Non-Gaussianity
as a Probe of the Physics of the Primordial Universe

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How Do We Test Inflation?

- How can we answer a simple question like this:
  - "*How were primordial fluctuations generated?*"

Power Spectrum

- A very successful explanation (Mukhanov & Chibisov; Guth & Pi; Hawking; Starobinsky; Bardeen, Steinhardt & Turner) is:
  - *Primordial fluctuations were generated by quantum fluctuations of the scalar field that drove inflation.*
  - The prediction: a nearly scale-invariant power spectrum in the curvature perturbation, ζ:
    - **$P_\zeta(k) = A/k^{4-n_s} \sim A/k^3$**
    - where $n_s \sim 1$ and A is a normalization.

Komatsu et al. (2009)

$n_s<1$ Observed

- The latest results from the WMAP 5-year data:
  - **$n_s=0.960 \pm 0.013$** (68%CL; for tensor modes = zero)
  - **$n_s=0.970 \pm 0.015$** (68%CL; for tensor modes $\neq$ zero)
    - tensor-to-scalar ratio **< 0.22** (95%CL)
- $n_s \neq 1$: another line of evidence for inflation
- Detection of non-zero tensor modes is a next important step

Anything Else?

- One can also look for other signatures of inflation. For example:
  - **Isocurvature perturbations**
    - Proof of the existence of multiple fields
  - **Non-zero spatial curvature**
    - Evidence for "Landscape," if curvature is negative. Rules out Landscape ideas if positive.
  - **Scale-dependent n_s (running index)**
    - Complex dynamics of inflation

Komatsu et al. (2009)

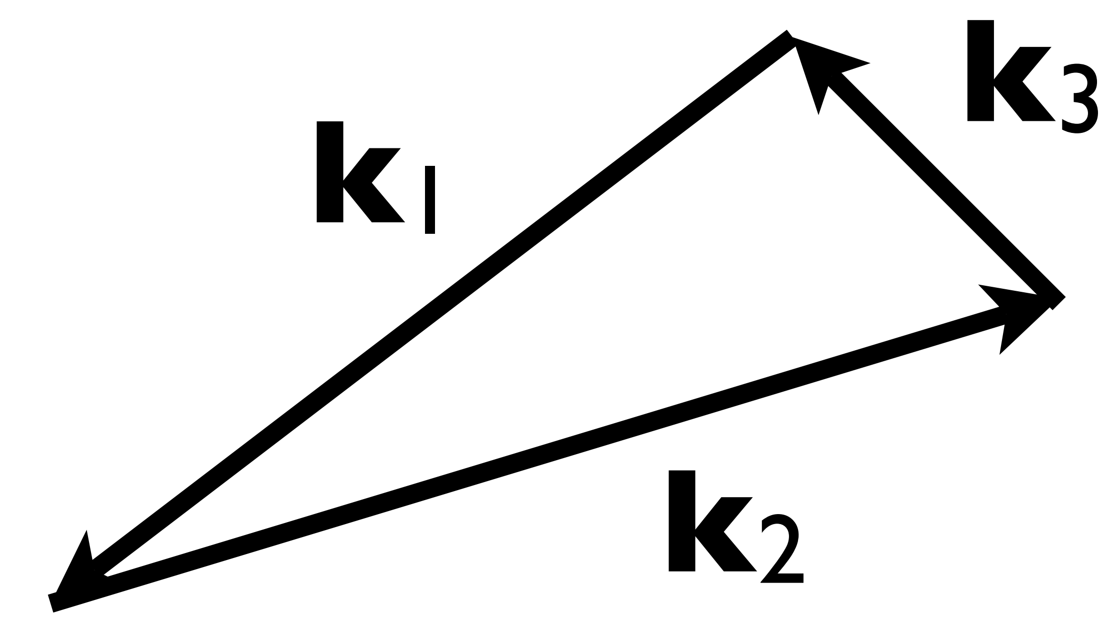
Anything Else?

- One can also look for other signatures of inflation. For example:
  - 95%CL limits on **Isocurvature perturbations**
    - **$S/(3\zeta)$ <0.089** (axion CDM); **<0.021** (curvaton CDM)
  - 95%CL limits on **Non-zero spatial curvature**
    - **$\Omega-1<0.018$** (for $\Omega>1$); **$1-\Omega<0.008$** (for $\Omega<1$)
      positive curvature — negative curvature
  - 95%CL limits on **Scale-dependent n_s**
    - **$-0.068 < dn_s/d\ln k < 0.012$**

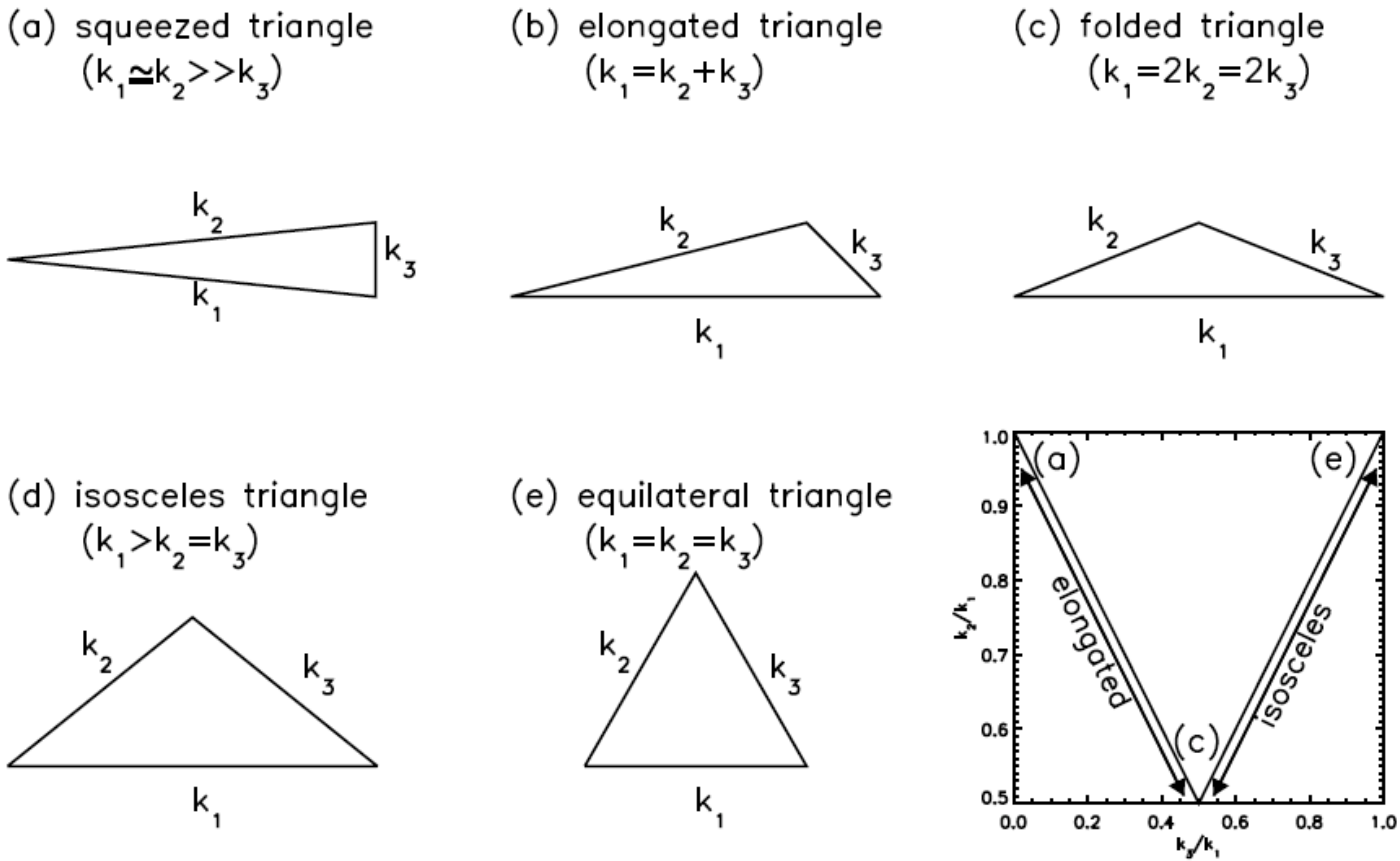
Beyond Power Spectrum

- All of these are based upon fitting the observed power spectrum.
- Is there any information one can obtain, beyond the power spectrum?

Bispectrum



- Three-point function!
- $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$
  $= \langle \zeta_{\mathbf{k}1}\zeta_{\mathbf{k}2}\zeta_{\mathbf{k}3} \rangle =$ (amplitude) x $(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)b(k_1,k_2,k_3)$
  model-dependent function



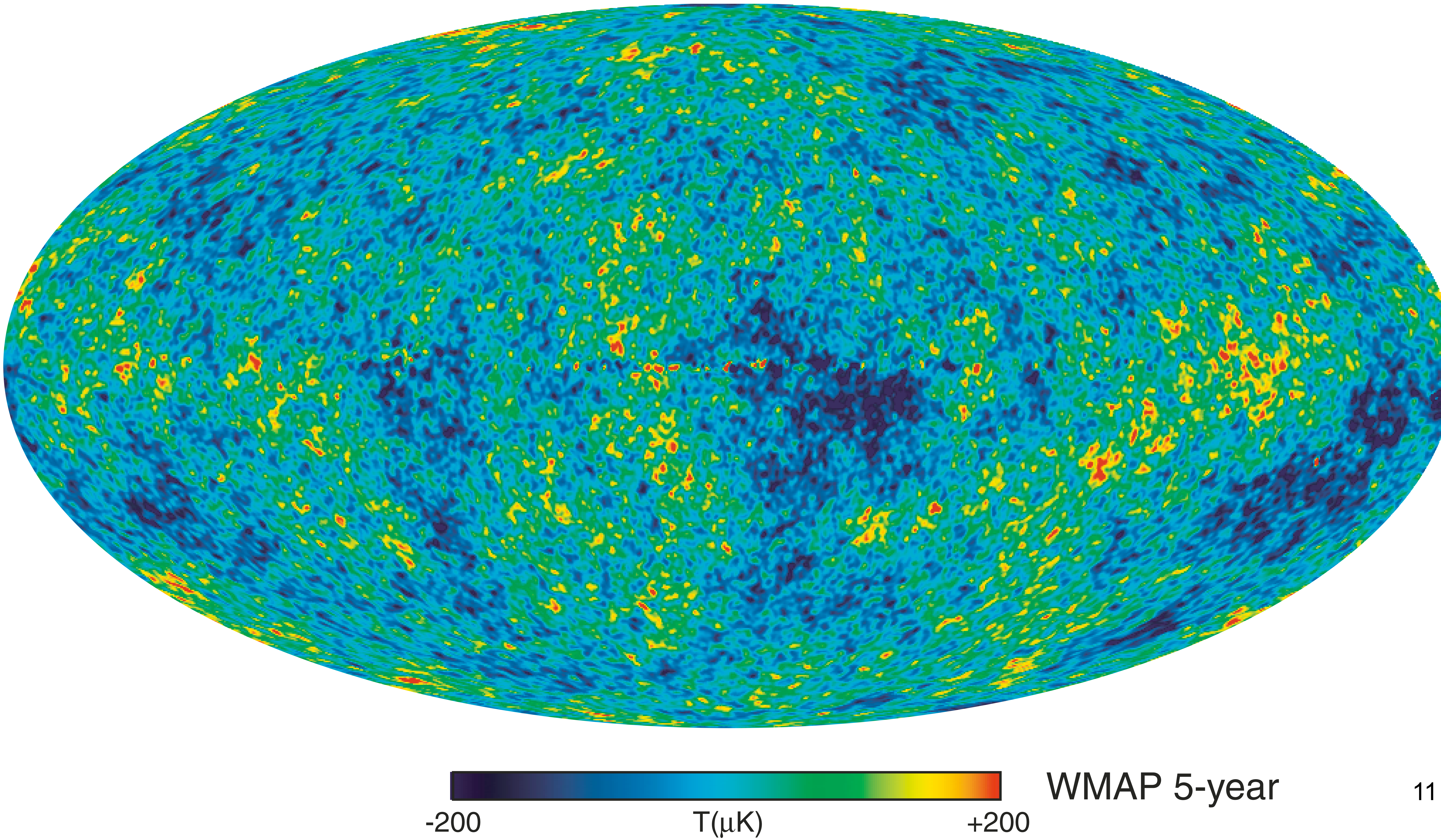
(a) squeezed triangle
($k_1 \simeq k_2 >> k_3$)
k_1
k_2
k_3
(b) elongated triangle
($k_1 = k_2 + k_3$)
k_1
k_2
k_3
(c) folded triangle
($k_1 = 2k_2 = 2k_3$)
k_1
k_2
k_3
(d) isosceles triangle
($k_1 > k_2 = k_3$)
k_1
k_2
k_3
(e) equilateral triangle
($k_1 = k_2 = k_3$)
k_1
k_2
k_3
k_2/k_1
k_3/k_1
1.0
0.9
0.8
0.7
0.6
0.5
0.0
0.2
0.4
0.6
0.8
1.0
(a)
(e)
(c)
elongated
isosceles

Why Study Bispectrum?

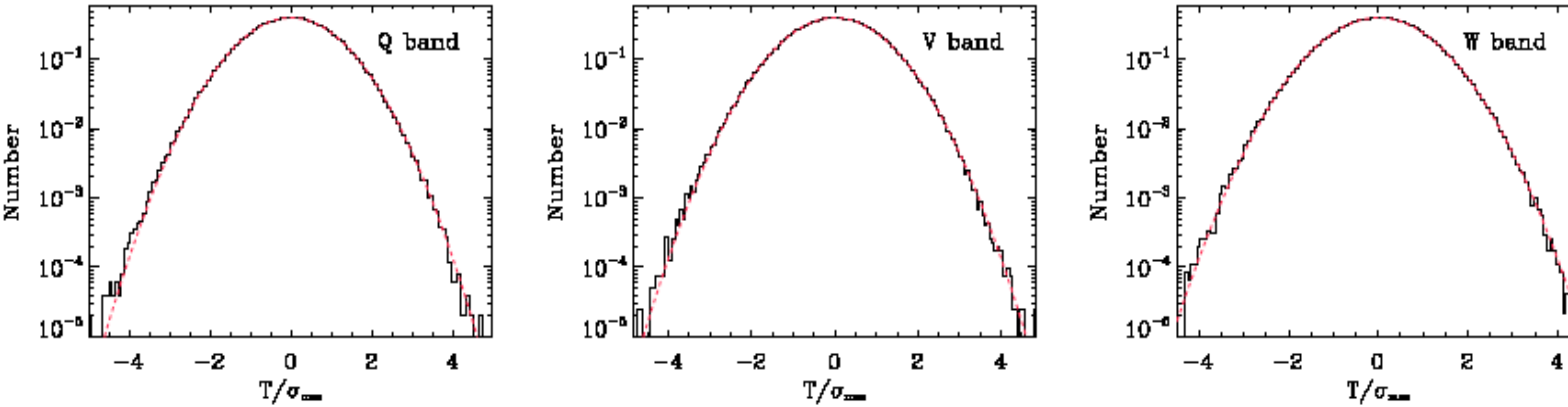
- It probes the interactions of fields - new piece of information that cannot be probed by the power spectrum
- But, above all, it provides us with a **critical test** of the simplest models of inflation: "***are primordial fluctuations Gaussian, or non-Gaussian?***"
- Bispectrum vanishes for Gaussian fluctuations.
- Detection of the bispectrum = detection of non-Gaussian fluctuations

Gaussian?

WMAP5



Take One-point Distribution Function



- The one-point distribution of WMAP map looks pretty Gaussian.
  - –Left to right: Q (41GHz), V (61GHz), W (94GHz).
- Deviation from Gaussianity is small, if any.

Inflation Likes This Result

- According to inflation (Mukhanov & Chibisov; Guth & Yi; Hawking; Starobinsky; Bardeen, Steinhardt & Turner), CMB anisotropy was created from **quantum fluctuations of a scalar field in Bunch-Davies vacuum** during inflation
- Successful inflation (with the expansion factor more than e^{60}) *demands* the scalar field be almost interaction-free
- The wave function of free fields in the ground state is a Gaussian!

But, Not Exactly Gaussian

- Of course, there are always corrections to the simplest statement like this.
- For one, inflaton field **does** have interactions. They are simply weak – they are suppressed by the so-called slow-roll parameter, $\varepsilon \sim O(0.01)$, relative to the free-field action.

A Non-linear Correction to Temperature Anisotropy

- The CMB temperature anisotropy, ΔT/T, is given by the curvature perturbation in the matter-dominated era, Φ.
  - One large scales (the Sachs-Wolfe limit), ΔT/T=–Φ/3.

For the Schwarzschild metric, Φ=+GM/R.

- Add a non-linear correction to Φ:
  - $\Phi(\mathbf{x}) = \Phi_g(\mathbf{x}) + f_{NL}[\Phi_g(\mathbf{x})]^2$ (Komatsu & Spergel 2001)
  - f_{NL} was predicted to be small (~0.01) for slow-roll models (Salopek & Bond 1990; Gangui et al. 1994)

f_{NL}: Form of B_ζ

- Φ is related to the primordial curvature perturbation, ζ, as $\Phi=(3/5)\zeta$.

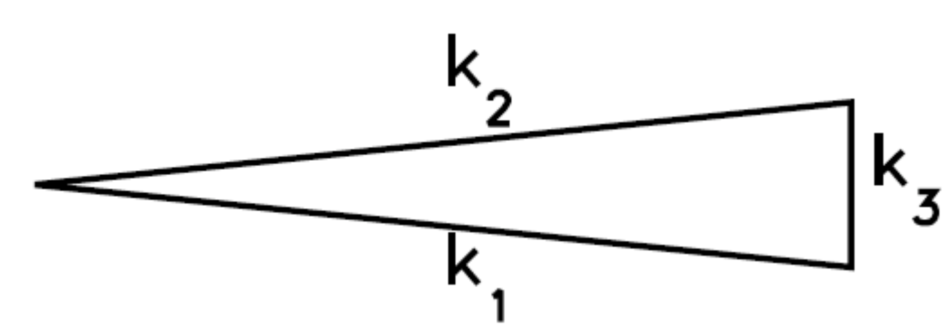
- $\zeta(\mathbf{x}) = \zeta_g(\mathbf{x}) + (3/5)f_{NL}[\zeta_g(\mathbf{x})]^2$

- $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)=(6/5)f_{NL} \times (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \times [P_\zeta(k_1)P_\zeta(k_2) + P_\zeta(k_2)P_\zeta(k_3) + P_\zeta(k_3)P_\zeta(k_1)]$

f_{NL}: Shape of Triangle

- For a scale-invariant spectrum, $P_\zeta(k)=A/k^3$,
  - $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)=(6A^2/5)f_{NL} \times (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \times [1/(k_1k_2)^3 + 1/(k_2k_3)^3 + 1/(k_3k_1)^3]$
- Let's order k_i such that $k_3 \le k_2 \le k_1$. For a given k_1, one finds the largest bispectrum when the smallest k, i.e., k_3, is very small.
  - $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$ peaks when $k_3 << k_2 \sim k_1$
  - Therefore, the shape of f_{NL} bispectrum is the squeezed triangle!

(Babich et al. 2004)



B_ζ in the Squeezed Limit

- In the squeezed limit, the f_{NL} bispectrum becomes:
  $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \approx (12/5)f_{NL} \times (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \times P_\zeta(k_1)P_\zeta(k_3)$

Maldacena (2003); Seery & Lidsey (2005); Creminelli & Zaldarriaga (2004)

Single-field Theorem (Consistency Relation)

- For **ANY** single-field models*, the bispectrum in the squeezed limit is given by
  - $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \approx (1-n_s) \times (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \times P_\zeta(k_1)P_\zeta(k_3)$
  - Therefore, all single-field models predict $f_{NL} \approx (5/12)(1-n_s)$.
  - With the current limit $n_s=0.96$, f_{NL} is predicted to be 0.017.

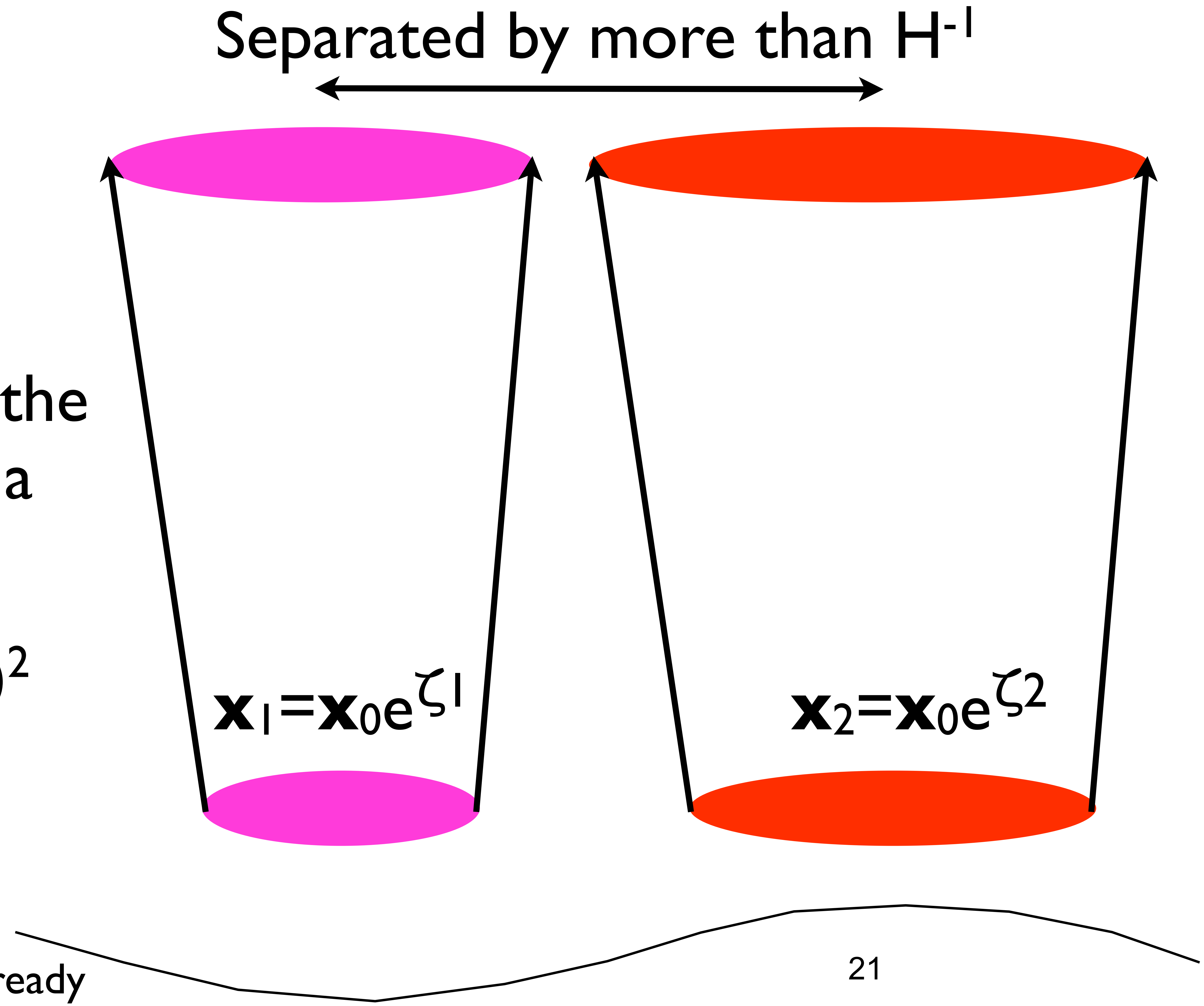
* for which the single field is solely responsible for driving inflation and generating observed fluctuations.

Understanding the Theorem

- First, the squeezed triangle correlates one very long-wavelength mode, k_L (=k_3), to two shorter wavelength modes, k_S (=$k_1 \approx k_2$):
  - $\langle \zeta_{\mathbf{k}1} \zeta_{\mathbf{k}2} \zeta_{\mathbf{k}3} \rangle \approx \langle (\zeta_{\mathbf{k}S})^2 \zeta_{\mathbf{k}L} \rangle$
- Then, the question is: "why should $(\zeta_{\mathbf{k}S})^2$ ever care about $\zeta_{\mathbf{k}L}$?"
  - The theorem says, "it doesn't care, if $\zeta_{\mathbf{k}}$ is exactly scale invariant."

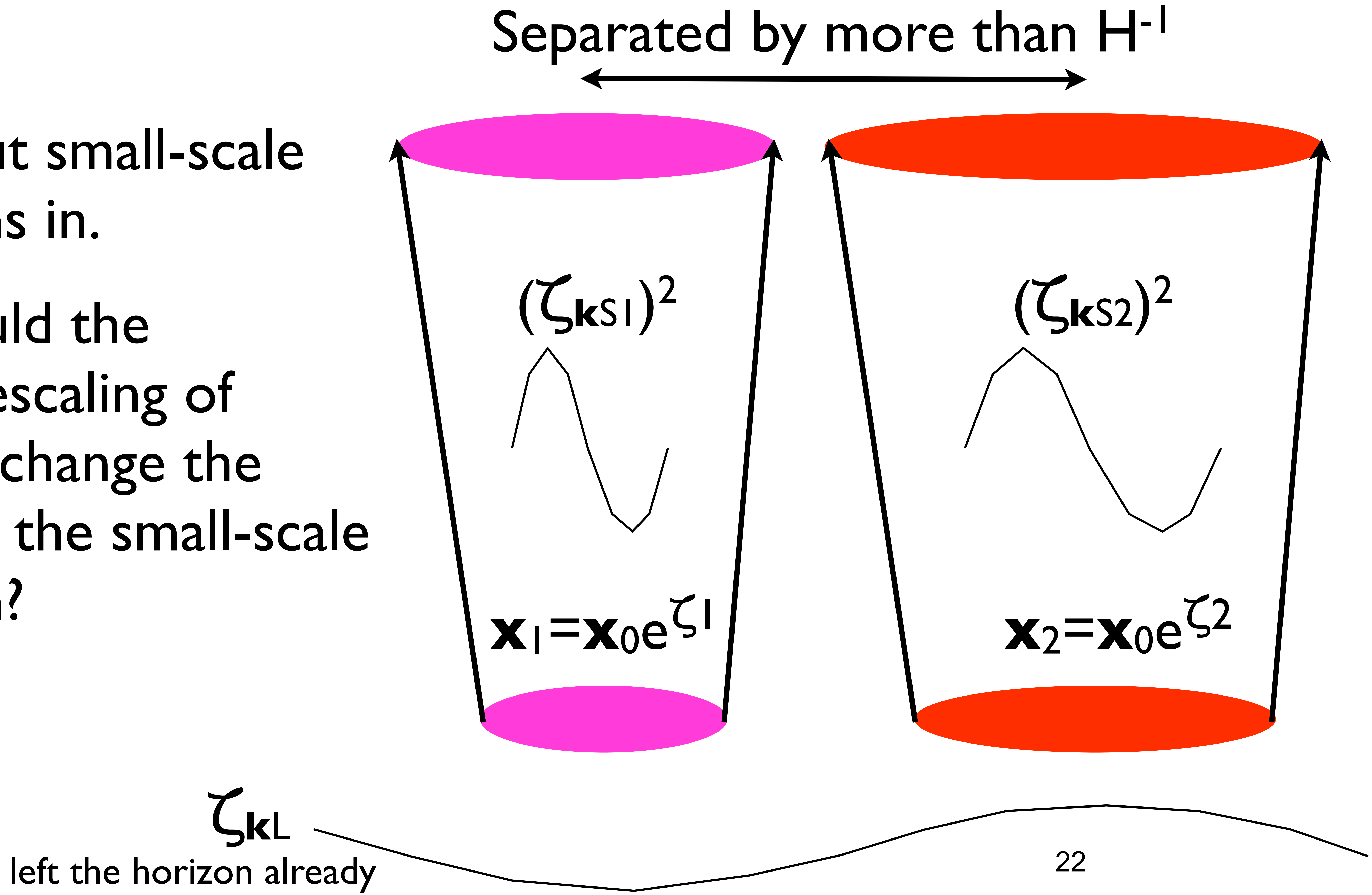
$\zeta_{\mathbf{k}L}$ rescales coordinates

- The long-wavelength curvature perturbation rescales the spatial coordinates (or changes the expansion factor) within a given Hubble patch:
  - $ds^2 = -dt^2 + [a(t)]^2 e^{2\zeta} (d\mathbf{x})^2$



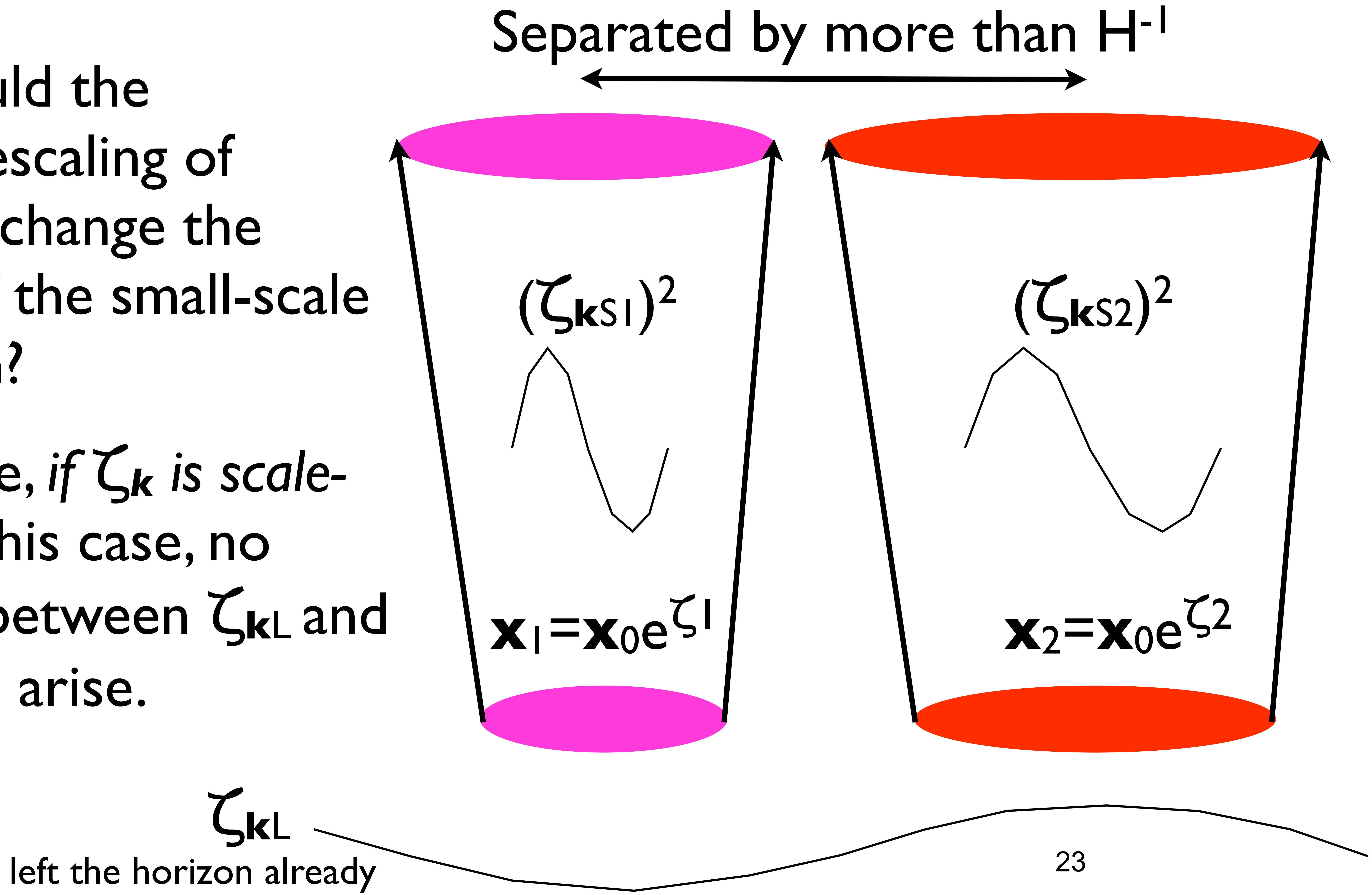
$\zeta_{\mathbf{k}L}$ rescales coordinates

- Now, let's put small-scale perturbations in.
- Q. How would the conformal rescaling of coordinates change the amplitude of the small-scale perturbation?



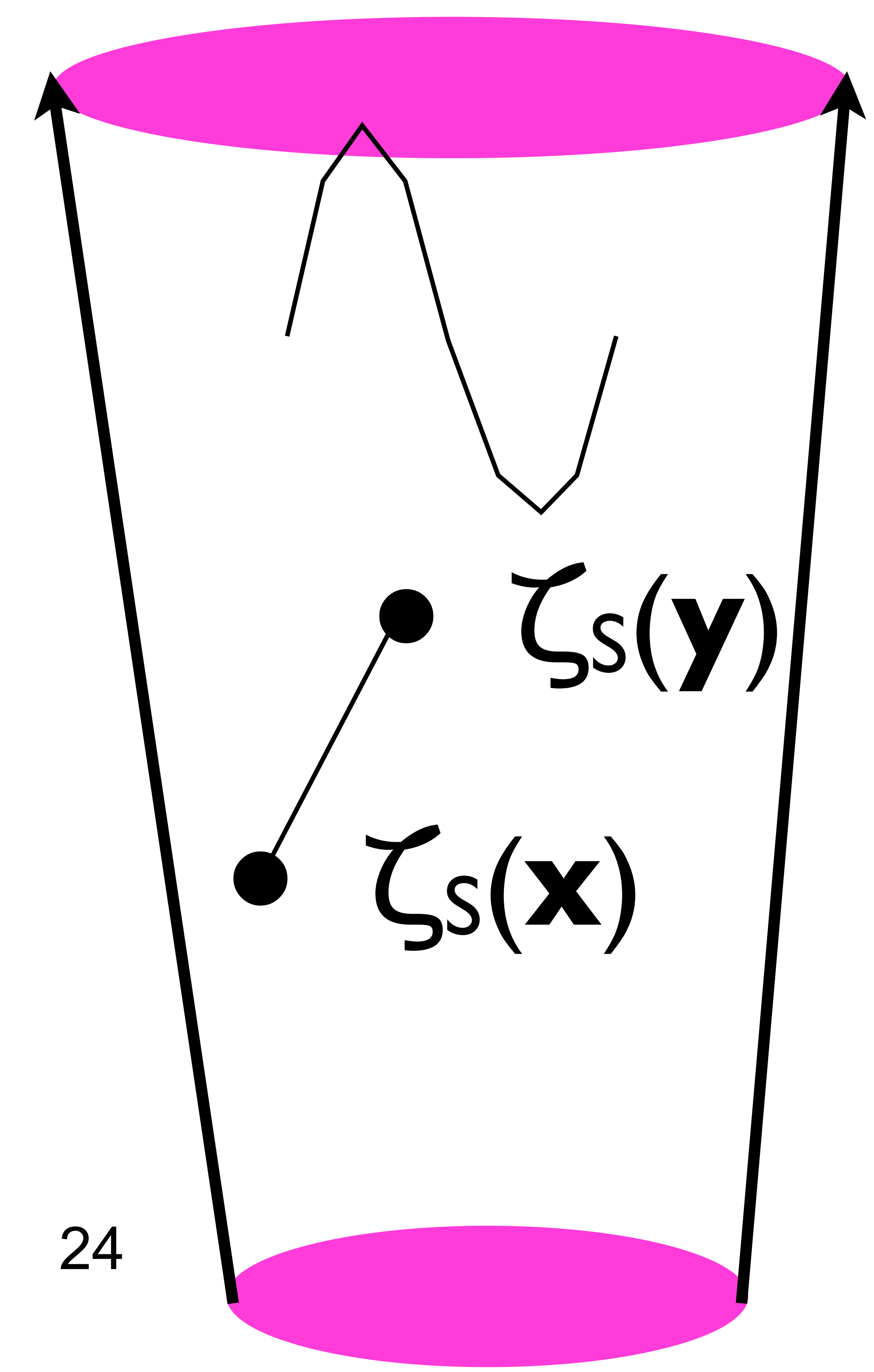
$\zeta_{\mathbf{k}L}$ rescales coordinates

- Q. How would the conformal rescaling of coordinates change the amplitude of the small-scale perturbation?
- A. No change, *if $\zeta_{\mathbf{k}}$ is scale-invariant*. In this case, no correlation between $\zeta_{\mathbf{k}L}$ and $(\zeta_{\mathbf{k}S})^2$ would arise.



Real-space Proof

- The 2-point correlation function of short-wavelength modes, $\xi = \langle\zeta_S(\mathbf{x})\zeta_S(\mathbf{y})\rangle$, within a given Hubble patch can be written in terms of its vacuum expectation value (in the absence of ζ_L), ξ_0, as:
  - $\xi_{\zeta L} \approx \xi_0(|\mathbf{x}-\mathbf{y}|) + \zeta_L\,[d\xi_0(|\mathbf{x}-\mathbf{y}|)/d\zeta_L]$
  - $\xi_{\zeta L} \approx \xi_0(|\mathbf{x}-\mathbf{y}|) + \zeta_L\,[d\xi_0(|\mathbf{x}-\mathbf{y}|)/d\ln|\mathbf{x}-\mathbf{y}|]$
  - $\xi_{\zeta L} \approx \xi_0(|\mathbf{x}-\mathbf{y}|) + \zeta_L\,(1-n_s)\xi_0(|\mathbf{x}-\mathbf{y}|)$



$$3\text{-pt func.} = \langle(\zeta_S)^2\zeta_L\rangle = \langle\xi_{\zeta L}\zeta_L\rangle$$
$$= (1-n_s)\xi_0(|\mathbf{x}-\mathbf{y}|)\langle\zeta_L^2\rangle$$

Where was “Single-field”?

- Where did we assume “single-field” in the proof?
- For this proof to work, it is crucial that there is only one dynamical degree of freedom, i.e., it is only ζ_L that modifies the amplitude of short-wavelength modes, and nothing else modifies it.
- Also, ζ must be constant outside of the horizon (otherwise anything can happen afterwards). This is also the case for single-field inflation models.

Therefore...

- A convincing detection of $f_{NL} > 1$ would rule out ***all*** of the single-field inflation models, regardless of:
  - the form of potential
  - the form of kinetic term (or sound speed)
  - the initial vacuum state
- A convincing detection of f_{NL} would be a breakthrough.

Large Non-Gaussianity from Single-field Inflation

- $S=(1/2)\int d^4x \sqrt{-g}\,[R-(\partial_\mu\varphi)^2-2V(\varphi)]$
- 2nd-order (which gives P_ζ)
  - $S_2=\int d^4x\ \varepsilon\ [a^3(\partial_t\zeta)^2-a(\partial_i\zeta)^2]$
- 3rd-order (which gives B_ζ)
  - $S_3=\int d^4x\ \boldsymbol{\varepsilon^2}\ [\ldots a^3(\partial_t\zeta)^2\zeta+\ldots a(\partial_i\zeta)^2\zeta+\ldots a^3(\partial_t\zeta)^3]+O(\varepsilon^3)$

Cubic-order interactions are suppressed by an additional factor of ε.
(Maldacena 2003)

Large Non-Gaussianity from Single-field Inflation

- $S=(1/2)\int d^4x \sqrt{-g}\,\{R-2P[(\partial_\mu\varphi)^2,\varphi]\}$ [general kinetic term]
- 2nd-order
  - $S_2=\int d^4x\ \varepsilon\ [a^3(\partial_t\zeta)^2/c_s^2-a(\partial_i\zeta)^2]$

"Speed of sound"
$c_s^2=P_{,X}/(P_{,X}+2XP_{,XX})$

- 3rd-order
  - $S_3=\int d^4x\ \boldsymbol{\varepsilon^2}\ [\ldots a^3(\partial_t\zeta)^2\zeta/\boldsymbol{c_s^2}+\ldots a(\partial_i\zeta)^2\zeta+\ldots a^3(\partial_t\zeta)^3/\boldsymbol{c_s^2}]+O(\varepsilon^3)$

Some interactions are enhanced for $c_s^2<1$.

(Seery & Lidsey 2005; Chen et al. 2007)

Large Non-Gaussianity from Single-field Inflation

- $S=(1/2)\int d^4x \sqrt{-g}\,\{R-2P[(\partial_\mu\varphi)^2,\varphi]\}$ [general kinetic term]
- 2nd-order
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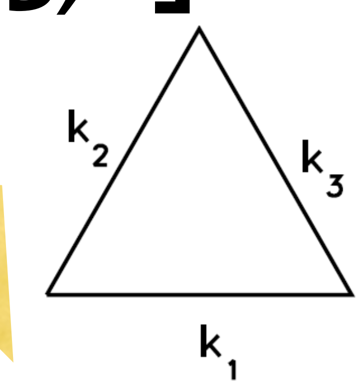
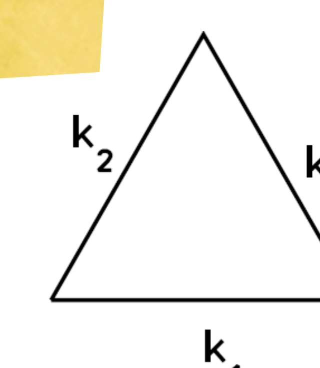
- 3rd-order
  - $S_3=\int d^4x\ \boldsymbol{\varepsilon^2}\ [\ldots a^3(\partial_t\zeta)^2\zeta/\boldsymbol{c_s^2}+\ldots a(\partial_i\zeta)^2\zeta+\ldots a^3(\partial_t\zeta)^3/\boldsymbol{c_s^2}]+O(\varepsilon^3)$

"Speed of sound"
$c_s^2=P_{,X}/(P_{,X}+2XP_{,XX})$



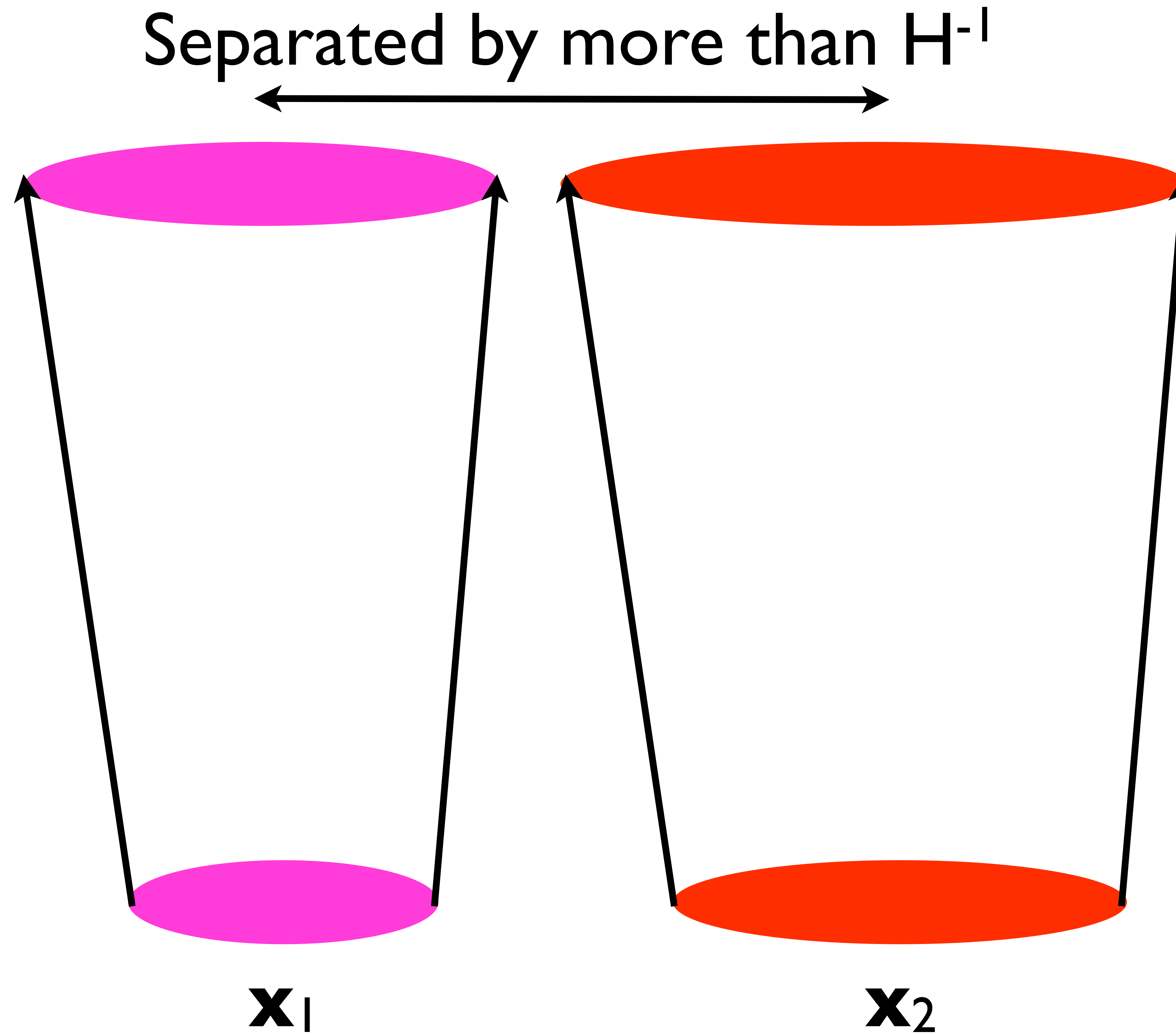


Some interactions are enhanced for $c_s^2<1$.

(Seery & Lidsey 2005; Chen et al. 2007)

Another Motivation For f_{NL}

- In multi-field inflation models, $\zeta_{\boldsymbol{k}}$ can evolve outside the horizon.
- This evolution can give rise to non-Gaussianity; however, causality demands that the form of non-Gaussianity must be local!



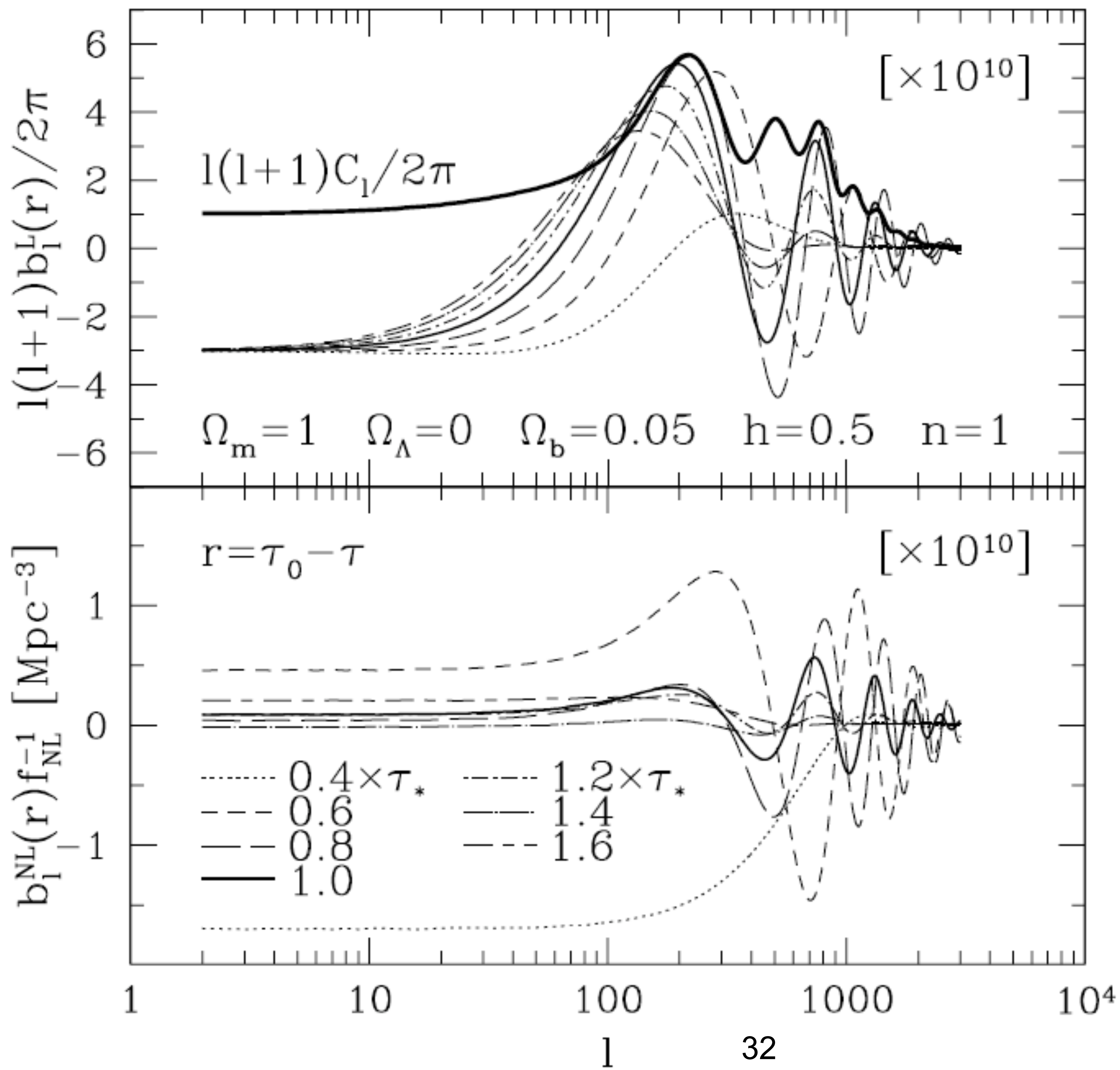
$$\zeta(\mathbf{x})=\zeta_g(\mathbf{x})+(3/5)f_{NL}[\zeta_g(\mathbf{x})]^2+A\chi_g(\mathbf{x})+B[\chi_g(\mathbf{x})]^2+\dots$$

Now:

- I hope that I could convince you that f_{NL} is a very powerful quantity for testing single-field inflation models.
- Let's look at the observational data!

Decoding Bispectrum

- Hydrodynamics at z=1090 generates acoustic oscillations in the bispectrum
- Well understood at the linear level (*Komatsu & Spergel 2001*)
- Non-linear extension?
  - *Nitta, Komatsu, Bartolo, Matarrese & Riotto*, arXiv: 0903.0894
  - f_{NL}^{local}~0.5



Measurement

- Use everybody's favorite: χ^2 minimization.
  - Minimize:

$$\chi^2 \equiv \sum_{2\le l_1\le l_2\le l_3} \frac{\left(B^{obs}_{l_1l_2l_3} - \sum_i A_i B^{(i)}_{l_1l_2l_3}\right)^2}{\sigma^2_{l_1l_2l_3}}$$

  - with respect to $A_i=(f_{NL}^{local}, f_{NL}^{equilateral}, b_{src})$
  - B^{obs} is the observed bispectrum
  - $B^{(i)}$ is the theoretical template from various predictions

Journal on f_{NL} (95%CL)

- $-3500 < f_{NL} < 2000$ [COBE 4yr, l_{max}=20] Komatsu et al. (2002)
- $-58 < f_{NL} < 134$ [WMAP 1yr, l_{max}=265] Komatsu et al. (2003)
- $-54 < f_{NL} < 114$ [WMAP 3yr, l_{max}=350] Spergel et al. (2007)
- **$-9 < f_{NL}^{local} < 111$ [WMAP 5yr, l_{max}=500]** Komatsu et al. (2008)

Latest on f_{NL}

(Fast-moving field!)

- CMB (WMAP5 + most optimal bispectrum estimator)
  - **$-4 < f_{NL} < 80$ (95%CL)**
  - $f_{NL} = 38 \pm 21$ (68%CL)

Smith, Senatore & Zaldarriaga (2009)

- Large-scale Structure (Using the SDSS power spectra)
  - **$-29 < f_{NL} < 70$ (95%CL)**
  - $f_{NL} = 31^{+16}_{-27}$ (68%CL)

Slosar et al. (2009)

Weak 2-σ "Hint"?

- So, currently we have something like f_{NL}~40±20 from the WMAP 5-year data, and 30±15 from WMAP5+LSS.
- Without a doubt, we need more data...
  - WMAP7 is coming up (early next year)
  - WMAP9 in ~2011–2012
- And...

Planck!

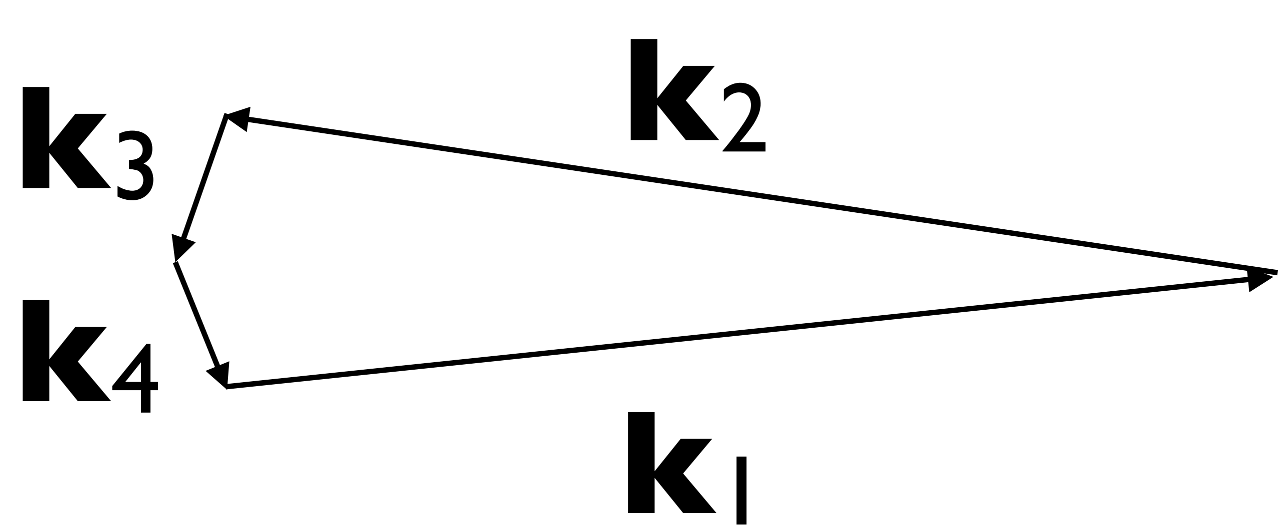
- Planck satellite is scheduled to be launched **TOMORROW**, from French Guiana.
- Planck's expected 68%CL errorbar is ~5.
  - Therefore, if f_{NL}~40, we would see it at 8σ. If ~30, 6σ. Either way, IF (big if) f_{NL}~30–40, we will see it unambiguously with Planck, which is expected to deliver the first-year results in ≥2012.

Trispectrum: Next Frontier?

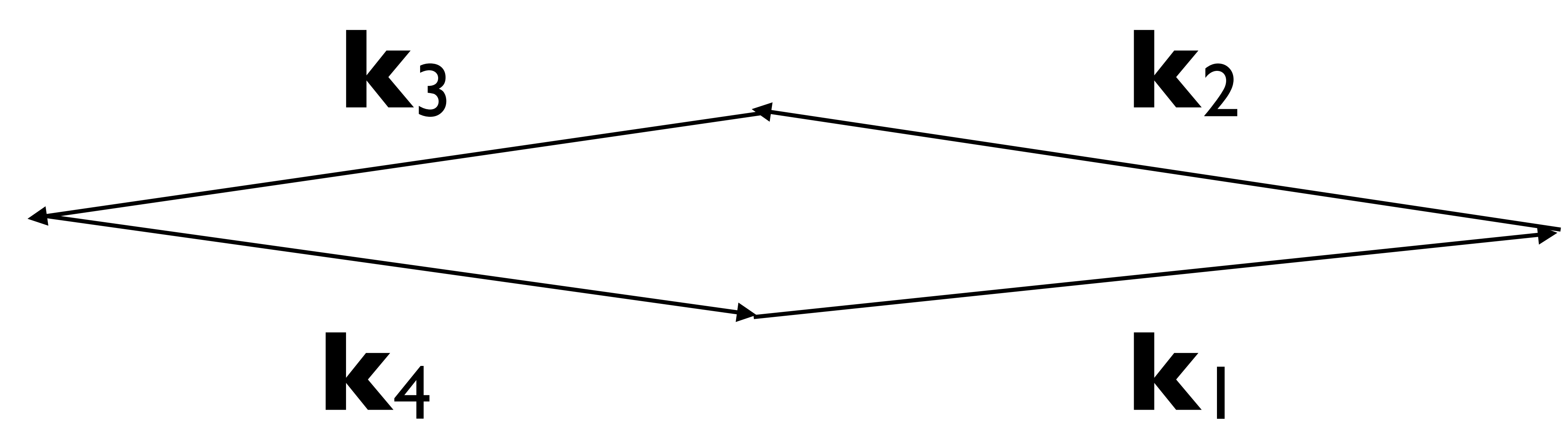
- The local form bispectrum, $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)=(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)f_{NL}[(6/5)P_\zeta(k_1)P_\zeta(k_2)+\text{cyc.}]$
- is equivalent to having the curvature perturbation in position space, in the form of:
  - $\zeta(\mathbf{x})=\zeta_g(\mathbf{x}) + (3/5)f_{NL}[\zeta_g(\mathbf{x})]^2$
    - This provides a useful model to parametrize non-Gaussianity, and generate initial conditions for, e.g., N-body simulations.
- This can be extended to higher-order:
  - $\zeta(\mathbf{x})=\zeta_g(\mathbf{x}) + (3/5)f_{NL}[\zeta_g(\mathbf{x})]^2 + (9/25)g_{NL}[\zeta_g(\mathbf{x})]^3$

Local Form Trispectrum

- For $\zeta(\mathbf{x})=\zeta_g(\mathbf{x}) + (3/5)f_{NL}[\zeta_g(\mathbf{x})]^2 + (9/25)g_{NL}[\zeta_g(\mathbf{x})]^3$, we obtain the trispectrum:
  - $T_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)=(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)$ $\{g_{NL}[(54/25)P_\zeta(k_1)P_\zeta(k_2)P_\zeta(k_3)+\text{cyc.}] +$ $(f_{NL})^2[(18/25)P_\zeta(k_1)P_\zeta(k_2)(P_\zeta(|\mathbf{k}_1+\mathbf{k}_3|)+P_\zeta(|\mathbf{k}_1+\mathbf{k}_4|))+\text{cyc.}]\}$



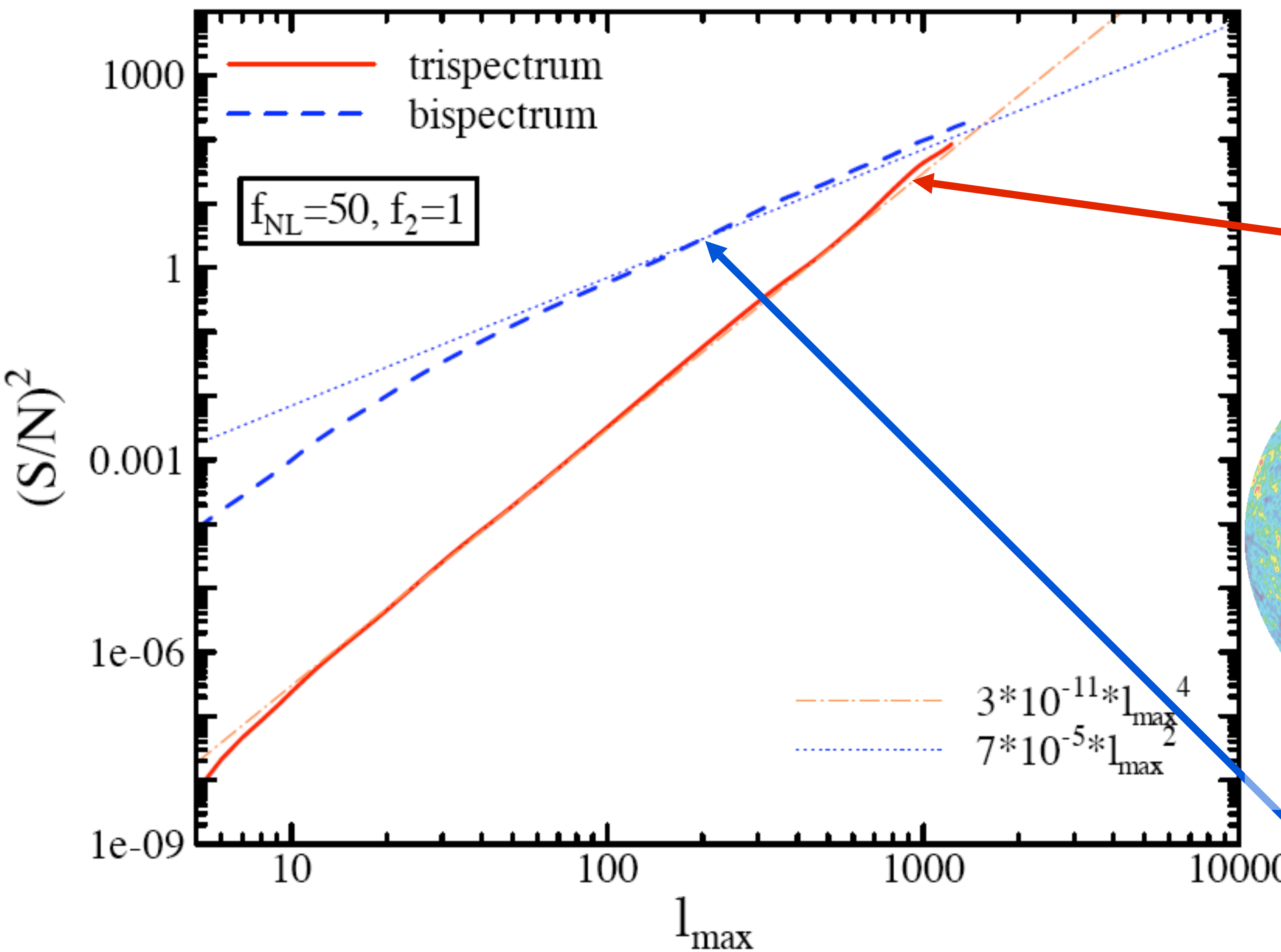
g_{NL}



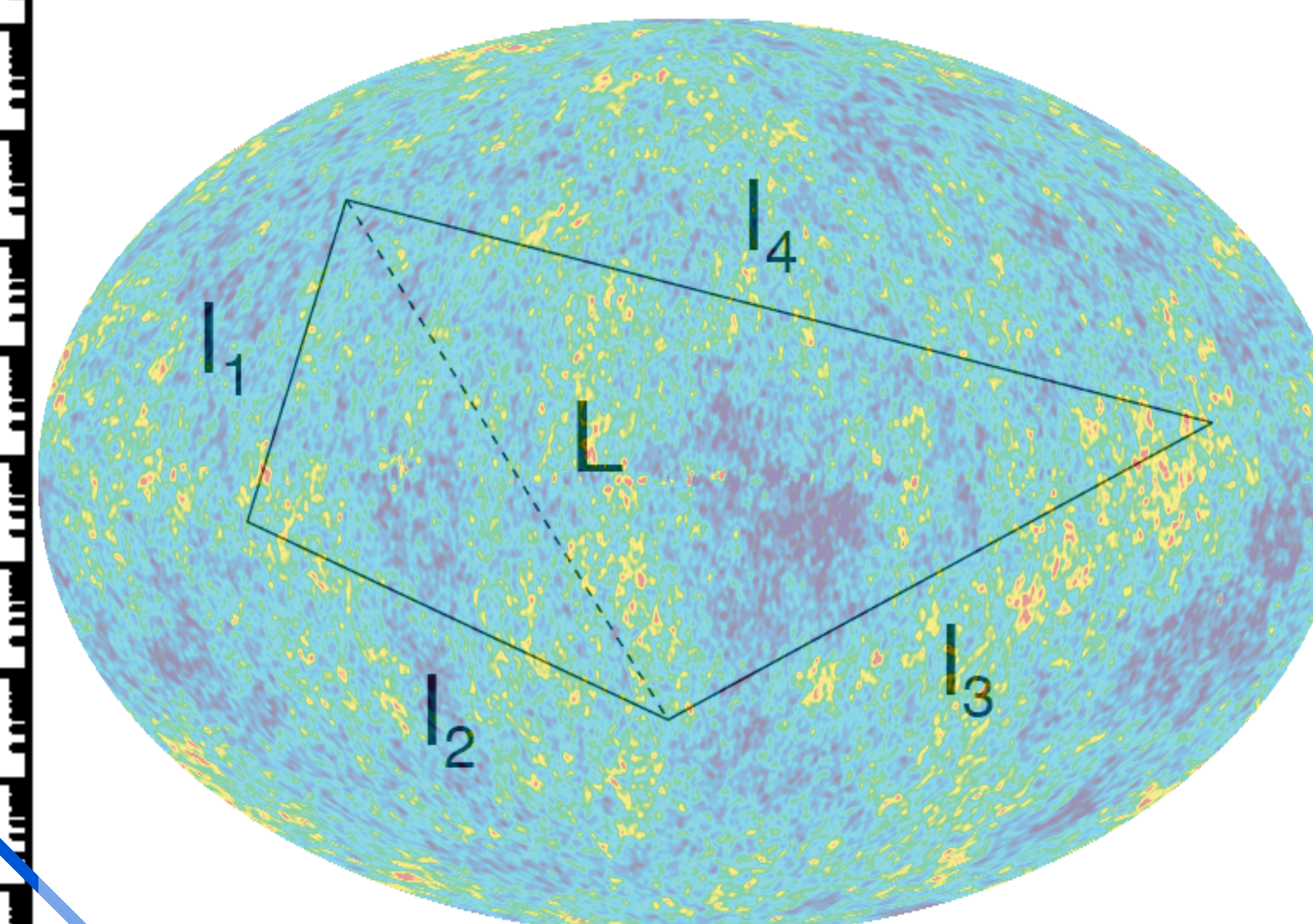
f_{NL}^2

Kogo & Komatsu (2006)

Trispectrum: if f_{NL} is ~50, excellent cross-check for Planck



Trispectrum (~f_{NL}^2)

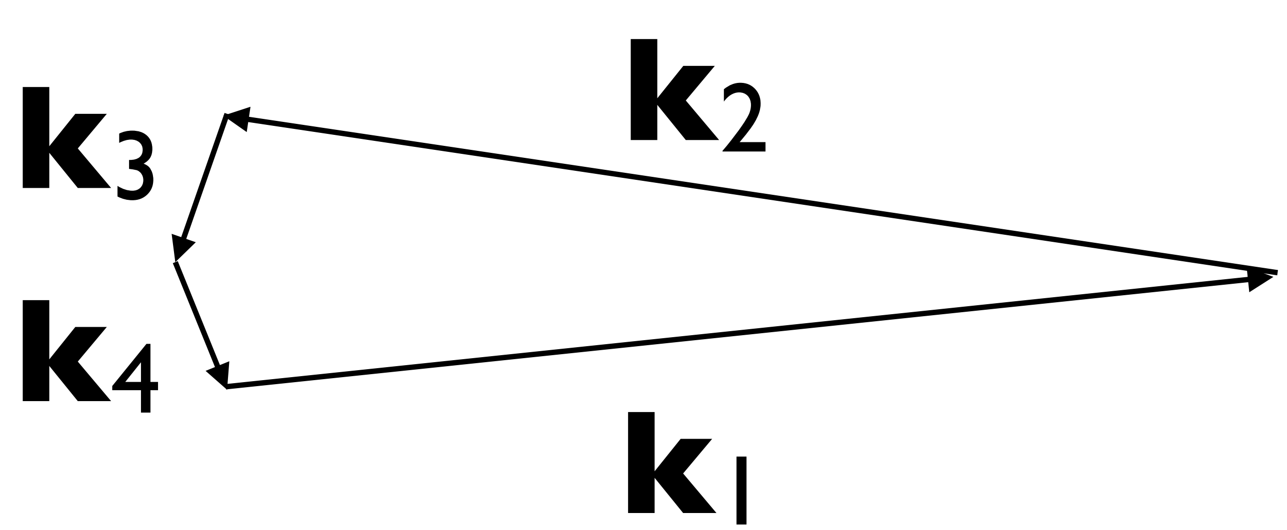


Bispectrum (~ f_{NL})

(Slightly) Generalized Trispectrum

- $T_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)=(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)$
  $\{g_{NL}[(54/25)P_\zeta(k_1)P_\zeta(k_2)P_\zeta(k_3)+\text{cyc.}]$
  $+\tau_{NL}[(18/25)P_\zeta(k_1)P_\zeta(k_2)(P_\zeta(|\mathbf{k}_1+\mathbf{k}_3|)+P_\zeta(|\mathbf{k}_1+\mathbf{k}_4|))+\text{cyc.}]\}$

The local form consistency relation, $\tau_{NL}=(f_{NL})^2$, may not be respected – additional test of multi-field inflation!



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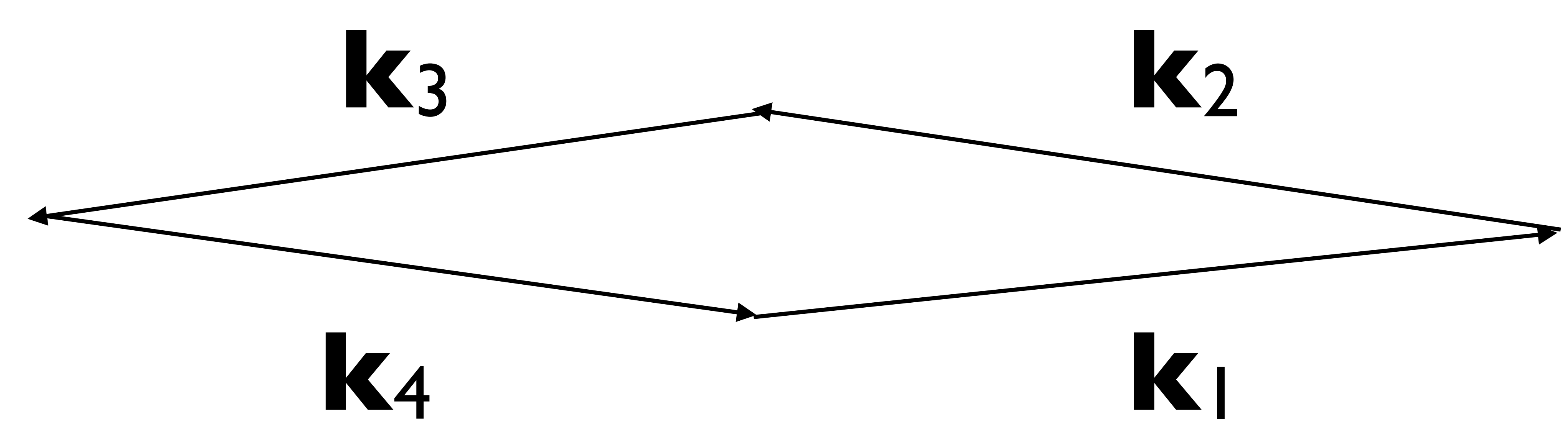


f_{NL}^2

Trispectrum: Next Frontier

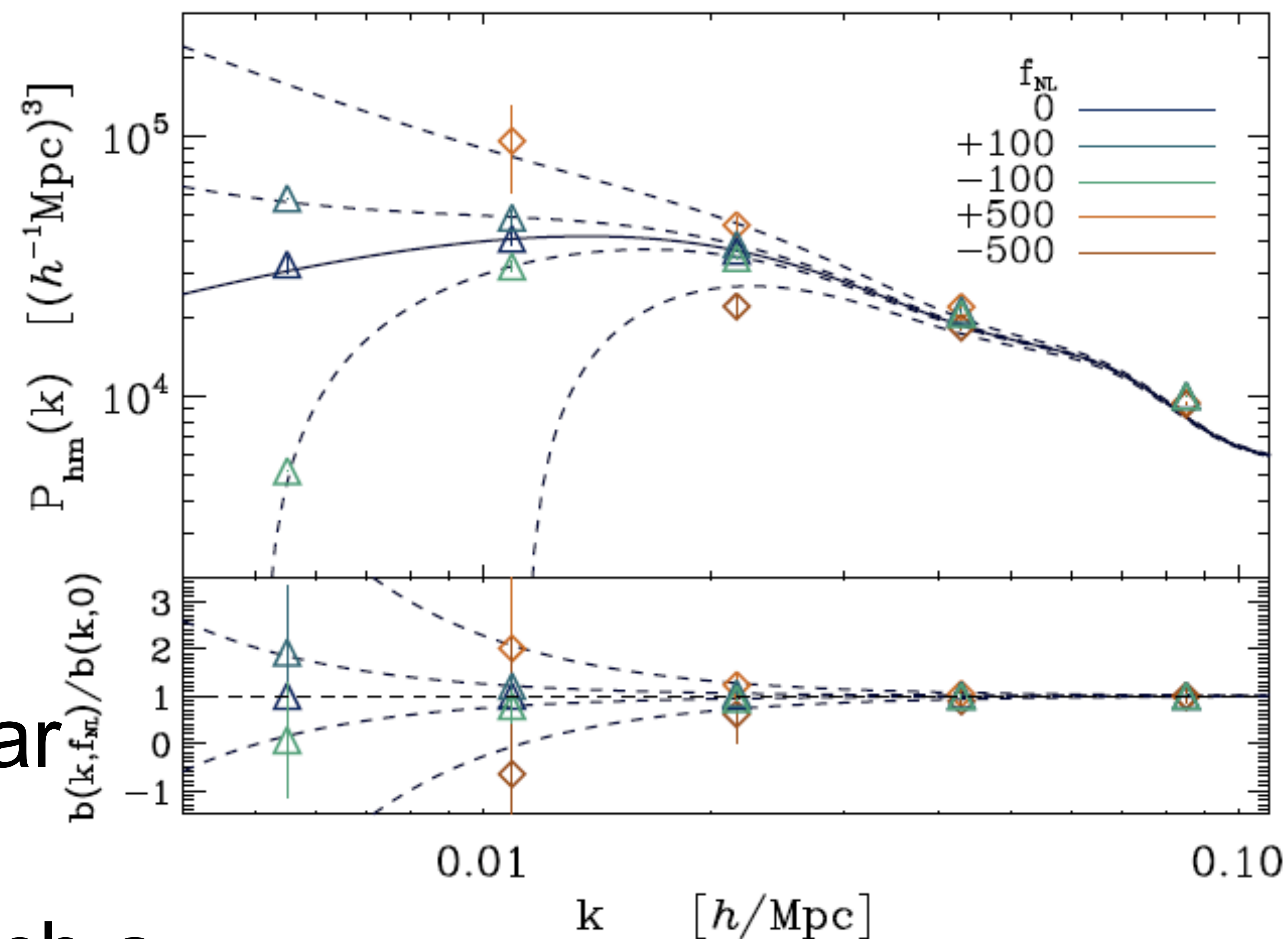
- A new phenomenon: many talks given at the IPMU non-Gaussianity workshop emphasized the importance of the trispectrum as a source of additional information on the physics of inflation.
- $\tau_{NL} \sim f_{NL}^2$; $\tau_{NL} \sim f_{NL}^{4/3}$; $\tau_{NL} \sim$ (isocurv.)*f_{NL}^2; $g_{NL} \sim f_{NL}$; $g_{NL} \sim f_{NL}^2$; or they are completely independent
- Shape dependence? (Squares from ghost condensate, diamonds and rectangles from multi-field, etc)

Large-scale Structure of the Universe

- New frontier: large-scale structure of the universe as a probe of primordial non-Gaussianity

New, Powerful Probe of f_{NL}

- f_{NL} modifies the power spectrum of galaxies on very large scales
  - –*Dalal et al.; Matarrese & Verde*
  - –*Mcdonald; Afshordi & Tolley*
- The statistical power of this method is **VERY** promising
  - –SDSS: $-29 < f_{NL} < 70$ (95%CL); Slosar et al.
  - –Comparable to the WMAP 5-year limit already
  - –Expected to beat CMB, and reach a sacred region: **f_{NL}~1**



Effects of f_{NL} on the statistics of PEAKS

- The effects of f_{NL} on the power spectrum of peaks (i.e., galaxies) are profound.
- **How about the bispectrum of galaxies?**

Previous Calculation

- Scoccimarro, Sefusatti & Zaldarriaga (2004); Sefusatti & Komatsu (2007)
  - Treated the distribution of galaxies as a *continuous distribution*, biased relative to the matter distribution:
    - $\delta_g = b_1\delta_m + (b_2/2)(\delta_m)^2 + \ldots$
  - Then, the calculation is straightforward. Schematically:
    - $<\delta_g^3> = (b_1)^3<\delta_m^3> + (b_1^2 b_2)<\delta_m^4> + \ldots$

Non-linear Gravity
Primordial NG

Non-linear Bias Bispectrum

Previous Calculation

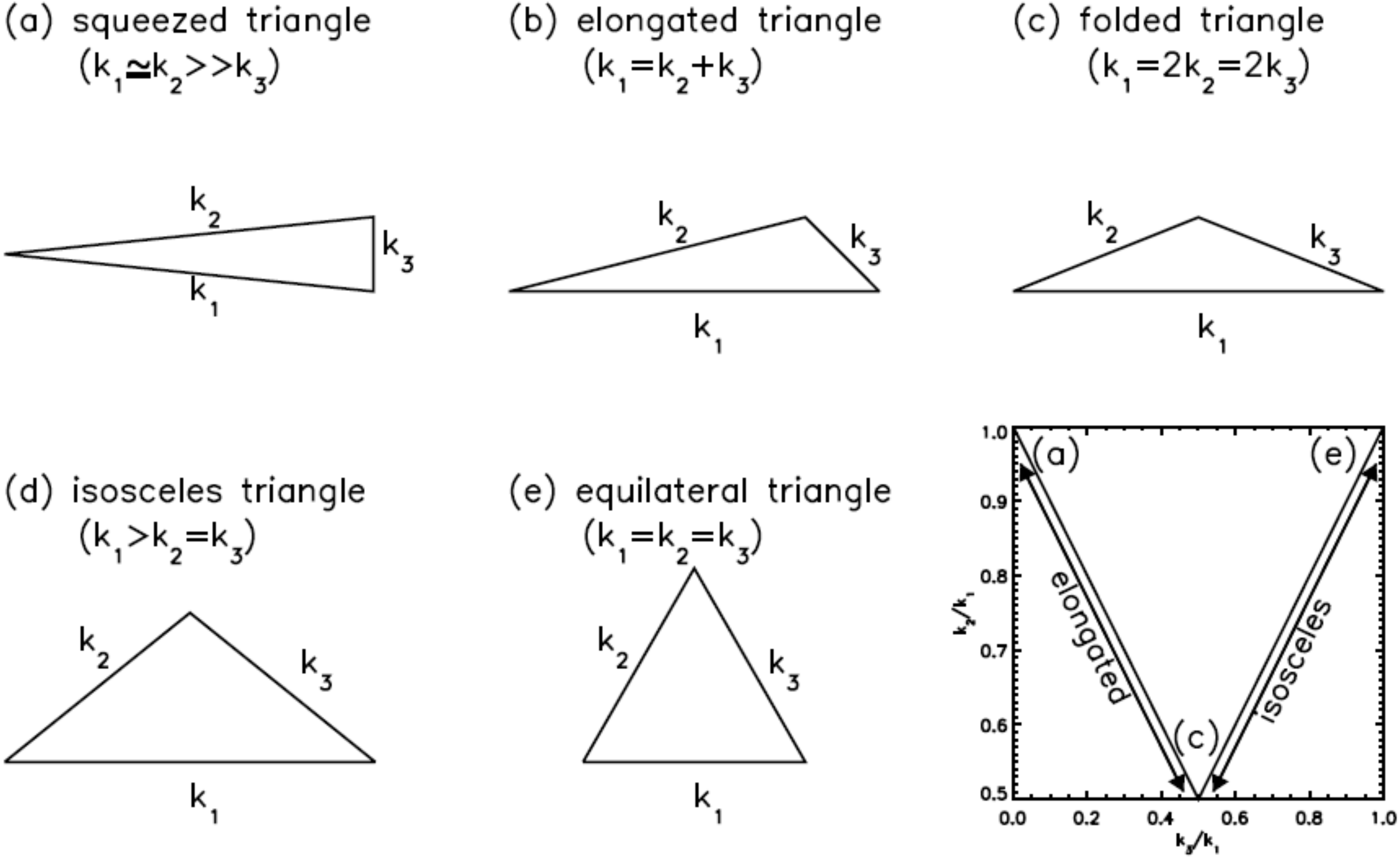
$$
\begin{aligned}
&B_g(k_1, k_2, k_3, z) \\
&= 3b_1^3 f_{\rm NL} \Omega_m H_0^2 \left[\frac{P_m(k_1, z)}{k_1^2 T(k_1)} \frac{P_m(k_2, z)}{k_2^2 T(k_2)} \frac{k_3^2 T(k_3)}{D(z)} + (\text{cyclic}) \right] \\
&+ 2b_1^3 \left[F_2^{(s)}(\boldsymbol{k}_1, \boldsymbol{k}_2) P_m(k_1, z) P_m(k_2, z) + (\text{cyclic}) \right] \\
&+ b_1^2 b_2 \left[P_m(k_1, z) P_m(k_2, z) + (\text{cyclic}) \right]
\end{aligned}
$$

Primordial NG

Non-linear Gravity

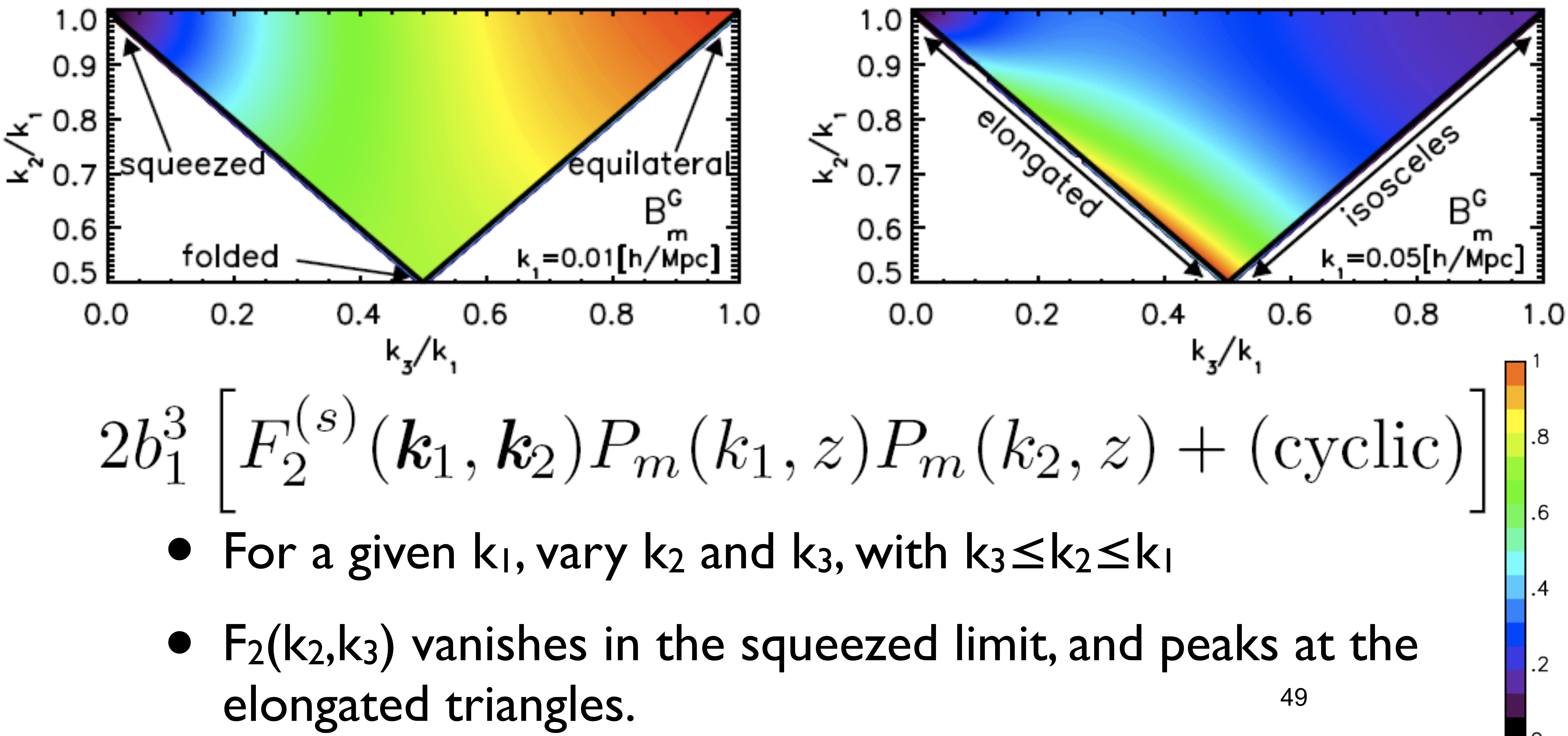
Non-linear Bias

- We find that this formula captures only a part of the full contributions. In fact, this formula is sub-dominant in the squeezed configuration, and the new terms are dominant.



(a) squeezed triangle
($k_1 \simeq k_2 >> k_3$)
(b) elongated triangle
($k_1 = k_2 + k_3$)
(c) folded triangle
($k_1 = 2k_2 = 2k_3$)
(d) isosceles triangle
($k_1 > k_2 = k_3$)
(e) equilateral triangle
($k_1 = k_2 = k_3$)
k_1
k_2
k_3
elongated
isosceles
k_2/k_1
k_3/k_1
1.0
0.9
0.8
0.7
0.6
0.5
0.0
0.2
0.4
0.6
0.8
1.0

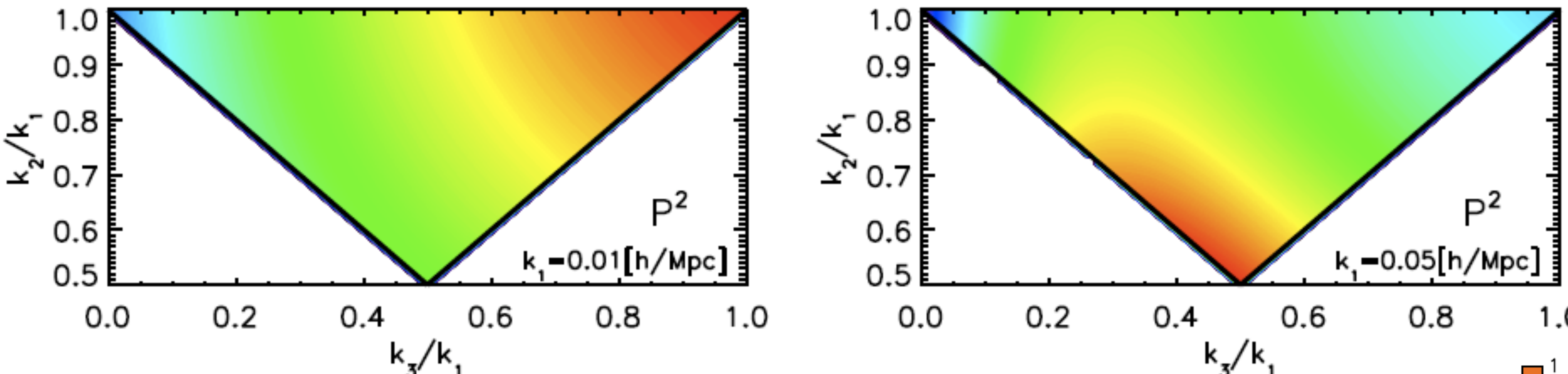
Non-linear Gravity



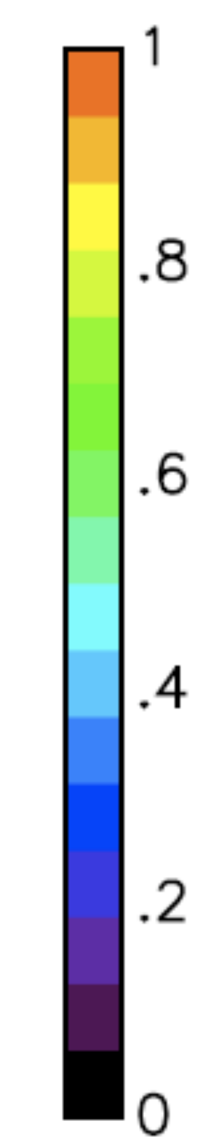
$$2b_1^3\left[F_2^{(s)}(\boldsymbol{k}_1,\boldsymbol{k}_2)P_m(k_1,z)P_m(k_2,z)+(\text{cyclic})\right]$$

- For a given k_1, vary k_2 and k_3, with $k_3 \leq k_2 \leq k_1$
- $F_2(k_2,k_3)$ vanishes in the squeezed limit, and peaks at the elongated triangles.

Non-linear Galaxy Bias

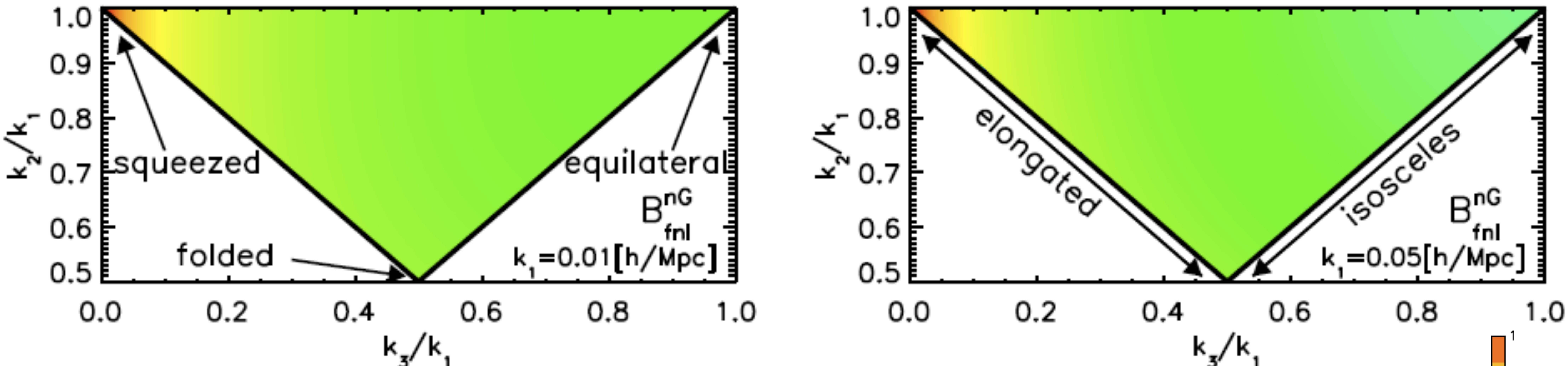


$$b_1^2 b_2 \left[P_m(k_1, z) P_m(k_2, z) + (\text{cyclic})\right]$$

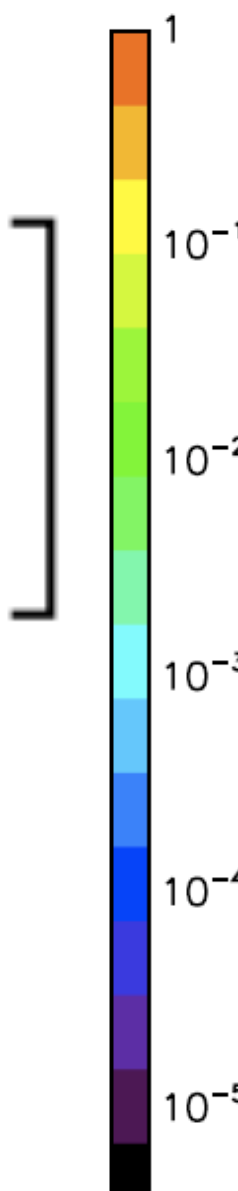


- There is no F_2: less suppression at the squeezed, and less enhancement along the elongated triangles.
- Still peaks at the equilateral or elongated forms.

Primordial NG (SK07)



$$3b_1^3 f_{\rm NL}\Omega_m H_0^2\left[\frac{P_m(k_1,z)}{k_1^2T(k_1)}\frac{P_m(k_2,z)}{k_2^2T(k_2)}\frac{k_3^2T(k_3)}{D(z)}+(\text{cyclic})\right]$$



- Notice the factors of k^2 in the denominator.
- This gives the peaks at the squeezed configurations.

New Terms

- But, it turns our that Sefusatti & Komatsu's calculation, which is valid only for the continuous field, misses the dominant terms that come from the statistics of PEAKS.
- Jeong & Komatsu, arXiv:0904.0497

Donghui Jeong

Matarrese, Lucchin & Bonometto (1986)

MLB Formula

$$1 + \xi_h(x_{12}) + \xi_h(x_{23}) + \xi_h(x_{31}) + \zeta_h(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3)$$

$$= \exp\left[\frac{1}{2}\frac{\nu^2}{\sigma_R^2}\sum_{i\neq j}\xi_R^{(2)}(x_{ij}) + \sum_{n=3}^{\infty}\left\{\sum_{m_1=0}^{n}\sum_{m_2=0}^{n-m_1}\frac{\nu^n\sigma_R^{-n}}{m_1!m_2!m_3!}\right.\right.$$

$$\times\xi_R^{(n)}\begin{pmatrix}\boldsymbol{x}_1,\cdots,\boldsymbol{x}_1, & \boldsymbol{x}_2,\cdots,\boldsymbol{x}_2, & \boldsymbol{x}_3,\cdots,\boldsymbol{x}_3 \\ m_1\text{ times} & m_2\text{ times} & m_3\text{ times}\end{pmatrix}$$

$$\left.\left.-3\frac{\nu^n\sigma_R^{-n}}{n!}\xi_R^{(n)}\begin{pmatrix}\boldsymbol{x},\cdots,\boldsymbol{x} \\ n\text{ times}\end{pmatrix}\right\}\right]$$

- N-point correlation function of peaks is the sum of M-point correlation functions, where M≥N.

Bottom Line

- **The bottom line is:**
- The power spectrum (2-pt function) of peaks is sensitive to the power spectrum of the underlying mass distribution, and the bispectrum, and the trispectrum, etc.
  - Truncate the sum at the bispectrum: sensitivity to f_{NL}
  - Dalal et al.; Matarrese&Verde; Slosar et al.; Afshordi&Tolley

Bottom Line

- **The bottom line is:**
- The bispectrum (3-pt function) of peaks is sensitive to the bispectrum of the underlying mass distribution, and the trispectrum, and the quadspectrum, etc.
  - Truncate the sum at the trispectrum: sensitivity to τ_{NL} ($\sim f_{NL}^2$) and g_{NL}!
  - This is the new effect that was missing in Sefusatti & Komatsu (2007).

Real-space 3pt Function

$$\zeta_h(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3) = \frac{\nu^3}{\sigma_R^3} \xi_R^{(3)}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3)$$
$$+ \frac{\nu^4}{\sigma_R^4} \left[\xi_R^{(2)}(x_{12}) \xi_R^{(2)}(x_{23}) + (\text{cyclic}) \right]$$
$$+ \frac{\nu^4}{2\sigma_R^4} \left[\xi_R^{(4)}(\boldsymbol{x}_1, \boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3) + (\text{cyclic}) \right]$$

- Plus 5-pt functions, etc...

New Bispectrum Formula

$$B_h(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$$

$$= b_1^3 \left[B_R(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) + \frac{b_2}{b_1} \{P_R(k_1)P_R(k_2) + (\text{cyclic})\} \right.$$

$$\left. + \frac{\delta_c}{2\sigma_R^2} \int \frac{d^3q}{(2\pi)^3} T_R(\boldsymbol{q}, \boldsymbol{k}_1 - \boldsymbol{q}, \boldsymbol{k}_2, \boldsymbol{k}_3) + (\text{cyclic}) \right].$$

- First: bispectrum of the underlying mass distribution.
- Second: non-linear bias
- Third: trispectrum of the underlying mass distribution.

Local Form Trispectrum

$\Phi=(3/5)\zeta$

$$\Phi(\boldsymbol{x}) = \phi(\boldsymbol{x}) + f_{\rm NL}\left[\phi^2(\boldsymbol{x}) - \langle\phi^2\rangle\right] + g_{\rm NL}\phi^3(\boldsymbol{x})$$

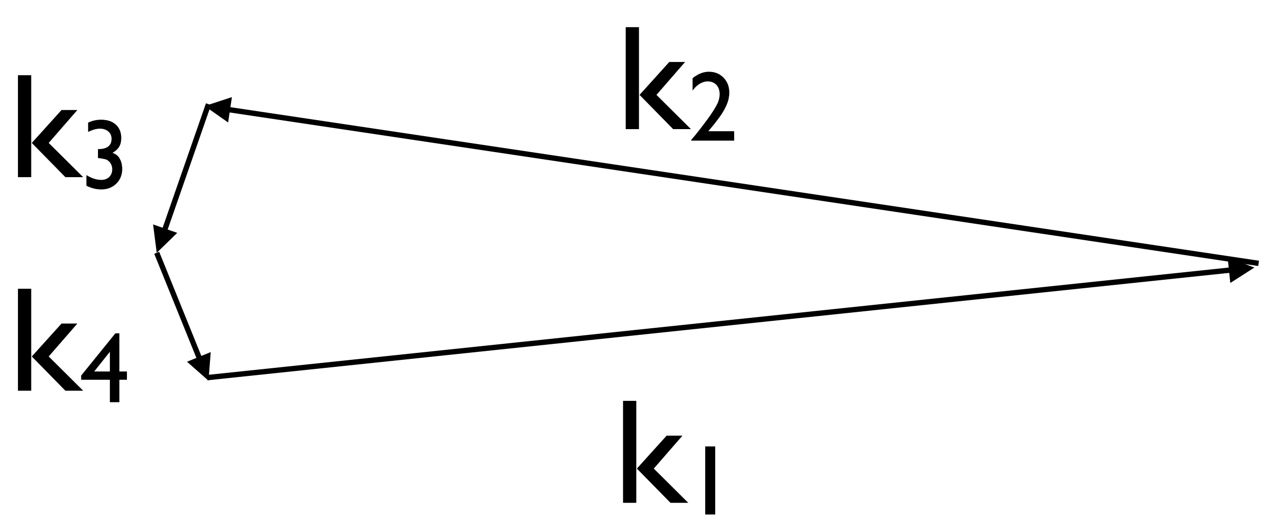
$$T_\Phi(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4)$$
$$= 6g_{\rm NL}\left[P_\phi(k_1)P_\phi(k_2)P_\phi(k_3) + ({\rm cyclic})\right] + 2f_{\rm NL}^2$$
$$\times\left[P_\phi(k_1)P_\phi(k_2)\left\{P_\phi(k_{13}) + P_\phi(k_{14})\right\} + ({\rm cyclic})\right]$$

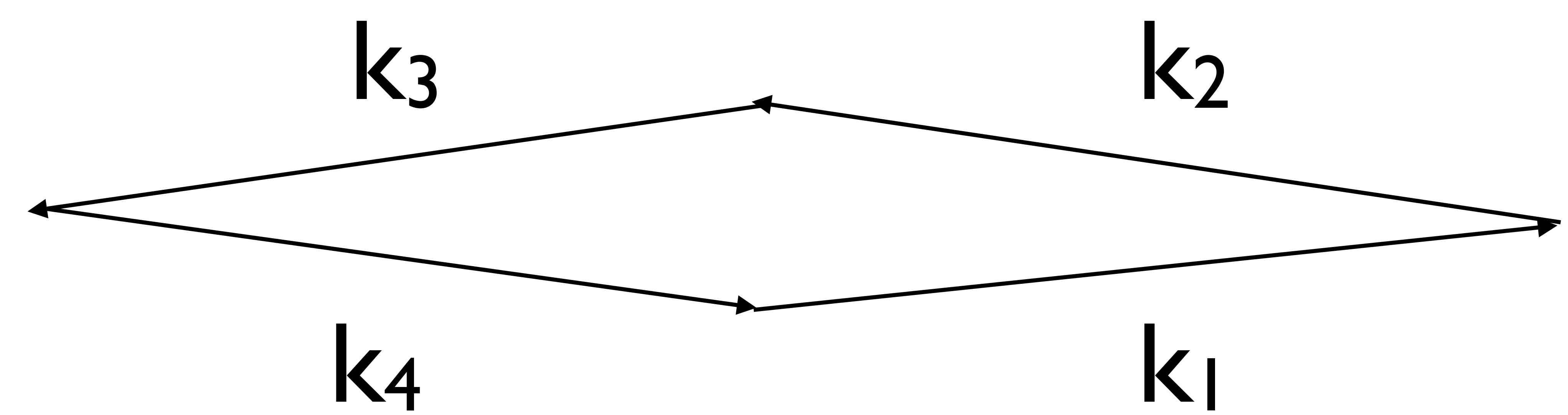
- For general multi-field models, f_{NL}^2 can be more generic: often called τ_{NL}.
- Exciting possibility for testing more about inflation!

Local Form Trispectrum

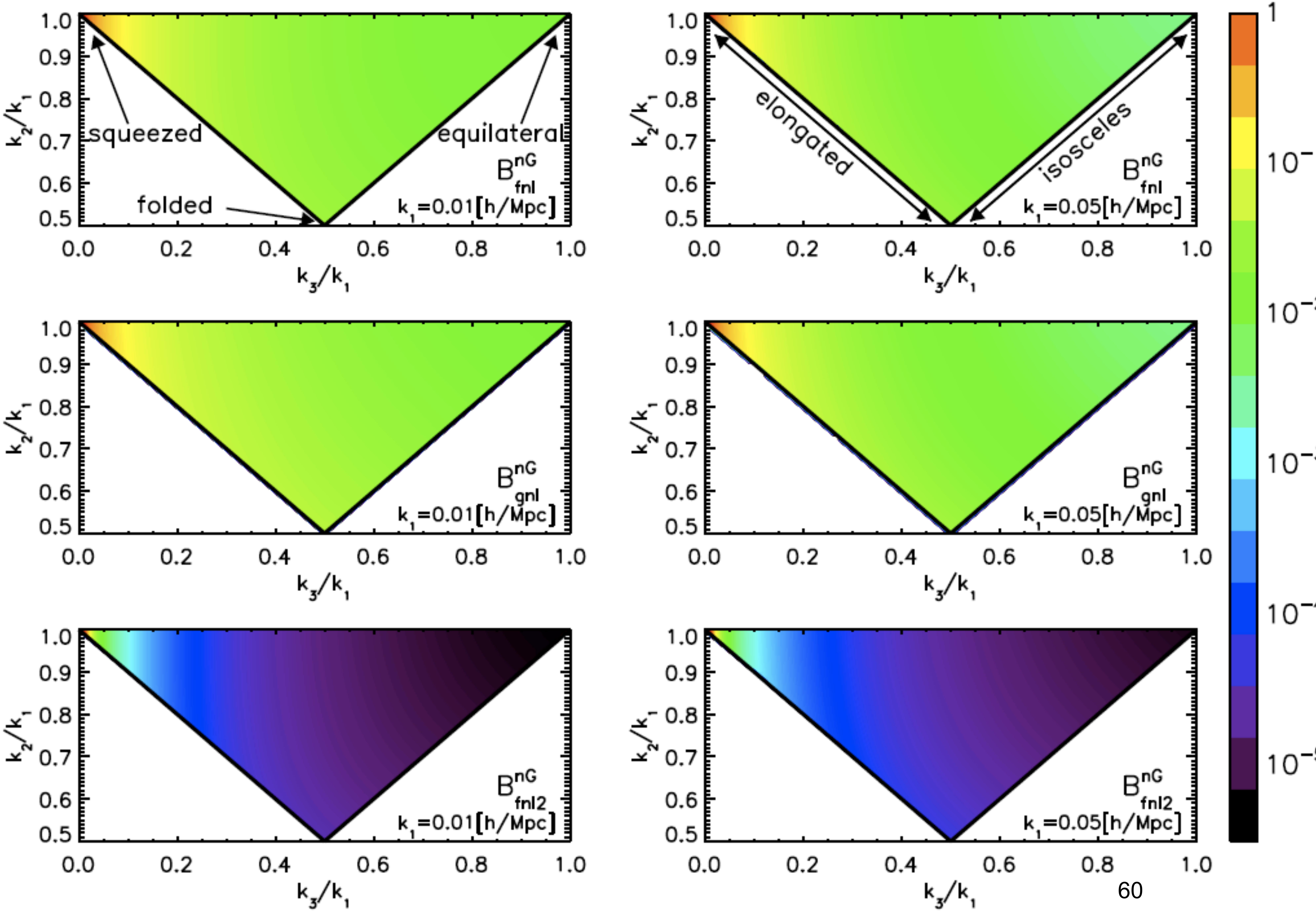
$\Phi=(3/5)\zeta$

$$T_\Phi(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4)$$
$$= 6 g_{\rm NL}\left[P_\phi(k_1)P_\phi(k_2)P_\phi(k_3) + ({\rm cyclic})\right] + 2 f_{\rm NL}^2$$
$$\times \left[P_\phi(k_1)P_\phi(k_2)\left\{P_\phi(k_{13}) + P_\phi(k_{14})\right\} + ({\rm cyclic})\right]$$





g_{NL} $\quad$ f_{NL}^2 (or τ_{NL})

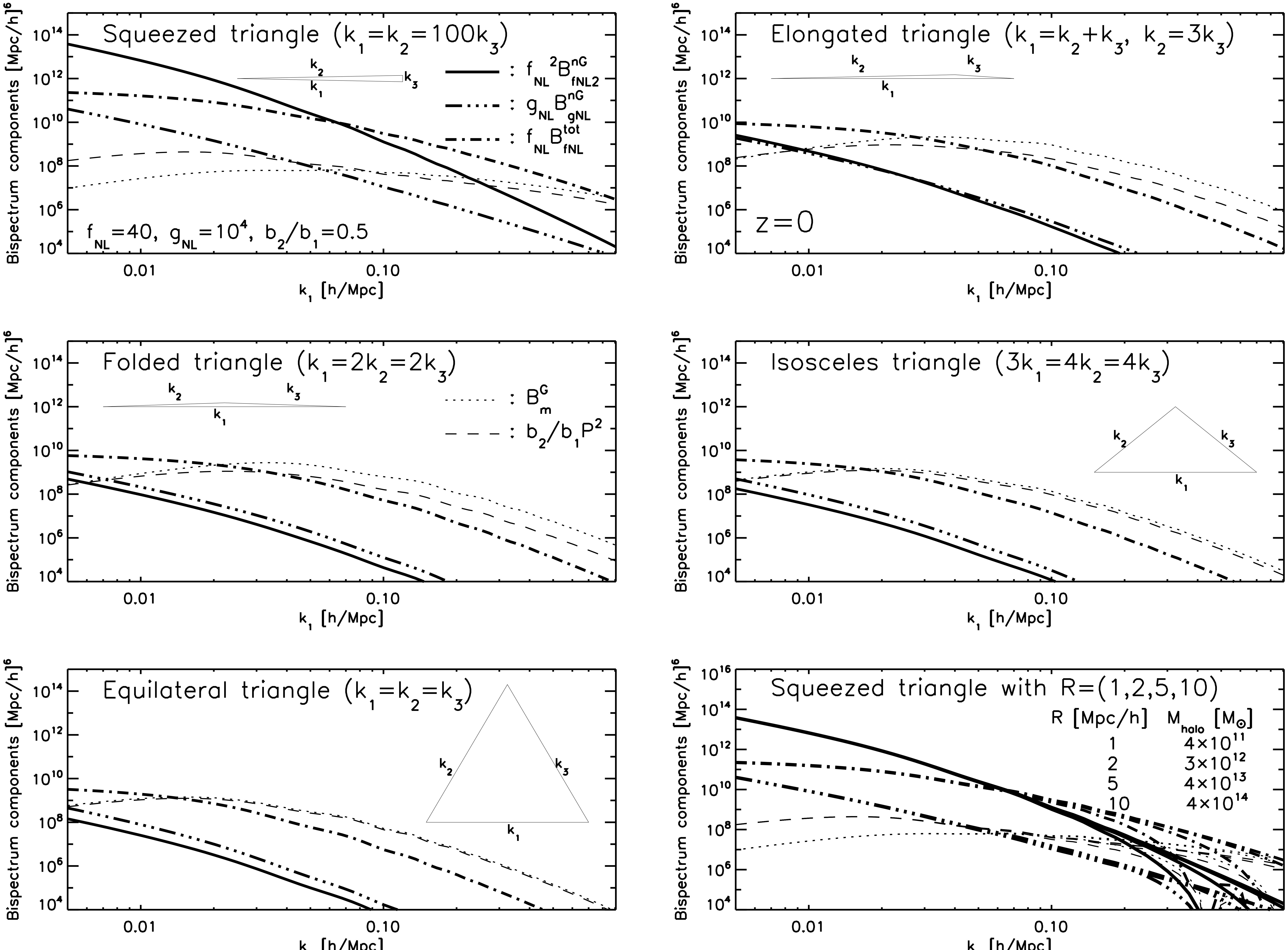

squeezed
equilateral
folded
elongated
isosceles
B^{nG}_{fnl}
k_1=0.01[h/Mpc]
k_1=0.05[h/Mpc]
B^{nG}_{gnl}
B^{nG}_{fnl2}
k_2/k_1
k_3/k_1

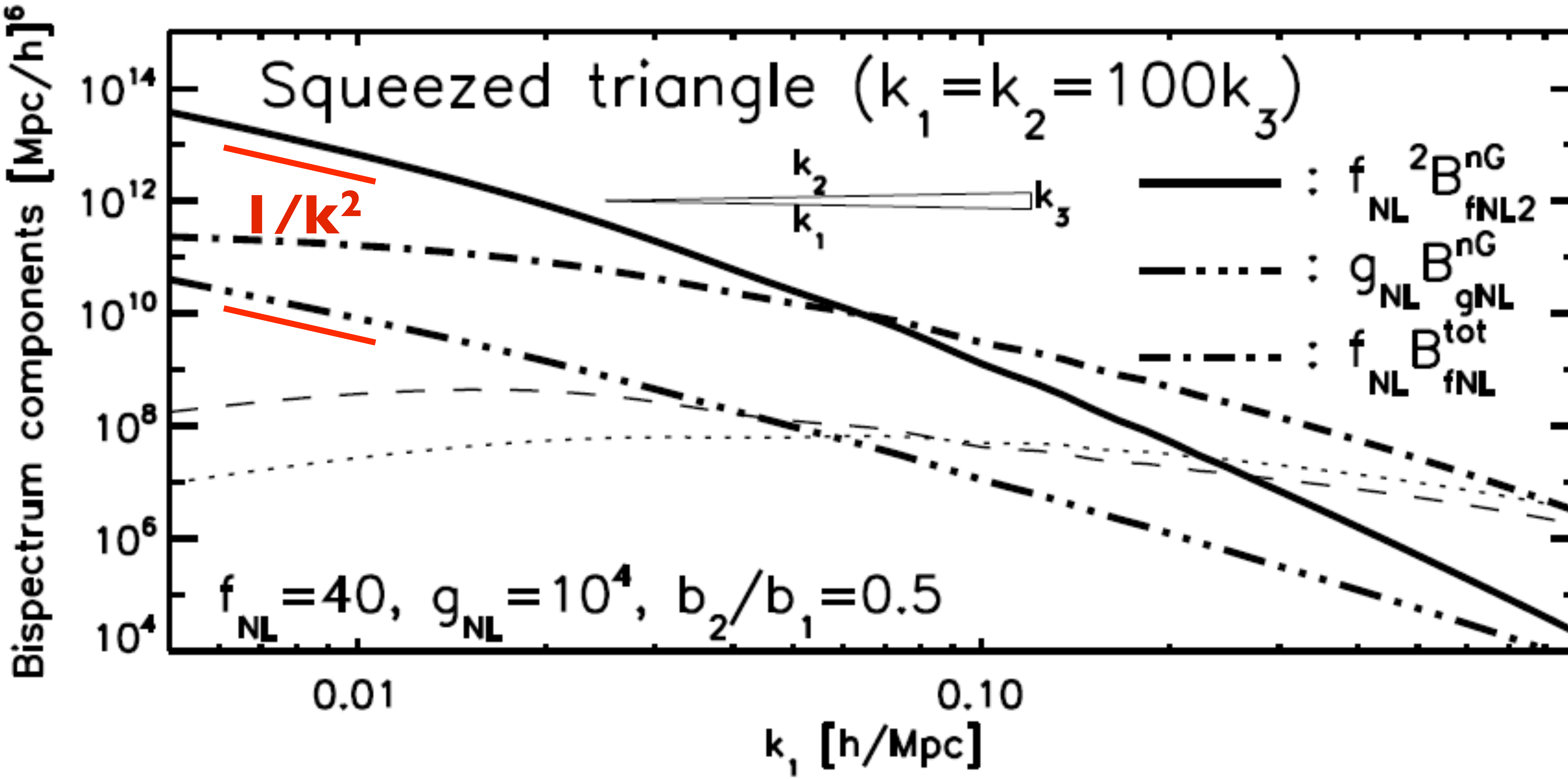
Shape Results

- The primordial non-Gaussianity terms peak at the squeezed triangle.
- f_{NL} and g_{NL} terms have the same shape dependence:
  - For $k_1=k_2=\alpha k_3$, (f_{NL} term)$\sim\alpha$ and (g_{NL} term)$\sim\alpha$
- f_{NL}^2 (τ_{NL}) is more sharply peaked at the squeezed:
  - (f_{NL}^2 term)$\sim\alpha^3$

Key Question

- Are g_{NL} or τ_{NL} terms important?


Squeezed triangle ($k_1=k_2=100k_3$)
k_1 k_2 k_3
: $f_{NL}{}^2 B^{nG}_{fNL2}$
: $g_{NL} B^{nG}_{gNL}$
: $f_{NL} B^{tot}_{fNL}$
$f_{NL}=40$, $g_{NL}=10^4$, $b_2/b_1=0.5$
Elongated triangle ($k_1=k_2+k_3$, $k_2=3k_3$)
$z=0$
Folded triangle ($k_1=2k_2=2k_3$)
: B^G_m
: $b_2/b_1 P^2$
Isosceles triangle ($3k_1=4k_2=4k_3$)
Equilateral triangle ($k_1=k_2=k_3$)
Squeezed triangle with R=(1,2,5,10)
R [Mpc/h] M_{halo} [$M_\odot$]
1 4×10^{11}
2 3×10^{12}
5 4×10^{13}
10 4×10^{14}
Bispectrum components [Mpc/h]^6
k_1 [h/Mpc]
0.01
0.10
10^4
10^6
10^8
10^{10}
10^{12}
10^{14}
10^{16}


Squeezed triangle (k1=k2=100k3)
Bispectrum components [Mpc/h]^6
k1 [h/Mpc]
1/k²
k2
k1
k3
: fNL 2 BnG fNL2
: gNL BnG gNL
: fNL Btot fNL
fNL=40, gNL=10^4, b2/b1=0.5
10^14
10^12
10^10
10^8
10^6
10^4
0.01
0.10

Summary

- **Non-Gaussianity is a *new, powerful* probe of physics of the early universe**
  - It has a best chance of ruling out all of the single-field inflation models at once.
- $f_{NL} \sim 2\sigma$ at the moment, wait for WMAP 9-year (2011) and Planck ($\geq$2012) for more σ's (if it's there!)
- To convince ourselves of detection, we need to see the acoustic oscillations, and the same signal in the bispectrum and trispectrum, of both CMB and the large-scale structure of the universe.

Now, let’s pray:

- May Planck succeed!

Now, let’s pray:

- May the signal be there!