(Personal) Summary of Solvay Workshop on "*Cosmological Frontiers in Fundamental Physics*"

Eiichiro Komatsu

Weinberg Theory Seminar, May 19, 2009



(Personal) Summary of *Belgium Beers*

Three Interesting Topics

- **Inflation & Bouncing Cosmology**
  - Mukhanov; Linde; Steinhardt; Khoury; McAllister
- **Blackhole and Cosmological Singularity Problem**
  - Horowitz; Turok; Damour; Nicolai; Blau; Trivedi; Verlinde
- **Horava-Lifshitz gravity**
  - Kiritsis
- **Other topics**: Dvali; Binetruy; de Boer; Kallosh; Sethi; Quevedo; Ross

Horava-Lifshitz Gravity

- Oh boy, is this hot...
- Horava wrote three papers on his new, *potentially renormalizable and UV complete*, theory of gravity, over the last 5 months (0812.4287; 0901.3775; 0902.3657).
- MANY papers have been written about this new theory.

Why Interesting?

- Who is not excited about a new idea about quantum gravity that could be renormalizable and could potentially be UV complete?
- For me, several results on cosmological implications are pretty interesting, too.

To mention a few...

- Solution to the horizon problem without inflation, Kiritsis & Kofinas (0904.1334)
- Scale-invariant spectrum without inflation, Mukohyama (0904.2190)
- Circular polarization of primordial gravitational waves, Takahashi & Soda (0904.0554)
- Non-singular bounce, Brandenberger (0904.2835); Calcaguni (0904.0829)

Basic Idea

- Seeking a "small" theory of quantum gravity in 3+1 dimensions, decoupled from strings.
- The basic idea comes from the condensed matter physics, in the theory of "quantum critical phenomena."

Most Important Ingredient

- Lorenz invariance dictates that space and time scale in the same way:
  - t’ = bt; x’ = bx
- In condensed matter physics, anisotropic scaling is also common:
  - t’ = b^{z}t; x’ = bx
- Horava formulates a theory of quantum gravity by having an anisotropic scaling with **z=3 in UV**.
- z “flows” from z=3 to z=1 as we go from UV to IR. assumption
  - Lorenz invariance is an emerging, accidental symmetry.

Scaling Dimensions

$$[\mathbf{x}] = -1, \qquad [t] = -z, \qquad [c] = z-1$$

- $z=1$ for GR; the speed of light is no longer dimensionless for $z\neq 1$ (so that $[ct]=[x]=-1$).

$$ds^2 = -N^2c^2dt^2 + g_{ij}(dx^i+N^idt)(dx^j+N^jdt)$$

$$\Rightarrow \quad [g_{ij}] = 0, \qquad [N_i] = z-1, \qquad [N] = 0.$$

WHY Z=3?

- The culprit of non-renormalizability of gravity is Newton's constant, which has the dimension of $[\text{mass}]^{-2}$
- **With z=3, the gravitational coupling constant becomes dimensionless!**
    - "Power-count renormalizable"

Kinetic Term of Gravity

$$S_K = \frac{2}{\kappa^2} \int dt d^D \mathbf{x} \sqrt{g} N (K_{ij} K^{ij} - \lambda K^2)$$

- ADM formalism is quite natural, as time and space do not scale in the same way anymore.

$$K_{ij} = \frac{1}{2N} (\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i) \quad \Rightarrow \quad [K] = z$$

Kinetic Term of Gravity

$$S_K = \frac{2}{\kappa^2} \int dt d^D \mathbf{x} \sqrt{g} N (K_{ij} K^{ij} - \lambda K^2)$$

$$[dt d^D \mathbf{x}] = -D - z, \qquad [K^2] = 2z$$

Since the action is dimensionless, we find

$[\kappa^2] = (z - D)/2$

For 3+1 gravity (D=3), z=3 is required to make the coupling dimensionless.

Another Coupling Constant

$$S_K = \frac{2}{\kappa^2} \int dt d^D \mathbf{x} \sqrt{g} N (K_{ij} K^{ij} - \lambda K^2)$$

- λ is dimensionless, and must be equal to 1 in IR to recover GR.

- λ should run, but beta function has not been computed yet: we don't even know whether λ=1 is a fixed point.

“Potential” Terms

- Now, consider the terms other than the kinetic term.
- Call these “potential” terms, and write down all terms (allowed by symmetry) with the dimension up to or equal to the dimension of the kinetic term, i.e., **$[K^2]=2z=6$** for z=3.

UV Terms

- In the UV limit, the most important terms have the dimension of 6. Examples include:

$$\nabla_k R_{ij} \nabla^k R^{ij}, \qquad \nabla_k R_{ij} \nabla^i R^{jk}, \qquad R \Delta R, \qquad R^{ij} \Delta R_{ij}$$

$$R^3, \qquad R^i_j R^j_k R^k_i, \qquad R R_{ij} R^{ij}$$

There are MANY such terms!

To make calculations practical,

Horava imposes an additional constraint...

“Detailed Balance”

$$S_V = \frac{\kappa^2}{8} \int dt d^D \mathbf{x} \sqrt{g} N E^{ij} \mathcal{G}_{ijk\ell} E^{k\ell}$$

$$\sqrt{g} E^{ij} = \frac{\delta W[g_{k\ell}]}{\delta g_{ij}}$$ where W is some action.

$\mathcal{G}_{ijk\ell}$ is the inverse of De Witt metric:

$$G^{ijk\ell} = \tfrac{1}{2}(g^{ik} g^{j\ell} + g^{i\ell} g^{jk}) - \lambda g^{ij} g^{k\ell}$$

In the context of condensed matter, the virtue of the detailed balance condition is in the simplification of the renormalization properties. Systems which satisfy the detailed balance condition with some D-dimensional action W typically exhibit a simpler quantum behavior than a generic theory in $D+1$ dimensions. Their renormalization can be reduced to the simpler renormalization of the associated theory described by W, followed by one additional step—the renormalization of the relative couplings between the kinetic and potential terms in S. Examples of this phenomenon include scalar fields [17] or Yang-Mills gauge theories [9,18].

Horava, 0901.3775

An Example (that doesn't work)

$$W = \frac{1}{\kappa_W^2} \int d^D \mathbf{x} \sqrt{g} (R - 2\Lambda_W).$$

- and obtains:

$$S_V = \frac{\kappa^2}{8\kappa_W^4} \int dt d^D \mathbf{x} \sqrt{g} N \left(R^{ij} - \frac{1}{2} R g^{ij} + \Lambda_W g^{ij} \right)$$

$$\times \mathcal{G}_{ijk\ell} \left(R^{k\ell} - \frac{1}{2} R g^{k\ell} + \Lambda_W g^{k\ell} \right).$$

These terms have the dimensions <=4.

So, Horava uses:

- "Cotton Tensor"

$$C^{ij} = \varepsilon^{ik\ell}\nabla_k\left(R^j_\ell - \tfrac{1}{4}R\delta^j_\ell\right).$$

- Symmetric, traceless, transverse, and conformal:

For

$$g_{ij} \to \exp\{2\Omega(\mathbf{x})\}g_{ij},$$

it transforms as

$$C^{ij} \to \exp\{-5\Omega(\mathbf{x})\}C^{ij},$$

- A product of the Cotton tensor has dimension=6.

Cotton Tensor From Action

- For the Cotton tensor to be compatible with the "detailed balance" form, it has to be derivable from an action. Such an action for the Cotton tensor exists:

Lastly, the Cotton tensor follows from a variational principle, with action

$$W = \frac{1}{w^2} \int_\Sigma \omega_3(\Gamma). \tag{36}$$

Here w^2 is a dimensionless coupling, and

$$\begin{aligned} \omega_3(\Gamma) &= \mathrm{Tr}(\Gamma \wedge d\Gamma + \tfrac{2}{3}\Gamma \wedge \Gamma \wedge \Gamma) \\ &\equiv \varepsilon^{ijk}(\Gamma^m_{i\ell}\partial_j\Gamma^\ell_{km} + \tfrac{2}{3}\Gamma^n_{i\ell}\Gamma^\ell_{jm}\Gamma^m_{kn})d^3\mathbf{x} \end{aligned} \tag{37}$$

is the gravitational Chern-Simons term

The Full Action (in UV)

$$S = \int dt d^3\mathbf{x}\sqrt{g}N\left\{\frac{2}{\kappa^2}(K_{ij}K^{ij} - \lambda K^2) - \frac{\kappa^2}{2w^4}C_{ij}C^{ij}\right\}$$

$$= \int dt d^3\mathbf{x}\sqrt{g}N\left\{\frac{2}{\kappa^2}(K_{ij}K^{ij} - \lambda K^2) - \frac{\kappa^2}{2w^4}\right.$$

$$\left.\times\left(\nabla_i R_{jk}\nabla^i R^{jk} - \nabla_i R_{jk}\nabla^j R^{ik} - \frac{1}{8}\nabla_i R\nabla^i R\right)\right\}.$$

- Recap: **[t]=−3 & [x]=−1**; detailed balance (not necessary)

Adds Lower-dimension Relevant Terms

- To have the proper IR limit (i.e., GR), we must also add lower-dimension operators. Horava wants to preserve the "detailed balance" form, so does it by adding

$$W = \frac{1}{w^2}\int \omega_3(\Gamma) \;\Rightarrow\; W = \frac{1}{w^2}\int \omega_3(\Gamma) + \mu \int d^3\mathbf{x}\sqrt{g}(R - 2\Lambda_W)$$

The Horava-Lifshitz Action

$$S = \int dt d^3\mathbf{x}\sqrt{g}N\Big\{\frac{2}{\kappa^2}(K_{ij}K^{ij} - \lambda K^2) - \frac{\kappa^2}{2w^4}C_{ij}C^{ij} + \frac{\kappa^2\mu}{2w^2}\varepsilon^{ijk}R_{i\ell}\nabla_j R^{\ell}_k - \frac{\kappa^2\mu^2}{8}R_{ij}R^{ij} + \frac{\kappa^2\mu^2}{8(1-3\lambda)}\Big(\frac{1-4\lambda}{4}R^2 + \Lambda_W R - 3\Lambda_W^2\Big)\Big\}.$$

- This has to be compatible with GR in the IR limit:

$$S_{\rm EH} = \frac{1}{16\pi G_N}\int d^4x\sqrt{g}N\{(K_{ij}K^{ij} - K^2) + R - 2\Lambda\}.$$

Emergent Parameter: *c*

- By comparing the full action and the IR action in the IR limit, Horava obtains:

In order to compare these two theories, it is natural to express our model in relativistic coordinates by rescaling t,

$$x^0 = ct, \tag{62}$$

with the emergent speed of light given by

$$c = \frac{\kappa^2 \mu}{4}\sqrt{\frac{\Lambda_W}{1-3\lambda}}. \tag{63}$$

Emergent Parameter: G_N

- By comparing the full action and the IR action in the IR limit, Horava obtains:

Newton constant is given by

$$G_N = \frac{\kappa^2}{32\pi c}$$

Emergent Parameter: Λ

- By comparing the full action and the IR action in the IR limit, Horava obtains:

the effective cosmological constant

$$\Lambda = \frac{3}{2}\Lambda_W.$$

Propagation of Gravitons

- The action for the transeverse-traceless tensor metric perturbation is:

$$S_K \approx \frac{1}{2\gamma^2}\int dtd^3\mathbf{x}\left\{\dot{\tilde{H}}_{ij}\dot{\tilde{H}}_{ij} + \frac{1-\lambda}{2(1-3\lambda)}\dot{H}^2\right\} + S_V \approx -\frac{\gamma^2}{8}\int dtd^3\mathbf{x}\tilde{H}_{ij}(\partial^2)^3\tilde{H}_{ij}$$

- The dispersion relation in the UV limit (dominated by S_V) is

$$\omega^2 = \frac{\gamma^4}{4}(\mathbf{k}^2)^3$$

Solution to the Horizon Problem?

$$\omega^2 = \frac{\gamma^4}{4}(\mathbf{k}^2)^3$$

- The speed of gravitons goes infinite as k->0.
- Trivial solution to the horizon problem...

Scalar Field in H-L Gravity

- Mukohyama (0904.2190) showed that you get a scale-invariant spectrum for a scalar field fluctuation for free!
- Scalar matter action, up to or equal to the dimension=6

$$I = \frac{1}{2}\int dt d^3\vec{x} a^3 N\sqrt{q}\left[\frac{1}{N^2}\left(\partial_t\Phi - N^i\partial_i\Phi\right)^2 + \Phi\mathcal{O}\Phi\right],$$

$$\mathcal{O} = \frac{1}{M^4}\Delta^3 - \frac{\lambda}{M^2}\Delta^2 + \Delta - m^2,$$

The scaling dimension of Φ has to be zero for z=3!

⇒ Φ is automatically scale invariant.

Generating Super-horizon Fluctuations

- In the UV limit,

the action in the UV limit is

$$I_{UV} = \frac{1}{2}\int dt d^3\vec{x} a^3 \left[(\partial_t \Phi)^2 + \frac{1}{M^4 a^6} \Phi (\delta^{ij}\partial_i \partial_j)^3 \Phi \right]$$

- The dispersion relation is given by:

$$\omega^2 \propto k_{phy}^6 = \frac{k^6}{a^6}$$ << H^2 → Freeze-out

Generating Super-horizon Fluctuations

$$\omega^2 \propto k_{phy}^6 = \frac{k^6}{a^6} \quad << H^2 \quad \Rightarrow \quad \text{Freeze-out}$$

- So, to have "initially sub-horizon fluctuations" go out of the horizon later, we need to have

$$\partial_t \left(a^6 H^2 \right) > 0$$

- This can be satisfied by a decelerating universe, $a(t) \sim t^p$, with $p>1/3$ - no need for inflation, $p>1$!!

Singularity Problem

- Not that I understood them, but some results seemed very interesting... So, I only mention their results.
- **Turok** (0905.0709) claimed that they could find one example where a bounce of **4d** universe through singularity was possible!
  - $AdS^4 \times S^7$; They studied 3d CFT dual to $AdS^4 \times S^7$
  - In 5d the particle production (back reaction) at singularity spoils bounce, but they found one solution in 4d where the particle production is suppressed by 1/N. "4d cosmology bounces whereas 5d doesn't!" (Turok)

Singularity Problem

- **Damour** and **Nicolai** gave talks on E_{10}, infinite-dimensional Lie algebra, which "nobody understands." (Nicolai)
- Nevertheless, they present some ideas: 11d supergravity gets replaced by $E_{10}/K(E_{10})$ (where $K(E_{10})$ is the maximally compact subgroup of E_{10})
  - "de-emergence of space-time"

D-brane Inflation

- **McAllister** (0808.2811) presented a systematic derivation of the general form of potential possible for the location of D3 brane in a warped throat (i.e., the form of potential for inflaton):
  - $V(\varphi)=V_0+c_1\varphi+c_2\varphi^{2/3}+c_3\varphi^2+\ldots$

Vector Inflation

- **Mukhanov** presented his “vector inflation” model (0802.2068), and showed how he killed it (0810.4304).

Motivation for Vector Inflation

- "*Can we mimic a minimally-coupled (to Ricci tensor), massive scalar field, using a vector field?*
  - To do this, one must break conformal invariance, and couple a vector field to Ricci in a specific way:

$$S = \int \mathrm{d}x^4 \sqrt{-g} \left(-\frac{R}{16\pi} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \left(m^2 + \frac{R}{6} \right) A_\mu A^\mu \right)$$

Equation of Motion

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}F^{\mu\nu}\right)+\left(m^2+\frac{R}{6}\right)A^\nu=0.$$

In the spatially flat Friedmann universe with the metric

$$\mathrm{d}s^2=\mathrm{d}t^2-a^2(t)\delta_{ik}\,\mathrm{d}x^i\,\mathrm{d}x^k,$$

these equations take the following form:

$$-\frac{1}{a^2}\Delta A_0+\left(m^2+\frac{R}{6}\right)A_0+\frac{1}{a^2}\partial_i\dot{A}_i=0,$$

→ $A_0=0$ (for $\partial_i A=0$)

$$\ddot{A}_i+\frac{\dot{a}}{a}\dot{A}_i-\frac{1}{a^2}\Delta A_i+\left(m^2+\frac{R}{6}\right)A_i-\partial_i\dot{A}_0-\frac{\dot{a}}{a}\partial_i A_0+\frac{1}{a^2}\partial_i\left(\partial_k A_k\right)=0$$

→ $\ddot{B}_i+3H\dot{B}_i+m^2B_i=0$ where $B_i\equiv A_i/a$

Exactly same as the massive scale field!

However...

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^{\mu}}\left(\sqrt{-g}F^{\mu\nu}\right)+\left(m^{2}+\frac{R}{6}\right)A^{\nu}=0.$$

In the spatially flat Friedmann universe with the metric

$$ds^{2}=dt^{2}-a^{2}(t)\delta_{ik}\,dx^{i}\,dx^{k},$$

these equations take the following form: **Can this happen?**

$$-\frac{1}{a^{2}}\Delta A_{0}+\left(m^{2}+\frac{R}{6}\right)A_{0}+\frac{1}{a^{2}}\partial_{i}\dot{A}_{i}=0,$$

→ $A_0=0$ (for $\partial_i A=0$)

$$\ddot{A}_{i}+\frac{\dot{a}}{a}\dot{A}_{i}-\frac{1}{a^{2}}\Delta A_{i}+\left(m^{2}+\frac{R}{6}\right)A_{i}-\partial_{i}\dot{A}_{0}-\frac{\dot{a}}{a}\partial_{i}A_{0}+\frac{1}{a^{2}}\partial_{i}\left(\partial_{k}A_{k}\right)=0$$

→ $\ddot{B}_{i}+3H\dot{B}_{i}+m^{2}B_{i}=0$ where $B_{i}\equiv A_{i}/a$

Exactly same as the massive scale field!

No, for a single A_μ

For a homogeneous vector field in a flat Friedmann universe we obtain

$$T^0_0 = \frac{1}{2}(\dot{B}_k^2 + m^2 B_k^2),$$

$$T^i_j = [-\frac{5}{6}(\dot{B}_k^2 - m^2 B_k^2) - \frac{2}{3} H \dot{B}_k B_k - \frac{1}{3}(\dot{H} + 3H^2) B_k^2]\delta^i_j + \dot{B}_i \dot{B}_j + H(\dot{B}_i B_j + \dot{B}_j B_i) + (\dot{H} + 3H^2 - m^2) B_i B_j,$$

- The off-diagonal term drives anisotropic expansion, and therefore the scale factor cannot be isotropic.
- This problem can be fixed by having multiple vector fields.

Multi-Vector Model

Let us first consider a triplet of mutually orthogonal vector fields $B_i^{(a)}$ [5], with the same magnitude $|B|$ each. Then from

$$\sum_i B_i^{(a)} B_i^{(b)} = |B|^2 \delta_b^a, \tag{7}$$

it follows that

$$\sum_a B_i^{(a)} B_j^{(a)} = |B|^2 \delta_j^i.$$

- Then the stress-energy tensor becomes...

Multi-Vector Model

$$T_0^0 = \varepsilon = \frac{3}{2}(\dot{B}_k^2 + m^2 B_k^2),$$

$$T_j^i = -p\delta_j^i = -\frac{3}{2}(\dot{B}_k^2 - m^2 B_k^2)\delta_j^i,$$

where B_k are the components of any field from the triplet which satisfy

$$\ddot{B}_i + 3H\dot{B}_i + m^2 B_i = 0,$$

and H is now given by

$$H^2 = 4\pi(\dot{B}_k^2 + m^2 B_k^2).$$

- Isotropic expansion!

Another Approach

- Instead of having orthogonal vector fields, have many vectors (N vectors) with random orientations:

$$\sum_{a=1}^{N} B_i^{(a)} B_j^{(a)} \simeq \frac{N}{3} B^2 \delta^i_j + \mathrm{O}(1)\sqrt{N} B^2$$

The energy–momentum tensor is

$$T^0_0 = \tfrac{1}{2}(\dot{B}_k^2 + V(B^2)),$$

$$T^i_j = [-\tfrac{5}{6}\dot{B}_k^2 + \tfrac{1}{2}V(B^2) - \tfrac{2}{3}H\dot{B}_k B_k - \tfrac{1}{3}(\dot{H} + 3H^2 - V'(B^2))B_k^2]\delta^i_j \\ + \dot{B}_i \dot{B}_j + H(\dot{B}_i B_j + \dot{B}_j B_i) + (\dot{H} + 3H^2 - V'(B^2))B_i B_j,$$

and after averaging over N fields we obtain

$$T^i_j = -p\delta^i_j \simeq \frac{N}{2}(-\dot{B}_k^2 + V(B^2))\delta^i_j.$$

However:

- In the following publication (0810.4304), he showed that this model leads to a disaster: gravitons become tachyons...

$$h''_{ik} + 2\left(\mathcal{H} + \frac{4\pi N B B'}{3 + 4\pi N B^2}\right) h'_{ik} - \triangle h_{ik} = -m_{\rm g}^2 h_{ik}.$$

$$m_{\rm g}^2 \approx 16\pi m^2 a^2 N B^2 \left(\frac{5 - 12\pi N B^2}{9 + 12\pi N B^2}\right)$$

- This is negative because $N > 1/B^2$ to have isotropic expansion...
- This problem occurs for m^2A^2 potential, but can be fixed by giving A_μ a different form of potential.

Bouncing Cosmology

- **Khoury** (0811.3633) gave a nice summary of the power spectrum of bispectrum that one can expect from a contracting universe (*assuming that going through singularity does not destroy it!*)

Spectrum of density perturbations determined by...

e.g. $$\mathcal{L} = P(X,\phi) + V(\phi)$$ $$X = -\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

- Equation of state w $$w = \frac{P - V}{2XP_{,X} - P + V}$$

And, equally important, ...

- Speed of sound c_s $$c_s^2 = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}}$$

What choice of w and $c_s(t)$ lead to nearly scale-invariant spectrum?

Standard case ($c_s = 1$)

Brandenberger, Feldman & Mukhanov

$$S = \int \mathrm{d}^3x \mathrm{d}\tau z^2 \left[\left(\frac{\mathrm{d}\zeta}{\mathrm{d}\tau} \right)^2 - (\vec{\nabla}\zeta)^2 \right]$$ where $z = a\sqrt{2\epsilon}$

(Here $\epsilon = -\frac{\dot{H}}{H^2} = \frac{3}{2}(1+w)$)

In terms of canonically-normalized $v = z\zeta$,

$$v_k'' + \left(k^2 - \frac{z''}{z} \right) v_k = 0$$

$\Longrightarrow$ Scale invariant spectrum for $z''/z = 2/\tau^2$

$\Longrightarrow$ $a = (-\tau)^{-1}$ OR $a = \tau^2$

de Sitter expansion (ε = 0)

dust contraction (ε = 3/2)

Wands ('99), Finelli & Brandenberger ('02)

For general sound speed

Garriga & Mukhanov (1999)

$$S = \int \mathrm{d}^3x \mathrm{d}\tau z^2 \left[\left(\frac{\mathrm{d}\zeta}{\mathrm{d}\tau} \right)^2 - \left(\vec{\nabla}\zeta \right)^2 \right]$$

$$\longrightarrow \quad S = \int \mathrm{d}^3x \mathrm{d}\tau \frac{z^2}{c_s^2} \left[\left(\frac{\mathrm{d}\zeta}{\mathrm{d}\tau} \right)^2 - c_s^2 \left(\vec{\nabla}\zeta \right)^2 \right]$$

To put in standard form, define sound horizon time: $\mathrm{d}y = c_s \mathrm{d}\tau$

$$S = \int \mathrm{d}^3x \mathrm{d}y q^2 \left[\left(\frac{\mathrm{d}\zeta}{\mathrm{d}y} \right)^2 - \left(\vec{\nabla}\zeta \right)^2 \right] \quad \text{where} \quad q = \frac{a\sqrt{2\epsilon}}{\sqrt{c_s}}$$

Again in terms of canonically-normalized $v = q\zeta$,

$$v_k'' + \left(k^2 - \frac{q''}{q}\right) v_k = 0$$ where $q = \frac{a\sqrt{2\epsilon}}{\sqrt{c_s}}$ and $' = \mathrm{d}/\mathrm{d}y$

$\Longrightarrow$ Exactly scale invariant spectrum for $q''/q = 2/y^2$

Expanding Branch

$$q = (-y)^{-1}$$

$$\Longrightarrow \quad \epsilon_s = -2\epsilon$$

Armendariz-Picon & Lim ('03)
Magueijo ('08)

(Includes slow-roll inf'n)

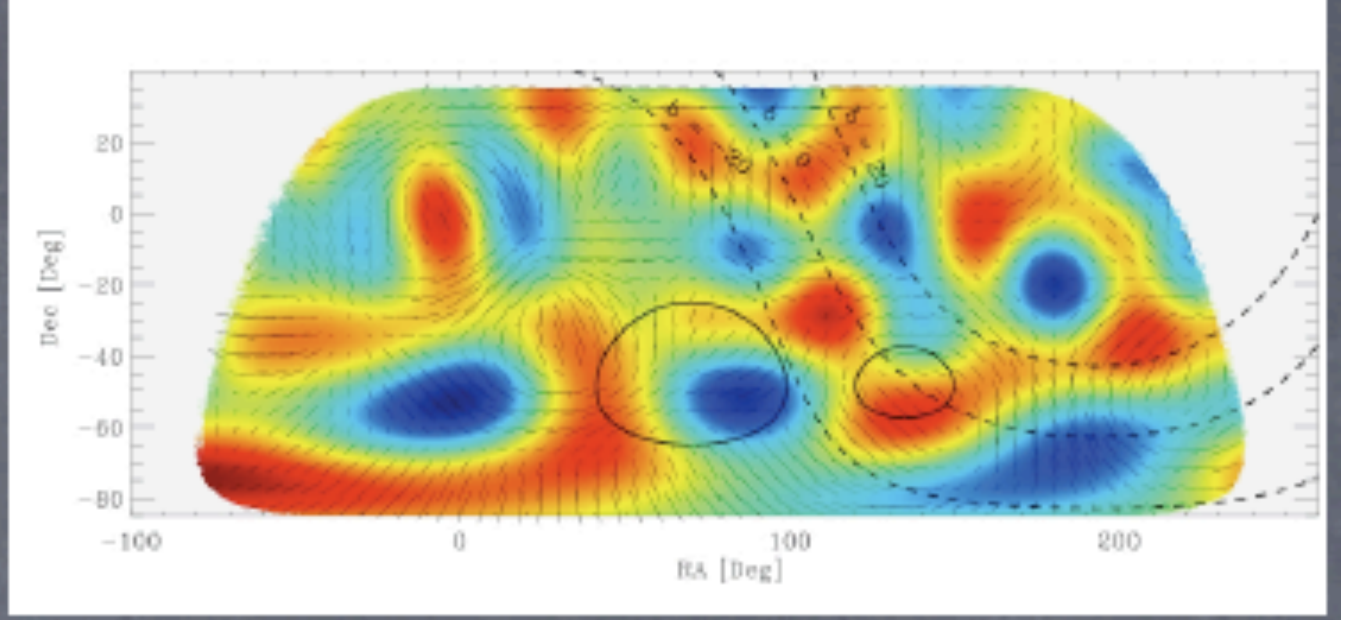
Contracting Branch

$$q = y^2$$

$$\Longrightarrow \quad \epsilon_s = \frac{2}{5}(3 - 2\epsilon)$$

$\therefore$ For any background, can compensate with $\epsilon_s = \frac{\dot{c}_s}{Hc_s}$

Gravity Waves Never Lie



Tensor modes evolve according to the actual cosmological background:

$$\frac{\mathrm{d}^2 h_k}{\mathrm{d}\tau^2} + \left(k^2 - \frac{1}{a}\frac{\mathrm{d}^2 a}{\mathrm{d}\tau^2}\right) h_k = 0$$

$\Longrightarrow$ generically far from scale invariant!

Expanding Branch

$$\Longrightarrow \quad n_{\mathrm{T}} = \frac{-2\epsilon}{1-\epsilon} < 0$$

- Amplitude peaks on large scales
- CMB $\Rightarrow$ $\epsilon < 0.3$

Contracting Branch

$$\Longrightarrow \quad n_{\mathrm{T}} = \frac{2\epsilon}{\epsilon - 1} > 0$$

- Amplitude peaks on small scales
- Unobservable on CMB scales

Stability and ζ

JK & Piazza (2008)

To assess stability of background, can consider the $k \to 0$ limit of perturbations:

$$ds^2 = a^2(\tau)\left[-\mathrm{d}\tau^2 + e^{2\zeta(x,\tau)}\mathrm{d}\vec{x}^2\right]$$

Canonically-normalized mode functions, $v = q\zeta$, satisfy standard eqn:

$$v_k'' + \left(k^2 - \frac{2}{y^2}\right) v_k = 0 \quad \Longrightarrow \quad v_k \sim 1/y$$

Expanding Branch

$$q = \frac{a\sqrt{2\epsilon}}{\sqrt{c_s}} = (-y)^{-1}$$

$$\Longrightarrow \quad \zeta = \frac{v}{q} \sim \text{const.}$$

Stable

Contracting Branch

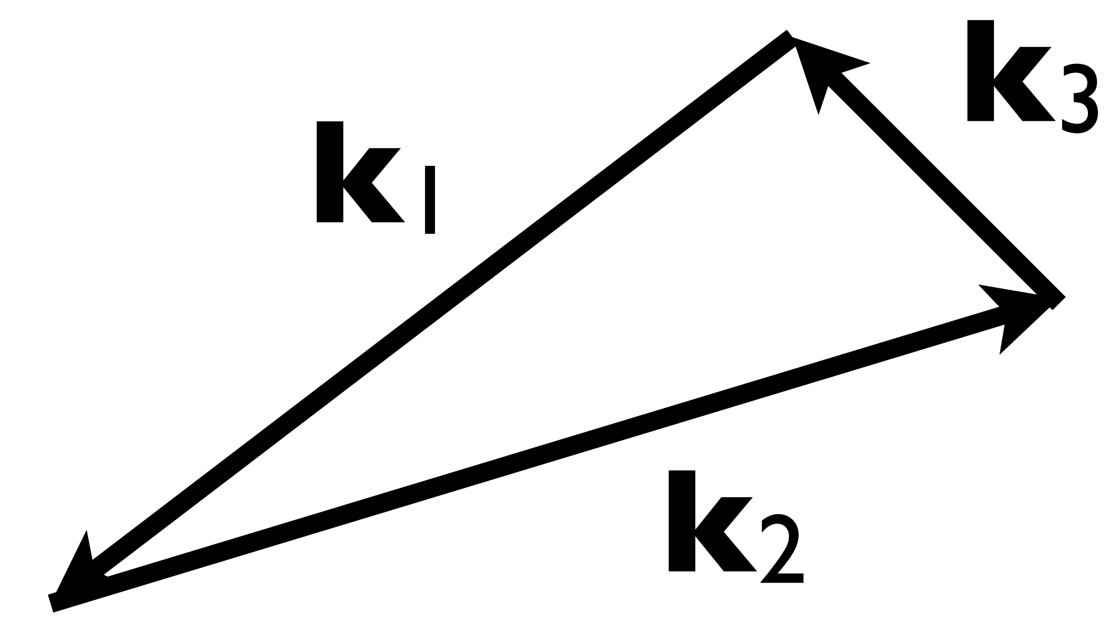
$$q = \frac{a\sqrt{2\epsilon}}{\sqrt{c_s}} = y^2$$

$$\Longrightarrow \quad \zeta = \frac{v}{q} \sim \frac{1}{y^3}$$

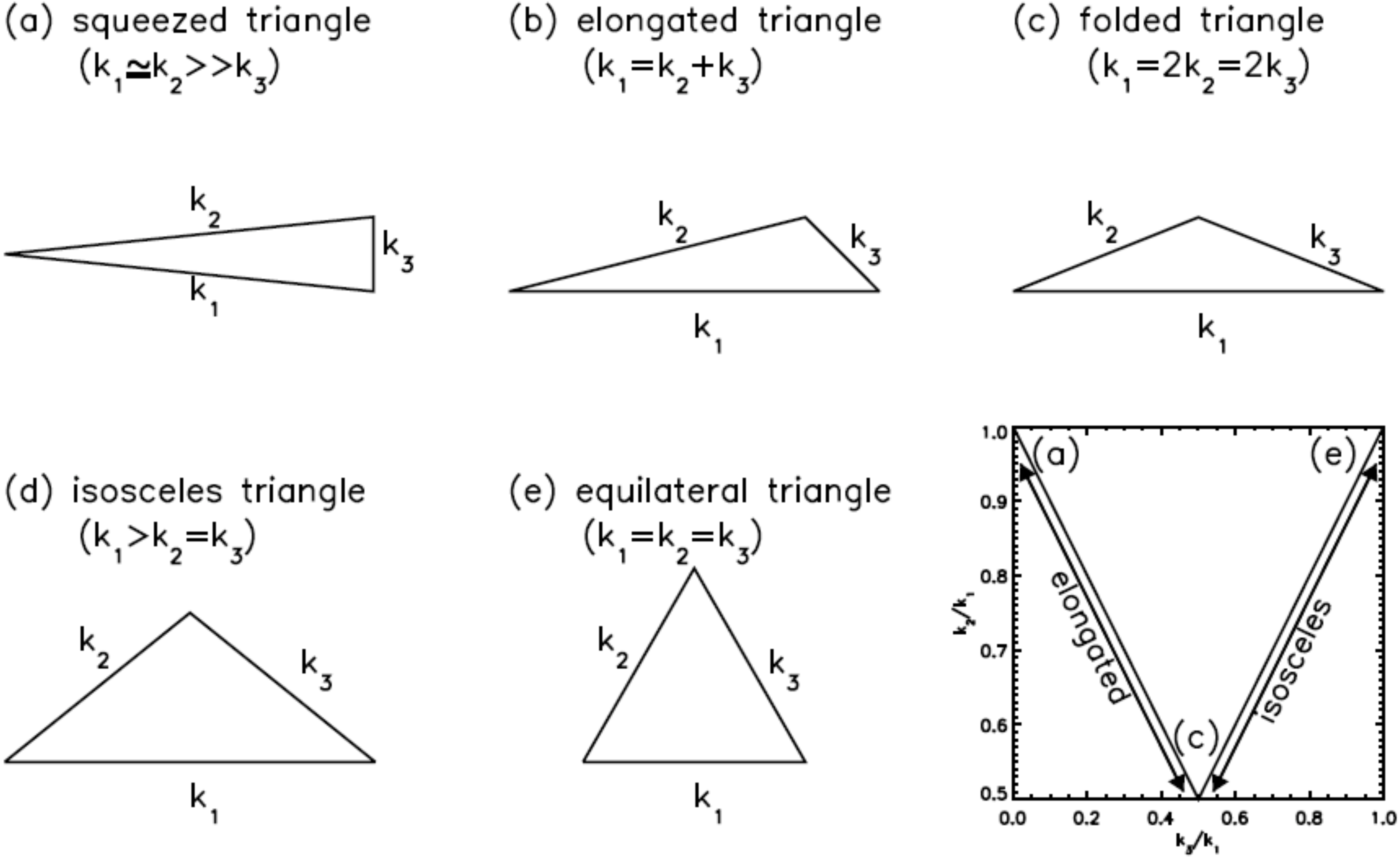
Unstable

A Few Slides From My Talk...

Bispectrum



- Three-point function!
- $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$
  $= \langle\zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle$ = (amplitude) x $(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)b(k_1,k_2,k_3)$
  model-dependent function



(a) squeezed triangle
($k_1 \simeq k_2 >> k_3$)
(b) elongated triangle
($k_1 = k_2 + k_3$)
(c) folded triangle
($k_1 = 2k_2 = 2k_3$)
(d) isosceles triangle
($k_1 > k_2 = k_3$)
(e) equilateral triangle
($k_1 = k_2 = k_3$)
k_1
k_2
k_3
elongated
isosceles
(a)
(c)
(e)
k_2/k_1
k_3/k_1
1.0
0.9
0.8
0.7
0.6
0.5
0.0
0.2
0.4
0.6
0.8
1.0

Why Study Bispectrum?

- It probes the interactions of fields - new piece of information that cannot be probed by the power spectrum
- But, above all, it provides us with a **critical test** of the simplest models of inflation: "***are primordial fluctuations Gaussian, or non-Gaussian?***"
- Bispectrum vanishes for Gaussian fluctuations.
- Detection of the bispectrum = detection of non-Gaussian fluctuations

A Non-linear Correction to Temperature Anisotropy

- The CMB temperature anisotropy, $\Delta T/T$, is given by the curvature perturbation in the matter-dominated era, Φ.
  - One large scales (the Sachs-Wolfe limit), $\Delta T/T = -\Phi/3$.
- Add a non-linear correction to Φ:

For the Schwarzschild metric, $\Phi = +GM/R$.

  - $\Phi(\mathbf{x}) = \Phi_g(\mathbf{x}) + f_{NL}[\Phi_g(\mathbf{x})]^2$ (Komatsu & Spergel 2001)
  - f_{NL} was predicted to be small (~0.01) for slow-roll models (Salopek & Bond 1990; Gangui et al. 1994)

f_{NL}: Form of B_ζ

- Φ is related to the primordial curvature perturbation, ζ, as $\Phi=(3/5)\zeta$.

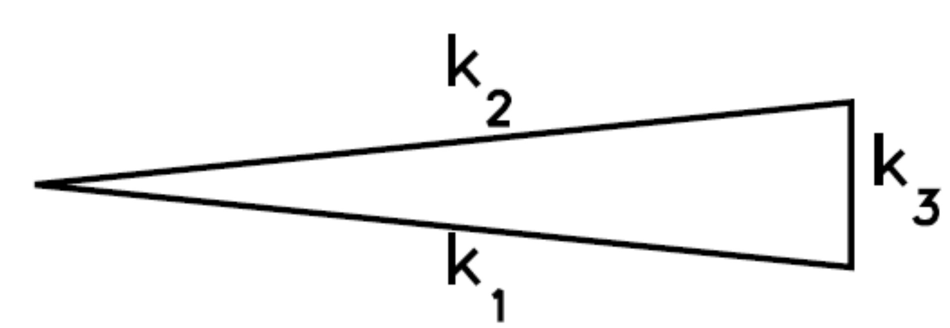
- $\zeta(\mathbf{x}) = \zeta_g(\mathbf{x}) + (3/5)f_{NL}[\zeta_g(\mathbf{x})]^2$

- $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)=(6/5)f_{NL} \times (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \times [P_\zeta(k_1)P_\zeta(k_2) + P_\zeta(k_2)P_\zeta(k_3) + P_\zeta(k_3)P_\zeta(k_1)]$

f_{NL}: Shape of Triangle

- For a scale-invariant spectrum, $P_\zeta(k)=A/k^3$,
  - $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)=(6A^2/5)f_{NL} \times (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \times [1/(k_1k_2)^3 + 1/(k_2k_3)^3 + 1/(k_3k_1)^3]$
- Let's order k_i such that $k_3 \leq k_2 \leq k_1$. For a given k_1, one finds the largest bispectrum when the smallest k, i.e., k_3, is very small.
  - $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$ peaks when $k_3 << k_2 \sim k_1$
  - Therefore, the shape of f_{NL} bispectrum is the squeezed triangle!

  (Babich et al. 2004)



B_ζ in the Squeezed Limit

- In the squeezed limit, the f_{NL} bispectrum becomes: $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \approx (12/5)f_{NL} \times (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \times P_\zeta(k_1)P_\zeta(k_3)$

Maldacena (2003); Seery & Lidsey (2005); Creminelli & Zaldarriaga (2004)

Single-field Theorem (Consistency Relation)

- For **ANY** single-field models*, the bispectrum in the squeezed limit is given by
  - $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \approx (1-n_s) \times (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \times P_\zeta(k_1)P_\zeta(k_3)$
  - Therefore, all single-field models predict $f_{NL}\approx(5/12)(1-n_s)$.
  - With the current limit $n_s=0.96$, f_{NL} is predicted to be 0.017.

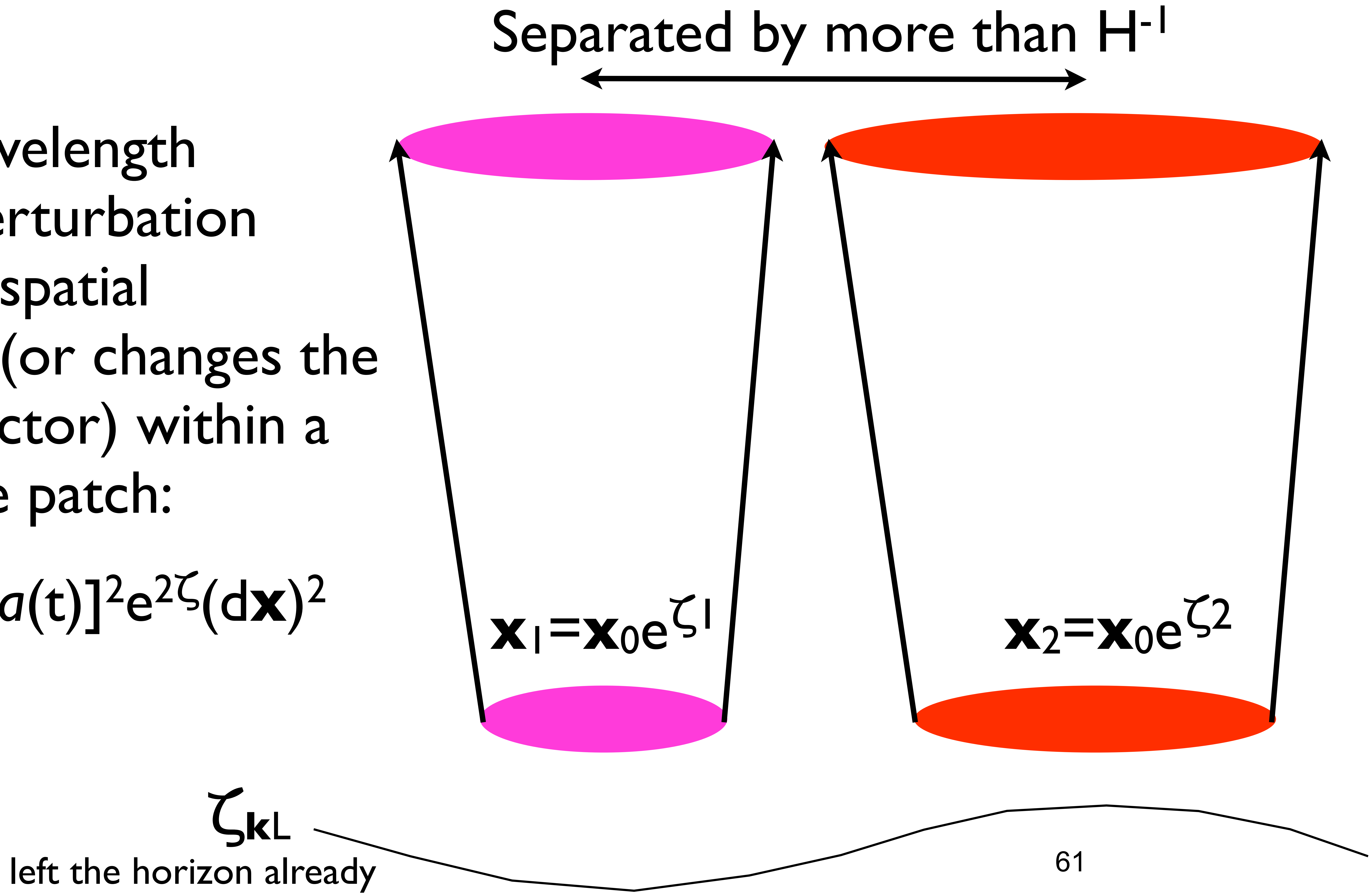
* for which the single field is solely responsible for driving inflation and generating observed fluctuations.

Understanding the Theorem

- First, the squeezed triangle correlates one very long-wavelength mode, k_L (=k_3), to two shorter wavelength modes, k_S (=$k_1 \approx k_2$):
  - $<\zeta_{\mathbf{k}1}\zeta_{\mathbf{k}2}\zeta_{\mathbf{k}3}> \approx <(\zeta_{\mathbf{k}S})^2\zeta_{\mathbf{k}L}>$
- Then, the question is: "why should $(\zeta_{\mathbf{k}S})^2$ ever care about $\zeta_{\mathbf{k}L}$?"
  - The theorem says, "it doesn't care, if $\zeta_{\mathbf{k}}$ is exactly scale invariant."

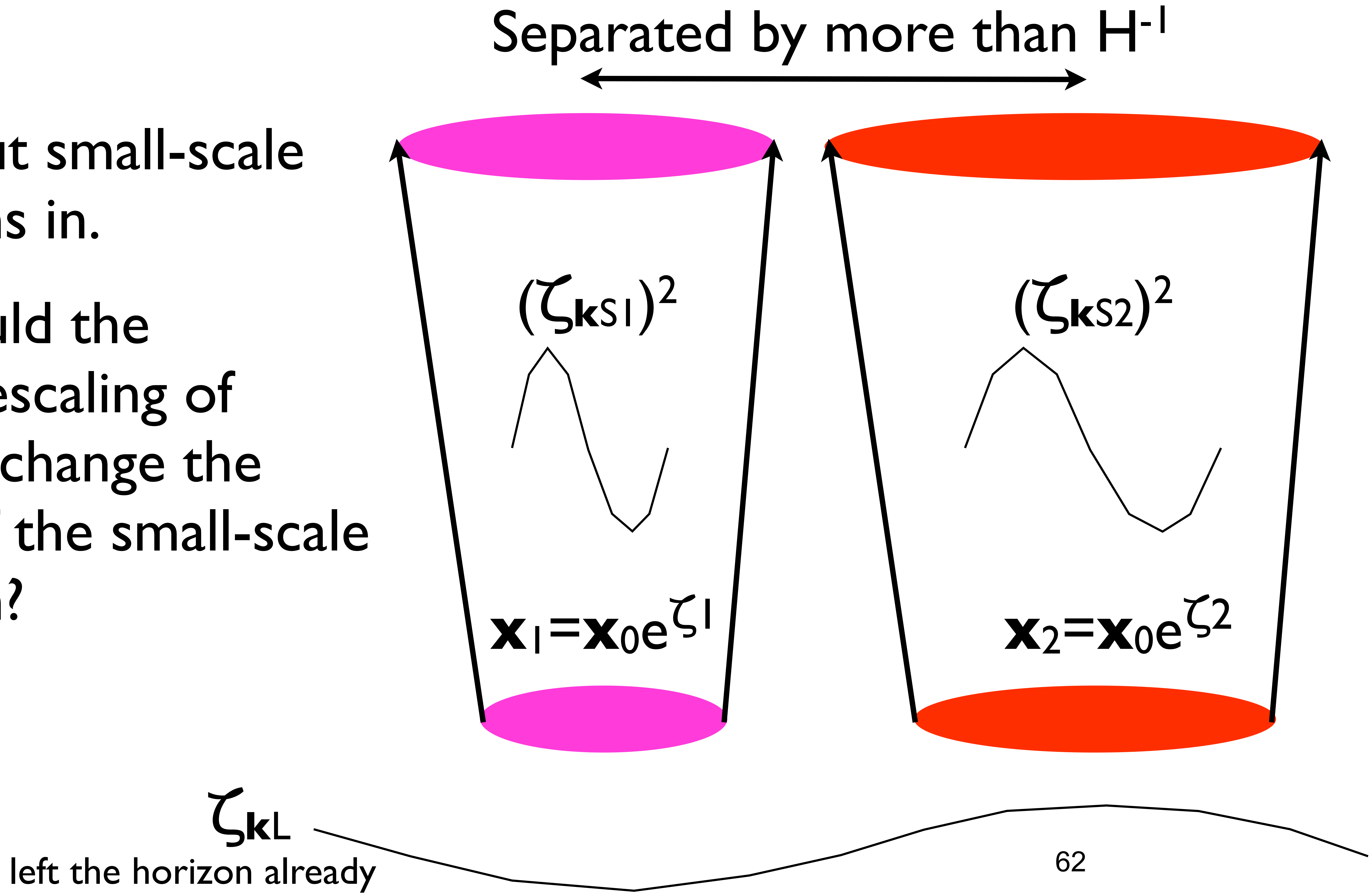
$\zeta_{\mathbf{k}L}$ rescales coordinates

- The long-wavelength curvature perturbation rescales the spatial coordinates (or changes the expansion factor) within a given Hubble patch:
  - $ds^2 = -dt^2 + [a(t)]^2 e^{2\zeta} (d\mathbf{x})^2$



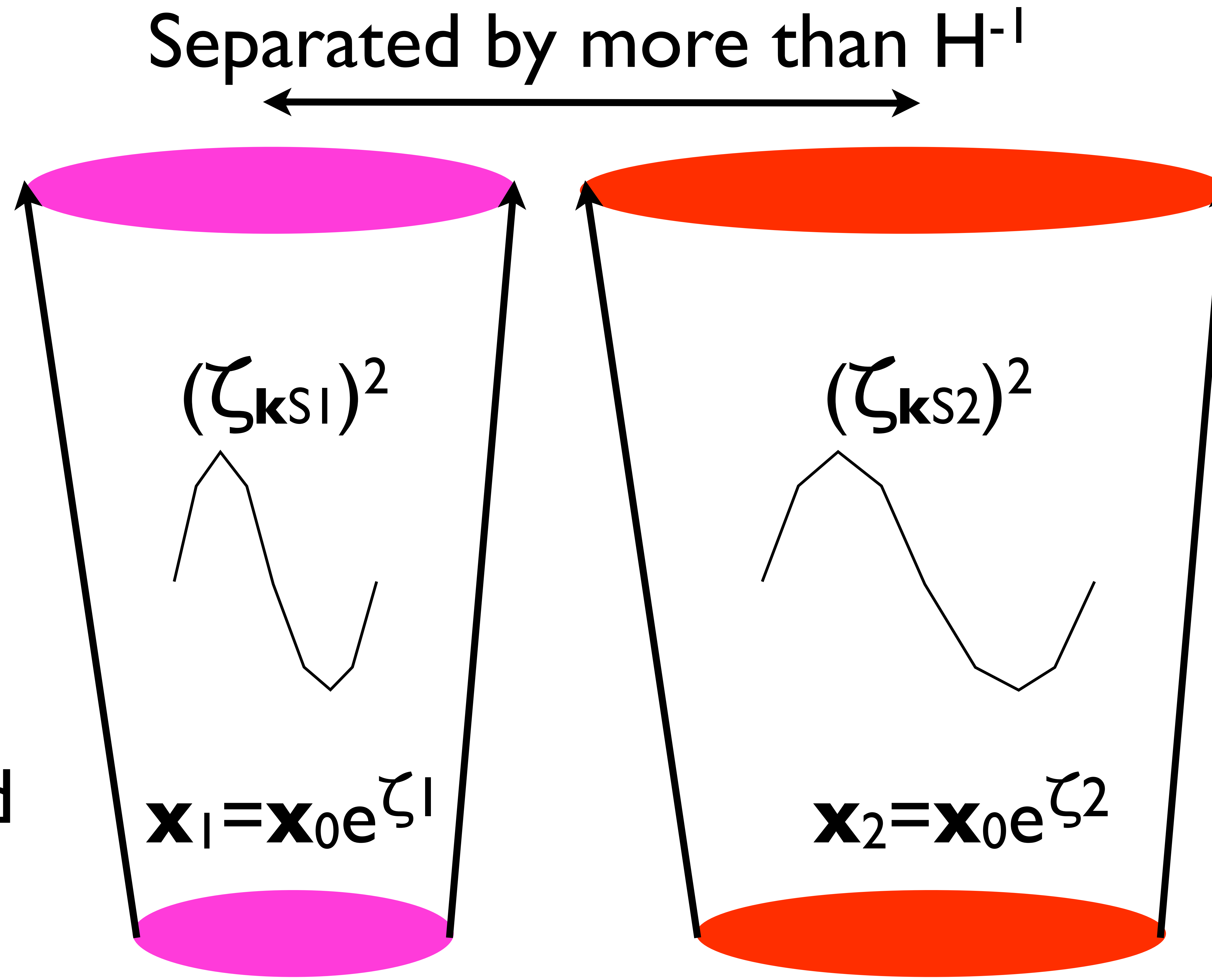
$\zeta_{\mathbf{k}L}$ rescales coordinates

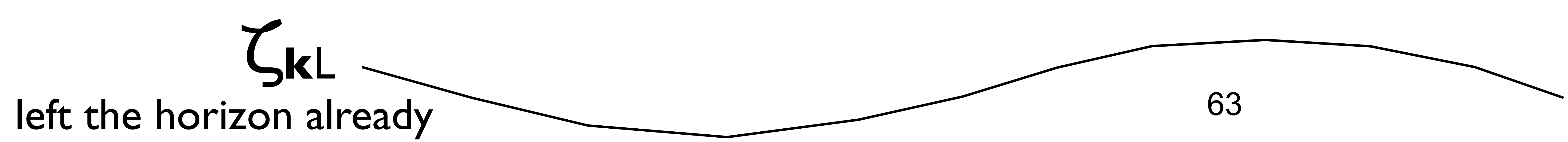
- Now, let's put small-scale perturbations in.
- Q. How would the conformal rescaling of coordinates change the amplitude of the small-scale perturbation?



$\zeta_{\mathbf{k}L}$ rescales coordinates

- Q. How would the conformal rescaling of coordinates change the amplitude of the small-scale perturbation?
- A. No change, *if $\zeta_{\mathbf{k}}$ is scale-invariant*. In this case, no correlation between $\zeta_{\mathbf{k}L}$ and $(\zeta_{\mathbf{k}S})^2$ would arise.

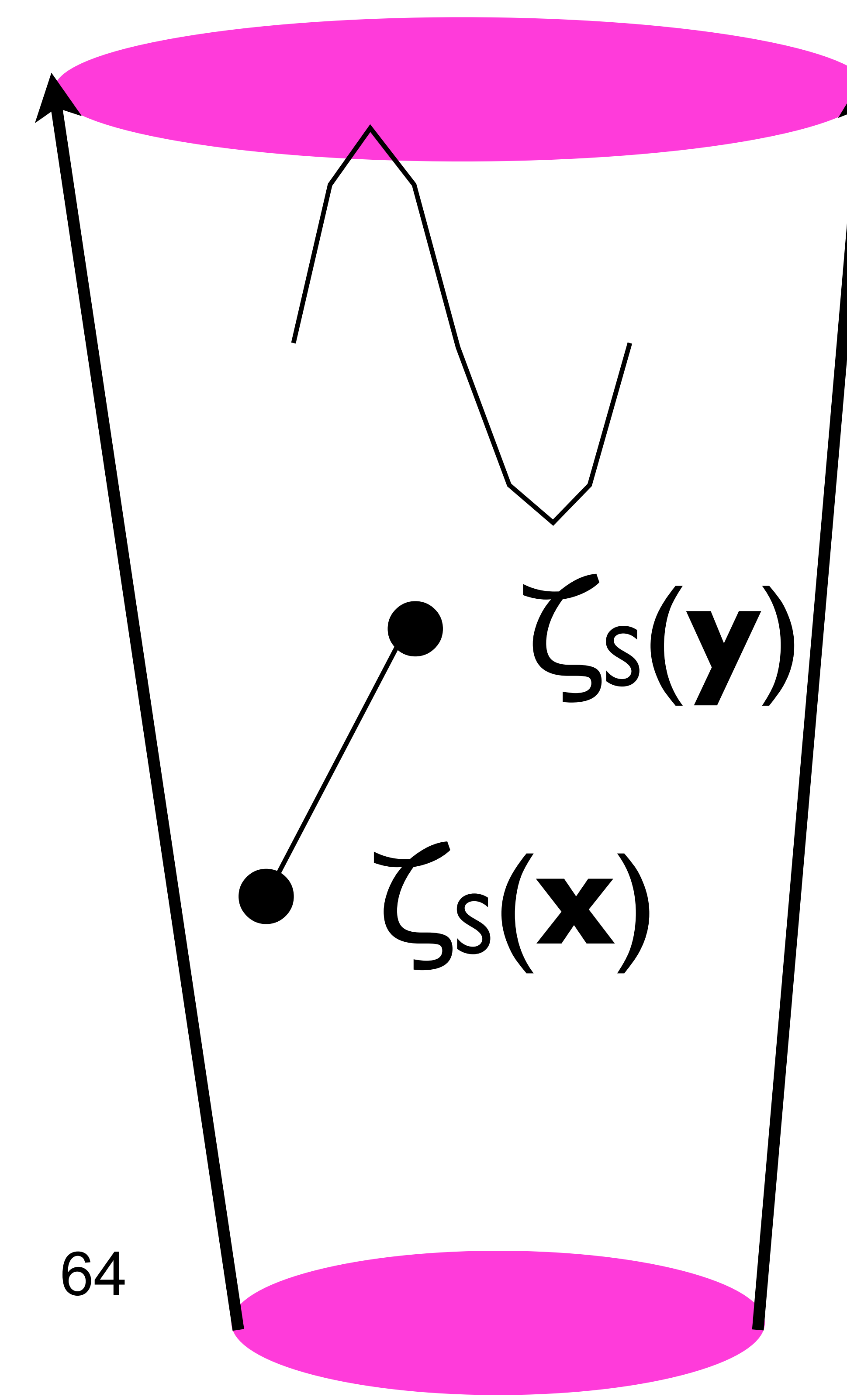




Real-space Proof

- The 2-point correlation function of short-wavelength modes, $\xi = \langle \zeta_S(\mathbf{x})\zeta_S(\mathbf{y}) \rangle$, within a given Hubble patch can be written in terms of its vacuum expectation value (in the absence of ζ_L), ξ_0, as:

  - $\xi_{\zeta L} \approx \xi_0(|\mathbf{x}-\mathbf{y}|) + \zeta_L\,[d\xi_0(|\mathbf{x}-\mathbf{y}|)/d\zeta_L]$
  - $\xi_{\zeta L} \approx \xi_0(|\mathbf{x}-\mathbf{y}|) + \zeta_L\,[d\xi_0(|\mathbf{x}-\mathbf{y}|)/d\ln|\mathbf{x}-\mathbf{y}|]$
  - $\xi_{\zeta L} \approx \xi_0(|\mathbf{x}-\mathbf{y}|) + \zeta_L\,(1-n_s)\xi_0(|\mathbf{x}-\mathbf{y}|)$



$$\text{3-pt func.} = \langle (\zeta_S)^2 \zeta_L \rangle = \langle \xi_{\zeta L} \zeta_L \rangle$$
$$= (1-n_s)\xi_0(|\mathbf{x}-\mathbf{y}|)\langle \zeta_L^2 \rangle$$

Where was "Single-field"?

- Where did we assume "single-field" in the proof?
- For this proof to work, it is crucial that there is only one dynamical degree of freedom, i.e., it is only ζ_L that modifies the amplitude of short-wavelength modes, and nothing else modifies it.
- Also, ζ must be constant outside of the horizon (otherwise anything can happen afterwards). This is also the case for single-field inflation models.

Therefore...

- A convincing detection of $f_{NL} > 1$ would rule out ***all*** of the single-field inflation models, regardless of:
  - the form of potential
  - the form of kinetic term (or sound speed)
  - the initial vacuum state
- A convincing detection of f_{NL} would be a breakthrough.

Large Non-Gaussianity from Single-field Inflation

- $S=(1/2)\int d^4x \sqrt{-g}\,[R-(\partial_\mu\varphi)^2-2V(\varphi)]$
- 2nd-order (which gives P_ζ)
  - $S_2=\int d^4x\, \varepsilon\, [a^3(\partial_t\zeta)^2-a(\partial_i\zeta)^2]$
- 3rd-order (which gives B_ζ)
  - $S_3=\int d^4x\, \boldsymbol{\varepsilon^2}\, [\ldots a^3(\partial_t\zeta)^2\zeta+\ldots a(\partial_i\zeta)^2\zeta+\ldots a^3(\partial_t\zeta)^3] + O(\varepsilon^3)$

Cubic-order interactions are suppressed by an additional factor of ε.
(Maldacena 2003)

Large Non-Gaussianity from Single-field Inflation

- $S=(1/2)\int d^4x \sqrt{-g}\,\{R-2P[(\partial_\mu\varphi)^2,\varphi]\}$ [general kinetic term]
- 2nd-order
  - $S_2=\int d^4x\,\varepsilon\,[a^3(\partial_t\zeta)^2/c_s^2-a(\partial_i\zeta)^2]$

"Speed of sound"
$c_s^2=P_{,X}/(P_{,X}+2XP_{,XX})$

- 3rd-order
  - $S_3=\int d^4x\,\boldsymbol{\varepsilon^2}\,[\ldots a^3(\partial_t\zeta)^2\zeta/\boldsymbol{c_s^2}+\ldots a(\partial_i\zeta)^2\zeta+\ldots a^3(\partial_t\zeta)^3/\boldsymbol{c_s^2}]+O(\varepsilon^3)$

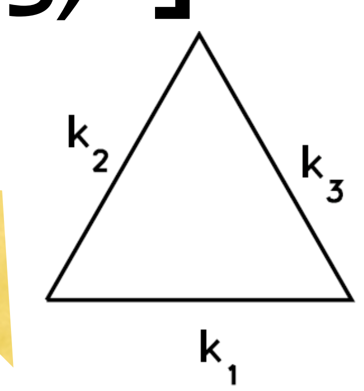
Some interactions are enhanced for $c_s^2<1$.

(Seery & Lidsey 2005; Chen et al. 2007)

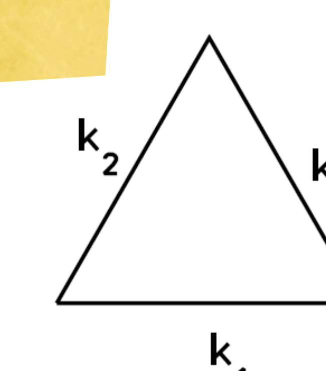
Large Non-Gaussianity from Single-field Inflation

- $S=(1/2)\int d^4x \sqrt{-g}\,\{R-2P[(\partial_\mu\varphi)^2,\varphi]\}$ [general kinetic term]
- 2nd-order
  - $S_2=\int d^4x\, \varepsilon\, [a^3(\partial_t\zeta)^2/c_s^2 - a(\partial_i\zeta)^2]$

"Speed of sound"
$c_s^2=P_{,X}/(P_{,X}+2XP_{,XX})$



- 3rd-order
  - $S_3=\int d^4x\, \boldsymbol{\varepsilon^2}\, [\ldots a^3(\partial_t\zeta)^2\zeta/\boldsymbol{c_s^2} + \ldots a(\partial_i\zeta)^2\zeta + \ldots a^3(\partial_t\zeta)^3/\boldsymbol{c_s^2}] + O(\varepsilon^3)$

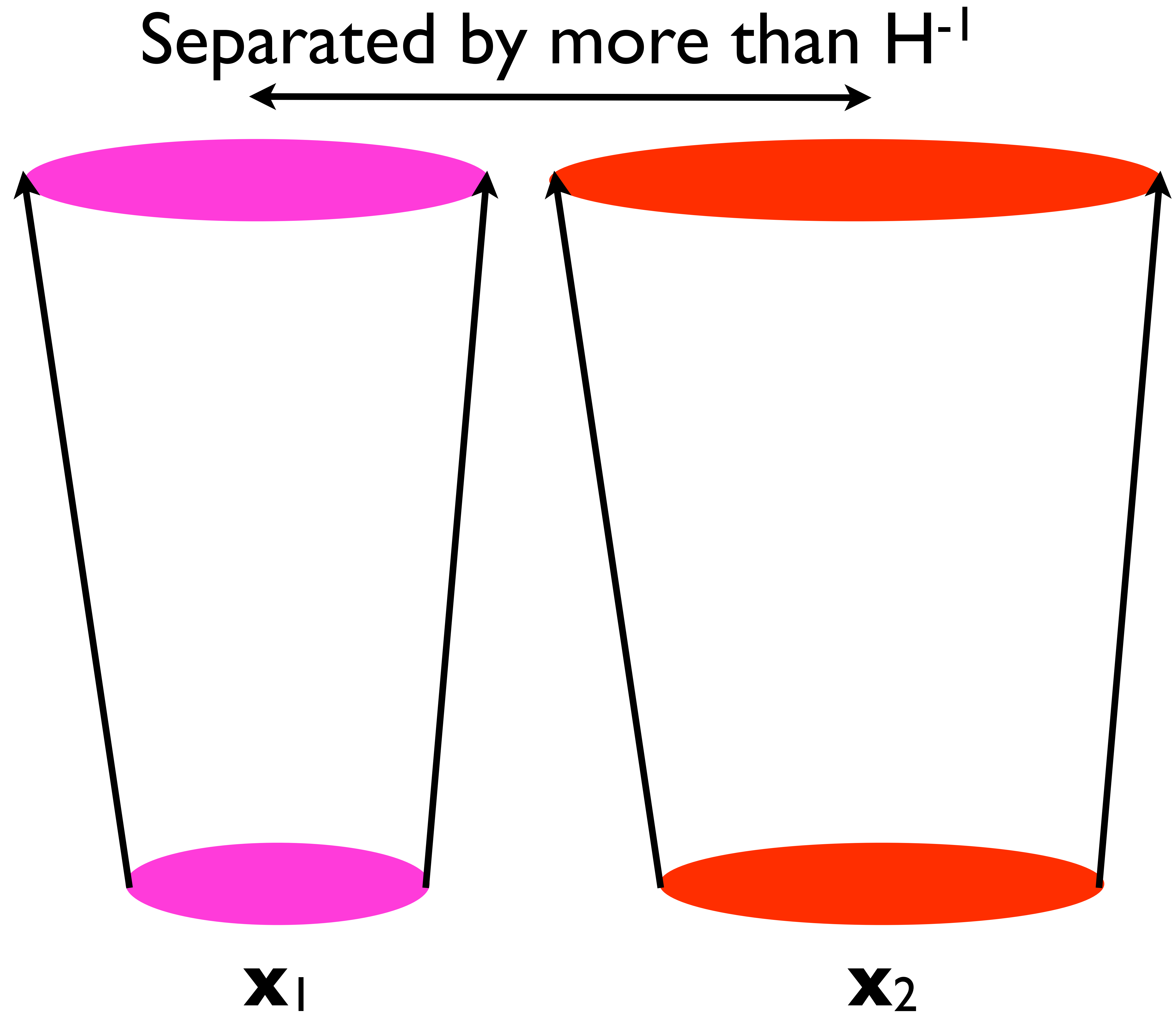


Some interactions are enhanced for $c_s^2<1$.

(Seery & Lidsey 2005; Chen et al. 2007)

Another Motivation For f_{NL}

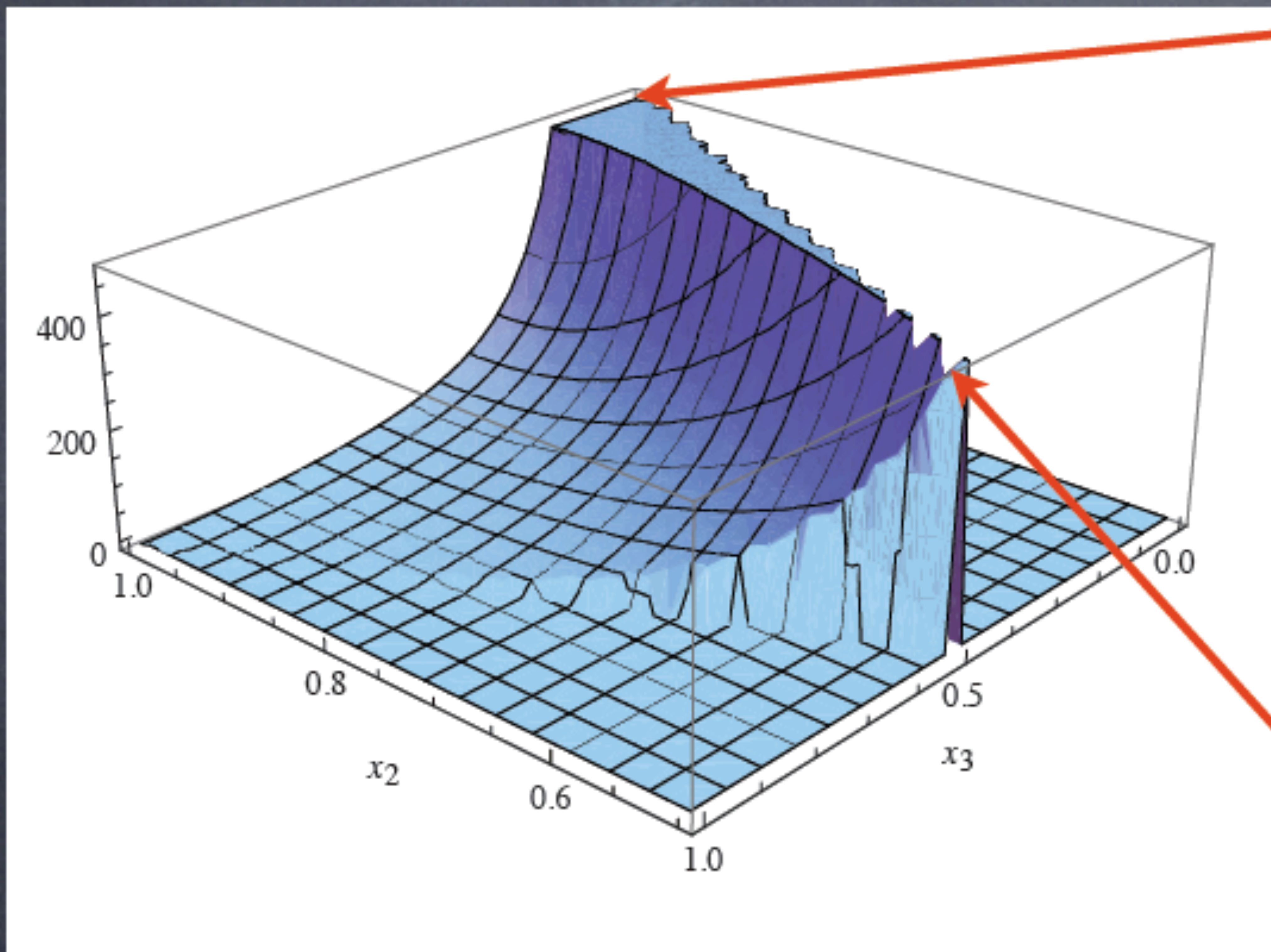
- In multi-field inflation models, $\zeta_{\boldsymbol{k}}$ can evolve outside the horizon.
- This evolution can give rise to non-Gaussianity; however, causality demands that the form of non-Gaussianity must be local!



$$\zeta(\mathbf{x})=\zeta_g(\mathbf{x})+(3/5)f_{NL}[\zeta_g(\mathbf{x})]^2+A\chi_g(\mathbf{x})+B[\chi_g(\mathbf{x})]^2+\ldots$$

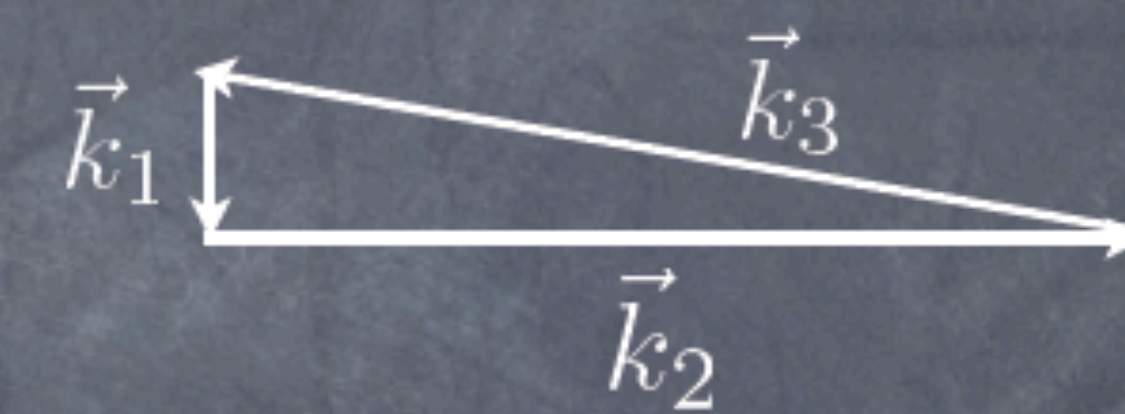
Back to Khoury's Talk

Contracting Branch

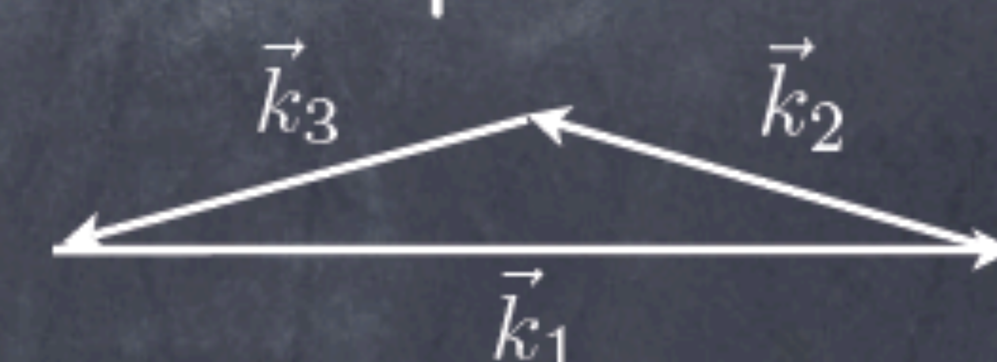


Predominantly <u>local</u> shape

(because ζ grows outside horizon)



Important "squashed" contribution



This is quite a unique "prediction" of contracting universe.