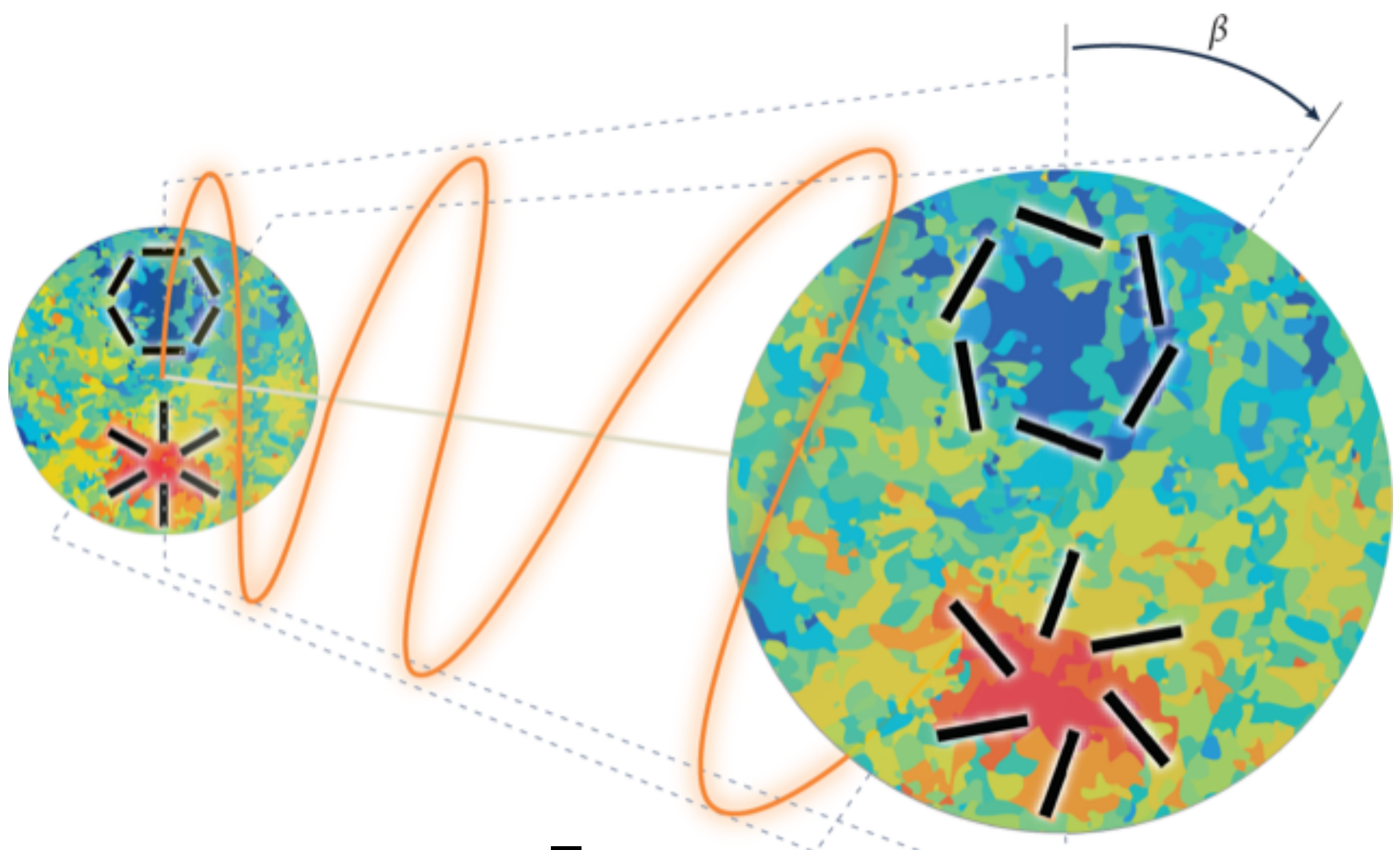
$$I_{\rm CS} = \int d^4x \sqrt{-g}\left(-\frac{\alpha}{4f}\chi F\tilde{F}\right)$$



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Parity Violation in Cosmology

In search of new physics for the Universe

The lecture slides are available at
https://wwwmpa.mpa-garching.mpg.de/~komatsu/lectures--reviews.html

Day 6

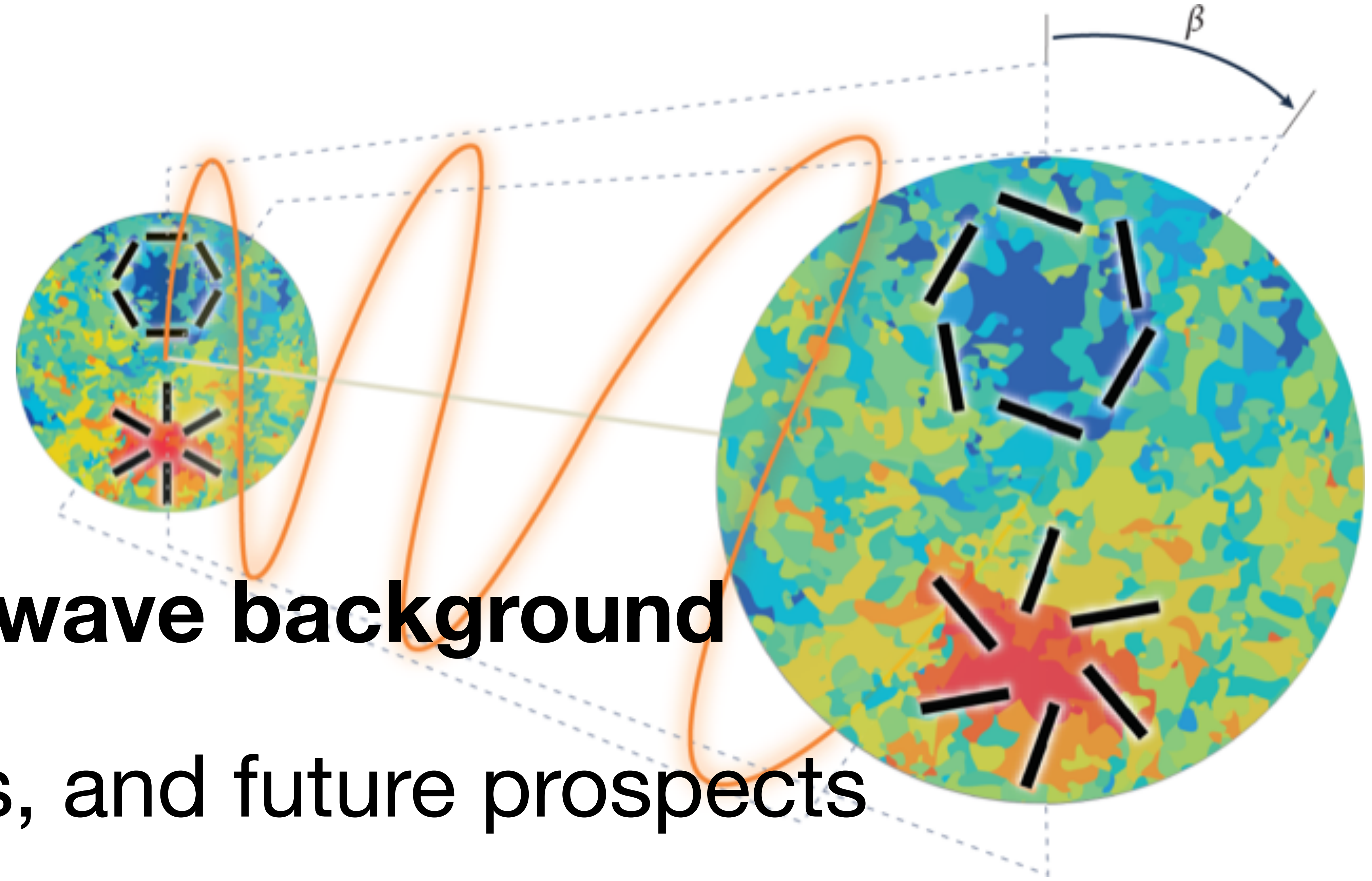
Eiichiro Komatsu (Max Planck Institute for Astrophysics)
Nagoya University, June 6–30, 2023

Topics

From the syllabus

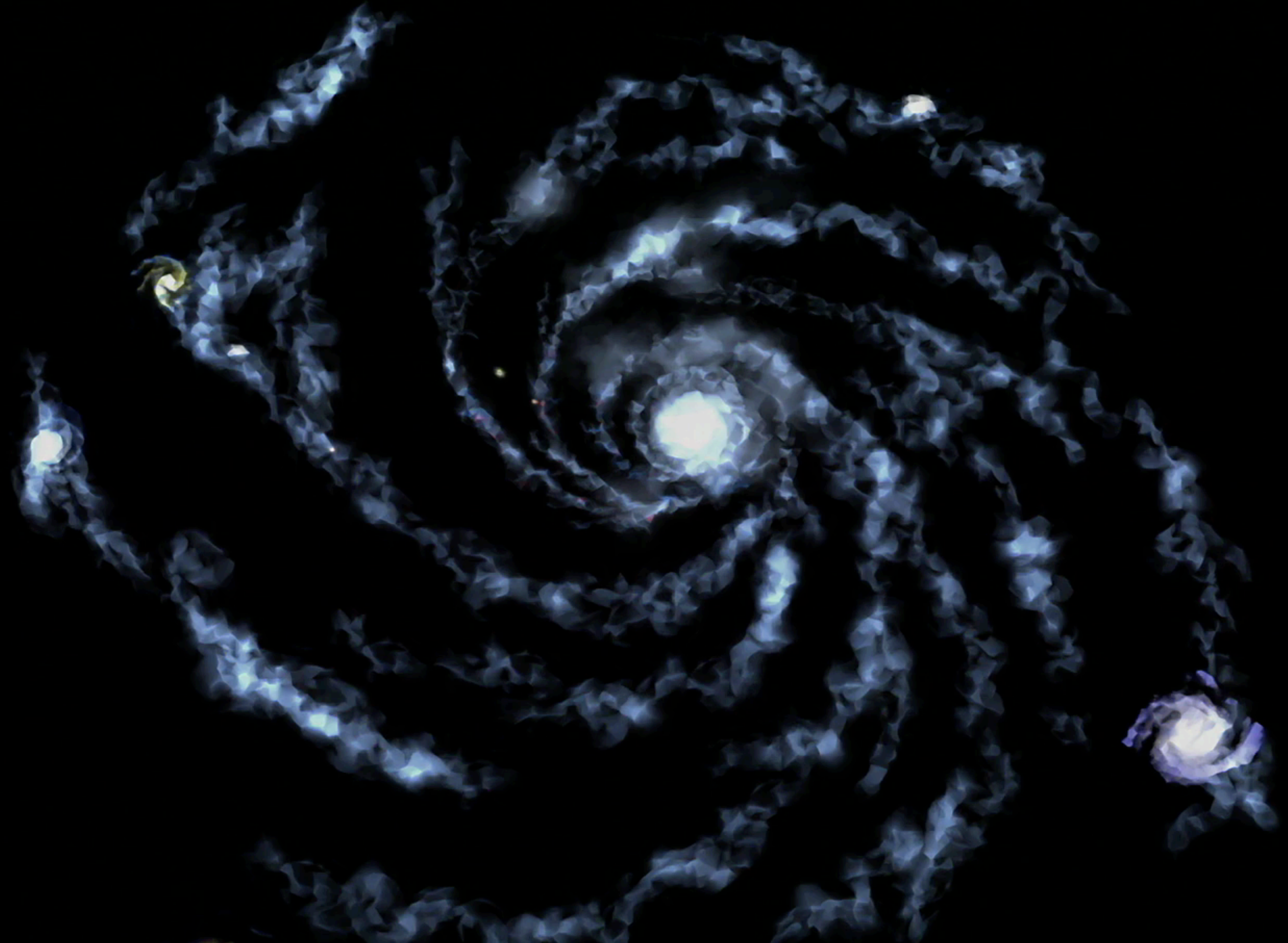
1. What is parity symmetry?
2. Chern-Simons interaction
3. Parity violation 1: Cosmic inflation
4. Parity violation 2: Dark matter
5. Parity violation 3: Dark energy
6. Light propagation: birefringence
7. **Physics of polarization of the cosmic microwave background**
8. Recent observational results, their implications, and future prospects

$$I_{\mathrm{CS}} = \int \mathrm{d}^4x \sqrt{-g}\left(-\frac{\alpha}{4f}\chi F\tilde{F}\right)$$



7.1 Generation of Polarization in the CMB

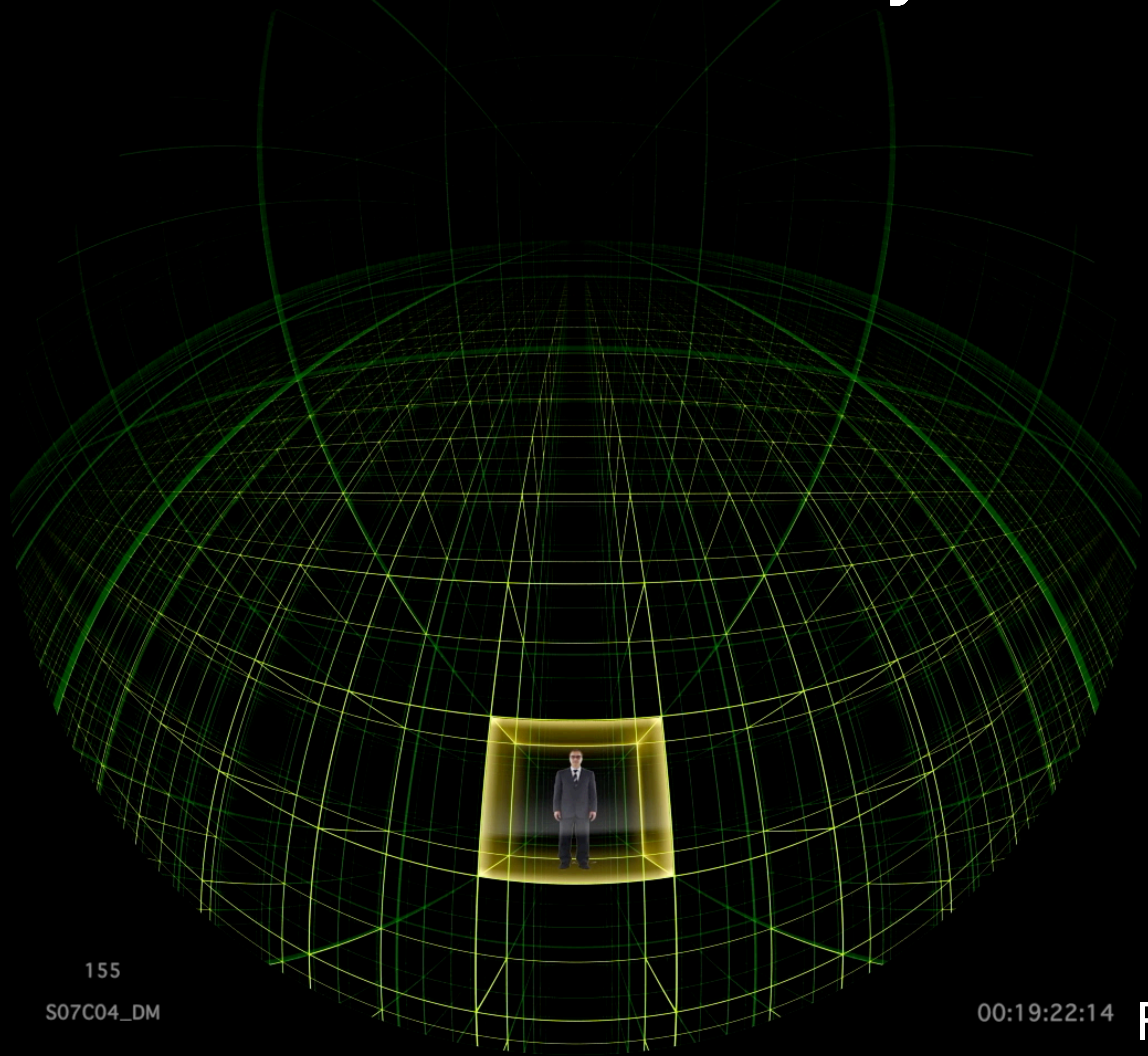
Credit: WMAP Science Team



The sky in various wavelengths

Visible -> Near Infrared -> Far Infrared -> Submillimeter -> Microwave

Where did the CMB we see today come from?

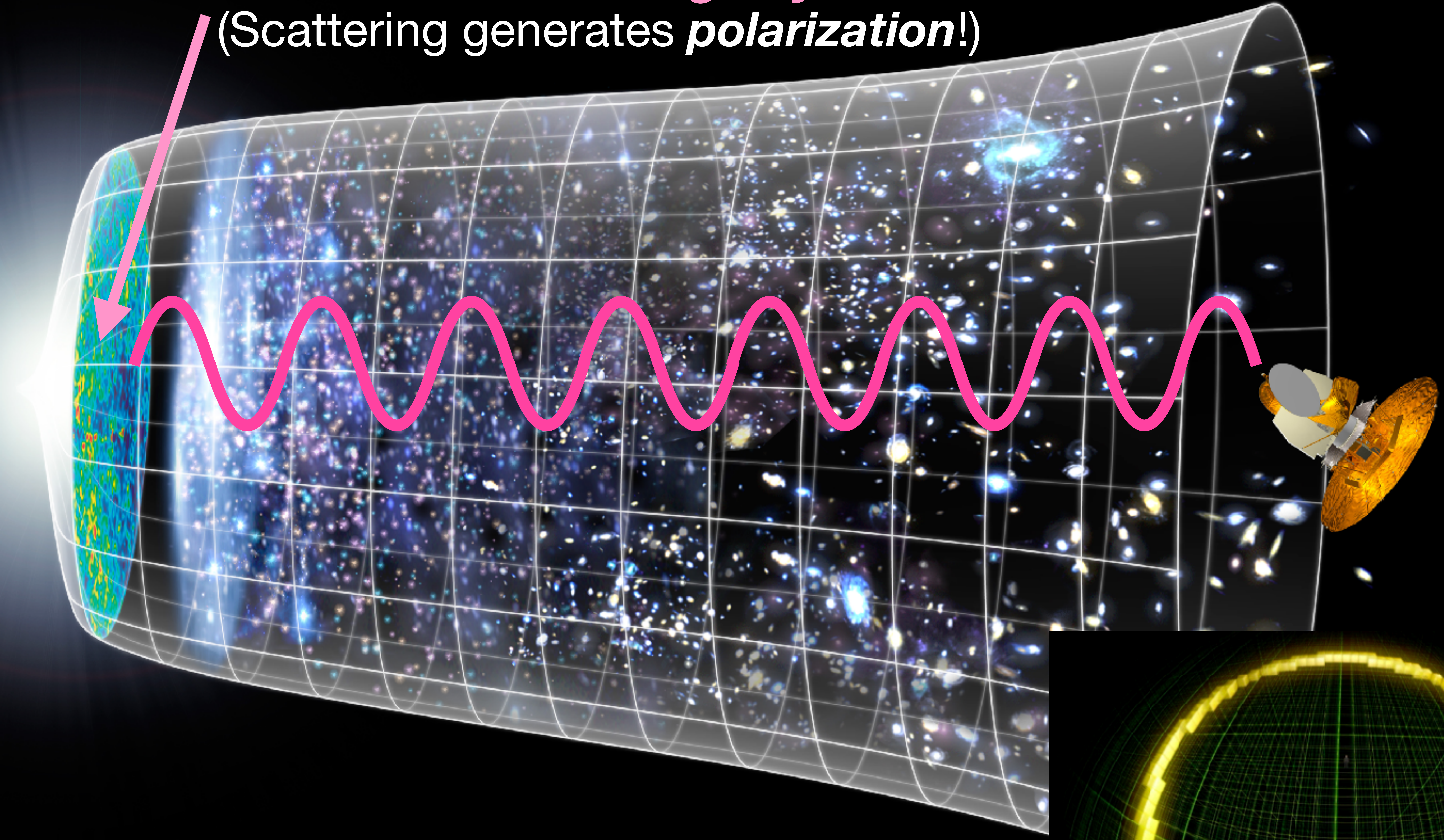


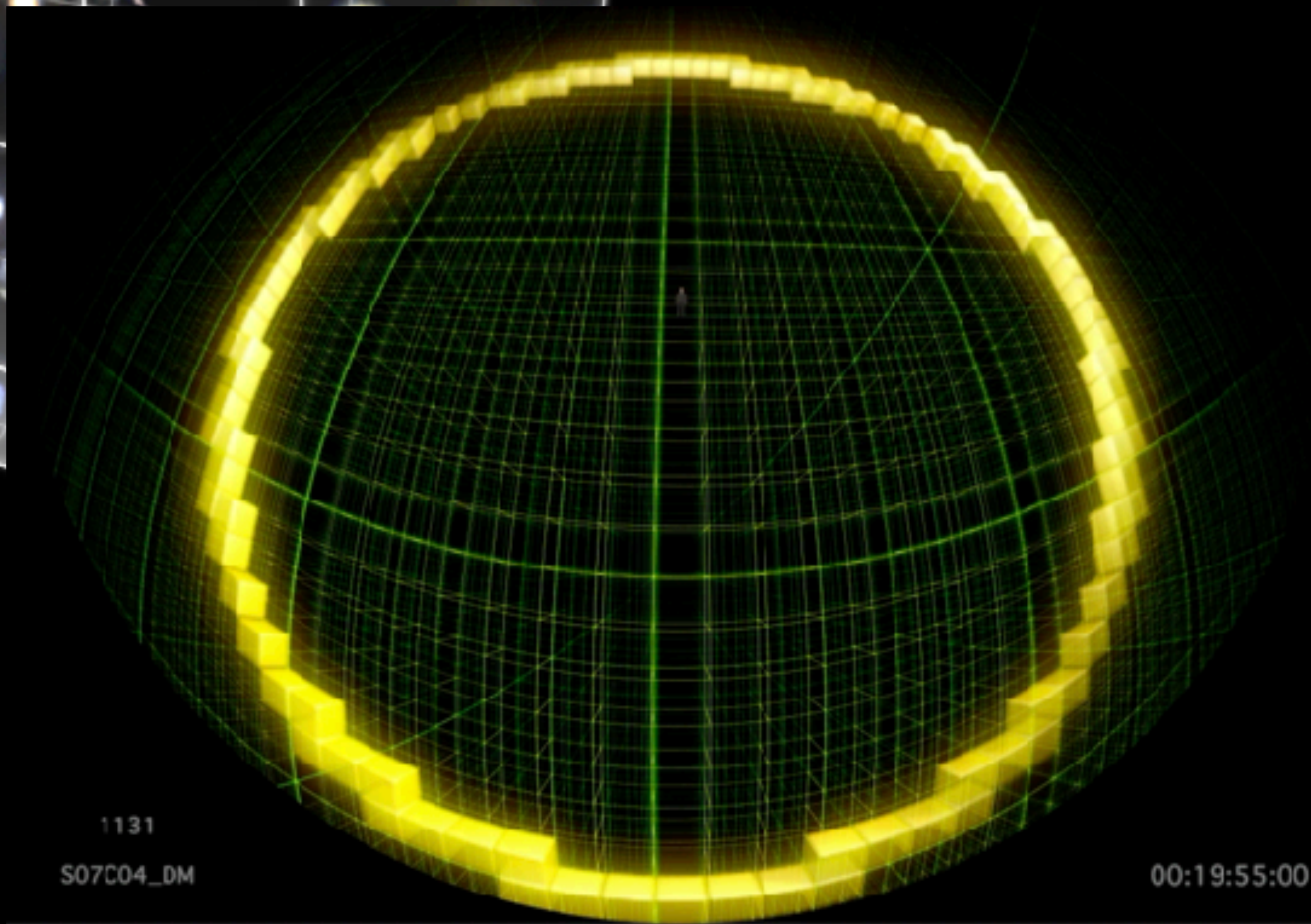
From “HORIZON”

Credit: WMAP Science Team

The surface of “last scattering” by electrons

(Scattering generates ***polarization***!)





Not shown: The cosmological redshift due to the expansion of the Universe


Credit: TALEX
Horizontally polarized

Credit: TALEX

Physics of CMB Polarization

Necessary and sufficient condition: Scattering and Quadrupole Anisotropy

Credit：Wayne Hu

Temperature (smoothed)

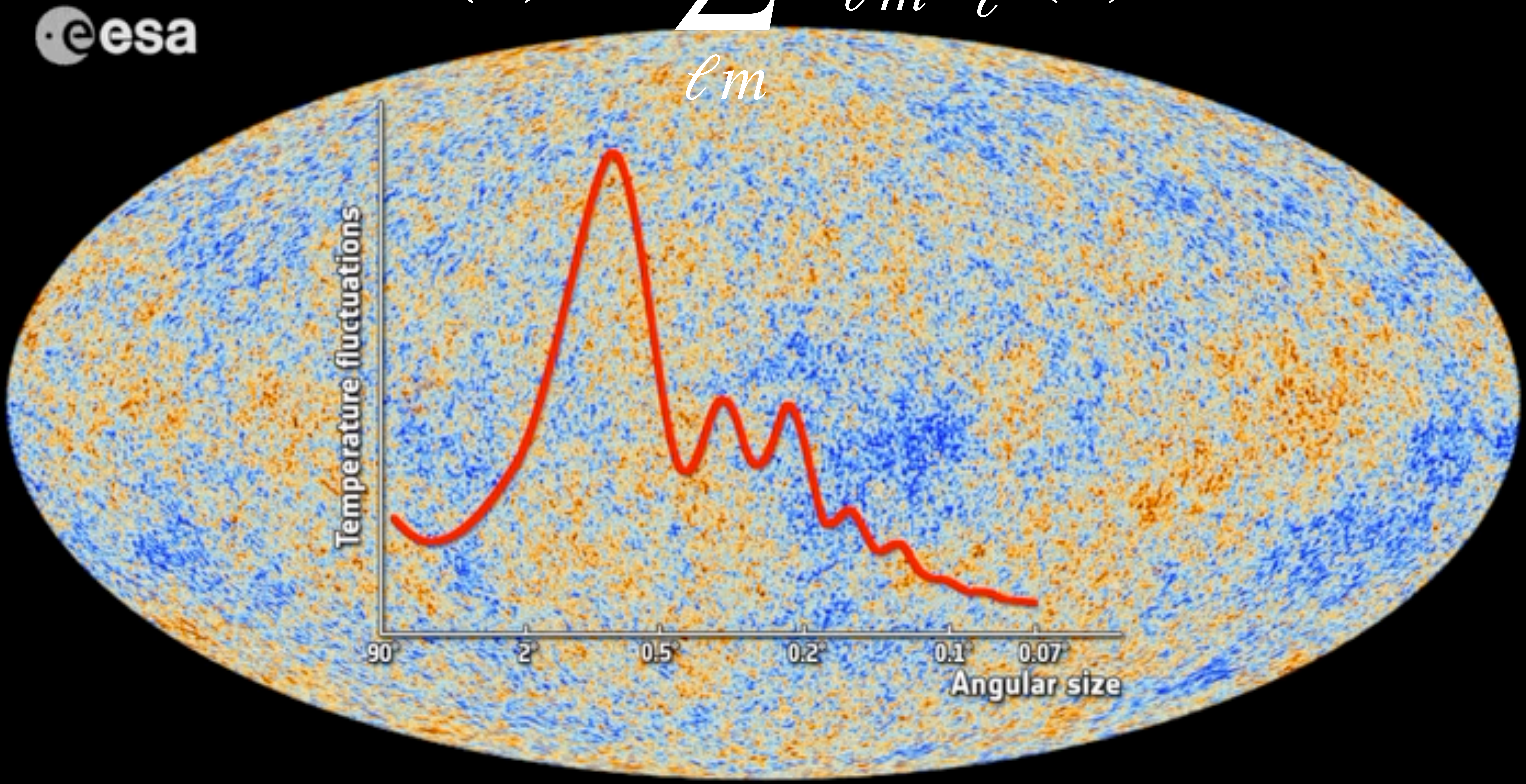
Credit: ESA

Credit: ESA

Temperature (smoothed) + Polarisation

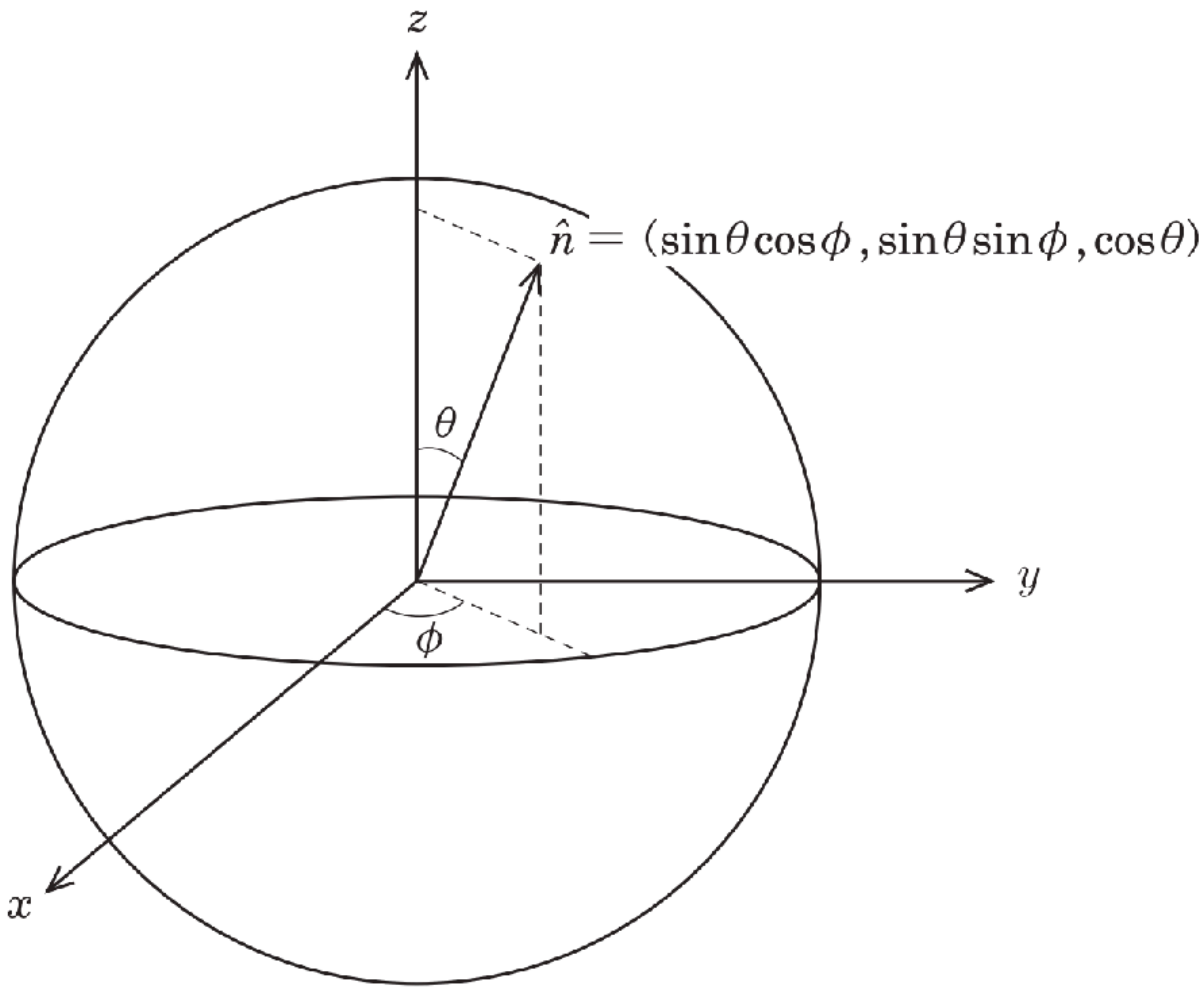
Spherical Harmonics Decomposition

$$\Delta T(\hat{n}) = \sum_{\ell m} a_{\ell m} Y_\ell^m(\hat{n})$$



Parity transformation of temperature anisotropy

- The line-of-sight unit vector is $\hat{n}$.
  - Parity transformation is $\hat{n} \rightarrow \hat{n}' = -\hat{n}$.
- The spherical harmonics transform as $Y_\ell^m(-\hat{n}) = (-1)^\ell Y_\ell^m(\hat{n})$. Thus,



$$\Delta T(\hat{n}) = \sum a_{\ell m} Y_\ell^m(\hat{n})$$

$$\rightarrow \Delta T(\hat{n}') = \sum a'_{\ell m} Y_\ell^m(-\hat{n})$$

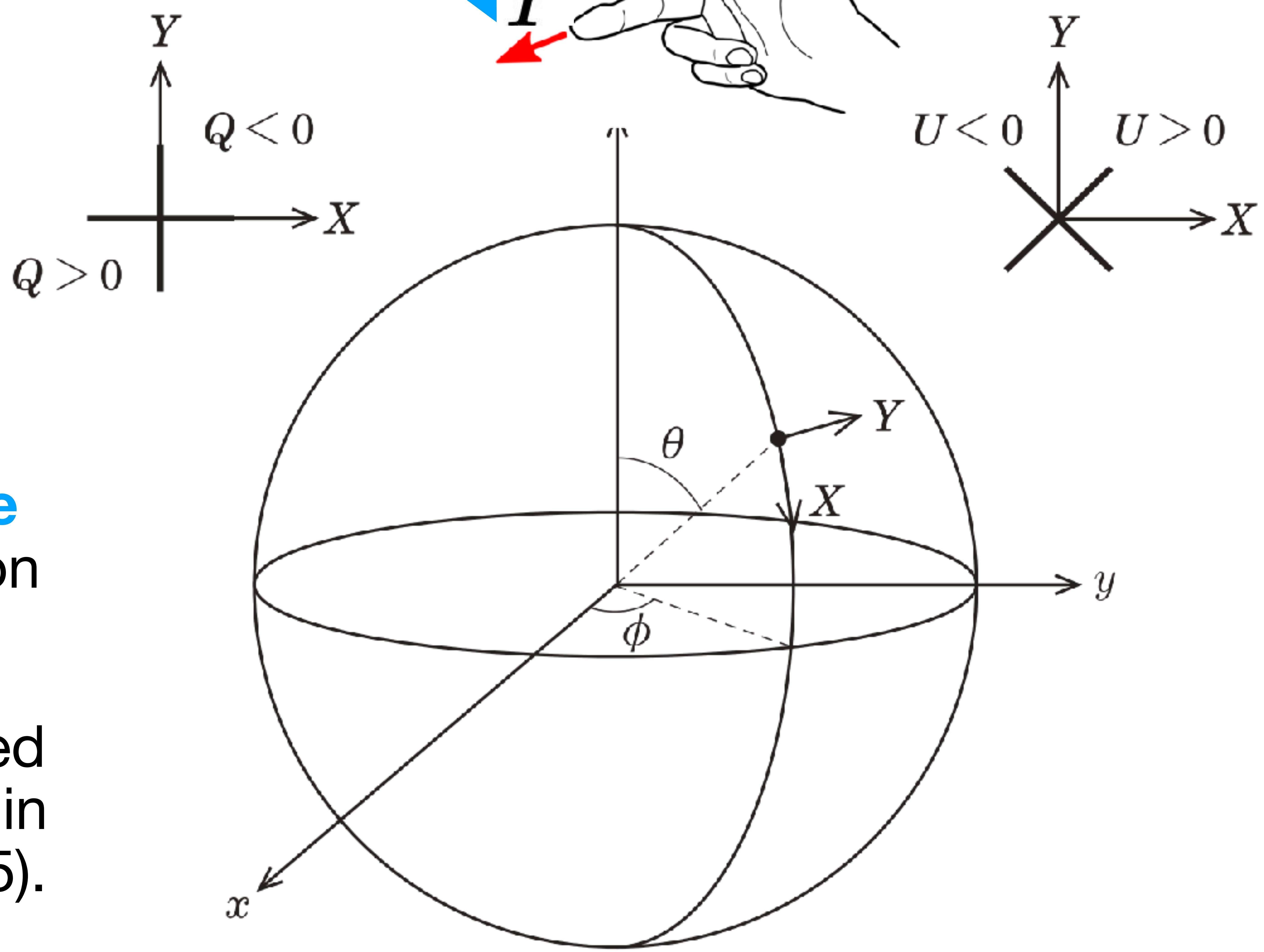
$$= \sum a'_{\ell m} (-1)^\ell Y_\ell^m(\hat{n})$$

$$\Rightarrow \boxed{a'_{\ell m} = (-1)^\ell a_{\ell m}}$$

Full-sky Stokes Parameters

In the CMB convention

- The line-of-sight unit vector is $\hat{n}$.
- In the CMB convention, *Q*, *U*, and the position angle (PA) are defined in **the right-handed coordinate system with the *z*-axis in the line of sight**, rather than in the direction of the photons.
- This is equivalent to the left-handed coordinate system with the *z*-axis in the direction of the photons (Day 5).



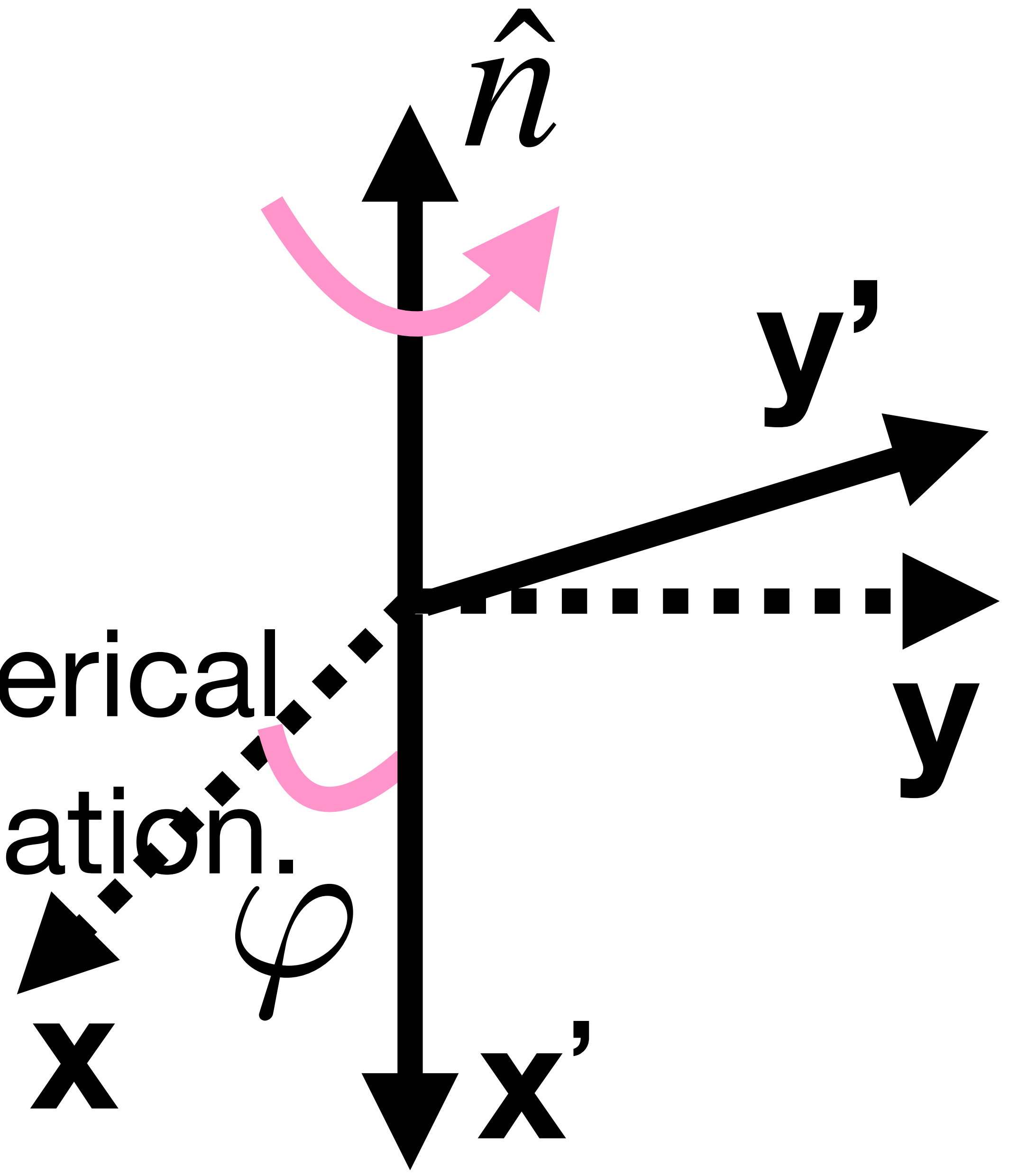
7.2 E- and B-mode Polarization

Spin-2 Spherical Harmonics

- To probe parity symmetry in the CMB polarization, Stokes parameters *Q* and *U* are not convenient because **they depend on the choice of coordinates**.
- If we write $Q \pm iU = Pe^{\pm 2i\beta}$ (Day 5) and rotate the coordinates by φ in the right-handed coordinate system with the z-axis in the line of sight, we find

$$\beta \to \beta' = \beta - \varphi$$

- Thus, $$Q' \pm iU' = e^{\mp 2i\varphi}(Q \pm iU)$$
- This means that we cannot expand $Q \pm iU$ using the usual spherical harmonics, as Y_ℓ^m does not transform as a spin-2 field under rotation.



Spin-2 Spherical Harmonics

- Spin-2 spherical harmonics, ${}_{\pm 2}Y_\ell^m(\hat{n})$, are constructed as follows.

1. Take two derivatives of Y_ℓ^m with respect to the directions perpendicular to the line of sight, $\tilde{\nabla}_i \tilde{\nabla}_j Y_\ell^m(\hat{n})$, where

$$\tilde{\nabla} = \hat{\theta}\frac{\partial}{\partial\theta} + \frac{\hat{\phi}}{\sin\theta}\frac{\partial}{\partial\phi}$$

with orthogonal unit vectors given by

$$\hat{\theta} = (\cos\phi\cos\theta, \sin\phi\cos\theta, -\sin\theta)$$

$$\hat{\phi} = (-\sin\phi, \cos\phi, 0)$$

$$\hat{n}\cdot\hat{\theta} = \hat{n}\cdot\hat{\phi} = \hat{\theta}\cdot\hat{\phi} = 0$$

Under parity transformation, $\hat{n} \to \hat{n}' = -\hat{n}$ ($\theta \to \pi - \theta$, $\phi \to \phi + \pi$):

$$\hat{\theta} \to \hat{\theta}, \quad \hat{\phi} \to -\hat{\phi}, \quad \tilde{\nabla} \to -\tilde{\nabla}$$

Spin-2 Spherical Harmonics

- Spin-2 spherical harmonics, ${}_{\pm 2}Y_\ell^m(\hat{n})$, are constructed as follows.

  2. Take the dot product of $\tilde{\nabla}_i\tilde{\nabla}_j Y_\ell^m(\hat{n})$ and two polarization vectors given by $\mathbf{e}_\pm = (\hat{\theta} \pm i\hat{\phi})/\sqrt{2}$, so that

$$ {}_{+2}Y_\ell^m(\hat{n}) \propto \sum_{ij} e_{+i}e_{+j}\tilde{\nabla}_i\tilde{\nabla}_j Y_\ell^m(\hat{n}) $$

$$ {}_{-2}Y_\ell^m(\hat{n}) \propto \sum_{ij} e_{-i}e_{-j}\tilde{\nabla}_i\tilde{\nabla}_j Y_\ell^m(\hat{n}) $$

These spherical harmonics transform as spin-2 fields.

Side note: $\sum e_{\pm,i}\tilde{\nabla}_i Y_\ell^m(\hat{n})$ is proportional to the spin-1 spherical harmonics, ${}_{\pm 1}Y_\ell^m(\hat{n})$.

Spin-2 Spherical Harmonics

- Spin-2 spherical harmonics, ${}_{\pm 2}Y_\ell^m(\hat{n})$, are constructed as follows.

  3. Determine the proportionality constant from the orthonormality condition given by

$$\int_{-1}^{1} d(\cos\theta) \int_0^{2\pi} d\phi \;{}_sY_\ell^m(\hat{n})\,{}_sY_{\ell'}^{m'*}(\hat{n}) = \delta_{\ell\ell'}\delta_{mm'}$$

  4. The results: $${}_{\pm 2}Y_\ell^m(\hat{n}) = 2\sqrt{\frac{(\ell-2)!}{(\ell+2)!}} \sum_{ij} e_{\pm i} e_{\pm j} \tilde{\nabla}_i \tilde{\nabla}_j Y_\ell^m(\hat{n})$$

Problem Set 6

Parity transformation of ${}_{\pm 2}Y_\ell^m(\hat{n})$

- Show that the polarization vectors transform as $\mathbf{e}_\pm(\hat{n}') = \mathbf{e}_\mp(\hat{n})$ under parity transformation, $\hat{n} \to \hat{n}' = -\hat{n}$.

- Show that the spin-2 spherical harmonics transform as

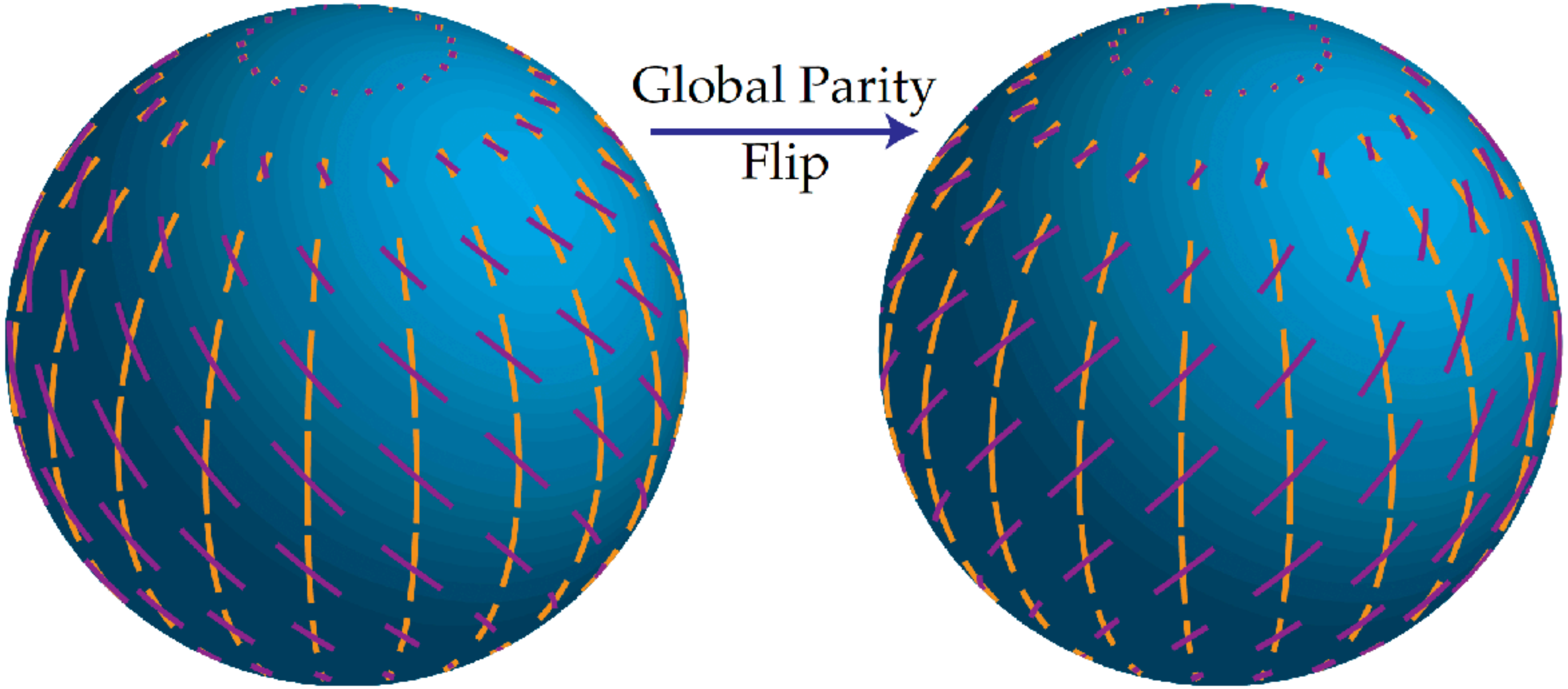
$$
{}_{+2}Y_\ell^m(\hat{n}') = (-1)^\ell \, {}_{-2}Y_\ell^m(\hat{n})
$$
$$
{}_{-2}Y_\ell^m(\hat{n}') = (-1)^\ell \, {}_{+2}Y_\ell^m(\hat{n})
$$

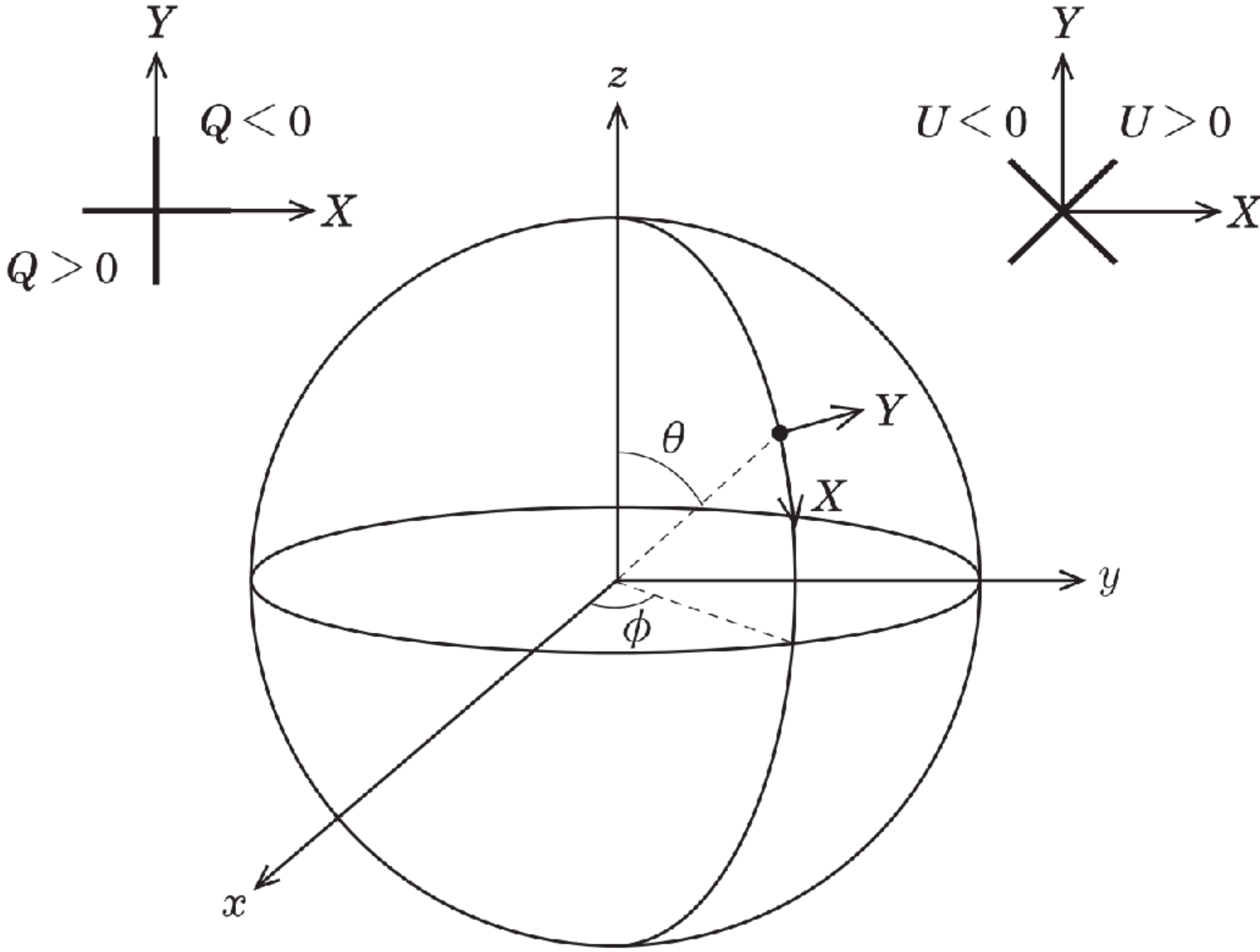
Hu, White (1997)

Parity transformation of *Q* and *U*

The sign of *U* changes.

- Under parity transformation, $\hat{n} \rightarrow \hat{n}' = -\hat{n}$, Stokes parameters Q and *U* transform as $Q(\hat{n}') = Q(\hat{n}), U(\hat{n}') = -U(\hat{n})$. **The sign of *U* changes.**





Zaldarriaga, Seljak (1997); Kamionkowski, Kosowsky, Stebbins (1997)

Eigenstates of parity: E and B modes

Expansion of *Q*±i*U* using the spin-2 spherical harmonics

- We expand Stokes parameters using the spin-2 spherical harmonics as

$$Q(\hat{n}) \pm iU(\hat{n}) = -\sum_{\ell m}(E_{\ell m} \pm iB_{\ell m})_{\pm 2}Y_\ell^m(\hat{n})$$

- Parity transformation, $\hat{n} \rightarrow \hat{n}' = -\hat{n}$, is

Hint: ${}_{\pm 2}Y_\ell^m(-\hat{n}) = (-1)^\ell {}_{\mp 2}Y_\ell^m(\hat{n})$ Problem Set 6

$$Q(\hat{n}') \pm iU(\hat{n}') = -\sum(E'_{\ell m} \pm iB'_{\ell m})(-1)^\ell{}_{\mp 2}Y_\ell^m(\hat{n})$$

$$= Q(\hat{n}) \mp iU(\hat{n}) = -\sum(E_{\ell m} \mp iB_{\ell m})_{\mp 2}Y_\ell^m(\hat{n})$$

Eigenstates of parity: E and B modes

Expansion of *Q*±i*U* using the spin-2 spherical harmonics

- We expand Stokes parameters using the spin-2 spherical harmonics as

$$Q(\hat{n}) \pm iU(\hat{n}) = -\sum_{\ell m} (E_{\ell m} \pm iB_{\ell m})\,{}_{\pm 2}Y_\ell^m(\hat{n})$$

- Parity transformation, $\hat{n} \to \hat{n}' = -\hat{n}$, is

Hint: ${}_{\pm 2}Y_\ell^m(-\hat{n}) = (-1)^\ell\,{}_{\mp 2}Y_\ell^m(\hat{n})$

$$E'_{\ell m} = (-1)^\ell E_{\ell m}$$
$$B'_{\ell m} = (-1)^{\ell+1} B_{\ell m}$$

E and B modes have the opposite parity!

$$-\sum (E'_{\ell m} \pm iB'_{\ell m})(-1)^\ell\,{}_{\mp 2}Y_\ell^m(\hat{n})$$
$$= -\sum (E_{\ell m} \mp iB_{\ell m})\,{}_{\mp 2}Y_\ell^m(\hat{n})$$

Temperature and Polarization Power Spectra

- For $\Delta T(\hat{n}) = \sum T_{\ell m} Y_\ell^m(\hat{n})$ and $Q(\hat{n}) + iU(\hat{n}) = -\sum (E_{\ell m} + iB_{\ell m})_{\pm 2}Y_\ell^m(\hat{n})$, the power spectra are given by

$$C_\ell^{XY} = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} \mathrm{Re}(X_{\ell m} Y_{\ell m}^*)$$

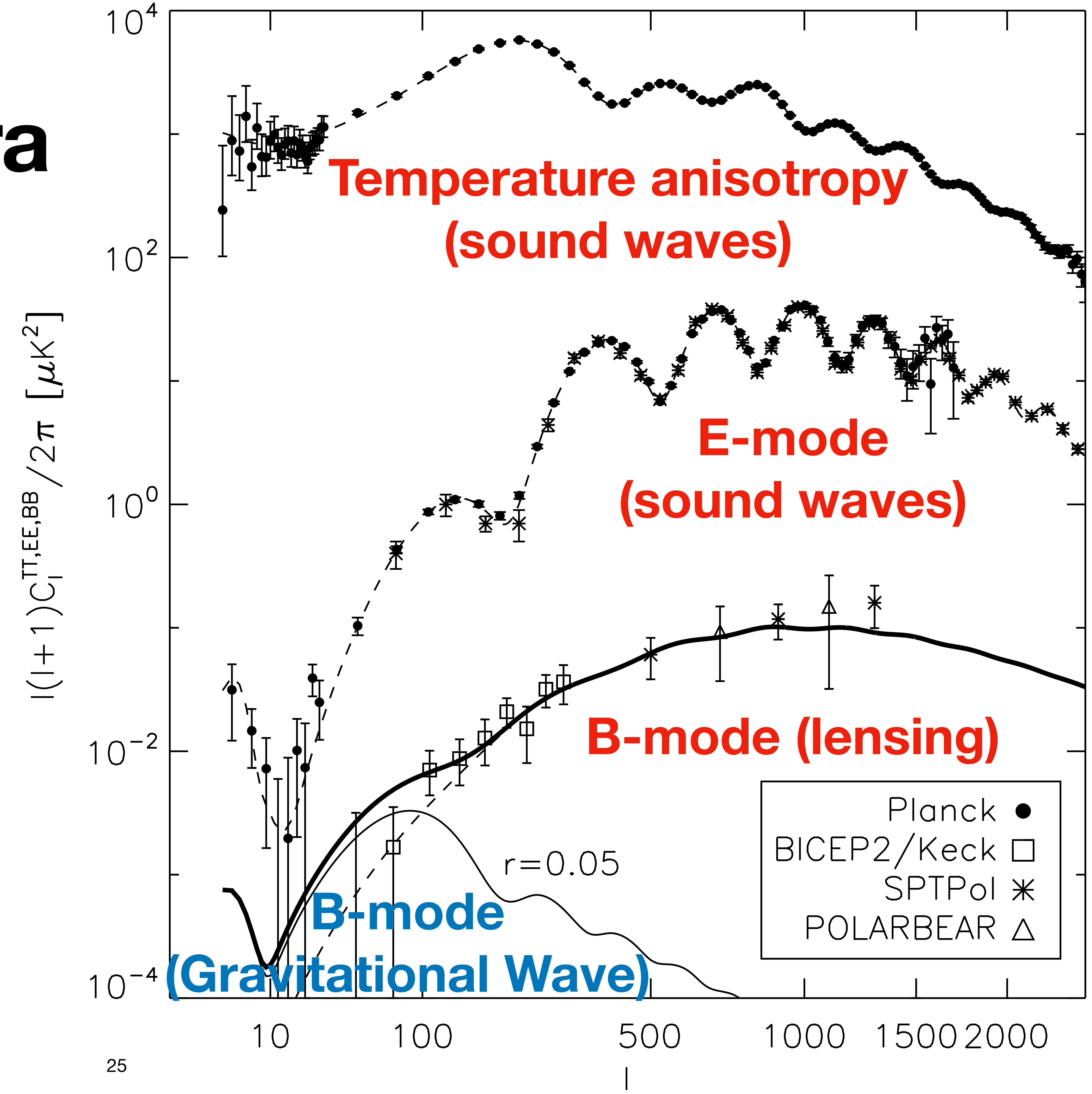
where (X,Y)=(T, E, B)

- C_ℓ^{TT}, C_ℓ^{TE}, C_ℓ^{EE}, and C_ℓ^{BB} have even parity, whereas $C_\ell^{TB}$and C_ℓ^{EB} have odd parity, which are **sensitive probes of violation of parity symmetry.**
  - 4 even-parity and 2 odd-parity combinations.

CMB Power Spectra

Progress over 30 years

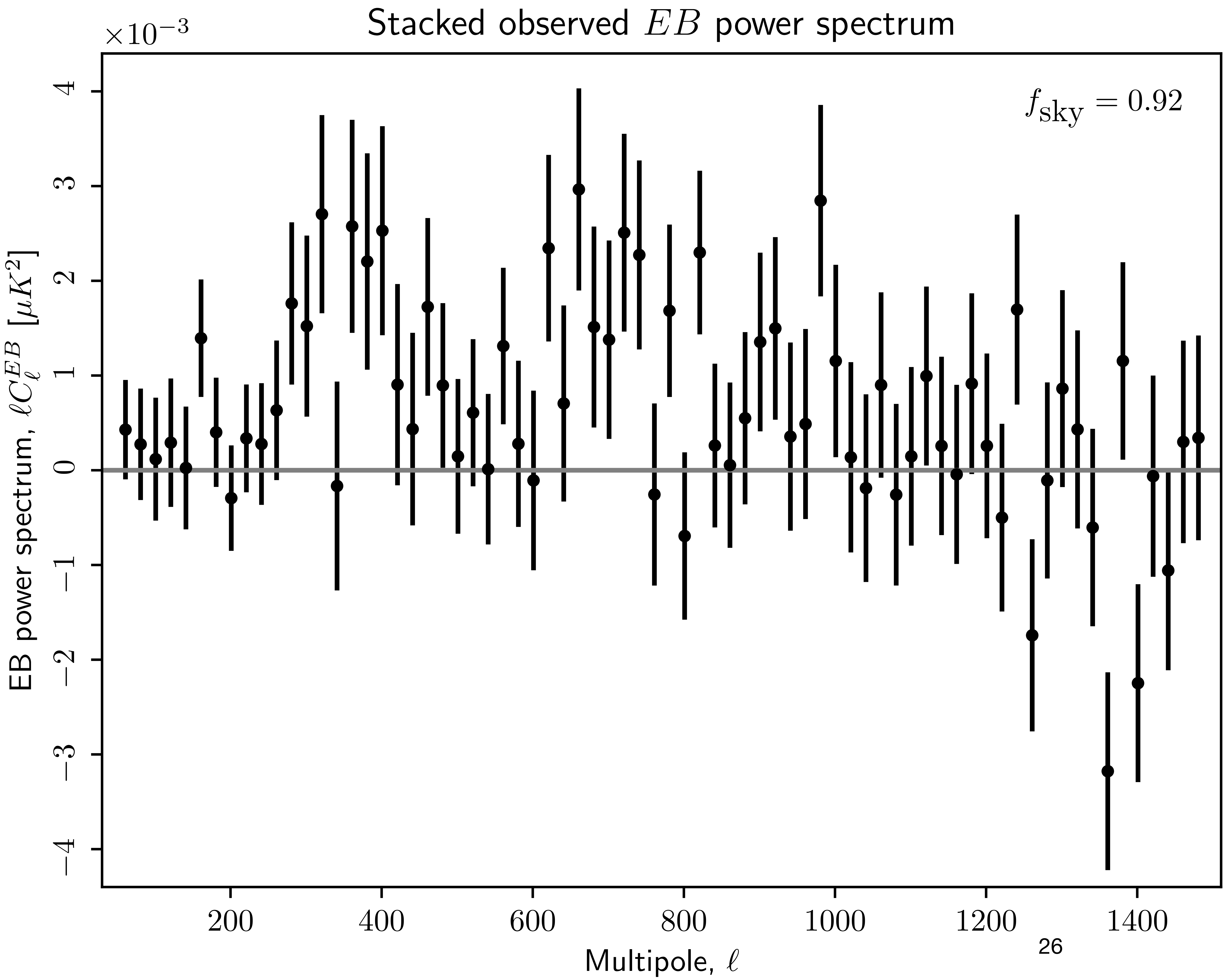
- This is the typical figure seen in talks and lectures on the CMB.
  - The temperature and the E- and B-mode polarization power spectra are well measured.
- **Parity violation appears in the TB and EB power spectra, not shown here.**

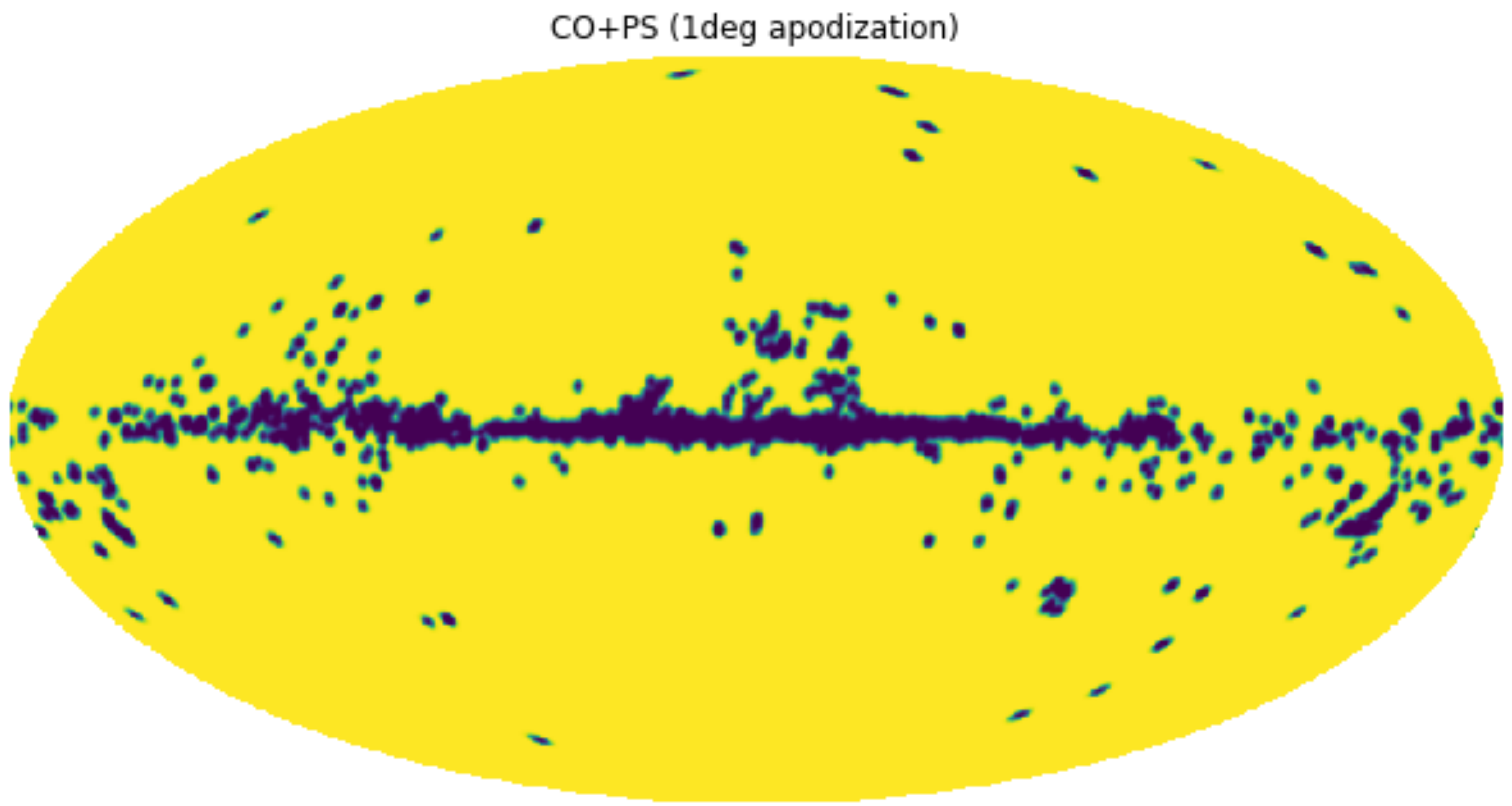


Eskilt, EK (2022)

This is the EB power spectrum (WMAP+Planck)

Nearly full-sky data (92% of the sky)



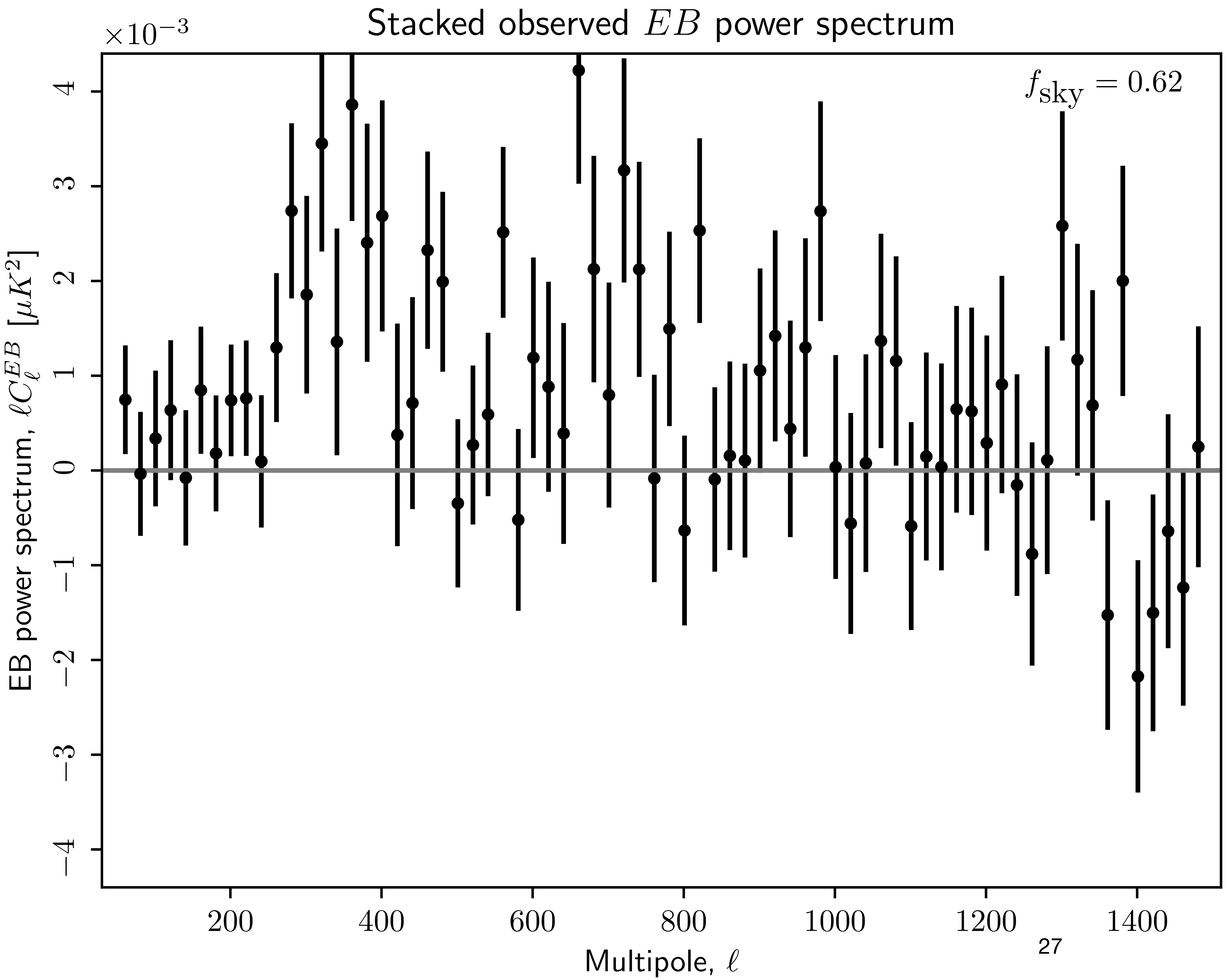


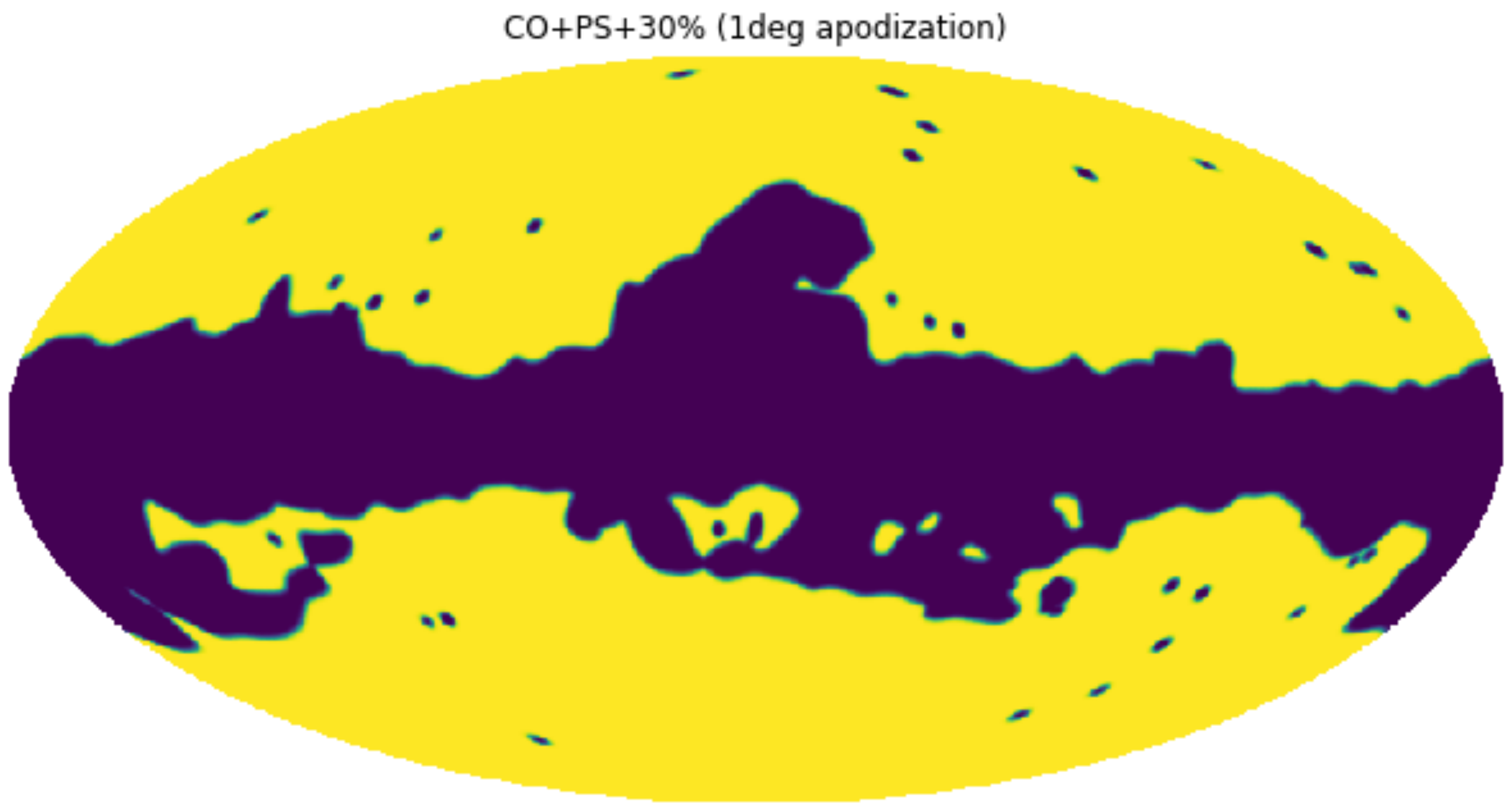
- χ^2 = 125.5 for DOF=72
    - Unambiguous signal of something!

Eskilt, EK (2022)

This is the EB power spectrum (WMAP+Planck)

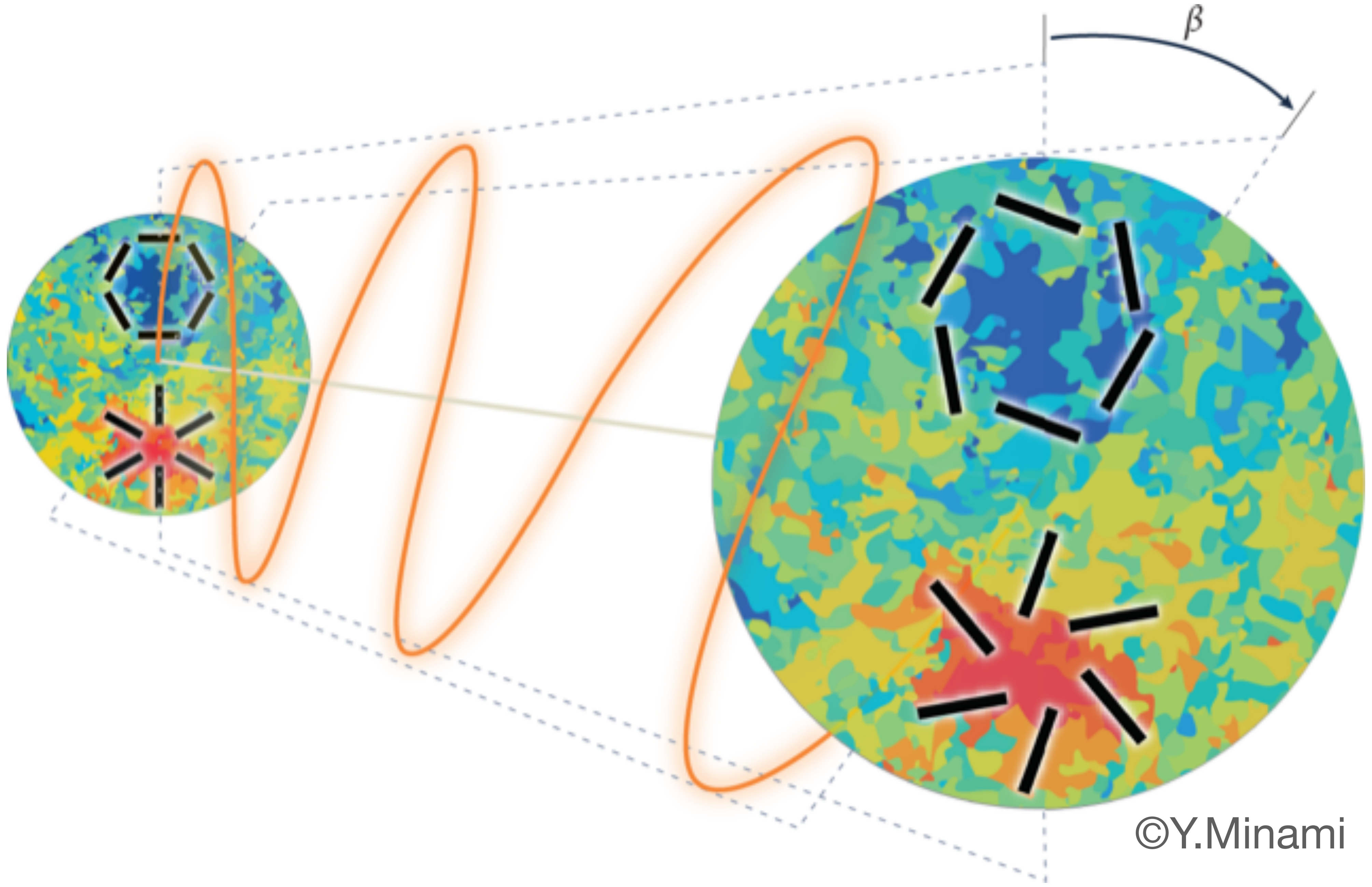
Galactic plane removed (62% of the sky)





- χ^2 = 138.4 for DOF=72
  - The signal exists regardless of the Galactic mask. This rules out the Galactic foreground.

7.3 Cosmic Birefringence in the CMB



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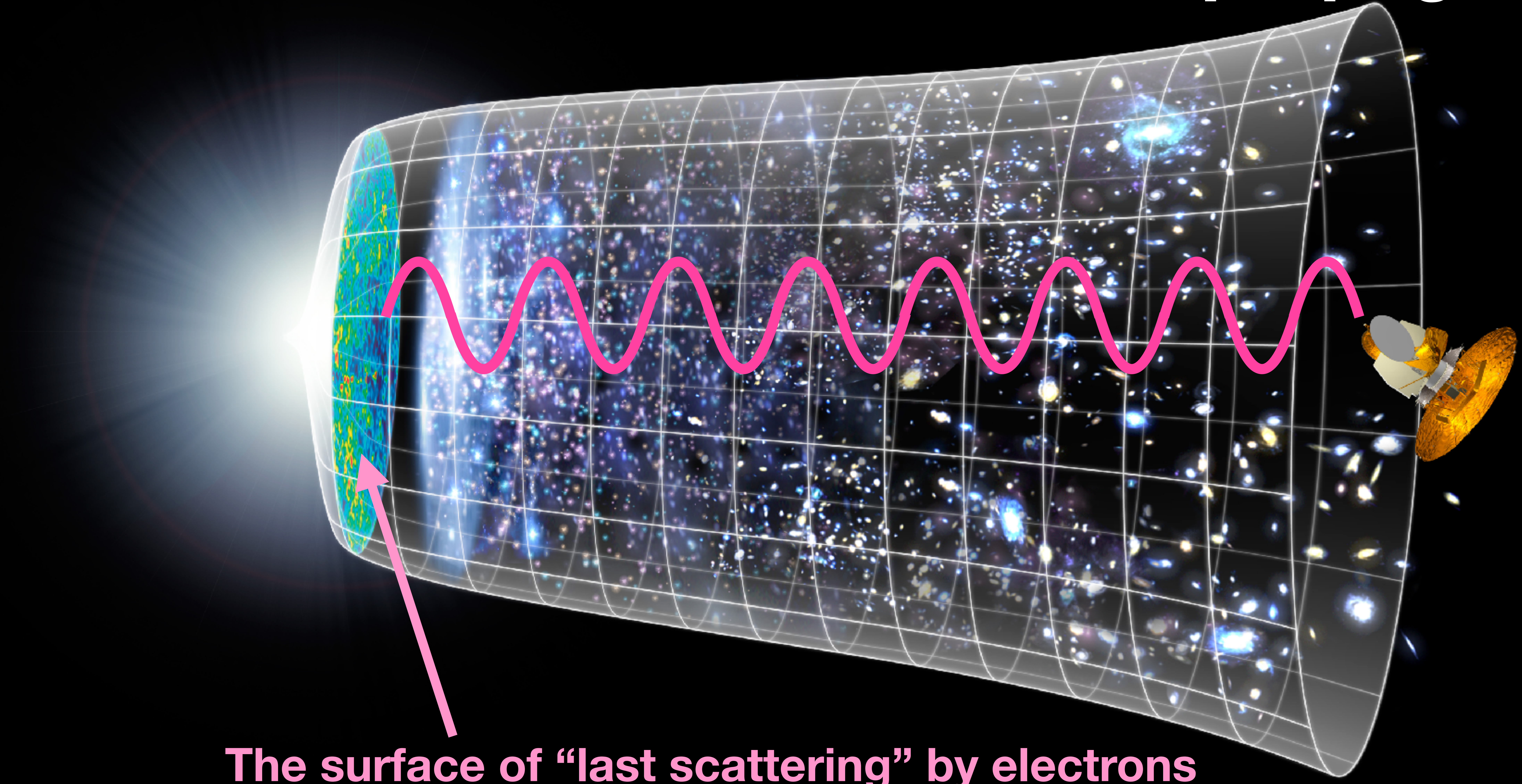
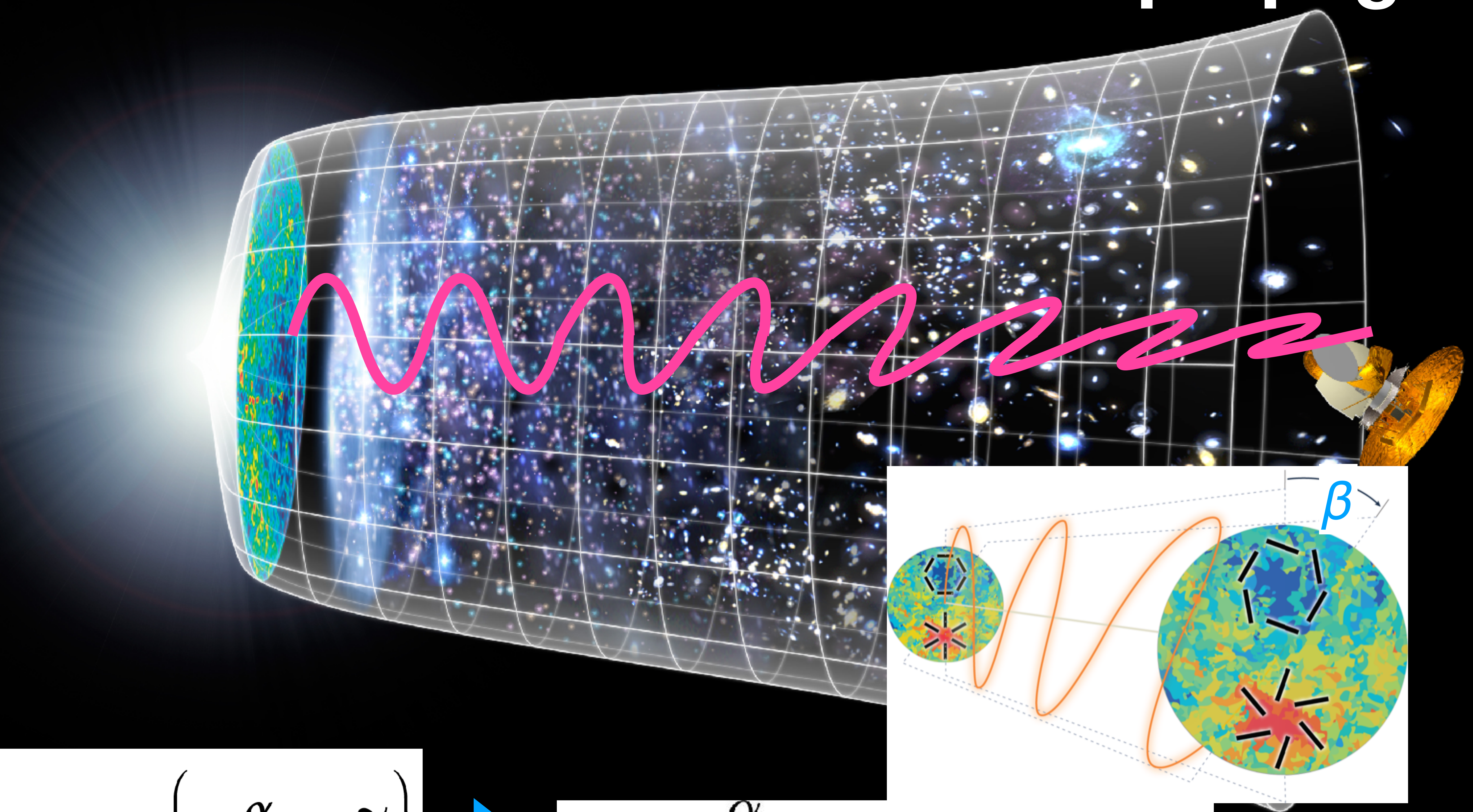
How does the EM wave of the CMB propagate?



Credit: WMAP Science Team

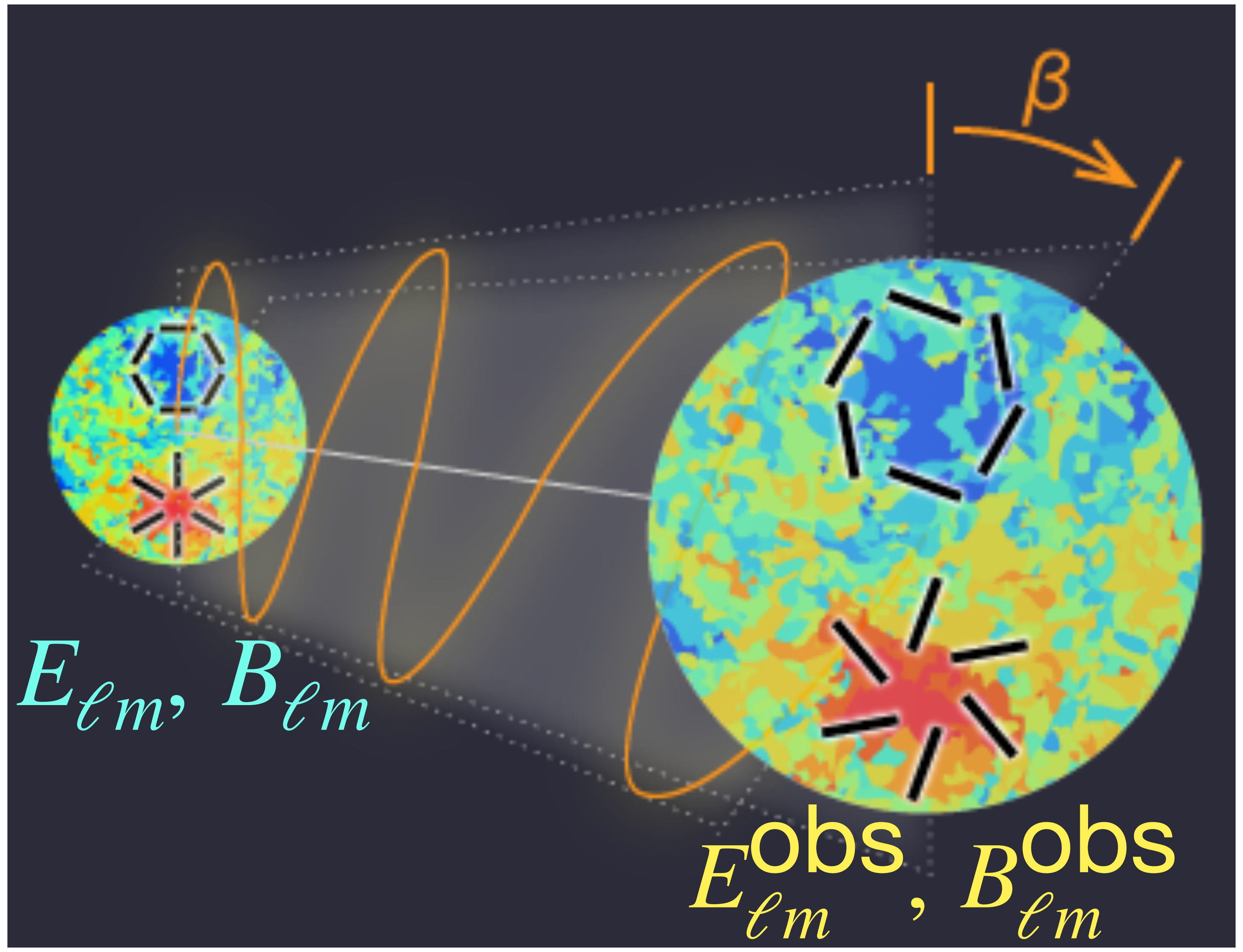
How does the EM wave of the CMB propagate?



$$I_{\mathrm{CS}} = \int \mathrm{d}^4x \sqrt{-g}\left(-\frac{\alpha}{4f}\chi F\widetilde{F}\right)$$

$$\beta = +\frac{\alpha}{2f}\left[\chi(\tau_{\mathrm{obs}}) - \chi(\tau_{\mathrm{em}})\right]$$

EB from rotation of the plane of linear polarization



- Stokes parameters can be written as (Day 5)

$$Q \pm iU = Pe^{\pm 2i\text{PA}}$$

- Cosmic birefringence shifts the position angle (PA) by PA –> PA + β. Thus, the observed E and B modes are related to those at the surface of last scattering as

$$E^{\rm obs}_{\ell m} \pm iB^{\rm obs}_{\ell m} = (E_{\ell m} \pm iB_{\ell m})e^{\pm 2i\beta}$$

$$\Rightarrow \quad E^{\rm obs}_{\ell m} = E_{\ell m}\cos(2\beta) - B_{\ell m}\sin(2\beta)$$

$$B^{\rm obs}_{\ell m} = E_{\ell m}\sin(2\beta) + B_{\ell m}\cos(2\beta)$$

Lue, Wang, Kamionkowski (1999); Feng et al. (2005); Liu, Lee, Ng (2006)
Zhao et al. (2015)

Searching for cosmic birefringence

- The observed polarization power spectra are given by

$$C_\ell^{EE,\mathrm{obs}} = C_\ell^{EE}\cos^2(2\beta) + C_\ell^{BB}\sin^2(2\beta) - C_\ell^{EB}\sin(4\beta)$$

$$C_\ell^{BB,\mathrm{obs}} = C_\ell^{EE}\sin^2(2\beta) + C_\ell^{BB}\cos^2(2\beta) + C_\ell^{EB}\sin(4\beta)$$

$$C_\ell^{EE,\mathrm{obs}} - C_\ell^{BB,\mathrm{obs}} = (C_\ell^{EE} - C_\ell^{BB})\cos(4\beta) - 2C_\ell^{EB}\sin(4\beta)$$

- We find

$$C_\ell^{EB,\mathrm{obs}} = \frac{1}{2}(C_\ell^{EE} - C_\ell^{BB})\sin(4\beta) + C_\ell^{EB}\cos(4\beta)$$

$$= \frac{1}{2}\underline{(C_\ell^{EE,\mathrm{obs}} - C_\ell^{BB,\mathrm{obs}})}\tan(4\beta) + \frac{C_\ell^{EB}}{\cos(4\beta)}$$

EB is given by the *difference* between EE and BB spectra.

Lue, Wang, Kamionkowski (1999); Feng et al. (2005); Liu, Lee, Ng (2006)
Zhao et al. (2015)

Searching for cosmic birefringence

- Similarly,

$$C_\ell^{TB,\mathrm{obs}} = C_\ell^{TE,\mathrm{obs}} \tan(2\beta) + \frac{C_\ell^{TB}}{\cos(2\beta)}$$

- We find

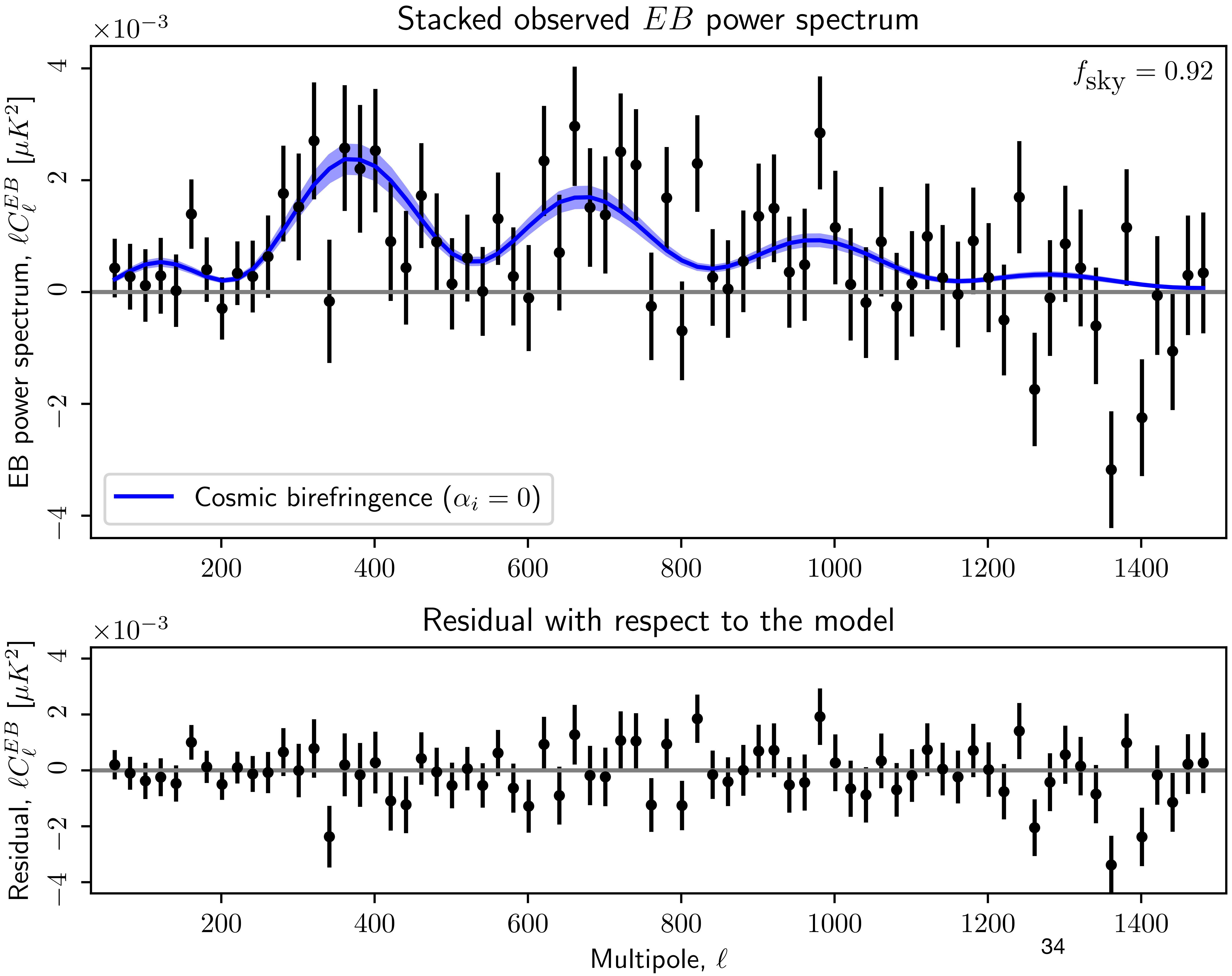
$$C_\ell^{EB,\mathrm{obs}} = \frac{1}{2}(C_\ell^{EE} - C_\ell^{BB}) \sin(4\beta) + C_\ell^{EB} \cos(4\beta)$$

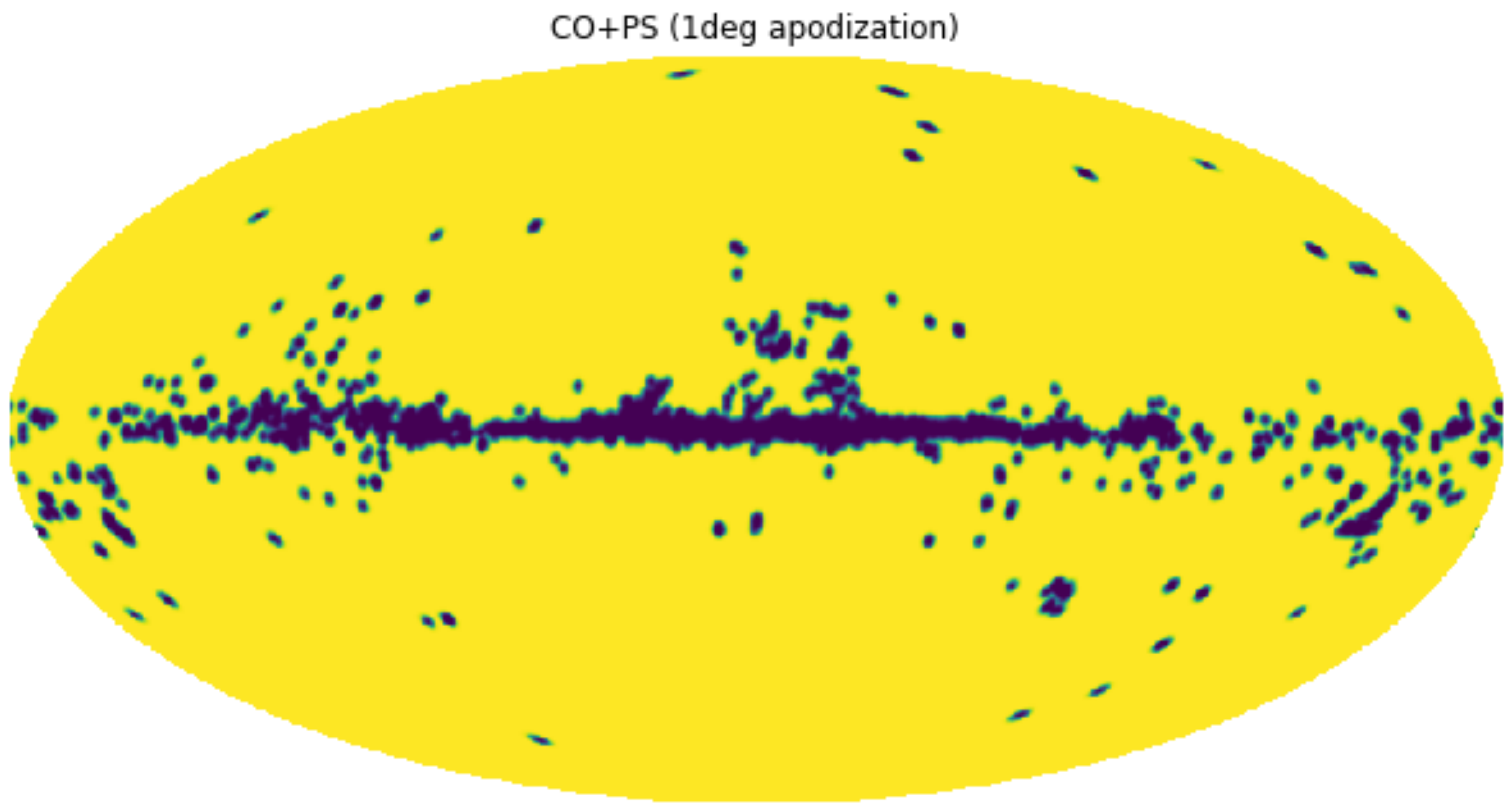
$$= \frac{1}{2}(C_\ell^{EE,\mathrm{obs}} - C_\ell^{BB,\mathrm{obs}}) \tan(4\beta) + \frac{C_\ell^{EB}}{\cos(4\beta)}$$

EB is given by the *difference* between EE and BB spectra.

Cosmic Birefringence fits well(?)

Nearly full-sky data (92% of the sky)

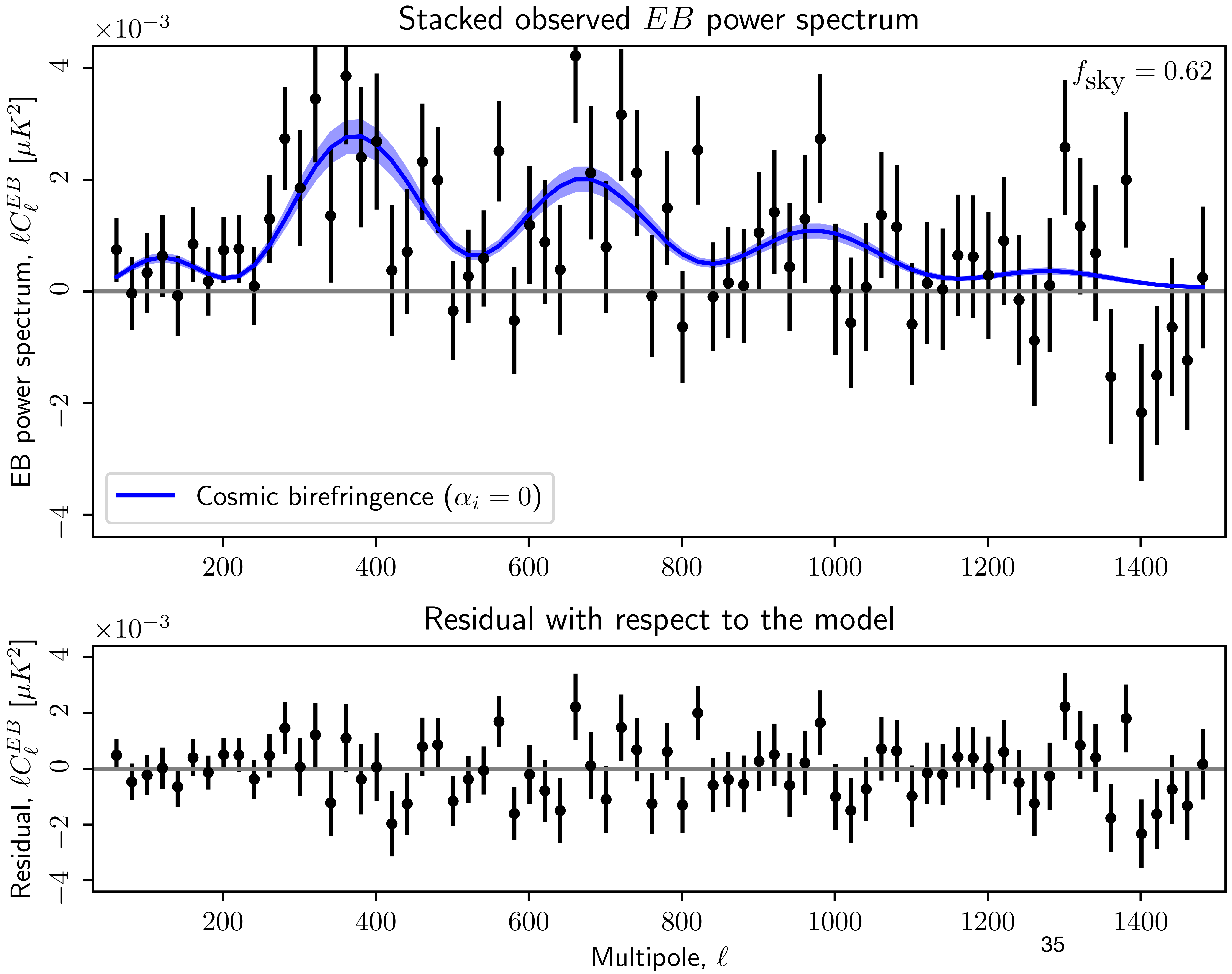


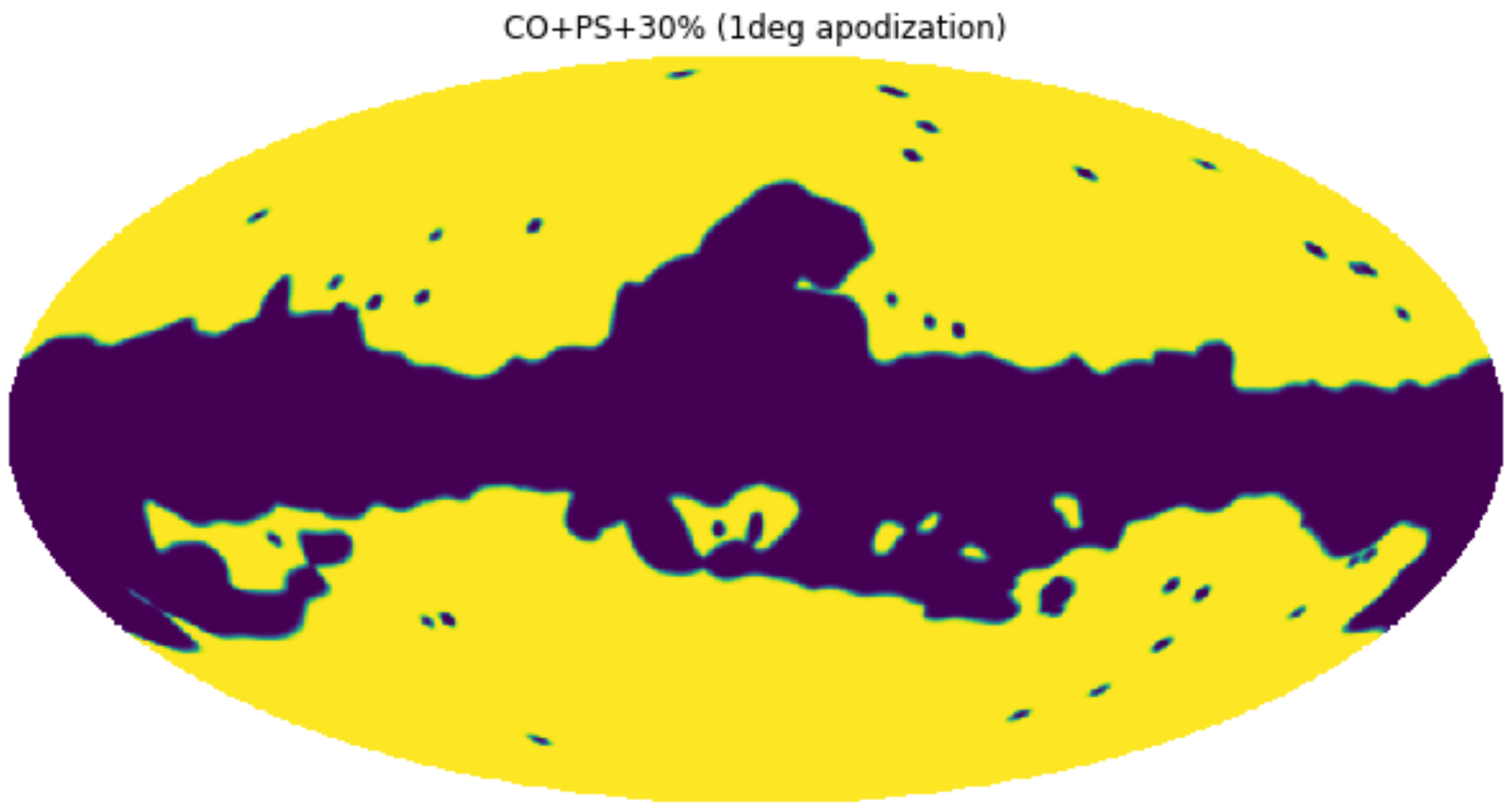


- β = 0.288 ± 0.032 deg
- $\chi^2 = 66.1$
  - Good fit! 9σ detection?

Cosmic Birefringence fits well(?)

Galactic plane removed (62% of the sky)



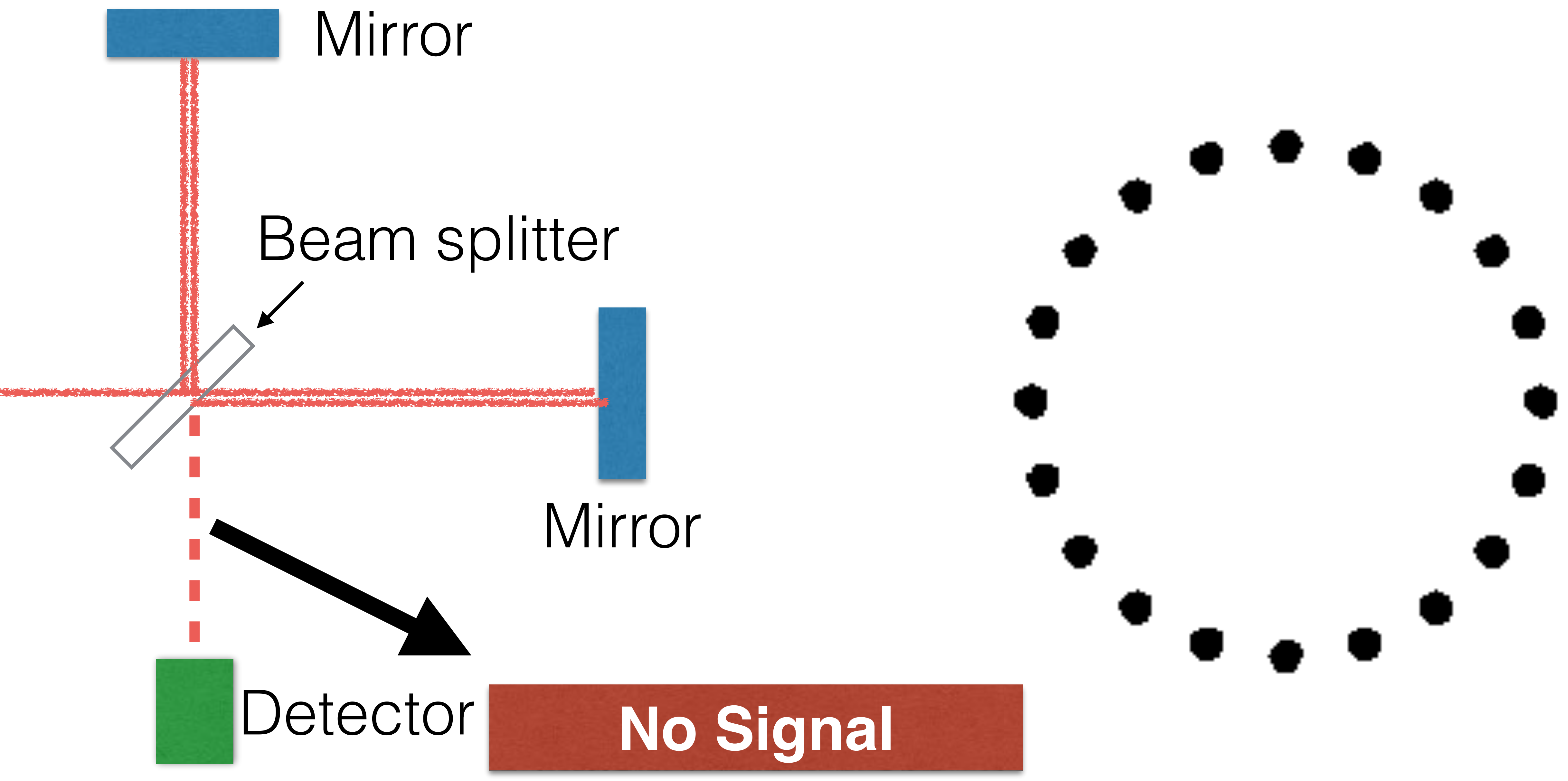


- β = 0.330 ± 0.035 deg
- $\chi^2 = 64.5$
- Signal is robust with respect to the Galactic mask.

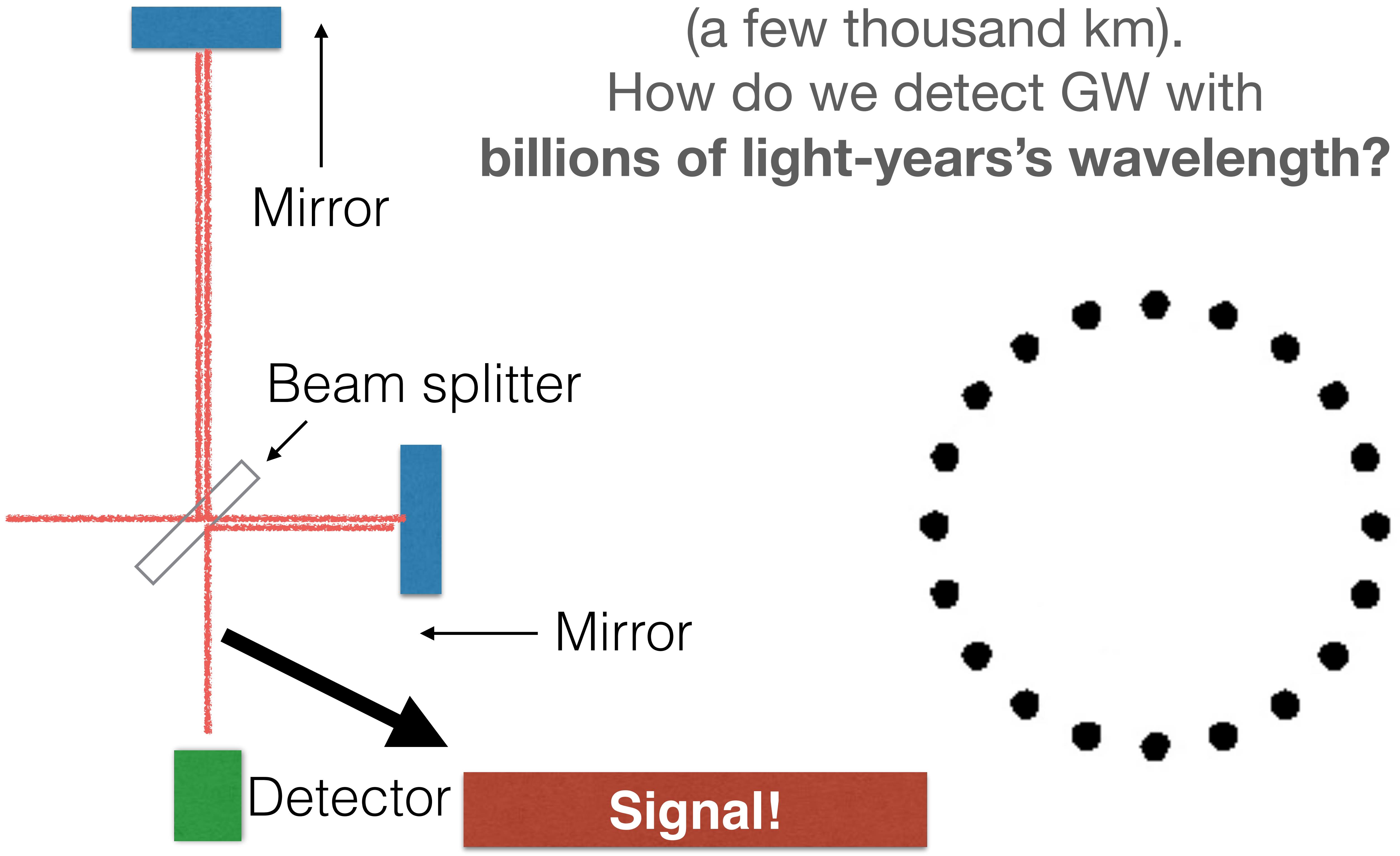
7.4 CMB Polarization from GW

How to detect GW?

Laser interferometer technique, used by LIGO and VIRGO

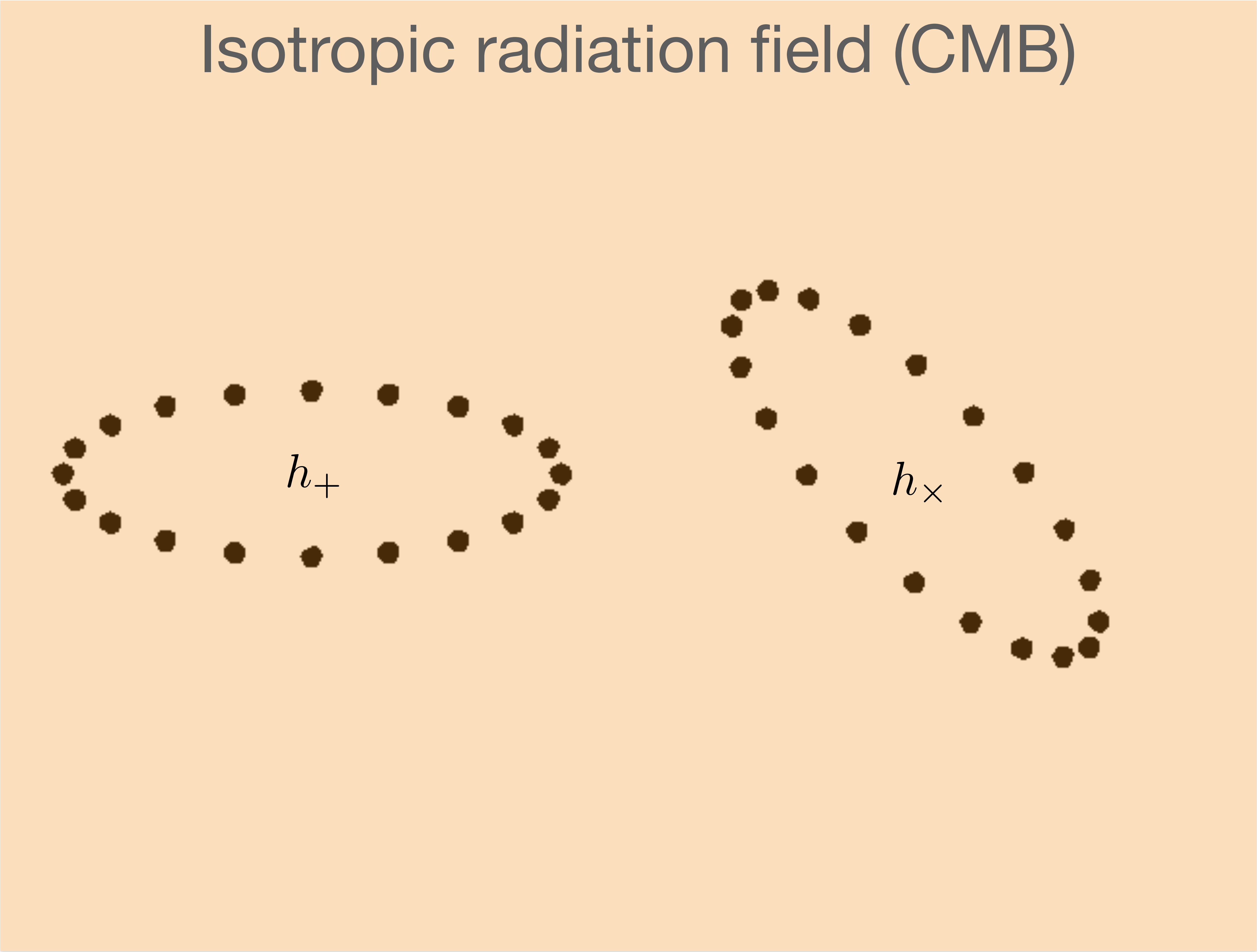


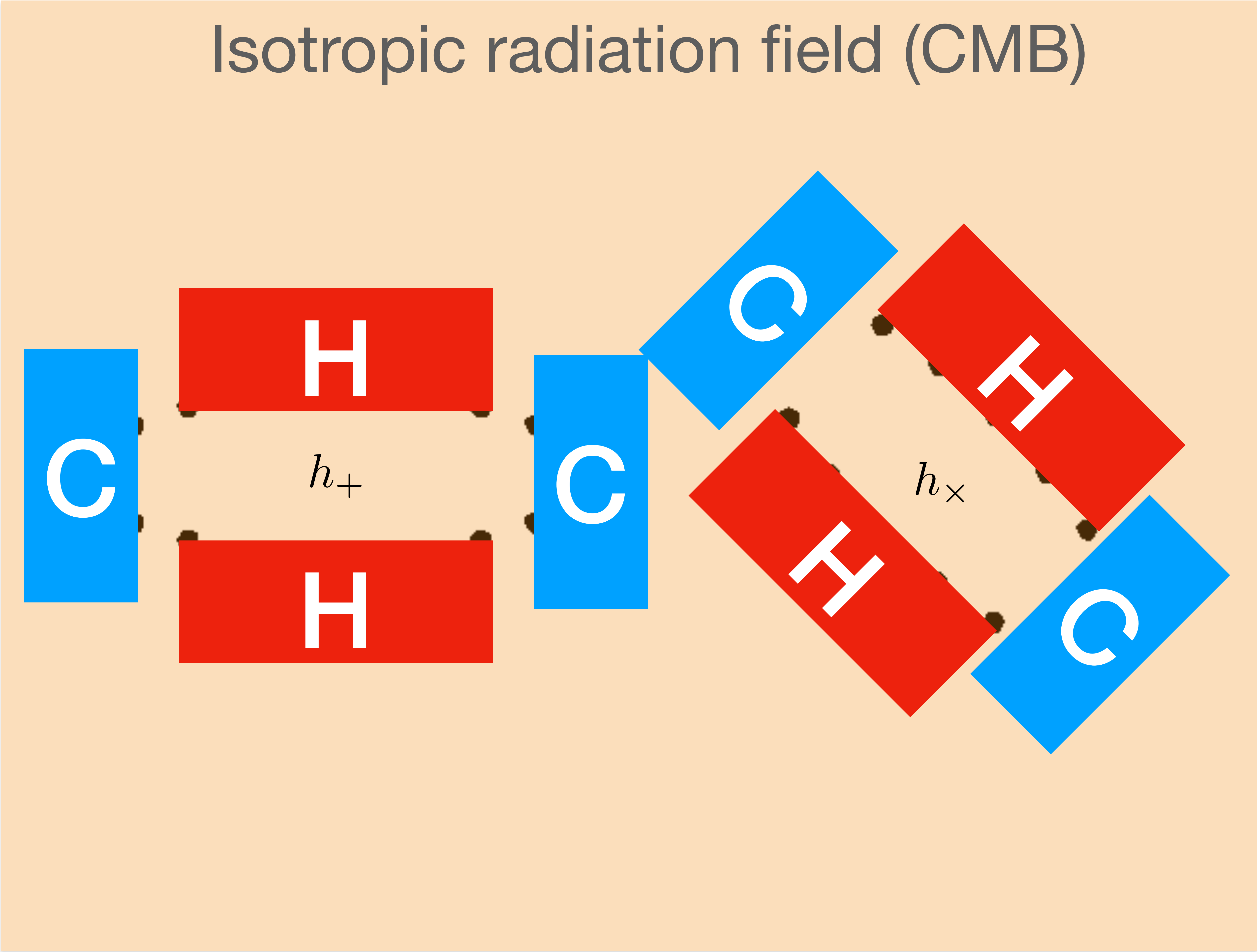
The wavelength of GW detectable by this method is the size of Earth (a few thousand km).
How do we detect GW with **billions of light-years's wavelength?**



Detecting GW by CMB

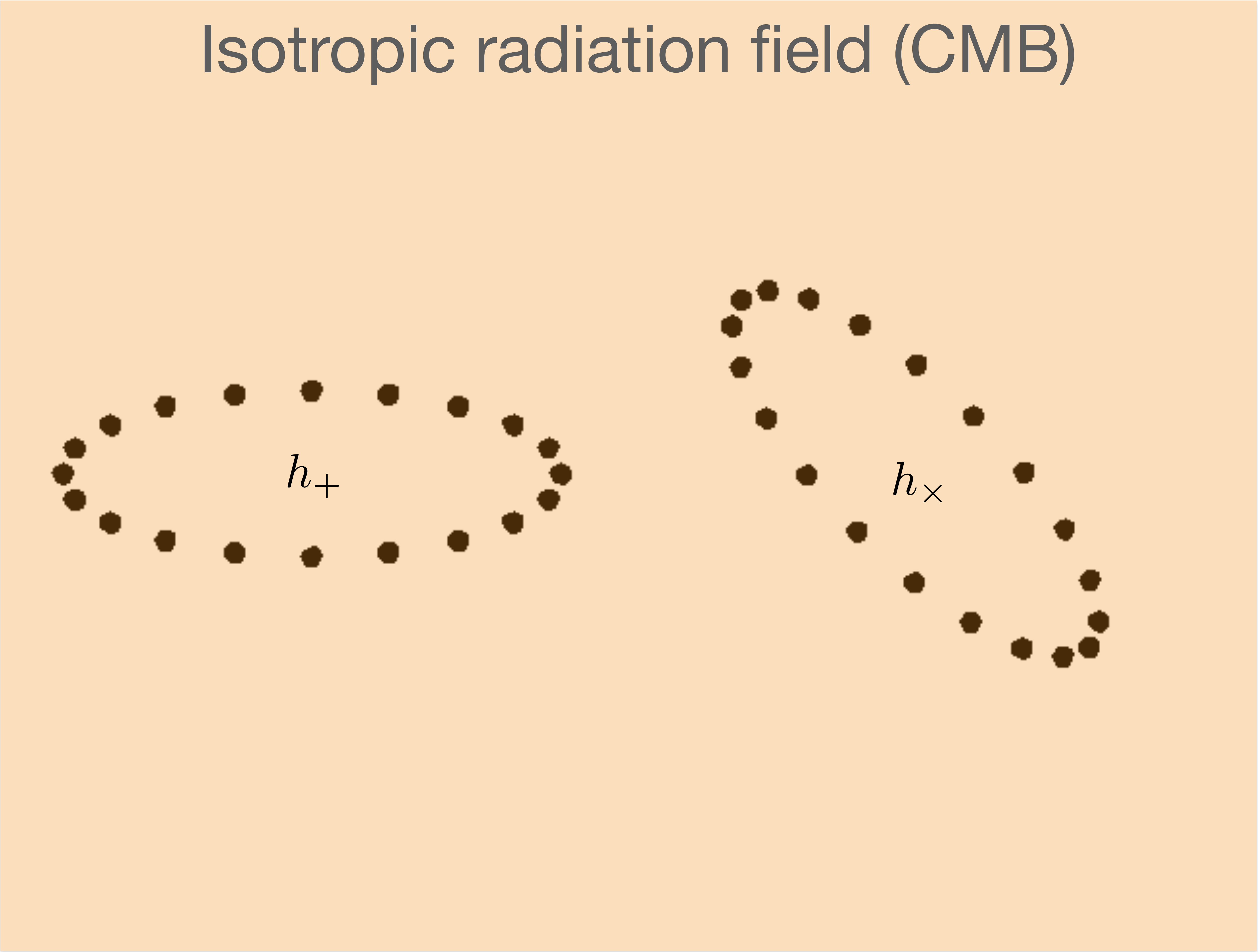
Quadrupole temperature anisotropy generated by red- and blue-shifting of photons

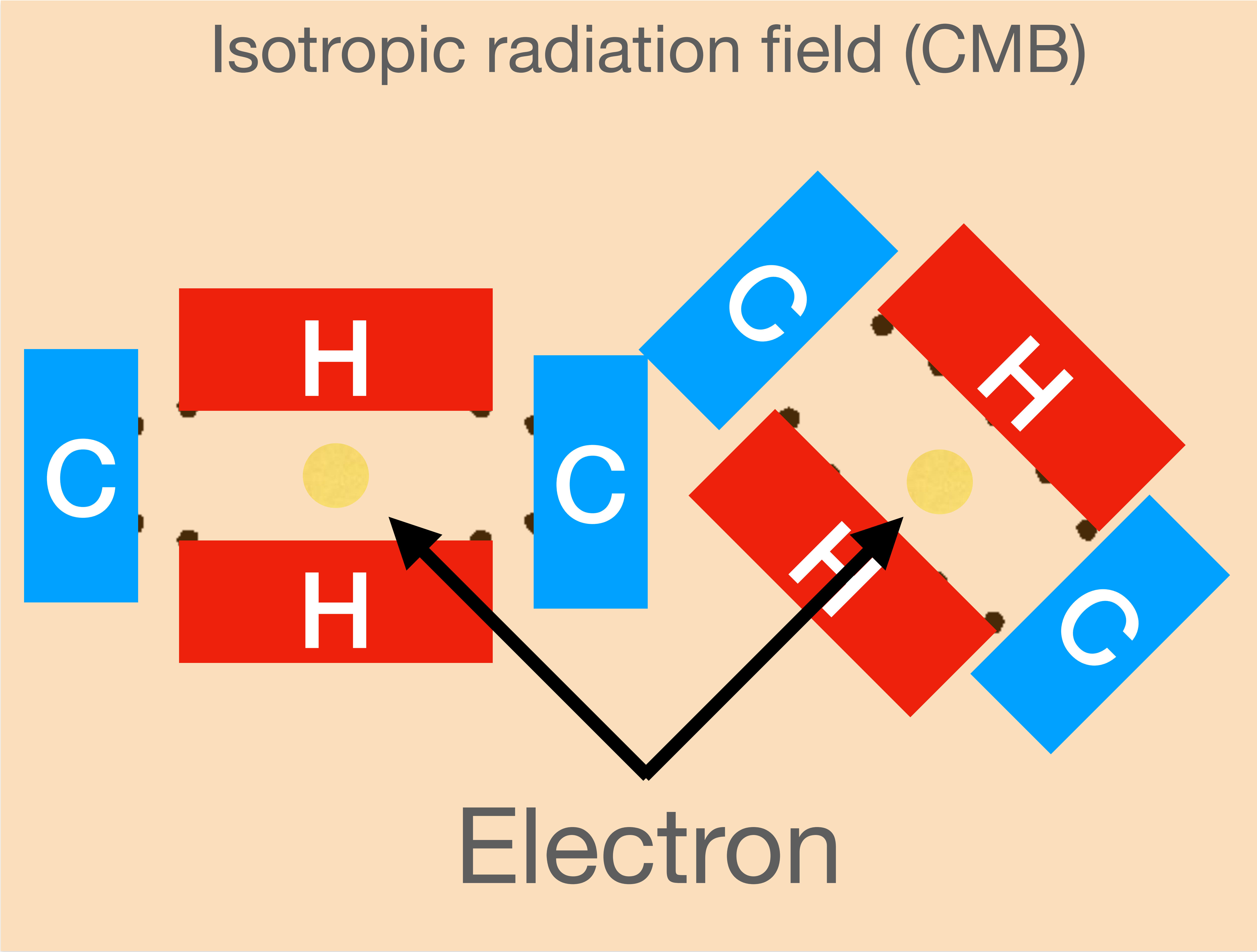




Detecting GW by CMB

Quadrupole temperature anisotropy generated by red- and blue-shifting of photons

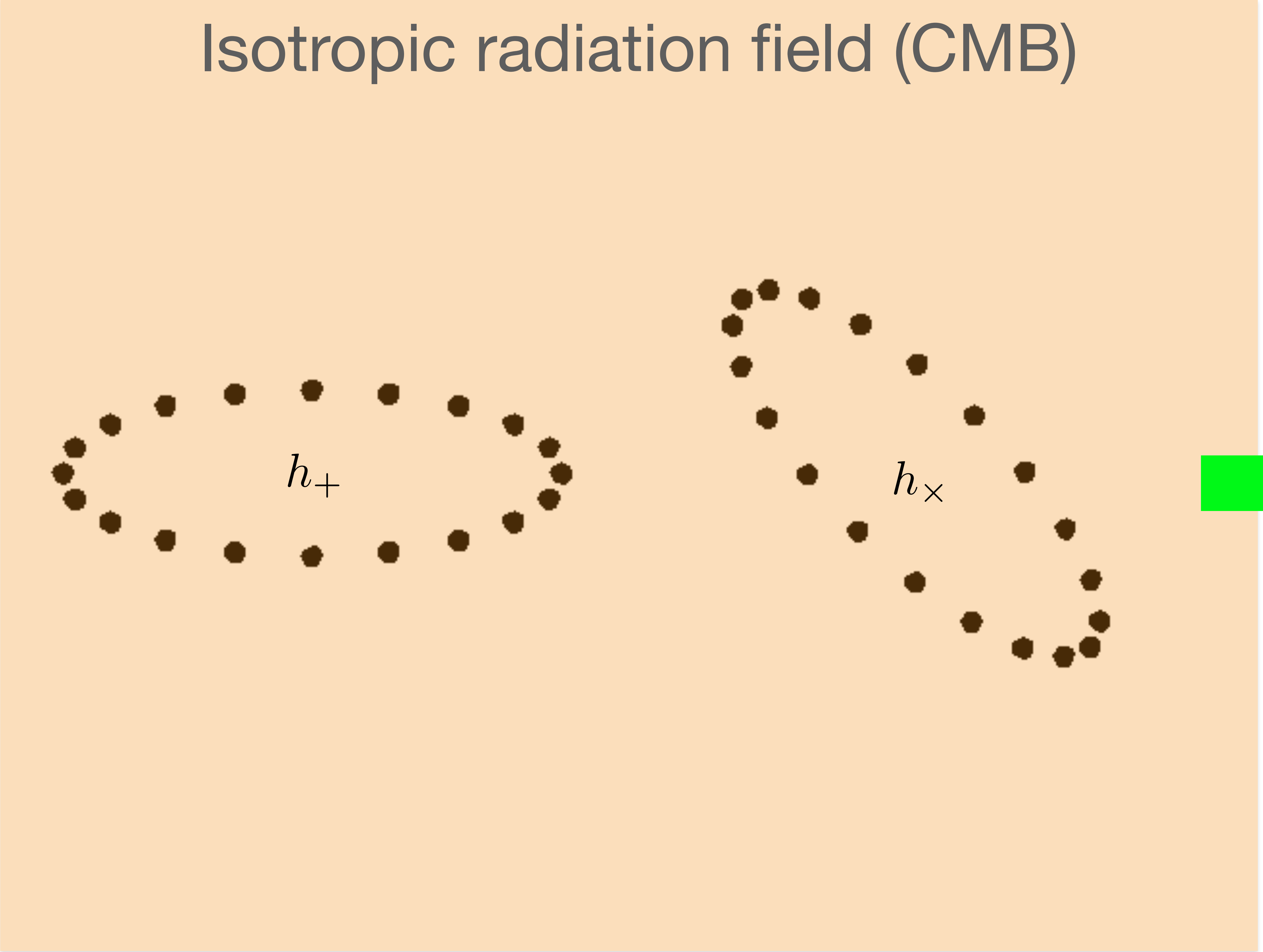


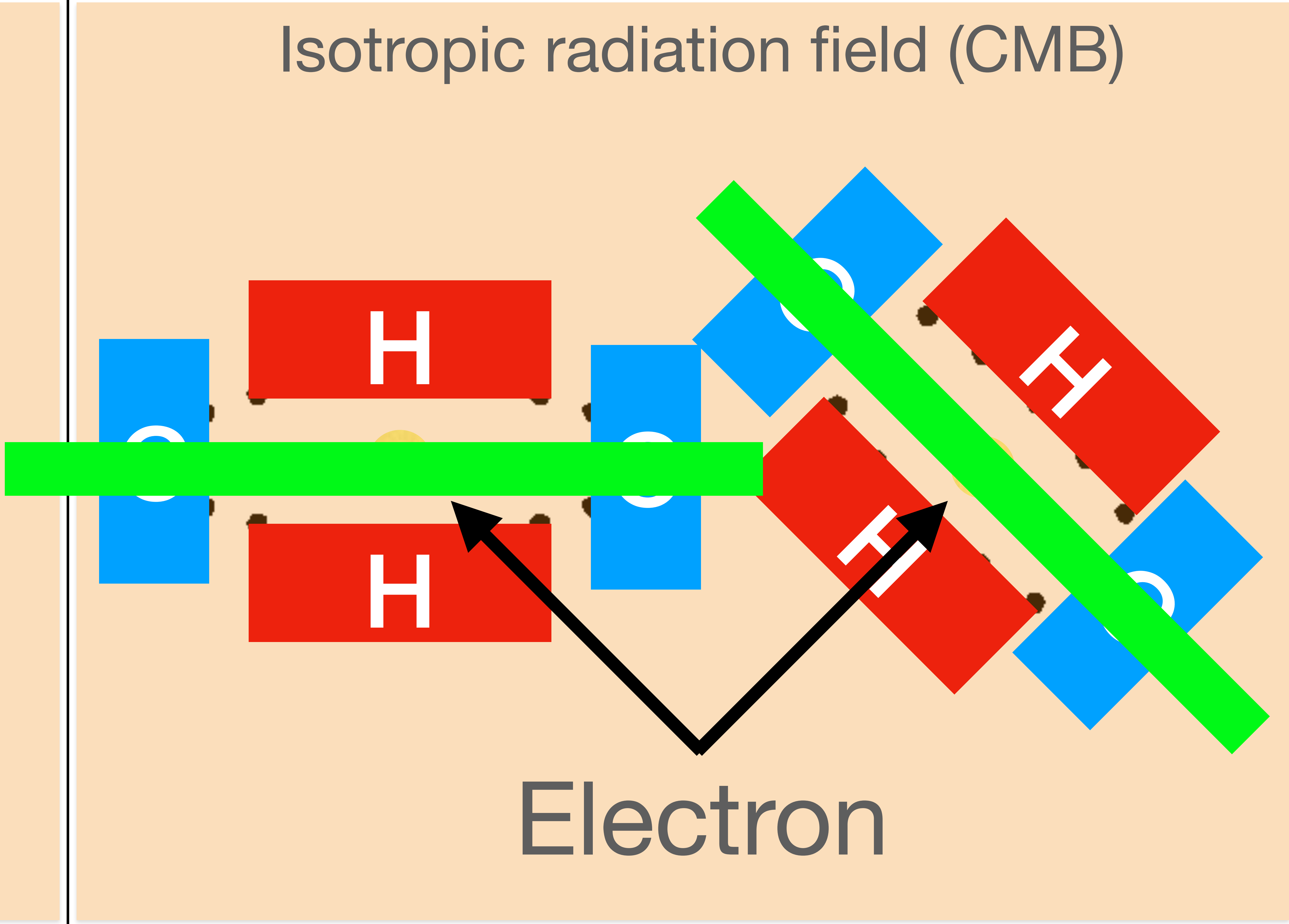


Polnarev (1985)

Detecting GW by CMB *Polarization*

Quadrupole temperature anisotropy scattered by an electron

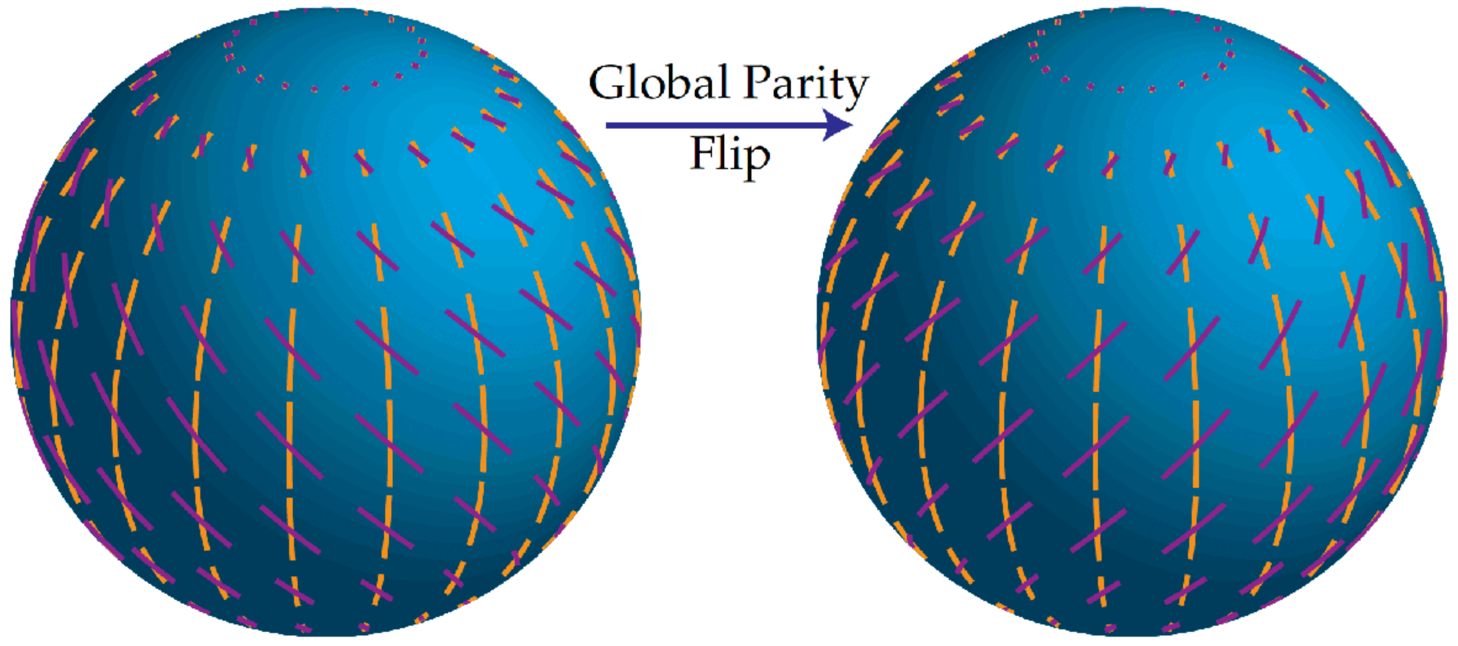


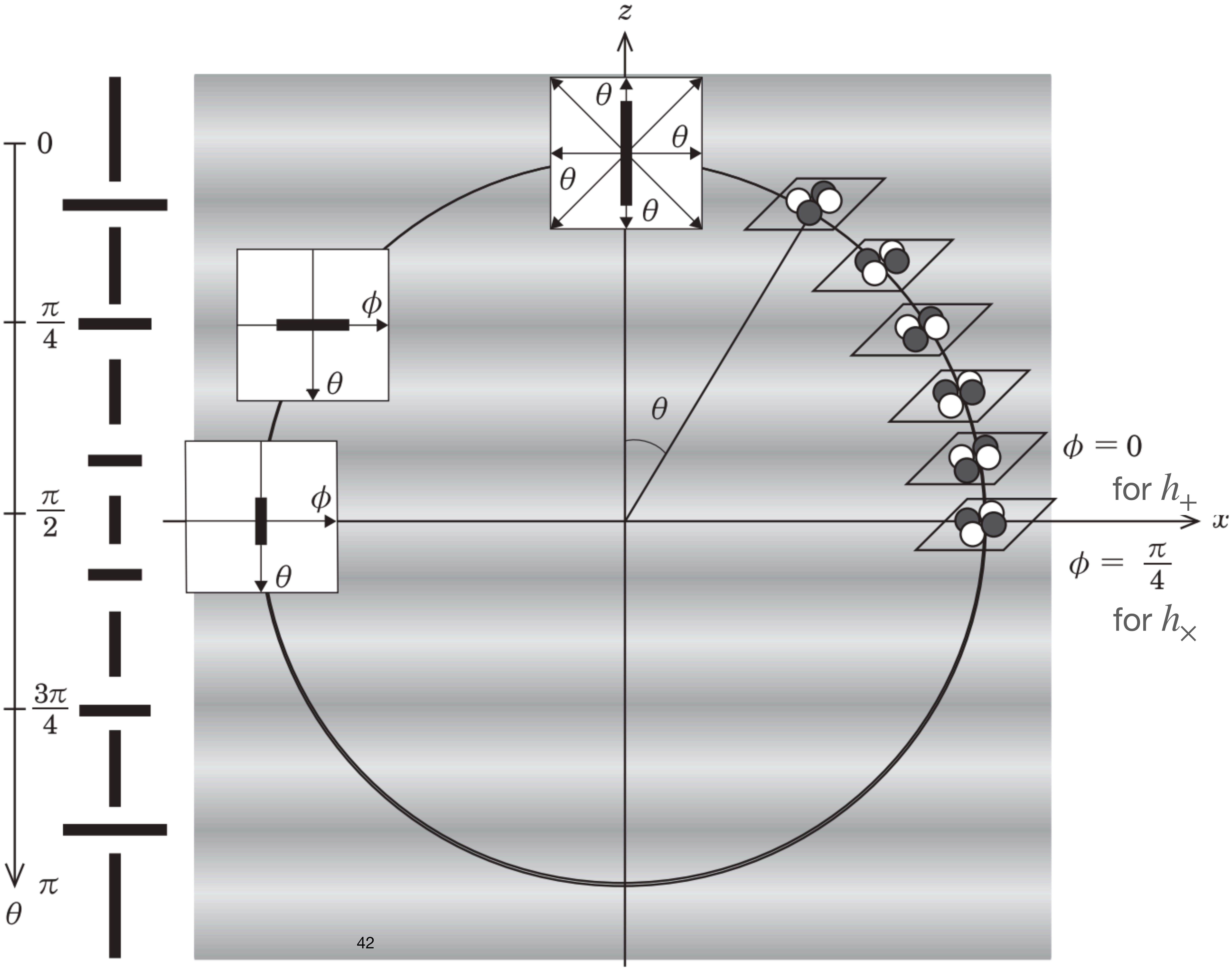


propagation direction of GW $\vec{k}$
z
$h_+=\cos(kz)$
$h_\times=\cos(kz)$

E modes from GW

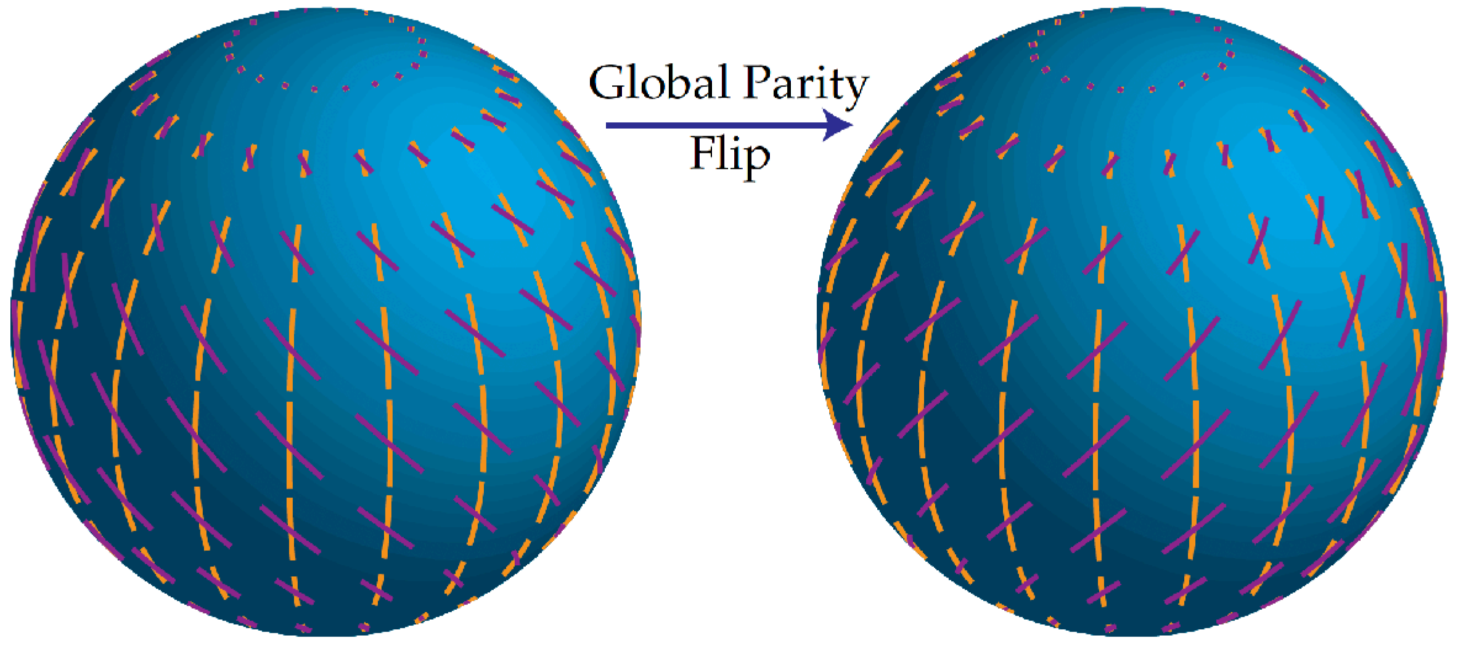
- GW is propagating in the z direction.
- This pattern has even parity.
  - **E-mode polarization.**

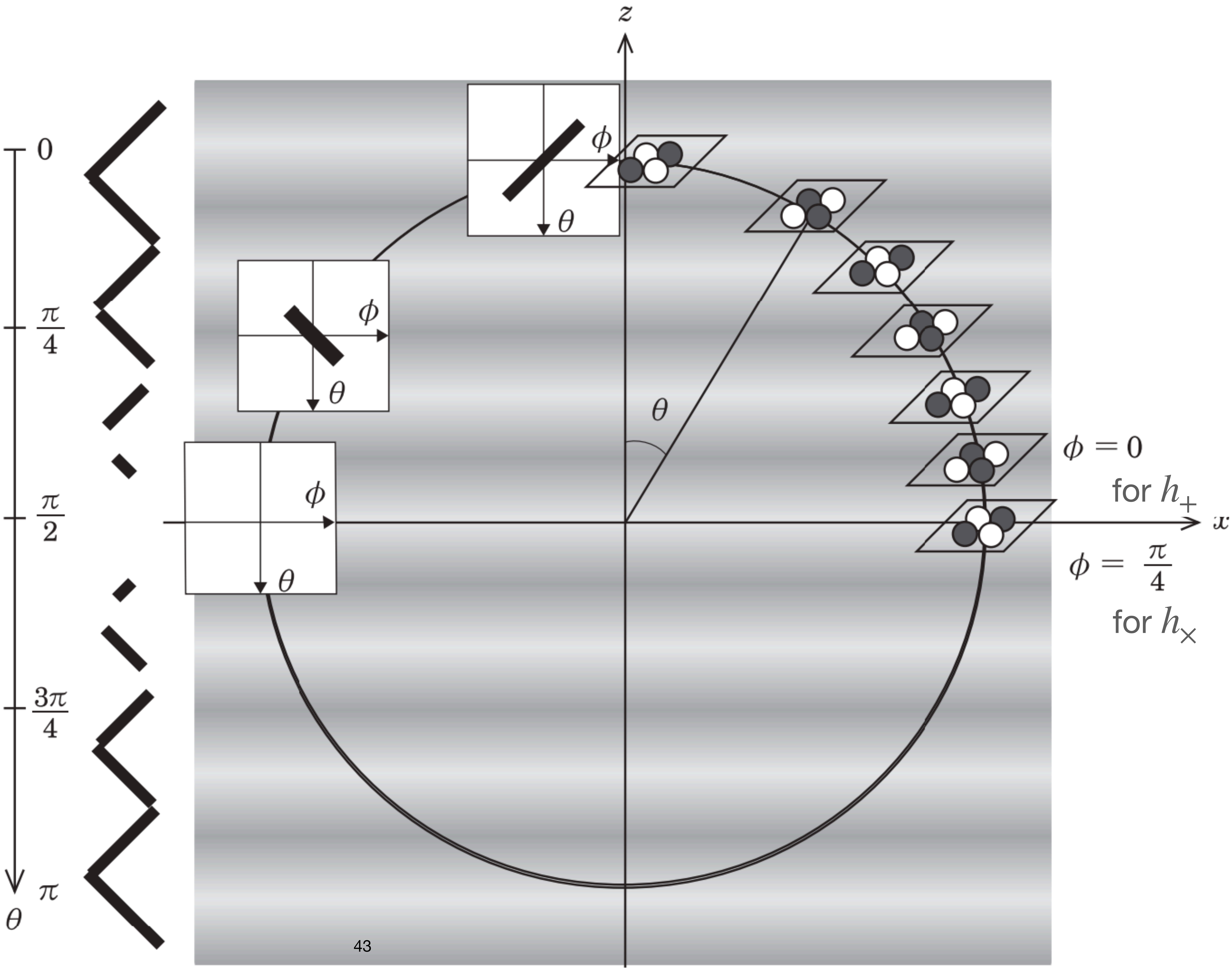




B modes from GW

- GW is propagating in the z direction.
- This pattern has odd parity.
    - **B-mode polarization.**

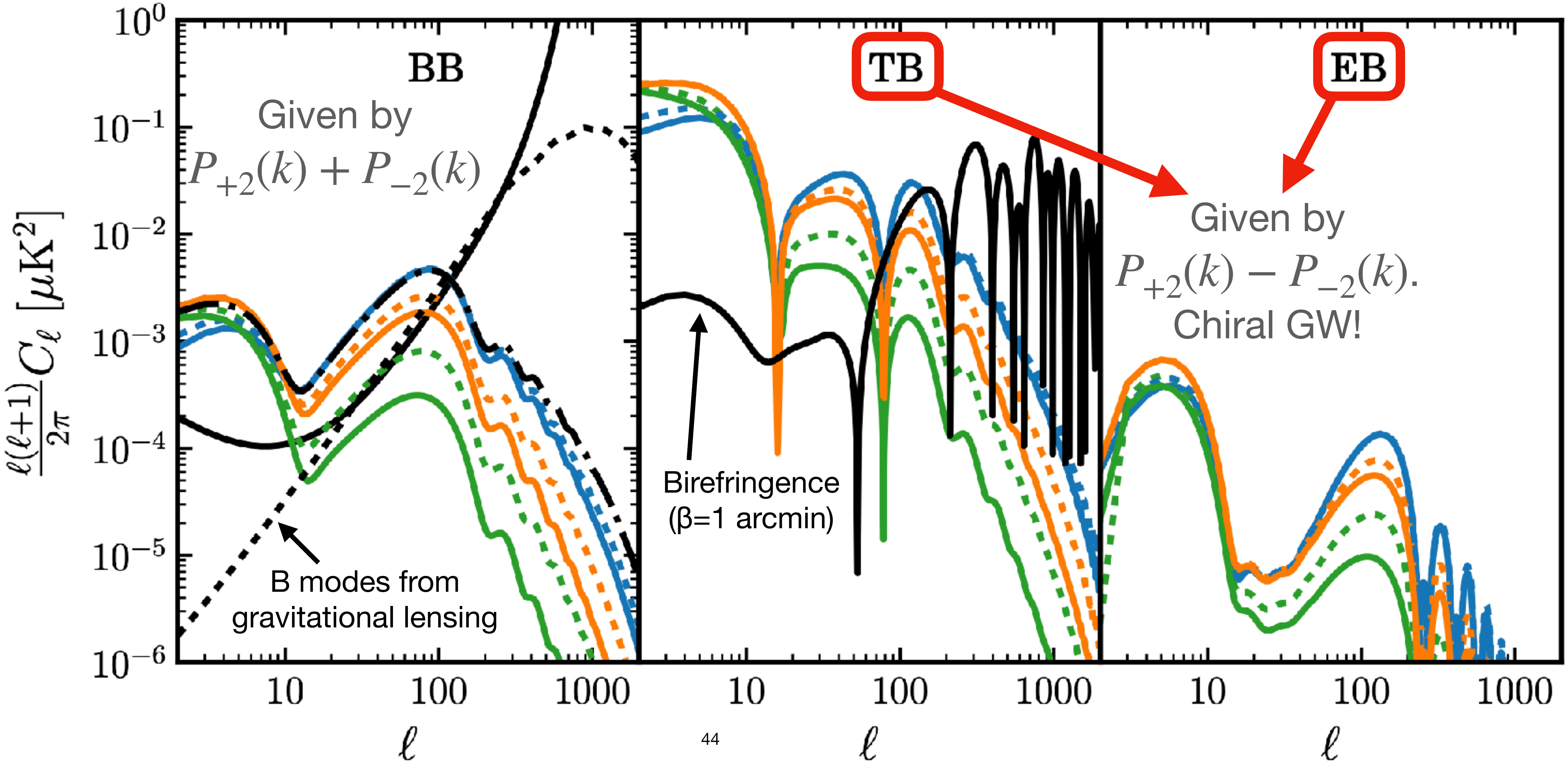




Lue et al. (1998); Saito, Ichiki, Taruya (2007); … ; Thorne et al. (2018)

TB and EB from Chiral Primordial GW

$$\Box h_{ij} = 16\pi G(E_i E_j + B_i B_j)^{\rm TT}$$



Recap: Day 6

- The CMB polarization is produced by Thomson scattering of a locally anisotropic temperature distribution around electrons at the surface of the last scattering.
- Both density fluctuations and GW generate a locally anisotropic temperature distribution around electrons.
- Using the spin-2 spherical harmonics, Stokes parameters $Q \pm iU$ can be decomposed into parity eigenstates called E and B modes with the opposite parity. GW can produce both E and B modes.
- The cross-correlation power spectra, TB and EB, are sensitive to parity-violating physics such as cosmic birefringence and chiral gravitational waves.

There is a signal in the EB power spectrum with a statistical significance of 9σ. What is the source of this signal?