Observational Constraints on Primordial Non-Gaussianity

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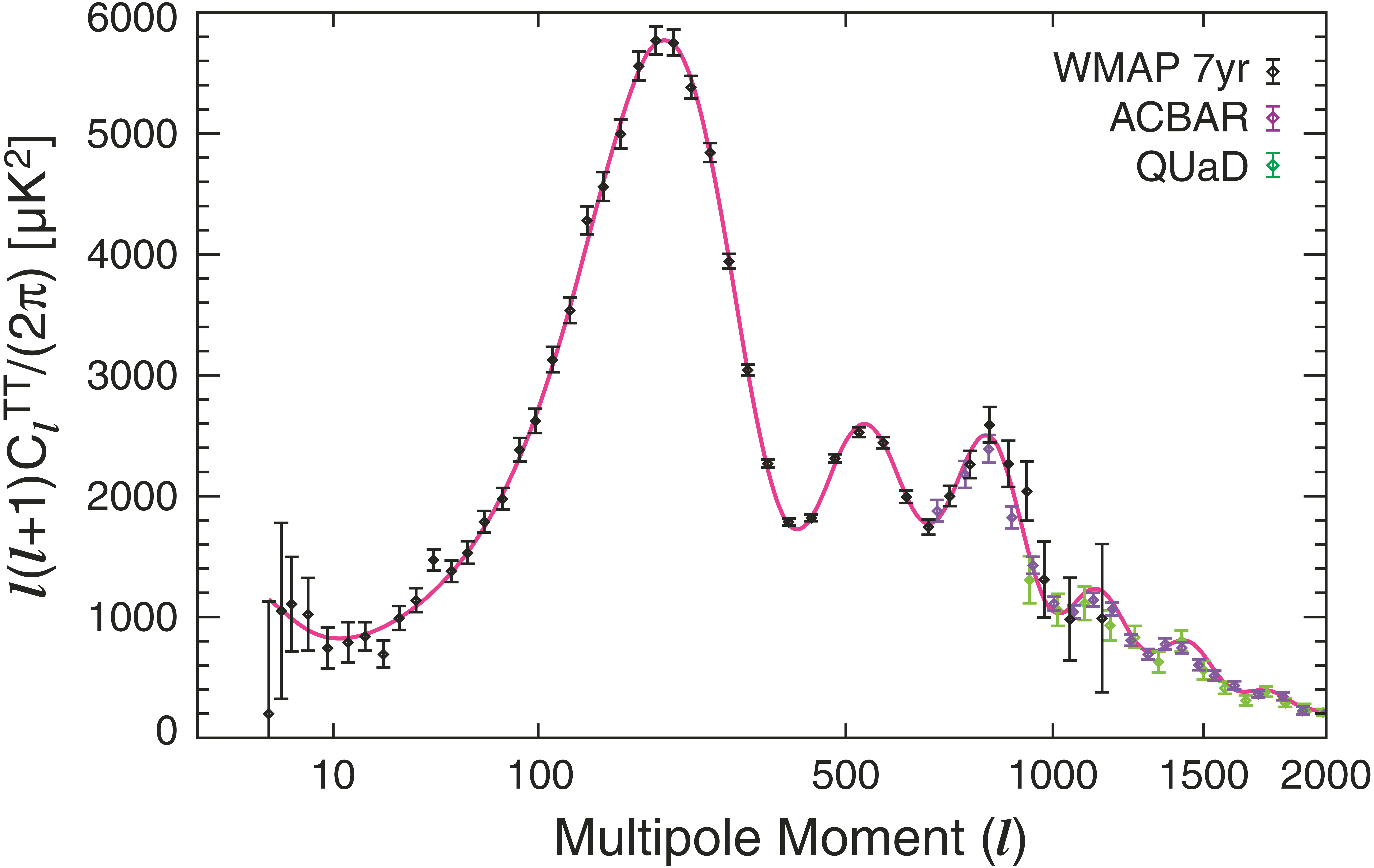
"Non-Gaussian Universe" workshop, YITP, March 26, 2010

Conclusion

- So far, no detection of primordial non-Gaussianity of *any kind* by *any method*.

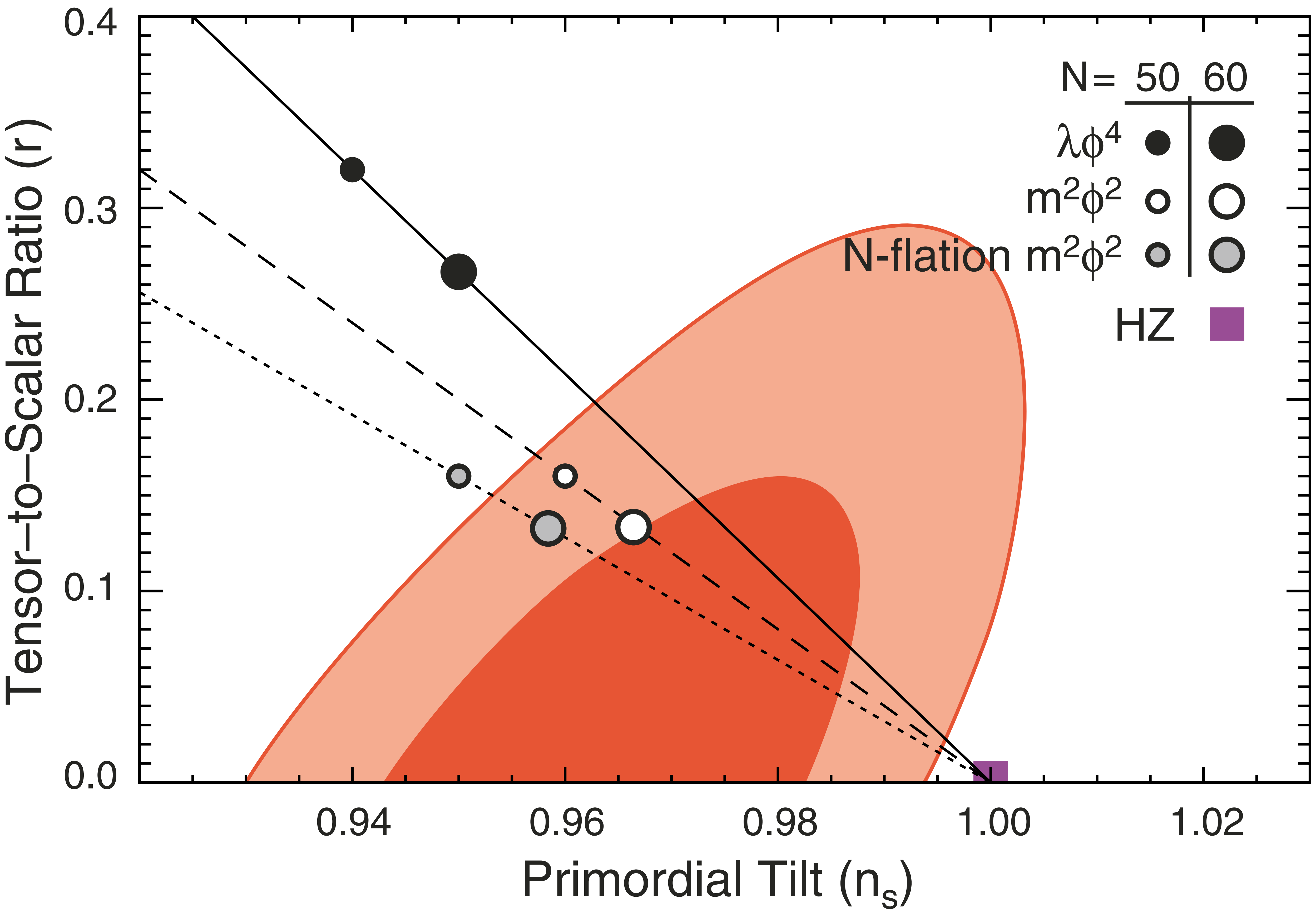
Komatsu et al. (2010)

The 7-year Power Spectrum



Komatsu et al. (2010)

Probing Inflation (Power Spectrum)



- Joint constraint on the primordial tilt, n_s, and the tensor-to-scalar ratio, r.
  - Not so different from the 5-year limit.
  - $r < 0.24$ (95%CL)

(Like many of you) I am writing a review article...

- What is the major progress that has been achieved since 2004 (when the review, Bartolo et al., was written)?

Discovery I:
Testing all single-field models

- $f_{NL}^{local} >> 1$ would rule out **all** single-field inflation models, regardless of the details of the models.
  - *Creminelli & Zaldarriaga* (2004)

Discovery II: Measuring f_{NL} optimally

- A general formula for THE optimal estimators for f_{NL} has been found and implemented.
  - The latest on this: *Smith, Senatore & Zaldarriaga* (2010)

Discovery III: Identifying the secondary

- The most serious contamination of f_{NL}^{local} due to the secondary anisotropy is the coupling between the gravitational lensing and the Integrated Sachs-Wolfe effect.
  - *Serra & Cooray* (2008) [This effect was first calculated by *Goldberg & Spergel* (1999)]

Discovery IV: Physics and Shapes

- Different shapes of the triangle configurations probe distinctly different aspects of the physics of the generation of primordial fluctuations.
  - *Creminelli* (2003); *Babich, Creminelli & Zaldarriaga* (2004); *Chen et al*. (2007)

Discovery V: Four-point Function

- The trispectrum can be as powerful as the bispectrum. Different models predict different relations (if any) between the bispectrum and trispectrum.
  - Tomo Takahashi's talk.
  - $\tau_{NL}<(25/36)f_{NL}^2$ would rule out **all** local-form non-Gaussianities. [Everyone agrees?]

Discovery VI: Large-scale Structure

- The effect of f_{NL}^{local} appears in the *power spectrum* of density peaks (corresponding to galaxies and clusters of galaxies).
  - *Dalal et al.* (2008)
  - Similarly, the effect of τ_{NL} and g_{NL} appears in the *bispectrum* of density peaks. (*Jeong & Komatsu* 2009)
    - Nishimichi's talk

Warm-up: Gaussian vs Non-Gaussian

- ΔT is Gaussian if and only if its PDF is given by

$$P(\Delta T) = \frac{1}{(2\pi)^{N_{\rm pix}/2}|\xi|^{1/2}} \exp\left[-\frac{1}{2}\sum_{ij} \Delta T_i (\xi^{-1})_{ij} \Delta T_j\right]$$

- In harmonic space:

$$P(a) = \frac{1}{(2\pi)^{N_{\rm harm}/2}|C|^{1/2}} \exp\left[-\frac{1}{2}\sum_{lm}\sum_{l'm'} a^*_{lm} (C^{-1})_{lm,l'm'} a_{l'm'}\right]$$

If isotropic, $C_{lm,l'm'} = C_l \delta_{ll'} \delta_{mm'}$, but
a violation of isotropy doesn't imply non-Gaussianity in general.

Warm-up: Gaussian vs Non-Gaussian

- For non-Gaussian fluctuations, what is the PDF?
  - We can't write it down for general cases; however, in the limit that non-Gaussianity is weak AND the bispectrum contribution is more important than the trispectrum or higher-order correlations, one can **expand** the PDF around a Gaussian:

bispectrum

$$P(a) = \left[1 - \sum_{\text{all } l_i m_j} \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle \frac{\partial}{\partial a_{l_1 m_1}} \frac{\partial}{\partial a_{l_2 m_2}} \frac{\partial}{\partial a_{l_3 m_3}}\right] \times \frac{e^{-\frac{1}{2}\sum_{lm}\sum_{l'm'} a^*_{lm} (C^{-1})_{lm,l'm'} a_{lm}}}{(2\pi)^{N_{\text{harm}}/2} |C|^{1/2}}.$$

Taylor & Watts (2001); *Babich* (2005)

Performing derivatives

$$P(a) = \frac{1}{(2\pi)^{N_{\rm harm}/2}|C|^{1/2}} \exp\left[-\frac{1}{2}\sum_{lm}\sum_{l'm'} a^*_{lm}(C^{-1})_{lm,l'm'}a_{l'm'}\right]$$

$$\times \left\{1 + \sum_{\text{all } l_i m_j} \langle a_{l_1m_1}a_{l_2m_2}a_{l_3m_3}\rangle \left[(C^{-1}a)_{l_1m_1}(C^{-1}a)_{l_2m_2}(C^{-1}a)_{l_3m_3}\right.\right.$$

$$- (C^{-1})_{l_1m_1,l_2m_2}(C^{-1}a)_{l_3m_3} - (C^{-1})_{l_3m_3,l_1m_1}(C^{-1}a)_{l_2m_2}$$

$$\left.\left. -(C^{-1})_{l_2m_2,l_3m_3}(C^{-1}a)_{l_1m_1}\right]\right\}.$$

- **This is great!** - now we have the full PDF (up to the bispectrum), which contains all the information about a_{lm} (up to the bispectrum).

Parameterization: $f_{NL}^{(i)}$

- In order to proceed, we need models for the bispectrum. Let's assume that we know the shape, but we don't know the amplitude:

$$\langle a_{l_1m_1} a_{l_2m_2} a_{l_3m_3} \rangle = \mathcal{G}_{l_1l_2l_3}^{m_1m_2m_3} \sum_i f_{\rm NL}^{(i)} b_{l_1l_2l_3}^{(i)}$$

amp. shape

Find the optimal estimators

- Now we have the PDF as a function of $f_{NL}^{(i)}$. Then, the estimator is given by maximizing the PDF:

$$d \ln P / df_{\rm NL}^{(i)} = 0$$

which gives the optimal estimator:

$$f_{\rm NL}^{(i)} = \sum_j (F^{-1})_{ij} S_j$$

covariance matrix (error matrix)

"skewness parameters" measured from the data

General formula for S_i

$$S_i \equiv \frac{1}{6}\sum_{\text{all } lm} \mathcal{G}^{m_1 m_2 m_3}_{l_1 l_2 l_3} b^{(i)}_{l_1 l_2 l_3}$$
$$\times \left[(C^{-1}a)_{l_1 m_1}(C^{-1}a)_{l_2 m_2}(C^{-1}a)_{l_3 m_3} - (C^{-1})_{l_1 m_1, l_2 m_2}(C^{-1}a)_{l_3 m_3} \right.$$
$$\left. -(C^{-1})_{l_3 m_3, l_1 m_1}(C^{-1}a)_{l_2 m_2} - (C^{-1})_{l_2 m_2, l_3 m_3}(C^{-1}a)_{l_1 m_1} \right],$$

- where "*a*" is the data (a_{lm}), and C is the covariance matrix of a_{lm} (which is a function of C_l and the noise model).
- This is the best (optimal) way of measuring the amplitudes of any (not just primordial) bispectra.
- This is what we used to measure f_{NL}^{local}, f_{NL}^{equil}, f_{NL}^{orthog}

Speaking of shapes...

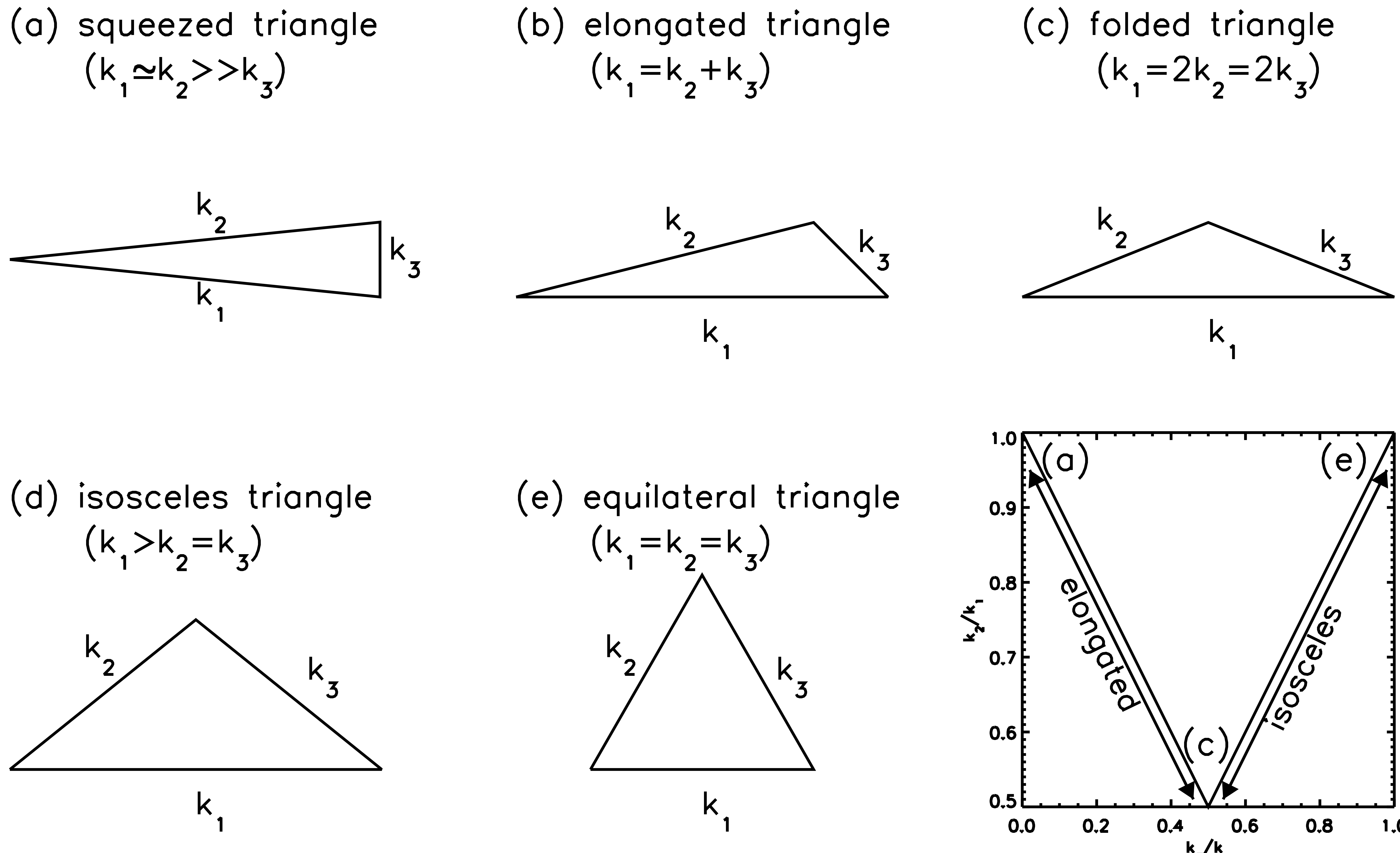
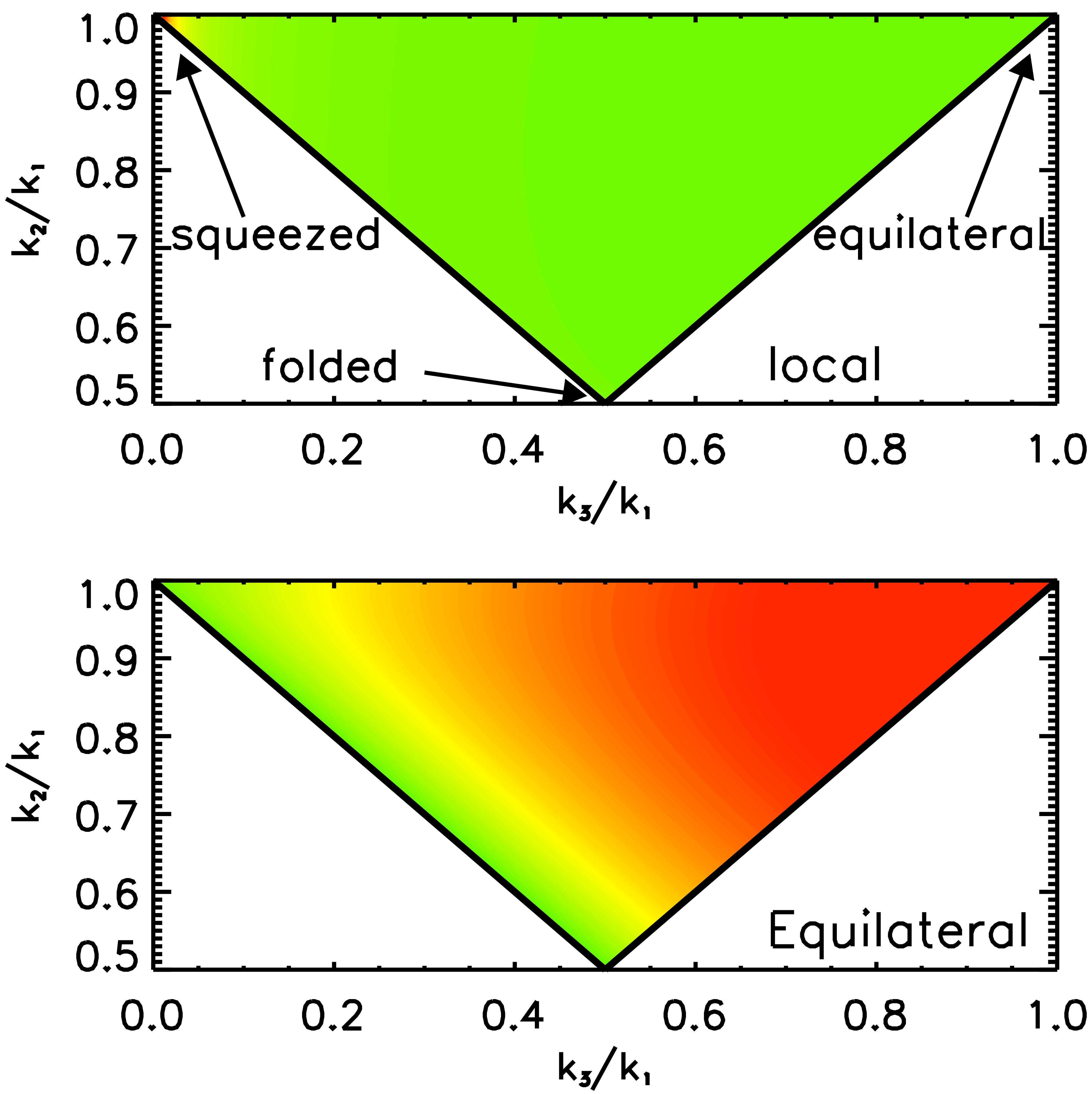
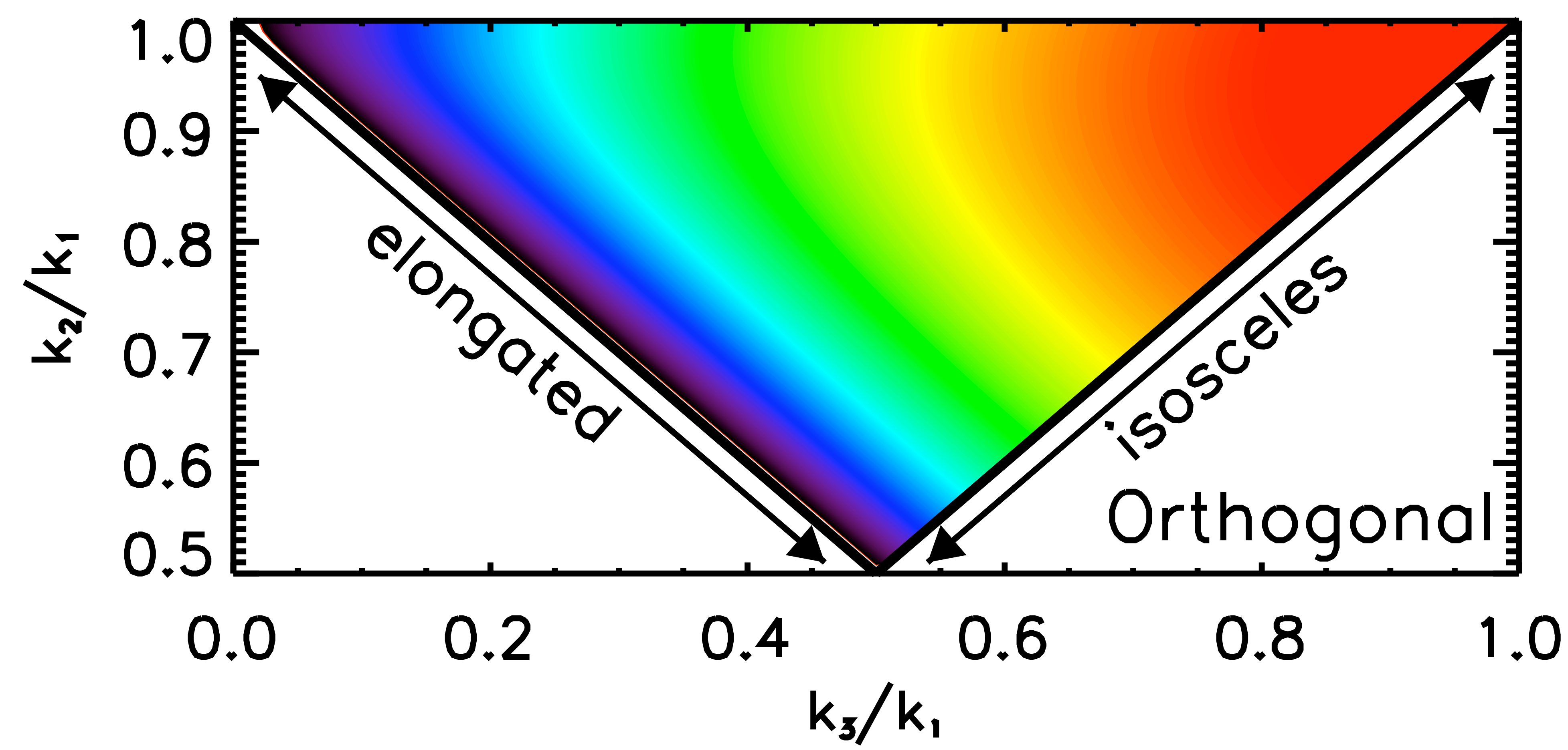
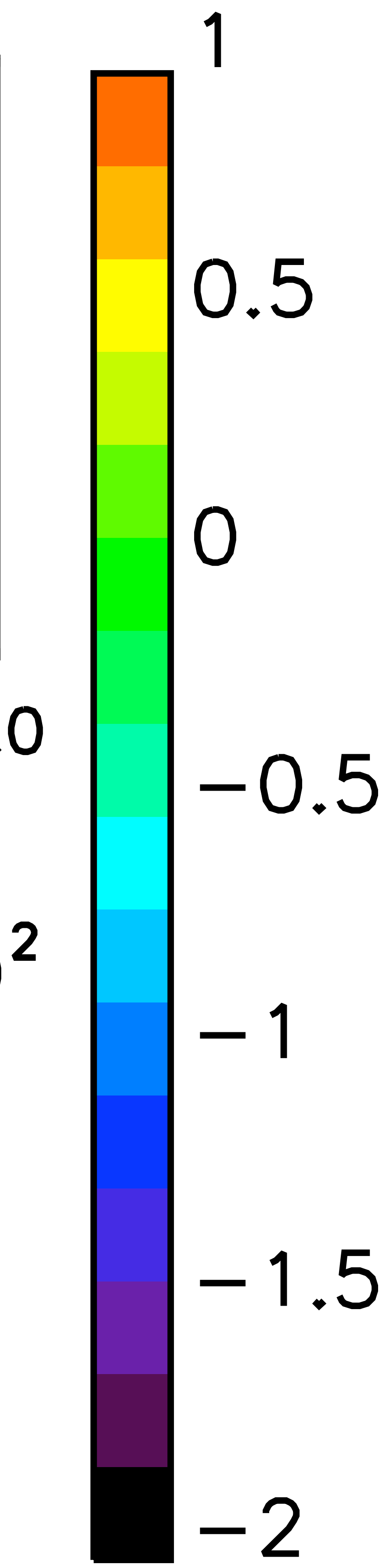


Figure made by Donghui Jeong

Local, Equil, Orthog







Shape of $F_x(1, k_2/k_1, k_3/k_1)(k_2/k_1)^2(k_3/k_1)^2$
where x=local, equilateral, orthogonal

WMAP 7-year Resutls

- No detection of 3-point functions of primordial curvature perturbations. The 95% CL limits are:
  - $-10 < f_{NL}^{local} < 74$
  - $-214 < f_{NL}^{equilateral} < 266$
  - $-410 < f_{NL}^{orthogonal} < 6$
- The WMAP data are consistent with the prediction of **simple single-inflation inflation** models:
  - $1-n_s \approx r \approx f_{NL}^{local}$, $f_{NL}^{equilateral} = 0 = f_{NL}^{orthogonal}$.

Komatsu et al. (2010)

Looking Closer

Band	Foreground[b]	f_{NL}^{local}	f_{NL}^{equil}	f_{NL}^{orthog}	b_{src}
V+W	Raw	59 ± 21	33 ± 140	-199 ± 104	N/A
V+W	Clean	42 ± 21	29 ± 140	-198 ± 104	N/A
V+W	Marg.[c]	32 ± 21	26 ± 140	-202 ± 104	-0.08 ± 0.12
V	Marg.	43 ± 24	64 ± 150	-98 ± 115	0.32 ± 0.23
W	Marg.	39 ± 24	36 ± 154	-257 ± 117	-0.13 ± 0.19

- The foreground contamination of $f_{NL}^{local} \sim 10$?
  - This could be a disaster for Planck: but we can hope that they would understand the foreground better because they have a lot more frequency channels.

Komatsu *et al.* (2010)

Looking Closer

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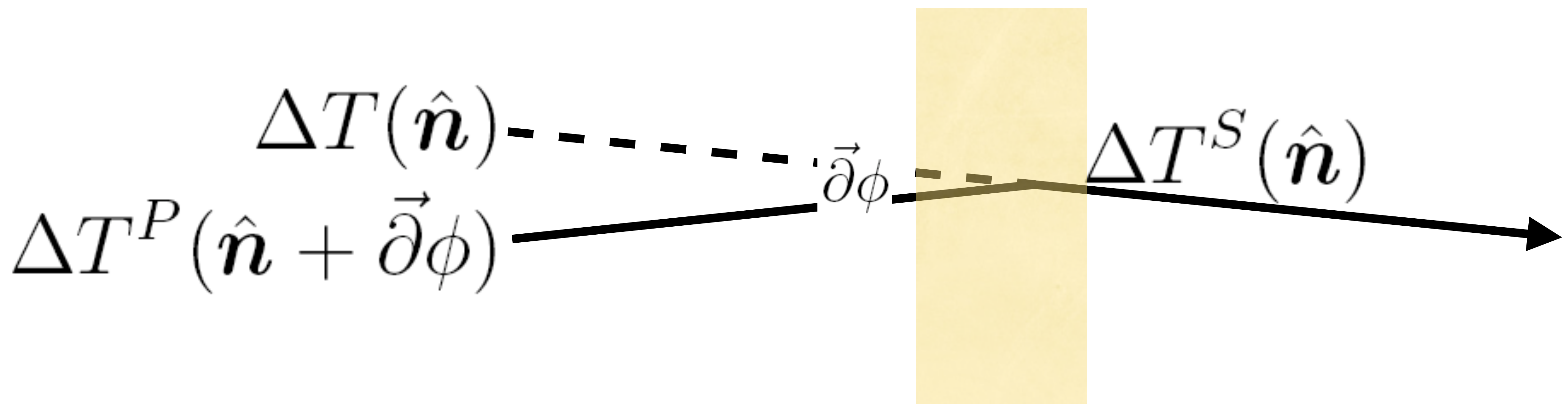
V	Marg.	43 ± 24	64 ± 150	-98 ± 115	0.32 ± 0.23
W	Marg.	39 ± 24	36 ± 154	-257 ± 117	-0.13 ± 0.19

- What is going on here?
    - No studies on the contamination of f_{NL}^{orthog} (due to point sources and secondaries) have been done.
    - Don't get too excited about f_{NL}^{orthog} just yet!

Speaking of Secondaries...

- The secondary anisotropies involving the gravitational lensing could be dangerous for f_{NL}^{local} because the lensing can couple small scales (matter clustering) to large scales (via deflection).

Lensing-secondary Coupling

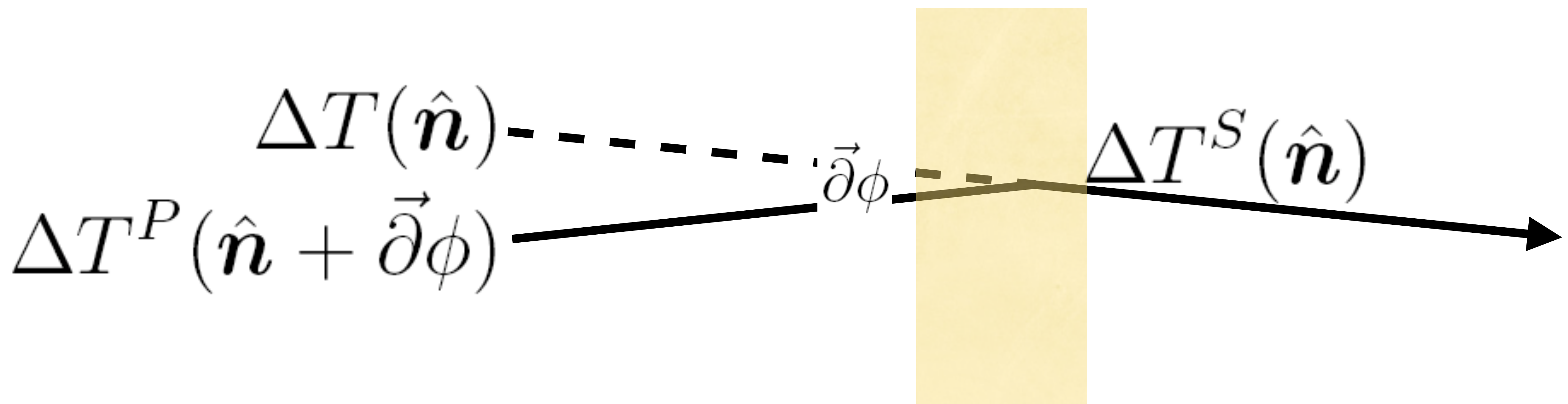


$$\begin{aligned}\Delta T(\hat{\boldsymbol{n}}) &= \Delta T^P(\hat{\boldsymbol{n}} + \vec{\partial}\phi) + \Delta T^S(\hat{\boldsymbol{n}}) \\ &\approx \Delta T^P(\hat{\boldsymbol{n}}) + [(\vec{\partial}\phi)\cdot(\vec{\partial}\Delta T^P)](\hat{\boldsymbol{n}}) + \Delta T^S(\hat{\boldsymbol{n}})\end{aligned}$$

$$b_{l_1 l_2 l_3}^{\mathrm{lens-S}} = \frac{l_1(l_1+1) - l_2(l_2+1) + l_3(l_3+1)}{2} C_{l_1}^P C_{l_3}^{\phi S} + (5\ \mathrm{perm.})$$

- This is a general formula for the lens-secondary bispectrum (*Goldberg & Spergel* 1999)

Lensing-ISW Coupling



where

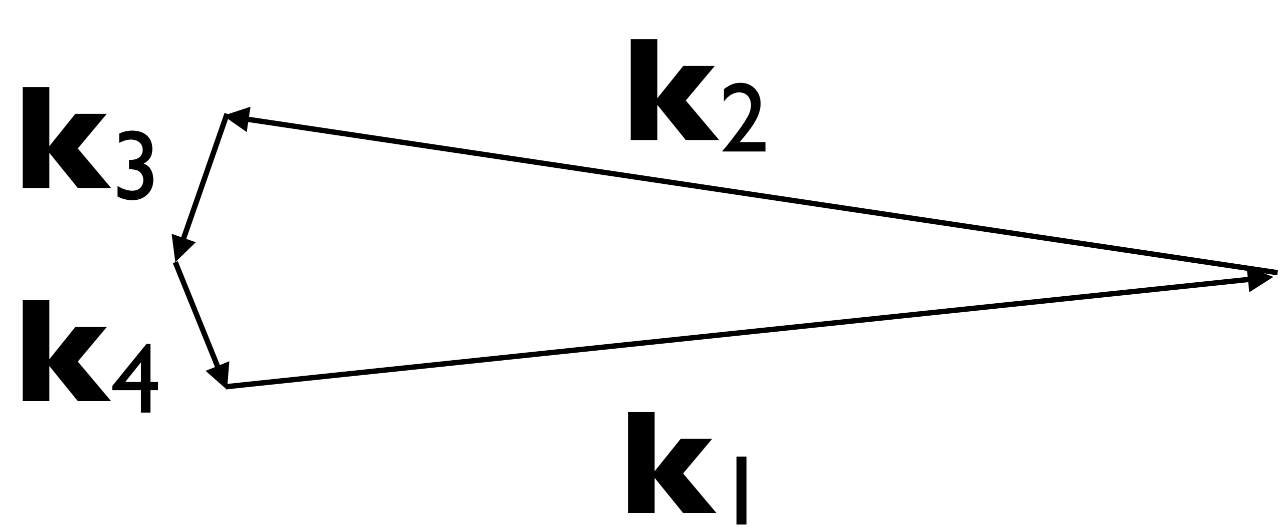
$$\frac{\Delta T^S(\hat{\boldsymbol{n}})}{T} = -2\int_0^{r_*} dr\ \frac{\partial\Phi}{\partial r}(r,\hat{\boldsymbol{n}}r)$$

$$\phi(\hat{\boldsymbol{n}}) = -2\int_0^{r_*} dr \frac{r_*-r}{rr_*}\Phi(r,\hat{\boldsymbol{n}}r)$$

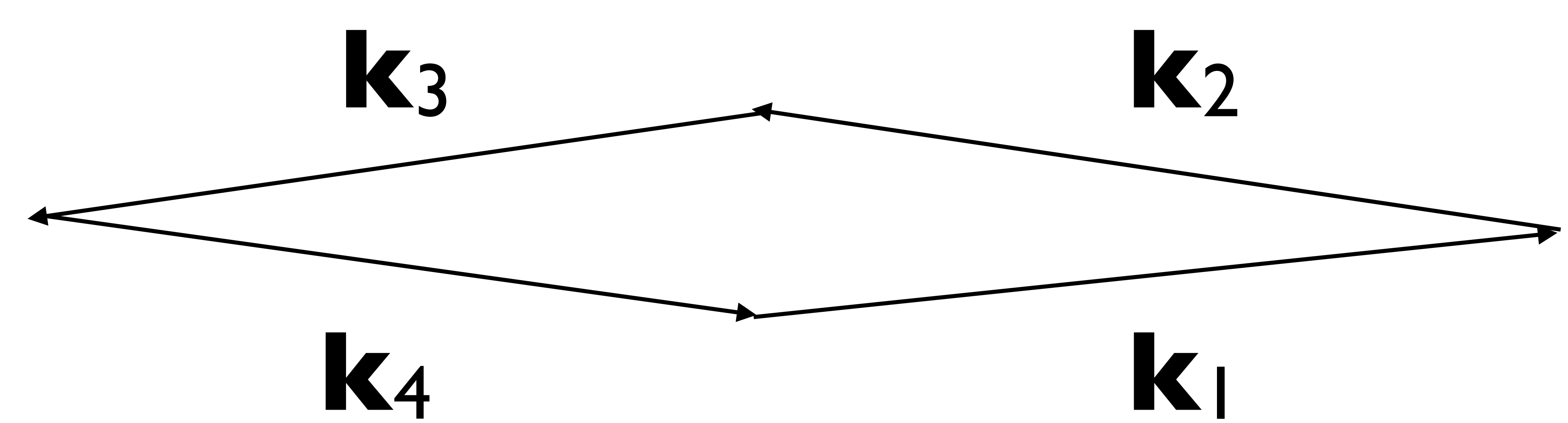
- Δf_{NL}~2.7 for WMAP, and ~10 for Planck (*Hanson et al.* 2009). **This must be included for Planck.**

Local Form Trispectrum

- For $\zeta(\mathbf{x})=\zeta_g(\mathbf{x}) + (3/5)f_{NL}[\zeta_g(\mathbf{x})]^2 + (9/25)g_{NL}[\zeta_g(\mathbf{x})]^3$, we obtain the trispectrum:
    - $T_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)=(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)$
      $\{g_{NL}[(54/25)P_\zeta(k_1)P_\zeta(k_2)P_\zeta(k_3)+\text{cyc.}]$
      $+\tau_{NL}[(18/25)P_\zeta(k_1)P_\zeta(k_2)(P_\zeta(|\mathbf{k}_1+\mathbf{k}_3|)+P_\zeta(|\mathbf{k}_1+\mathbf{k}_4|))+\text{cyc.}]\}$



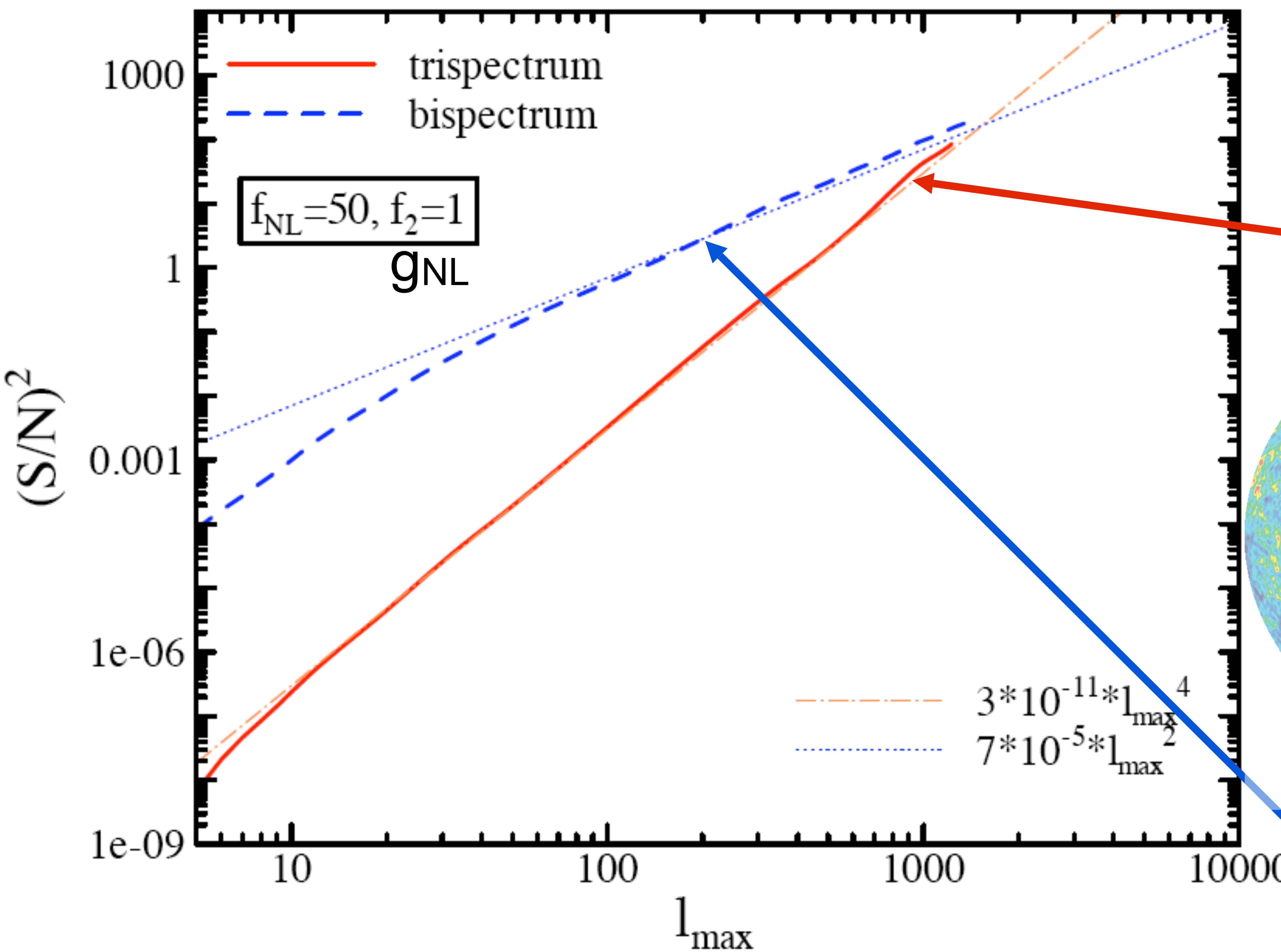
g_{NL}

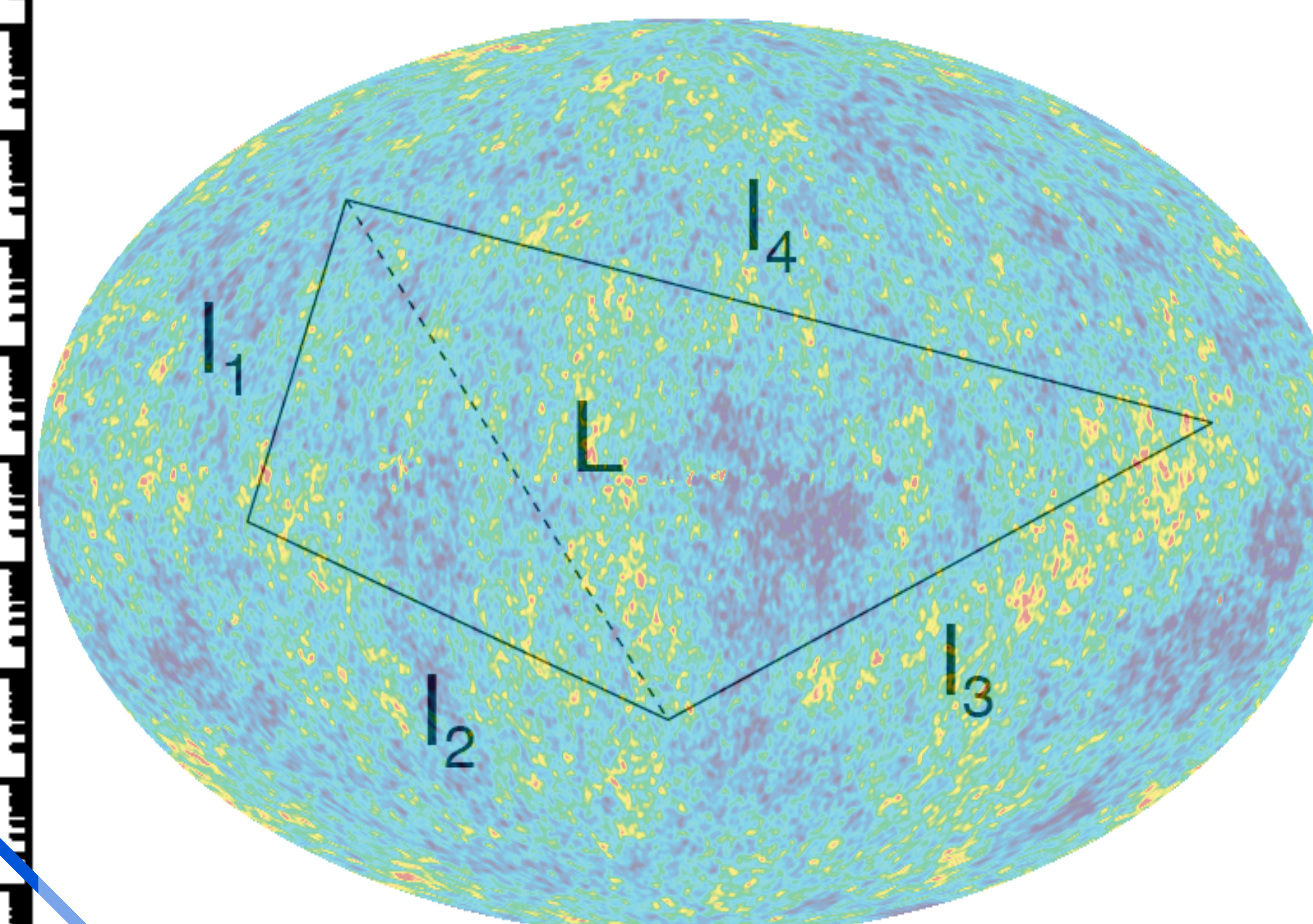


τ_{NL}

Kogo & Komatsu (2006)

Trispectrum: if f_{NL} is ~50, excellent cross-check for Planck





Trispectrum ($\sim f_{NL}^2$)

Bispectrum ($\sim f_{NL}$)

Current Limits and Forecasts

- Using the WMAP 5-year data, Smidt et al. (2010) found:
  - $-3.2\times10^5 < \tau_{NL} < 3.3\times10^5$ (95%CL)
  - The error bar is 100x larger than expected for WMAP; thus, there is a lot of room for improvement!
  - $-3.8\times10^6 < g_{NL} < 3.9\times10^6$ (95%CL)
  - The expectation is yet to be calculated, but probably this error is ~10x too large.
- Planck: $\Delta\tau_{NL} = 560$ (95%CL); Δg_{NL} = (not known; ~10^4?)

2nd-order Effects

- So far, the primordial curvature perturbations, ζ, has been propagated to ΔT using the linearized Boltzmann equation.

1st-order source

$$\Delta^{(1)\prime} + ik\mu\Delta^{(1)} - \tau'\Delta^{(1)} = S^{(1)}(k,\mu,\eta)$$

Formal solution for $\Delta = \sum a_{lm} Y_{lm}$

$$a_{lm}^{(1)} = 4\pi(-i)^l \int \frac{d^3k}{(2\pi)^3} g_l(k) Y_{lm}^* \zeta(\mathbf{k})$$

1st-order radiation transfer function

2nd-order Effects

- The second-order Boltzmann equation:

2nd-order source

$$\Delta^{(2)\prime} + ik\mu\Delta^{(2)} - \tau'\Delta^{(2)} = S^{(2)}(\mathbf{k}, \hat{\mathbf{n}}, \eta)$$

Formal solution for $\Delta = \sum a_{lm} Y_{lm}$

$$\tilde{a}^{(2)}_{lm} = \frac{4\pi}{8}(-i)^l \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3k'}{(2\pi)^3} \int d^3k'' \delta^3(\mathbf{k}' + \mathbf{k}'' - \mathbf{k}) \times \sum_{l'm'} F^{l'm'}_{lm}(\mathbf{k}', \mathbf{k}'', \mathbf{k}) Y^*_{l'm'}(\hat{\mathbf{k}}) \zeta(\mathbf{k}')\zeta(\mathbf{k}'')$$

2nd-order radiation transfer function

2nd-order Source

$$ds^2 = a^2(\eta)\left[-e^{2\Phi}d\eta^2 + 2\omega_i dx^i d\eta + (e^{-2\Psi}\delta_{ij} + \chi_{ij})dx^i dx^j\right],$$

"intrinsic 2nd order"

$$S_{lm}(\mathbf{k},\eta) = (4\Psi^{(2)\prime} - \tau'\Delta_{00}^{(2)})\delta_{l0}\delta_{m0} + 4k\Phi^{(2)}\delta_{l1}\delta_{m0} - 8\omega'_m\delta_{l1} - 4\tau' v_m^{(2)}\delta_{l1} - \frac{\tau'}{10}\Delta_{lm}^{(2)}\delta_{l2} - 4\chi'_m\delta_{l2}$$

"products of 1st order"

$$+\int \frac{d^3k_1}{(2\pi)^3}\Big\{ -2\tau'[(\delta_e^{(1)} + \Phi^{(1)})(\mathbf{k_1})\Delta_0^{(1)}(\mathbf{k_2}) + 2iv_0^{(1)}(\mathbf{k_1})\Delta_1^{(1)}(\mathbf{k_2})]\delta_{l0}\delta_{m0}$$
$$+4k\Phi^{(1)}(\mathbf{k_1})\Phi^{(1)}(\mathbf{k_2})\delta_{l1}\delta_{m0} + \tau'[(\delta_e^{(1)} + \Phi^{(1)})(\mathbf{k_1})\Delta_2^{(1)}(\mathbf{k_2}) - 2v^{(1)}(\mathbf{k_1})\Delta_1^{(1)}(\mathbf{k_2})]\delta_{l2}\delta_{m0}$$
$$+[8\Psi^{(1)\prime}(\mathbf{k_1}) + 2\tau'(\delta_e^{(1)} + \Phi^{(1)})(\mathbf{k_1})]\Delta_{l0}^{(1)}(\mathbf{k_2})\delta_{m0}\Big\}$$

+[other (1st)x(1st) terms]

- f_{NL}^{local}~0.5 from products of 1st-order terms (*Nitta, Komatsu et al.* 2009). But...

Intrinsic 2nd-order Dominates?!

"intrinsic 2nd order"

$$S_{lm}(\mathbf{k},\eta) = (4\Psi^{(2)\prime} - \tau'\Delta^{(2)}_{00})\delta_{l0}\delta_{m0} + 4k\Phi^{(2)}\delta_{l1}\delta_{m0} - \text{[stuff]}$$

- *Pitrou et al.* reported a surprising result that the terms above produce f_{NL}^{local}~5.
- Why surprising? The intrinsic 2nd-order terms are sourced by the products of 1st-order terms via the causal mechanism (i.e., gravity).
- The causal mechanism usually produces the equilateral configuration, *not* the local.

Newtonian $\Phi^{(2)}$

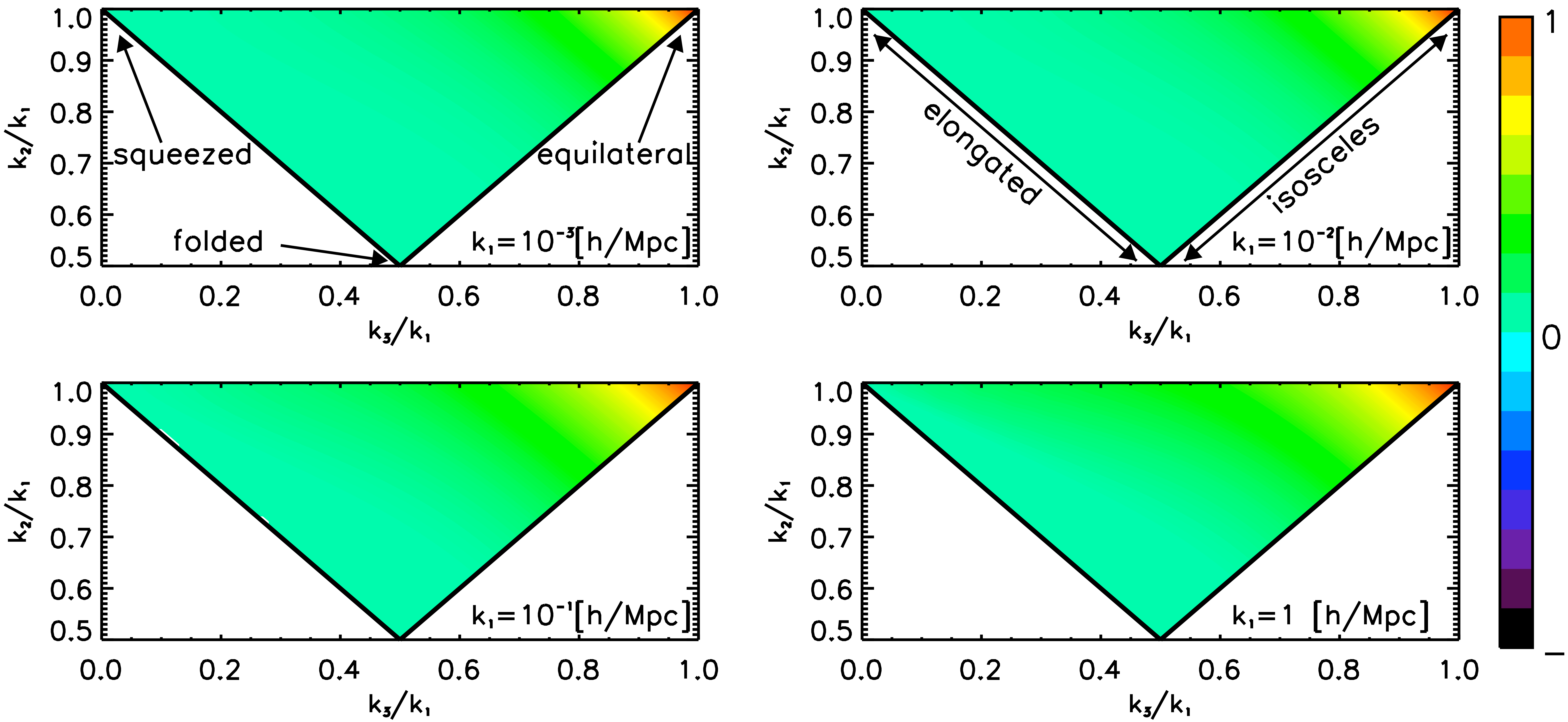
- The 2nd-order perturbation theory of Newtonian equations (continuity, Euler, Poisson) gives
  - $\delta^{(2)}(\mathbf{k})=F_2^{(s)}(\mathbf{k}_1,\mathbf{k}_2)\delta^{(1)}(\mathbf{k}_1)\delta^{(1)}(\mathbf{k}_2)$, where

$$F_2^{(s)}(\boldsymbol{k}_1, \boldsymbol{k}_2) = \frac{5}{7} + \frac{\boldsymbol{k}_1 \cdot \boldsymbol{k}_2}{2k_1k_2}\left(\frac{k_1}{k_2} + \frac{k_2}{k_1}\right) + \frac{2}{7}\left(\frac{\boldsymbol{k}_1 \cdot \boldsymbol{k}_2}{k_1k_2}\right)^2$$

This function vanishes in the squeezed limit, $\boldsymbol{k}_1 = -\boldsymbol{k}_2$

Figure made by Donghui Jeong

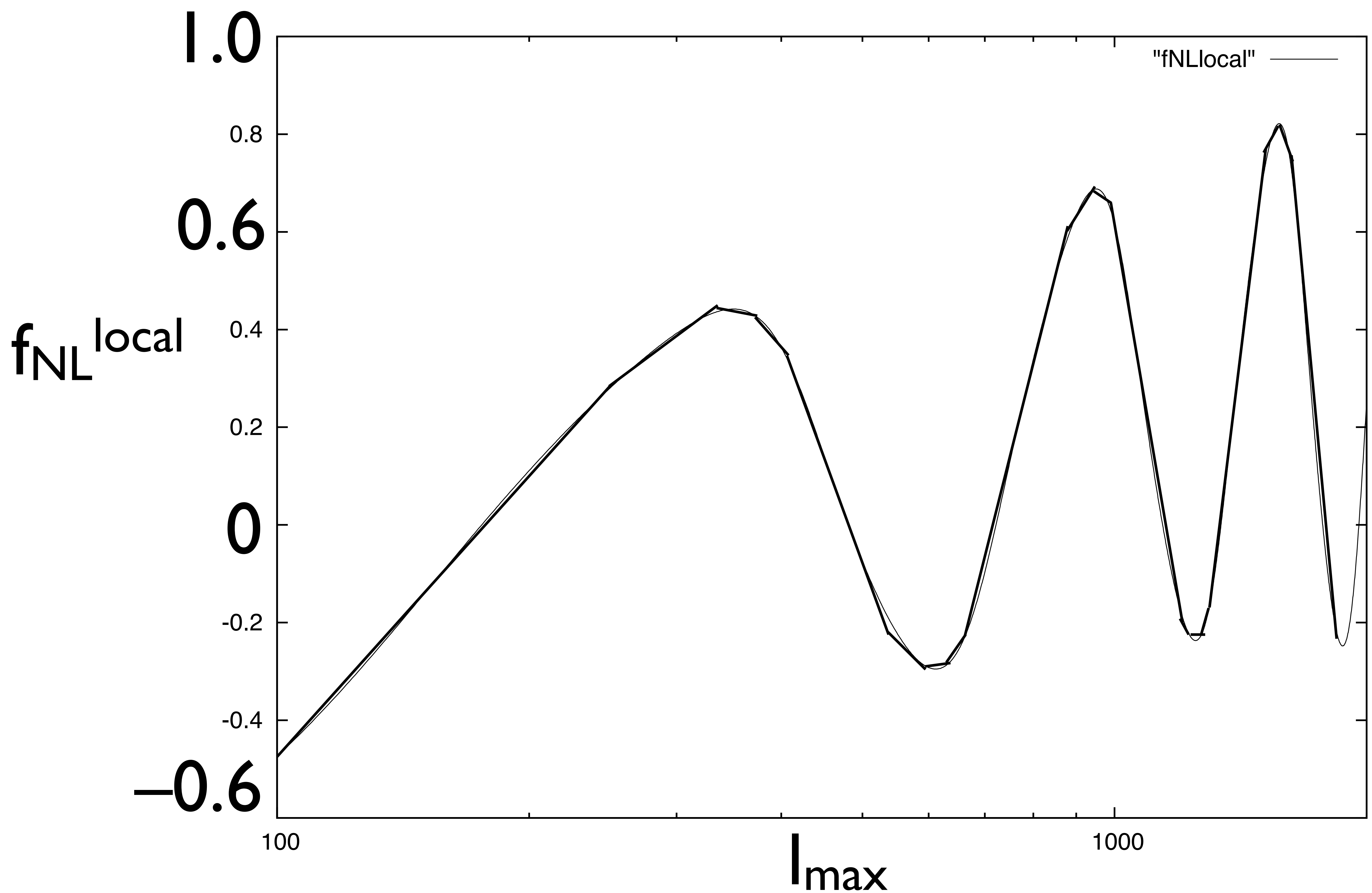
Shape: Newtonian $\Phi^{(2)}$



- Equilateral!

Daisuke Nitta's calculation

f_{NL}^{local}: Newtonian $\Phi^{(2)}$



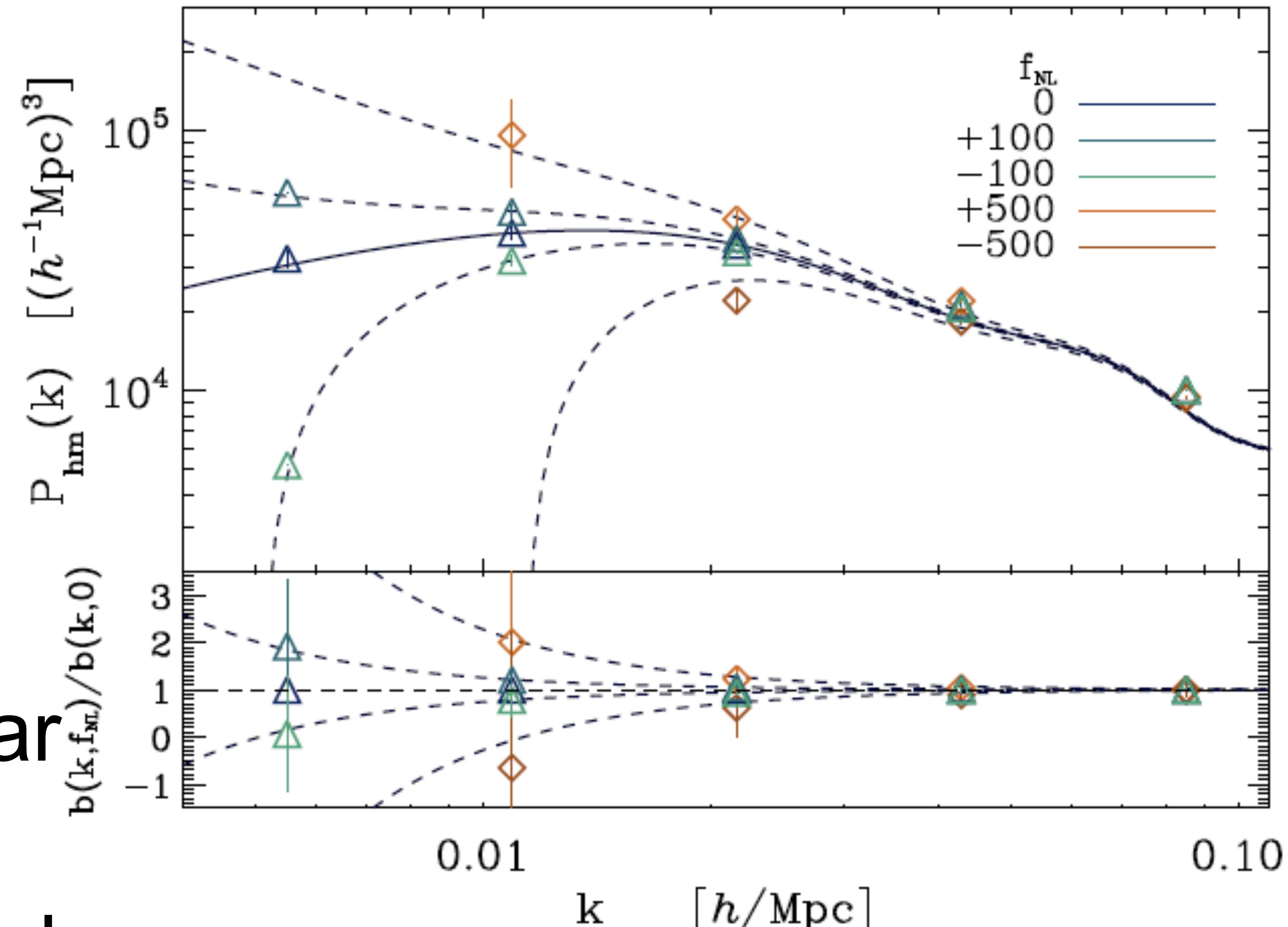
- $f_{NL}^{local} < 1$!

Current Situation

- So, according to *Pitrou et al.*'s results, the GR (post-Newtonian) evolution of $\Phi^{(2)}$ is responsible for f_{NL}^{local}~5. [The Newtonian contribution is equilateral.]
  - It would be nice to confirm this using a simpler method (instead of the full numerical integration).
- While it is rather shocking that the 2nd-order Boltzmann gives f_{NL}^{local}~5, a good news is that it comes from only a few terms in the 2nd-order source; thus, creating a template would probably be easy.

New, Powerful Probe of f_{NL}

- f_{NL} modifies the power spectrum of galaxies on very large scales
  - *–Dalal et al.; Matarrese & Verde*
  - *–Mcdonald; Afshordi & Tolley*
- The statistical power of this method is **VERY** promising
  - –SDSS: $-29 < f_{NL} < 70$ (95%CL); Slosar et al.
  - –Comparable to the WMAP 7-year limit already
  - –Expected to beat CMB, and reach a sacred region: **$f_{NL}^{local} \sim 1$**



Effects of f_{NL} on the statistics of PEAKS

- The effects of f_{NL} on the power spectrum of peaks (i.e., galaxies) are profound.
- **How about the bispectrum of galaxies?**

Previous Calculation

- *Scoccimarro, Sefusatti & Zaldarriaga* (2004); *Sefusatti & Komatsu* (2007)
  - Treated the distribution of galaxies as a *continuous distribution*, biased relative to the matter distribution:
    - $\delta_g = b_1\delta_m + (b_2/2)(\delta_m)^2 + \ldots$
  - Then, the calculation is straightforward. Schematically:
    - $<\delta_g^3> = (b_1)^3<\delta_m^3> + (b_1^2 b_2)<\delta_m^4> + \ldots$

Non-linear Gravity *Non-linear Bias Bispectrum*

Primordial NG

Previous Calculation

$$
\begin{aligned}
&B_g(k_1,k_2,k_3,z) \\
&= 3b_1^3 f_{\rm NL}\Omega_m H_0^2 \left[\frac{P_m(k_1,z)}{k_1^2 T(k_1)}\frac{P_m(k_2,z)}{k_2^2 T(k_2)}\frac{k_3^2 T(k_3)}{D(z)} + \text{(cyclic)}\right] \\
&+ 2b_1^3 \left[F_2^{(s)}(\boldsymbol{k}_1,\boldsymbol{k}_2)P_m(k_1,z)P_m(k_2,z) + \text{(cyclic)}\right] \\
&+ b_1^2 b_2 \left[P_m(k_1,z)P_m(k_2,z) + \text{(cyclic)}\right]
\end{aligned}
$$

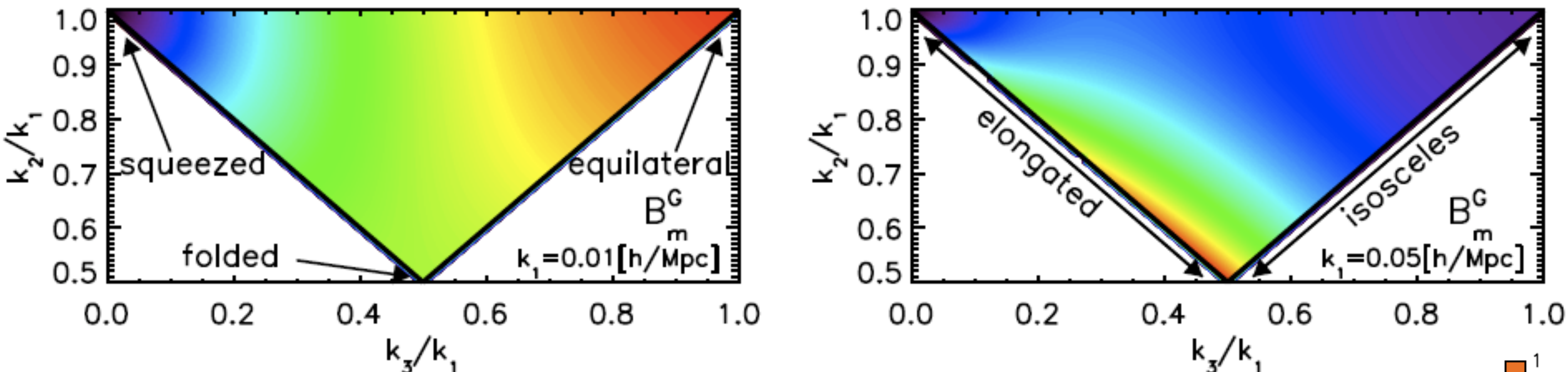
Primordial NG

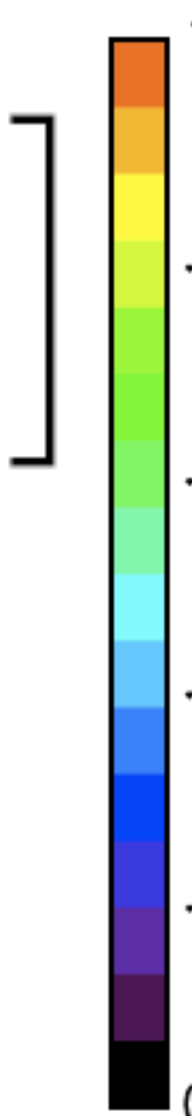
Non-linear Gravity

Non-linear Bias

- We find that this formula captures only a part of the full contributions. In fact, this formula is sub-dominant in the squeezed configuration, and the new terms are dominant.

Non-linear Gravity

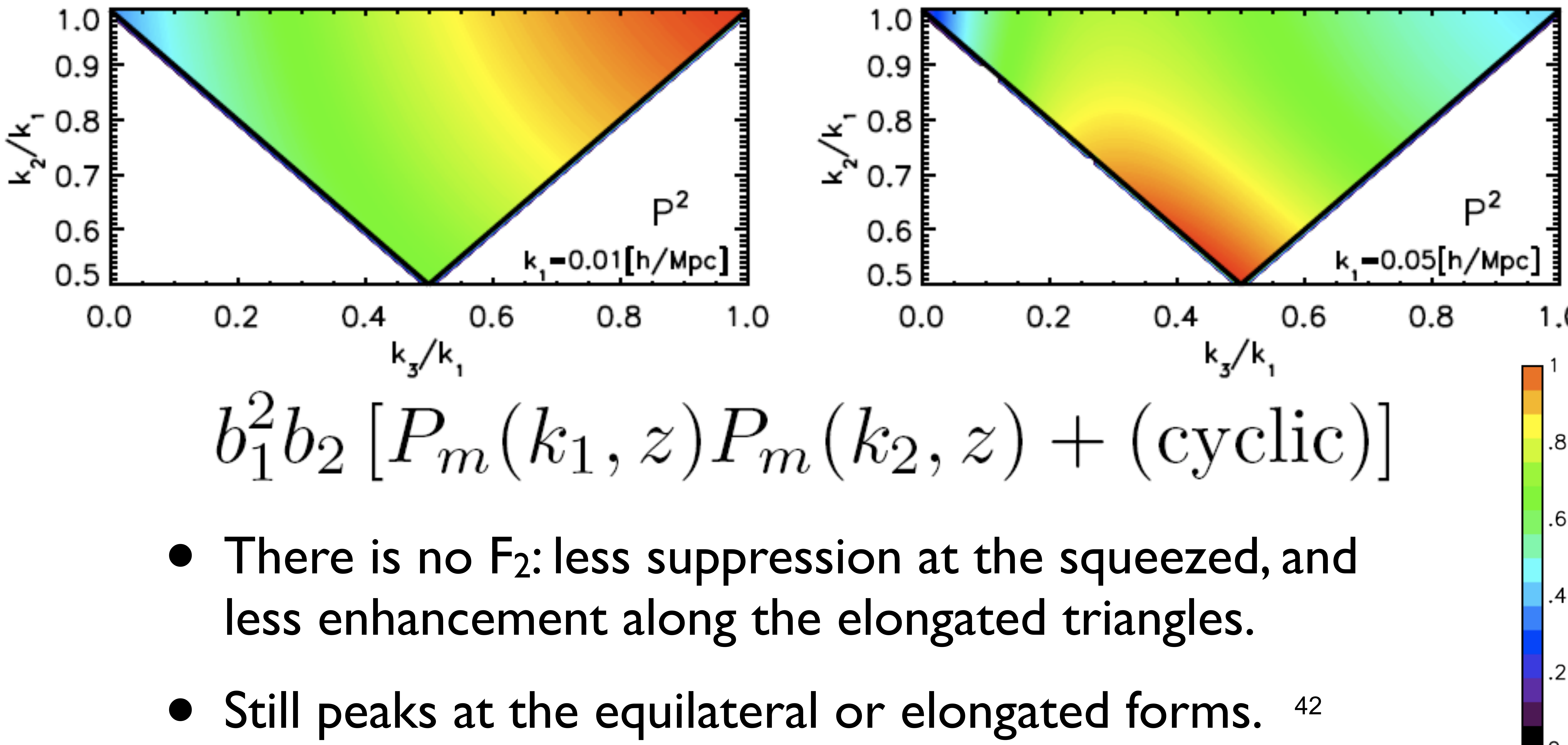




$$2b_1^3\left[F_2^{(s)}(\boldsymbol{k}_1,\boldsymbol{k}_2)P_m(k_1,z)P_m(k_2,z)+\text{(cyclic)}\right]$$

- For a given k_1, vary k_2 and k_3, with $k_3 \le k_2 \le k_1$
- $F_2(k_2,k_3)$ vanishes in the squeezed limit, and peaks at the elongated triangles.

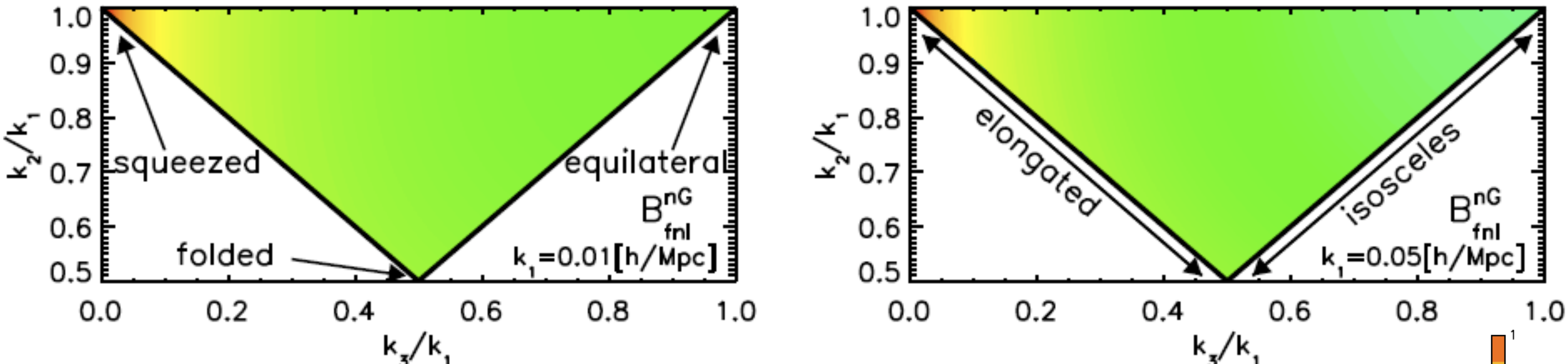
Non-linear Galaxy Bias

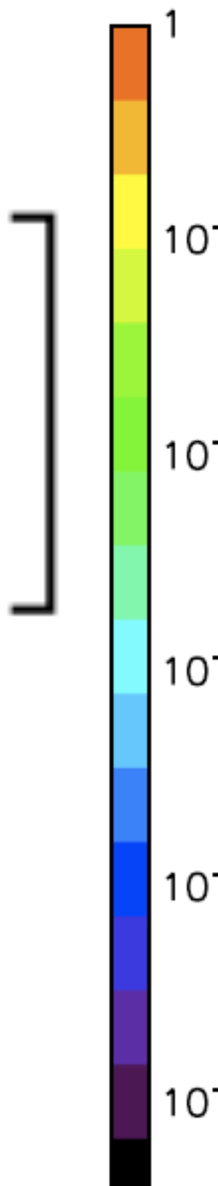


$$b_1^2 b_2 \left[P_m(k_1, z)P_m(k_2, z) + (\text{cyclic})\right]$$

- There is no F_2: less suppression at the squeezed, and less enhancement along the elongated triangles.
- Still peaks at the equilateral or elongated forms.

Primordial NG (SK07)





$$3b_1^3 f_{\rm NL}\Omega_m H_0^2 \left[\frac{P_m(k_1,z)}{k_1^2 T(k_1)}\frac{P_m(k_2,z)}{k_2^2 T(k_2)}\frac{k_3^2 T(k_3)}{D(z)} + ({\rm cyclic})\right]$$

- Notice the factors of k^2 in the denominator.
- This gives the peaks at the squeezed configurations.

Matarrese, Lucchin & Bonometto (1986)

MLB Formula

$$
\begin{aligned}
&1+\xi_h(x_{12})+\xi_h(x_{23})+\xi_h(x_{31})+\zeta_h(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3) \\
&=\exp\left[\frac{1}{2}\frac{\nu^2}{\sigma_R^2}\sum_{i\neq j}\xi_R^{(2)}(x_{ij})+\sum_{n=3}^{\infty}\left\{\sum_{m_1=0}^{n}\sum_{m_2=0}^{n-m_1}\frac{\nu^n\sigma_R^{-n}}{m_1!m_2!m_3!}\right.\right. \\
&\quad\times\xi_R^{(n)}\begin{pmatrix}\boldsymbol{x}_1,\cdots,\boldsymbol{x}_1, & \boldsymbol{x}_2,\cdots,\boldsymbol{x}_2, & \boldsymbol{x}_3,\cdots,\boldsymbol{x}_3 \\ m_1\,\text{times} & m_2\,\text{times} & m_3\,\text{times}\end{pmatrix} \\
&\quad\left.\left.-3\frac{\nu^n\sigma_R^{-n}}{n!}\xi_R^{(n)}\begin{pmatrix}\boldsymbol{x},\cdots,\boldsymbol{x} \\ n\,\text{times}\end{pmatrix}\right\}\right]
\end{aligned}
$$

- N-point correlation function of peaks is the sum of M-point correlation functions, where M≥N.

Bottom Line

- **The bottom line is:**
- The power spectrum (2-pt function) of peaks is sensitive to the power spectrum of the underlying mass distribution, and the bispectrum, and the trispectrum, etc.
    - Truncate the sum at the bispectrum: sensitivity to f_{NL}
    - Dalal et al.; Matarrese&Verde; Slosar et al.; Afshordi&Tolley

Bottom Line

- **The bottom line is:**
- The bispectrum (3-pt function) of peaks is sensitive to the bispectrum of the underlying mass distribution, and the trispectrum, and the quadspectrum, etc.
  - Truncate the sum at the trispectrum: sensitivity to τ_{NL} ($\sim f_{NL}^2$) and g_{NL}!
  - This is the new effect that was missing in Sefusatti & Komatsu (2007).

Real-space 3pt Function

$$\zeta_h(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3) = \frac{\nu^3}{\sigma_R^3}\xi_R^{(3)}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3) + \frac{\nu^4}{\sigma_R^4}\left[\xi_R^{(2)}(x_{12})\xi_R^{(2)}(x_{23}) + (\text{cyclic})\right] + \frac{\nu^4}{2\sigma_R^4}\left[\xi_R^{(4)}(\boldsymbol{x}_1, \boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3) + (\text{cyclic})\right]$$

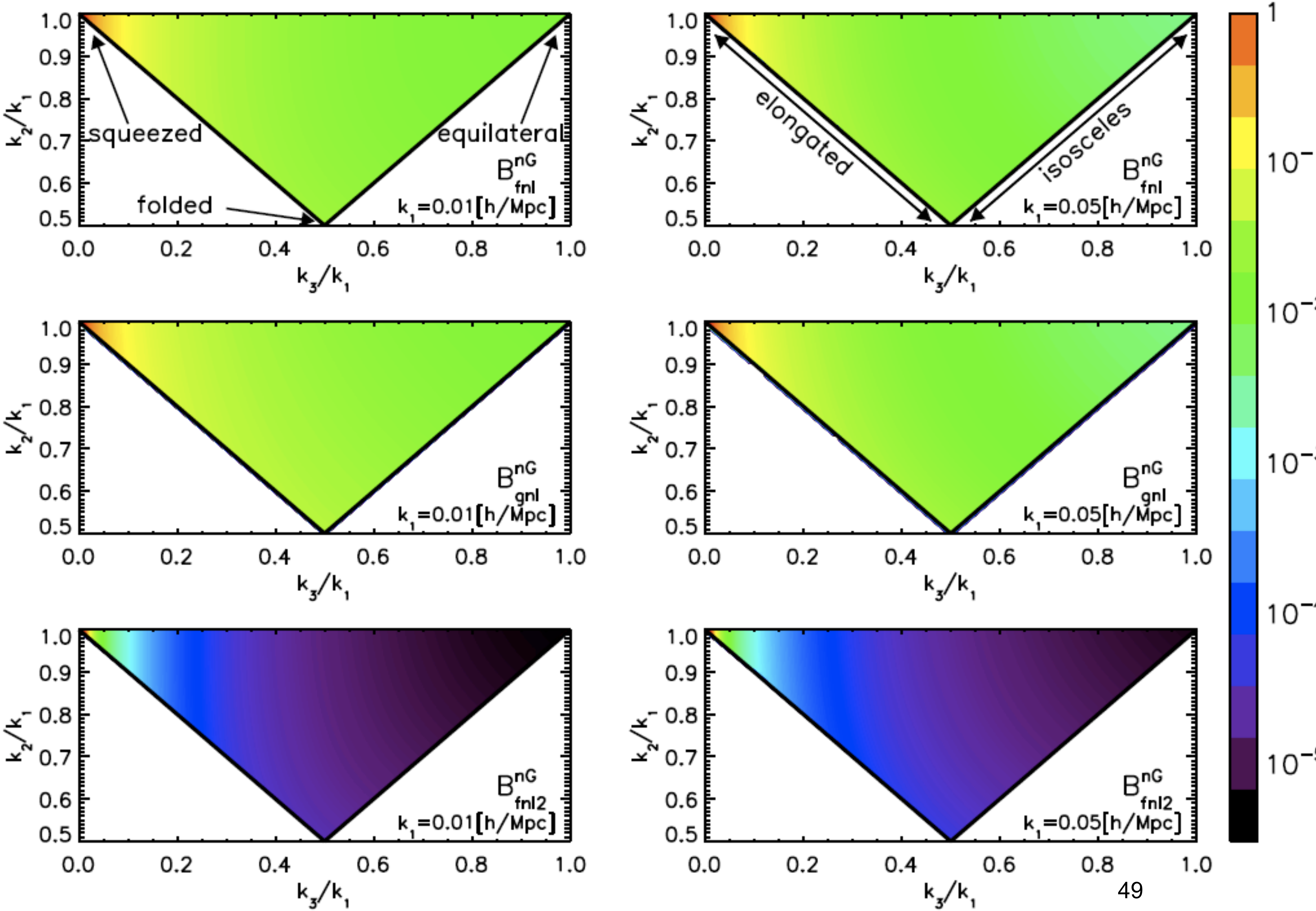
- Plus 5-pt functions, etc...

Jeong & Komatsu (2009)

New Bispectrum Formula

$$
\begin{aligned}
&B_h(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \\
&= b_1^3 \left[B_R(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) + \frac{b_2}{b_1} \{P_R(k_1)P_R(k_2) + (\text{cyclic})\} \right. \\
&\left. + \frac{\delta_c}{2\sigma_R^2} \int \frac{d^3q}{(2\pi)^3} T_R(\boldsymbol{q}, \boldsymbol{k}_1 - \boldsymbol{q}, \boldsymbol{k}_2, \boldsymbol{k}_3) + (\text{cyclic}) \right].
\end{aligned}
$$

- First: bispectrum of the underlying mass distribution.
- Second: non-linear bias
- Third: trispectrum of the underlying mass distribution.

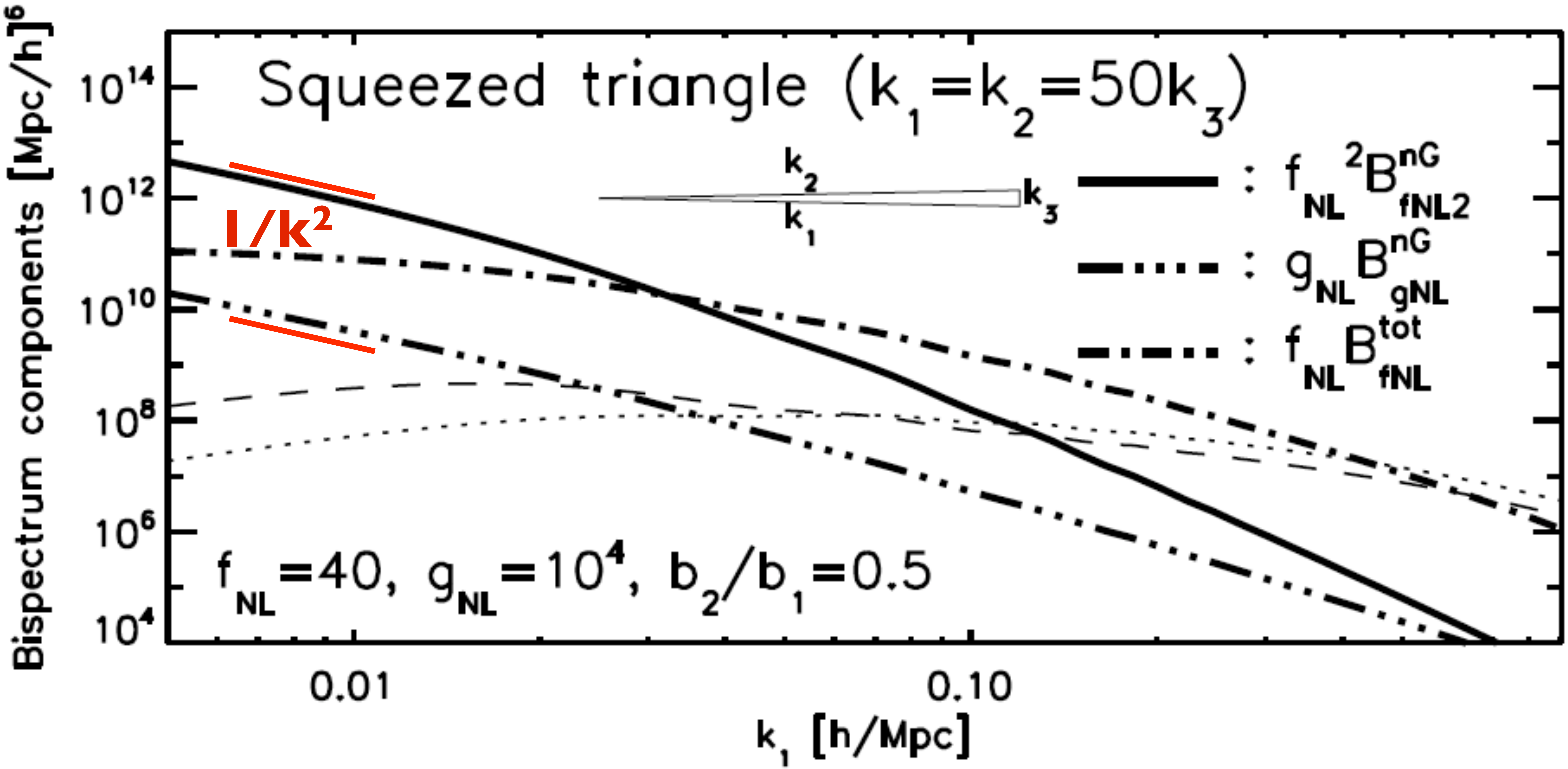

squeezed
equilateral
folded
elongated
isosceles
B^{nG}_{fnl}
k_1=0.01[h/Mpc]
k_1=0.05[h/Mpc]
B^{nG}_{gnl}
B^{nG}_{fnl2}
k_2/k_1
k_3/k_1
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}

Shape Results

- The primordial non-Gaussianity terms peak at the squeezed triangle.
- f_{NL} and g_{NL} terms have the same shape dependence:
  - For $k_1=k_2=\alpha k_3$, (f_{NL} term)~α and (g_{NL} term)~α
- f_{NL}^2 (τ_{NL}) is more sharply peaked at the squeezed:
  - (f_{NL}^2 term)~α^3

Key Question

- Are g_{NL} or τ_{NL} terms important?



Squeezed triangle ($k_1=k_2=50k_3$)
k_2
k_1
k_3
$1/k^2$
: $f_{NL}{}^2B^{nG}_{fNL2}$
: $g_{NL}B^{nG}_{gNL}$
: $f_{NL}B^{tot}_{fNL}$
$f_{NL}=40$, $g_{NL}=10^4$, $b_2/b_1=0.5$
Bispectrum components $[\mathrm{Mpc}/h]^6$
10^{14}
10^{12}
10^{10}
10^{8}
10^{6}
10^{4}
0.01
0.10
k_1 [h/Mpc]

Summary (f_{NL})

- No detection of f_{NL} of any kind.
  - The optimal estimators are in our hand.
    - f_{NL}^{local}=32±21 (68%CL)
      - Foreground may be an issue for Planck?
    - f_{NL}^{orthog}=–202±104 (68%CL)
      - Effects of point sources and secondaries on the orthogonal shape?
- f_{NL}^{local}=2.7 (WMAP) and 10 (Planck) from the lens-ISW: scary, but we know the shape.
- f_{NL}^{local}~5 (Planck) from the 2nd order? Look at PN $\Phi^{(2)}$

Summary (τ_{NL} & g_{NL})

Smidt et al. (2010) [WMAP 5-year]

- $-3.2\times10^5 < \tau_{NL} < 3.3\times10^5$ (95%CL)
    - The error 100x too large -> room for improvement
    - Planck: 560 (95%CL)
- $-3.8\times10^6 < g_{NL} < 3.9\times10^6$ (95%CL)
    - We don't have a forecast yet. (Someone is lazy.)
- Large-scale structure!
    - IMHO, the galaxy bispectrum is probably the best probe of τ_{NL} (and possibly g_{NL} as well).

Any Rumors?

- Planck has completed the first full-sky observation.
  - They have seen the power spectrum already (many peaks have been detected).
- This means that they may soon start measuring f_{NL}.
  - Do you have friends in the Planck collaboration?
  - Take them to a nice restaurant, let them drink like the hell (or heaven, whatever).
  - Gently ask, "*have you found it?*"