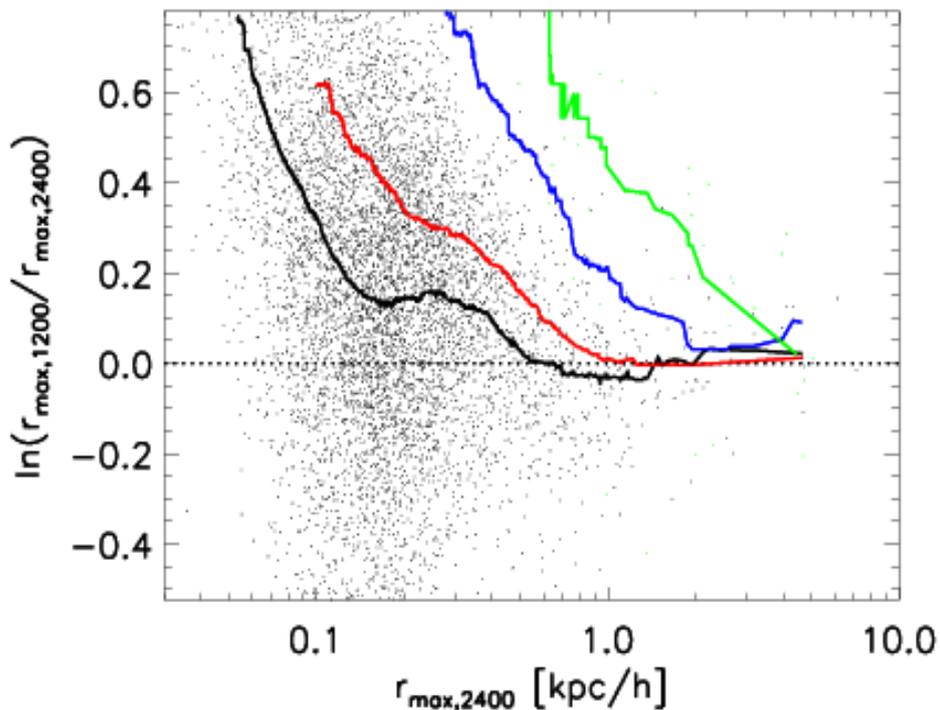
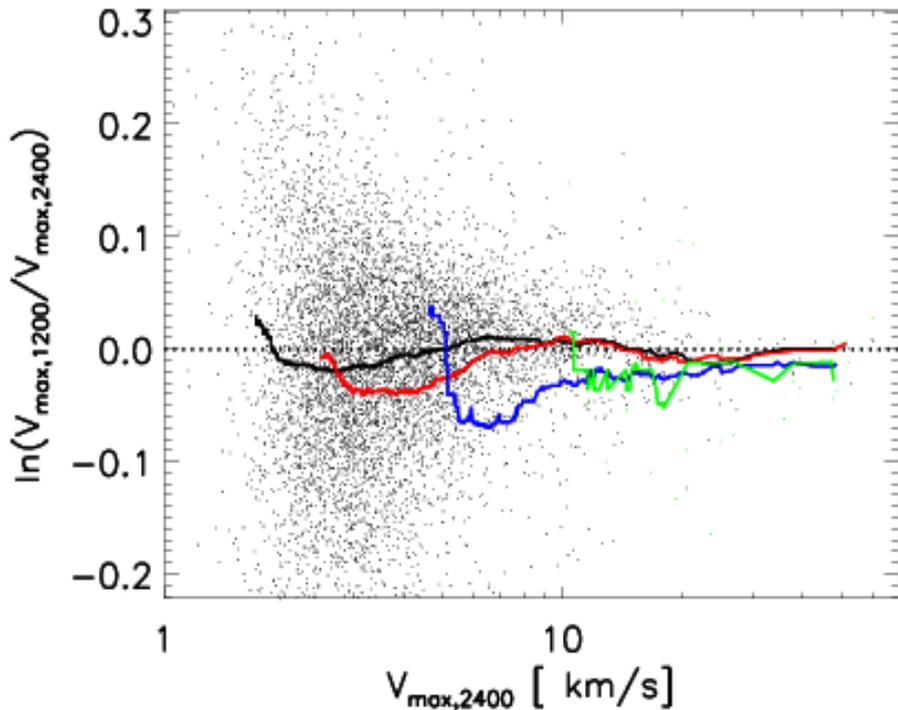
Convergence in the size and maximum circular velocity for individual subhalos cross-matched between simulation pairs.

Biggest simulation gives convergent results for

$$V_{\max} > 1.5 \text{ km/s}$$

$$r_{\max} > 165 \text{ pc}$$

Much smaller than the halos inferred for even the faintest dwarf galaxies



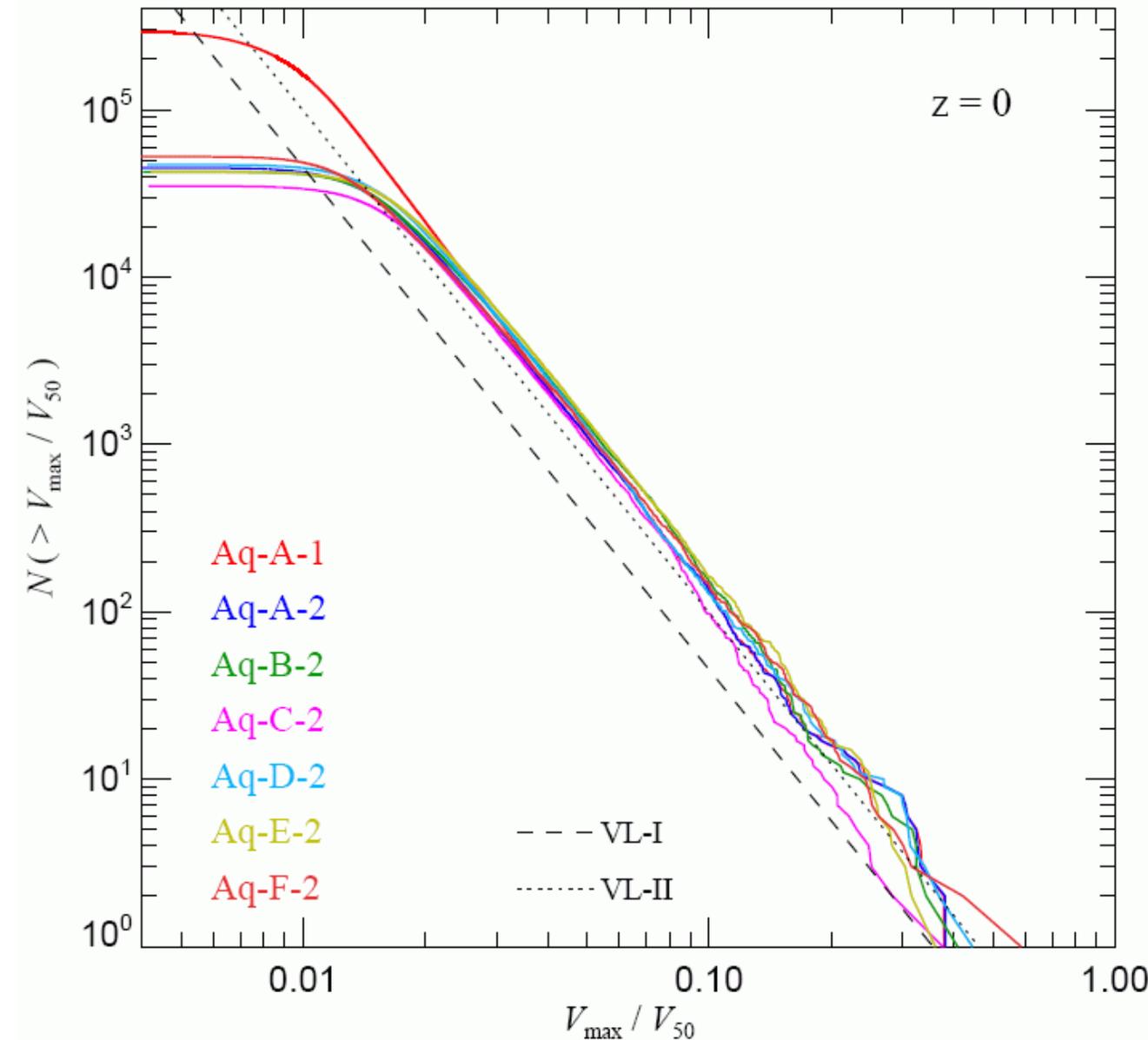
# How uniform are subhalo populations?

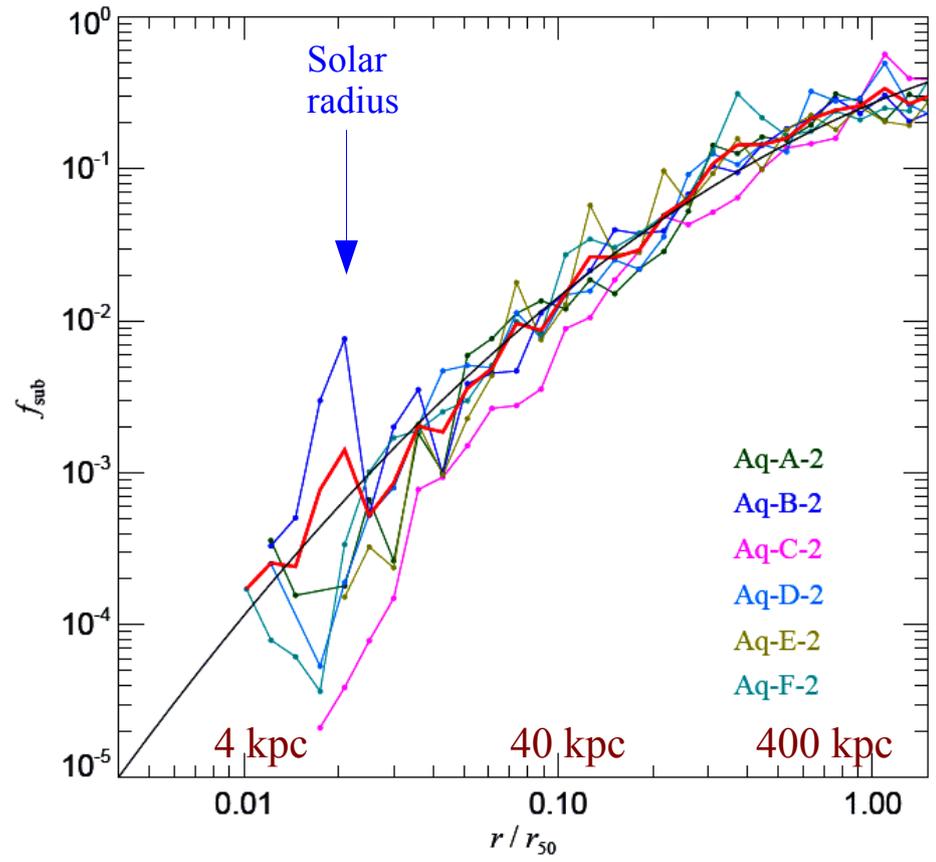
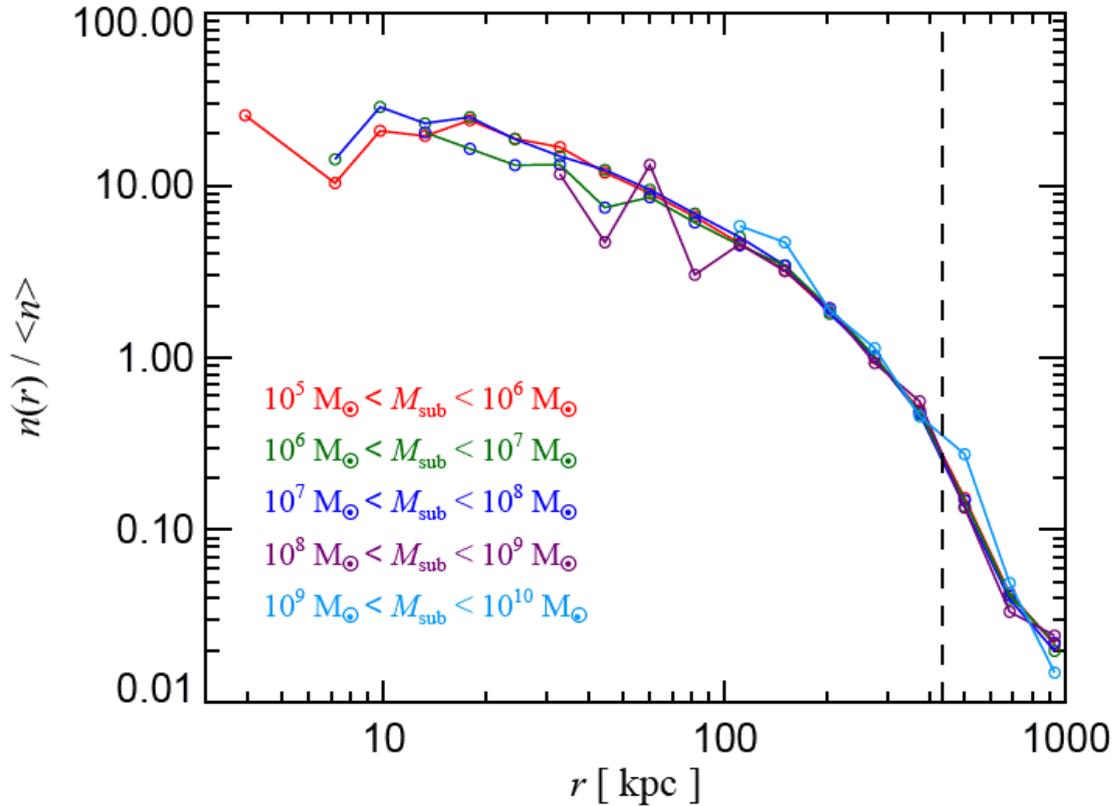
Springel et al 2008

For the six Aquarius halos, the scatter in subhalo abundance is Poisson at high mass and  $\sim 20\%$  at low mass

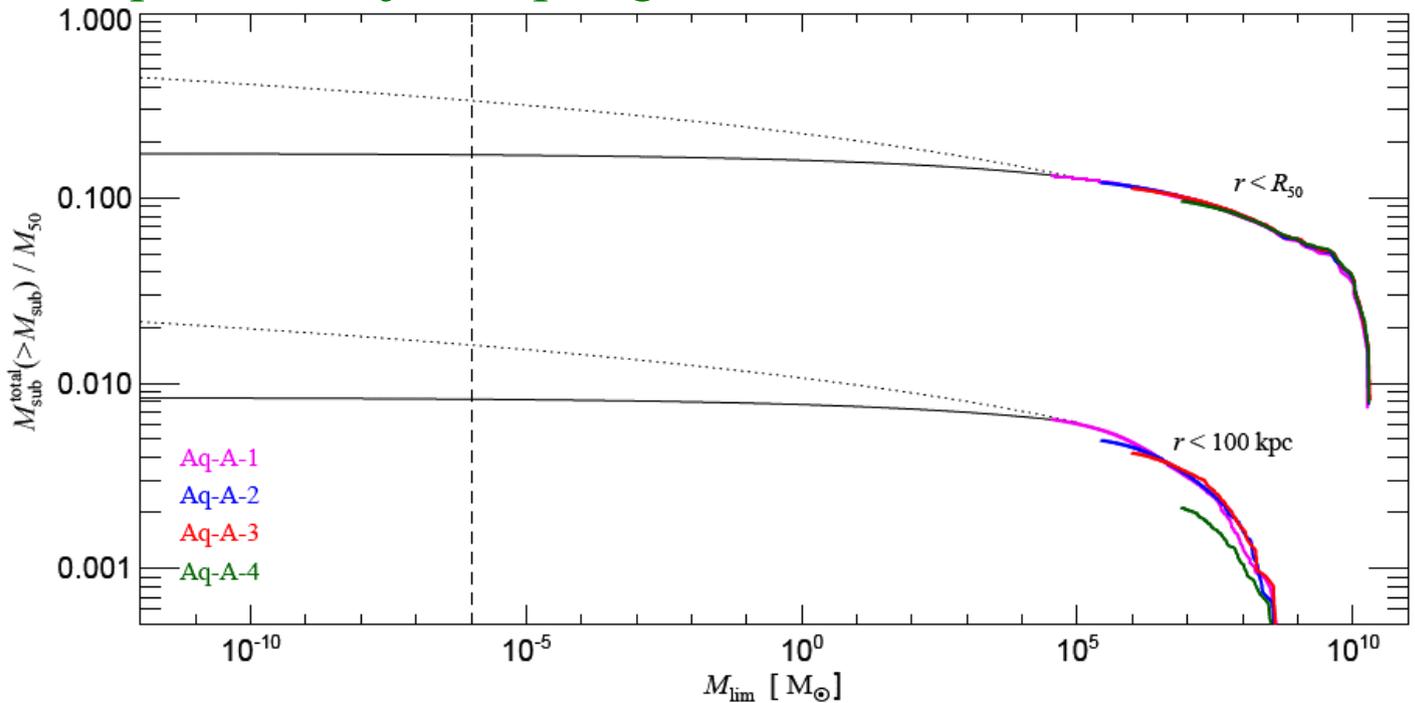
All “Milky Way” halos have similar amounts of substructure

The Via Lactea simulations differ significantly





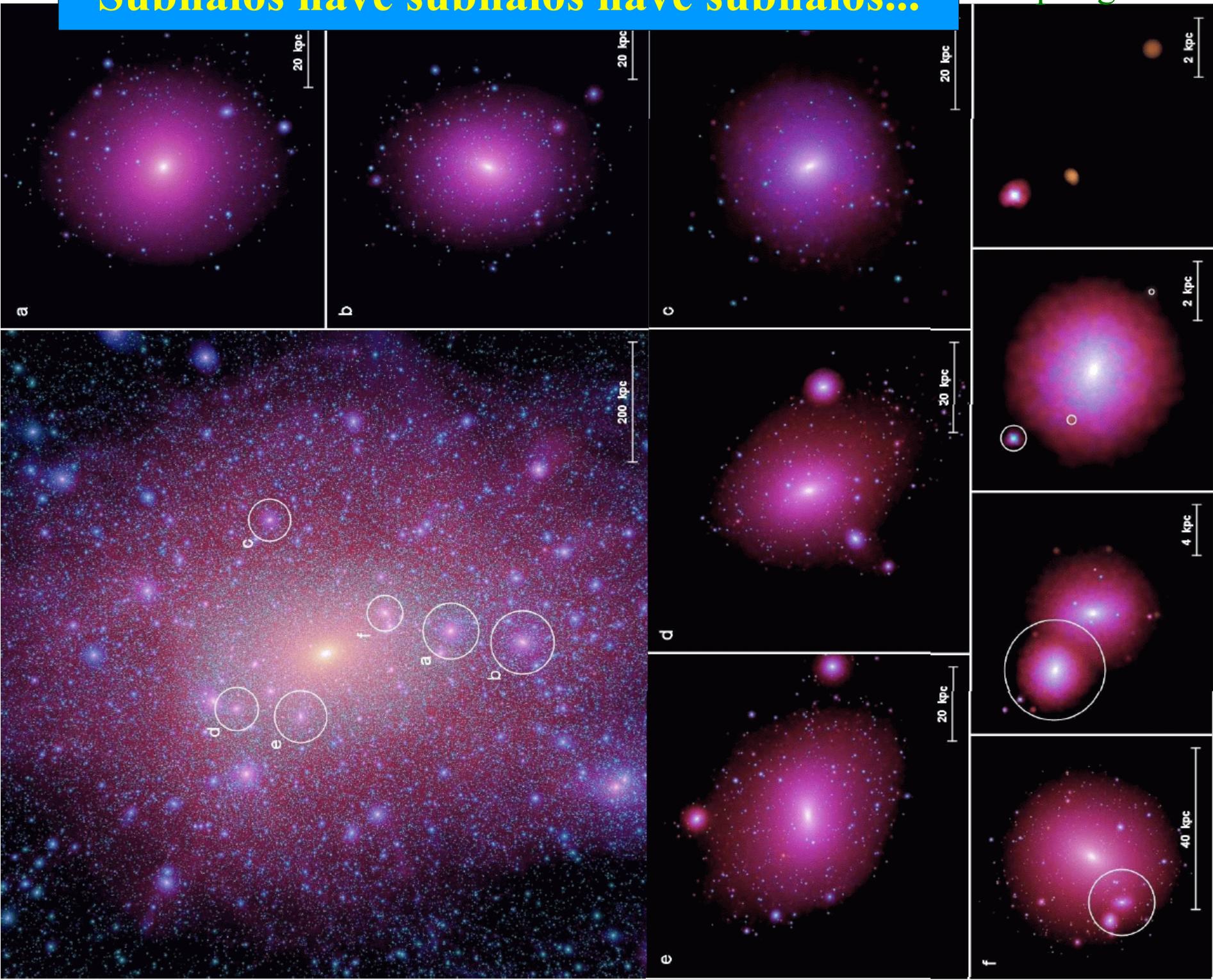
Aquarius Project: Springel et al 2008



- All mass subhalos are similarly distributed
- A small fraction of the inner mass in subhalos
- $\ll 1\%$  of the mass near the Sun is in subhalos

# Subhalos have subhalos have subhalos...

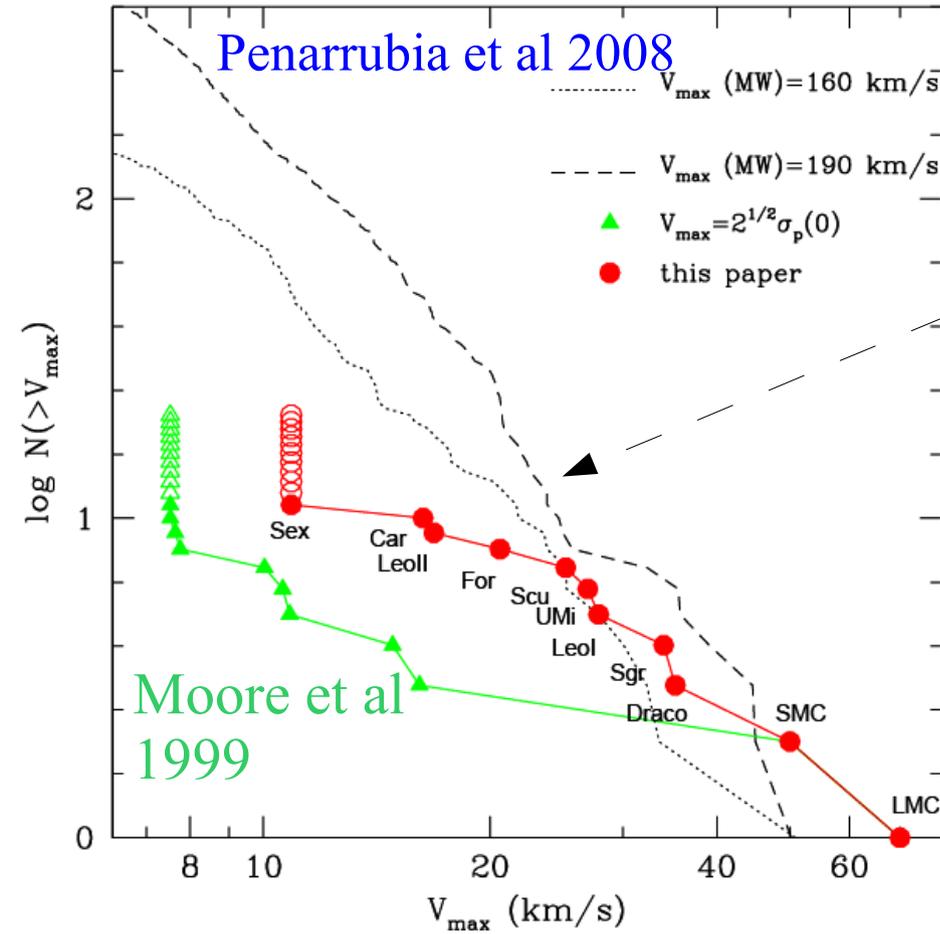
Springel et al 2008



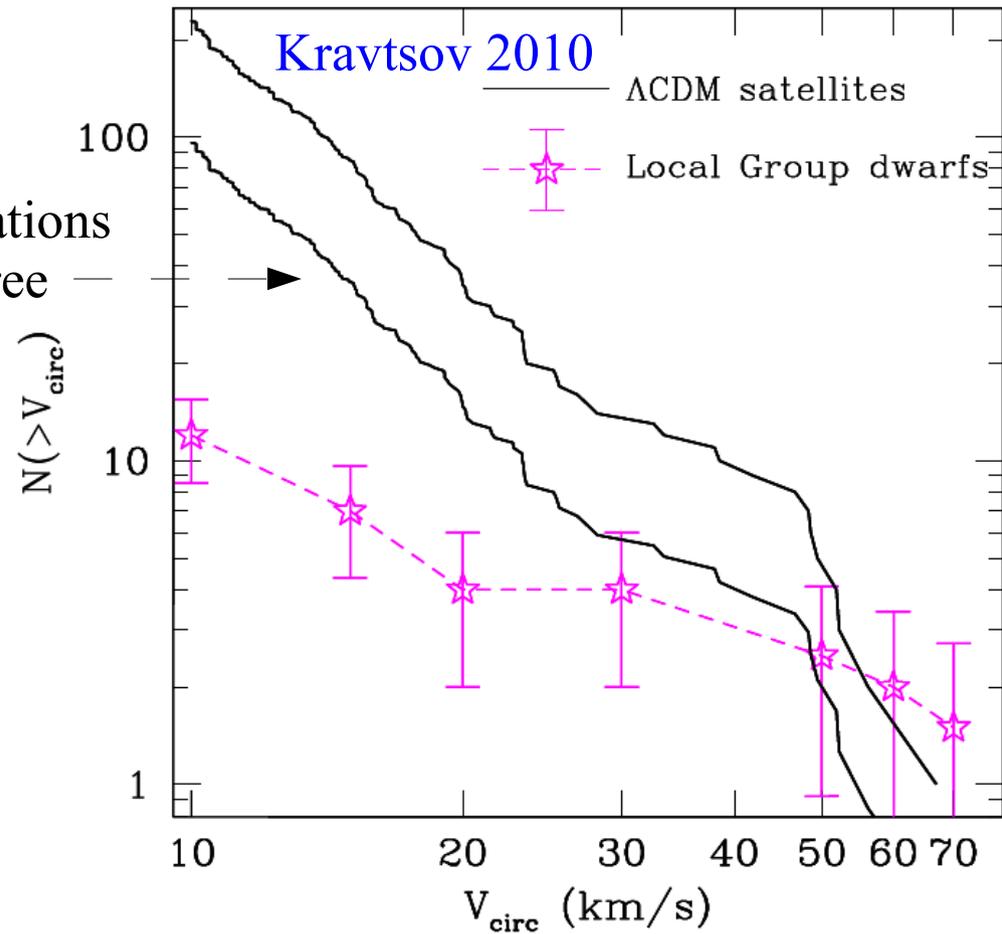
# Substructure: conclusions

- Substructure is primarily in the outermost parts of halos
- The radial distribution of subhalos is almost mass-independent
- Subhalo populations scale (almost) with the mass of the host
- The subhalo mass distribution converges only weakly at small  $m$
- Subhalos contain a small fraction of the mass in the inner halo

# A Milky Way satellite problem?



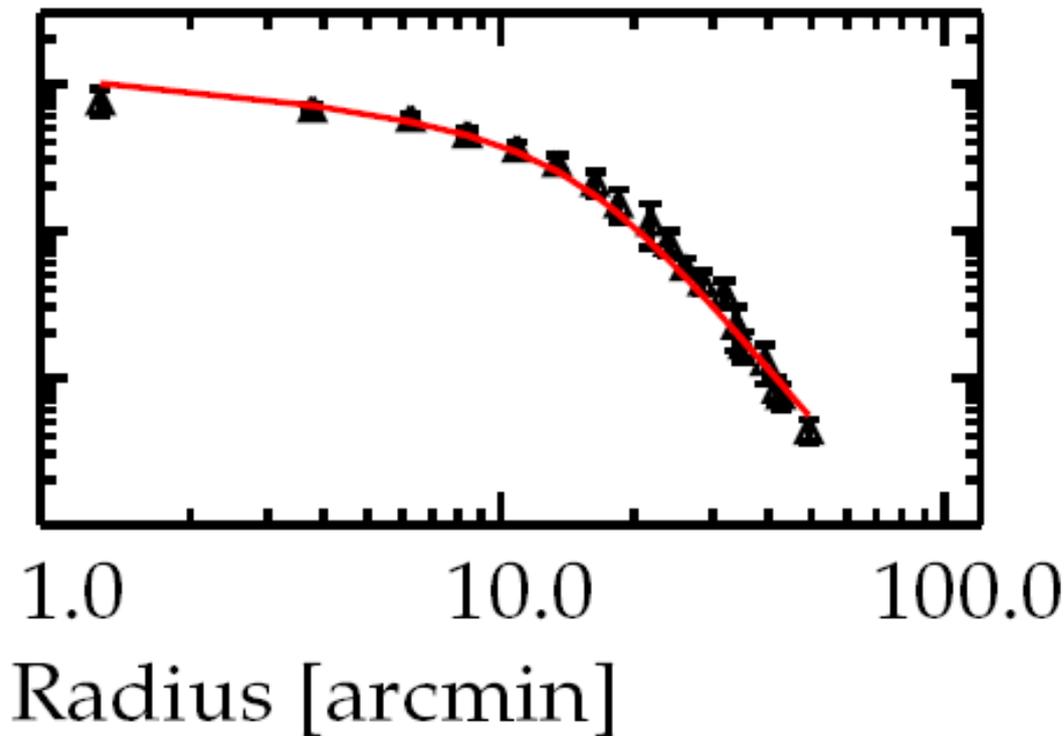
simulations  
agree



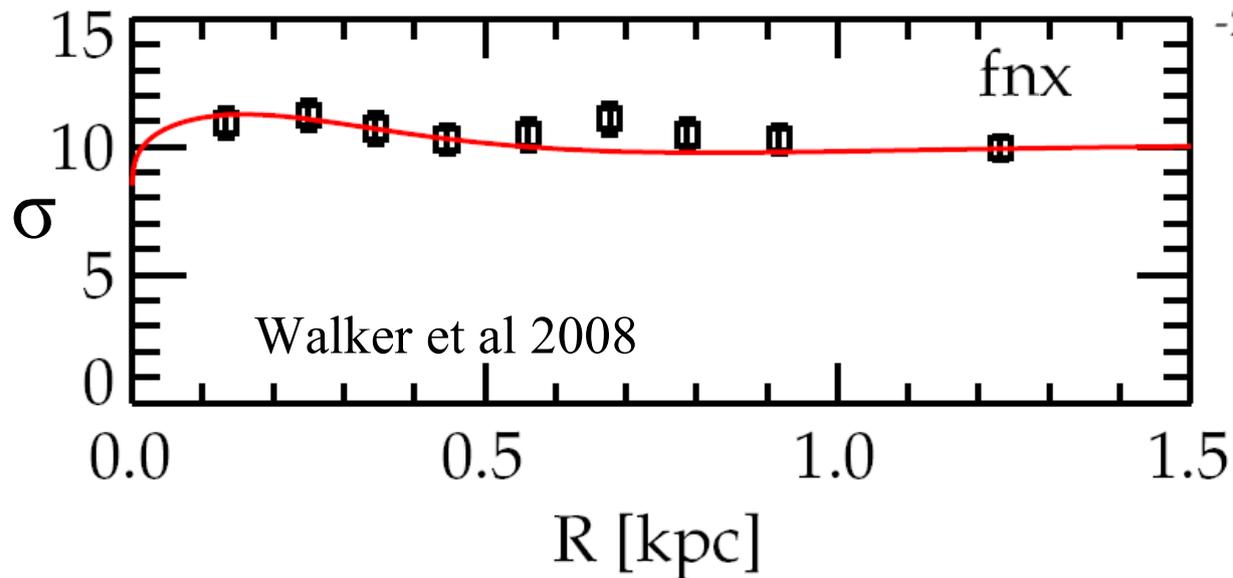
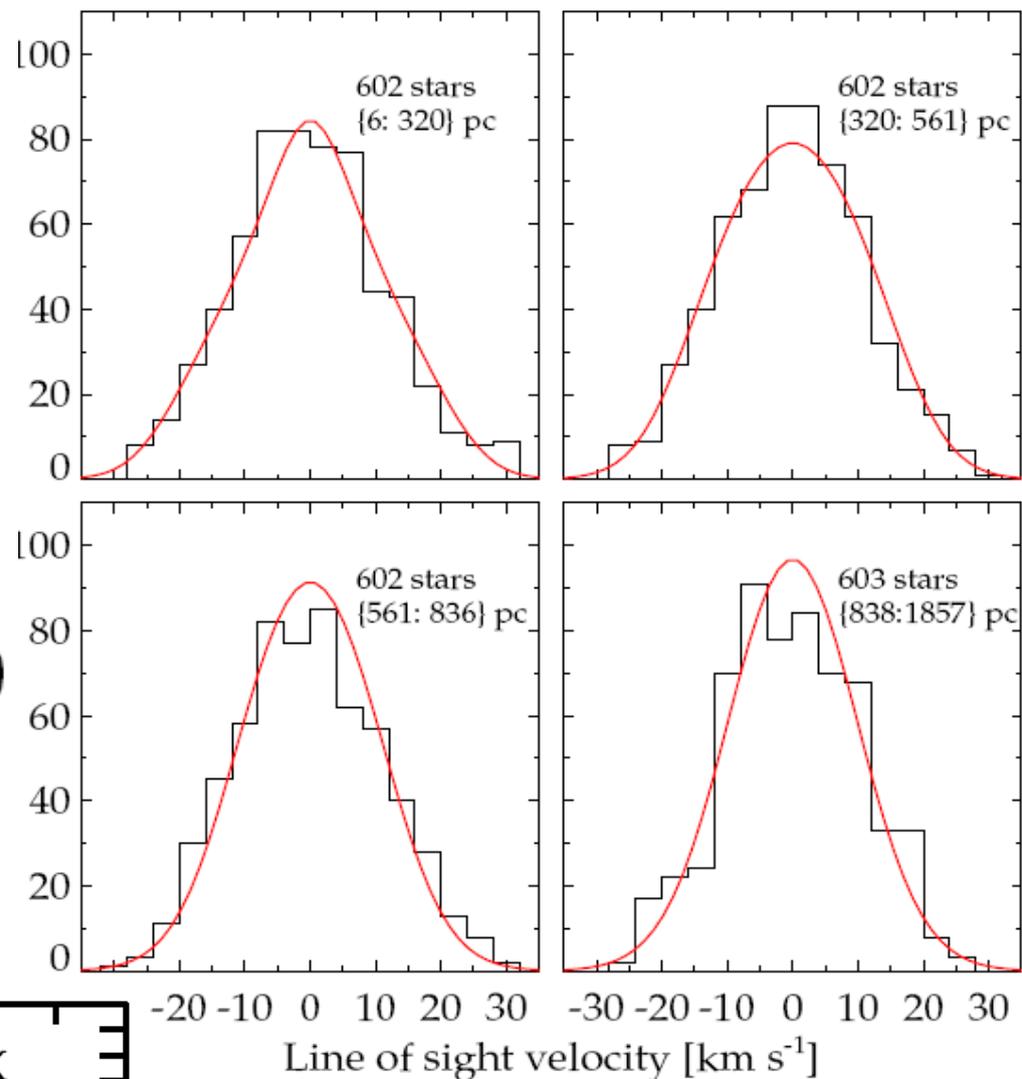
# of small subhalos in  $\Lambda\text{CDM}$  halos  $\gg$  # of MW satellites

Maybe most of them just didn't make stars?

Subhalos with stars must be less massive than many without stars?



Strigari, Frenk & White 2010

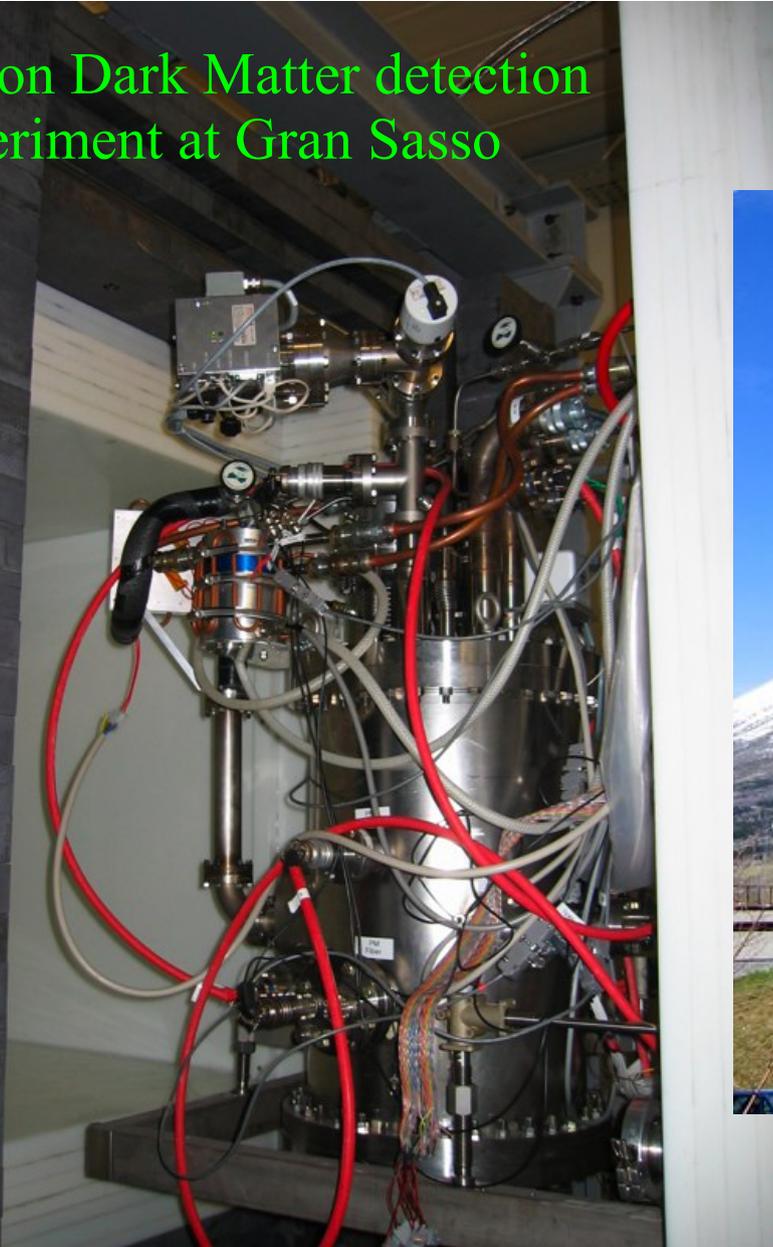


Fornax data are **consistent** with living in an Aquarius CDM subhalo with isotropic velocity dispersions

→ a cusp is not excluded

# Maybe Dark Matter can be detected in a laboratory

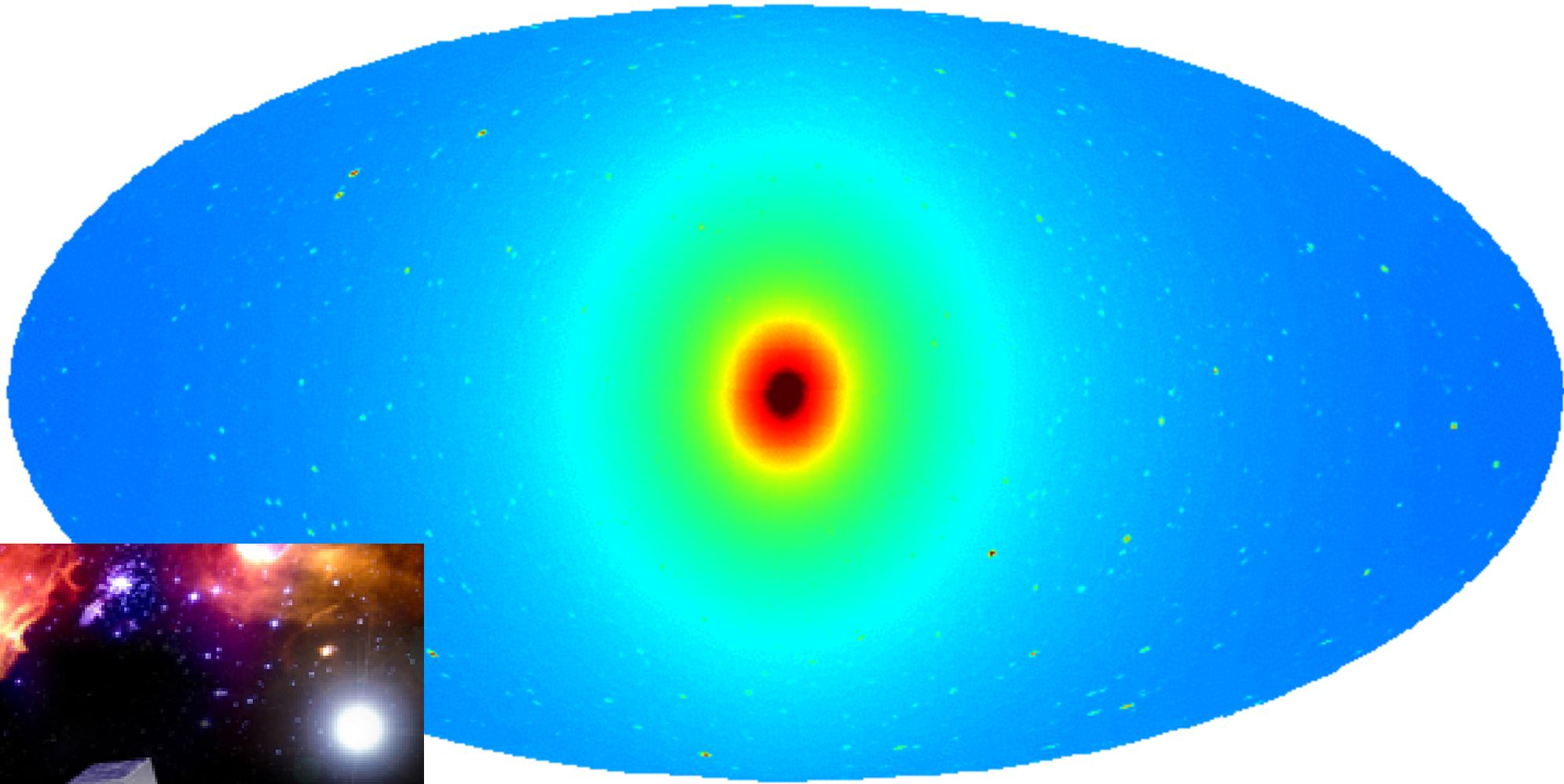
Xenon Dark Matter detection experiment at Gran Sasso



External view of Gran Sasso Laboratory



*total emission*



-0.50  2.0 Log(Intensity)



**Maybe the annihilation of Dark Matter will be seen by Fermi?**

**Fermi  $\gamma$ -ray observatory**

# Small-scale structure and DM detection

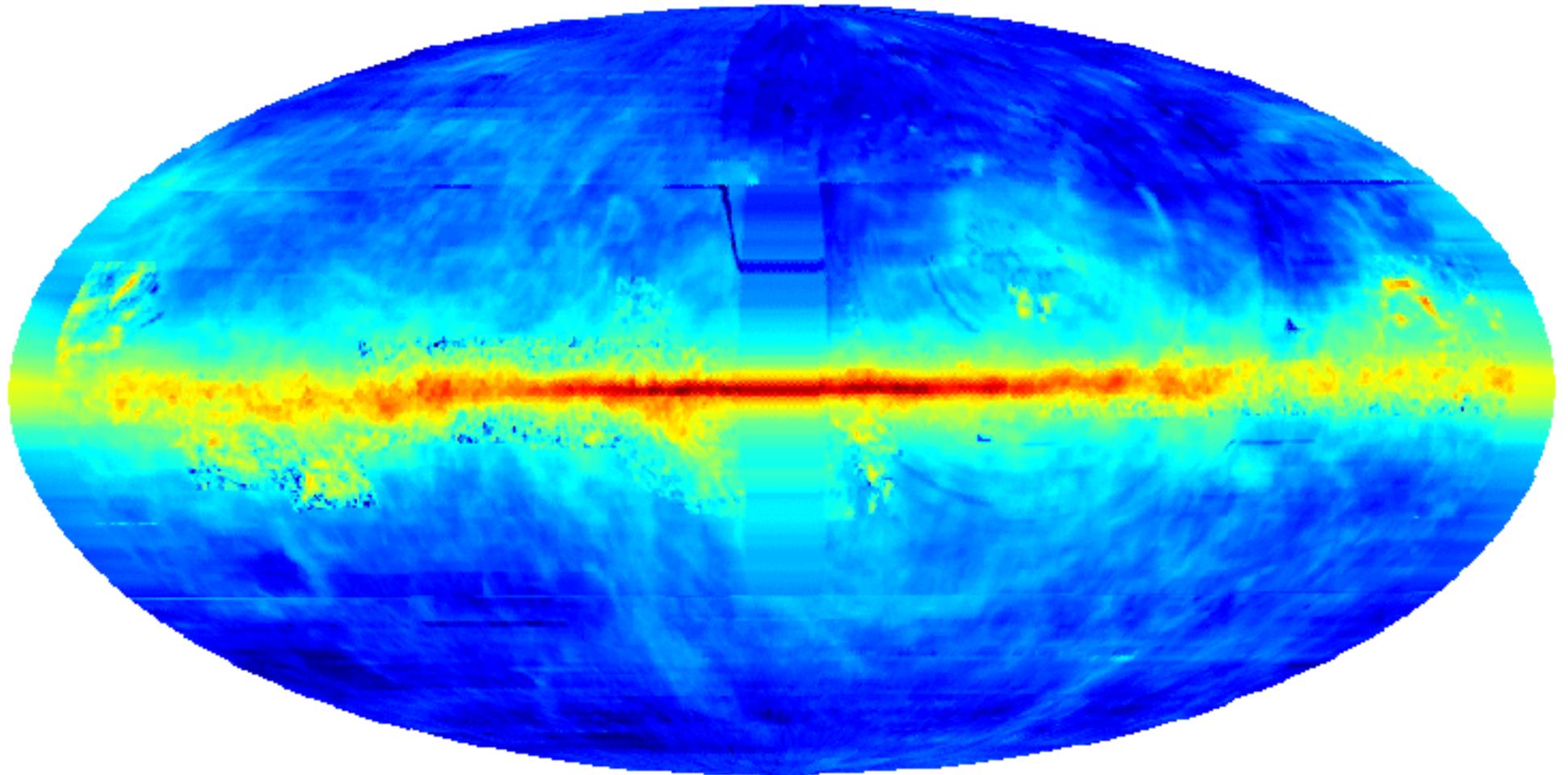
- Direct detection involves bolometers/cavities of meter scale which are sensitive to particle momentum
  - what is the density structure between m and kpc scales?
  - how many streams intersect the detector at any time?
- Indirect detection involves annihilation radiation
$$L \propto \int \rho^2(\mathbf{x}) \langle \sigma v \rangle dV$$
  - what is the density distribution around individual CDM particles on the annihilation interaction scale?

Predictions for detection experiments depend on the CDM distribution on scales far below those accessible to simulation

→ We require a good theoretical understanding of mixing and small-scale structure

# Predicted distribution of $\gamma$ -rays from the MW

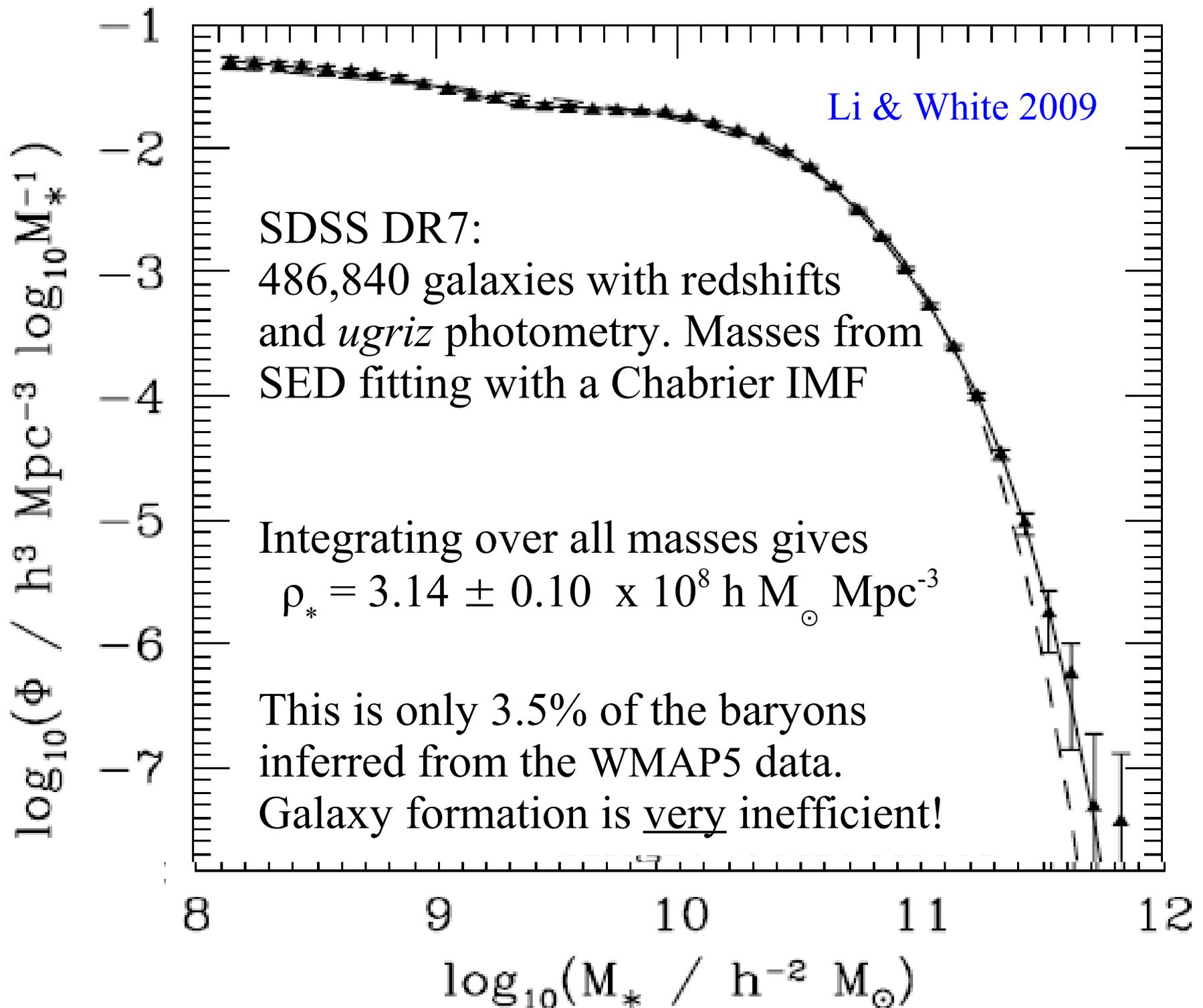
*GALPROP, optimized*



This is much brighter than the expected signal from DM annihilation

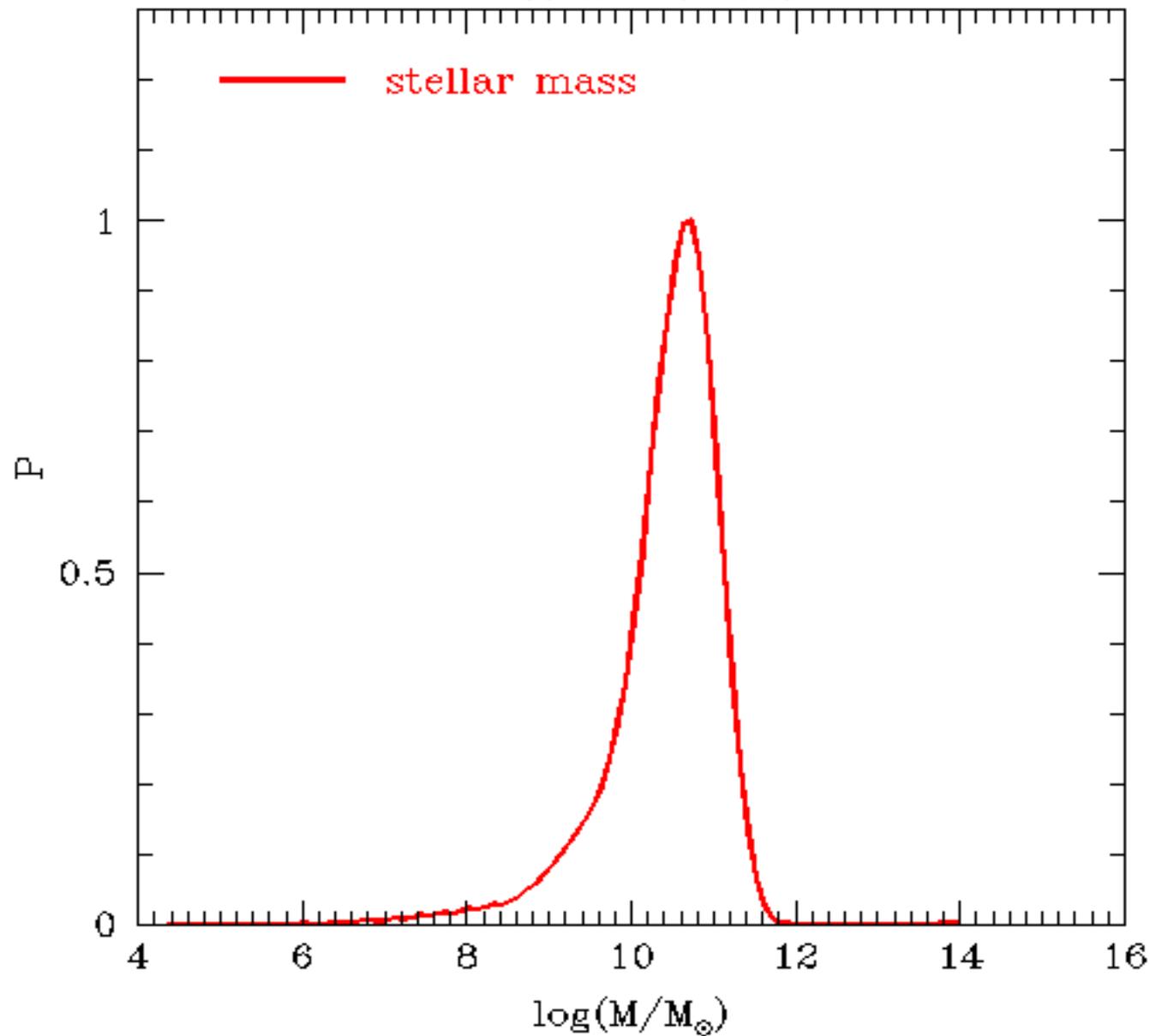
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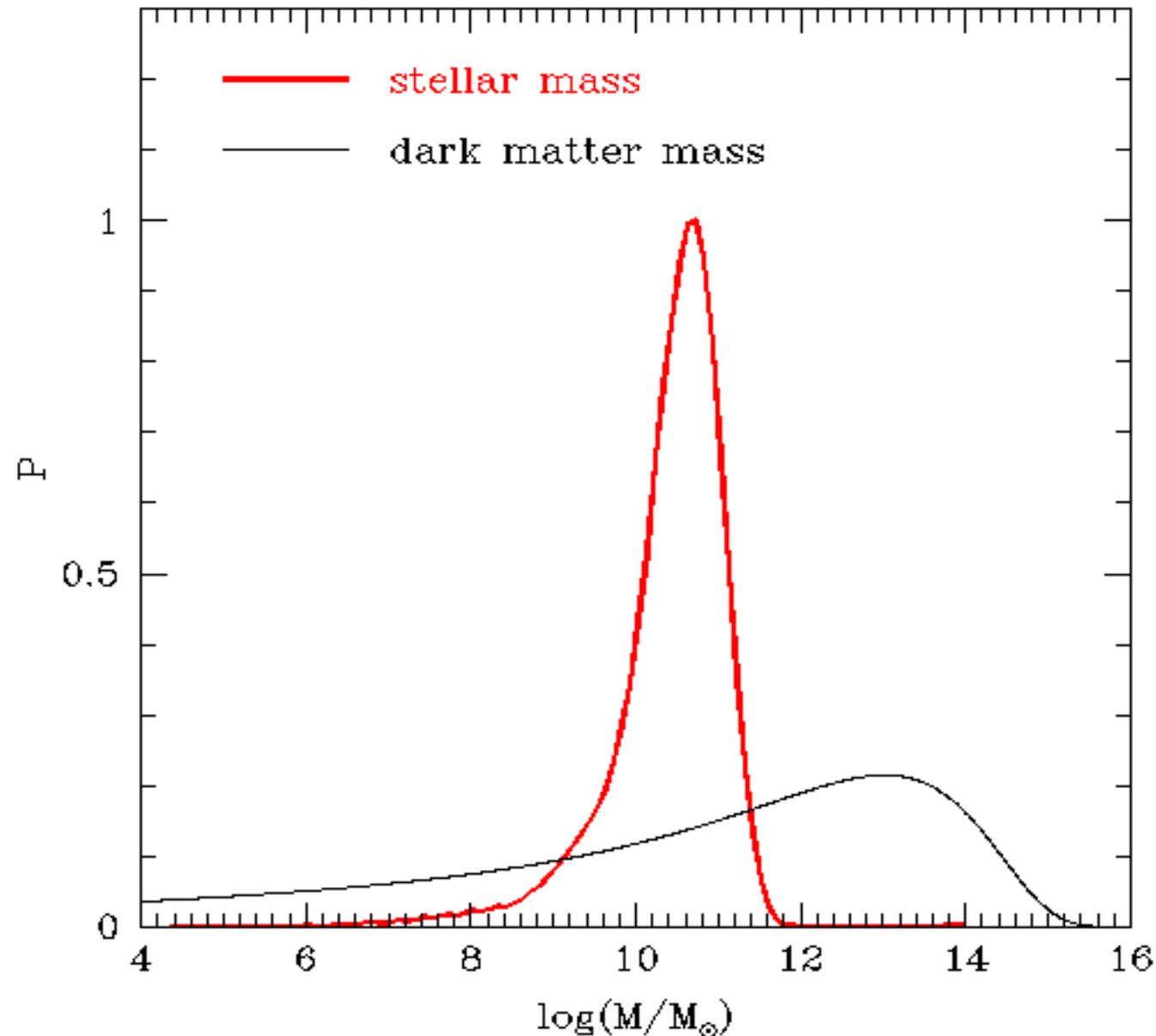
Most stars are in galaxies with similar stellar mass to the Milky Way

Li & White 2009



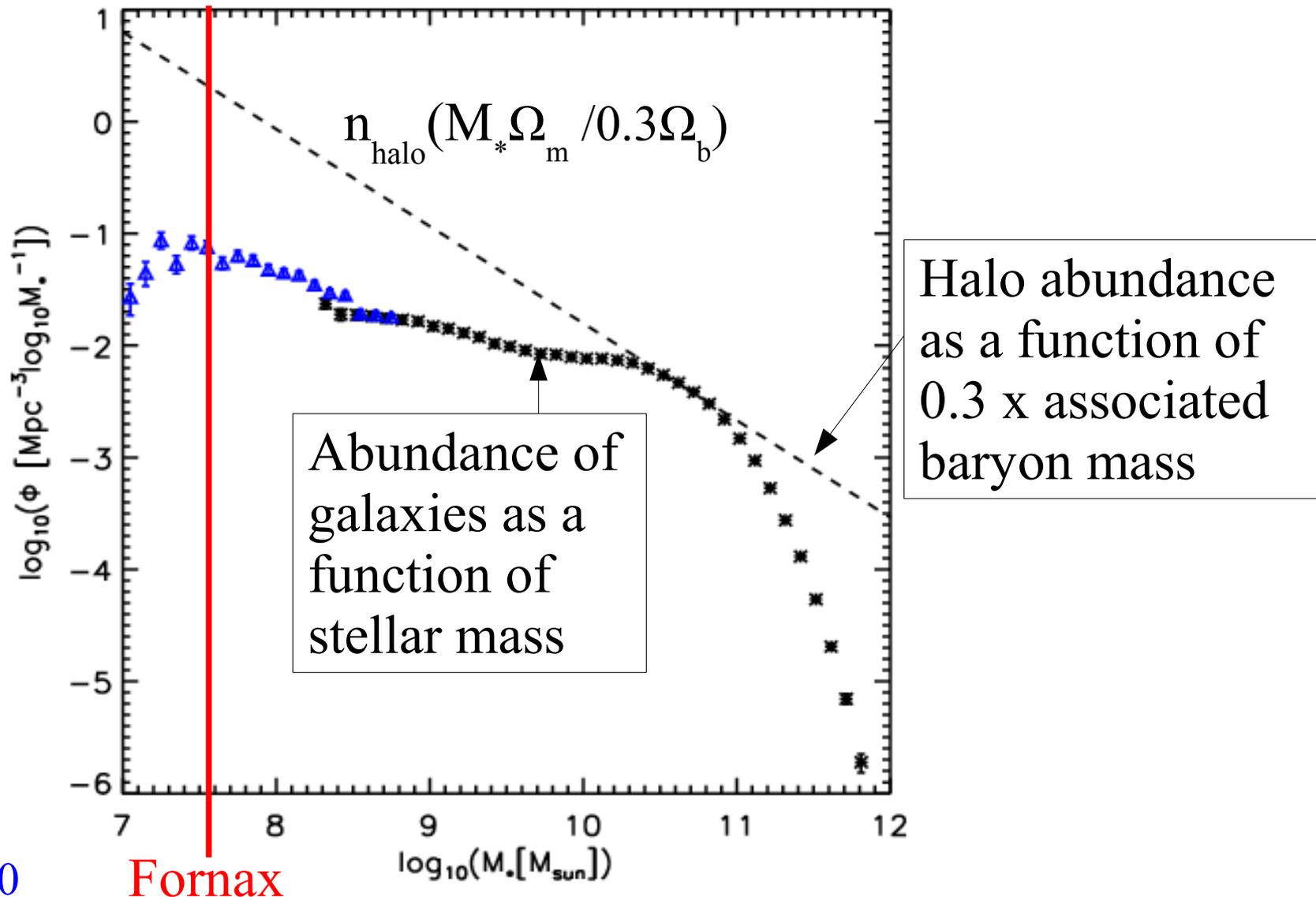
Most stars are in galaxies with similar stellar mass to the Milky Way  
Dark matter (and baryons) are *much* more broadly distributed across  
halo mass in the WMAP5 cosmology

Li & White 2009



# The problem with matching dwarfs in $\Lambda$ CDM

A formation efficiency which matches abundance of “Milky Ways” overproduces the number of “Fornax's” by a factor of 30!



# A counting argument relating halo and galaxy masses

The SDSS/DR7 data give a precise measurement of the abundance of galaxies as a function of stellar mass threshold,  $n(> M_*)$

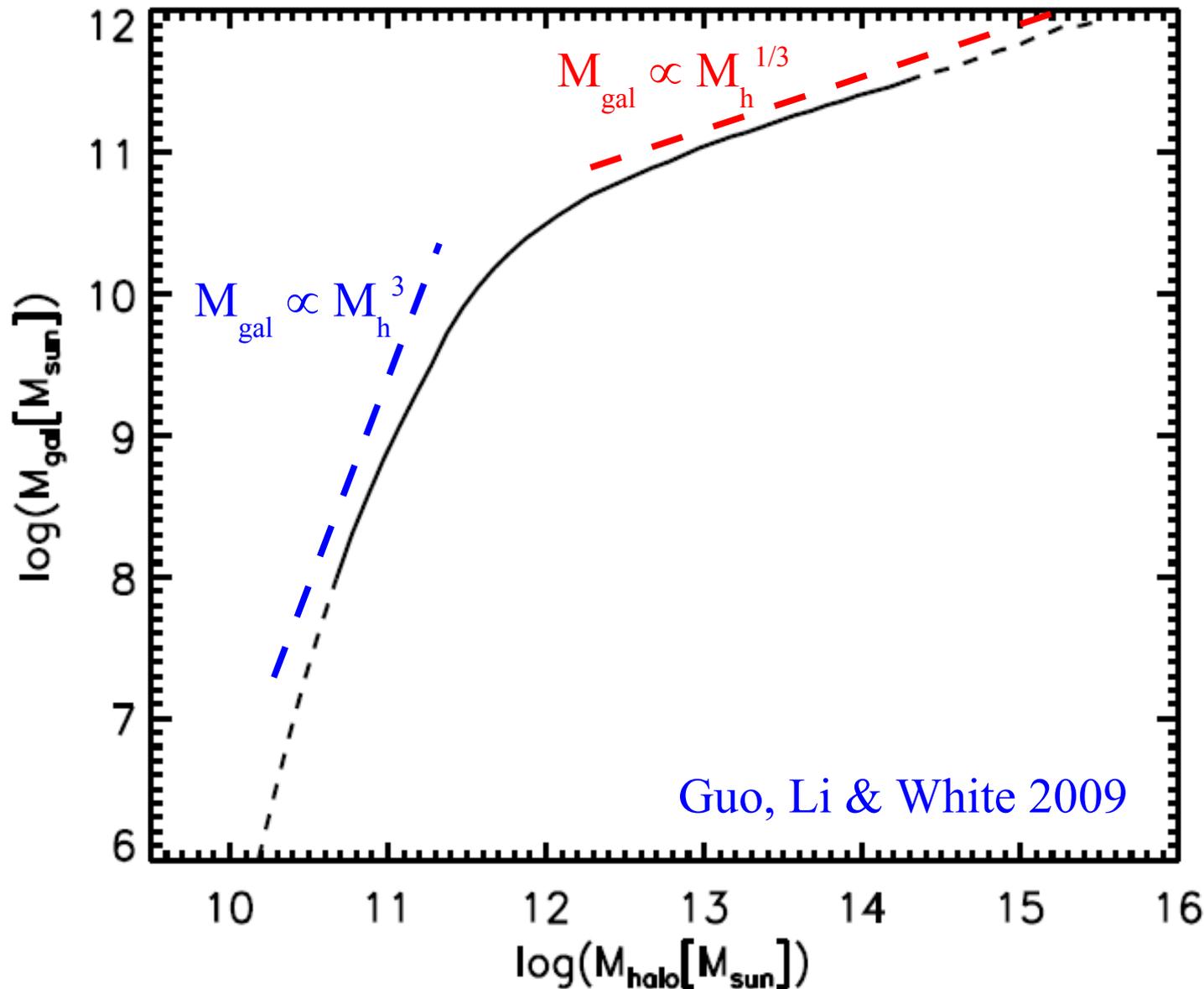
The Millennium and MS-II simulations allow all halos/subhalos massive enough to host  $z=0$  galaxies to be identified

Define  $M_{h,max}$  as the maximum mass *ever* attained by a halo/subhalo

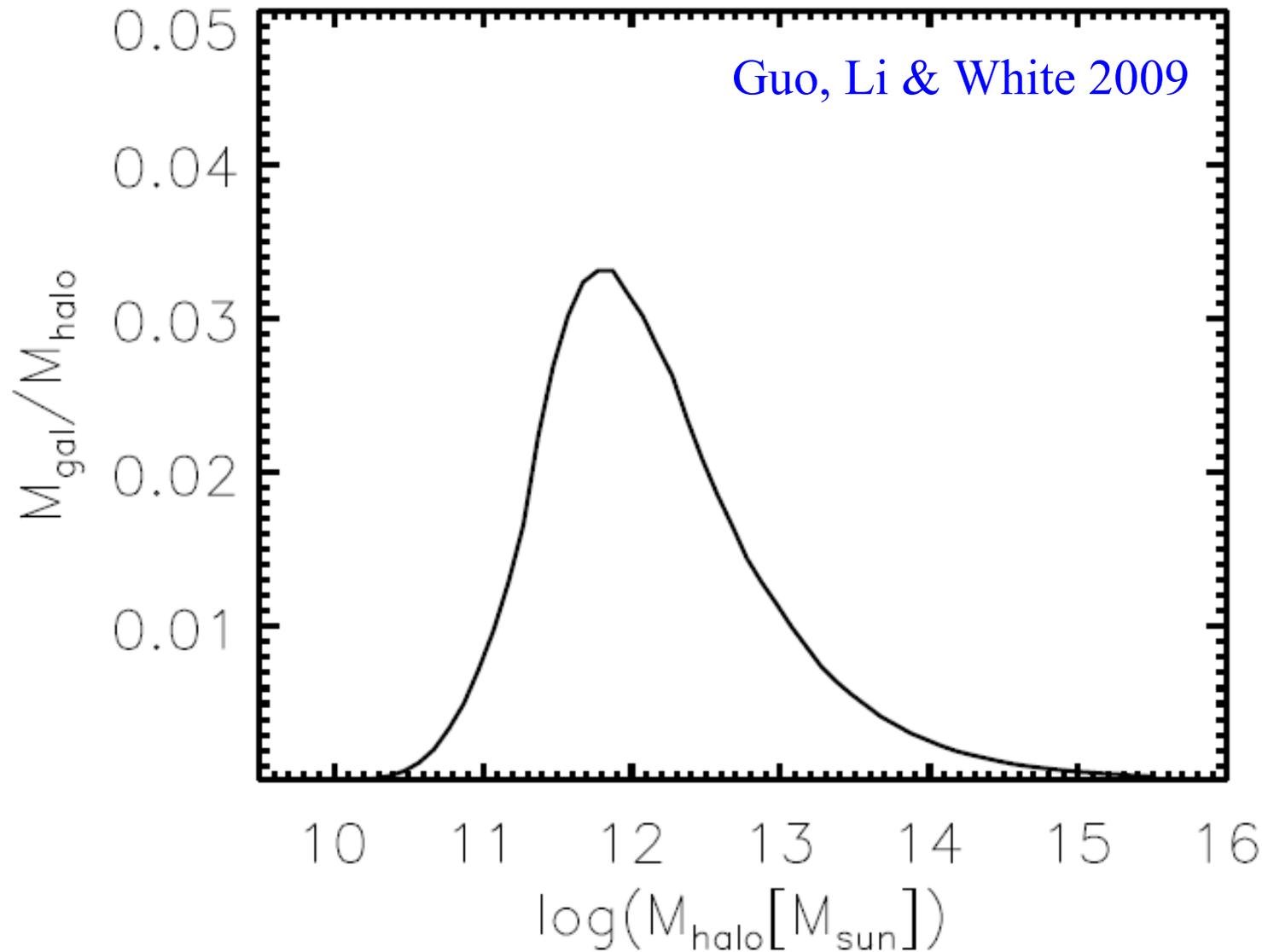
The simulations then give the halo/subhalo abundance,  $n(> M_{h,max})$

**Ansatz:** Assume the stellar mass of a galaxy to be a monotonically increasing function of the maximum mass ever attained by its halo

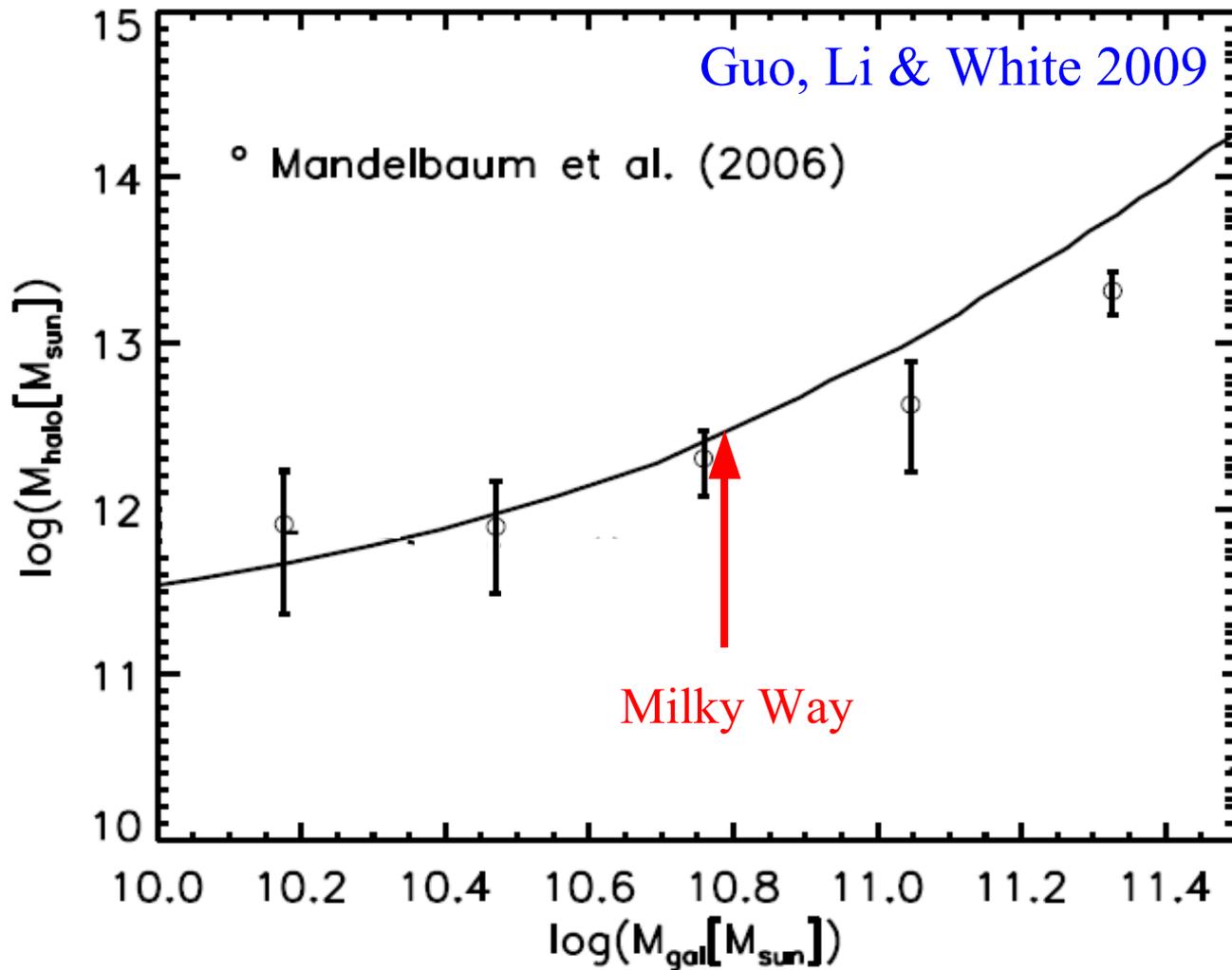
We can then derive  $M_*(M_{h,max})$  by setting  $n(> M_*) = n(> M_{h,max})$



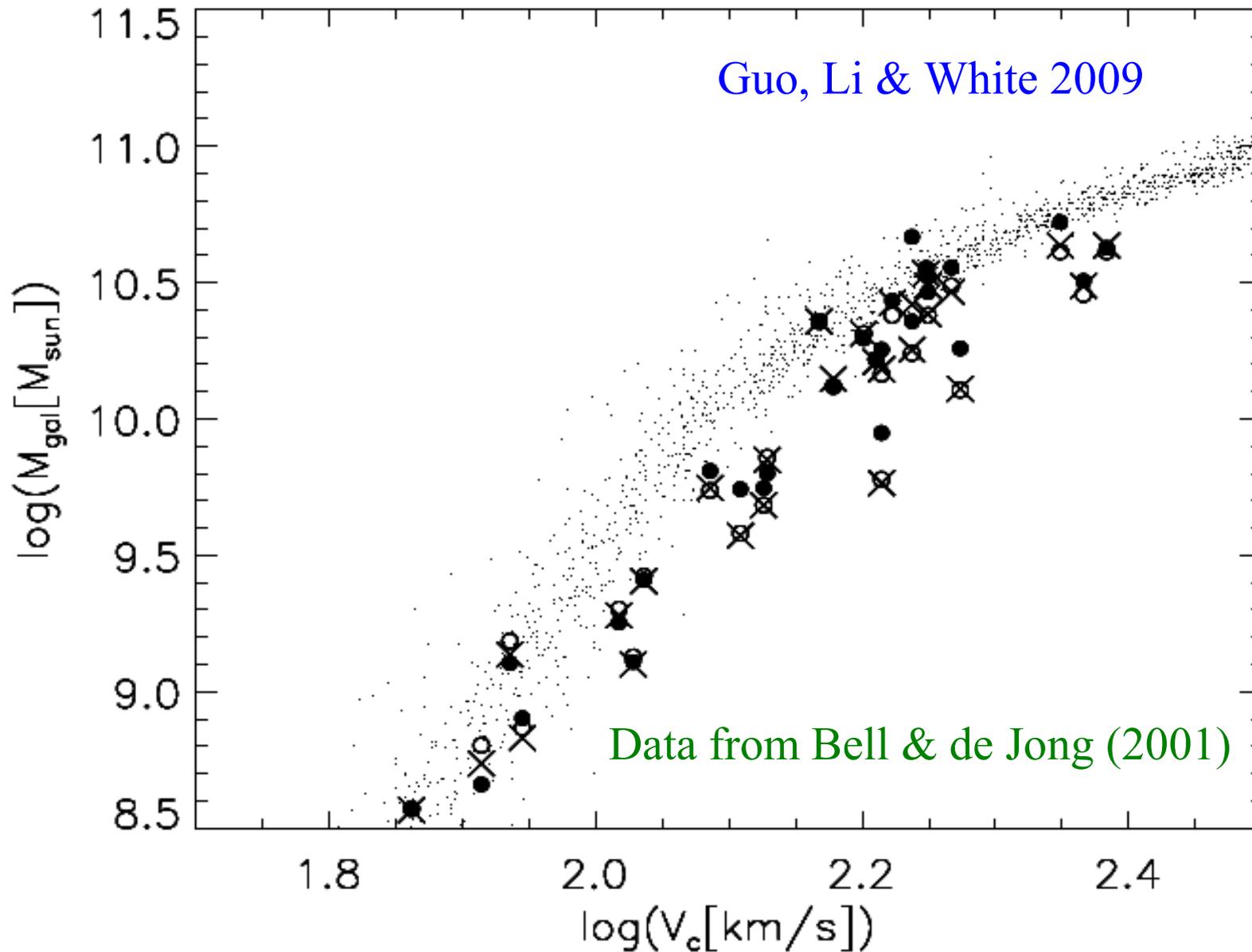
- The stellar mass of the central galaxy increases rapidly with halo mass at small halo mass, but slowly at large halo mass
- The characteristic halo mass at the bend is  $5 \times 10^{11} M_{\odot}$



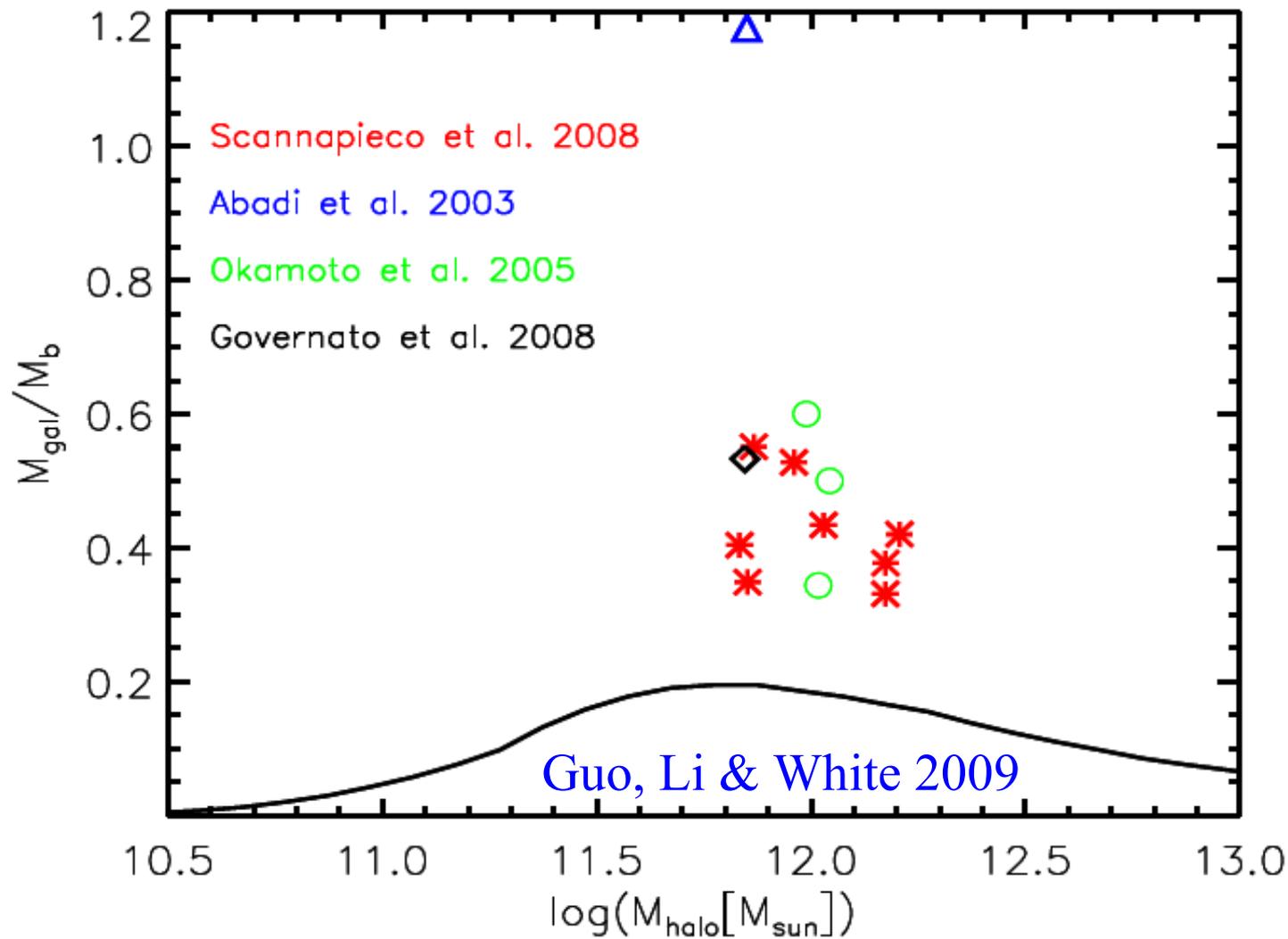
- The maximum halo mass fraction in central galaxy stars is 3.5%
- This is attained for halos similar in mass to the Milky Way's halo
- The fraction drops very rapidly to higher and lower masses



- The (maximum) halo masses inferred as a function of stellar mass agree well with those inferred from galaxy-galaxy lensing
- For  $M_* = 6 \times 10^{10} M_{\odot}$  the Milky Way should have  $M_h = 2 \times 10^{12} M_{\odot}$
- For  $M_h = 1.0 \times 10^{12} M_{\odot}$  it should have  $M_* = 3.5 \times 10^{10} M_{\odot}$

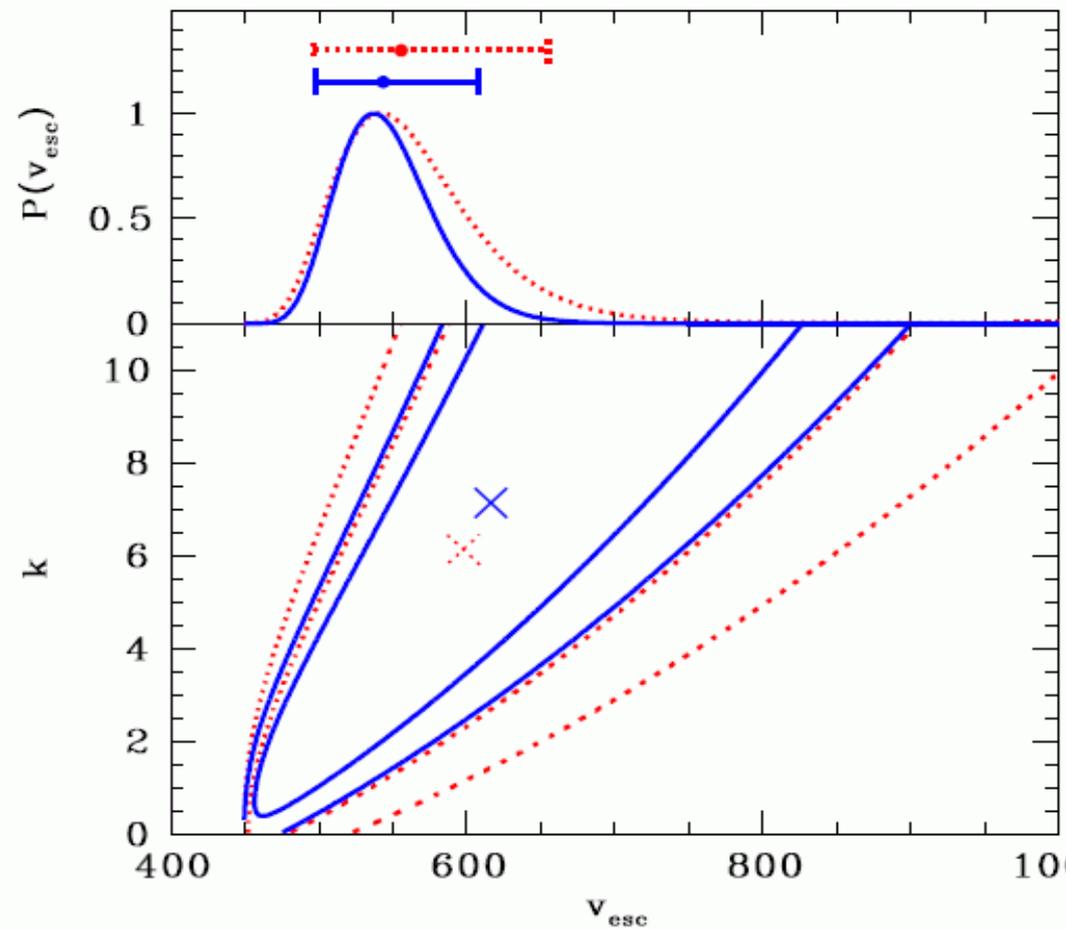
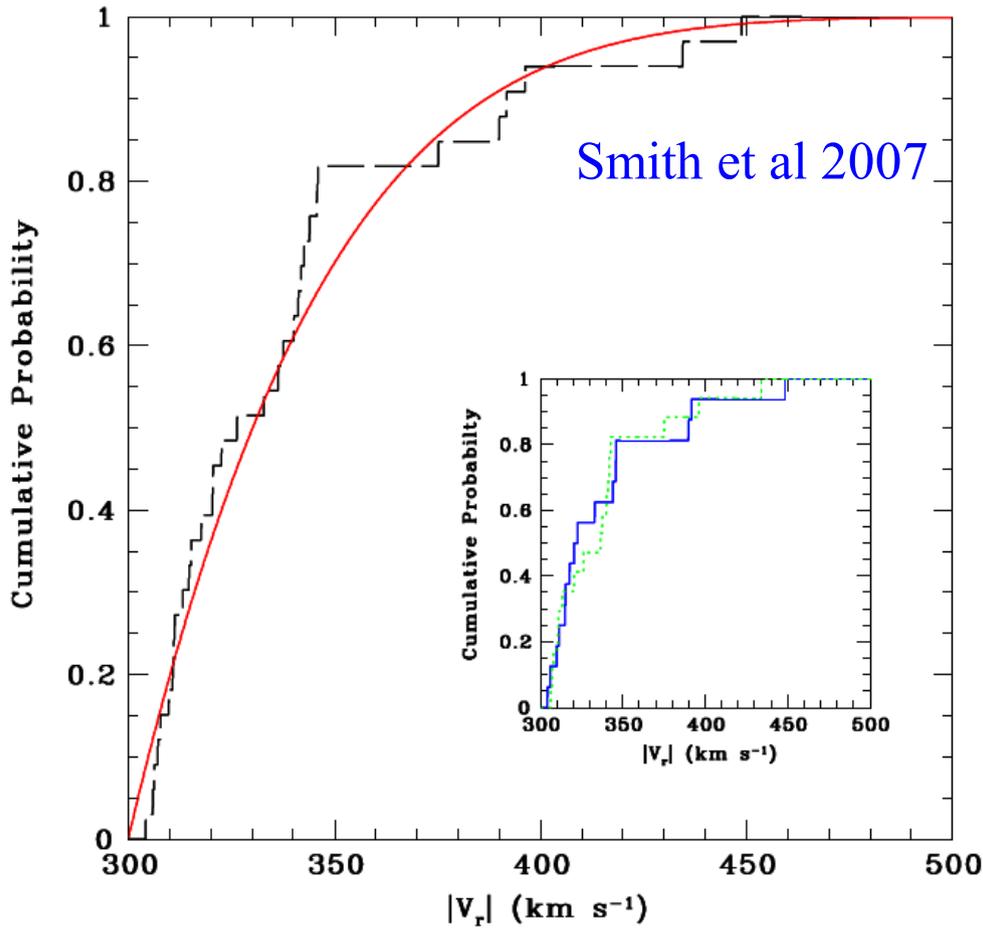


- The inferred relation between stellar mass and halo maximum circular velocity is consistent with the  $M_*$  “Tully-Fisher” relation



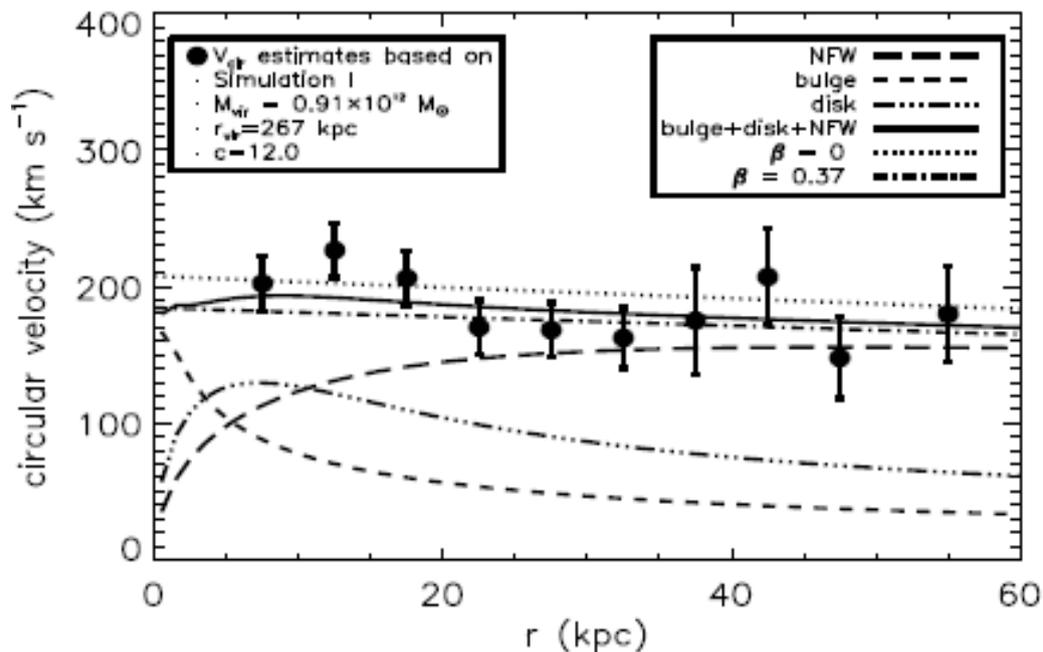
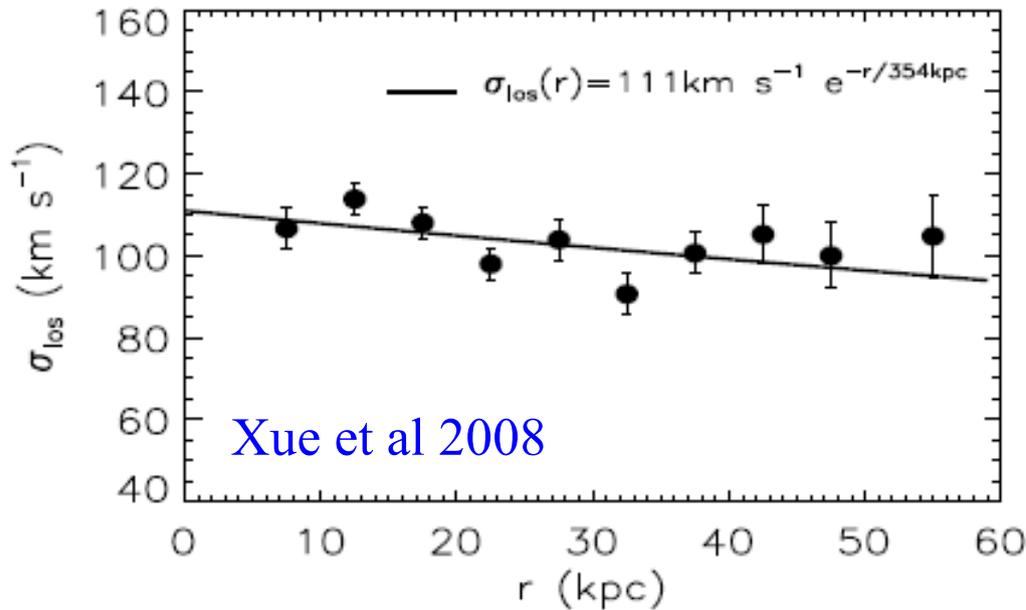
- Galaxy formation efficiency is:  $\epsilon = M_* / (\Omega_b M_{h,max} / \Omega_m)$
- This *maximises* at about 20%
- It is much lower than in all current galaxy formation simulations
- In the Milky Way about  $2 \times 10^{11} M_\odot$  of baryons are “missing”

# Milky Way mass from local escape velocity



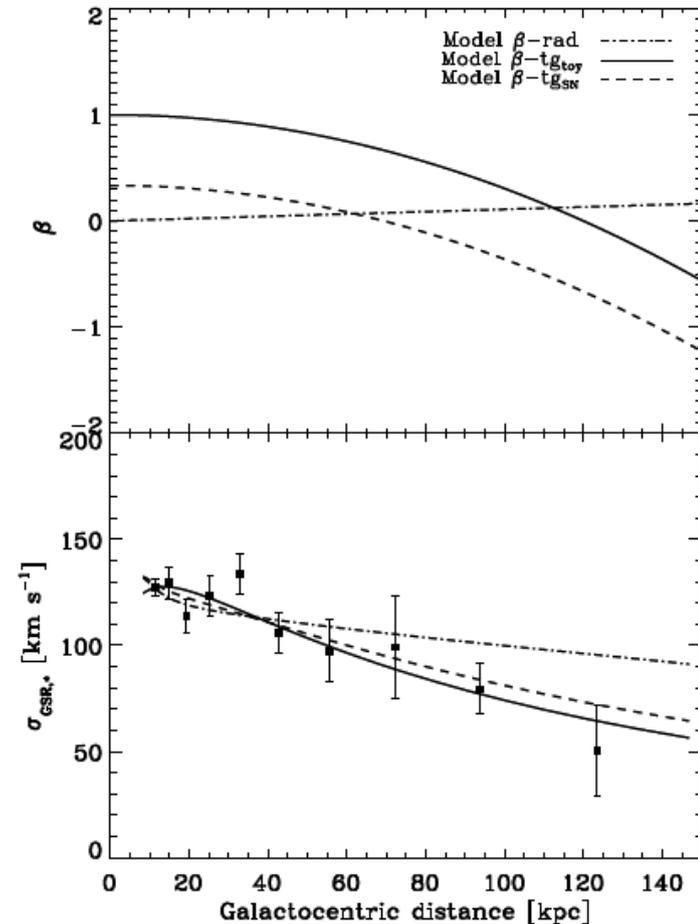
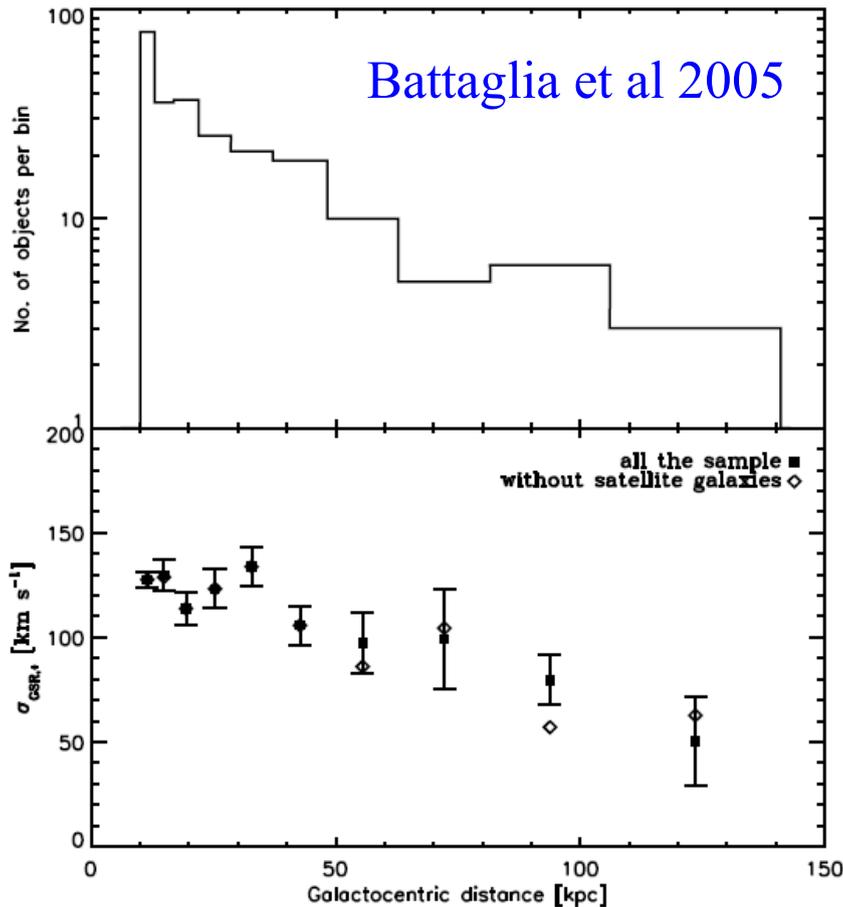
- Estimate based on 16 RAVE+ 17 archival stars with  $V > 300$  km/s  
 $498 \text{ km/s} < V_{\text{ESC}} < 608 \text{ km/s}$   
 $\rightarrow 9 \times 10^{11} M_{\odot} < M_{\text{NFW}} < 2.5 \times 10^{12} M_{\odot}$  (90% confidence)
- Sensitive to assumptions about shape and cut-off of high-velocity tail, as well as about shape (NFW, isothermal...) of the potential.

# Milky Way mass from distant tracer velocities



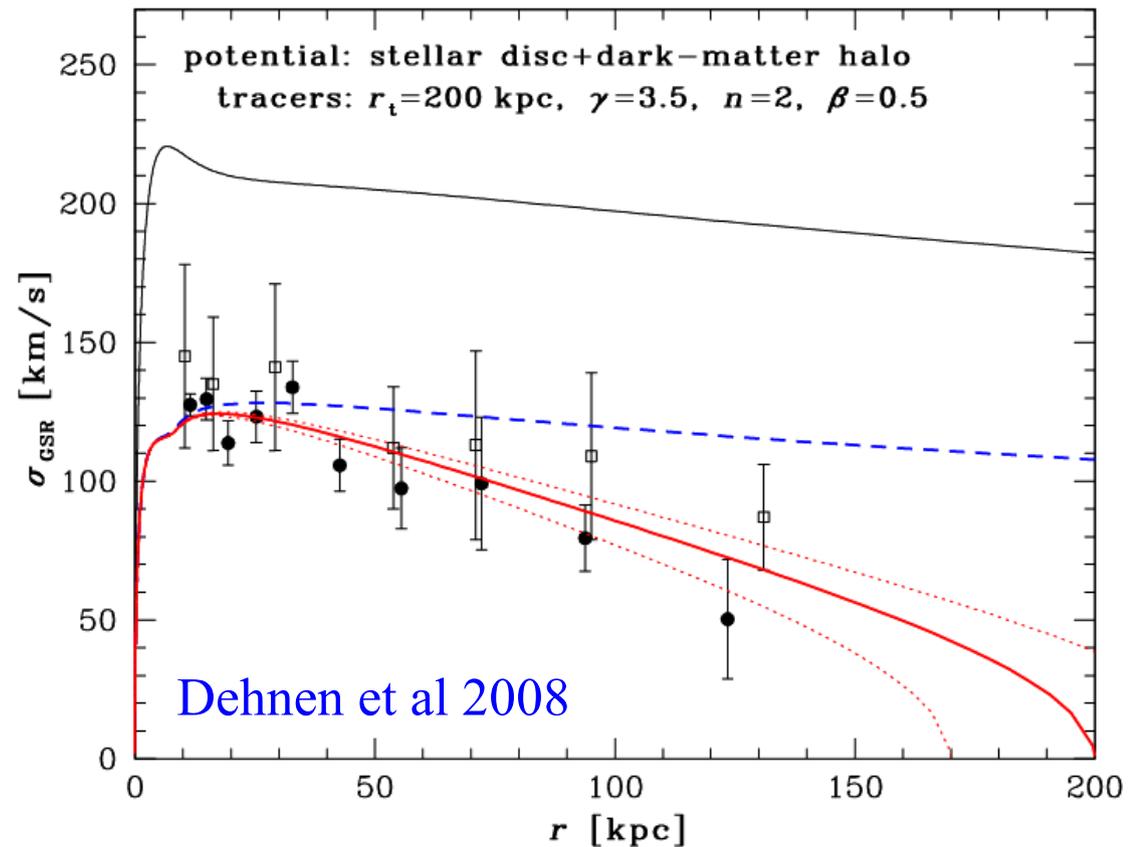
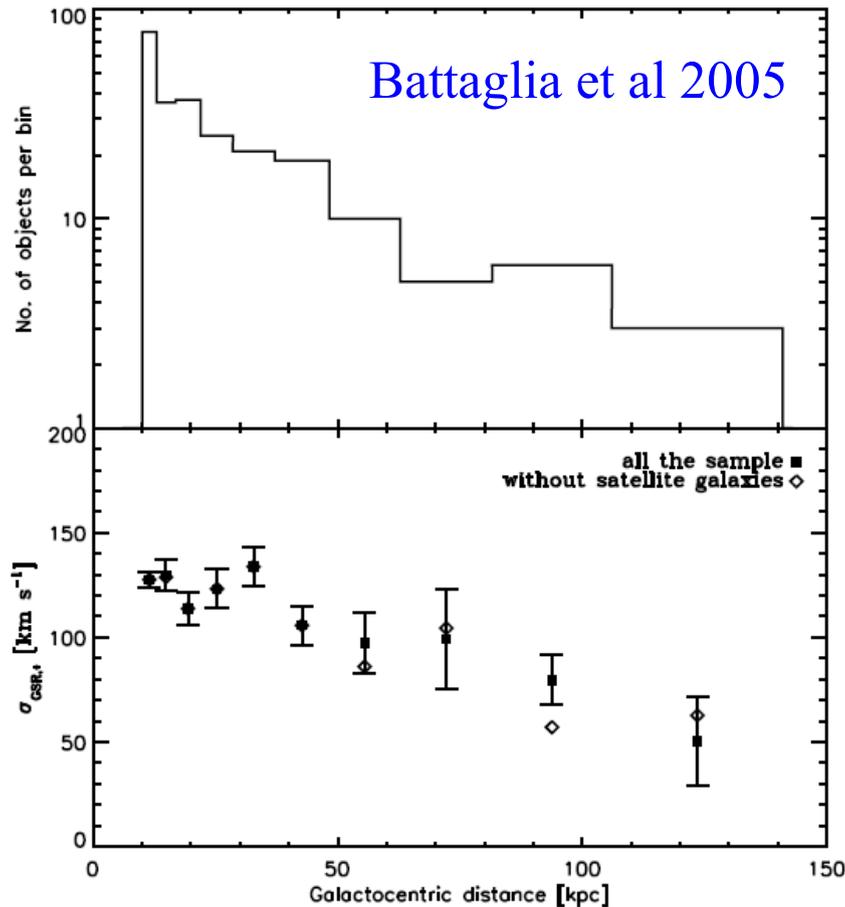
- Dispersions based on 2401 BHB stars from SDSS with  $|z| > 4 \text{ kpc}$
- Fit to CDM simulations of galaxy formation, adjusted using Jeans equations for differences in halo tracer profile and in  $V_{\text{circ}}$
- Good fits to NFW+disk for halo masses (at 68% confidence)  
 $8 \times 10^{11} M_{\odot} < M_{\text{NFW}} < 1.6 \times 10^{12} M_{\odot}$
- Sensitive to anisotropy assumption

# Milky Way mass from distant tracer velocities



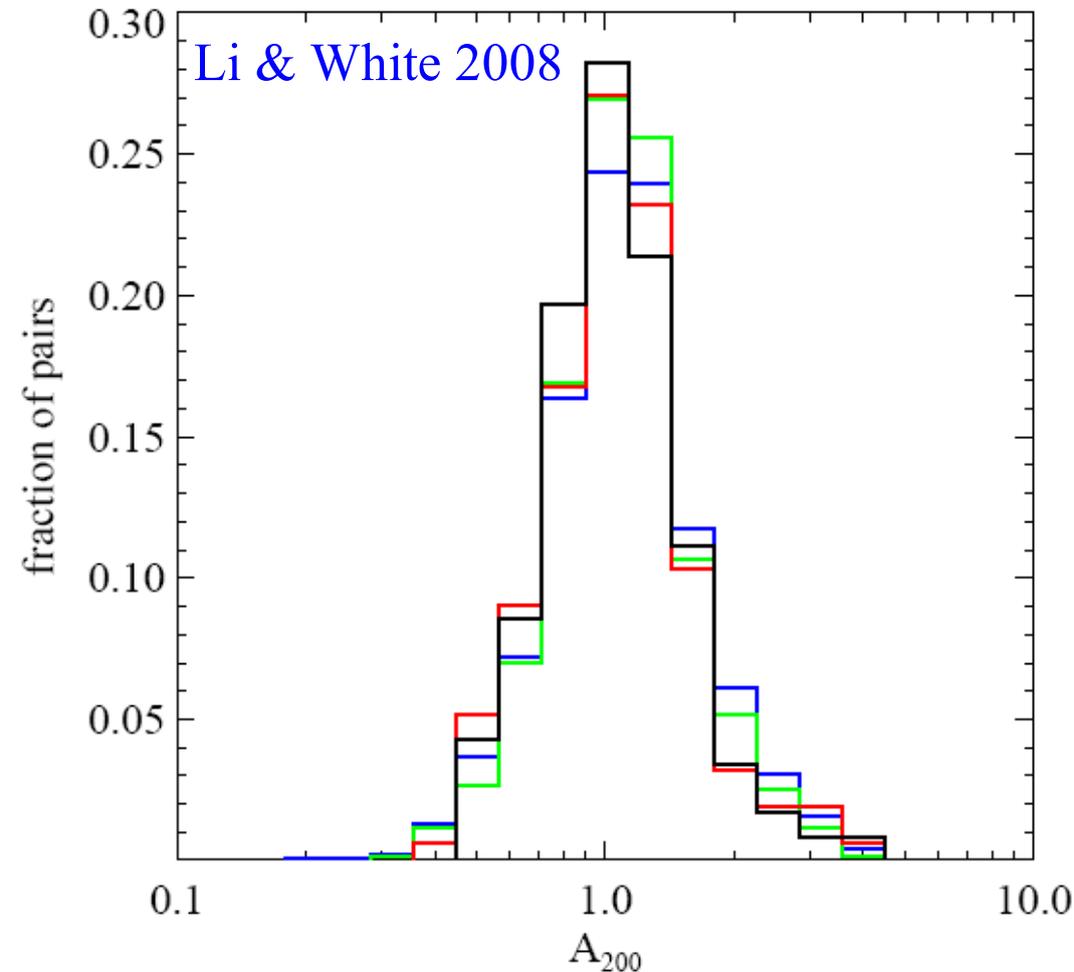
- Velocity dispersion from 240 halos stars + glob.clusters + satellites
- Jeans equations assuming  $\rho \propto r^{-3.5}$
- Tangentially biased velocities at large  $r$  needed to match fall in  $\sigma$   
 $6 \times 10^{11} M_{\odot} < M_{\text{NFW}} < 2.0 \times 10^{12} M_{\odot}$  (at 68% confidence)

# Milky Way mass from distant tracer velocities



- Velocity dispersion from 240 halo stars + glob.clusters + satellites
- Jeans equations assuming a cut-off in tracer density at  $r \sim 200$  kpc
- Radially anisotropic models now fit and there is *no* strong constraint on  $M_{\text{NFW}}$  from the data

# Timing Argument masses in the Local Group



The Kahn & Woltjer timing argument estimates the mass of the Local Group from the age of the Universe and the separation and relative radial velocity of the MW and M31

Calibrating using the Millennium Simulation gives (at 90% conf.)  
 $1.9 \times 10^{12} M_{\odot} < M_{LG} < 1.0 \times 10^{13} M_{\odot}$

A similar argument using Leo I gives  $M_{MW} \sim 2.4 \times 10^{12} M_{\odot}$  with  
 $M_{MW} > 8 \times 10^{11} M_{\odot}$  at 95% conf.

$M_{\text{Timing Argument}} / (M_{200}(\text{MW}) + M_{200}(\text{M31}))$   
for  $\sim 10^3$  Local Group analogues in the Millennium Simulation

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# Physics for Galaxy Formation Modelling

## Gas Cooling and Condensation

Sensitive to metal content, phase structure, UV background...

## Star Formation

No *a priori* understanding -- efficiency? IMF?

## Stellar Feedback

SF regulation, metal enrichment, galactic winds

## Stellar Aging

Population synthesis  luminosities, colours, spectra, (dust?)

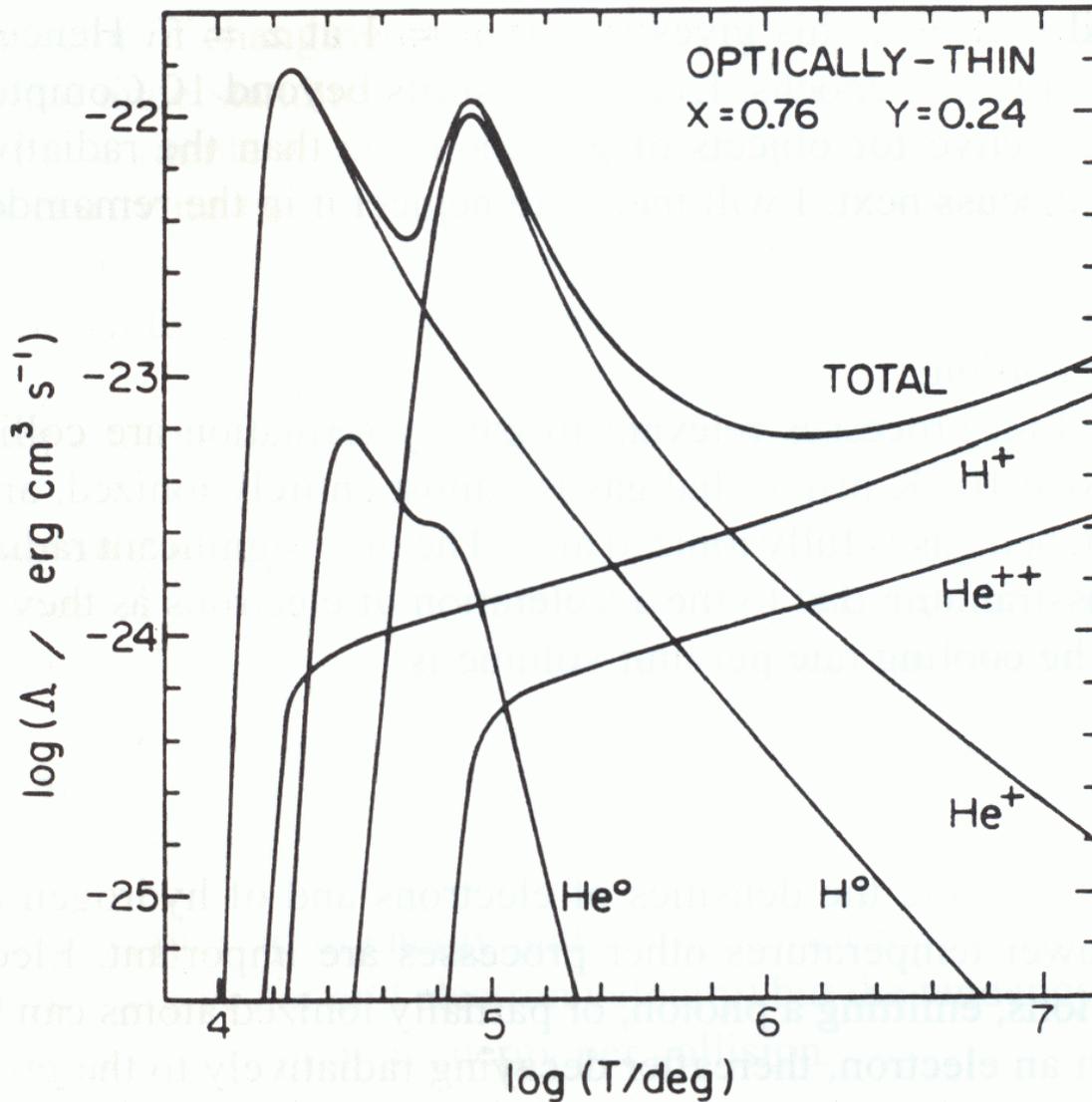
## AGN physics

Black hole formation, feeding, AGN phenomenology, feedback

## Environment interactions

Galaxy mergers, tidal effects, ram pressure effects

# Cooling curve for metal-free, optically thin gas in collisional ionisation equilibrium.



Luminosity/unit volume is

$$\mathcal{L} = n_e^2 \Lambda(T)$$

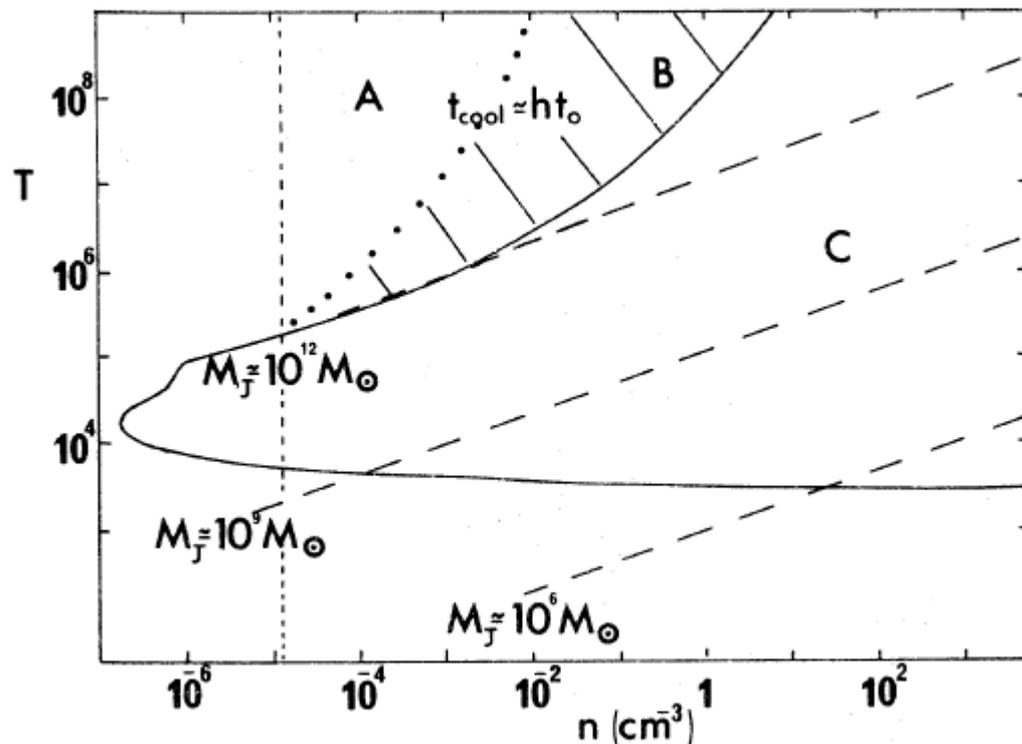
No cooling occurs below  $10^4\text{K}$  unless  $\text{H}_2$  can form

Addition of heavy elements increases cooling in the range  $10^5\text{K}$  to  $10^7\text{K}$

→ Optically thin cooling time  $t_{\text{cool}} \propto n_e T / \mathcal{L} \propto T / n_e \Lambda(T)$

c.f. gravitational collapse time  $t_{\text{dyn}} \propto (G \rho)^{-1/2} \propto n_e^{-1/2}$

# Radiative processes in galaxy formation



Rees & Ostriker 1977  
Silk 1977  
Binney 1977

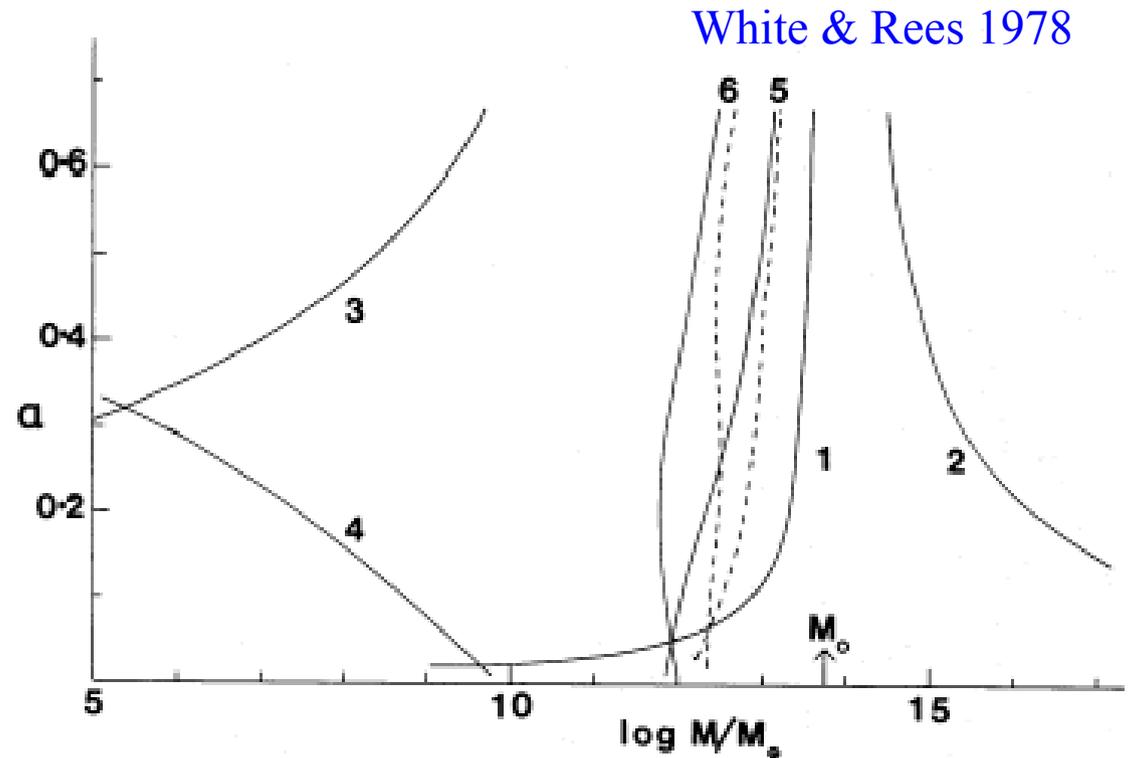
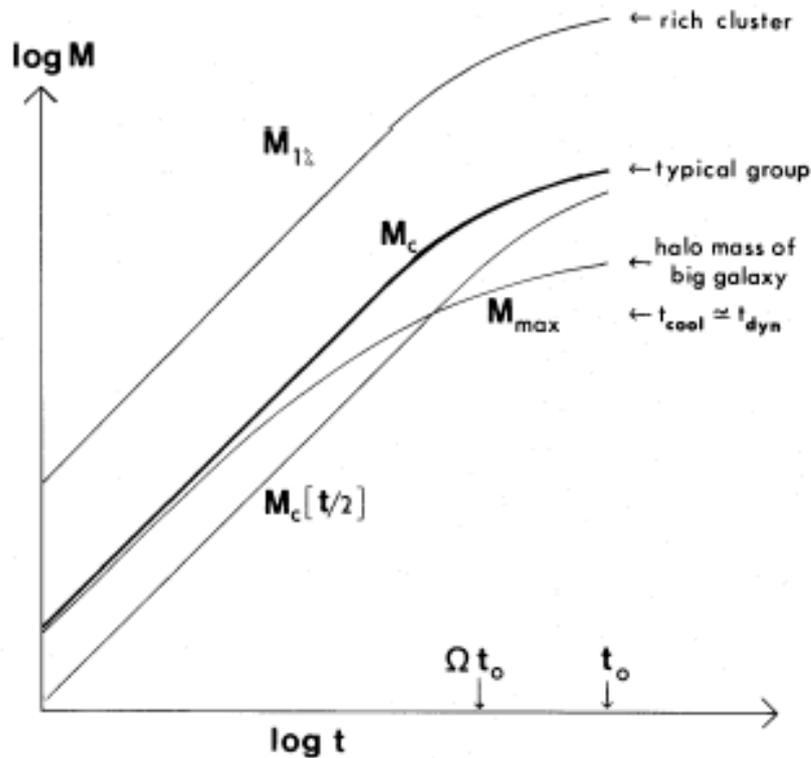
- When gas clouds of galactic mass collapse:
  - (i) shocks are radiative and collapse unimpeded, when  $t_{\text{cool}} < t_{\text{dyn}}$
  - (ii) shocks are non-radiative and collapse arrested, when  $t_{\text{cool}} > t_{\text{dyn}}$

where quantities are estimated at virial equilibrium

- Galaxies form in case (i) since fragmentation is possible

- Primordial cooling curve  $\longrightarrow$  characteristic mass  $10^{12} M_{\odot}$

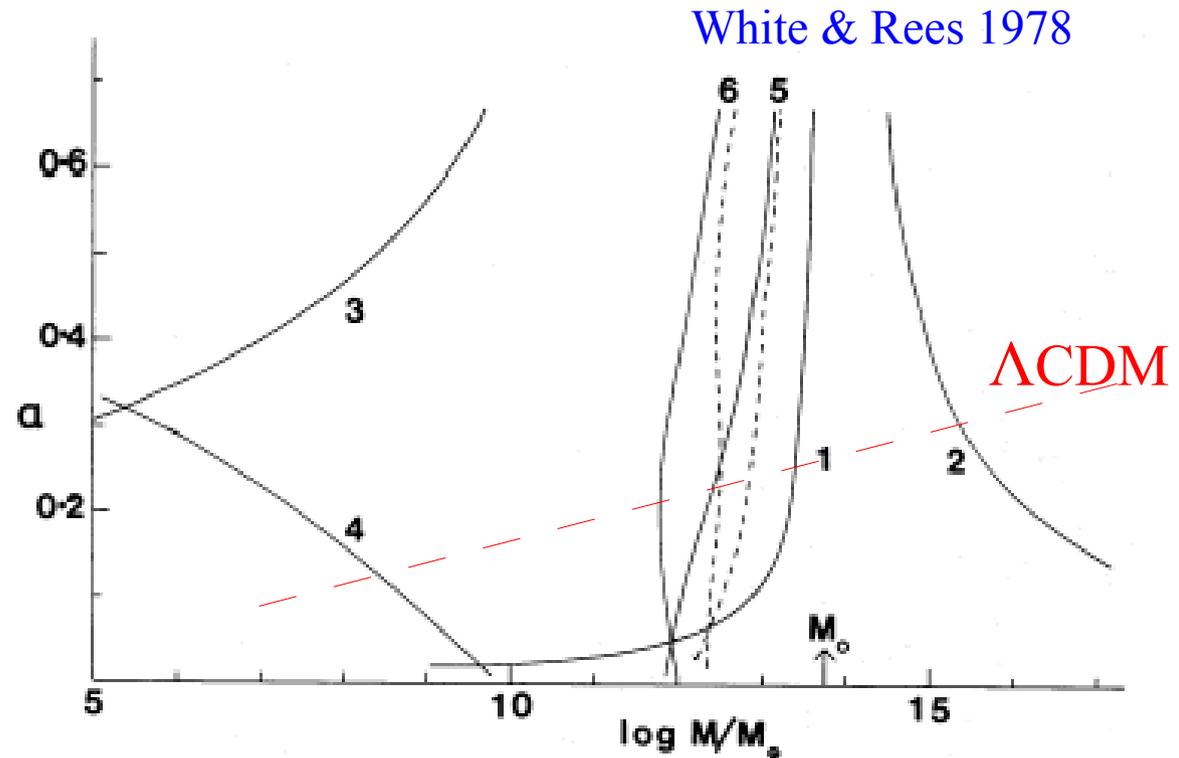
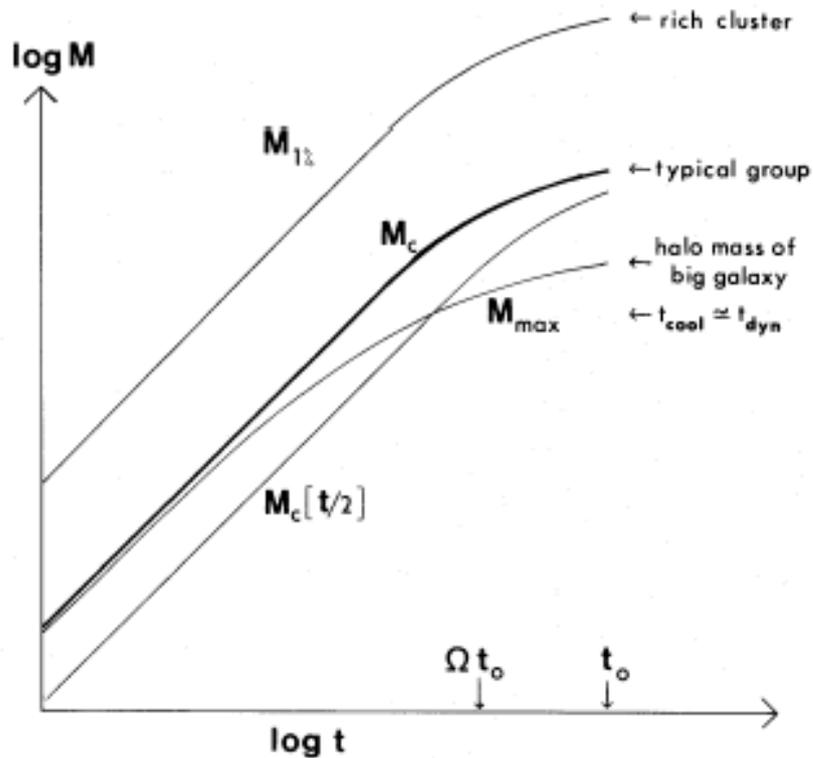
# Towards a “modern” theory



- Adding : (i) dark matter, (ii) hierarchical clustering, (iii) feedback
  - cooling always rapid for small masses and early times
  - only biggest galaxies sit in cooling flows
  - feedback *à la* Larson (1974) needed to suppress small galaxies

- A good model:  $\Omega_m = 0.20$ ,  $\Omega_{gas} / \Omega_{DM} = 0.20$ ,  $\alpha = 1/3$  ( $n = -1$ )

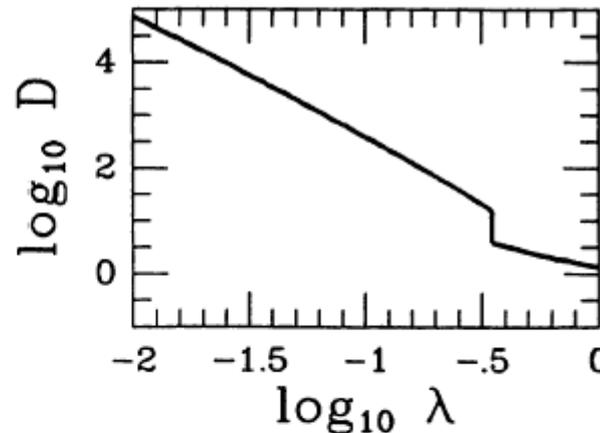
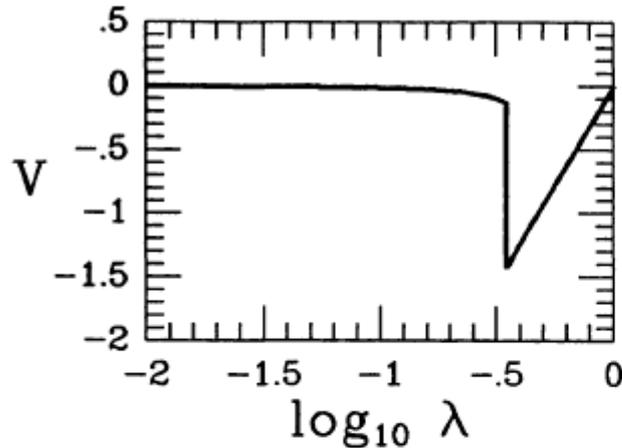
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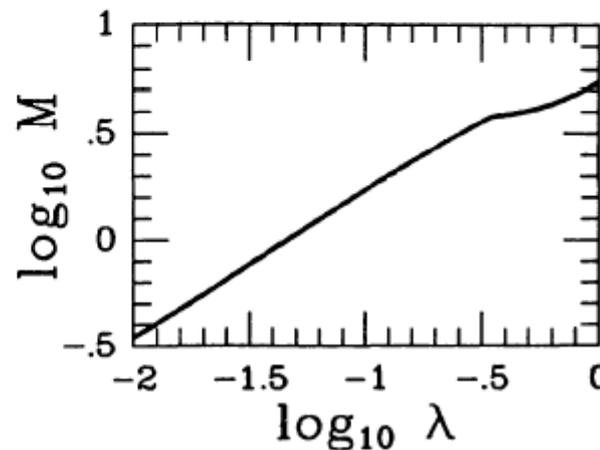
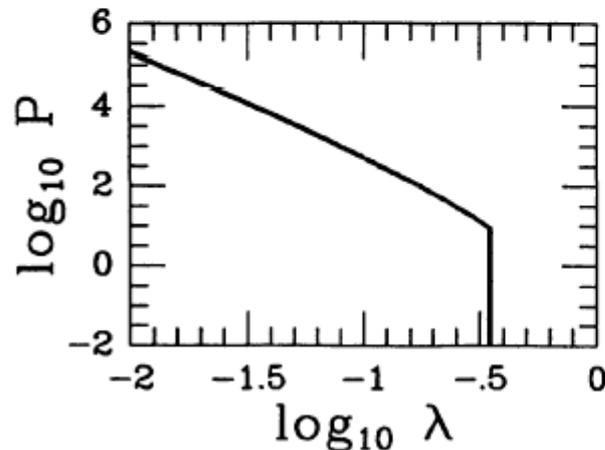
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# Spherical similarity solutions for infall



Bertschinger 1985

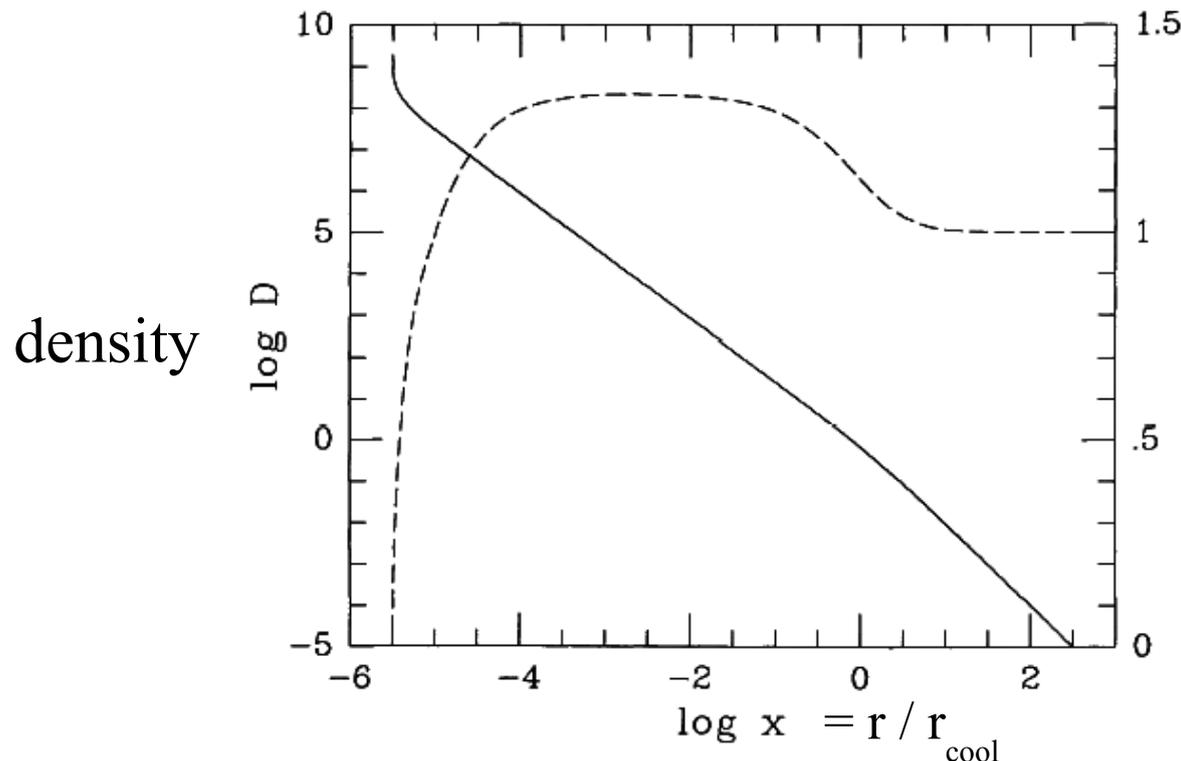
No cooling



- Infall of DM +  $\gamma = 5/3$  gas onto a point mass in an EdS universe
  - accretion shock at  $\sim 1/3$  of turn-round radius
  - gas almost static inside shock
  - pre-shock gas has density about 4 times the cosmic mean
  - $kT(r) / \mu \sim GM(r) / r = V_c^2$ ;  $R \sim V_c t$ ,  $M \sim V_c^3 t / G$

# Spherical similarity solutions for cooling

Bertschinger 1989



$\theta$  temperature

$$V_c^2 \propto T \propto \text{const.}$$

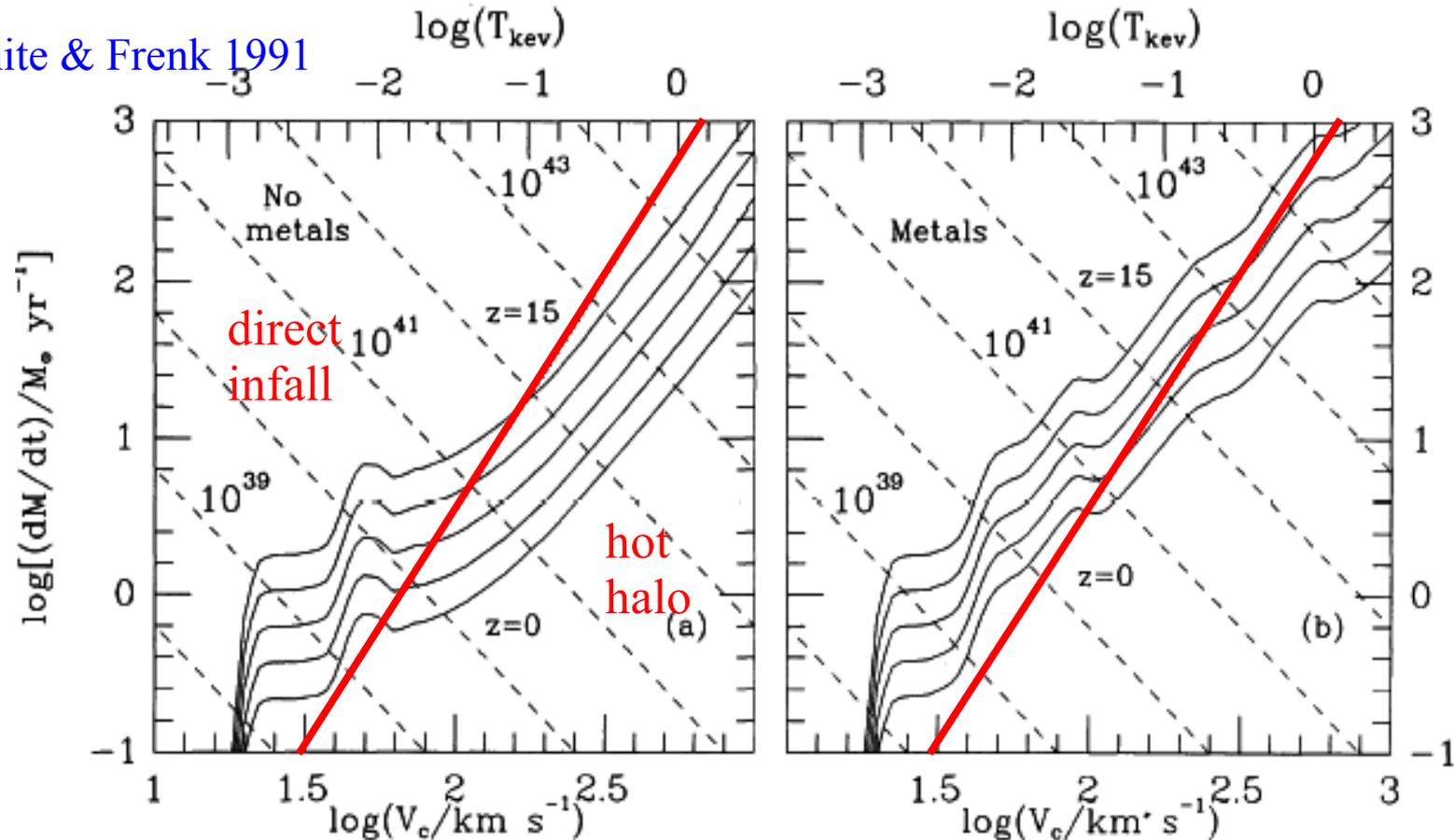
$$M = r V_c^2 / G \propto r$$

$$\rho \propto r^{-2}$$

- Cooling wave in equilibrium gas in an isothermal DM potential
  - $\rho \propto r^{-2}$  at large radius  $r > r_{\text{cool}}$  where  $t_{\text{cool}}(r_{\text{cool}}) = t$
  - $\rho \propto r^{-1.5}$  and  $T = 1.33 T_\infty$  at  $r_{\text{sonic}} < r < r_{\text{cool}}$
  - $\rho \propto r^{-1.5}$ , flow is supersonic free-fall, and  $T \rightarrow 0$  at  $r < r_{\text{sonic}}$
- Inflow rate  $\propto t^{-1/2}$ , cooling radius and cold mass  $\propto t^{+1/2}$
- $r_{\text{sonic}} \sim r_{\text{cool}} \sim r_{\text{shock}}$  in protogalaxies  $\longrightarrow$  no static atmosphere?

# Putting it together in a $\Lambda$ CDM universe

White & Frenk 1991



- Assuming  $r_{\text{cool}} < r_{\text{shock}}$  for a hot atmosphere and taking  $f_{\text{baryon}} = 0.1$ 
  - direct infall (i.e. no hot atmosphere – a “cold flow”) for  $V_{\text{circ}} < 80 \text{ km/s}$  at  $z=3$  when there is no chemical mixing, and for  $V_{\text{circ}} < 250 \text{ km/s}$  at  $z=3$  when efficient mixing is assumed

# Adding the baryons: hydrodynamics

- After recombination the baryons are in the form of a diffuse, near-uniform mixture of neutral H and He – no stars, no heavier elements, no magnetic fields (?)
- Need to solve hydrodynamics equations for the gas in addition to N-body equations for the DM
- $\partial\rho/\partial t + \nabla\cdot(\rho\mathbf{u}) = 0$  Mass conservation  
 $\partial(\rho\mathbf{u})/\partial t + \mathbf{u}\cdot\nabla(\rho\mathbf{u}) + \nabla p + \rho\nabla\Phi = 0$  Momentum cons.  
+ Energy conservation
- Main solution techniques
  - discretise on a regular fixed mesh (Eulerian hydro)
  - discretise on a variable, adaptive mesh (AMR)
  - discretise using a finite set of fluid elements  
(Smoothed Particle Hydrodynamics: SPH)

# Hydrodynamics and cluster formation

- Structure formation produces shocks which transform the K.E. of fluid motions into heat
- Bremsstrahlung and line emission from cluster gas is observable in X-rays

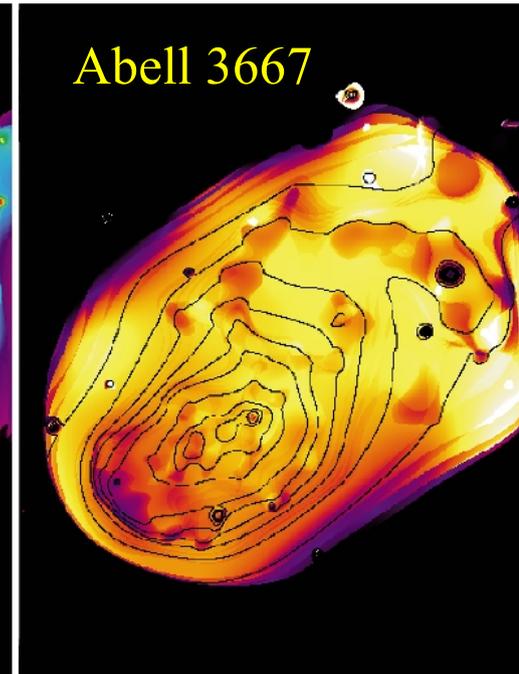
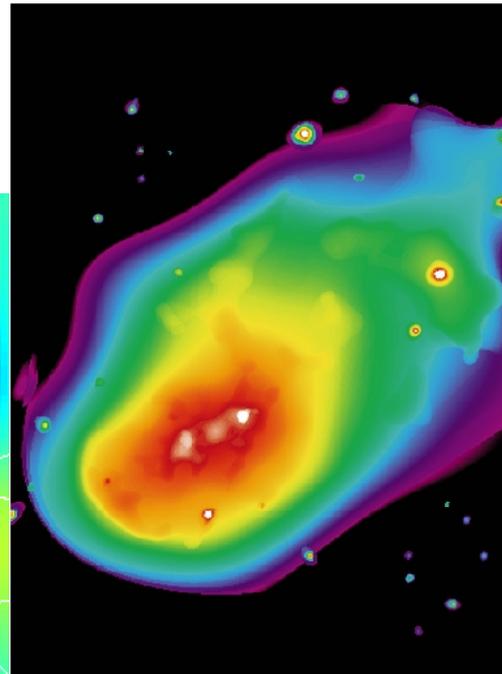
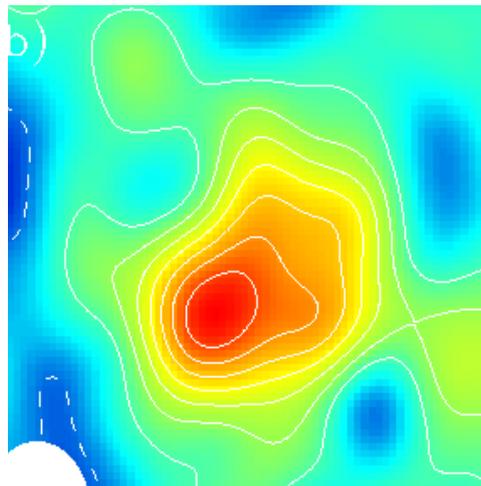
$$S = \int dl \rho^2 \Lambda(T)$$

- Cluster “shadows” are observable against the CMB through the Sunyaev-Zeldovich effect

$$\Delta T \propto \int dl p$$

- Hierarchical growth of structure produces the phenomenology observed in images of clusters

- asymmetries and sublumps
- shocks
- cold fronts
- cold cores



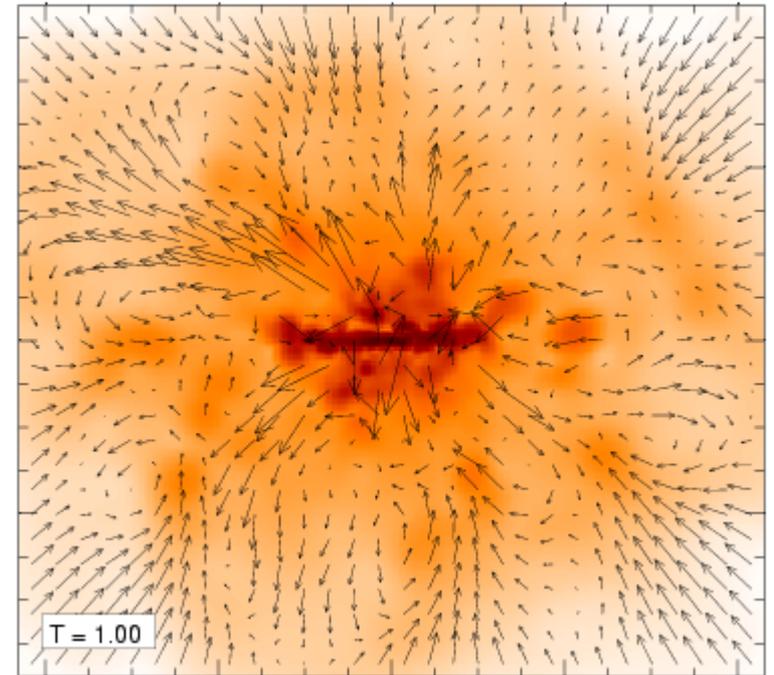
# Gas cooling and galaxy formation

- Bremsstrahlung and line emission cause shocked gas to radiate away its internal energy
- Gas settles into the centre of DM potential wells and starts to form the stellar populations of galaxies
- Gas cooling times are  $t_{cool} \propto T / \rho \Lambda(T)$  while objects at time  $t$  have typical density  $\rho \propto t^{-2}$  so  $t_{cool} / t \propto t T / \Lambda(T)$ 
  - rapid cooling in lower mass objects and at early times
  - efficient galaxy formation with no hot gas.  
i.e., as in Rees & Ostriker (1977) and White & Rees (1978)  
and subsequently included in all semianalytic models

This is now often referred to as the “new paradigm” of cold flows!

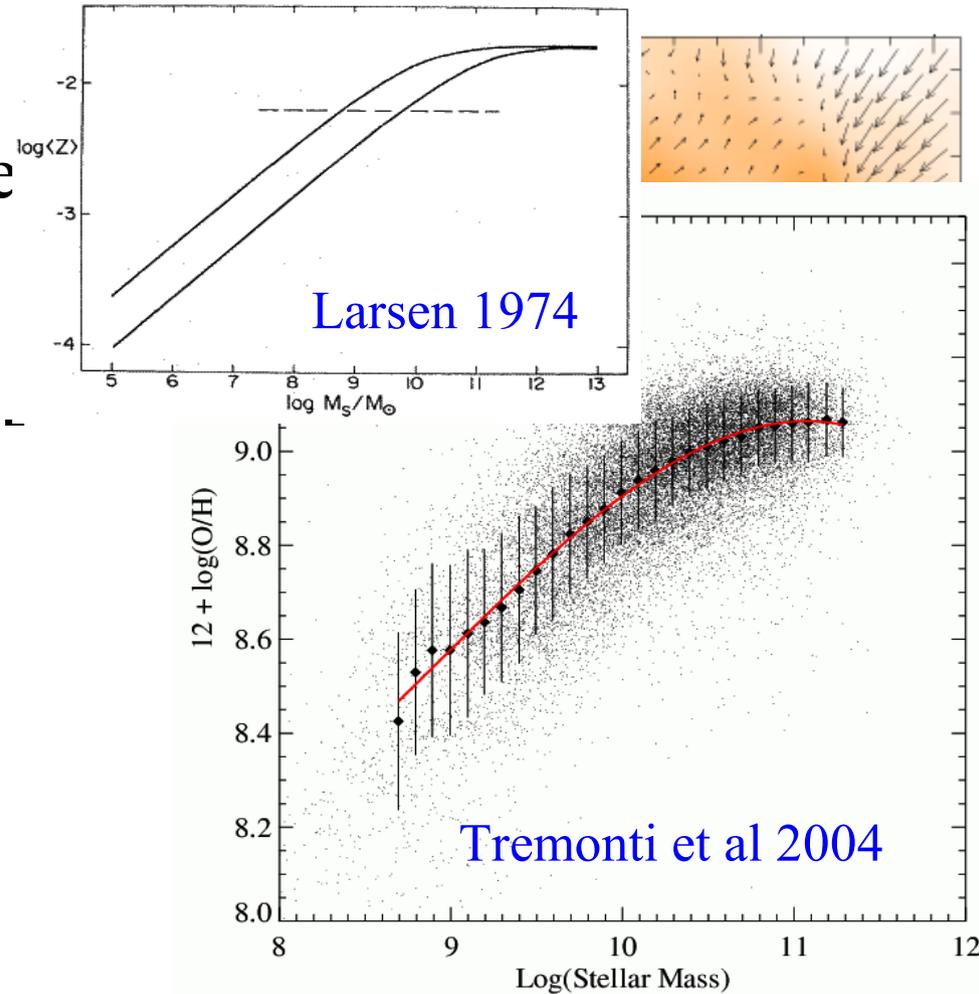
# Feedback/galactic wind issues

- Can supernova feedback drive galactic winds?
- Can these reproduce the mass-element abundance relation?
- Can they enrich intergalactic gas with heavy elements?
- Can these enhance formation of disks over bulges?
- What about feedback from Active Galactic Nuclei?



# Feedback/galactic wind issues

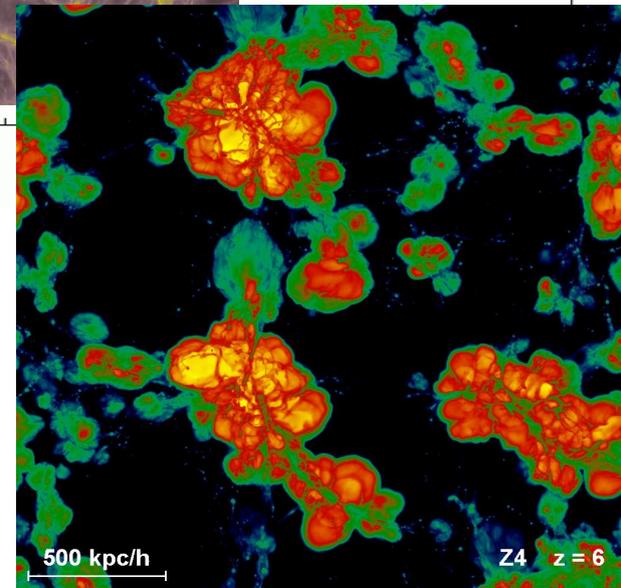
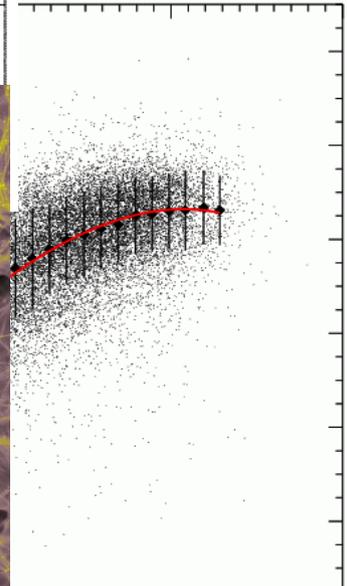
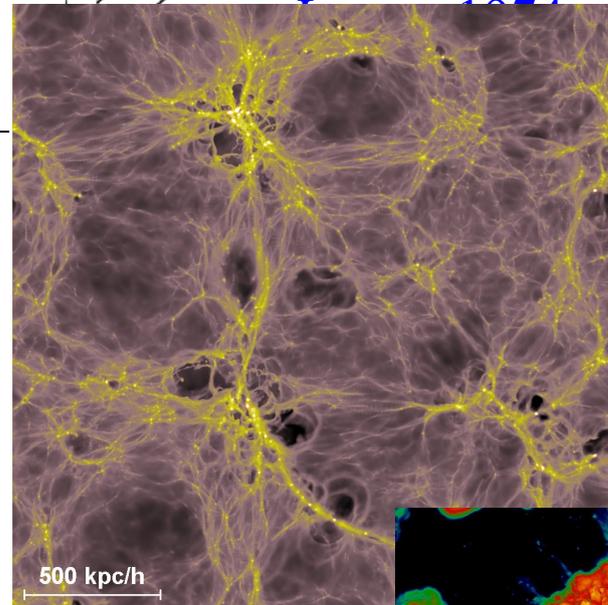
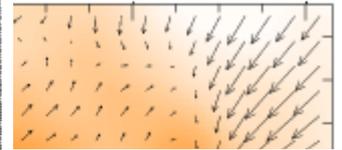
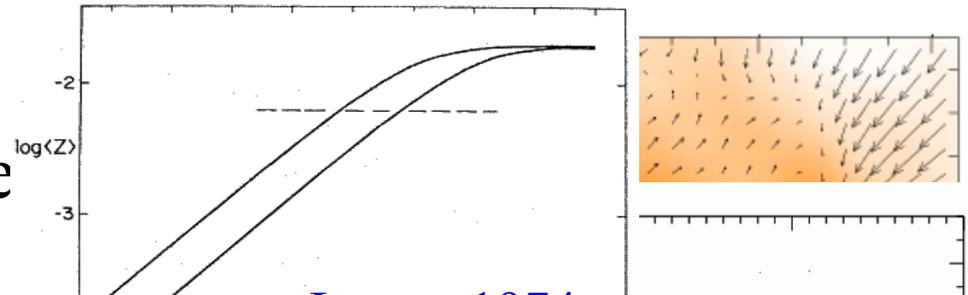
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$$\begin{aligned} \dot{M}_{\text{wind}} &\propto \dot{E}_{\text{SN}} / V_{\text{esc}}^2 \\ &\propto \dot{M}_* / V_c^2 \end{aligned}$$

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# Including the formation and evolution of stars

- Stars form where gas is dense, cold and self-gravitating
- Star formation is *not* resolved in galaxy formation simulations  
→ a simple *sub-grid* prescription is needed, for example
$$\dot{\rho}_* = \alpha \rho_{\text{gas}} t_{\text{dyn}}$$
- Aging of stars affects their brightness and colour
- Supernovae, explosions of massive stars put energy and heavy elements into surrounding gas – *feedback*, *chemical enrichment*
- Feedback processes are also not resolved and must also be implemented with phenomenological recipes
- Implementation details *strongly* affect galaxy formation models

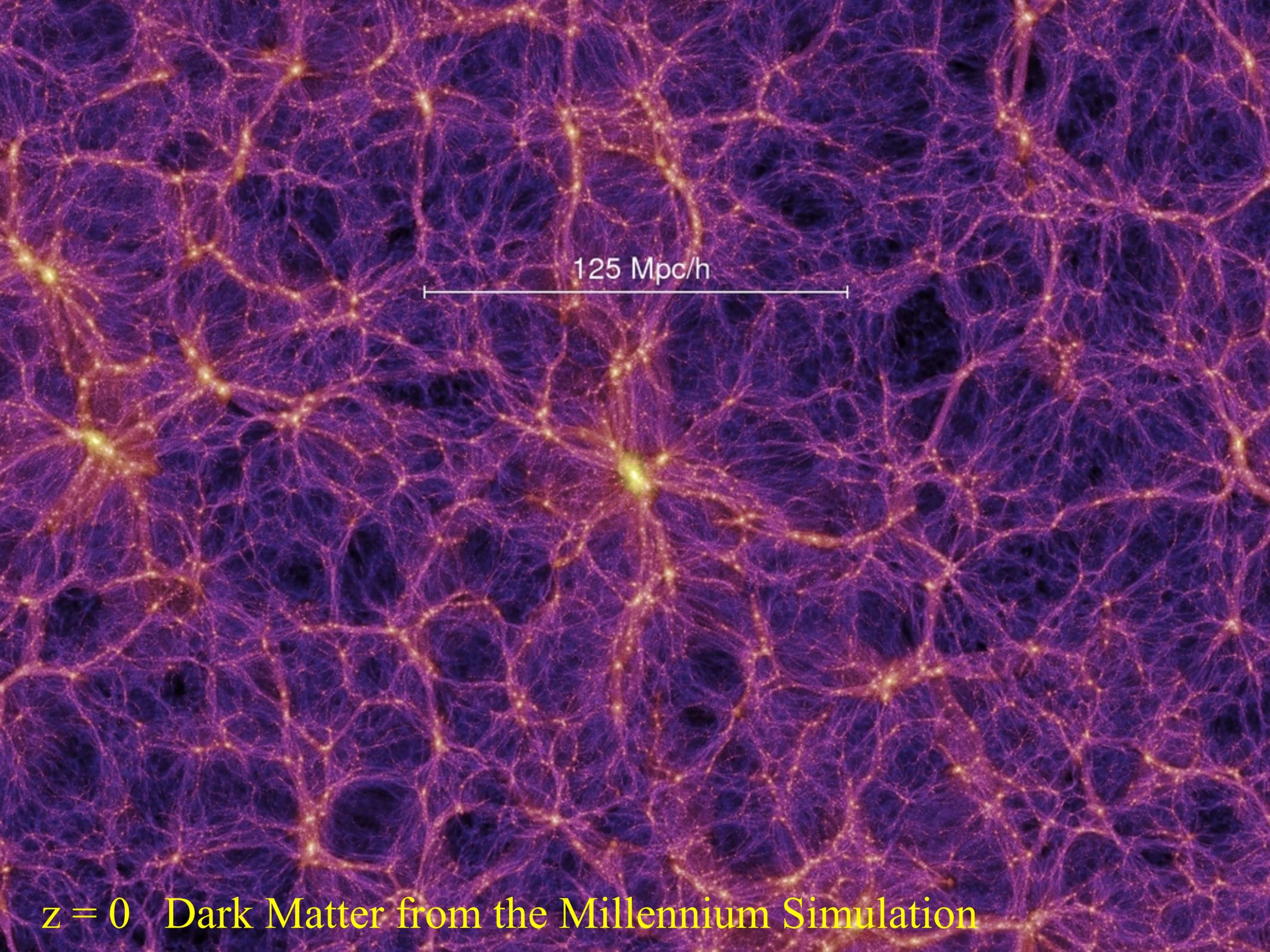
# Simulating the formation of individual galaxies

- Systems which have a *major merger* after most of their stars have formed end up looking like **Elliptical galaxies**
- Systems which have no major merger at late times end up having a substantial disk and looking like **Spiral galaxies**  
Disks grow inside-out and late,
- It seems very hard to make spiral galaxies with small bulges



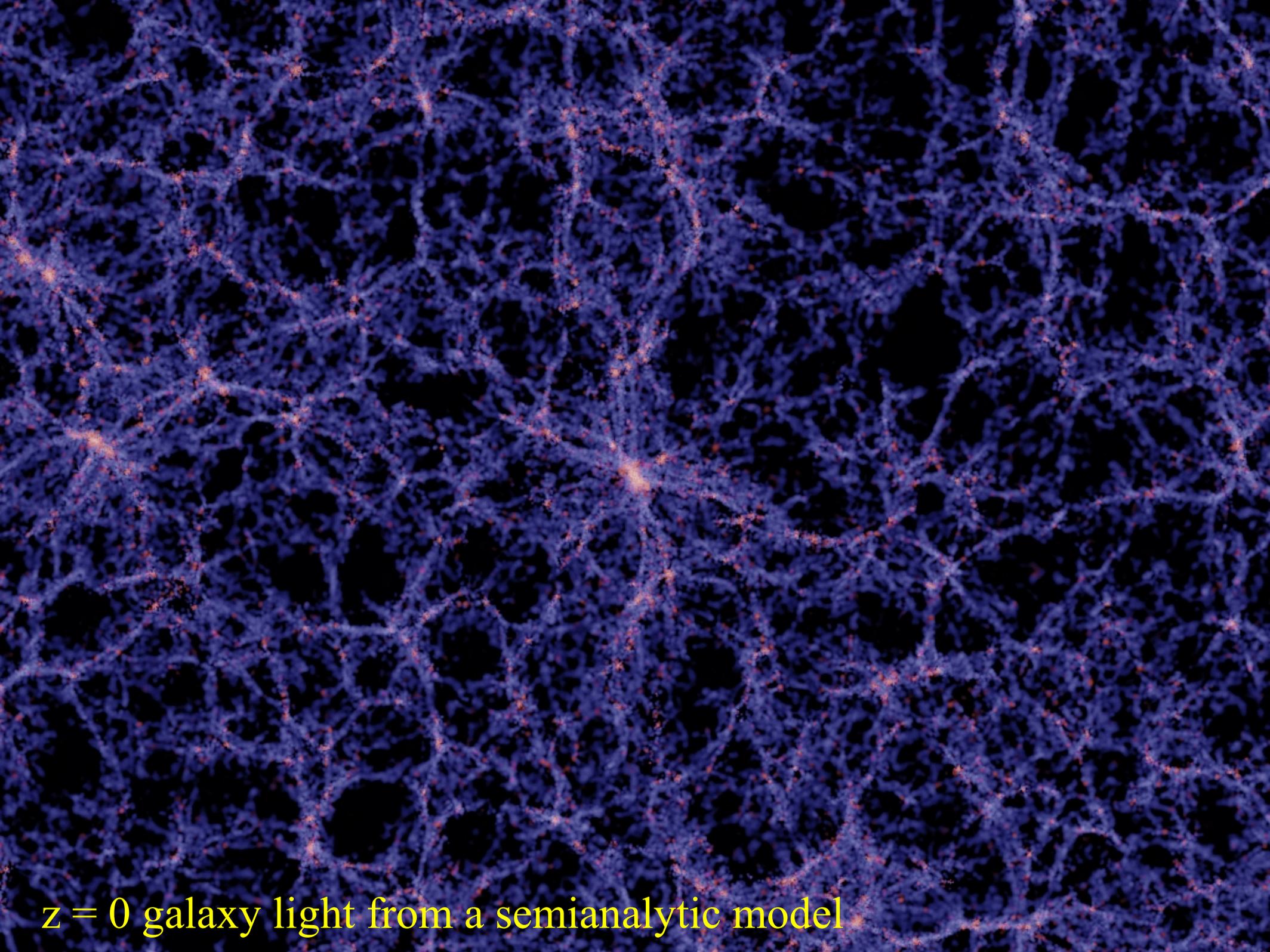
- It's also hard to match the low formation efficiencies required by abundance matching....
- ..or to get get enough heavy elements out of galaxies

**Problems with feedback recipes?**



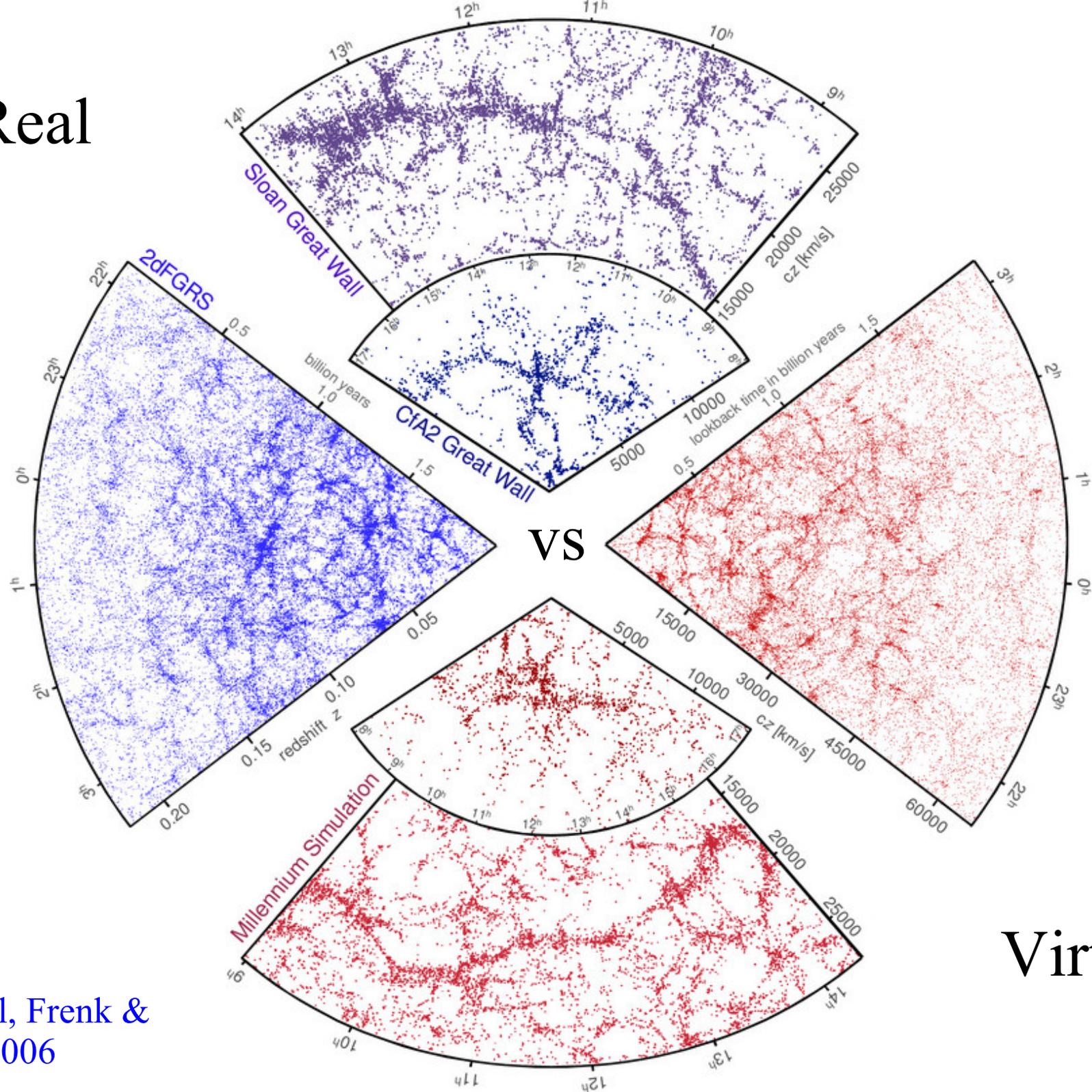
125 Mpc/h

$z = 0$  Dark Matter from the Millennium Simulation



$z = 0$  galaxy light from a semianalytic model

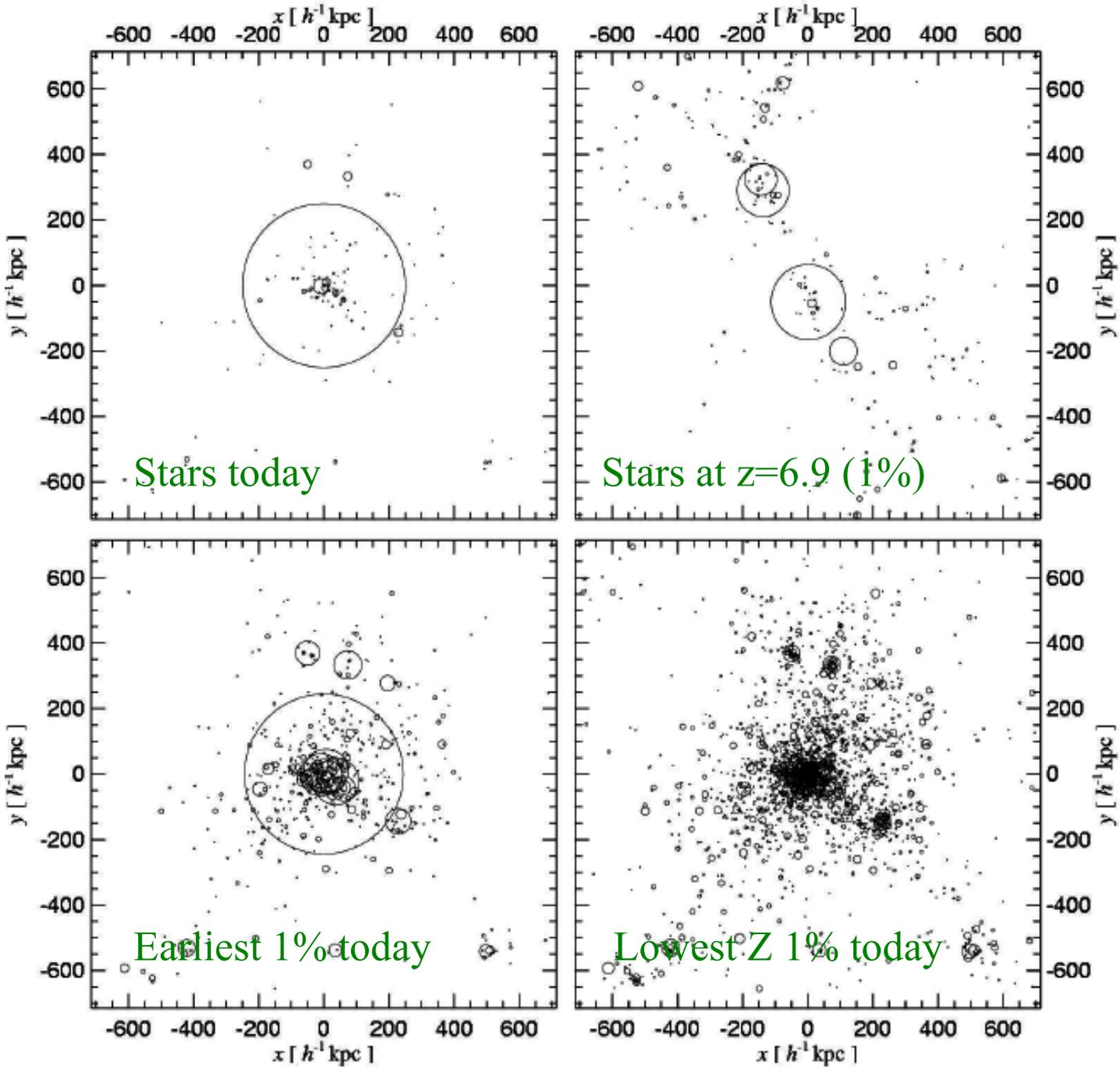
Real



Virtual

Springel, Frenk &  
White 2006

# Where are the first (lowest Z) stars now?



Hi-res simulation of the formation of a “Milky Way” and its satellites

“First stars” have little correlation with “lowest Z” stars.

Most “old stars are in the bulge

Most lo-Z stars are in satellites (60%) or their debris (30%)

White & Springel 1999

The first stars were metal-poor

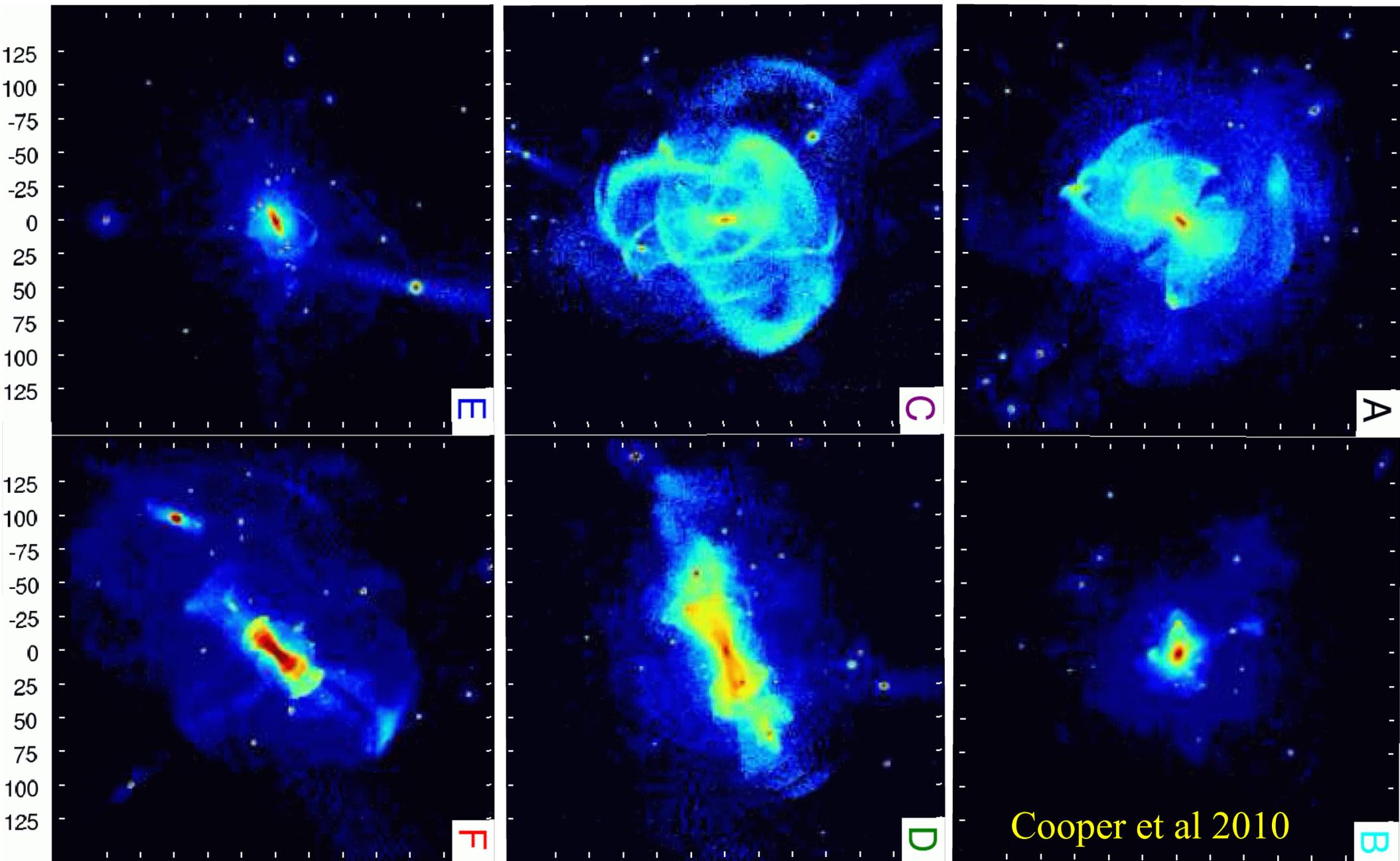
but

Today's most metal-poor stars are not the oldest stars  
-- rather they formed in the smallest systems

While metal-poor stars are in all objects, metal-rich stars  
are only in massive objects

→ It is the mean metallicity of the MW halo which  
betrays its progenitors, not the low-Z tail

# Aquarius stellar halos



# Structure formation simulations with gas

- Formation of a cluster, DM+ nonradiative gas (Springel)
- Formation of a spiral, “all” physics (Steinmetz)
- Formation of a spiral, “all” physics (Governato)
- Spiral merger with black holes (Springel/Di Matteo)