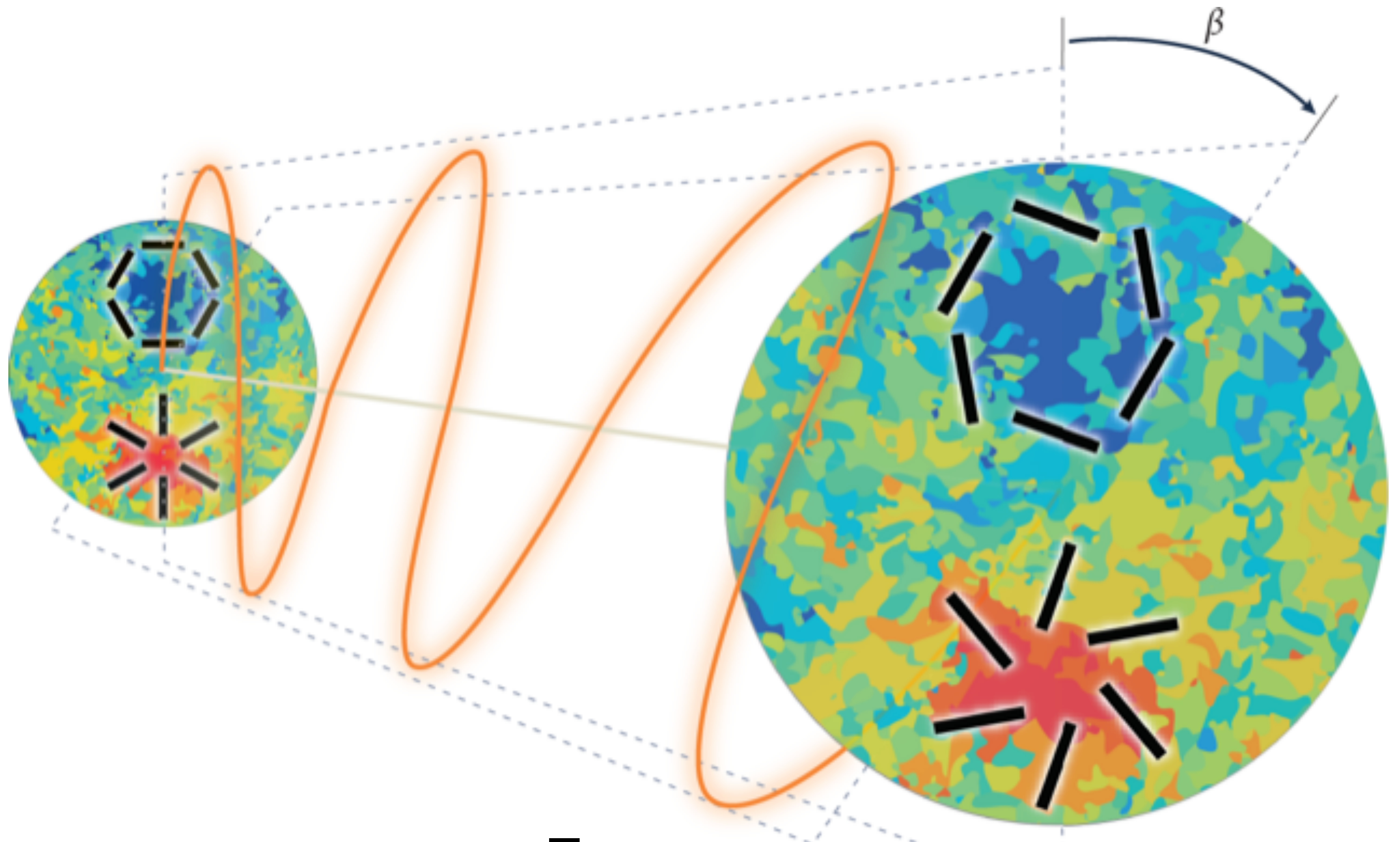
$$I_{\rm CS} = \int \mathrm{d}^4x \sqrt{-g} \left(-\frac{\alpha}{4f} \chi F \tilde{F} \right)$$



Parity Violation in Cosmology

In search of new physics for the Universe

The lecture slides are available at
https://wwwmpa.mpa-garching.mpg.de/~komatsu/lectures--reviews.html

Eiichiro Komatsu (Max Planck Institute for Astrophysics)
Nagoya University, June 6–30, 2023

Overarching Theme

Let's find new physics!

- The current cosmological model (*flat ΛCDM*) **requires** new physics beyond the standard model of elementary particles and fields.
    - What is dark matter (*CDM*)?
    - What is dark energy (*Λ*)?
    - Why is the spatial geometry of the Universe Euclidean (*flat*)?
    - What powered the Big Bang? What is the fundamental physics behind cosmic inflation?

Overarching Theme

There are many ideas

- The current cosmological model (*flat ΛCDM*) **requires** new physics beyond the standard model of elementary particles and fields.
    - What is dark matter (*CDM*)? => CDM, WDM, FDM, …
    - What is dark energy (*Λ*)? => Dynamical field, modified gravity, quantum gravity, …
    - Why is the spatial geometry of the Universe Euclidean (*flat*)? => Inflation, contracting universe, …
    - What powered the Big Bang? What is the fundamental physics behind cosmic inflation? => Scalar field, gauge field, …

Overarching Theme

There are many ideas

New in cosmology!

Violation of parity symmetry may hold the answer to these fundamental questions.

- The current cosmological model (*fl*
  the standard model of elementary
    - What is dark matter (*CDM*)? => CDM, WDM, FDM, …
    - What is dark energy (Λ)? => Dynamical field, modified gravity, quantum gravity, …
    - Why is the spatial geometry of the Universe Euclidean (*flat*)? => Inflation, contracting universe, …
    - What powered the Big Bang? What is the fundamental physics behind cosmic inflation? => Scalar field, gauge field, …

Reference:

nature reviews physics

Available also at arXiv:2202.13919

Review Article | Published: 18 May 2022

New physics from the polarized light of the cosmic microwave background

Eiichiro Komatsu ✉

Nature Reviews Physics **4**, 452–469 (2022) | Cite this article

Key Words:

1. Cosmic Microwave Background (CMB)
2. Polarization
3. Parity Symmetry

Structure of Lectures

7 x 85 minutes

- Each 85-min block roughly consists of
  - 30 minutes of lecture
  - 30 minutes of problem solving (so that I know you attended each lecture)
  - 25 minutes of lecture
- You can ask questions at any time during the lecture and problem solving. **You do not have to wait until the end of the lecture!**

The syllabus is available at https://syllabus.adm.nagoya-u.ac.jp/data/2023/26_2023_Y010802680538.html

Conventions and Units

- We will set $c = 1$ and $\hbar = 1$, unless otherwise noted.
- The distance between two points in the Minkowski spacetime is given by $ds^2 = -dt^2 + d\mathbf{x}^2$.
- Fourier transformation of a function $f(t, \mathbf{x})$ is given by $f(t, \mathbf{x}) = (2\pi)^{-3} \int d^3\mathbf{k}\, f_{\mathbf{k}}(t) e^{i\mathbf{k}\cdot\mathbf{x}}$.
- However, when quantizing the field with creation ($\hat{a}_{\mathbf{k}}$) and annihilation ($\hat{a}^{\dagger}_{\mathbf{k}}$) operators, we write $f(t, \mathbf{x}) = (2\pi)^{-3/2} \int d^3\mathbf{k} \left[u_k(t)\hat{a}_{\mathbf{k}} + u_k^*(t)\hat{a}^{\dagger}_{-\mathbf{k}} \right] e^{i\mathbf{k}\cdot\mathbf{x}}$, with the commutation relation given by $\left[\hat{a}_{\mathbf{k}}, \hat{a}^{\dagger}_{\mathbf{k}'}\right] = \delta_D(\mathbf{k} - \mathbf{k}')$. Here, $\delta_D(\mathbf{k})$ is Dirac's delta function.

Topics

From the syllabus

1. What is parity symmetry?

2. Chern-Simons interaction

3. Parity violation 1: Cosmic inflation

4. Parity violation 2: Dark matter

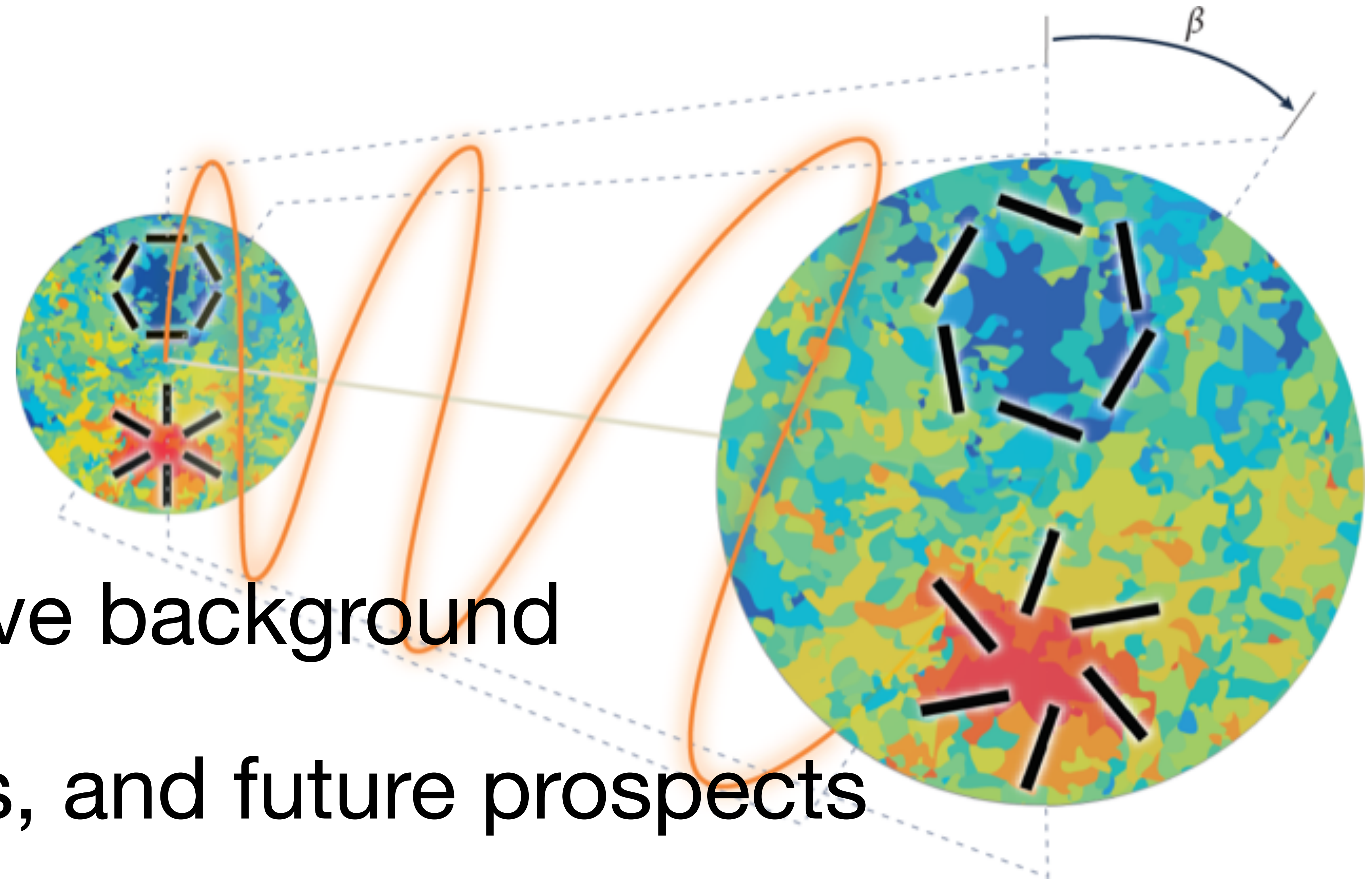
5. Parity violation 3: Dark energy

6. Light propagation: birefringence

7. Physics of polarization of the cosmic microwave background

8. Recent observational results, their implications, and future prospects

$$I_{\mathrm{CS}} = \int \mathrm{d}^4x \sqrt{-g} \left(-\frac{\alpha}{4f} \chi F \widetilde{F} \right)$$



1.1 Parity

What is “parity symmetry”?

Definition

- **Parity transformation = Inversion of spatial coordinates**
  - (x, y, z) –> (-x, -y, -z)
- Parity symmetry in physics states:
  - *The laws of physics are invariant under inversion of spatial coordinates.*
- Violation of parity symmetry = The laws of physics are **not** invariant under…
- To understand this intuitively, ask “*Can I tell if I am looking at a mirror image or the original image?*”
  - With a caveat that parity transformation is (x, y, z) –> (-x, -y, -z), whereas a mirror flips only one of (x,y,z), e.g., (x, y, z) –> (-x, y, z).

This is confusing!

Transformation of coordinates

- You may say, "*Coordinates are just a convenient mathematical tool. Physics should not depend on how we chart the world with coordinates.*"
    - Yes, that is absolutely correct.
- Coordinate **transformation** is different. The underlying physical principle does not depend on the choice of coordinates. However, "**how a physical system appears to change from one coordinate system to another**" often carries useful information.

Continuous Coordinate Transformation - 1

Spatial translation and homogeneity

- We do an experiment in Nagoya, and repeat it in Munich. We find the same answer (to within the uncertainty).
- This is evidence for **invariance under spatial translation**. We shift spatial coordinates by a constant vector $\boldsymbol{c}$, $\boldsymbol{x} \to \boldsymbol{x} + \boldsymbol{c}$, and the physics relevant to the experiment does not change.
  - There is no special location in space => **homogeneity**.
  - This even implies that the total momentum is conserved!

Continuous Coordinate Transformation - 2

Spatial rotation and isotropy

- We do an experiment. We repeat it a few times after rotating the experimental apparatus at different angles. We find the same answer (to within the uncertainty).
- This is evidence for **invariance under spatial rotation**. We rotate spatial coordinates by $\boldsymbol{x} \rightarrow R\boldsymbol{x}$, where R is a 3-dimensional rotation matrix, and the physics relevant to the experiment does not change.
  - There is no special direction in space => **isotropy**.
  - This even implies that the total angular momentum is conserved!

Parity and Rotation

Difference

- Parity transformation (***x*** –> -***x***) and 3d rotation (***x*** –> R***x***) are different.
    - R is a continuous transformation and the determinant of R is det(R) = +1.
    - Parity is a discrete transformation and the **determinant is –1**, as

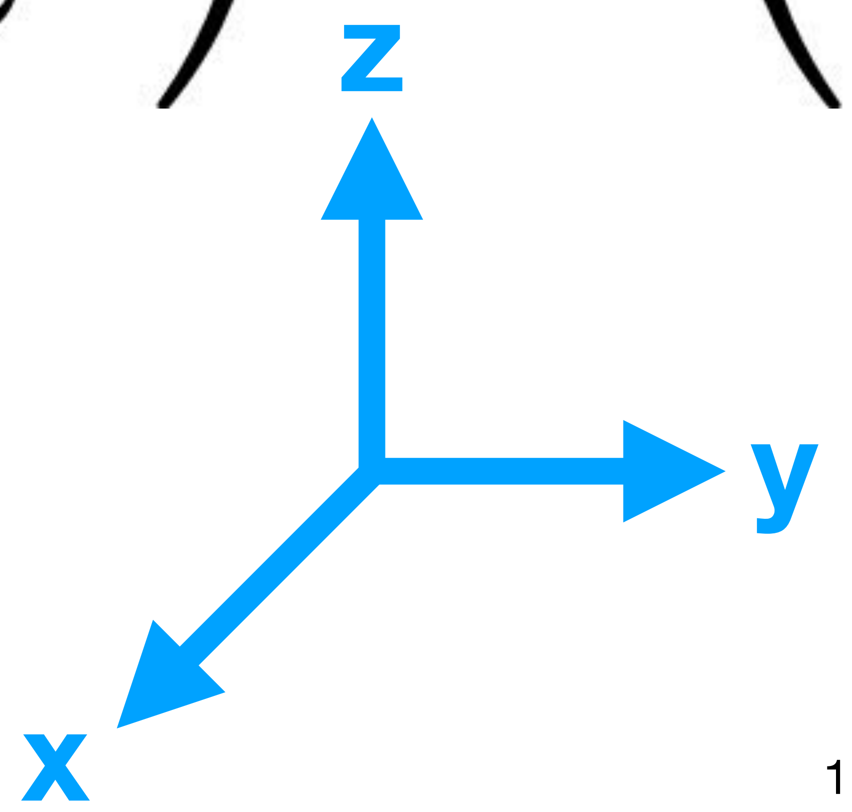
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Parity = Mirror + 2d Rotation

Relationship

- One may think of parity transformation as a mirror in one of the coordinates (e.g., x –> -x) and **2d** rotation by π in the others, e.g.,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \to \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$



$$R_2 = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

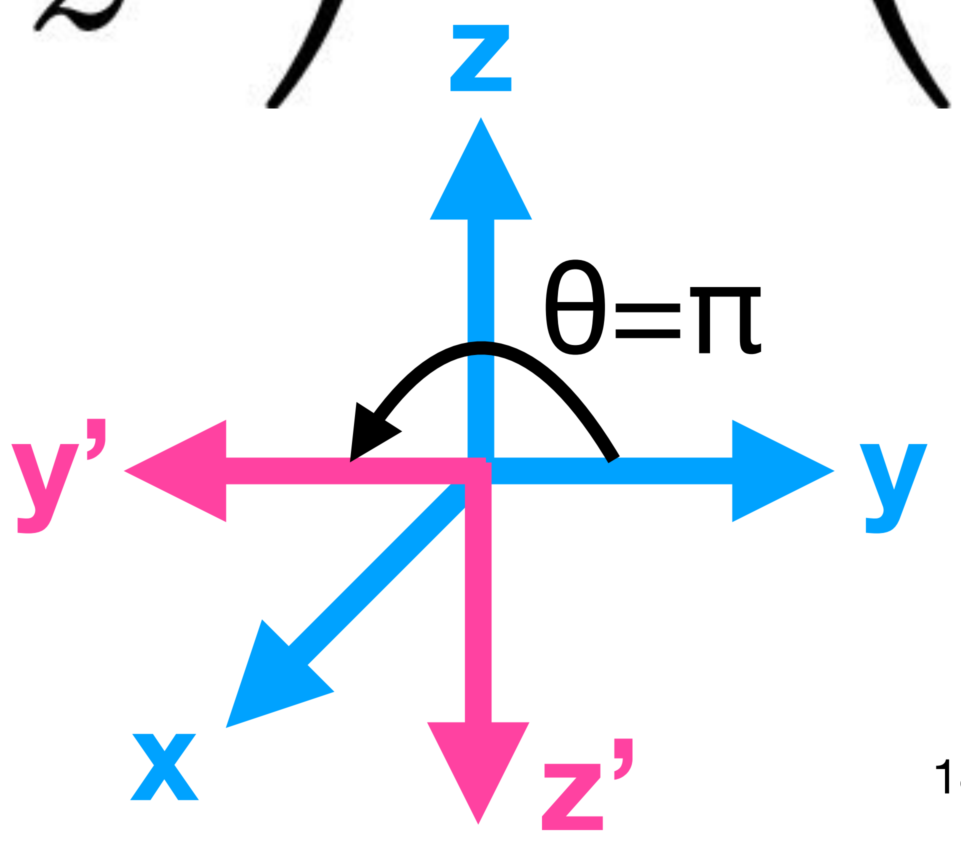
with θ = π

Parity = Mirror + 2d Rotation

Rotation

- One may think of parity transformation as a mirror in one of the coordinates (e.g., x –> -x) and **2d** rotation by π in the others, e.g.,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \to \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$



$$R_2 = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

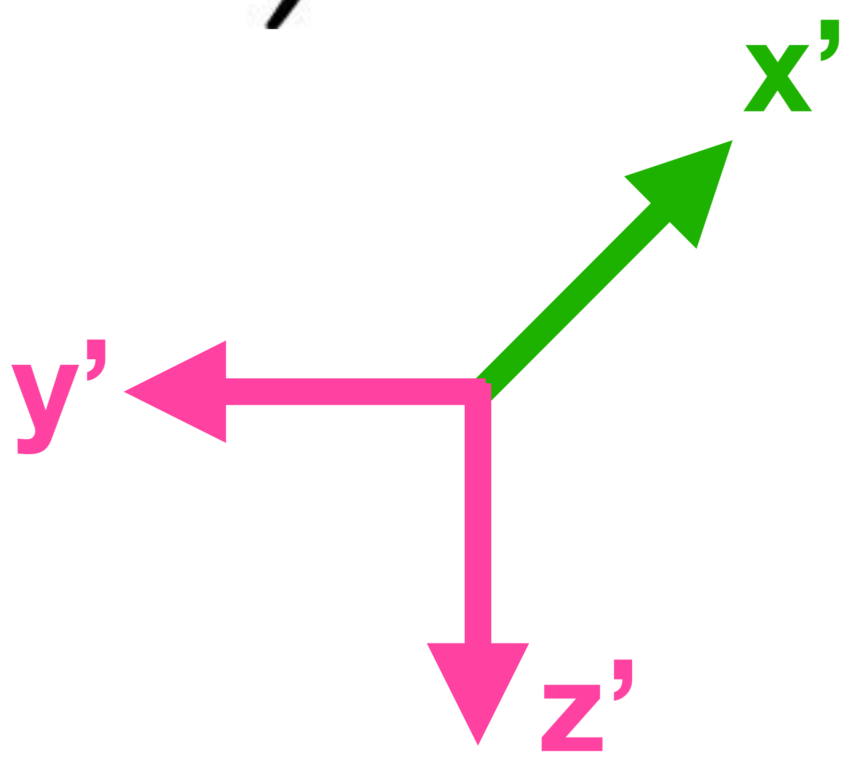
with θ = π

Parity = Mirror + 2d Rotation

Mirror

- One may think of parity transformation as a mirror in one of the coordinates (e.g., **x –> -x**) and **2d** rotation by π in the others, e.g.,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \to \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

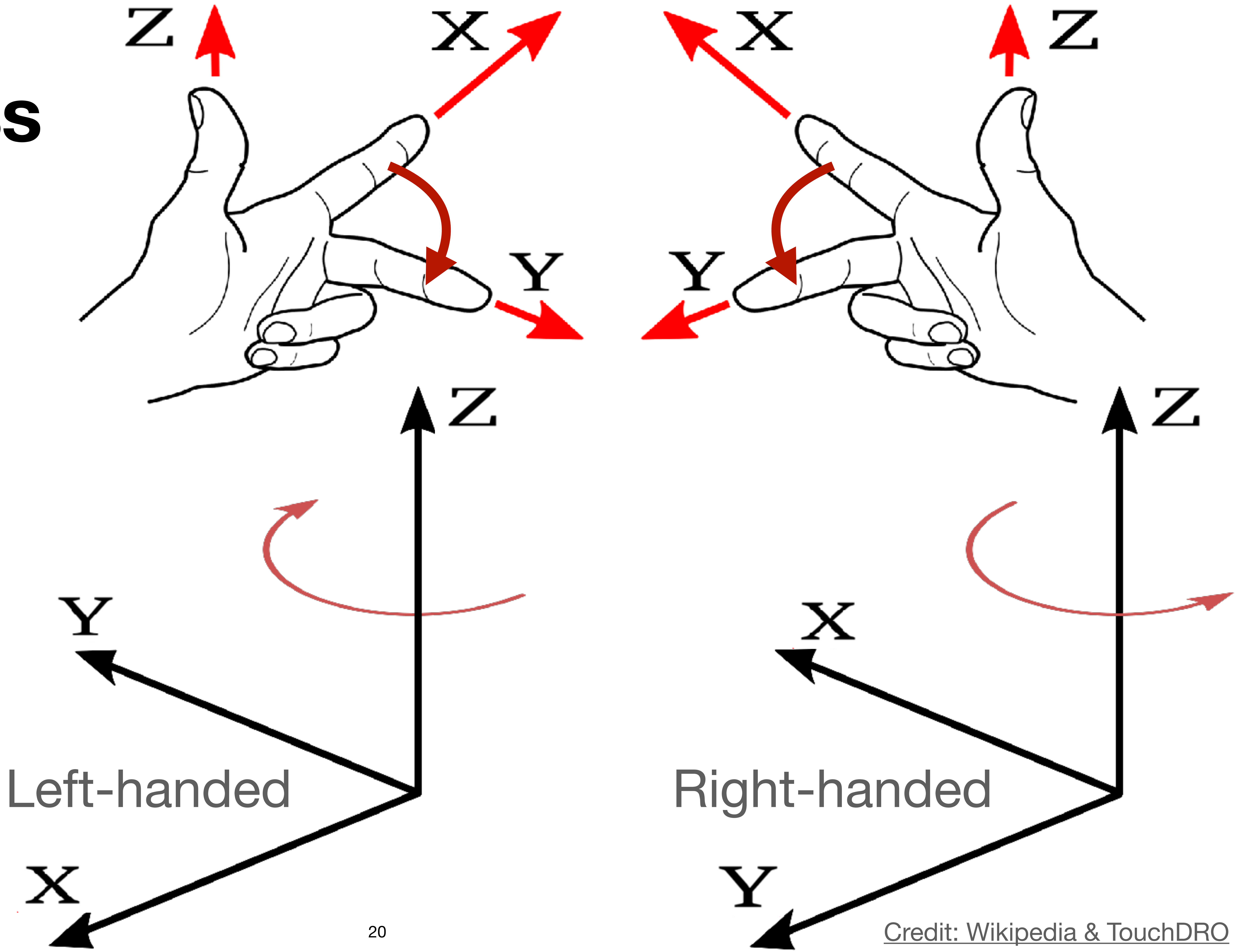


$$R_2 = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

with θ = π

Handedness

- Parity changes "right-handed system" to "left-handed system" and vice versa.



Credit: Wikipedia & TouchDRO

The most important concept in this lecture

Parity transformation changes handedness

- Unlike spatial translation and rotation, parity is not easy to picture. What is changing, really? What happens if parity symmetry is violated?
  - A short answer, which will be relevant to this lecture: Parity transformation changes “right-handed” to “left-handed” and vice versa, and **violation of parity symmetry implies that right- and left-handed states behave differently**.
    - The goal of today’s lecture is to understand this.
  - **Parity Violation in Cosmology = The Universe distinguishes between right- and left-handed states.**

To clarify terminology

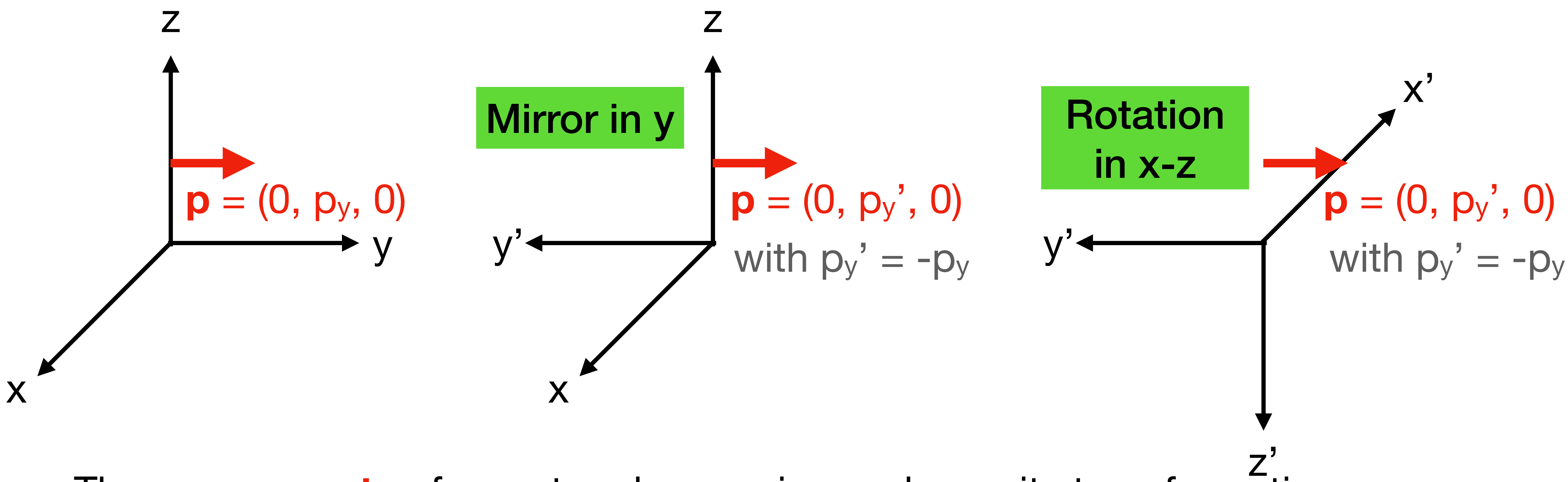
Looking in a mirror…?

- We often hear a statement, “*Parity transformation is like looking in a mirror*”.
    - Of course, this is not true. One has to invert all of (x,y,z).
    - The correct way to say is, “*Parity transformation is like looking in a mirror, followed by a rotation*”.

1.2 Vector and Pseudovector

Vector

Passive transformation: Transform coordinates

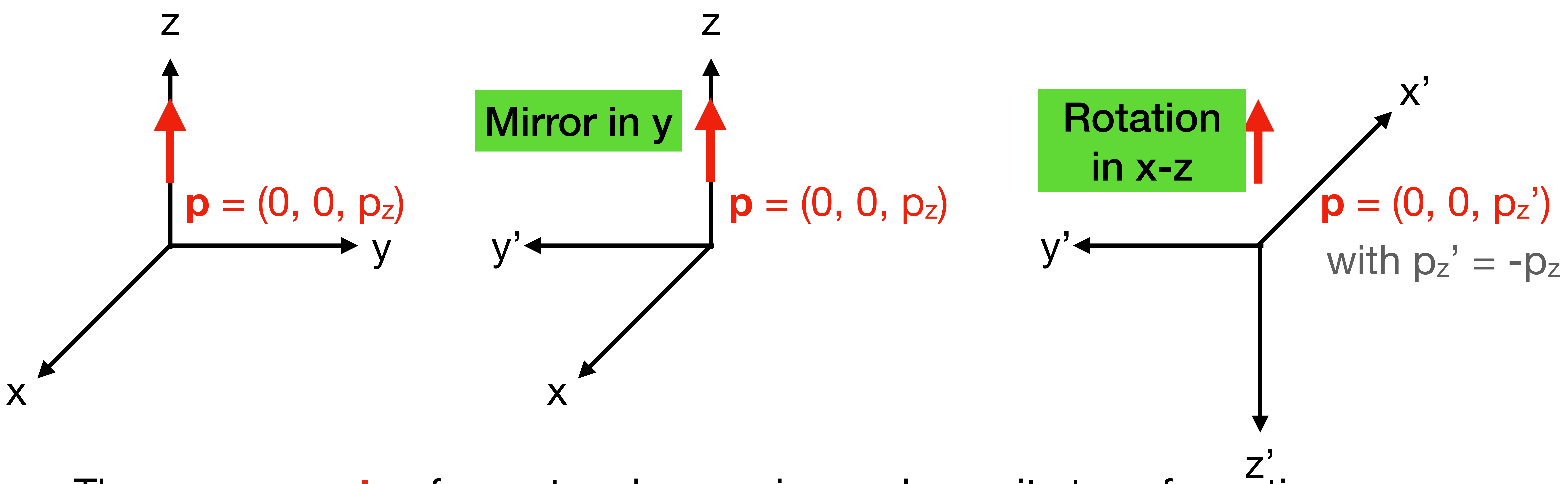


- The **components** of a vector change sign under parity transformation.

$$\mathbf{p} = (p_x, p_y, p_z) \rightarrow (-p_x, -p_y, -p_z)$$

Vector

Passive transformation: Transform coordinates

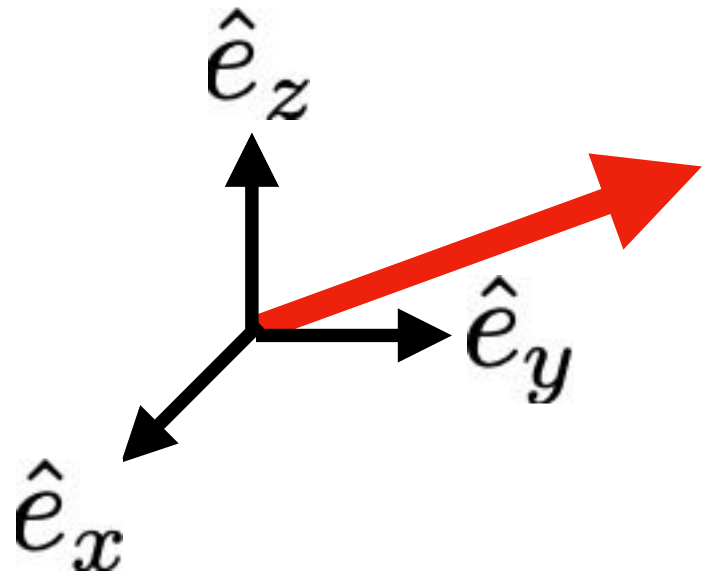


- The **components** of a vector change sign under parity transformation.

$$\mathbf{p} = (p_x, p_y, p_z) \rightarrow (-p_x, -p_y, -p_z)$$

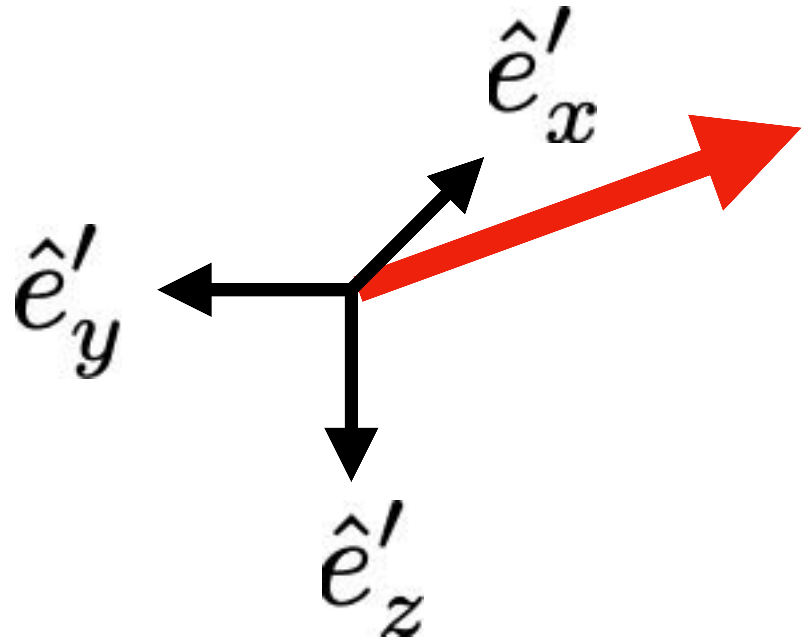
Vector

More general derivation using a basis vector



$$\mathbf{p} = p_x \hat{e}_x + p_y \hat{e}_y + p_z \hat{e}_z$$

$\hat{e}_i$ is a unit vector.

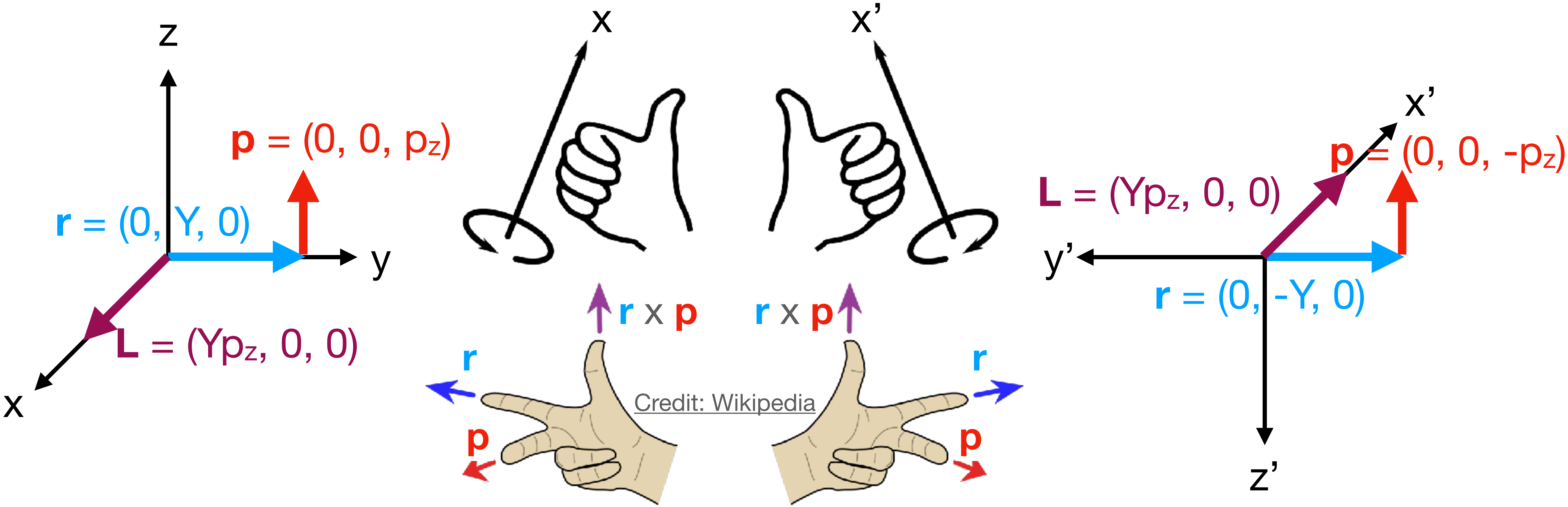


$$\mathbf{p} = p'_x \hat{e}'_x + p'_y \hat{e}'_y + p'_z \hat{e}'_z$$
$$= -p'_x \hat{e}_x - p'_y \hat{e}_y - p'_z \hat{e}_z$$

- **p** is the same vector, written using two different basis vectors.
- Therefore, **p**'s *components* transform as $(p'_x, p'_y, p'_z) = (-p_x, -p_y, -p_z)$

Pseudovector (axial vector)

Angular momentum, magnetic fields, …



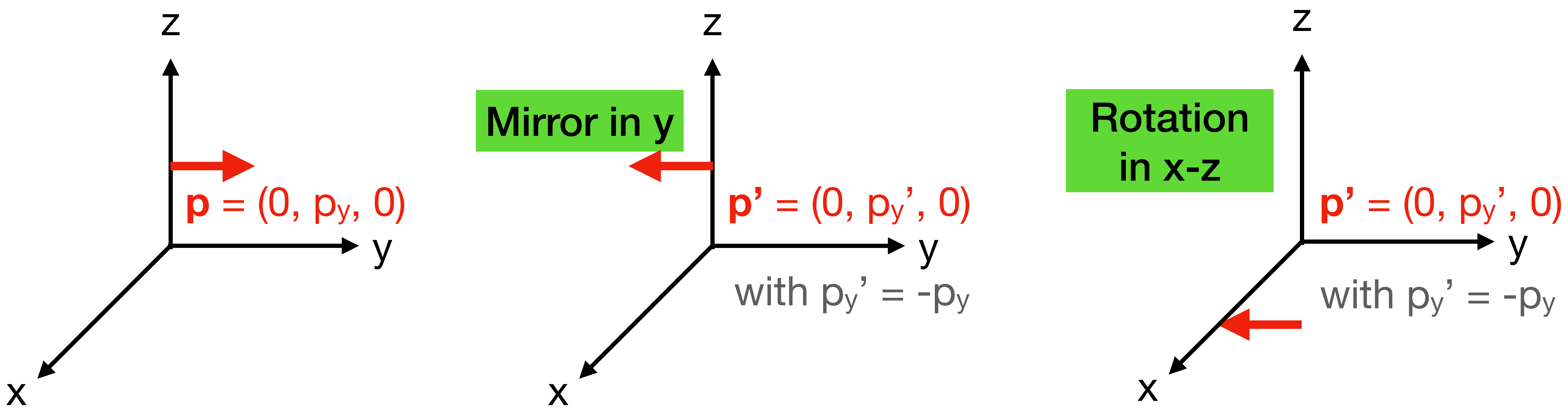
- Orbital angular momentum, **L** = **r** x **p**, is a *pseudovector*. Its *components* do **<u>not</u>** change sign under parity transformation: $(L'_x, L'_y, L'_z) = (L_x, L_y, L_z)$

What? What just happened?

If you do not like mixing right-handed to left-handed coordinate systems…

Vector

Active transformation: Transform the object

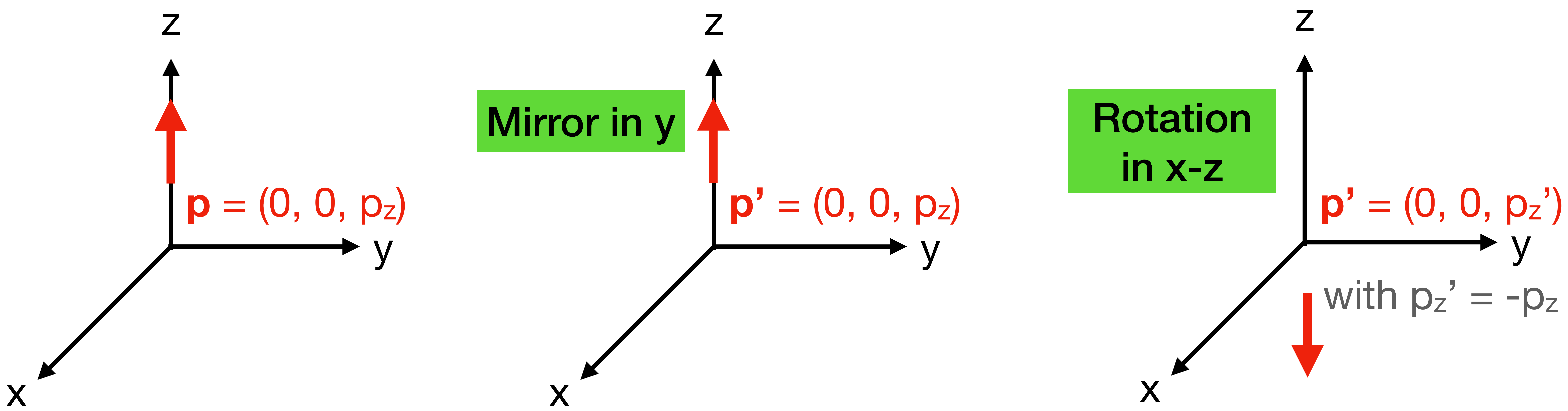


- A vector changes sign under parity transformation.

$$\mathbf{p} \to \mathbf{p}'(\mathbf{x}') = -\mathbf{p}(-\mathbf{x})$$

Vector

Active transformation: Transform the object



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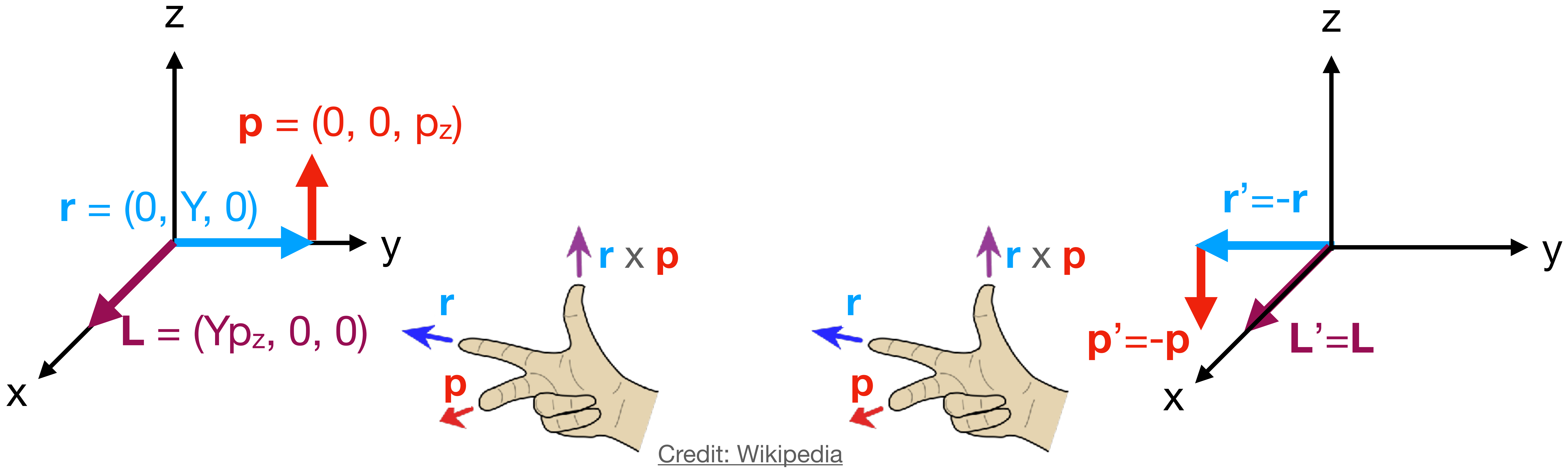
Pseudovector (axial vector)

Active transformation: Transform the object



- A *pseudovector* does **not** change sign under parity transformation.

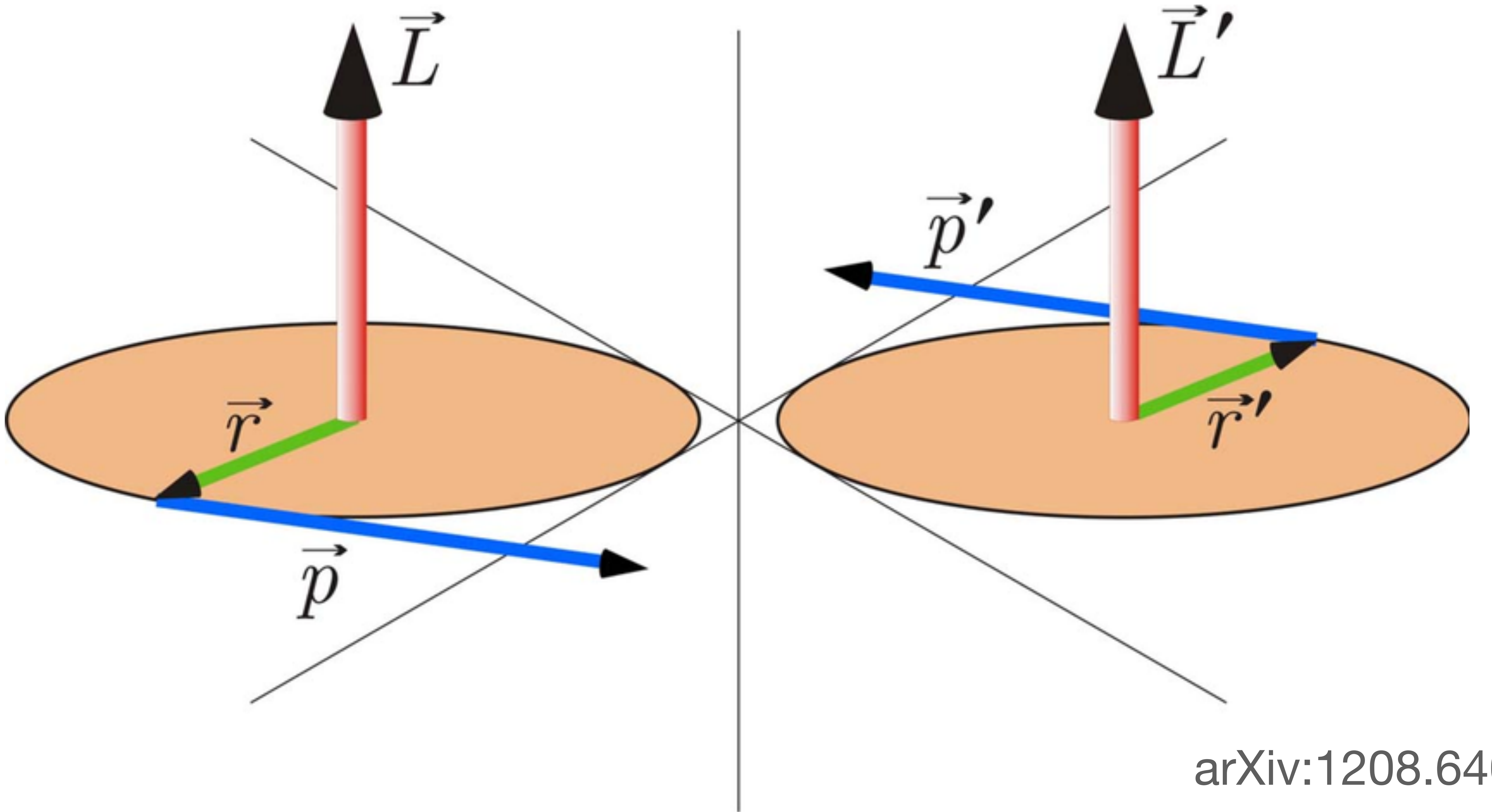
$$\mathbf{L}(\mathbf{x}) \rightarrow \mathbf{L}'(\mathbf{x}') = (-\mathbf{r}) \times (-\mathbf{p}) = +\mathbf{L}(-\mathbf{x})$$

Credit: Wikipedia

Pseudovector (axial vector)

Active transformation: Transform the object

$$\mathbf{L}(\mathbf{x}) \to \mathbf{L}'(\mathbf{x}') = (-\mathbf{r}) \times (-\mathbf{p}) = +\mathbf{L}(-\mathbf{x})$$



arXiv:1208.6409 [physics.class-ph]

Problem Set 1

Playing with pseudovectors

1. Using a basis vector, show that the components of the angular momentum vector, (L_x,L_y,L_z) for **L** = **r** x **p**, do not change under parity transformation.

- $\mathbf{r} = X\hat{e}_x + Y\hat{e}_y + Z\hat{e}_z$
- $\mathbf{p} = p_x\hat{e}_x + p_y\hat{e}_y + p_z\hat{e}_z$

Hint: Use

$$\left\{ \begin{array}{l} \hat{e}_i \times \hat{e}_j = -\hat{e}_j \times \hat{e}_i \\ \hat{e}_2 \times \hat{e}_3 = \hat{e}_1 \\ \hat{e}_3 \times \hat{e}_1 = \hat{e}_2 \\ \hat{e}_1 \times \hat{e}_2 = \hat{e}_3 \end{array} \right.$$

2. Show that the magnetic field, **B**, is also a pseudovector.

- You can use any arguments and methods, including equations, illustrations, and words. Feel free to use any material available to you.

1.3 Discovery of Parity Violation in β-decay (weak interaction)

Letters to the Editor,
Physical Review, 105, 1413 (1957)

Experimental Test of Parity Conservation in Beta Decay*

C. S. Wu, *Columbia University, New York, New York*

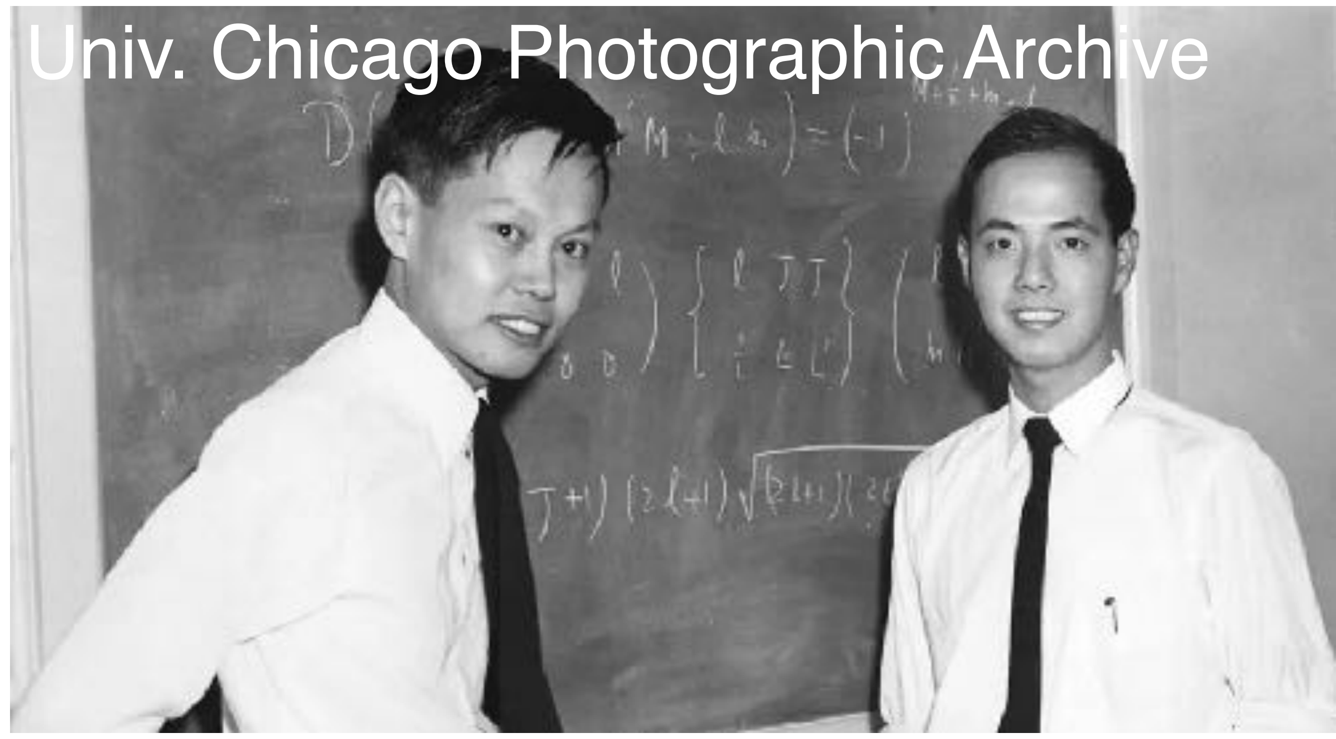
AND

E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson, *National Bureau of Standards, Washington, D. C.*

(Received January 15, 1957)

In a recent paper[1] on the question of parity in weak interactions, Lee and Yang critically surveyed the experimental information concerning this question and reached the conclusion that there is no existing evidence either to support or to refute parity conservation in weak interactions. They proposed a number of experiments on beta decays and hyperon and meson decays which would provide the necessary evidence for parity conservation or nonconservation. In beta decay, one could measure the angular distribution of the electrons coming from beta decays of polarized nuclei. If an asymmetry in the

Chien-Shiung Wu



Chen-Ning Yang   Tsung-Dao Lee

The Wu Experiment of β-decay

$^{60}Co \rightarrow {}^{60}Ni + e^- + \bar{\nu}_e + 2\gamma$

- Electrons must be emitted with equal probability in all directions relative to **J**, if parity symmetry is respected in β-decay.
  - This was not observed –> **Parity symmetry is violated in β-decay!**

The Wu Experiment of β-decay

$^{60}Co \rightarrow {}^{60}Ni + e^- + \bar{\nu}_e + 2\gamma$

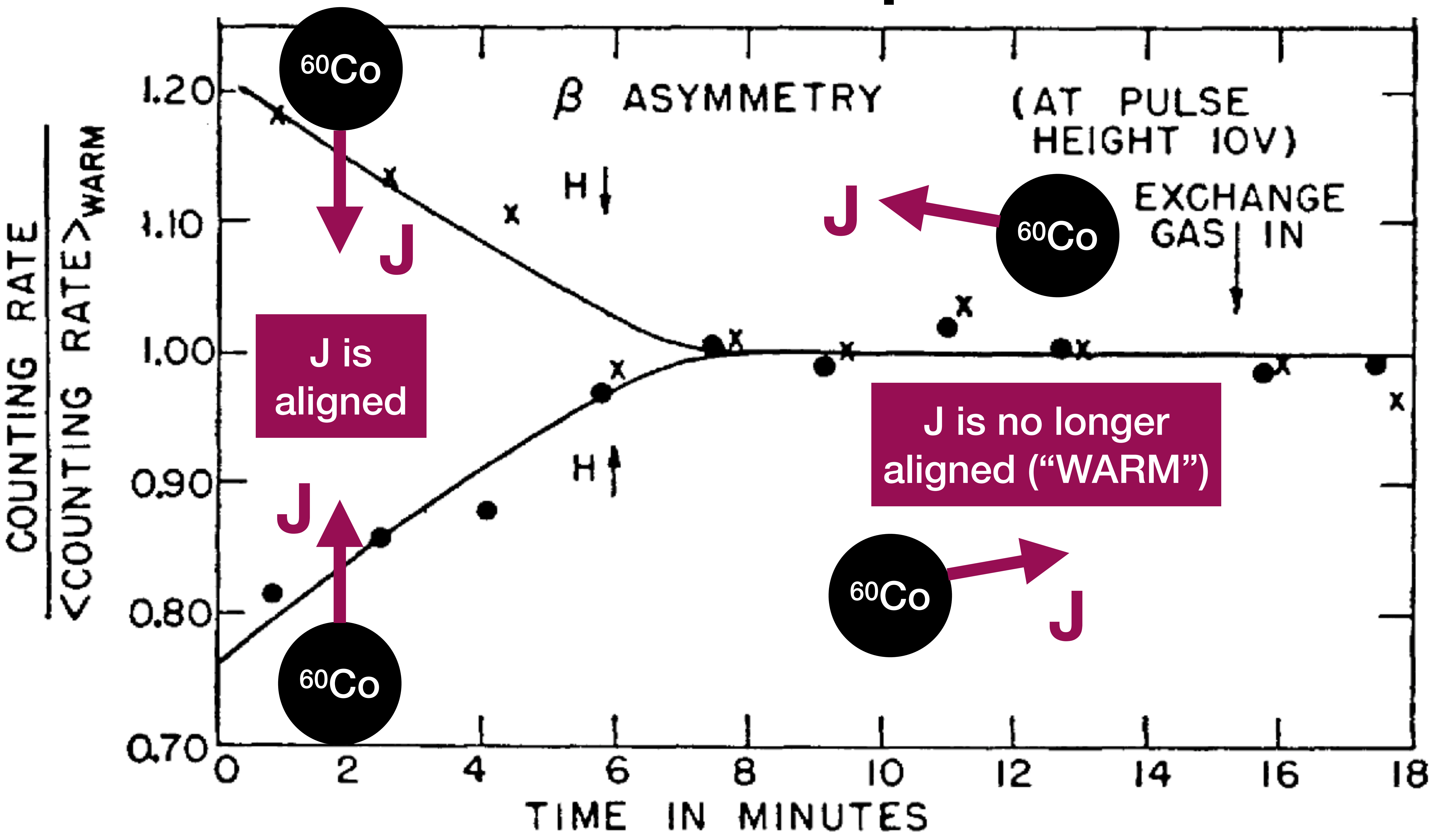
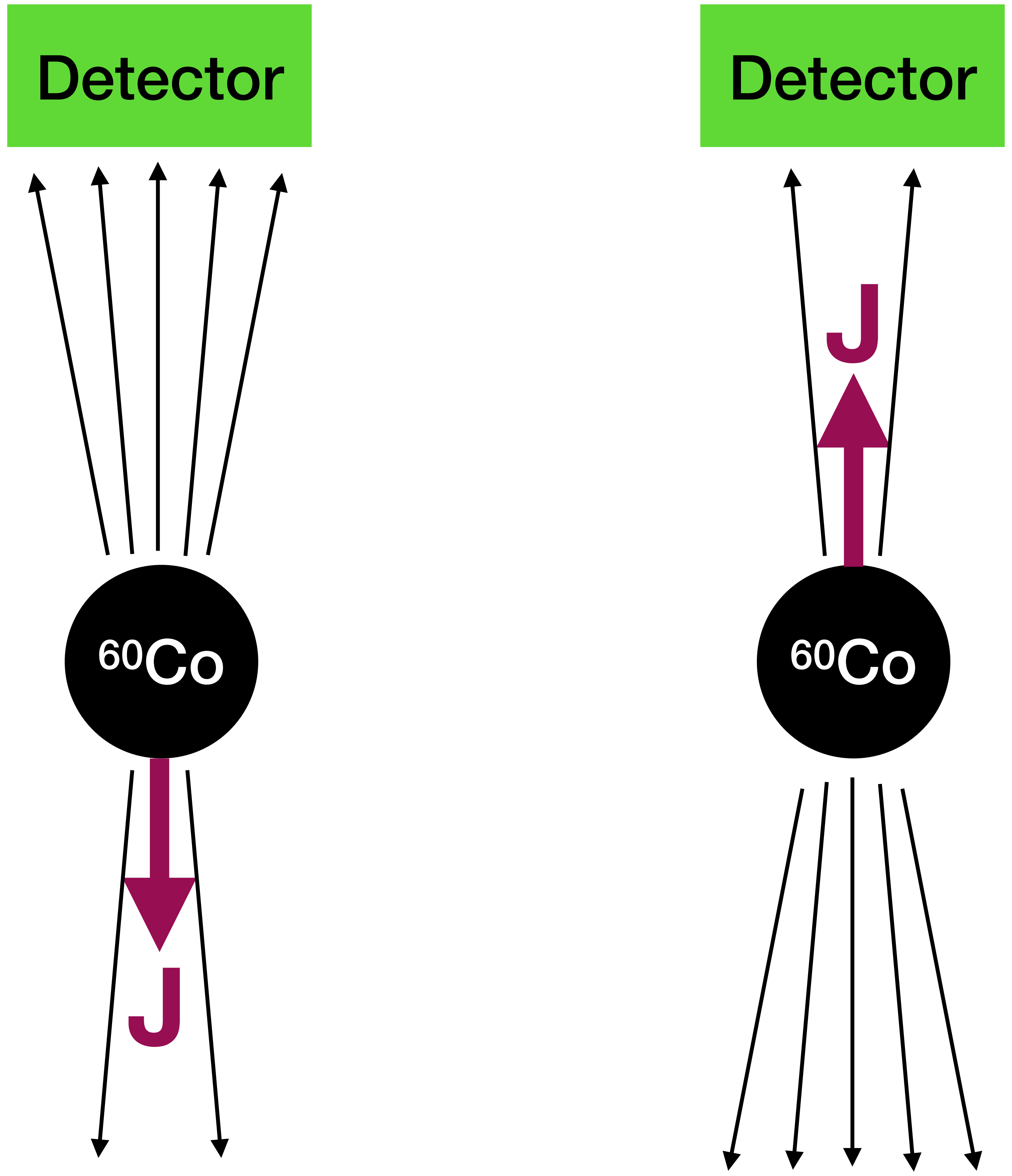


FIG. 2. Gamma anisotropy and beta asymmetry for polarizing field pointing up and pointing down.



- More electrons are emitted in the opposite direction of **J**.

Initial reaction

Many physicists did not believe it initially.

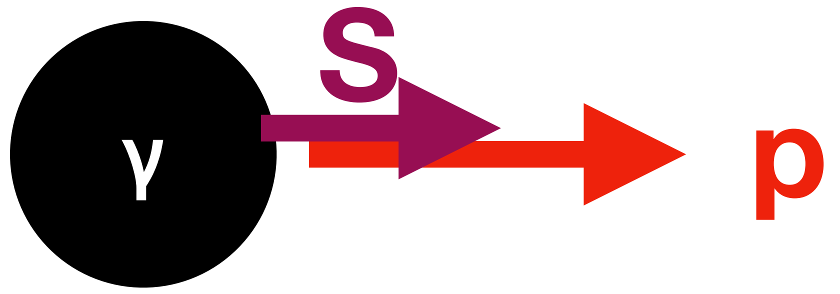
- To Lee and Yang’s theoretical paper on parity violation in β-decay:
    - Wolfgang Pauli said, “*Ich glaube aber nicht, daß der Herrgott ein schwacher Linkshänder ist*” (I do not believe that the Lord is a weak left-hander).
- To Wu’s discovery paper:
    - Wolfgang Pauli said, “*Sehr aufregend. Wie sicher ist die Nachricht?*” (Very exciting. How sure is this news?)
- **This was shocking news. The weak interaction distinguishes between left and right!**
- In this lecture we ask, “Does the Universe distinguish between left and right?” Most scientists answer, “No, of course it doesn’t”. Only experiments will decide.

1.4 Helicity

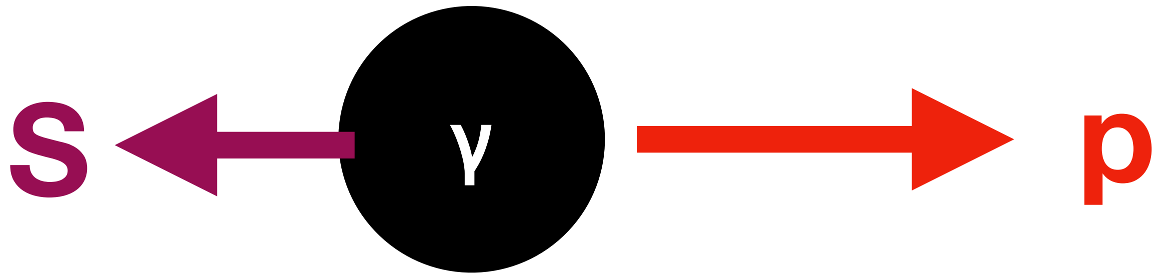
Momentum and spin of a massless particle

- Imagine a photon (γ) traveling at the speed of light with a momentum vector **p**.

- The photon is a spin-1 particle. The spin angular momentum vector **S** is a pseudovector. We then define “parallel” (right-handed) and “antiparallel” (left-handed) states as



“Parallel” (right-handed): $\mathbf{S} \cdot \mathbf{p} > 0$

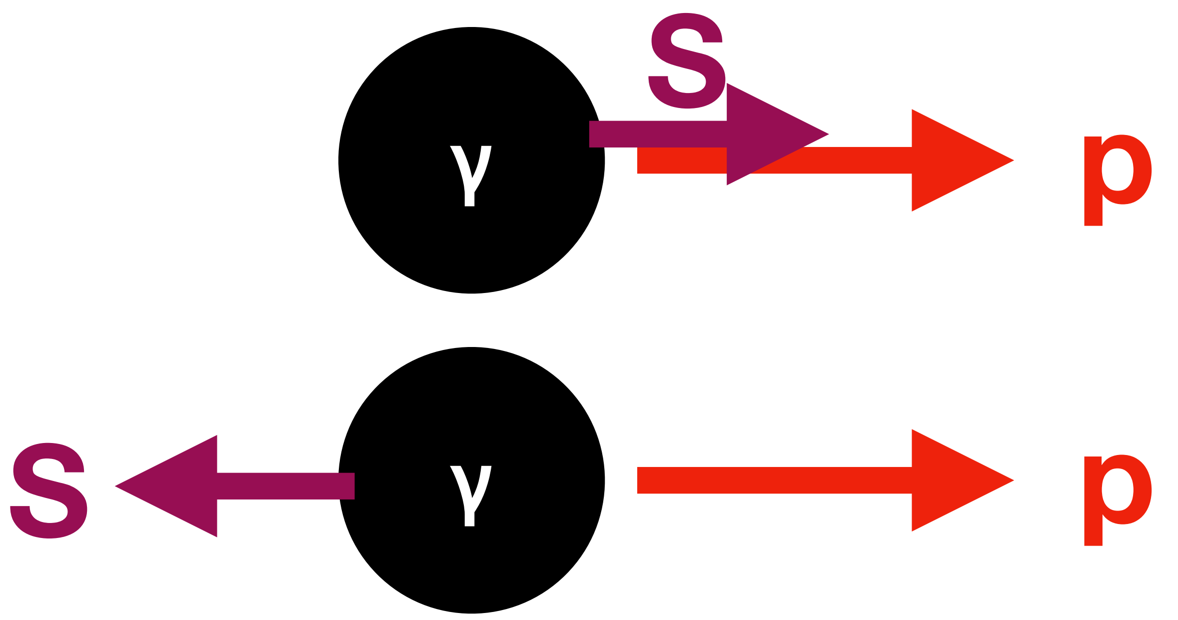


“Antiparallel” (left-handed): $\mathbf{S} \cdot \mathbf{p} < 0$

Momentum and spin of a massless particle

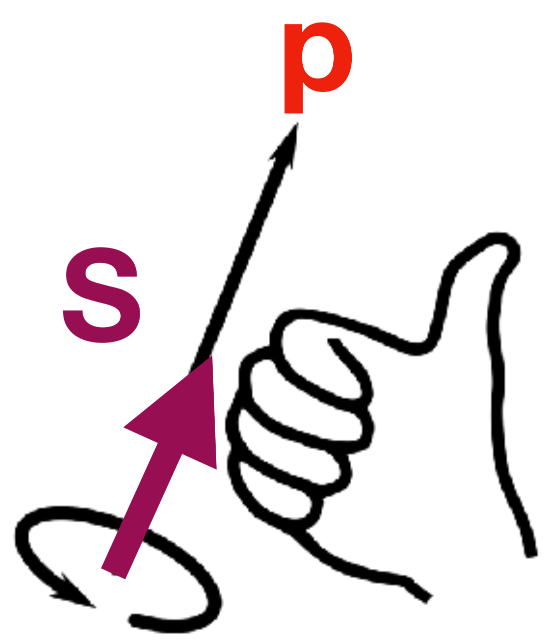
- Imagine a photon (γ) traveling at the speed of light with a momentum vector **p**.

- The photon is a spin-1 particle. The spin angular momentum vector **S** is a pseudovector. We then define “parallel” (right-handed) and “antiparallel” (left-handed) states as



“Parallel” (right-handed): $\mathbf{S} \cdot \mathbf{p} > 0$

“Antiparallel” (left-handed): $\mathbf{S} \cdot \mathbf{p} < 0$

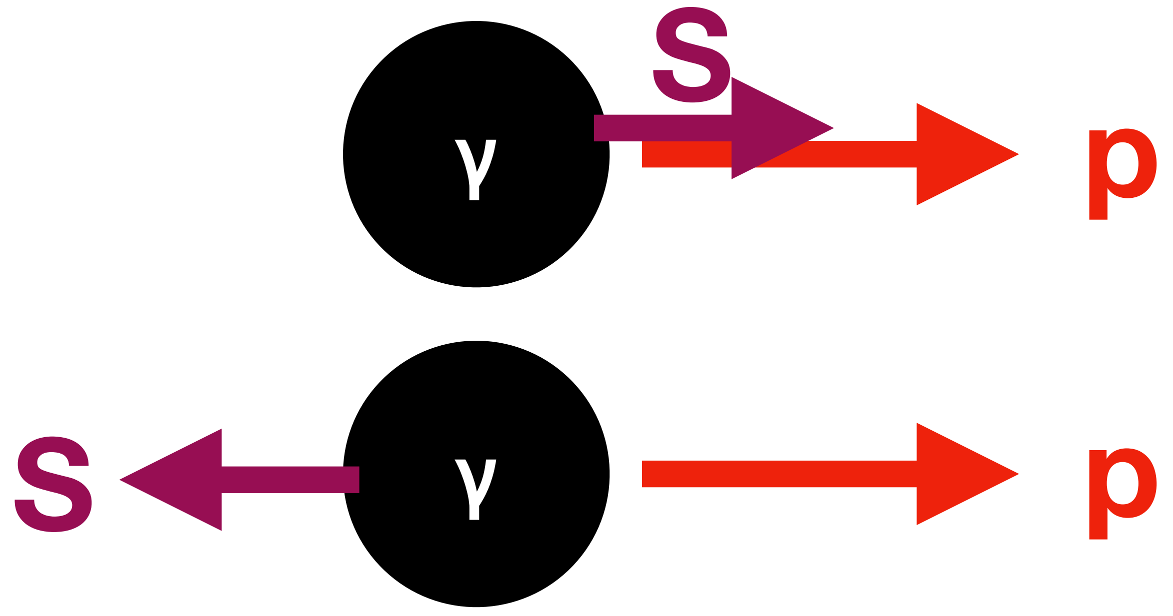


Right-handed

Momentum and spin of a massless particle

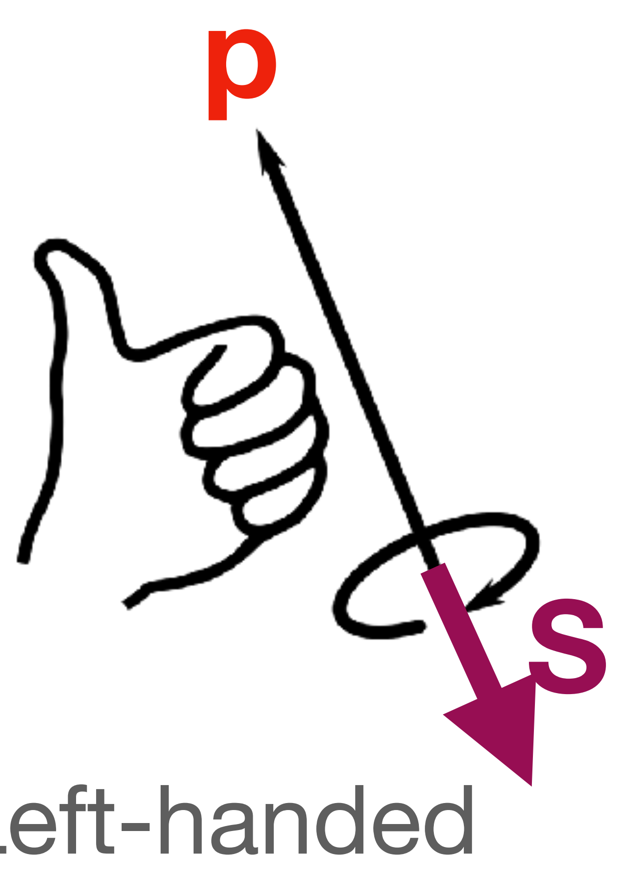
- Imagine a photon (γ) traveling at the speed of light with a momentum vector **p**.

- The photon is a spin-1 particle. The spin angular momentum vector **S** is a pseudovector. We then define “parallel” (right-handed) and “antiparallel” (left-handed) states as



“Parallel” (right-handed): $\mathbf{S} \cdot \mathbf{p} > 0$

“Antiparallel” (left-handed): $\mathbf{S} \cdot \mathbf{p} < 0$



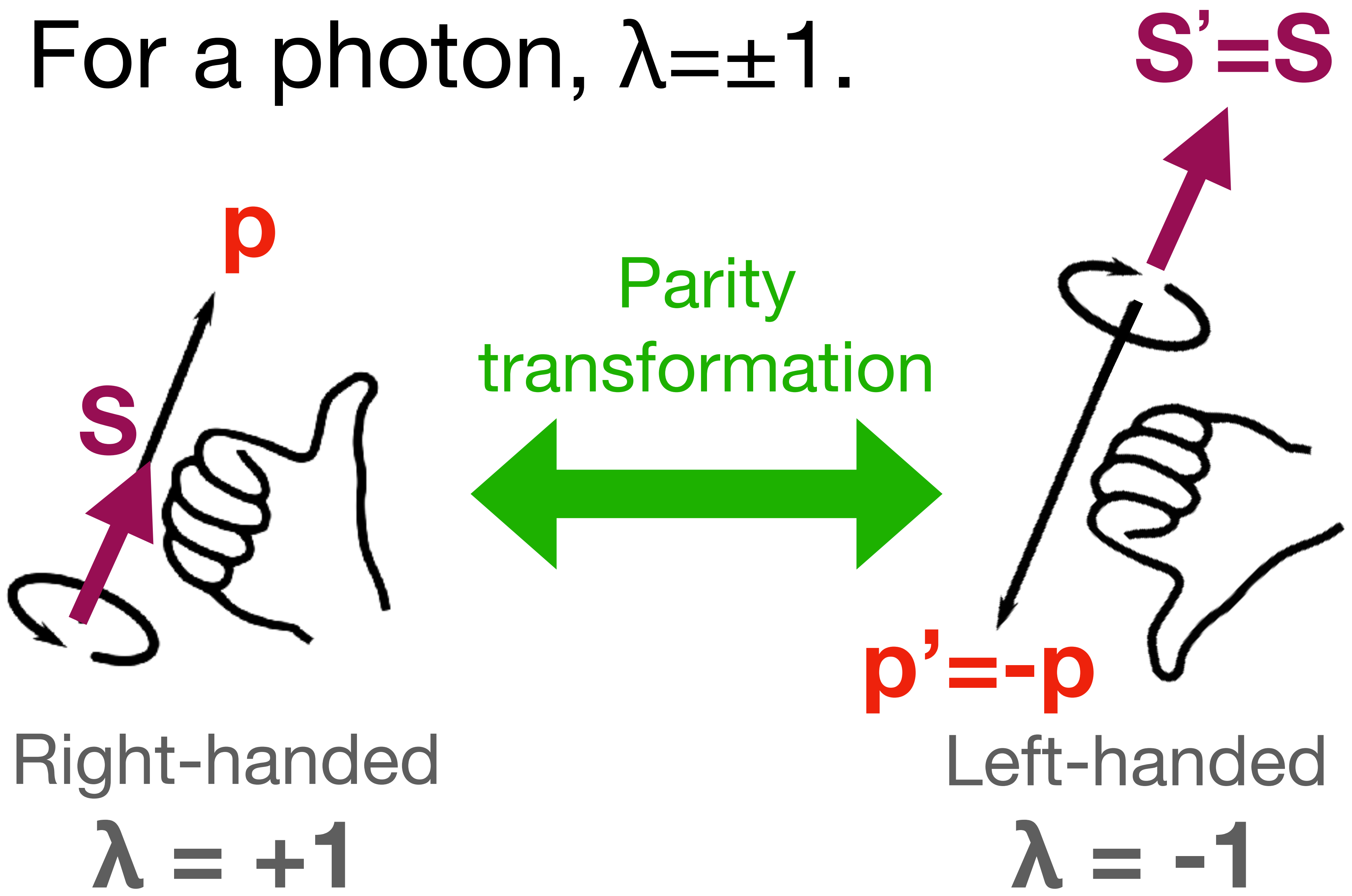
Helicity is a pseudoscalar

Party transformation changes “right-handed” to “left-handed” and vice versa

- For massless particles, we define the “helicity”, λ, as

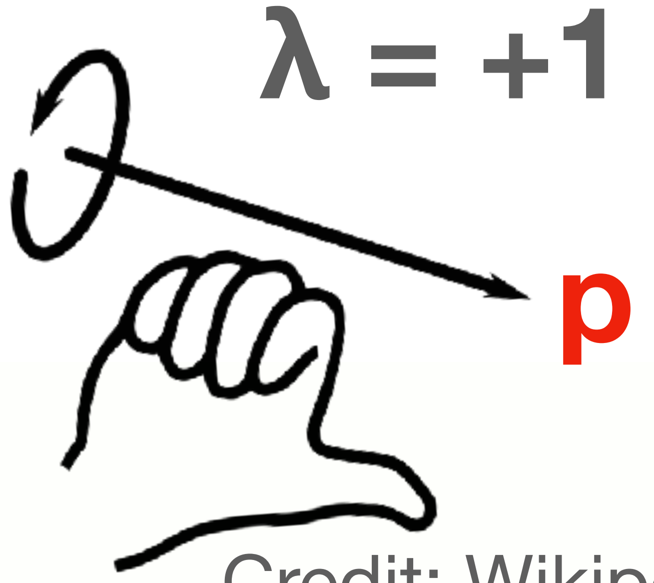
$$\mathbf{S} \cdot \frac{\mathbf{p}}{|\mathbf{p}|} = \lambda \hbar$$

- For a photon, λ=±1.



- As λ is the product of a vector (**p**) and a pseudovector (**S**), it changes sign under parity transformation.
    - For this reason, λ is called a “**pseudoscalar**”.
- On the other hand, “scalar”, such as $\mathbf{p}^2$ and $\mathbf{S}^2$, does not change sign.
- For a graviton, λ=±2.

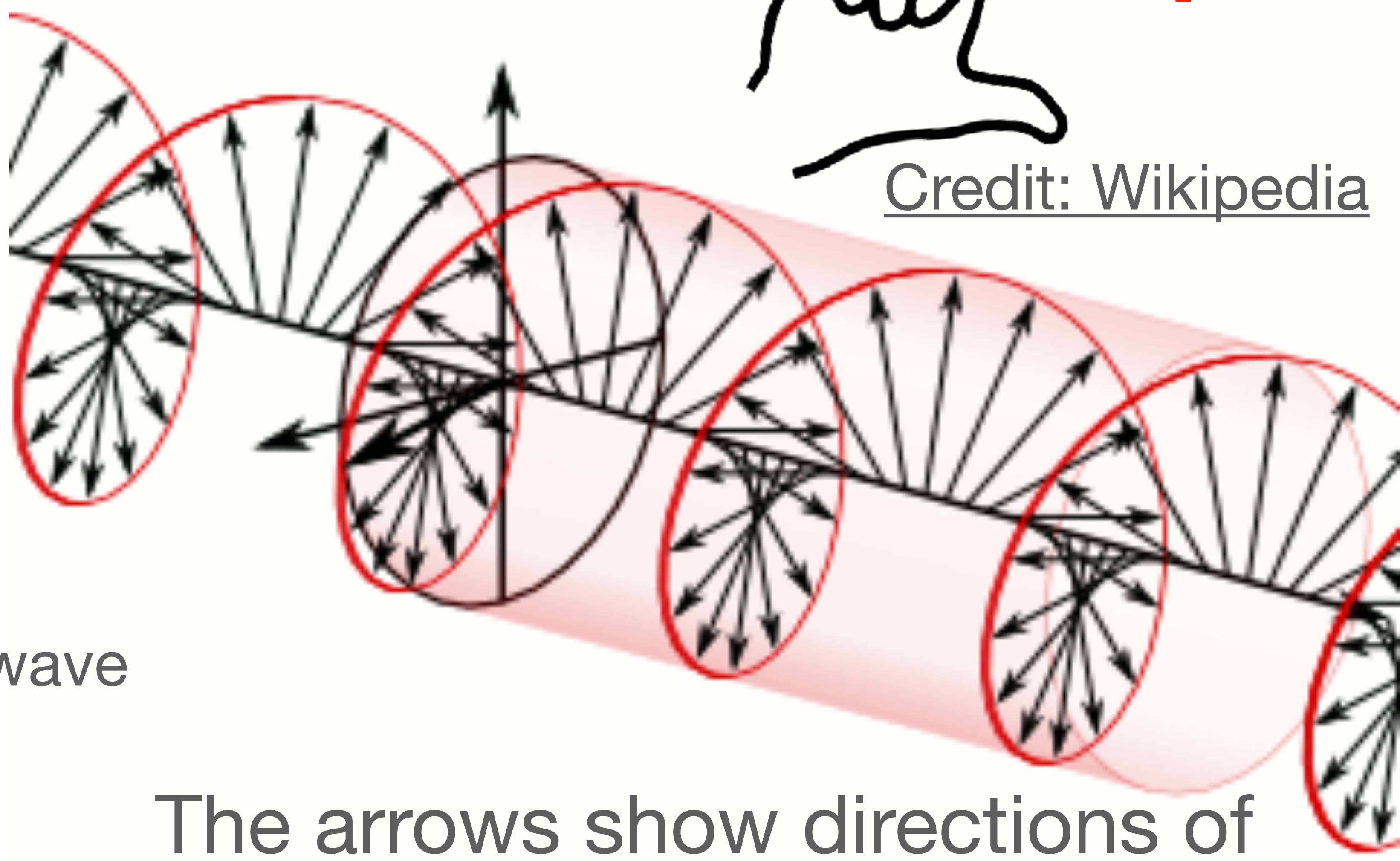
Helicity and circular polarization



Credit: Wikipedia

- λ=±1 states of a photon describe circular polarization.
  - λ=+1: Right-handed circular polarization
  - λ=–1: Left-handed circular polarization

Many photons
= Electromagnetic wave



The arrows show directions of the electric field vector **E**.

- Linear polarization is given by a super position of λ=±1 states with the equal number of photons. **Linear polarization carries no angular momentum!**
  - This means that circular polarization is a more fundamental state for photons. Circularly polarized light carries angular momentum, which is equal to $(N_+ - N_-)\hbar$ where $N_\pm$ is the number of λ=±1 photons.

Recap: Day 1

- Parity transformation changes “right-handed” to “left-handed” and vice versa.
    - Violation of parity symmetry: Nature distinguishes right- and left-handed states, which has been observed in β-decay. ***How about the Universe?***
- Transformation properties:
    - **Vector** (such as momentum) changes sign under parity transformation.
    - **Pseudovector** (such as angular momentum and spin) does not.
    - **Pseudoscalar** (such as helicity) does.