

# **Finding Gravitational Waves from the Early Universe**

**Finding the signature of gravitational waves in polarised light of  
the cosmic microwave background**

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***Innsbruck Physics Colloquium, Universität Innsbruck, May 24, 2023***

# Let's find Gravitational Waves (GW)!

*But how? The detection method depends on the GW frequency.*

- **Laser interferometers on the ground: deca- to kilo Hz** (*LIGO, VIRGO, ..., ET*)

- The wavelength  $\sim$  the size of Earth

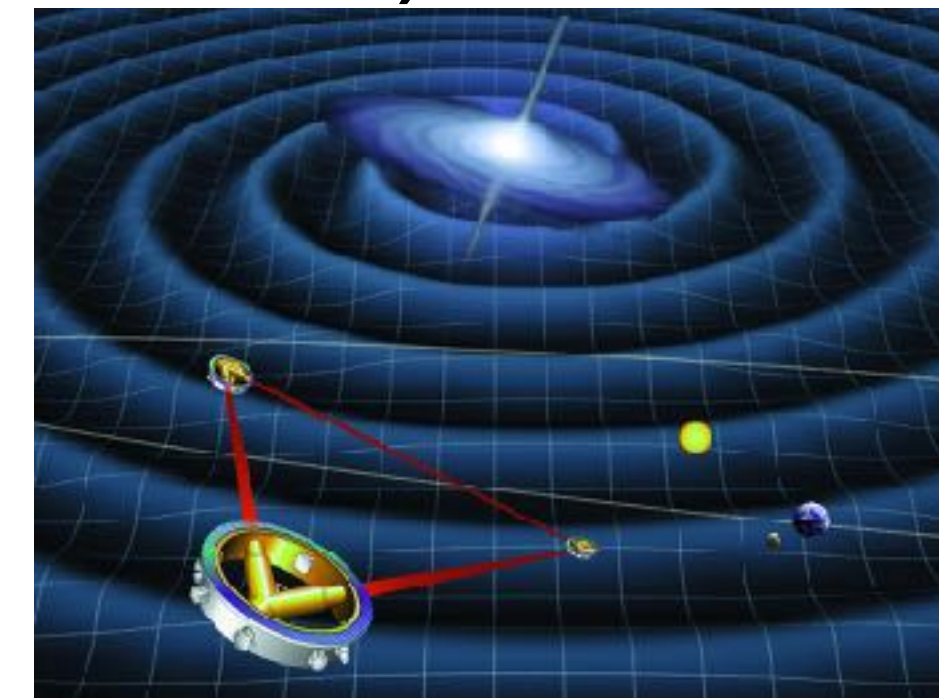


- **Laser interferometers in space: milli Hz** (*LISA*), deci Hz (future mission?)

- The wavelength  $\sim$  Astronomical Unit

- **Pulsar timing arrays: nano Hz** (*EPTA, SKA*)

- The wavelength  $\sim$  the size of the Milky Way



- **Cosmic microwave background: atto Hz** (*WMAP, Planck, LiteBIRD*)

- The wavelength  $\sim$  **billions of light years!**

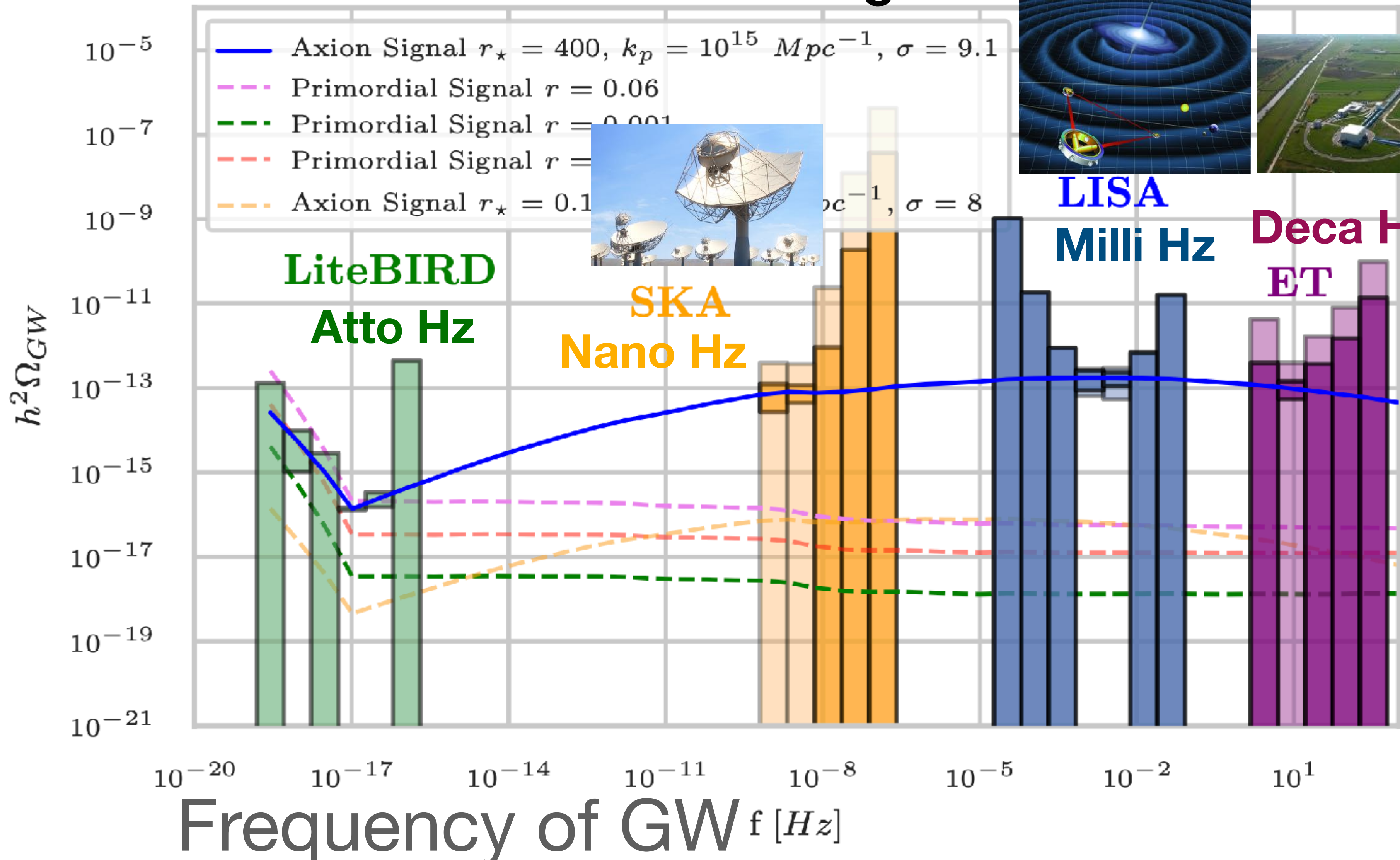




# GWs from the early Universe are everywhere!

We can measure it across 21 orders of magnitude in the GW frequency

Energy Density of GW  
today





[nature](#) > [nature reviews physics](#) > [review articles](#) > article

Review Article | [Published: 18 May 2022](#)

## New physics from the polarized light of the cosmic microwave background

[Eiichiro Komatsu](#) 

[Nature Reviews Physics](#) (2022) | [Cite this article](#)

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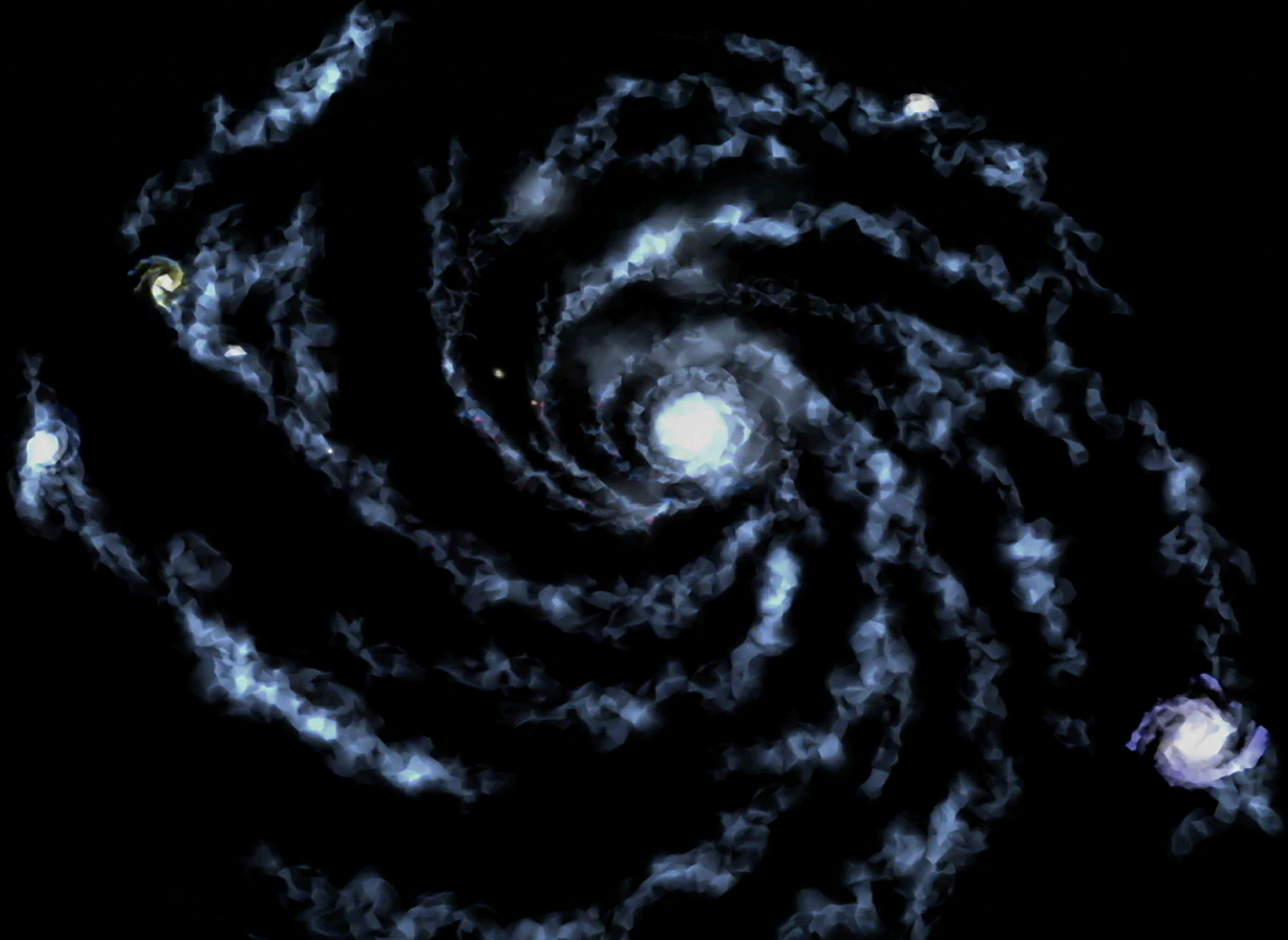
Available also at  
arXiv:2202.13919

### Key Words:

1. Cosmic Microwave Background (CMB)
2. Polarization
3. Parity Symmetry



Credit: WMAP Science Team

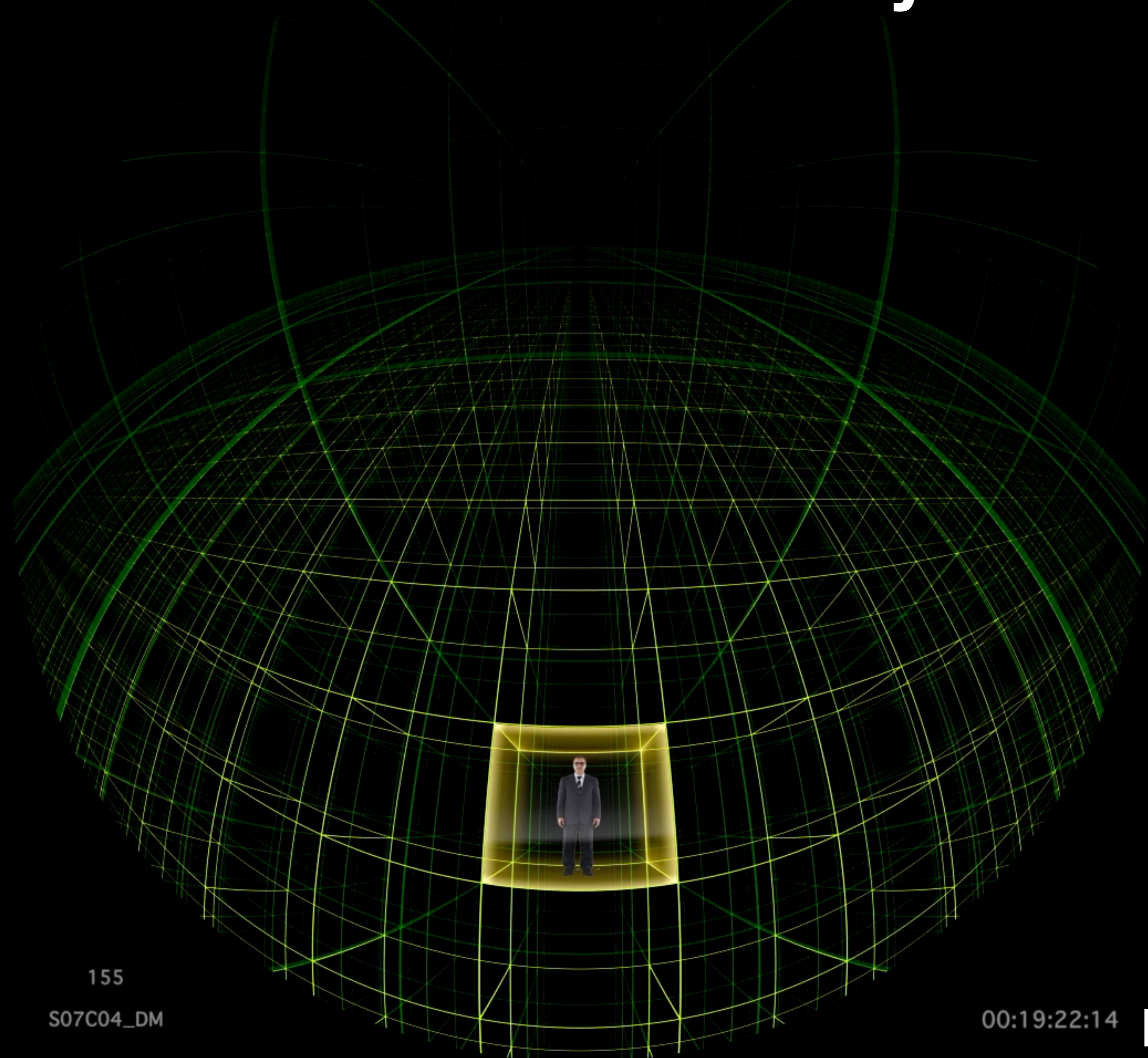


# The sky in various wavelengths

Visible -> Near Infrared -> Far Infrared -> Submillimeter -> Microwave



# Where did the CMB we see today come from?



155

S07C04\_DM

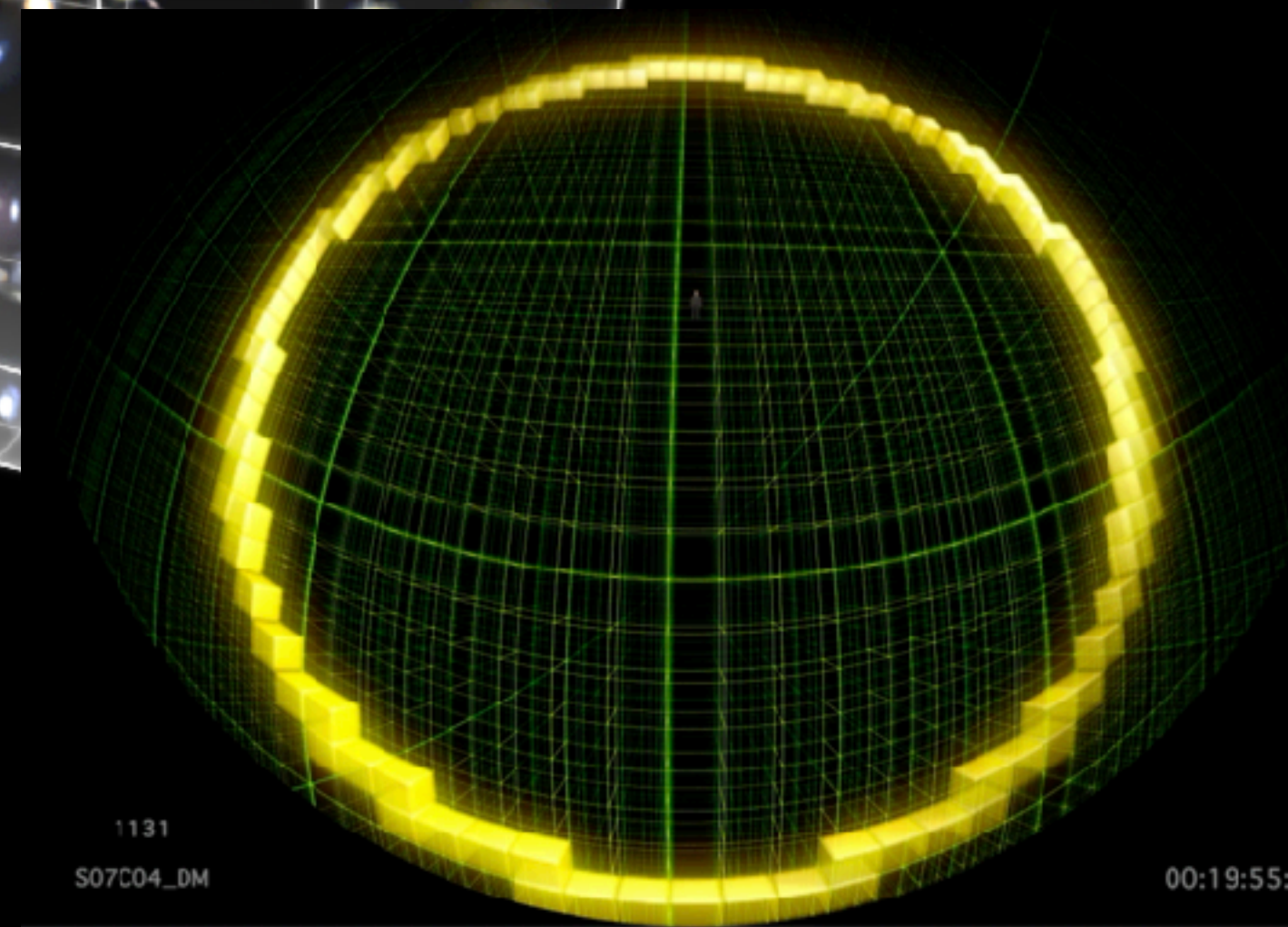
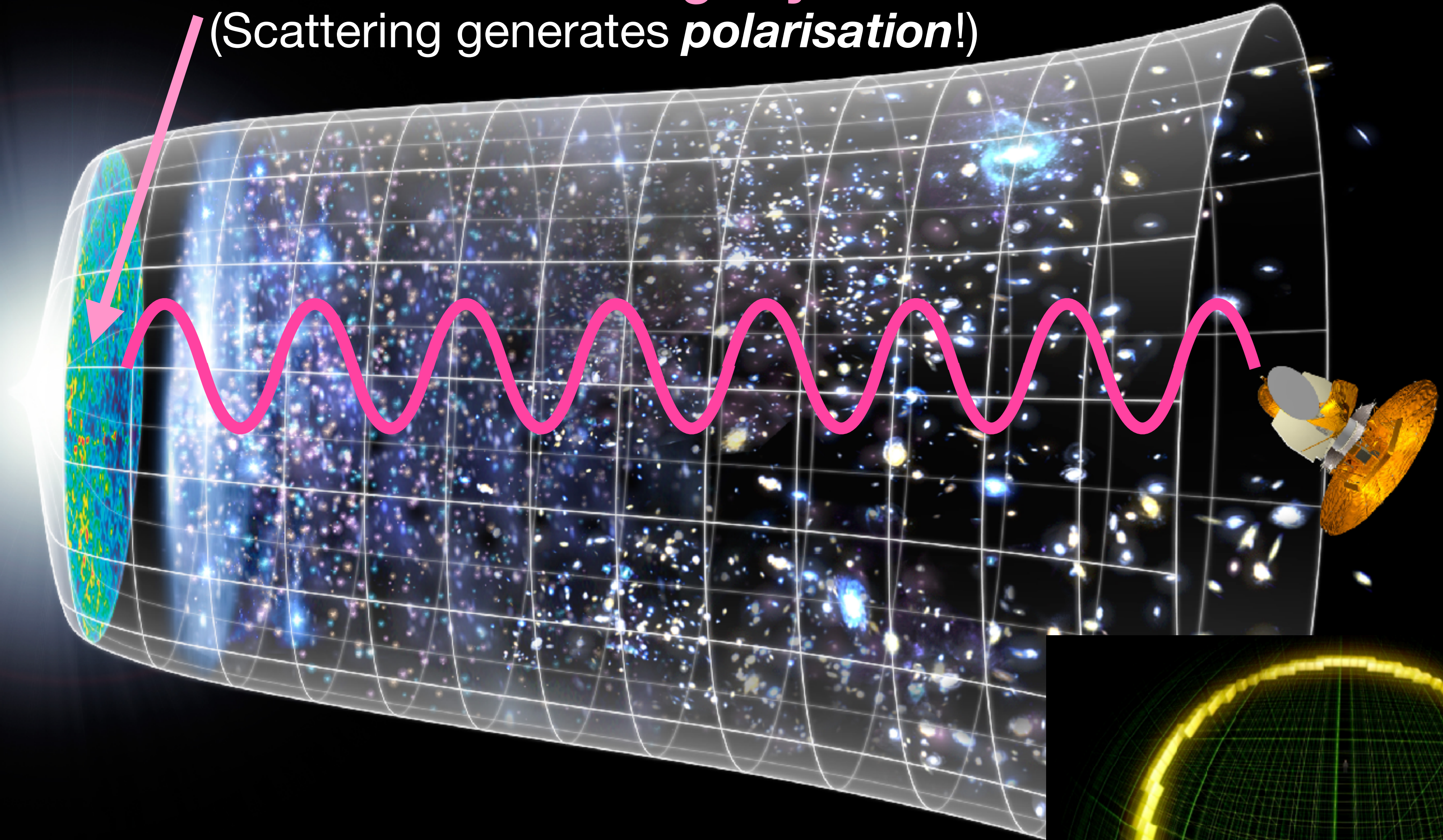
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From "HORIZON"



# The surface of “last scattering” by electrons

(Scattering generates *polarisation!*)

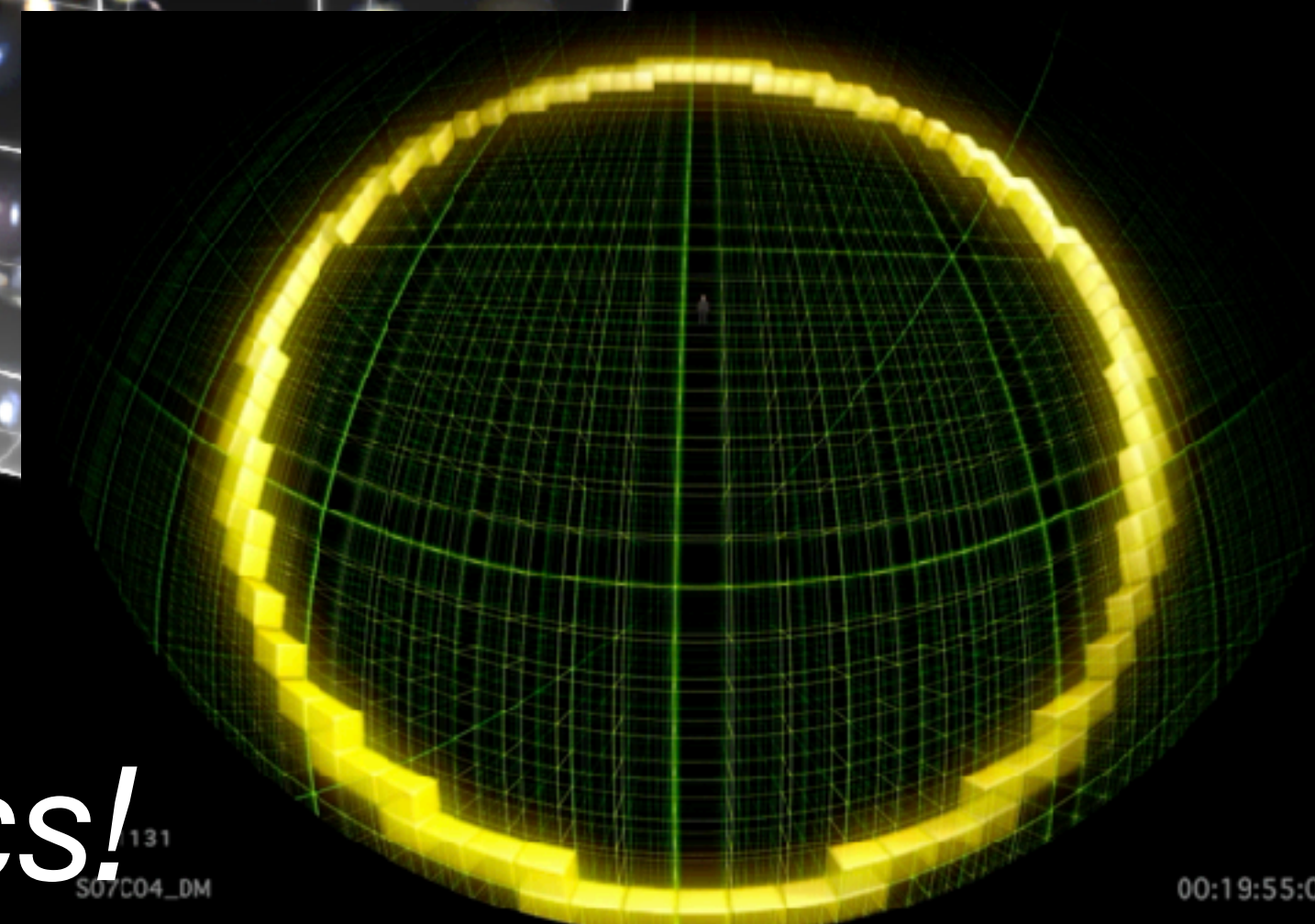
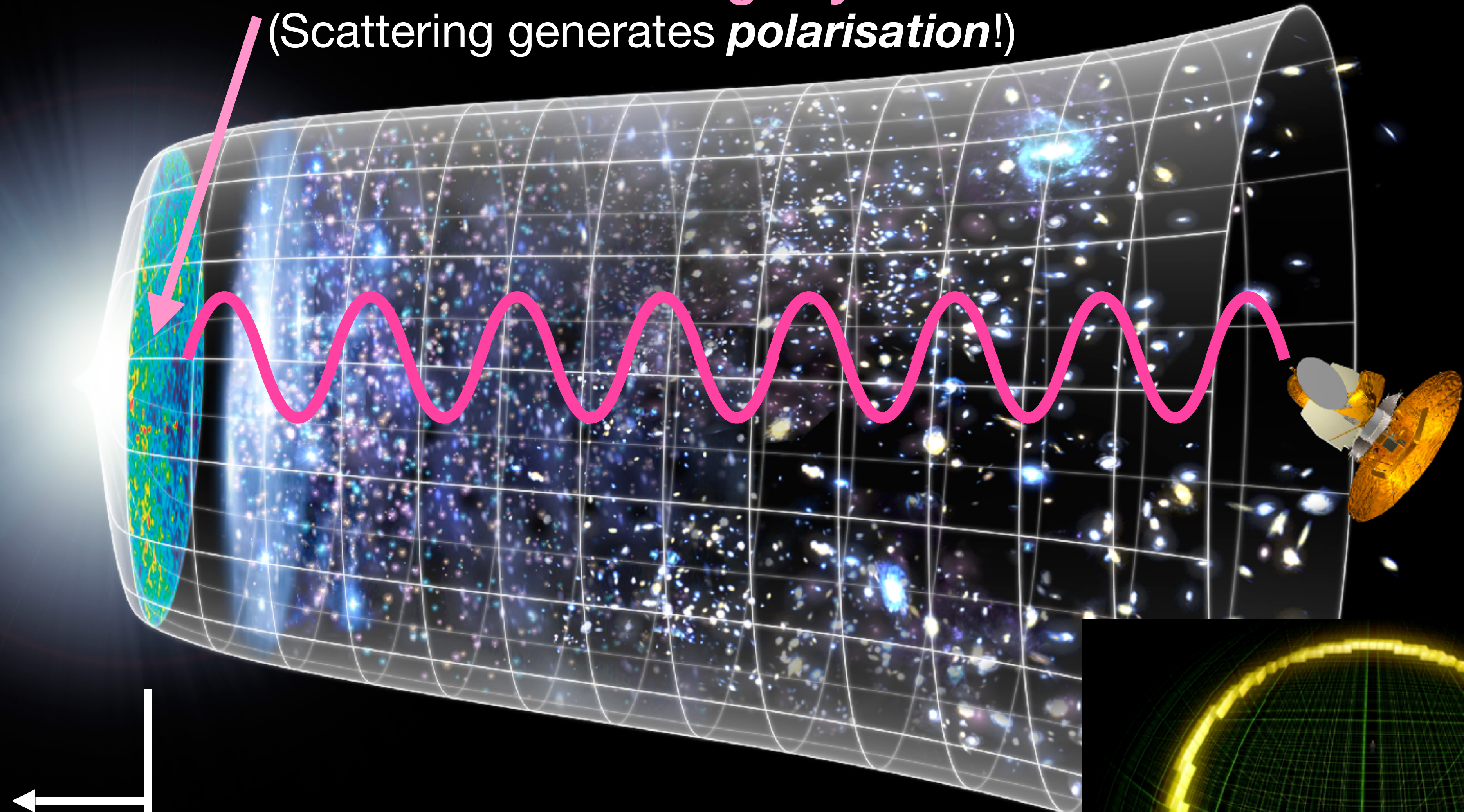


Not shown: The cosmological redshift due to the expansion of the Universe



# The surface of "last scattering" by electrons

(Scattering generates *polarisation!*)



How do we "see" beyond this "wall"? *Laws of physics!*



**Before we talk about the GW,  
let's talk about the sound waves  
(scalar modes)**



# Gravitational Field Equations (Einstein's Eq.)

$$\nabla^2 \Psi = 4\pi G a^2 \sum_{\alpha} \left[ \delta\rho_{\alpha} - \frac{3\dot{a}}{a} (\bar{\rho}_{\alpha} + \bar{P}_{\alpha}) \delta u_{\alpha} \right],$$

$$\partial_i \partial_j (\Phi - \Psi) = -8\pi G a^2 \partial_i \partial_j \sum_{\alpha} \pi_{\alpha},$$

## Energy Conservation

$$\frac{\partial}{\partial t} (\delta\rho_{\gamma} / \bar{\rho}_{\gamma}) - \frac{4q^2}{3a^2} \delta u_{\gamma} = 4\dot{\Psi},$$

$$\frac{\partial}{\partial t} (\delta\rho_B / \bar{\rho}_B) - \frac{q^2}{a^2} \delta u_B = 3\dot{\Psi},$$

## Momentum Conservation

$$\frac{4}{3} \frac{\partial}{\partial t} (\bar{\rho}_{\gamma} \delta u_{\gamma}) + \frac{4\dot{a}}{a} \bar{\rho}_{\gamma} \delta u_{\gamma} + \frac{4}{3} \bar{\rho}_{\gamma} \Phi + \frac{1}{3} \delta\rho_{\gamma} = \frac{4}{3} \sigma_T \bar{n}_e \bar{\rho}_{\gamma} (\delta u_B - \delta u_{\gamma}),$$

$$\frac{\partial}{\partial t} (\bar{\rho}_B \delta u_B) + \frac{3\dot{a}}{a} \bar{\rho}_B \delta u_B + \bar{\rho}_B \Phi = -\frac{4}{3} \sigma_T \bar{n}_e \bar{\rho}_{\gamma} (\delta u_B - \delta u_{\gamma}),$$

*Laws of physics!*



Gravitational Field Equations

+

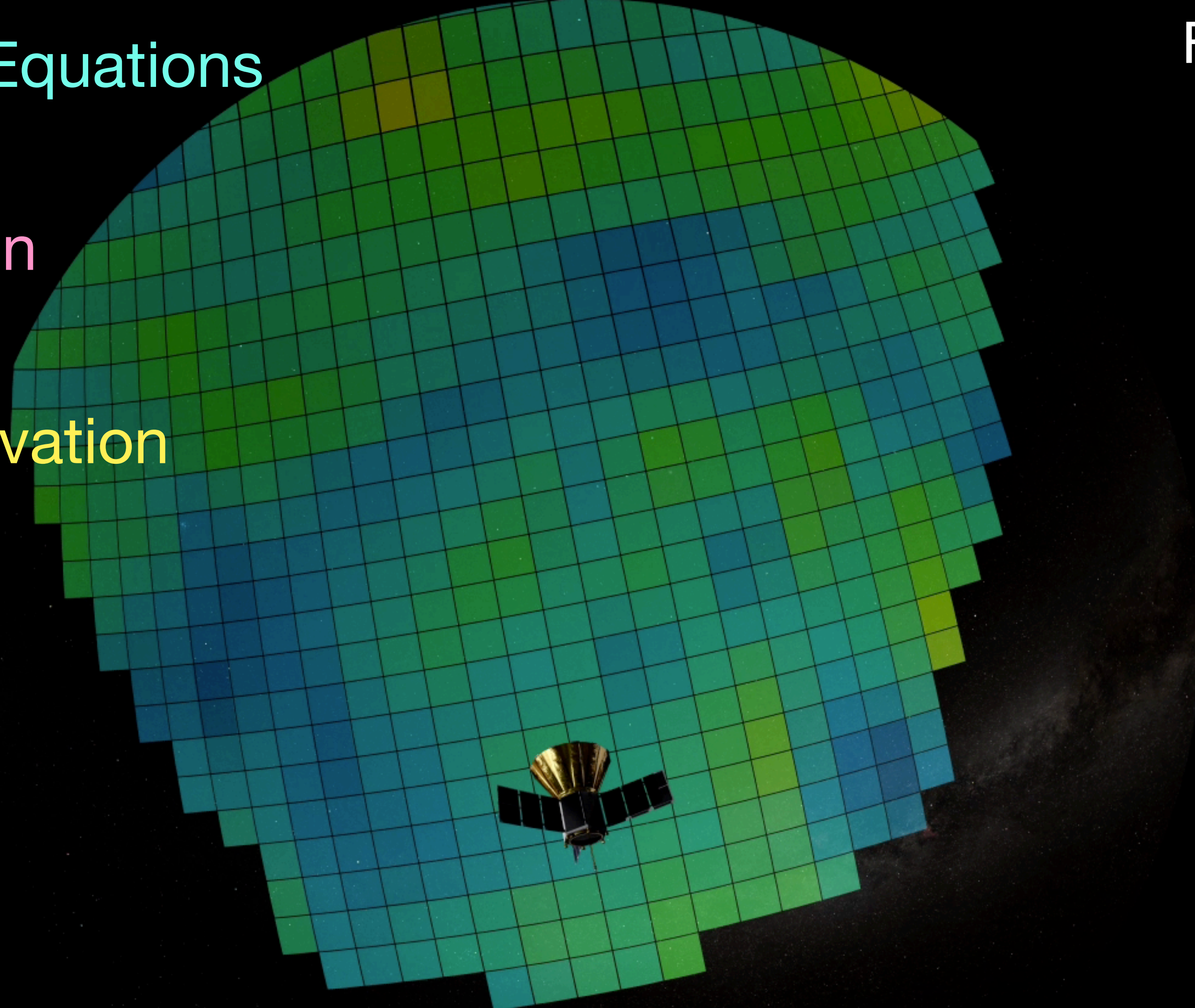
Energy Conservation

+

Momentum Conservation

||

**Sound Waves!**







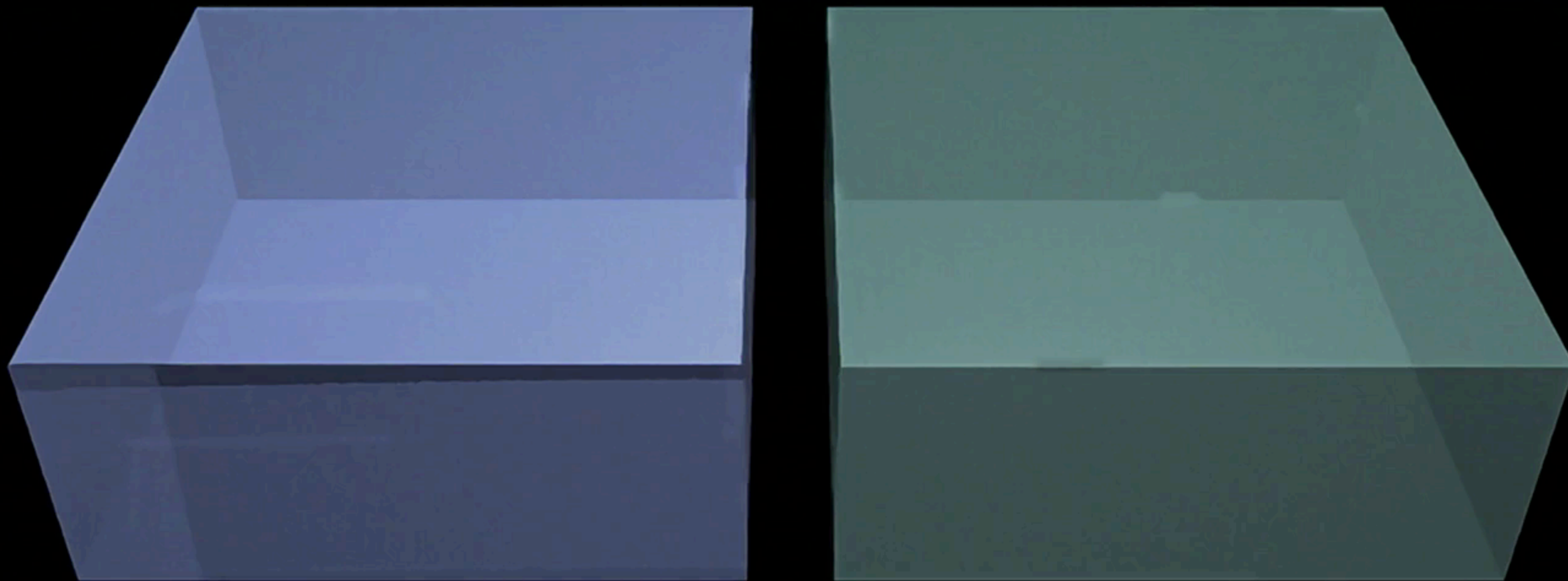


# Kosmische Miso-Suppe

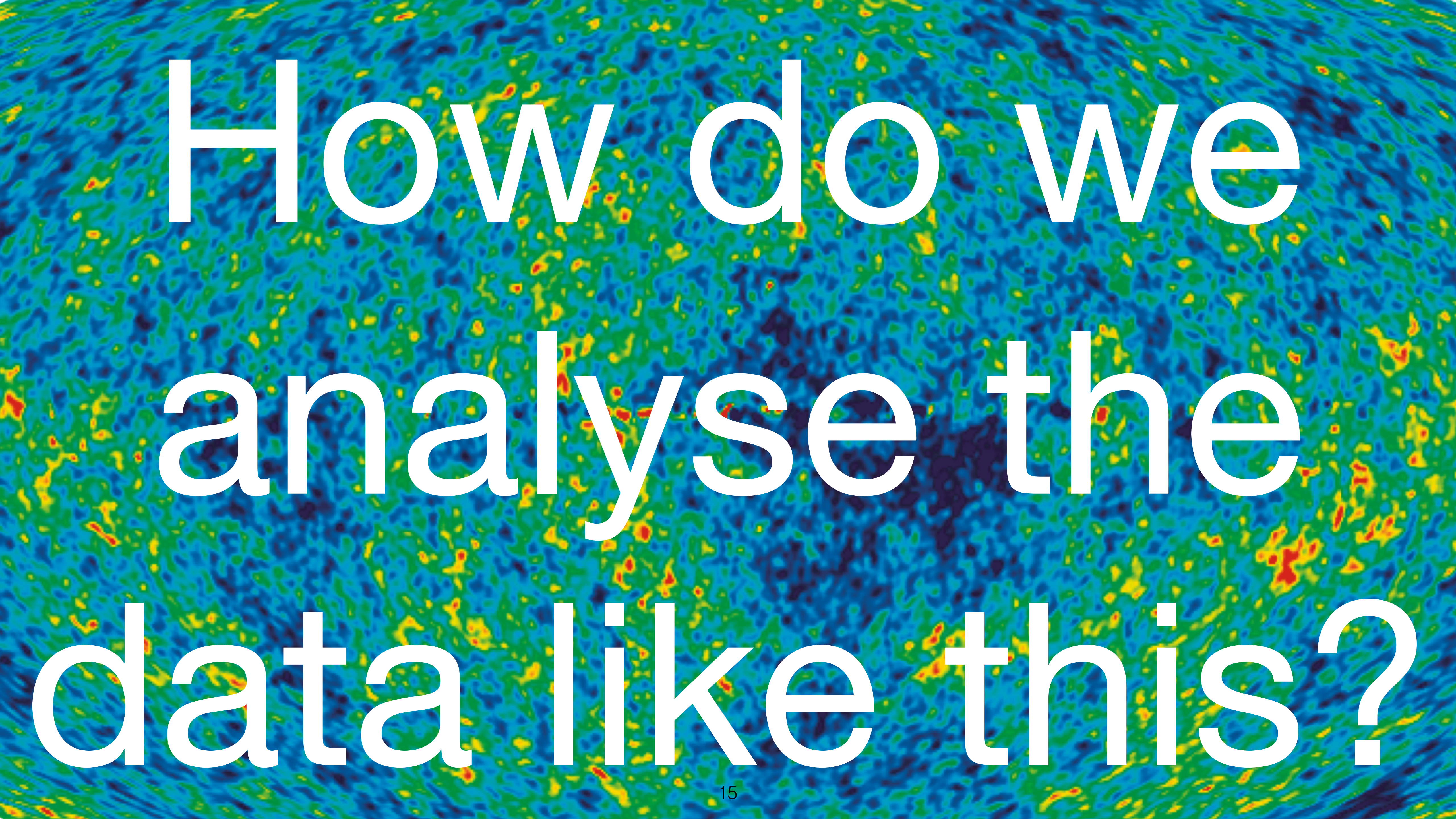
- When matter and radiation were hotter than 3000 K, matter was completely ionised. The Universe was filled with plasma, which behaves just like a soup
- Think about a Miso soup (if you know what it is). Imagine throwing Tofus into a Miso soup, while changing the density of Miso
- And imagine watching how ripples are created and propagate throughout the soup



Credit: WMAP Science Team





The background of the slide is a Cosmic Microwave Background (CMB) fluctuation map. It shows a complex, grainy pattern of temperature variations across the sky, with colors ranging from dark blue (cooler) to red and yellow (warmer). The pattern is roughly circular, representing the observable universe.

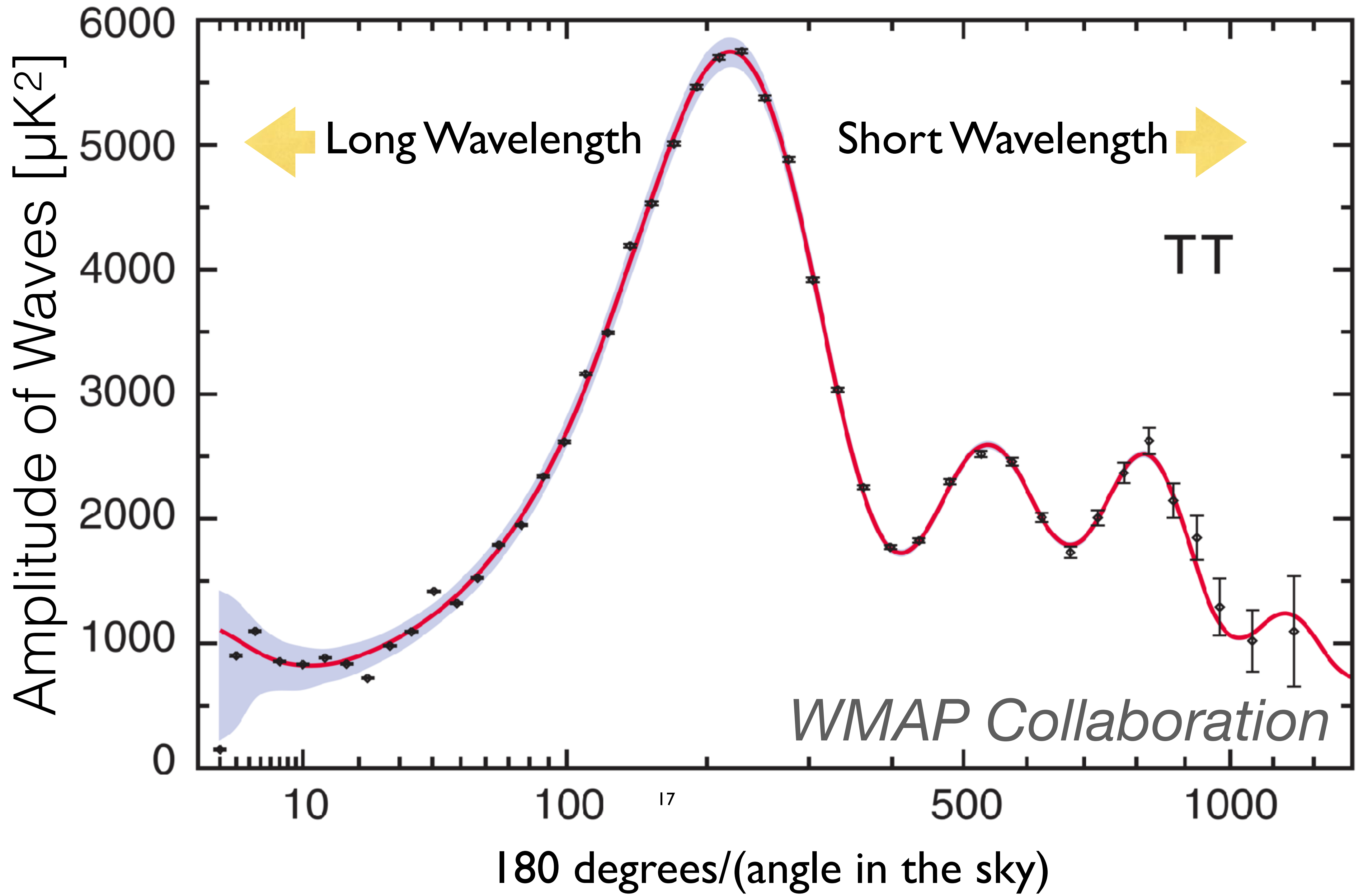
How do we  
analyse the  
data like this?



# Data Analysis

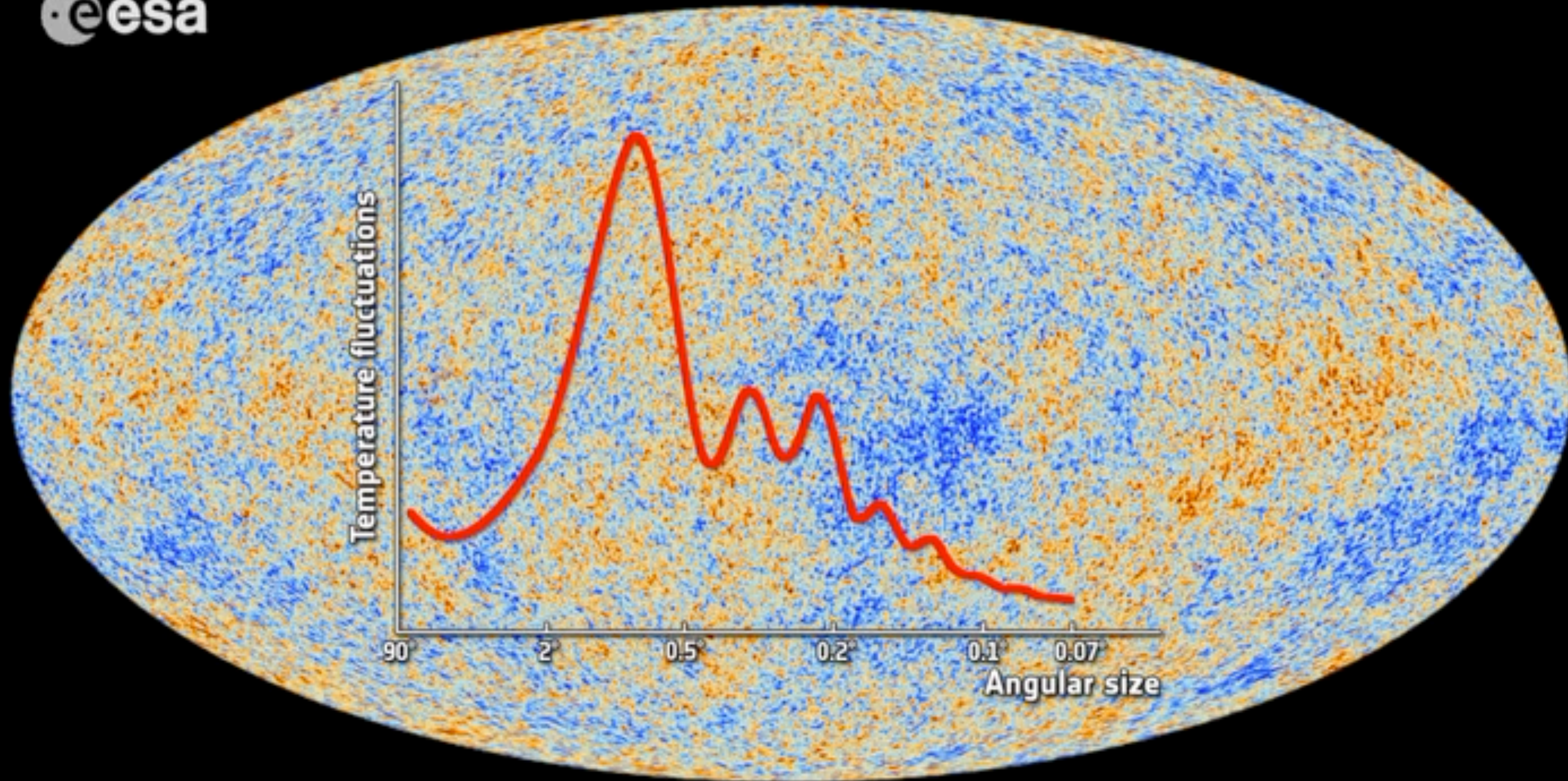
- Decompose temperature fluctuations in the sky into a set of waves with various wavelengths
- Make a diagram showing the strength of each wavelength: **Power Spectrum**







# Power Spectrum, Explained







The Royal Swedish Academy of Sciences has decided to award the 2019 Nobel Prize in Physics to

**JAMES PEEBLES**

"for theoretical discoveries in physical cosmology"

# Sound waves in the fireball Universe, predicted in 1970

## James Peebles Facts



James Peebles  
The Nobel Prize in Physics 2019

Born: 1935, Winnipeg, Canada

Affiliation at the time of the award: I  
Princeton, NJ, USA

Prize motivation: "for theoretical dis  
cosmology."

Prize share: 1/2

**THE ASTROPHYSICAL JOURNAL**, 162:815–836, December 1970

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### PRIMEVAL ADIABATIC PERTURBATION IN AN EXPANDING UNIVERSE\*

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Joseph Henry Laboratories, Princeton University

AND

J. T. YU‡

Goddard Institute for Space Studies, NASA, New York

*Received 1970 January 5; revised 1970 April 1*

Ill. Niklas Elmedhed. © Nobel





**At the *ICGC2011* conference, Goa, India**



# Sound waves in the fireball Universe, predicted in 1970

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## SMALL-SCALE FLUCTUATIONS OF RELIC RADIATION\*

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*Institute of Applied Mathematics, Academy of Sciences of the U.S.S.R., Moscow, U.S.S.R.*

(Received 11 September, 1969)

The Franklin Institute  
of Physics





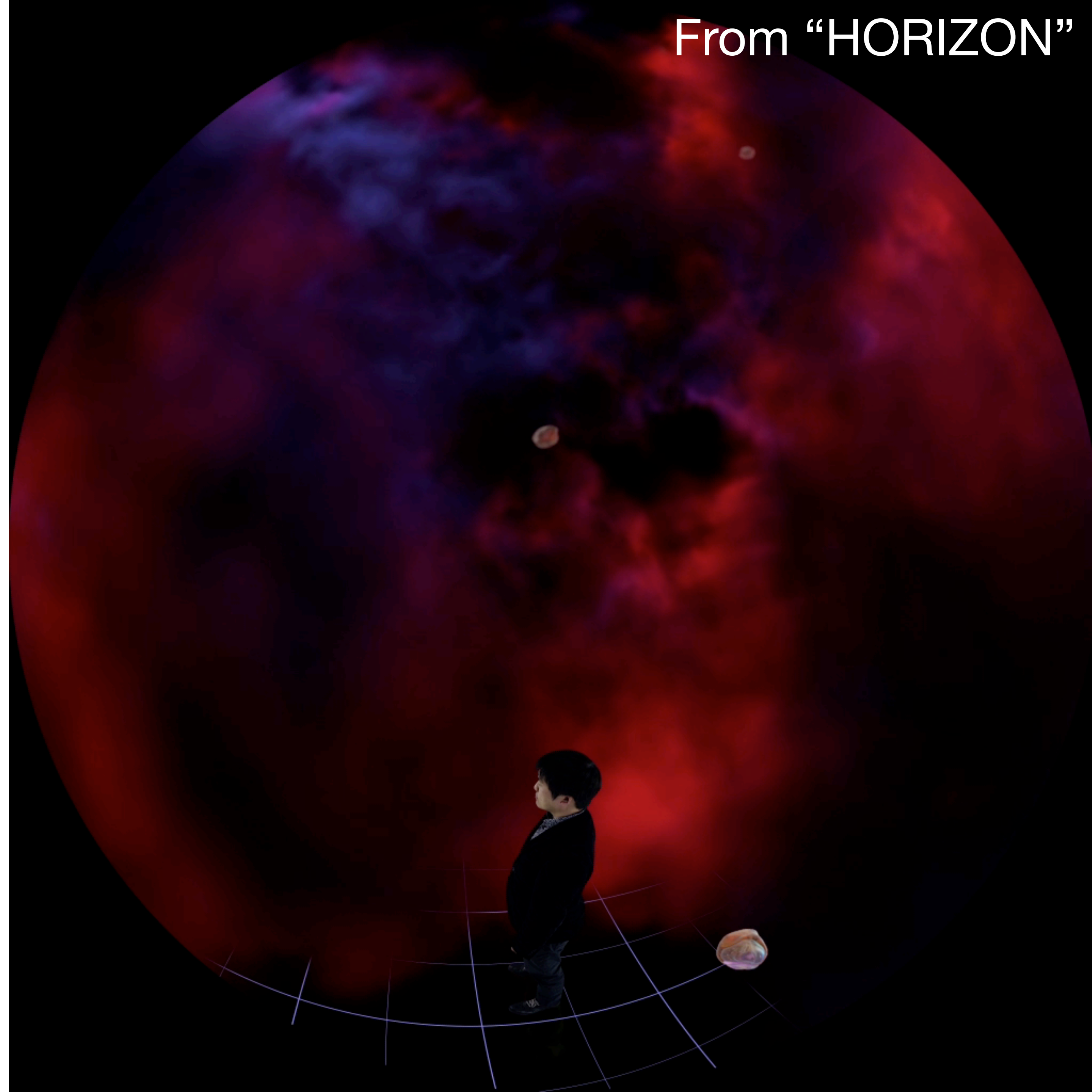
# Determine the composition of the Universe

## The Universe as a "hot soup"

- The power spectrum allows us to determine the composition of the Universe, such as the density of atoms, dark matter, and dark energy.



- **Definitive evidence for non-baryonic nature of dark matter!**





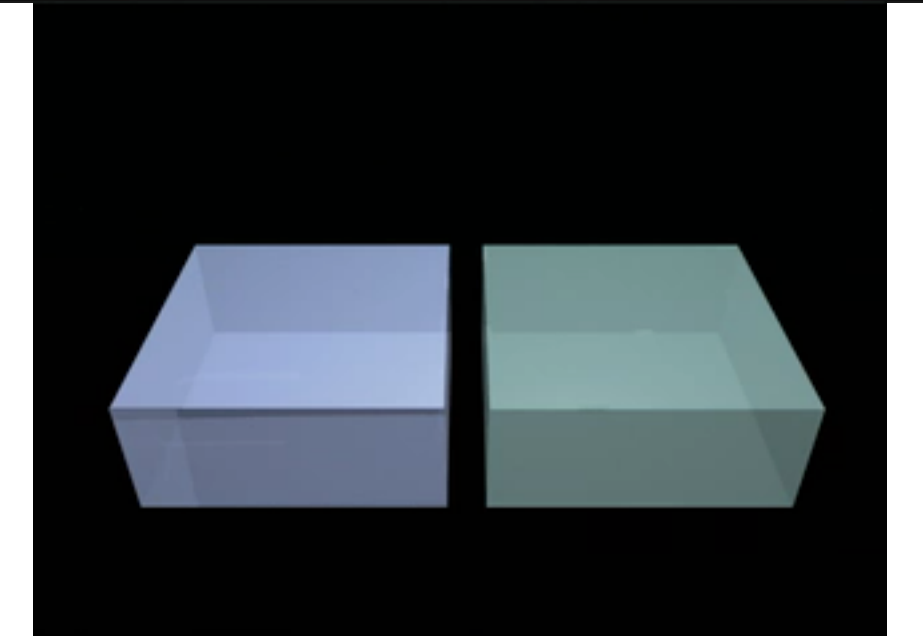
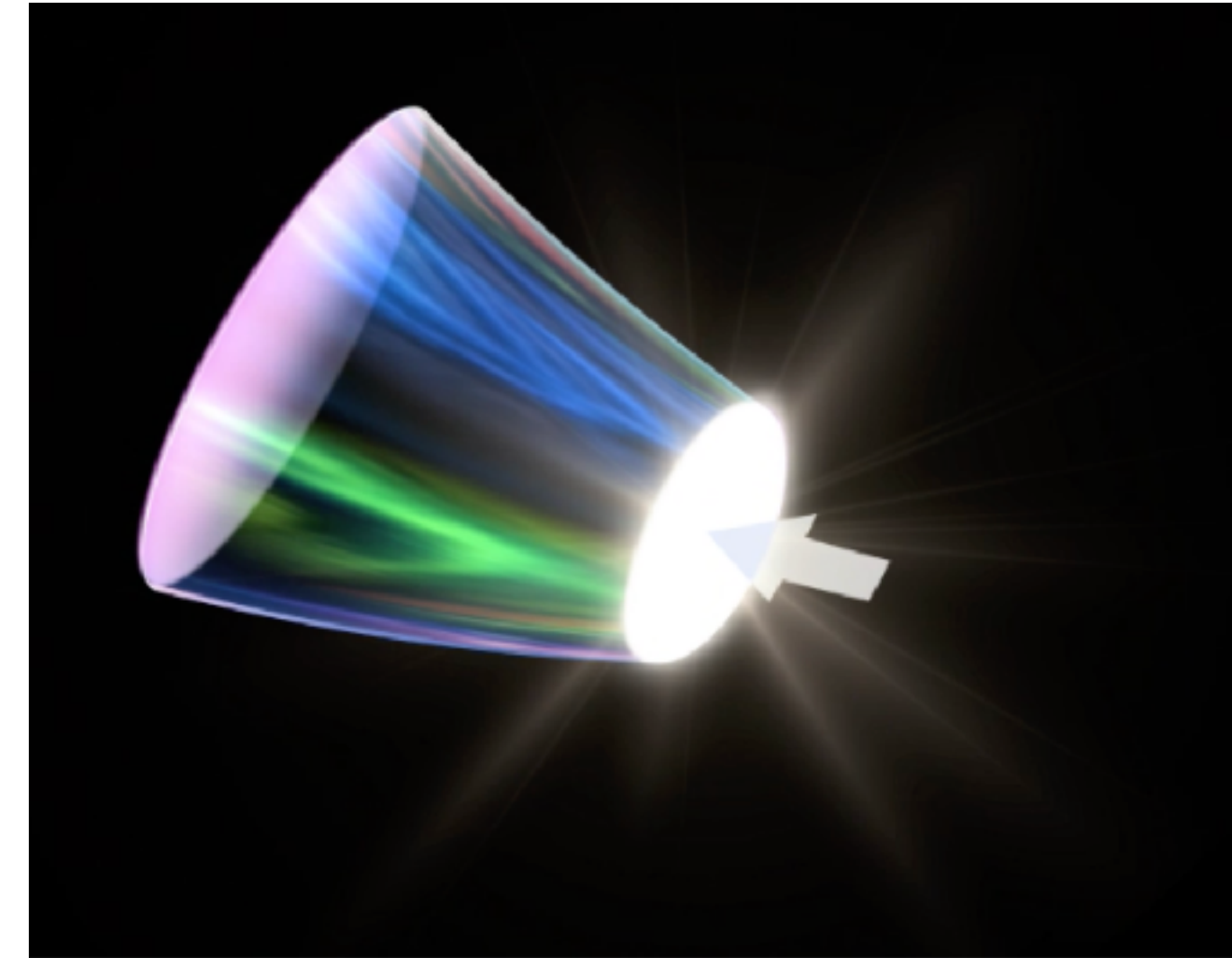
# “Let’s give some impact to the beginning of this model”

- What gave the initial fluctuation to the cosmic hot soup?

*Mukhanov & Chibisov (1981); Hawking (1982); Starobinsky (1982); Guth & Pi (1982);  
Bardeen, Turner & Steinhardt (1983)*

## Leading Idea:

- Quantum mechanics at work in the early Universe
  - “*We all came from quantum fluctuations*”
- But, how did the quantum fluctuation on the *microscopic* scale become *macroscopic* over large distances?
- **What is the missing link between the small and large scales?**





**Gravity + Quantum**

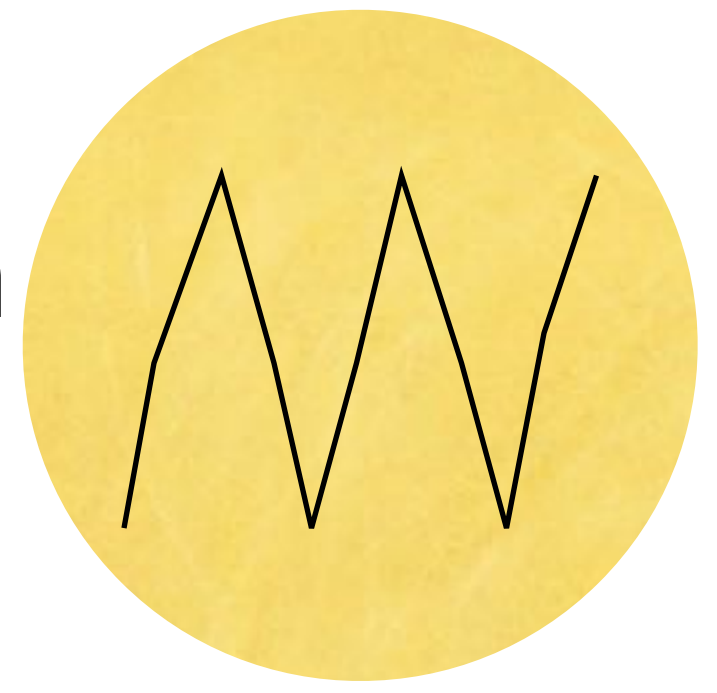
**= The origin of all the structures  
we see in the Universe**



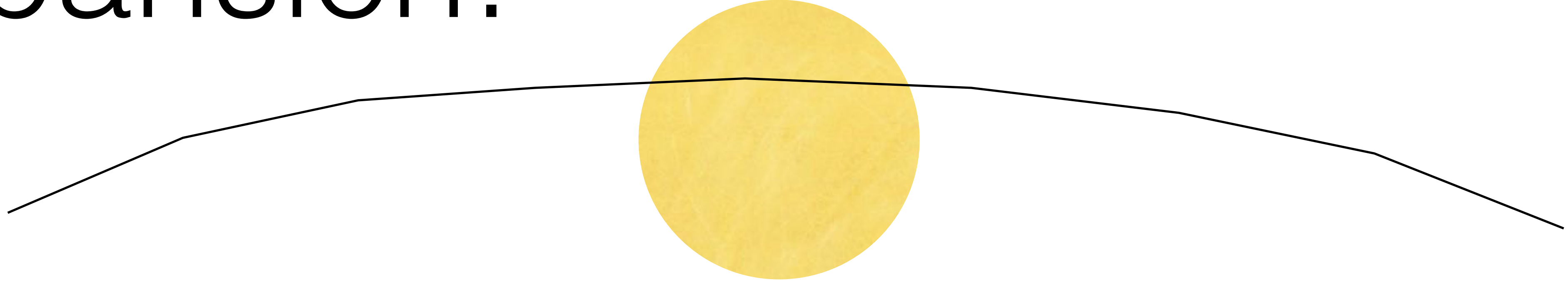
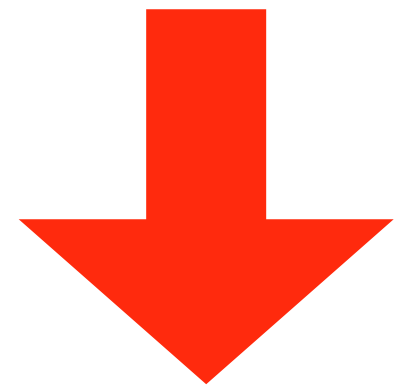
*Starobinsky (1980); Sato (1981); Guth (1981); Linde (1982); Albrecht & Steinhardt (1982)*

# Cosmic Inflation

Quantum mechanical fluctuation  
on microscopic scales



Exponential  
Expansion!



- Exponential expansion (inflation) stretches the wavelength of quantum fluctuations to cosmological scales



**What? How can we believe such a statement?**

***Only the data will decide!***



# Finding Cosmic Inflation

## What does inflation predict?

- Due to expansion of space, the distance between two points is stretched in proportion to  $a(t)$ .
- **The Hubble expansion rate** is defined as  $H(t) = a^{-1} (da/dt)$ . This has the units of [1/time].
  - In other words,  $a(t) = \exp[ \int H(t) dt ]$ .
  - During inflation, the distance between two points expands exponentially. This means  $H(t) \sim \text{constant}$ , which gives  $a(t) \sim \exp(Ht)$ .
- However, inflation must end. This means that  $H(t)$  is a slowly decreasing function of time.

How can we test this?



# Finding Cosmic Inflation

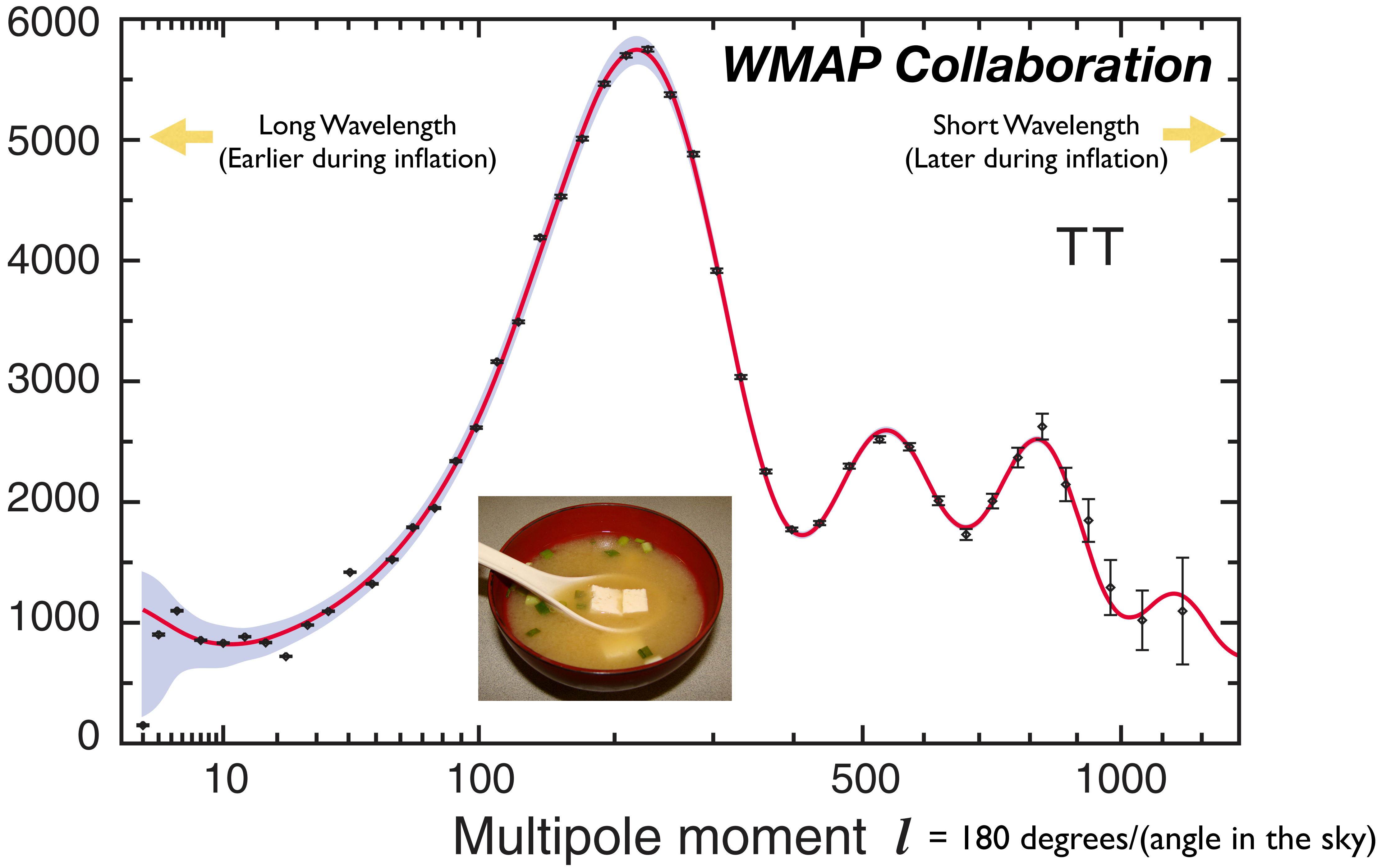
## What does inflation predict for the density fluctuation?

- During inflation, the density fluctuation is produced quantum mechanically.
- According to Quantum Mechanics during inflation,
  - **The strength of density fluctuation is proportional to  $H$**
- **THE KEY:** The earlier the fluctuations are generated, the more its wavelength is stretched, and thus the bigger the angles they subtend in the sky. **Because  $H(t)$  is a decreasing function of time, inflation predicts that the amplitude of fluctuations on large angular scales is slightly larger than that on small angular scales!**



Amplitude of Waves

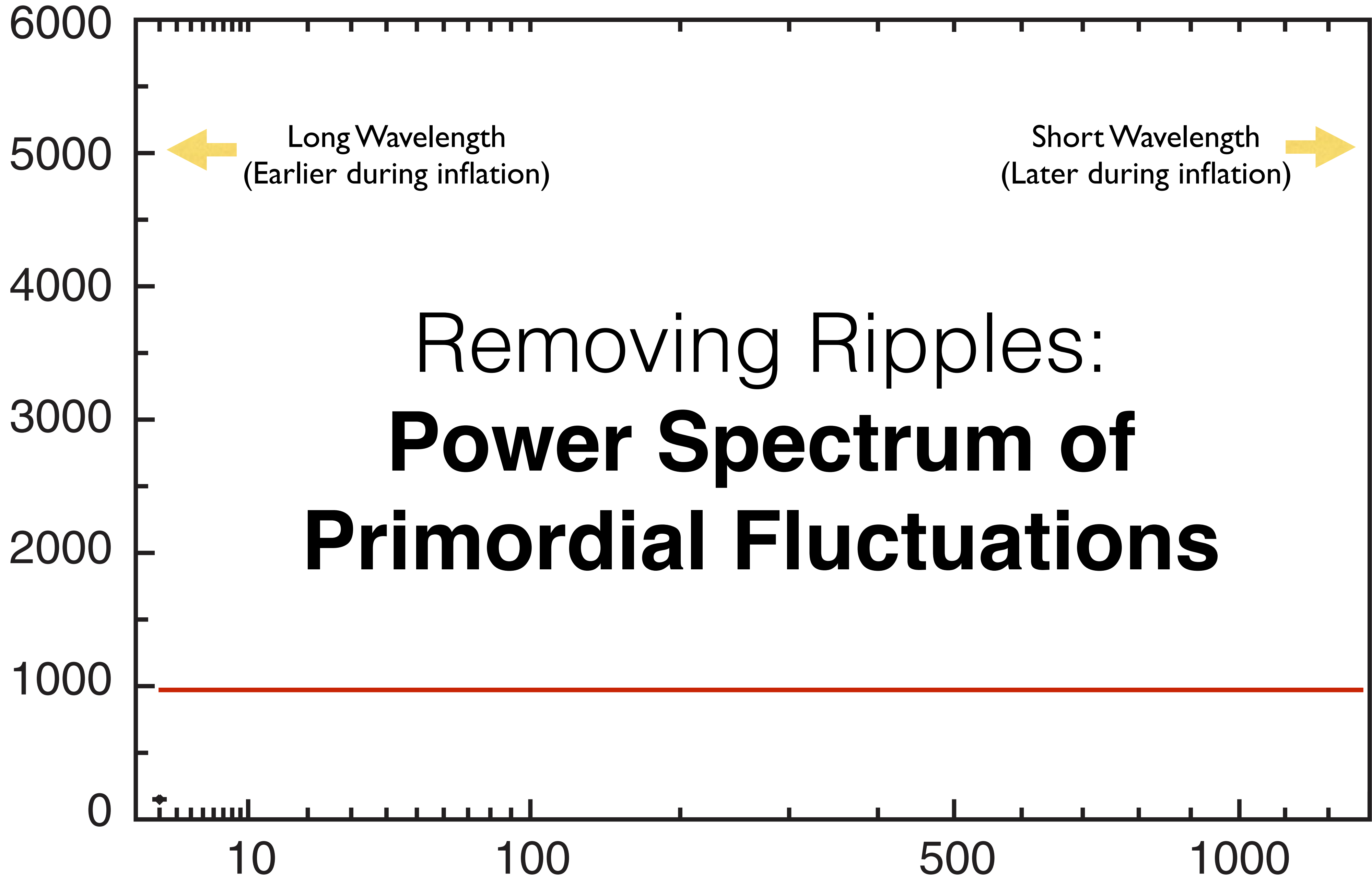
$[\mu\text{K}^2]$





Amplitude of Waves

[ $\mu\text{K}^2$ ]



Multipole moment  $l = 180 \text{ degrees}/(\text{angle in the sky})$



Amplitude of Waves

[ $\mu\text{K}^2$ ]

6000  
5000  
4000  
3000  
2000  
1000  
0

Long Wavelength  
(Earlier during inflation)

Short Wavelength  
(Later during inflation)

Removing Ripples:  
**Power Spectrum of  
Primordial Fluctuations**

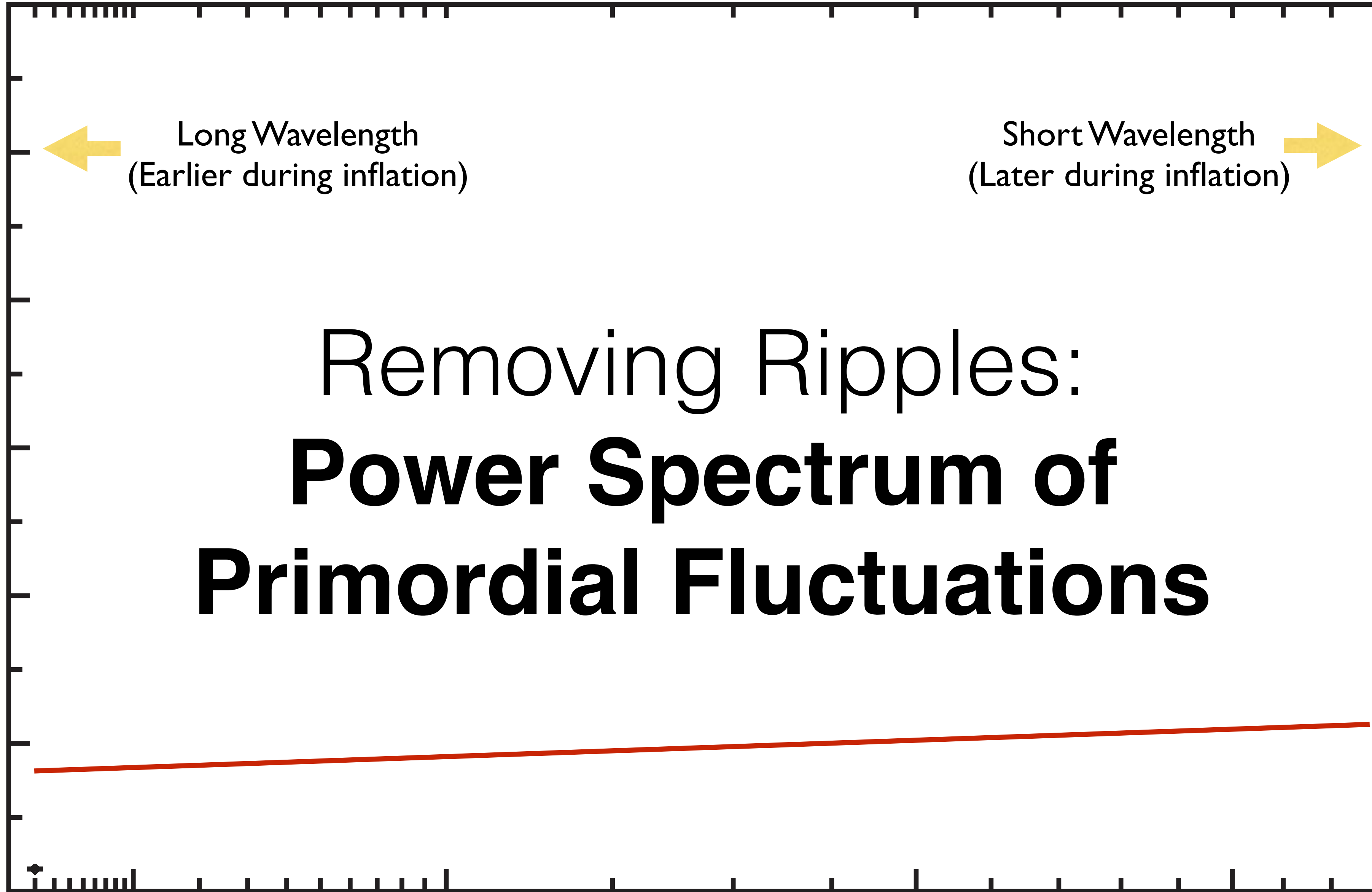
10

100

500

1000

Multipole moment  $l = 180 \text{ degrees}/(\text{angle in the sky})$





Amplitude of Waves

[ $\mu\text{K}^2$ ]

6000  
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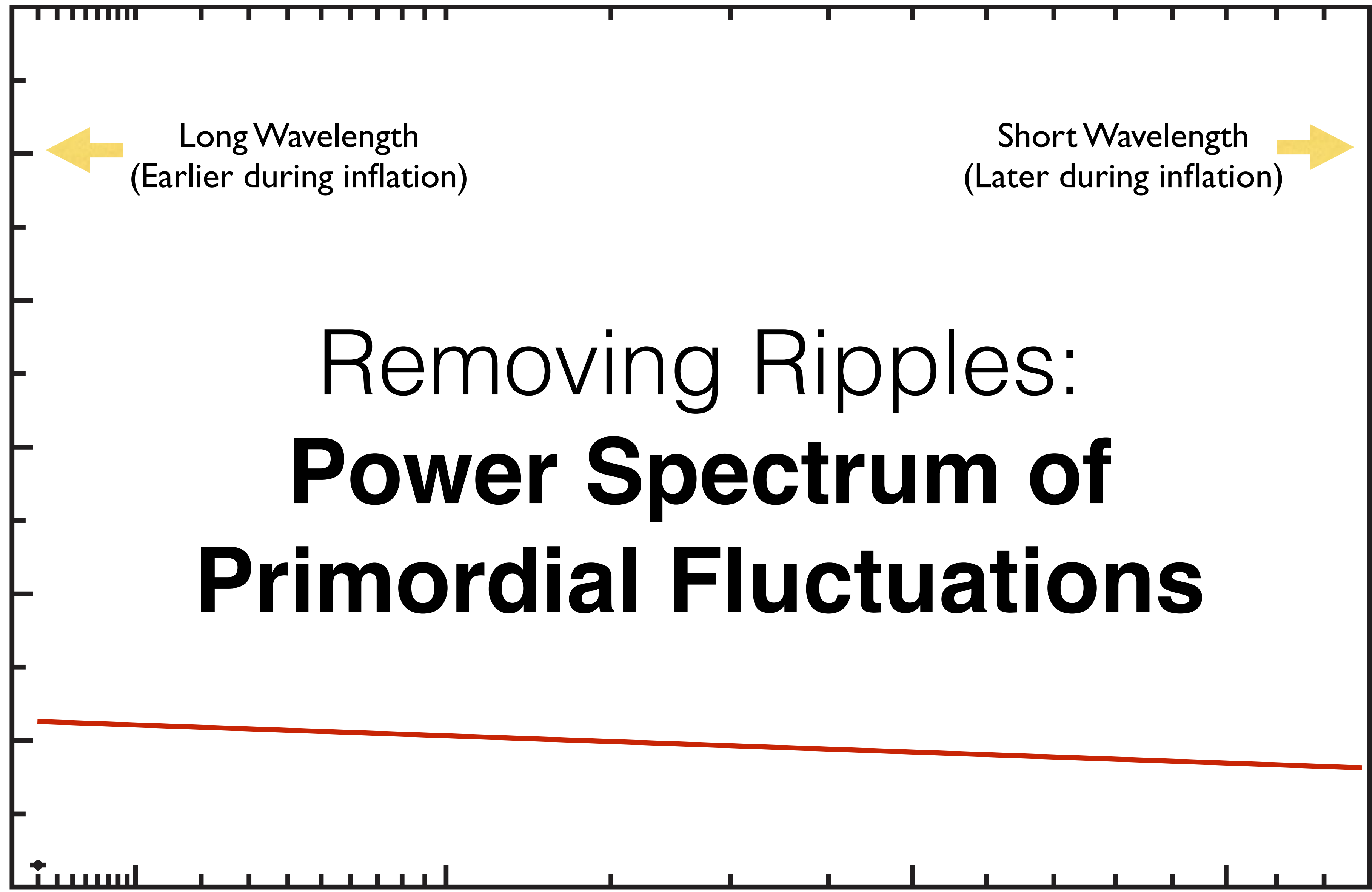
10

100

500

1000

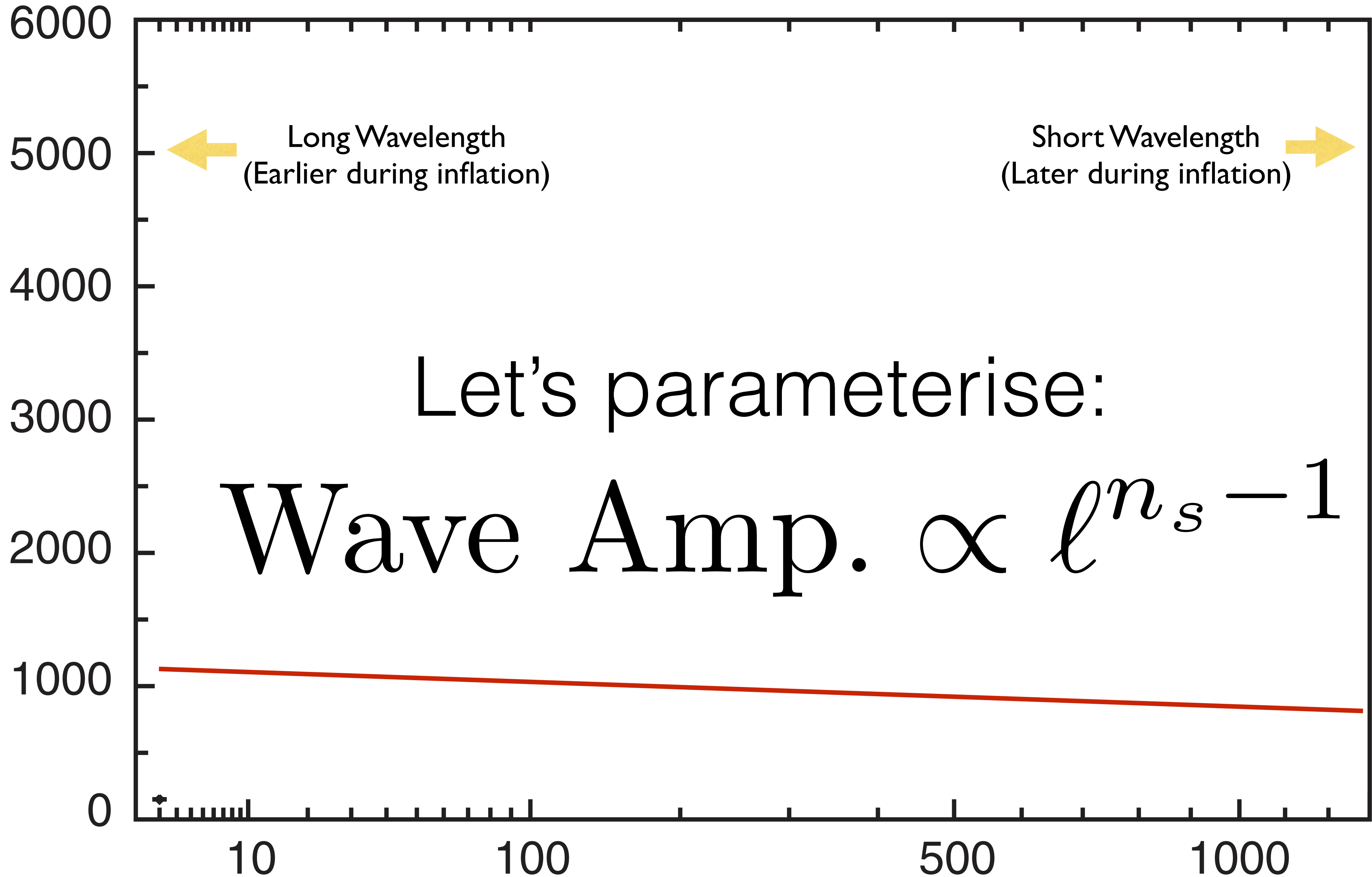
Multipole moment  $l = 180 \text{ degrees}/(\text{angle in the sky})$





Amplitude of Waves

[ $\mu\text{K}^2$ ]



Multipole moment  $l = 180 \text{ degrees}/(\text{angle in the sky})$

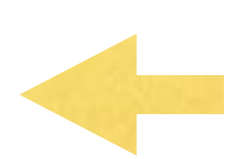


Amplitude of Waves

[ $\mu\text{K}^2$ ]

6000  
5000  
4000  
3000  
2000  
1000  
0

**Wright, Smoot, Bennett & Lubin (1994)**



Long Wavelength  
(Earlier during inflation)



Short Wavelength  
(Later during inflation)



**In 1994:**

**COBE 2-Year Limit!**

**$n_s = 1.25^{+0.4}_{-0.45}$  (68% CL)**

**$l=3-30$**

**Wave Amp.  $\propto l^{n_s - 1}$**

10 100 500 1000

Multipole moment  $l = 180 \text{ degrees}/(\text{angle in the sky})$

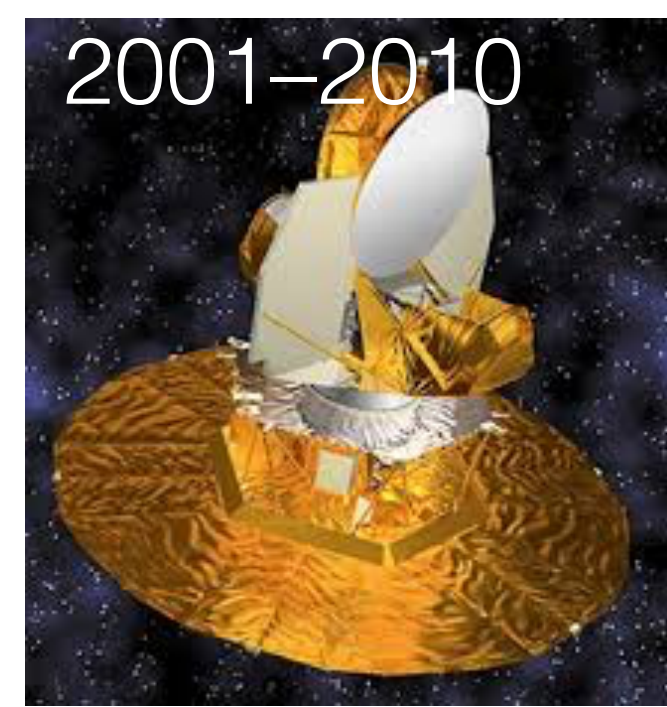


Amplitude of Waves

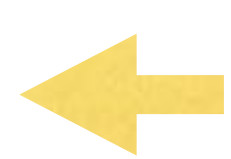
[ $\mu\text{K}^2$ ]

6000  
5000  
4000  
3000  
2000  
1000  
0

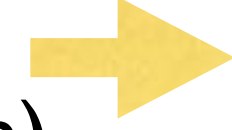
**WMAP Collaboration**



**In 2012:**



Long Wavelength  
(Earlier during inflation)



Short Wavelength  
(Later during inflation)

WMAP 9-Year Only:  
 **$n_s = 0.972 \pm 0.013$  (68%CL)**

Wave Amp.  $\propto \ell^{n_s - 1}$

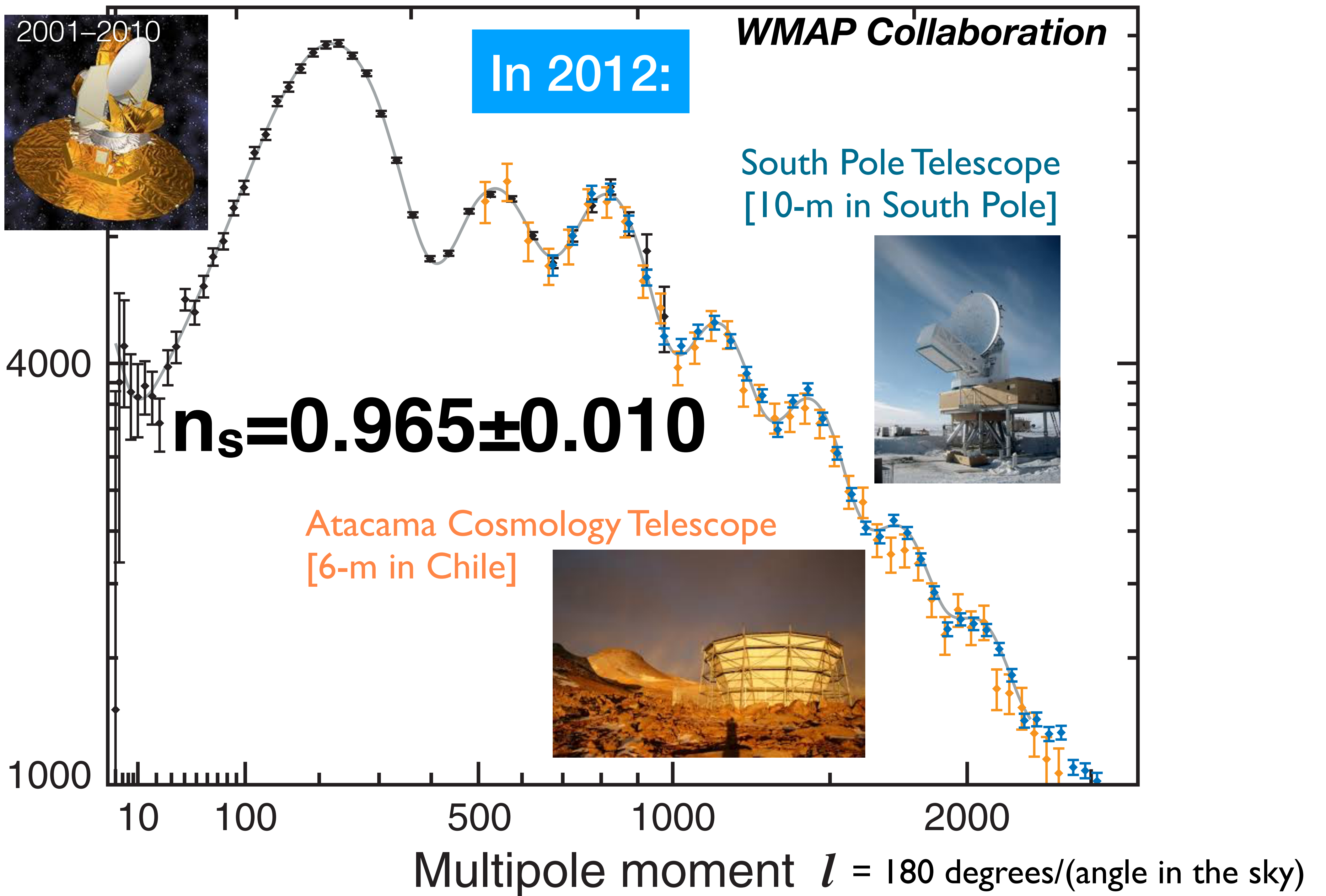
10 100 500 1000

Multipole moment  $l = 180 \text{ degrees}/(\text{angle in the sky})$



# Amplitude of Waves

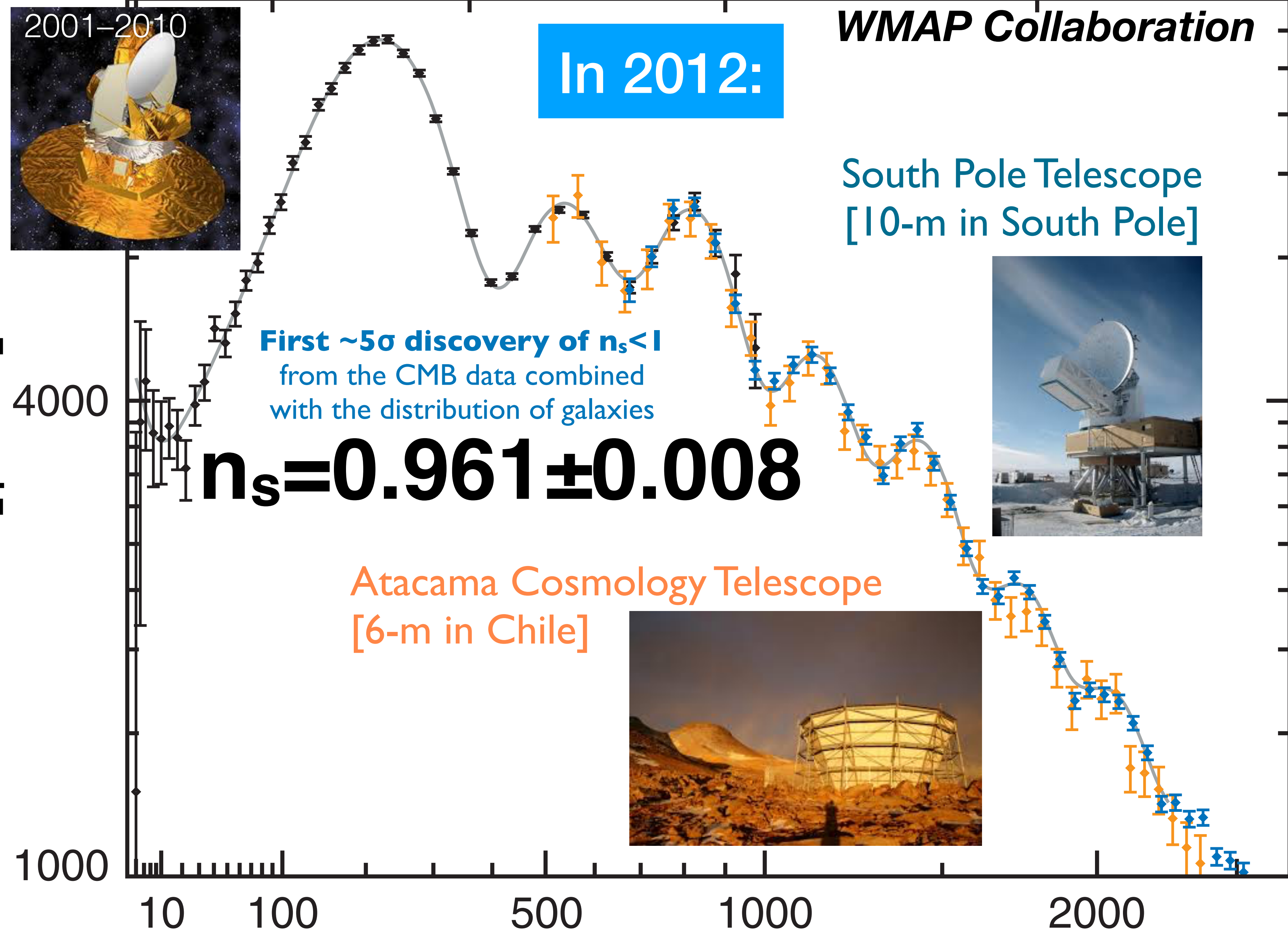
$[\mu\text{K}^2]$





# Amplitude of Waves

[ $\mu\text{K}^2$ ]



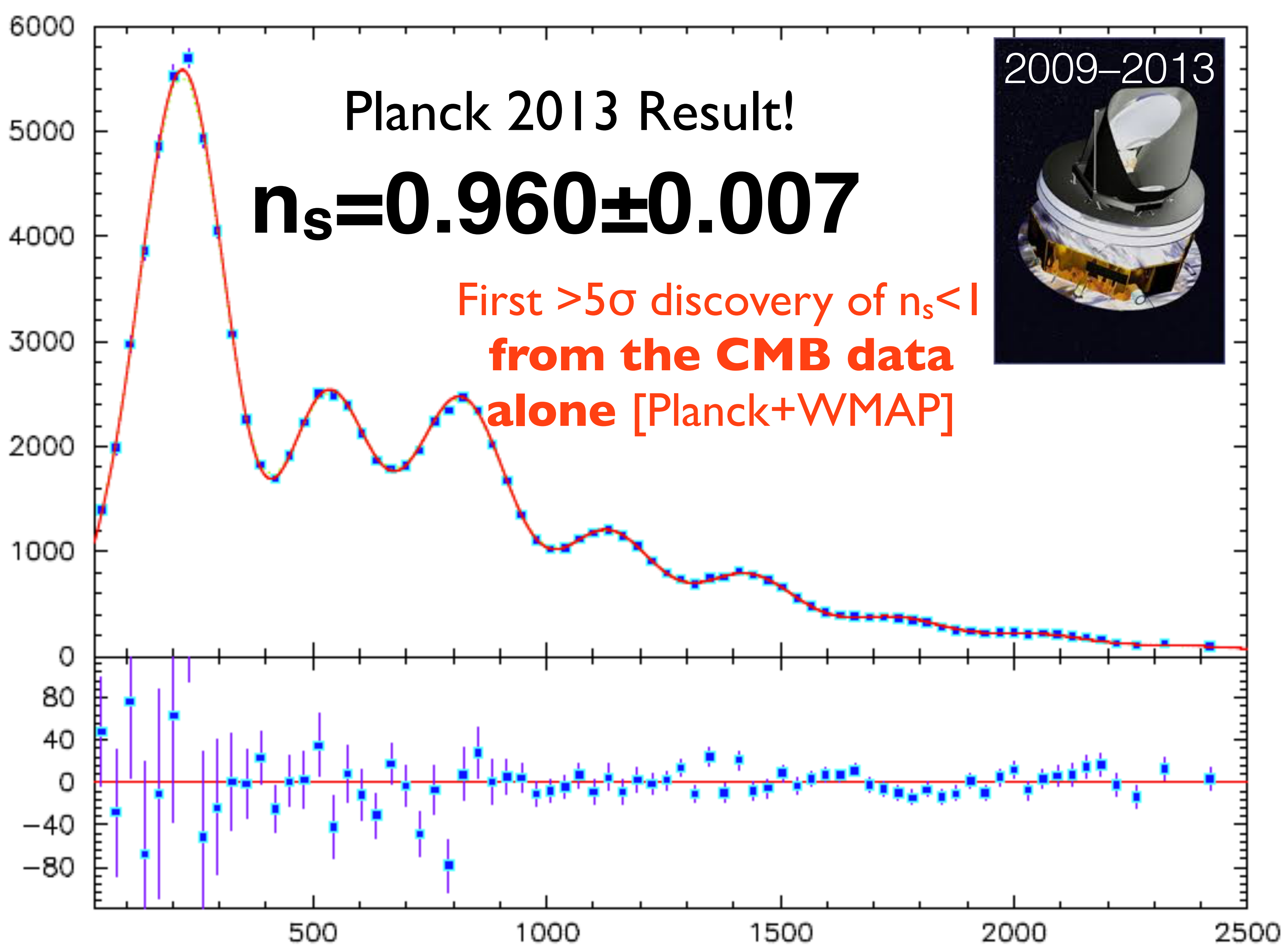
Multipole moment  $l = 180 \text{ degrees}/(\text{angle in the sky})$



Amplitude of Waves

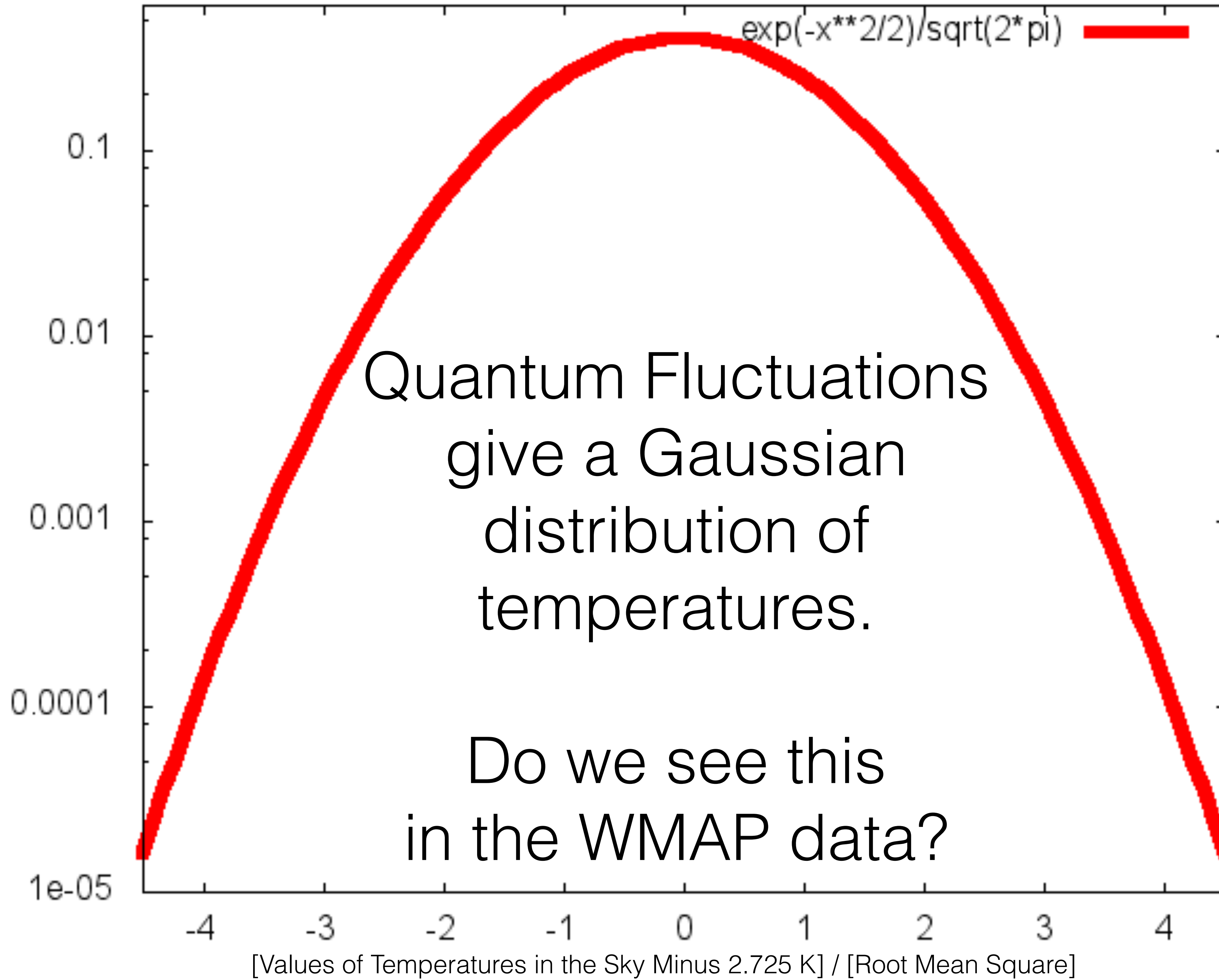
[ $\mu\text{K}^2$ ]

Residual



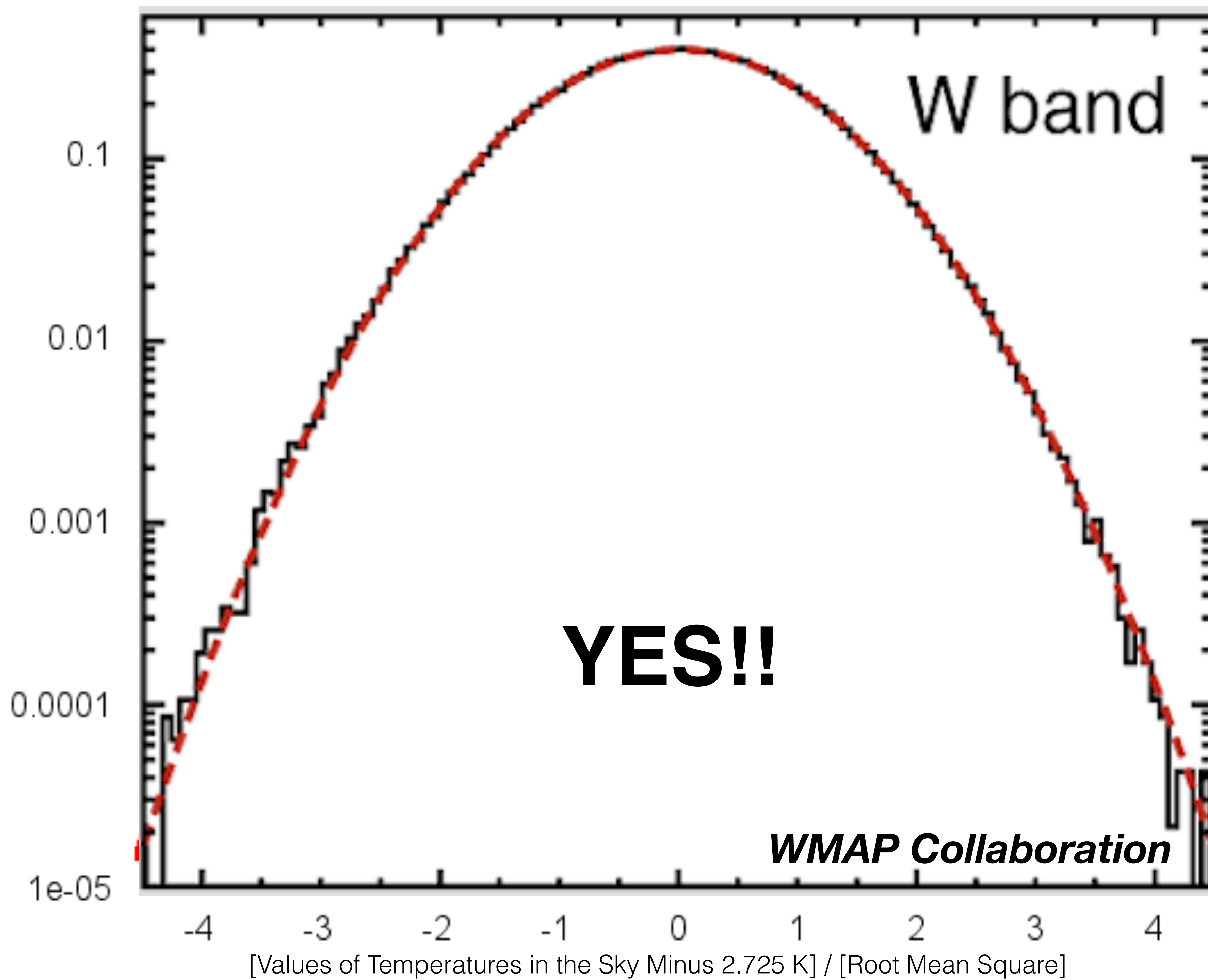


# Fraction of the Number of Pixels Having Those Temperatures





Fraction of the Number of Pixels  
Having Those Temperatures





# So, have we found inflation?

A lot of evidence in support of inflation exist already.

- Single-field slow-roll inflation looks very good:
  - ✓ •  $n_s < 1$
  - ✓ • Gaussian fluctuations
  - ✓ • Adiabatic fluctuations [no time to explain this today]
  - ✓ • Super-horizon fluctuations [no time to explain this today]
- What more do we want? **Primordial gravitational waves**
- Why more evidence? Because “***extraordinary claim requires extraordinary evidence***” (Carl Sagan)

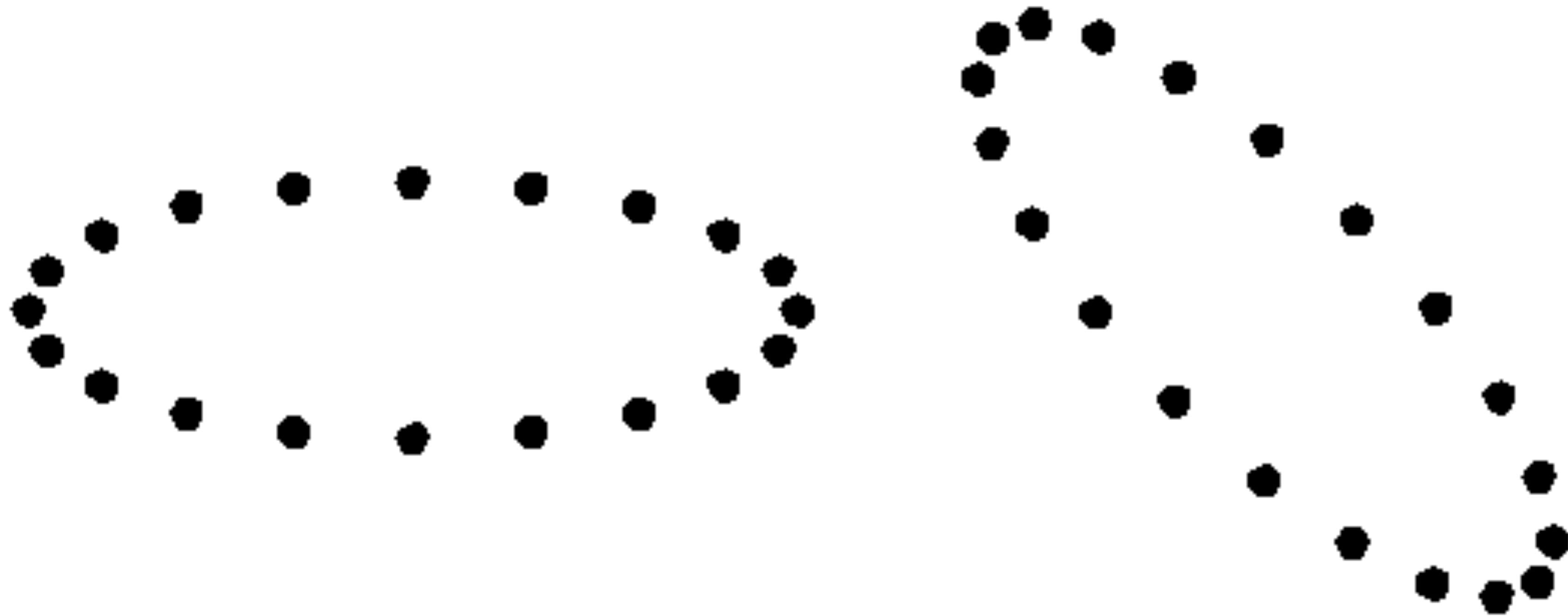


**Let's talk about the GW  
(tensor modes)**



# Gravitational waves are coming towards you!

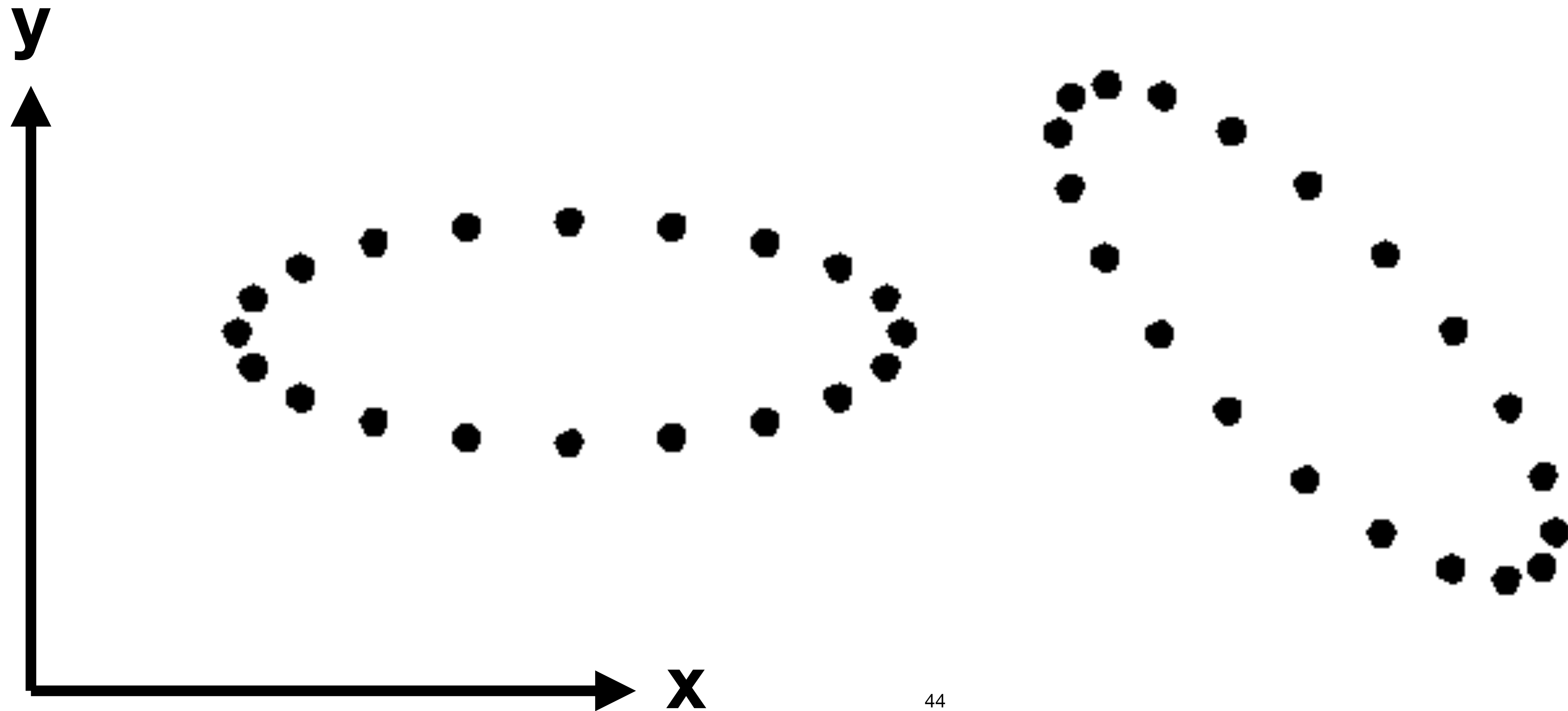
To visualise the waves, watch motion of test particles.





# Gravitational waves are coming towards you!

To visualise the waves, watch motion of test particles.





# Distance between two points

- In Cartesian coordinates, the distance between two points in Euclidean space is

$$ds^2 = dx^2 + dy^2 + dz^2$$

- To include the isotropic expansion of space,

$$ds^2 = a^2(t) (dx^2 + dy^2 + dz^2)$$

**Scale Factor**

**y**



**x**



# Distortion in space

$x^2$

- Compact notation using Kronecker's delta symbol:

$$ds^2 = a^2(t) \sum_{i=1}^3 \sum_{j=1}^3 \delta_{ij} dx^i dx^j$$

$\mathbf{x} = (x, y, z)$

$$\begin{aligned} \delta_{ij} &= 1 \text{ for } i=j; \\ \delta_{ij} &= 0 \text{ otherwise} \end{aligned}$$

- To include distortion in space,

$$ds^2 = a^2 \sum_{i=1}^3 \sum_{j=1}^3 (\delta_{ij} + \boxed{h_{ij}}) dx^i dx^j$$

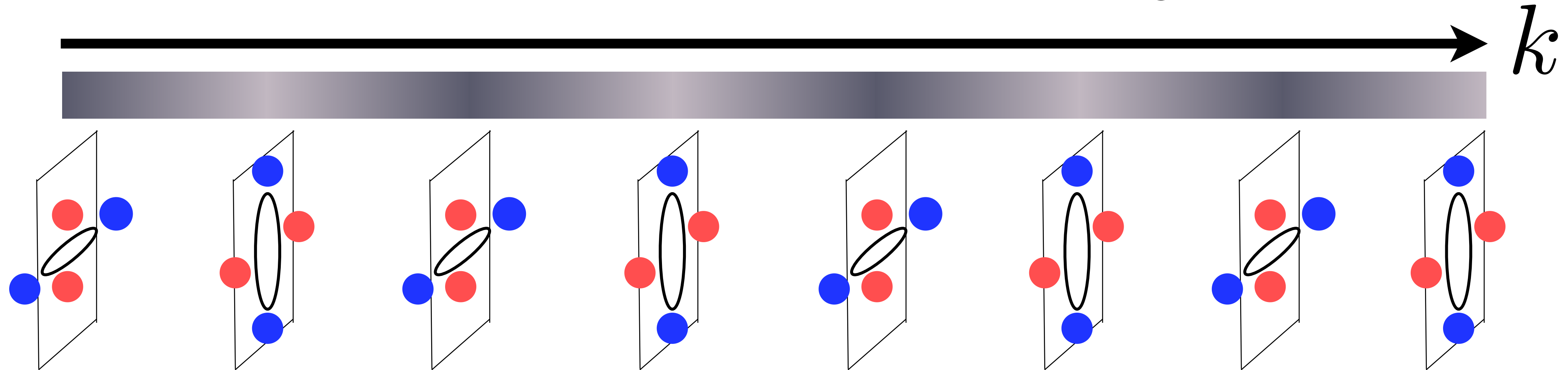
**Distortion in space!**



# Four conditions for gravitational waves

- The gravitational wave shall be transverse.

- The direction of distortion is perpendicular to the propagation direction  $\vec{k}$



Thus, 
$$\sum_{i=1}^3 k^i h_{ij} = 0$$

**3 conditions for  $h_{ij}$**

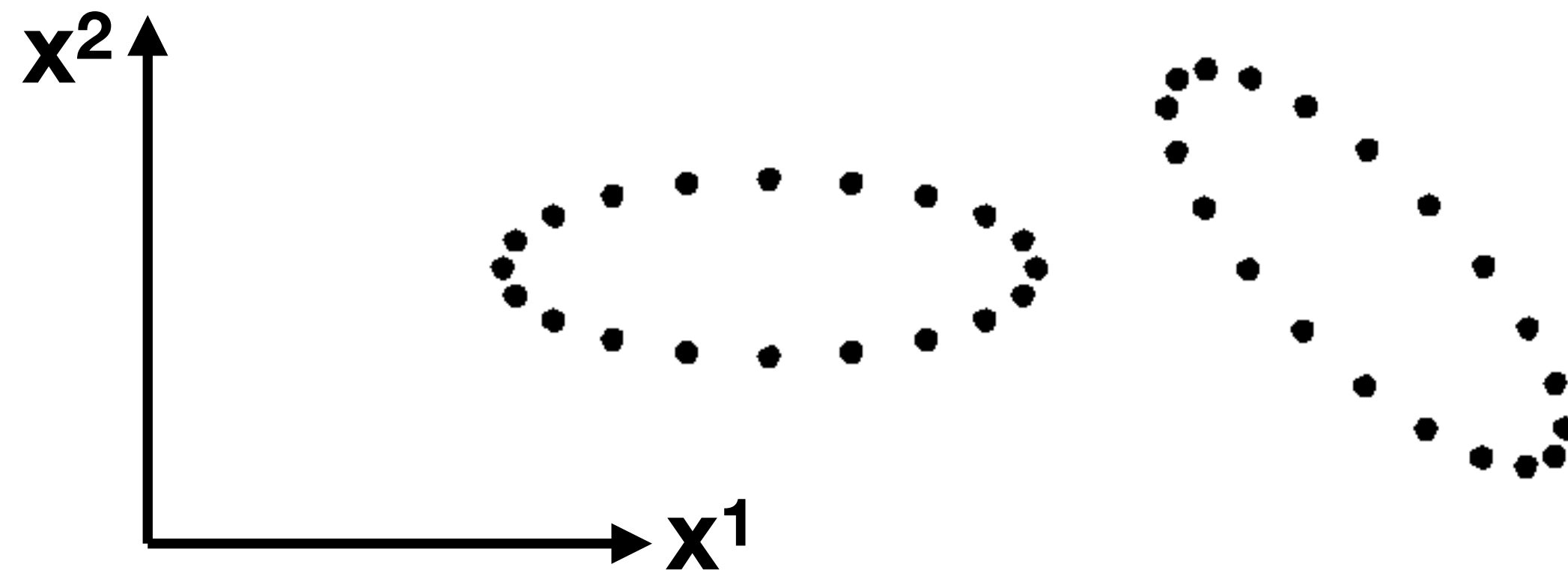


# Four conditions for gravitational waves

- The gravitational wave shall not change the area

- The determinant of  $\delta_{ij}+h_{ij}$  is 1

$$ds^2 = a^2 \sum_{i=1}^3 \sum_{j=1}^3 (\delta_{ij} + h_{ij}) dx^i dx^j$$



Thus, 
$$\sum_{i=1}^3 h_{ii} = 0$$

1 condition for  $h_{ij}$

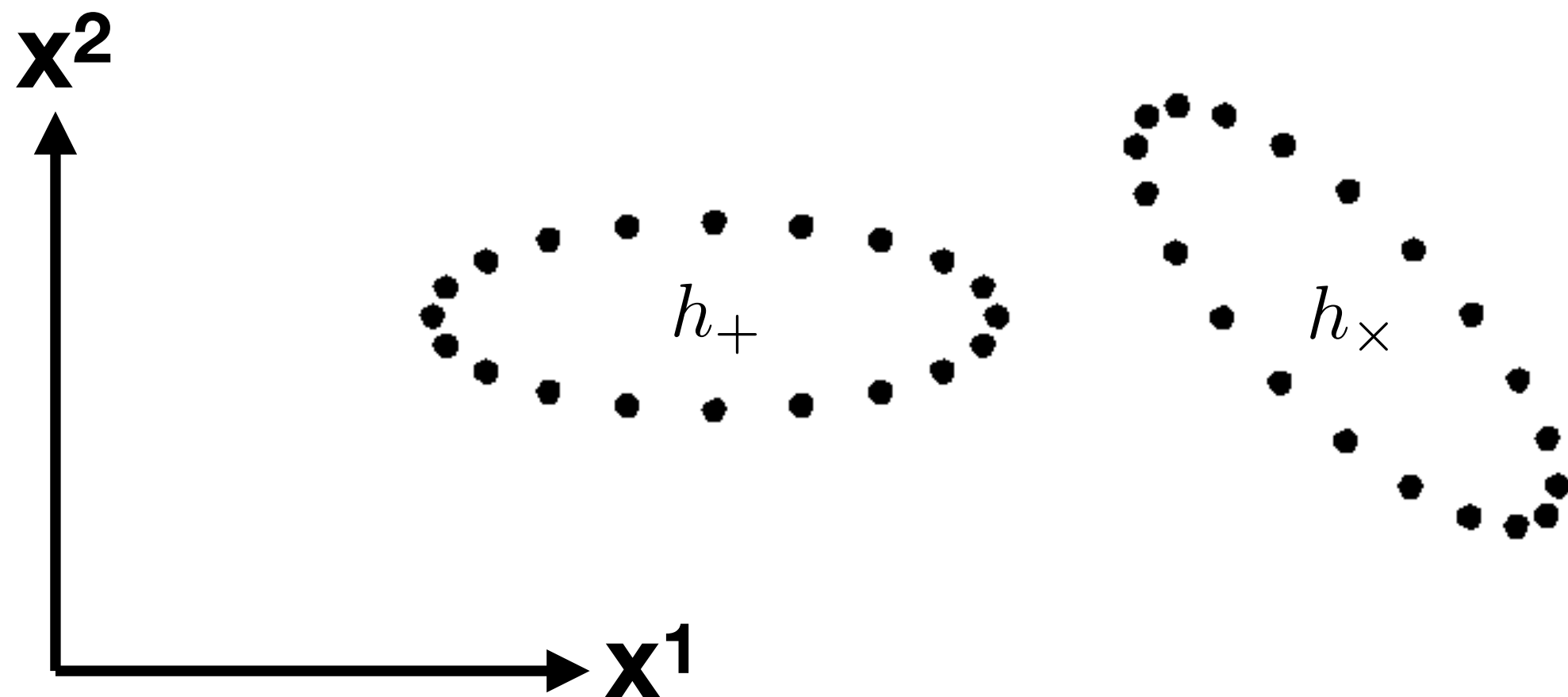


# 6 – 4 = 2 degrees of freedom for GW

We call them “plus” and “cross” modes

- The symmetric matrix  $h_{ij}$  has 6 components, but there are 4 conditions. Thus, we have two degrees of freedom.
- If the GW propagates in the  $x^3=z$  axis, non-vanishing components of  $h_{ij}$  are

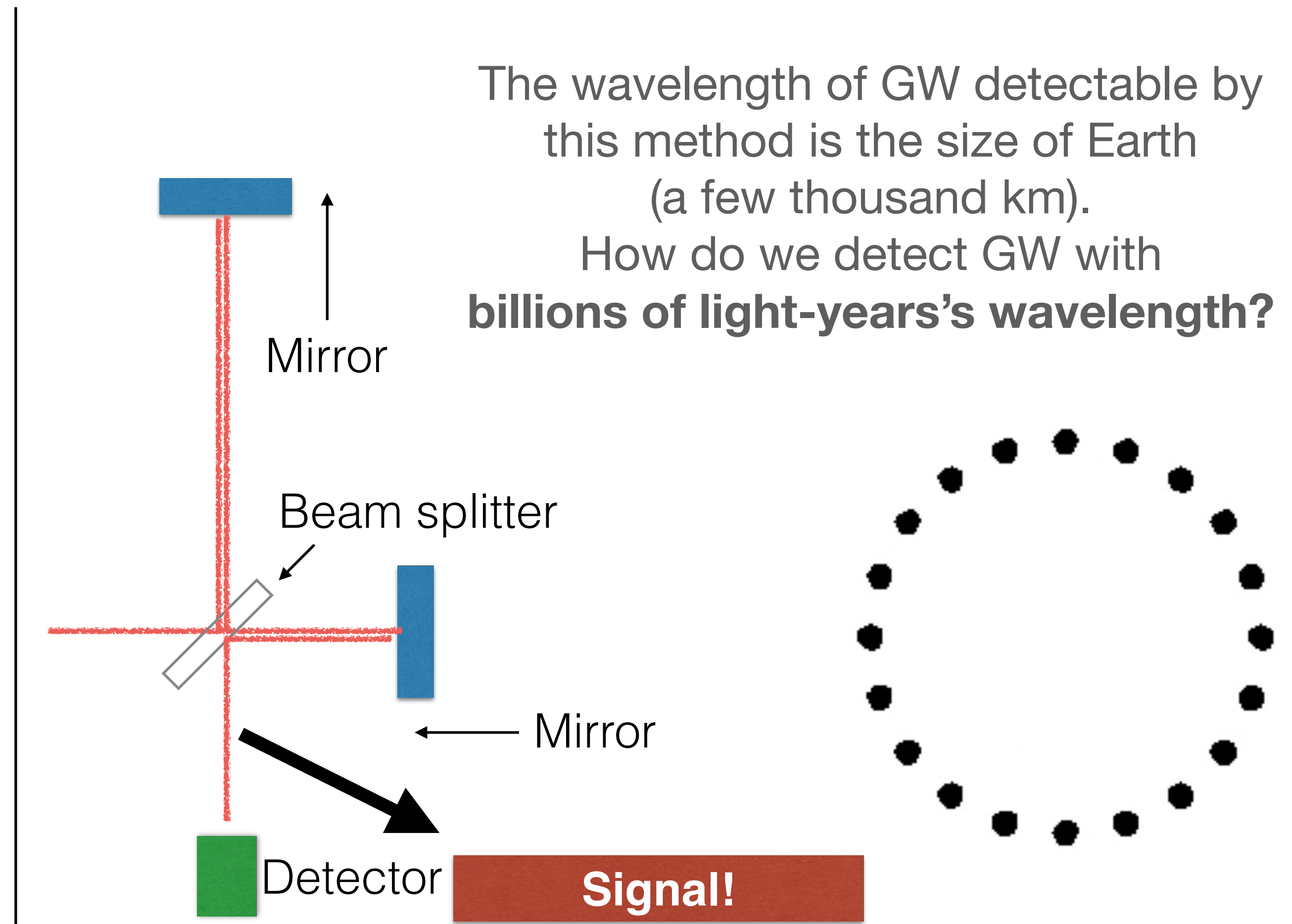
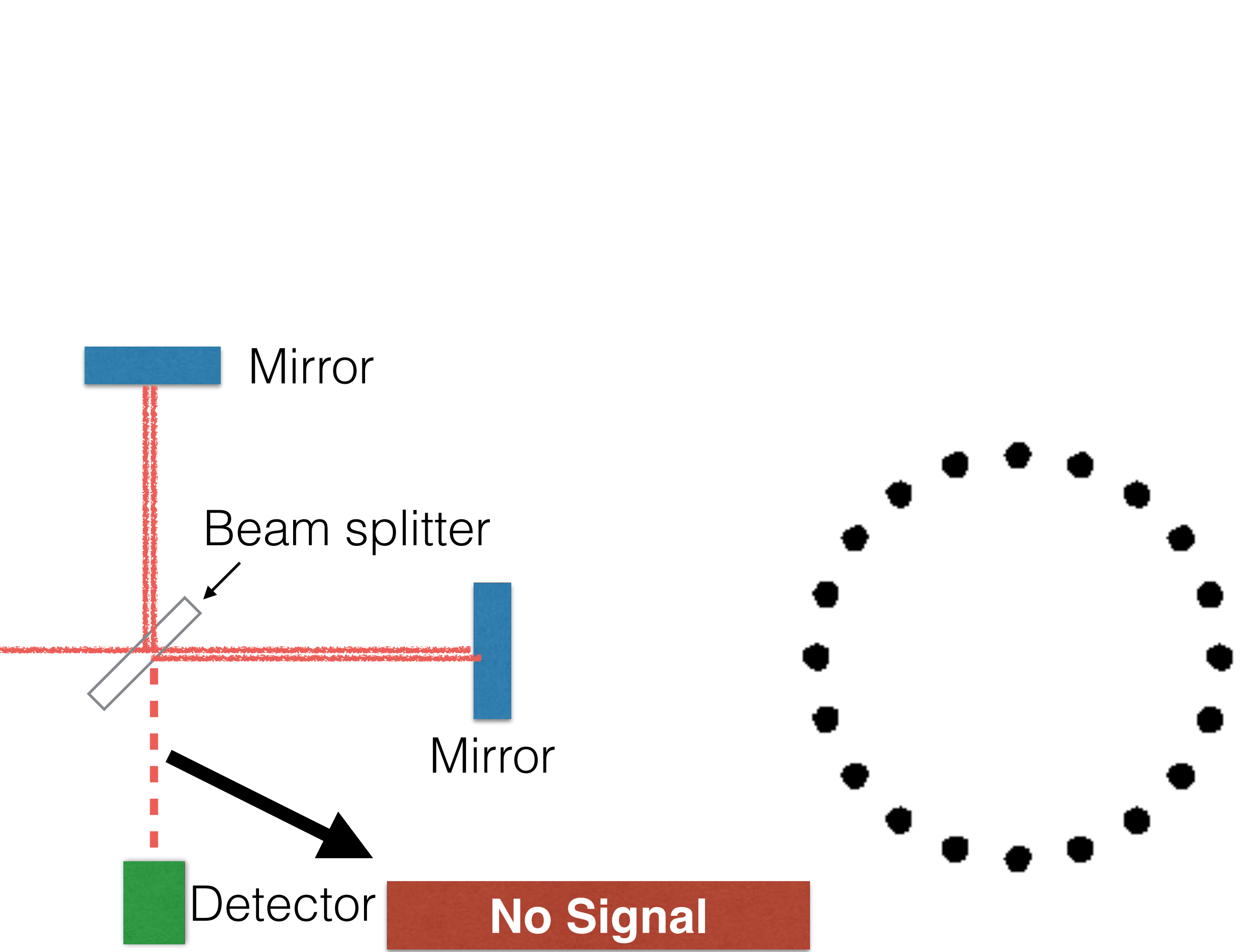
$$h_{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$





# How to detect GW?

## Laser interferometer technique, used by LIGO and VIRGO

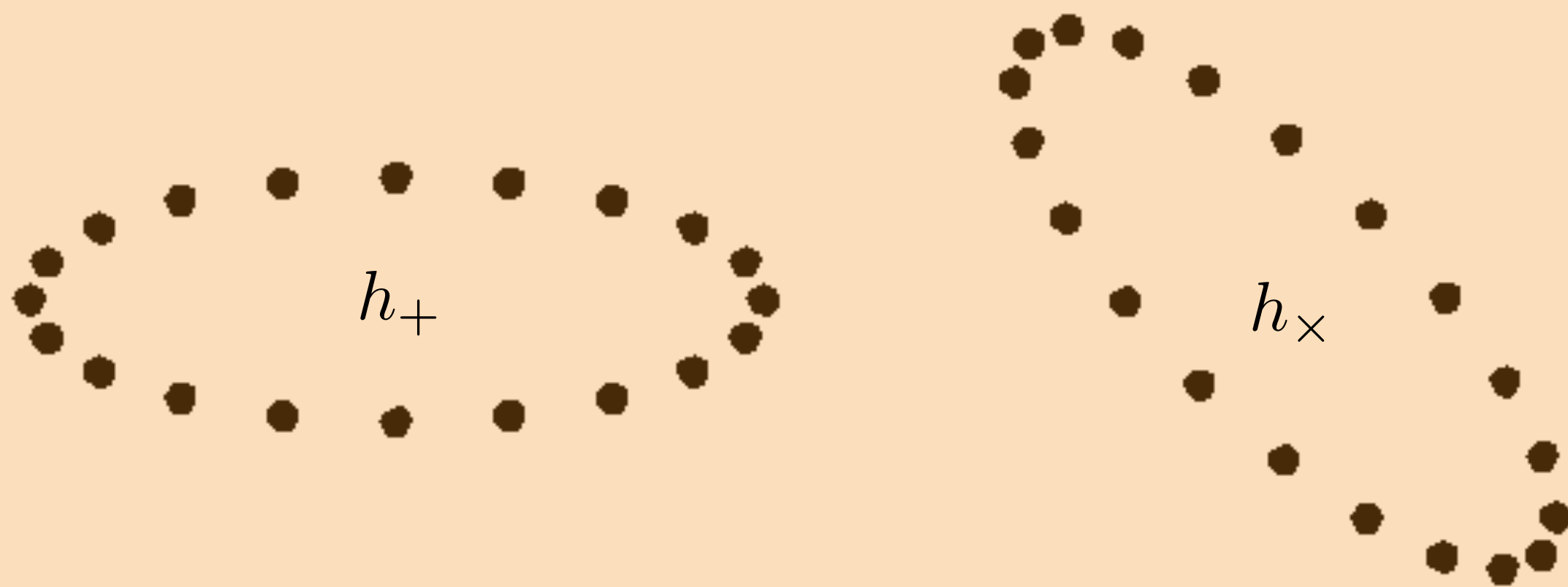




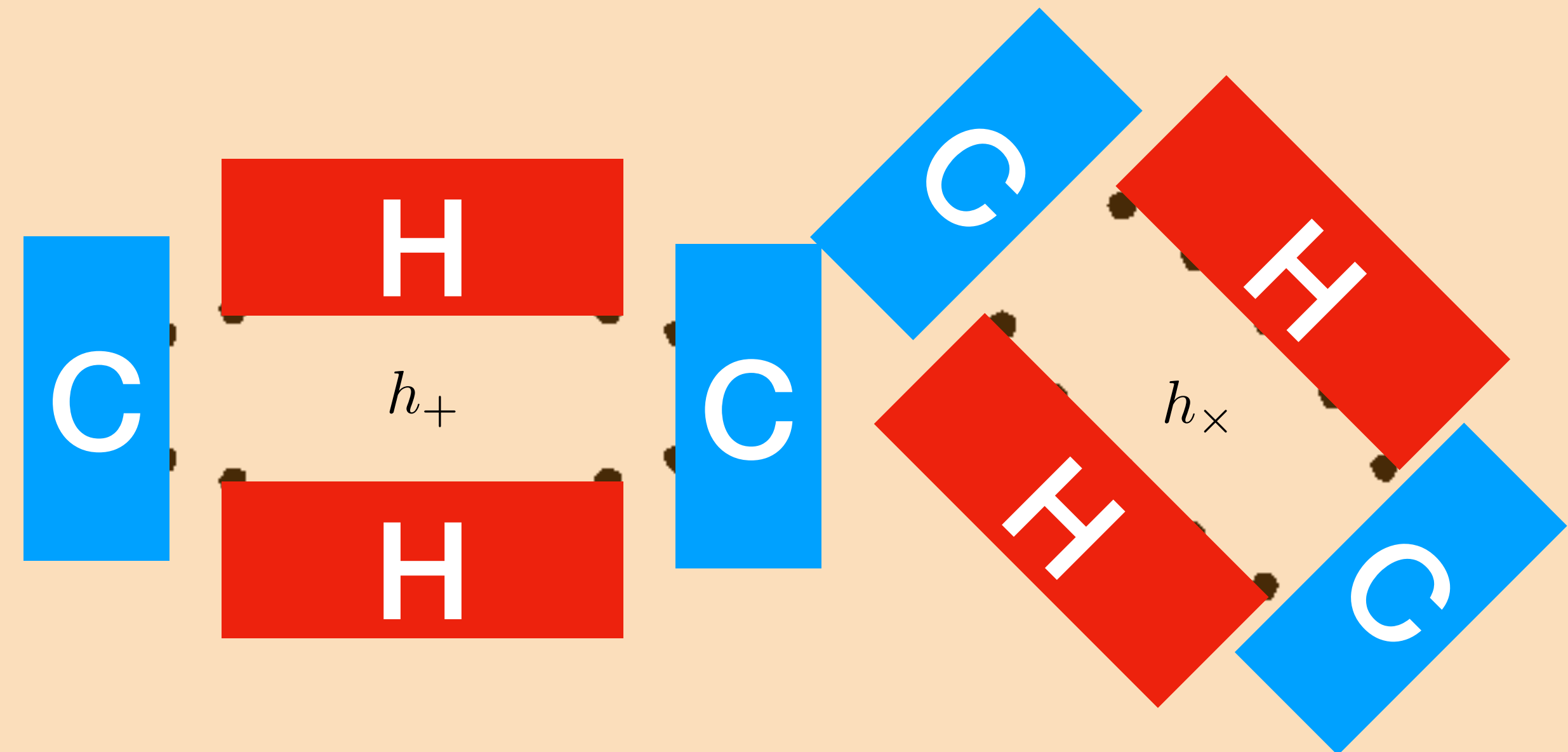
# Detecting GW by CMB

Quadrupole temperature anisotropy generated by red- and blue-shifting of photons

Isotropic radiation field (CMB)



Isotropic radiation field (CMB)

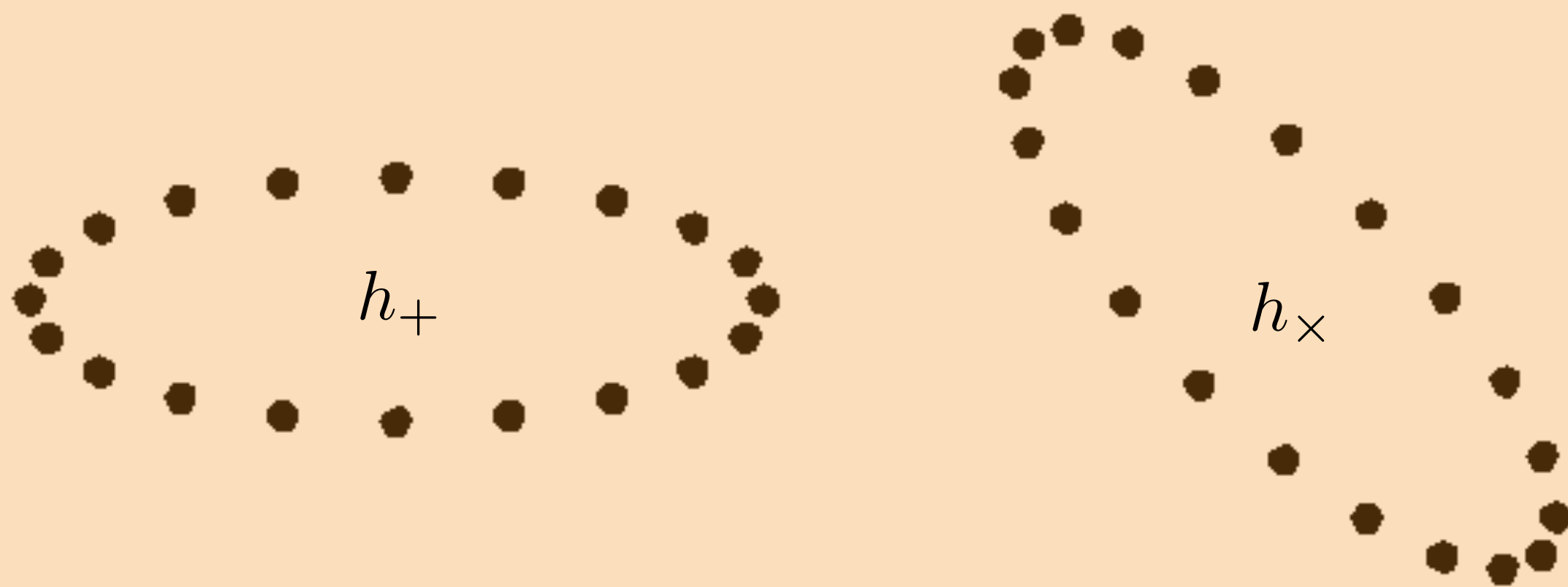




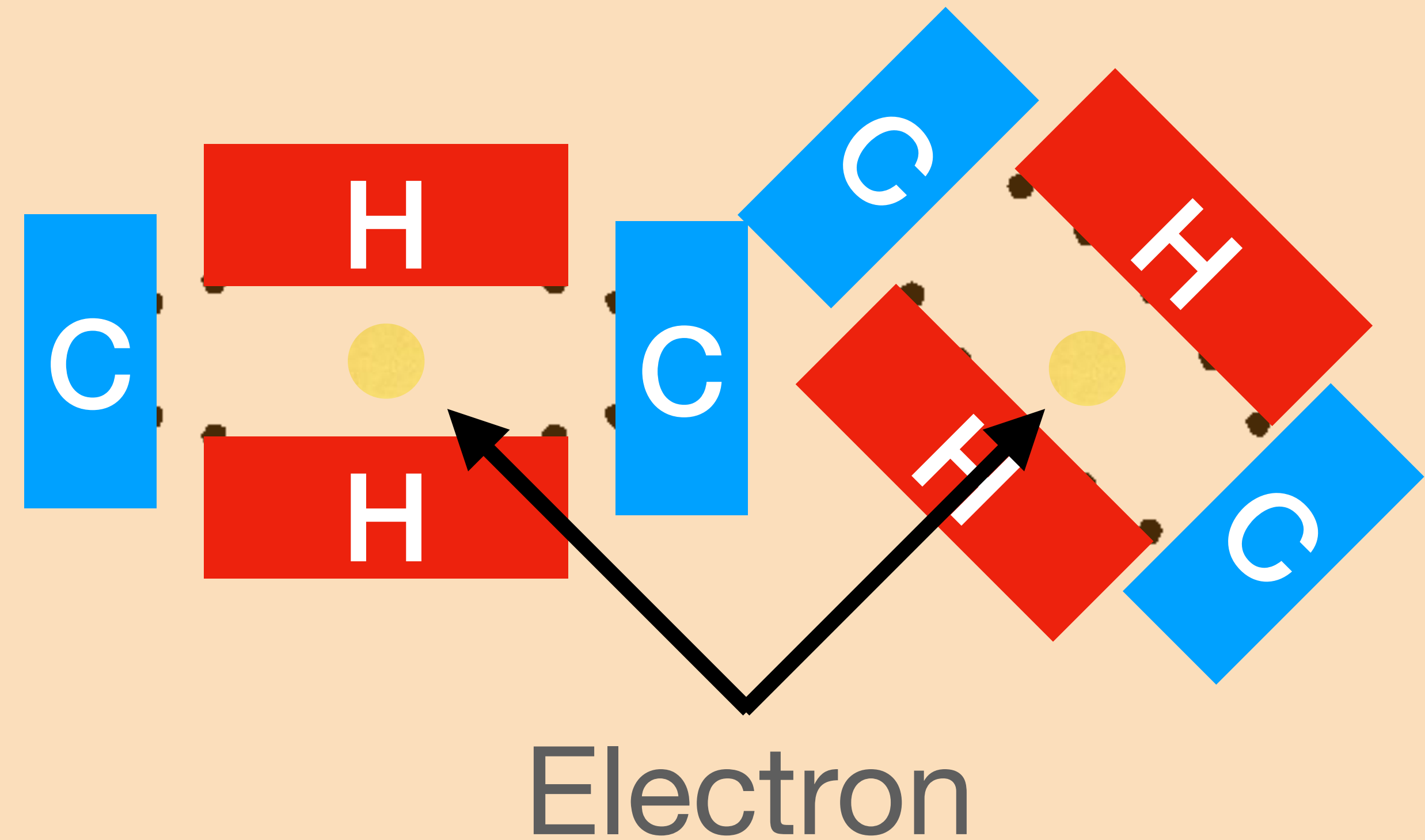
# Detecting GW by CMB

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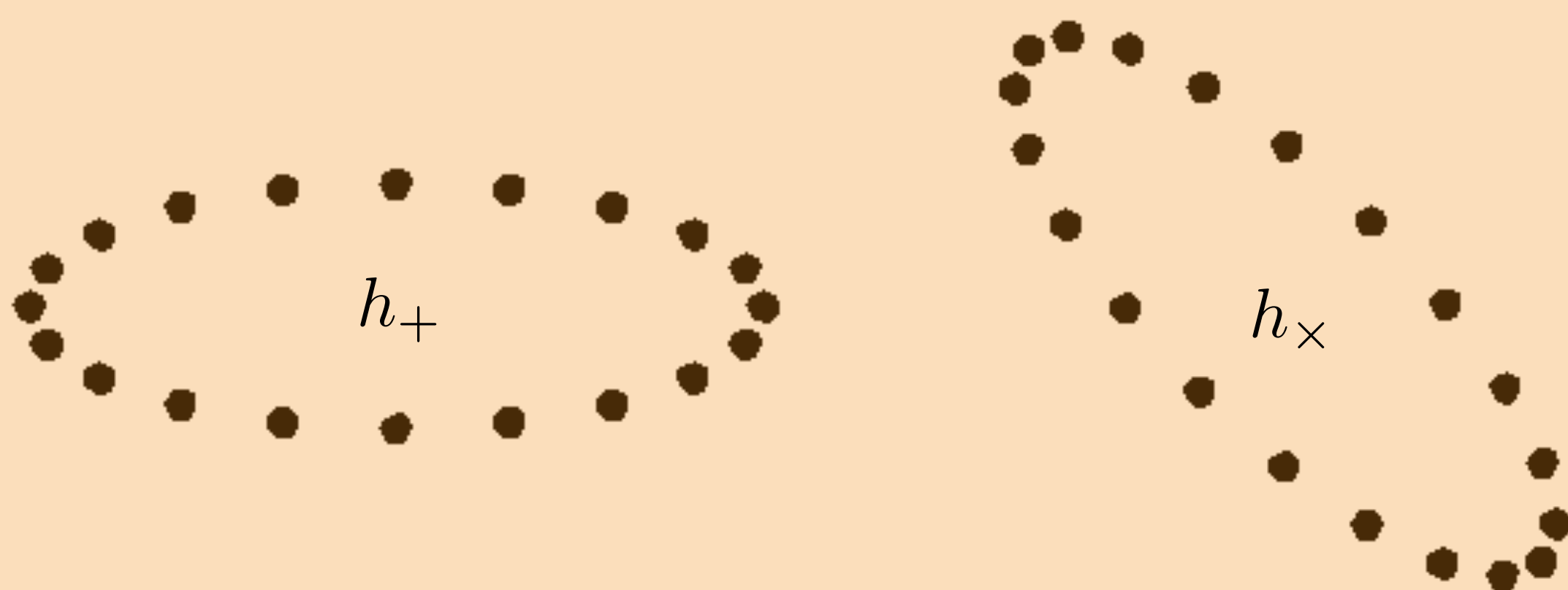




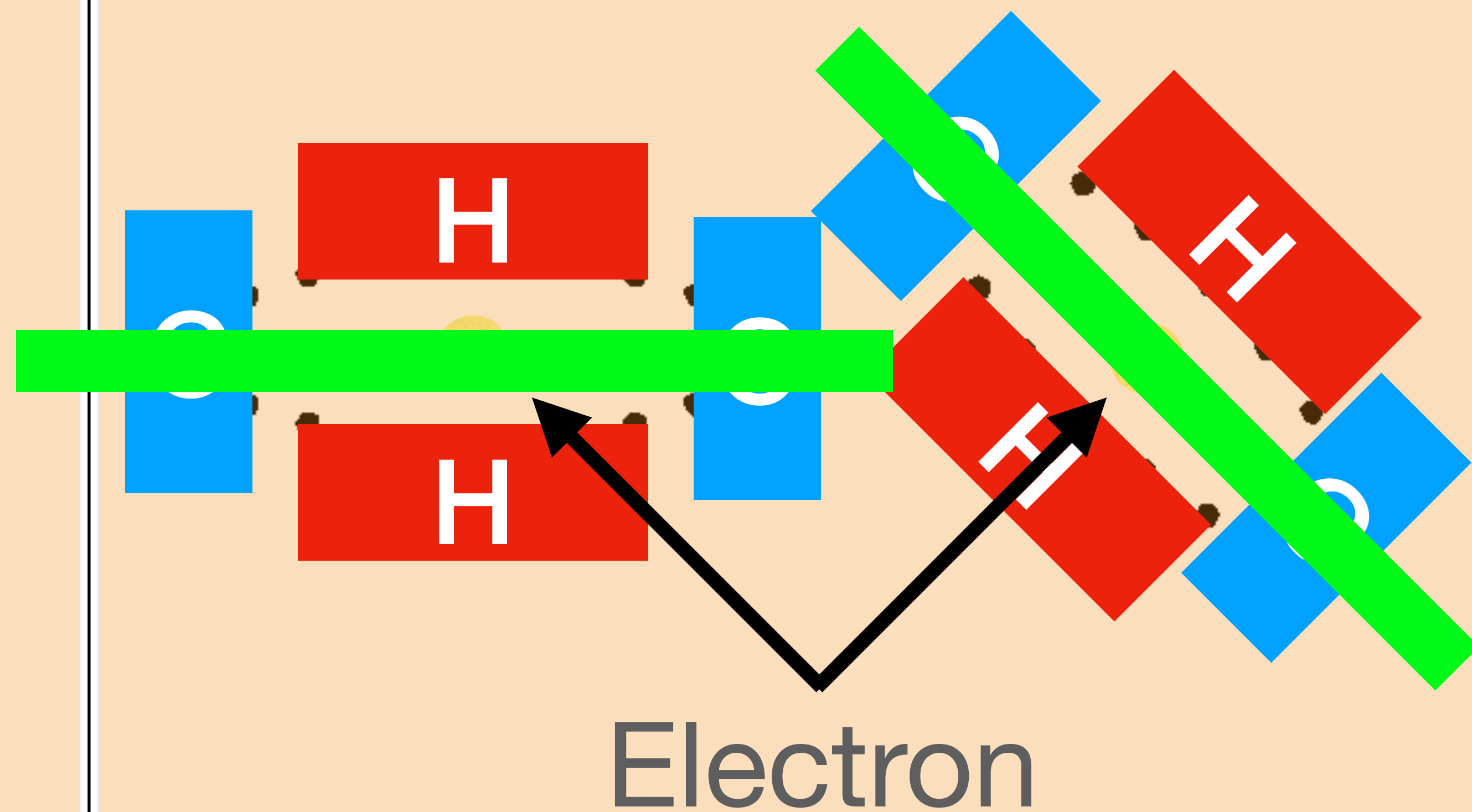
# Detecting GW by CMB *Polarisation*

Quadrupole temperature anisotropy scattered by an electron

Isotropic radiation field (CMB)



Isotropic radiation field (CMB)









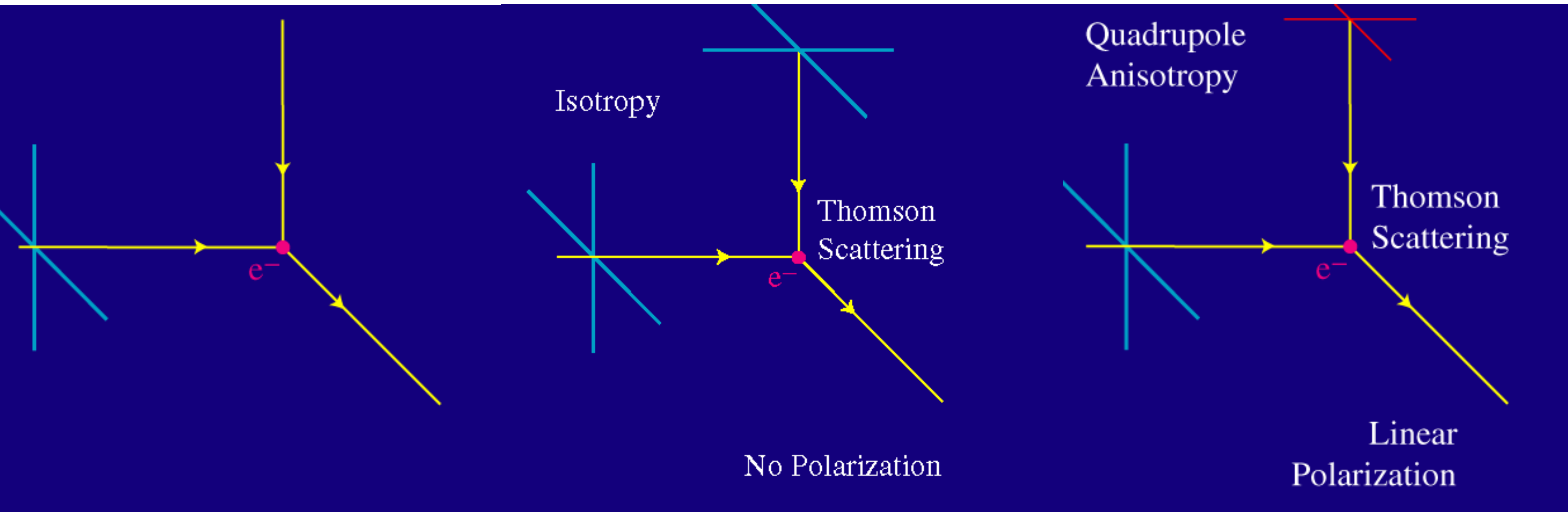
Credit: TALEX





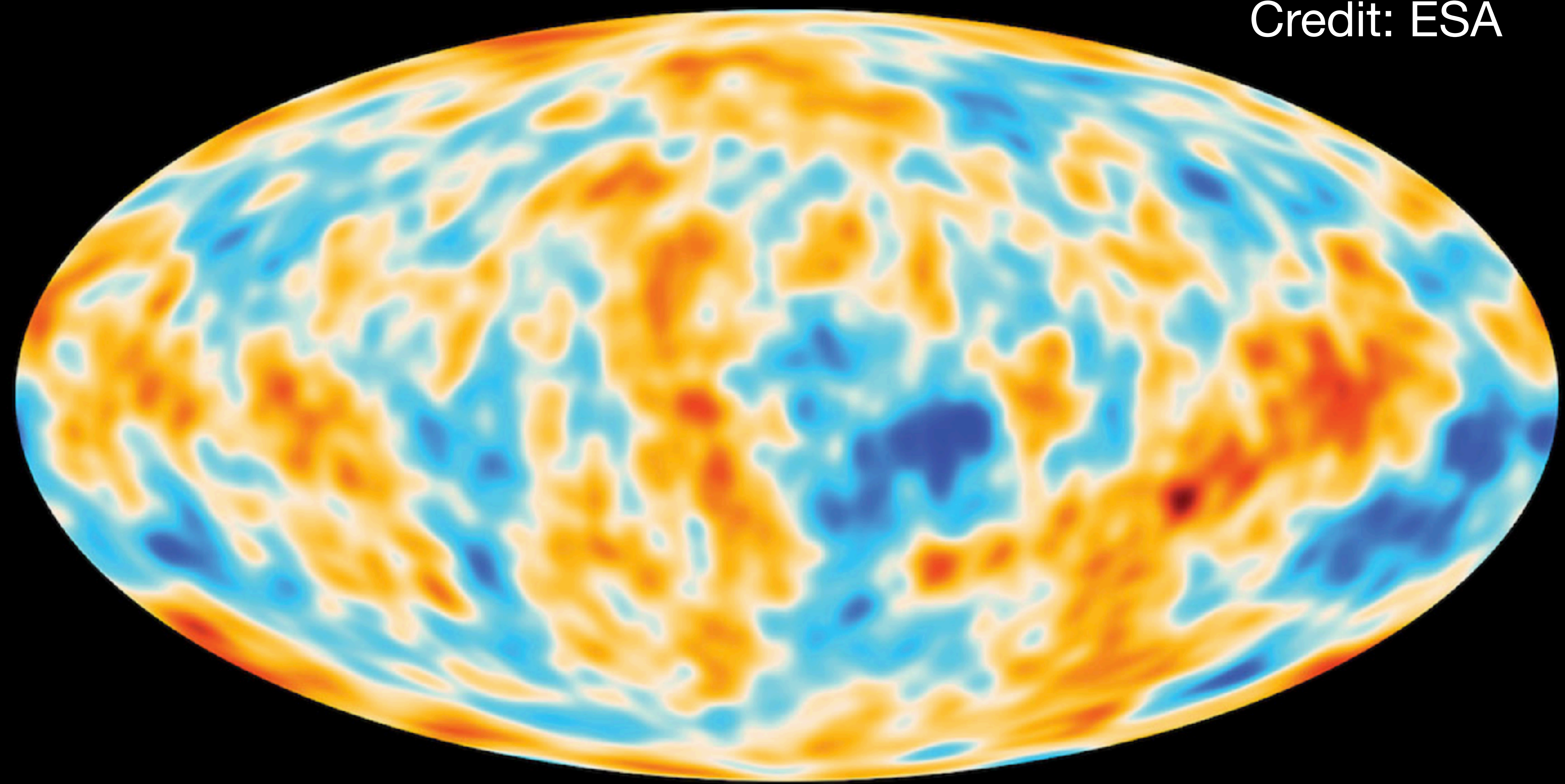
# Physics of CMB Polarisation

Necessary and sufficient condition: Scattering and Quadrupole Anisotropy





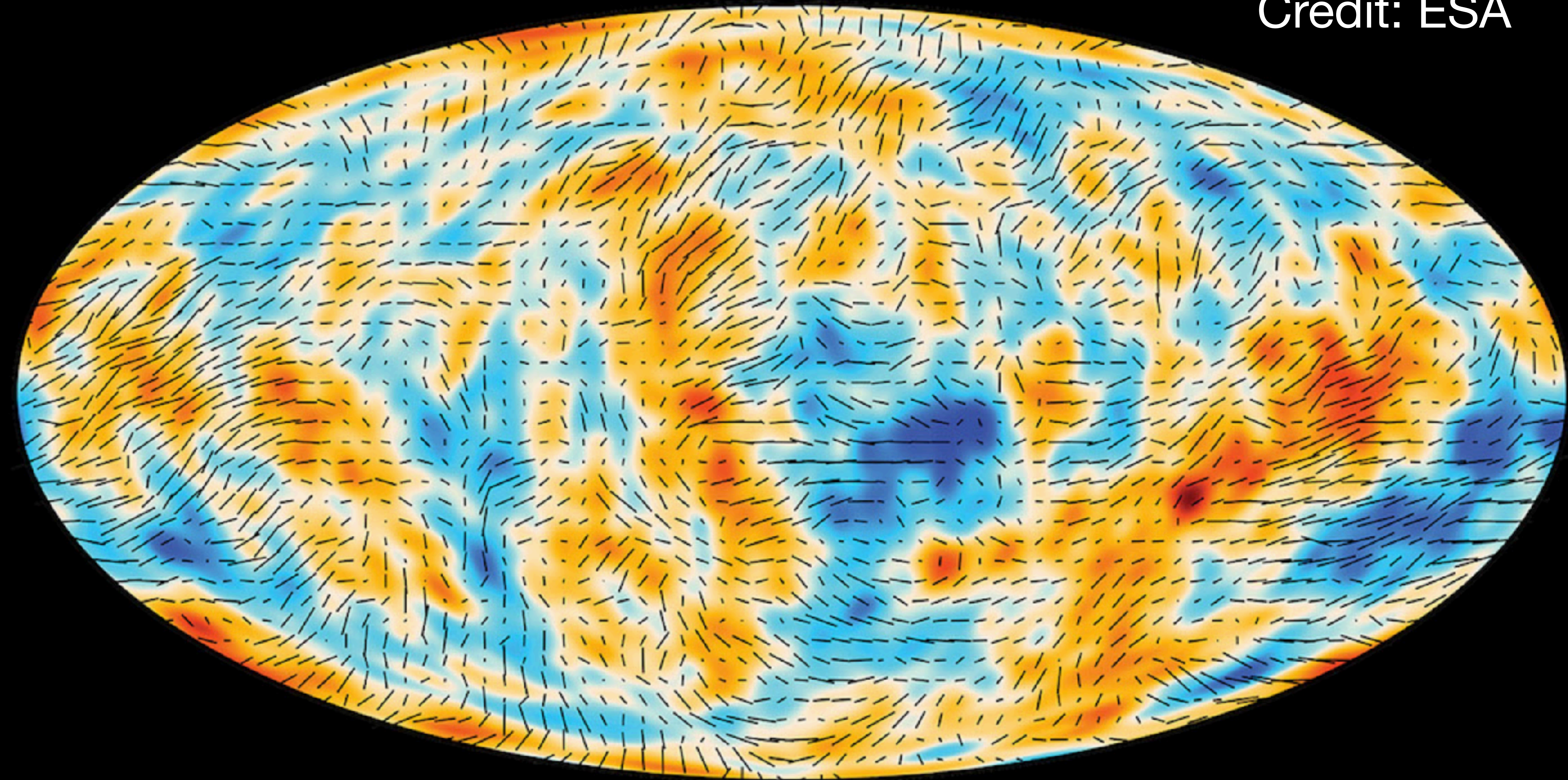
Credit: ESA



Temperature (smoothed)



Credit: ESA

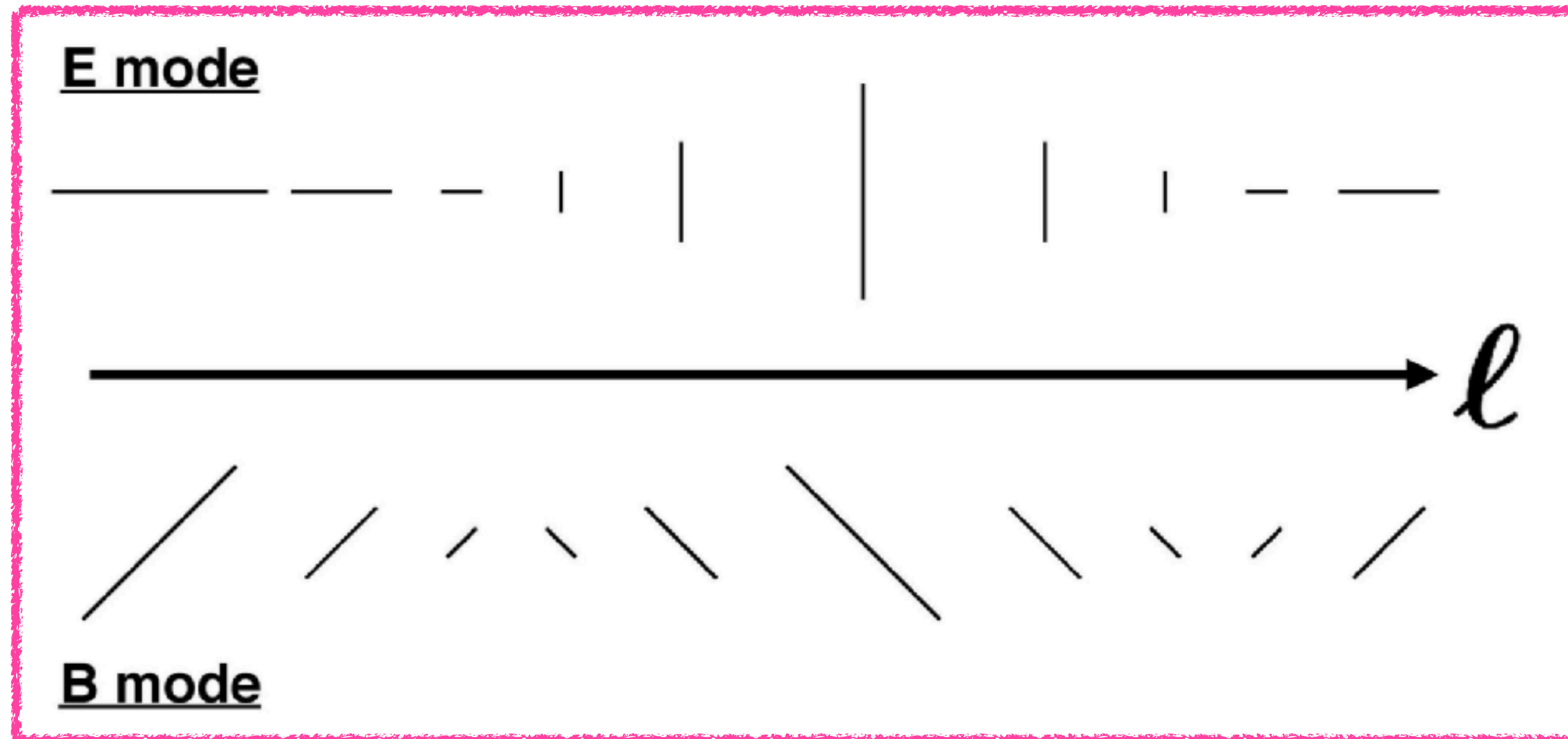


Temperature (smoothed) + Polarisation



# E- and B-mode decomposition

Concept defined in Fourier space



Direction of the Fourier wavenumber vector

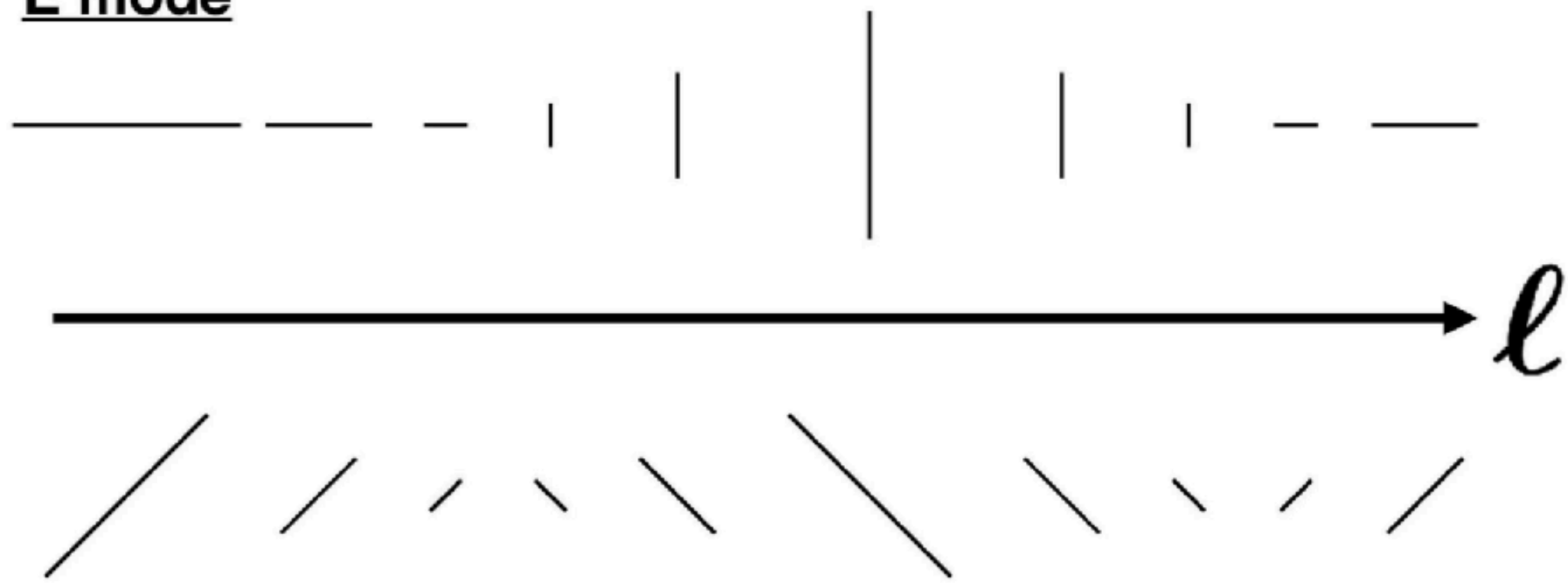
- **E-mode** : Polarisation directions are **parallel or perpendicular** to the wavenumber direction
- **B-mode** : Polarisation directions are **45 degrees tilted** w.r.t the wavenumber direction



# Parity Flip

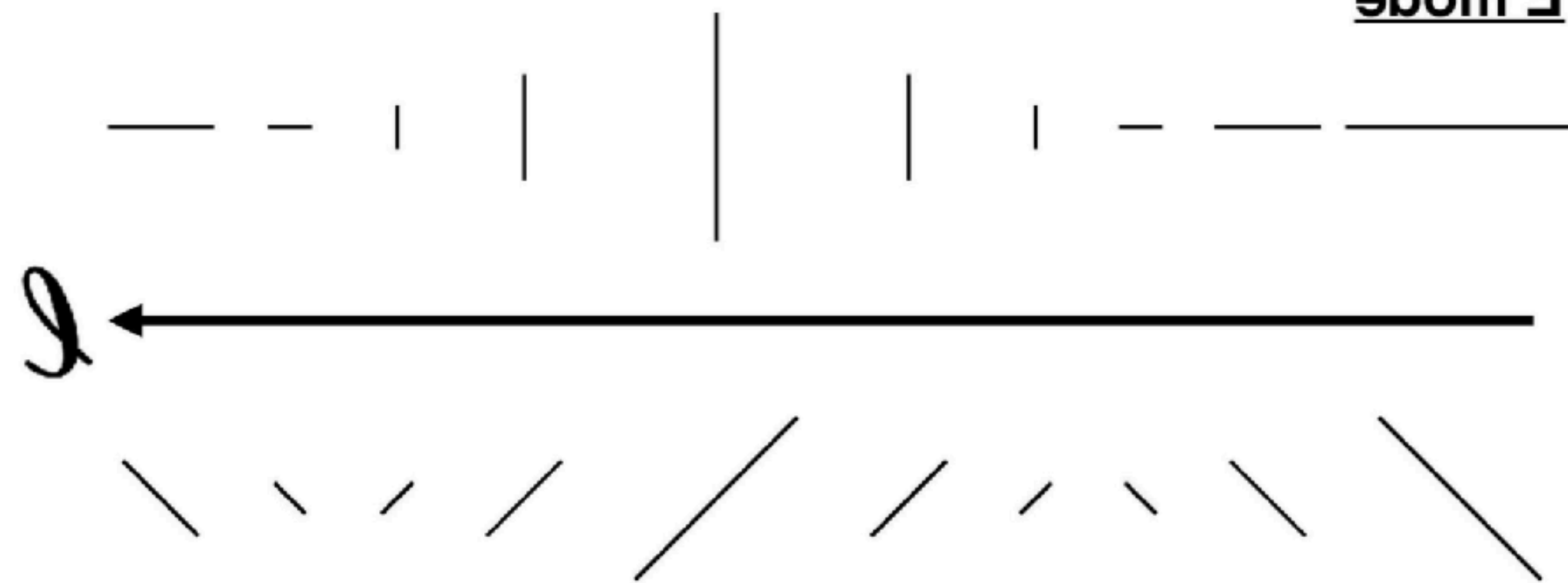
**E-mode remains the same, whereas B-mode changes the sign**

E mode



B mode

E mode



B mode

- Two-point correlation functions invariant under the parity flip are

$$\langle E_{\ell} E_{\ell'}^* \rangle = (2\pi)^2 \delta_D^{(2)}(\ell - \ell') C_{\ell}^{EE}$$

$$\langle B_{\ell} B_{\ell'}^* \rangle = (2\pi)^2 \delta_D^{(2)}(\ell - \ell') C_{\ell}^{BB}$$

$$\langle T_{\ell} E_{\ell'}^* \rangle = \langle T_{\ell'}^* E_{\ell} \rangle = (2\pi)^2 \delta_D^{(2)}(\ell - \ell') C_{\ell}^{TE}$$

- The other combinations  $\langle TB \rangle$  and  $\langle EB \rangle$  are not invariant under the parity flip.

- **[Side Note] We can use these combinations to probe parity-violating physics (e.g., axions)**

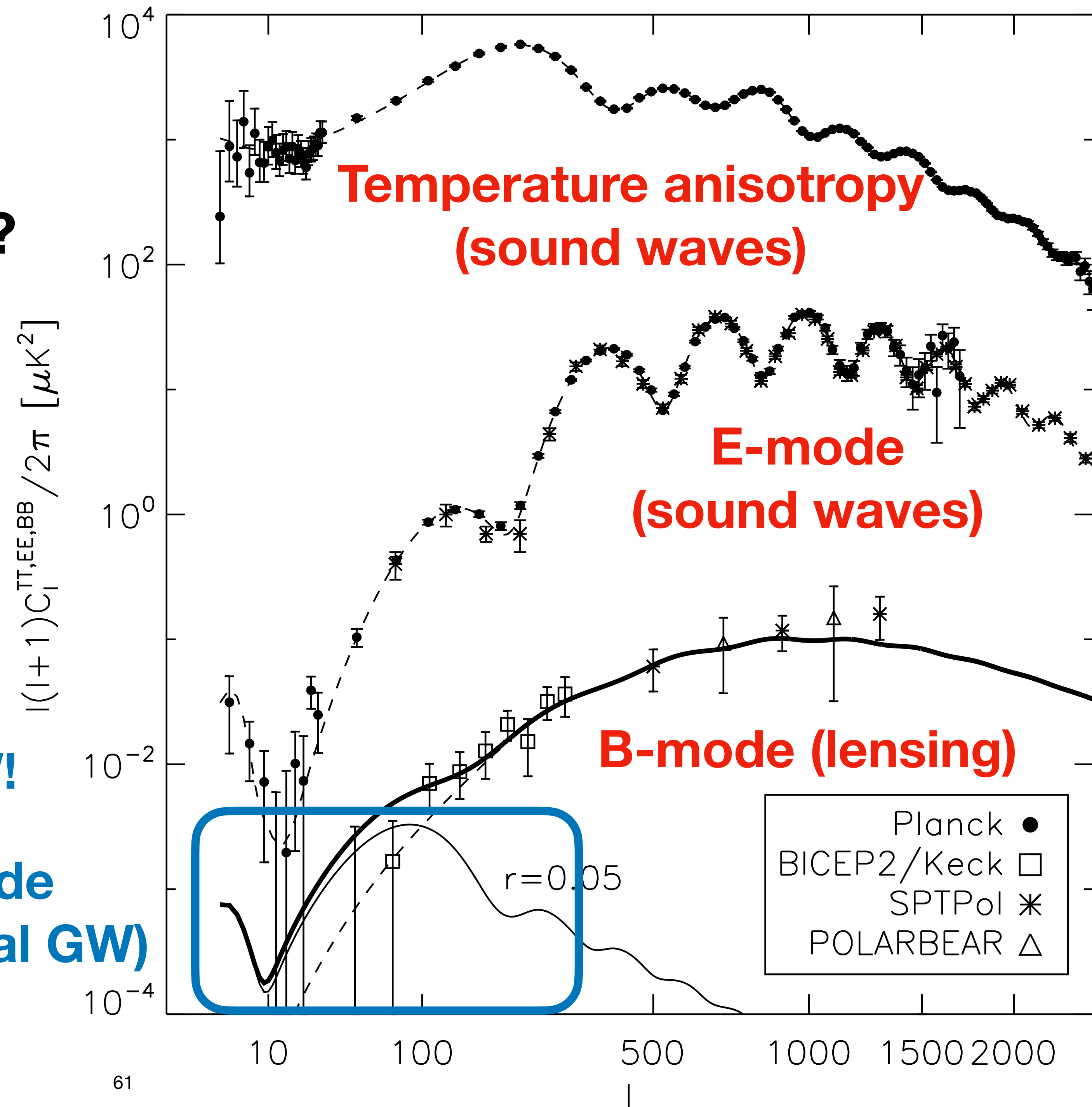


# Power Spectra

Where are we? What is next?

- The temperature and polarisation power spectra originating from **the scalar (density) fluctuation** have been measured.
- The next quest: **B-mode power spectrum from the primordial GW!**

**B-mode  
(Primordial GW)**





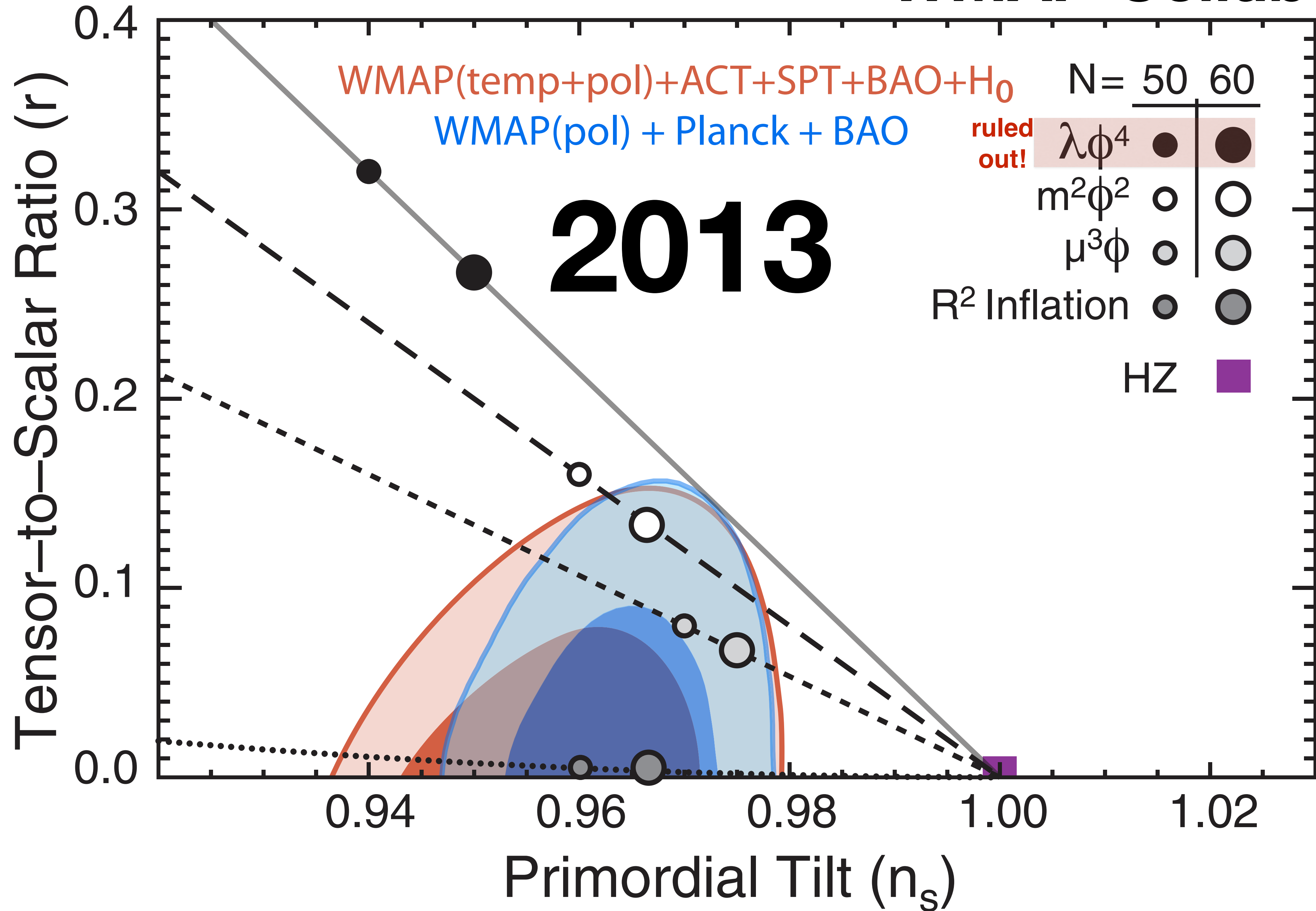
# Tensor-to-scalar Ratio

$$r \equiv \frac{\langle h_{ij} h^{ij} \rangle}{\langle \zeta^2 \rangle}$$

Scalar mode

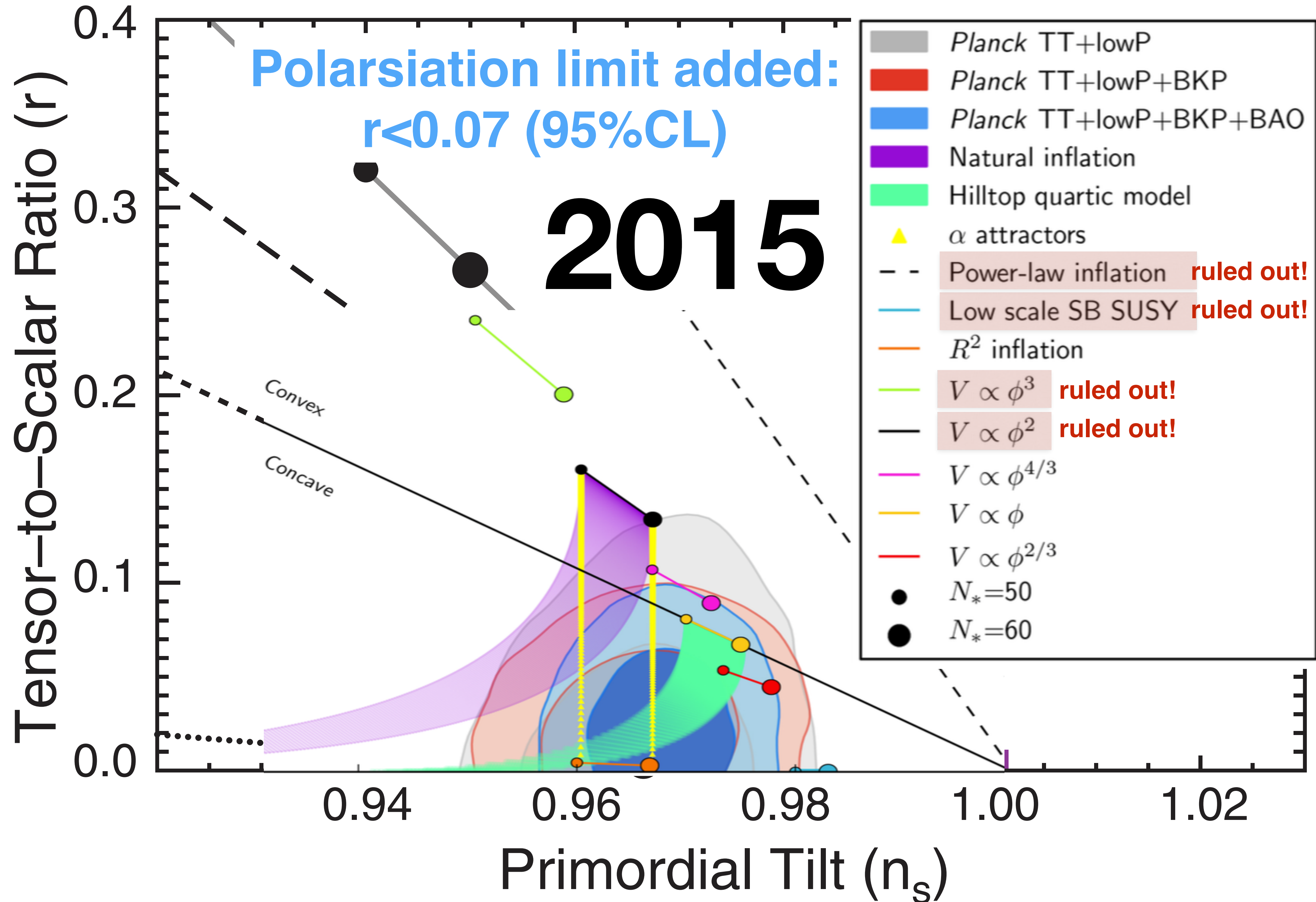
- We really want to find this! The current upper bound is  **$r < 0.036$**  [95%CL; *BICEP2/Keck Array Collaboration (2021)*]





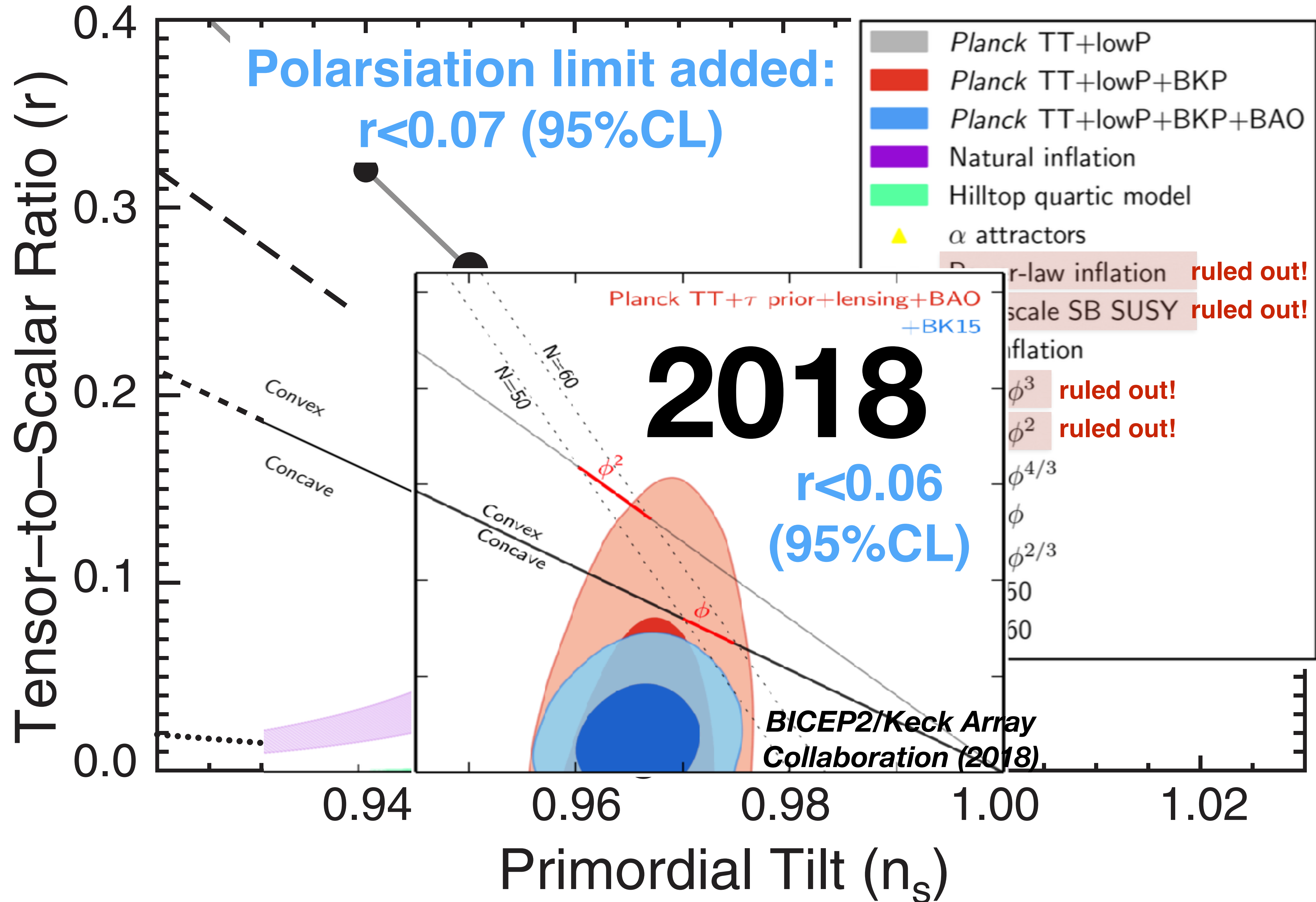


**Planck Collaboration (2015); BICEP2/Keck Array Collaboration (2016)**



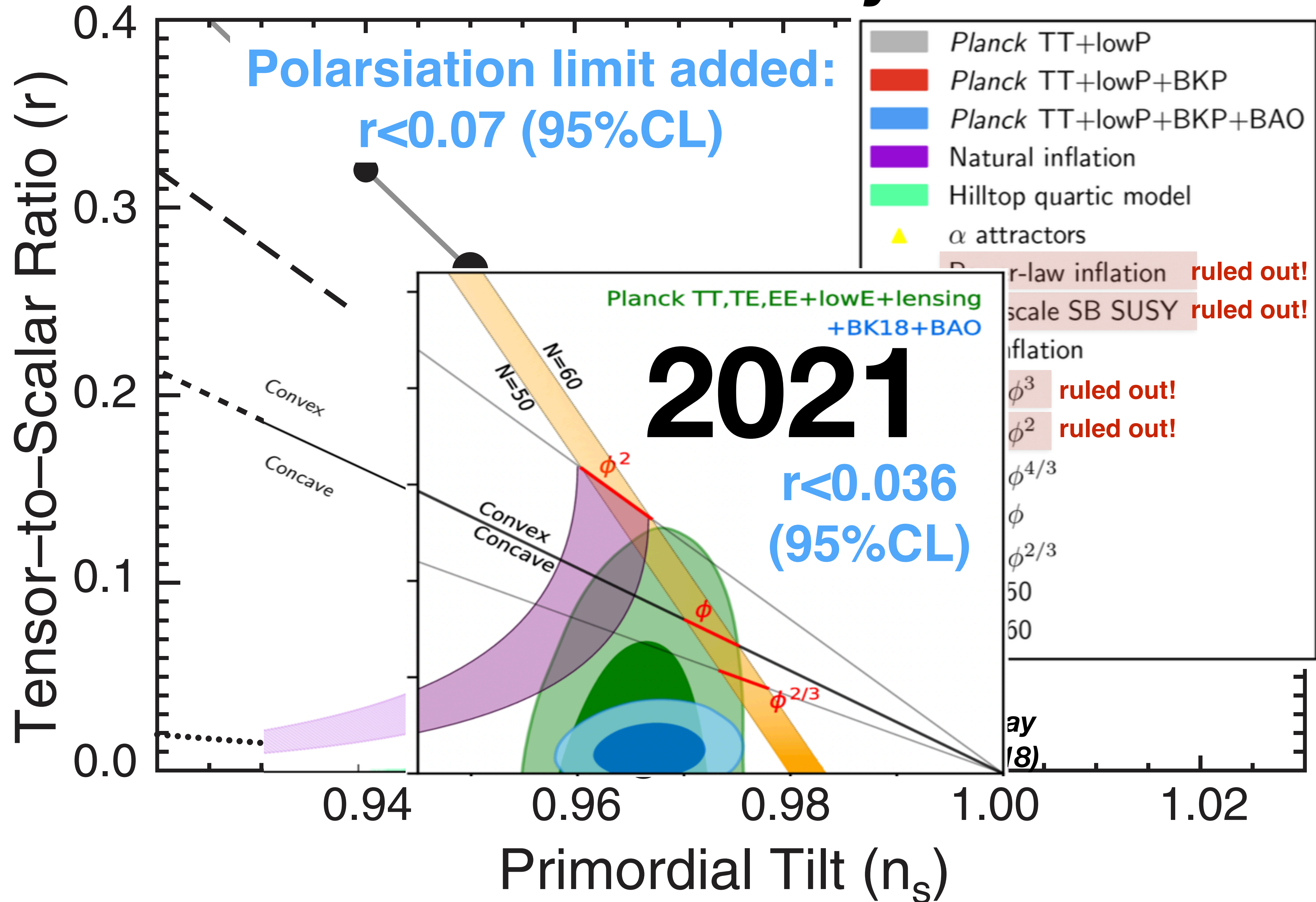


**Planck Collaboration (2015); BICEP2/Keck Array Collaboration (2016)**





# BICEP2/Keck Array Collaboration (2021)





# Experimental Landscape



# CMB Stages

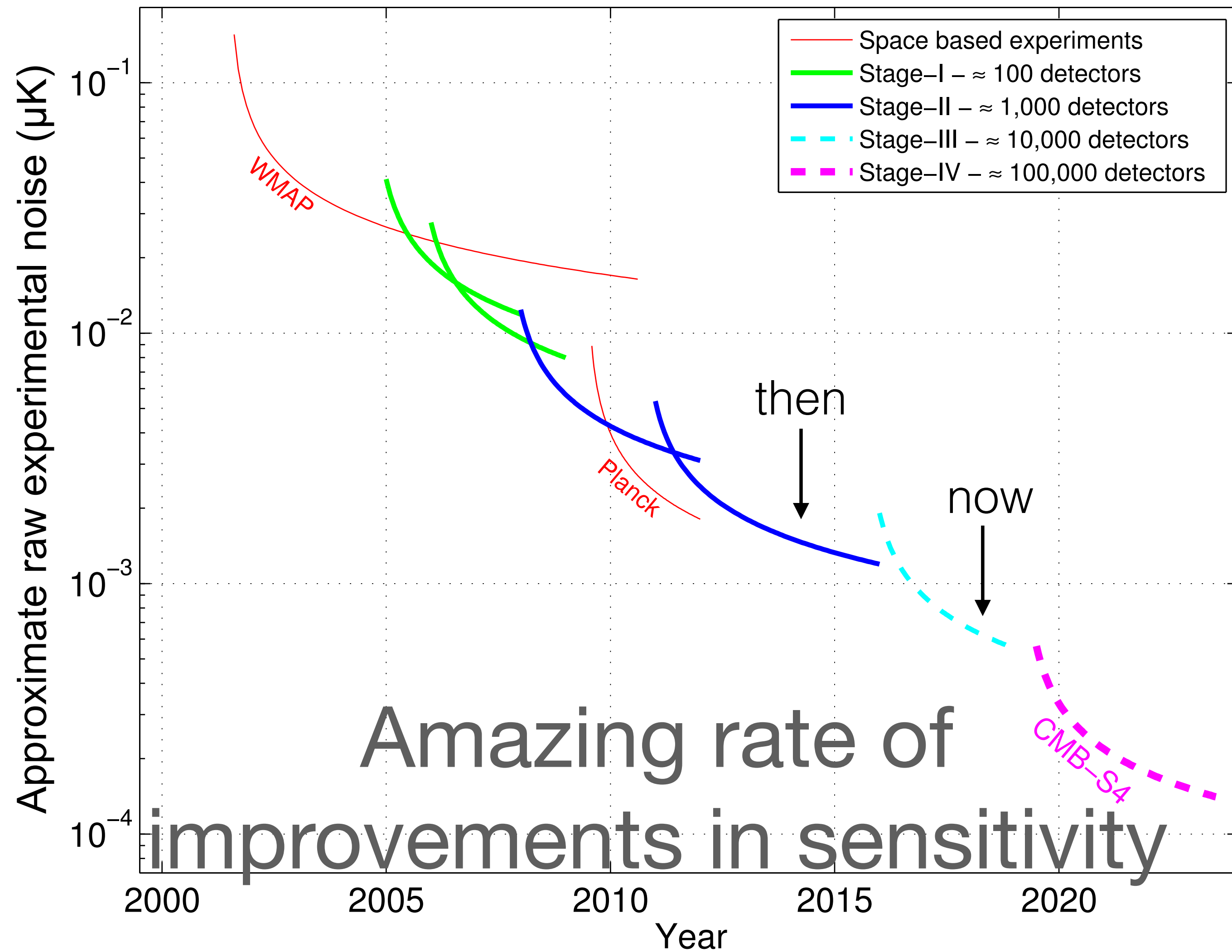


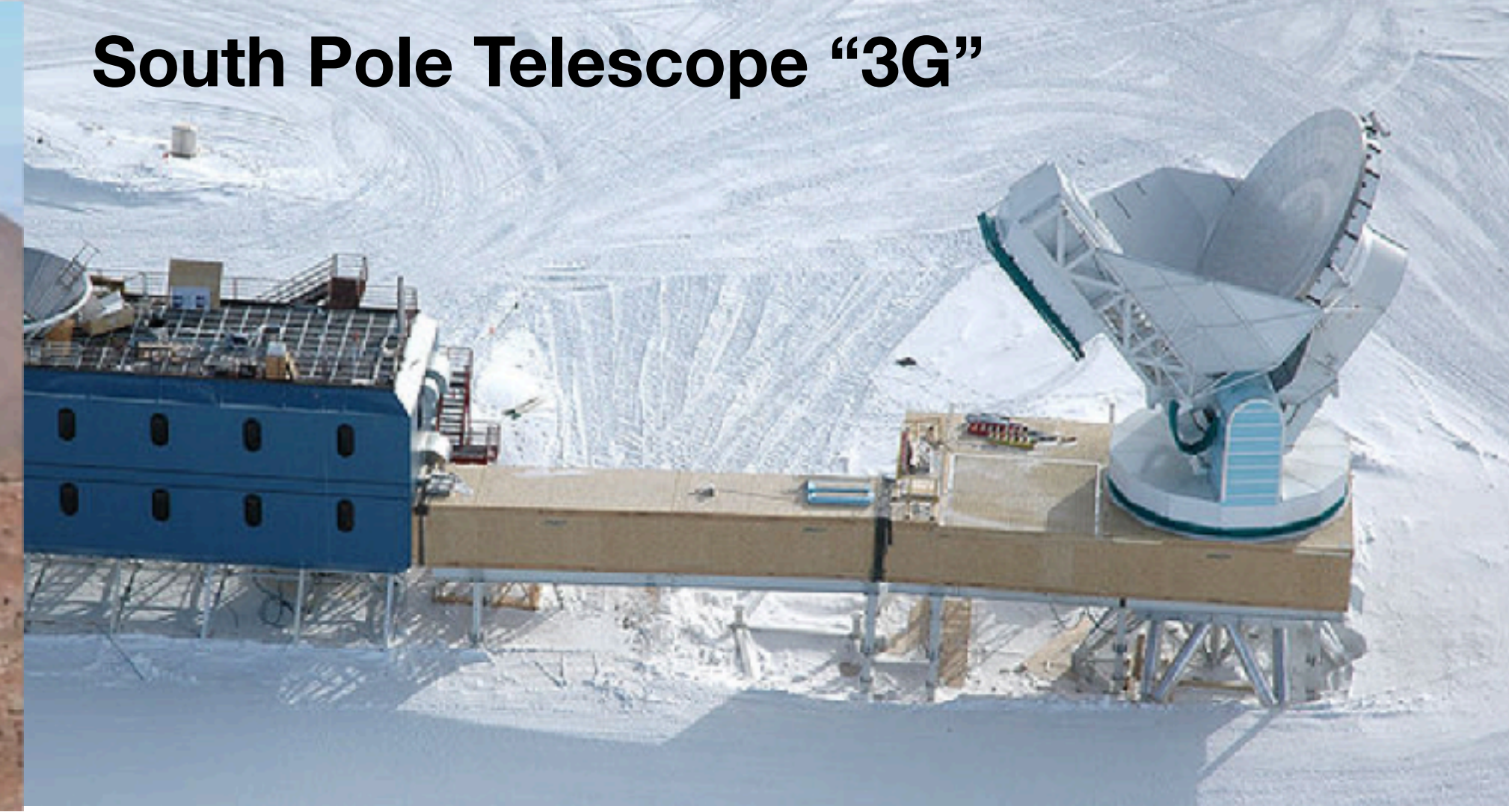
Figure by Clem Pryke for 2013 Snowmass documents



**Advanced Atacama  
Cosmology Telescope**

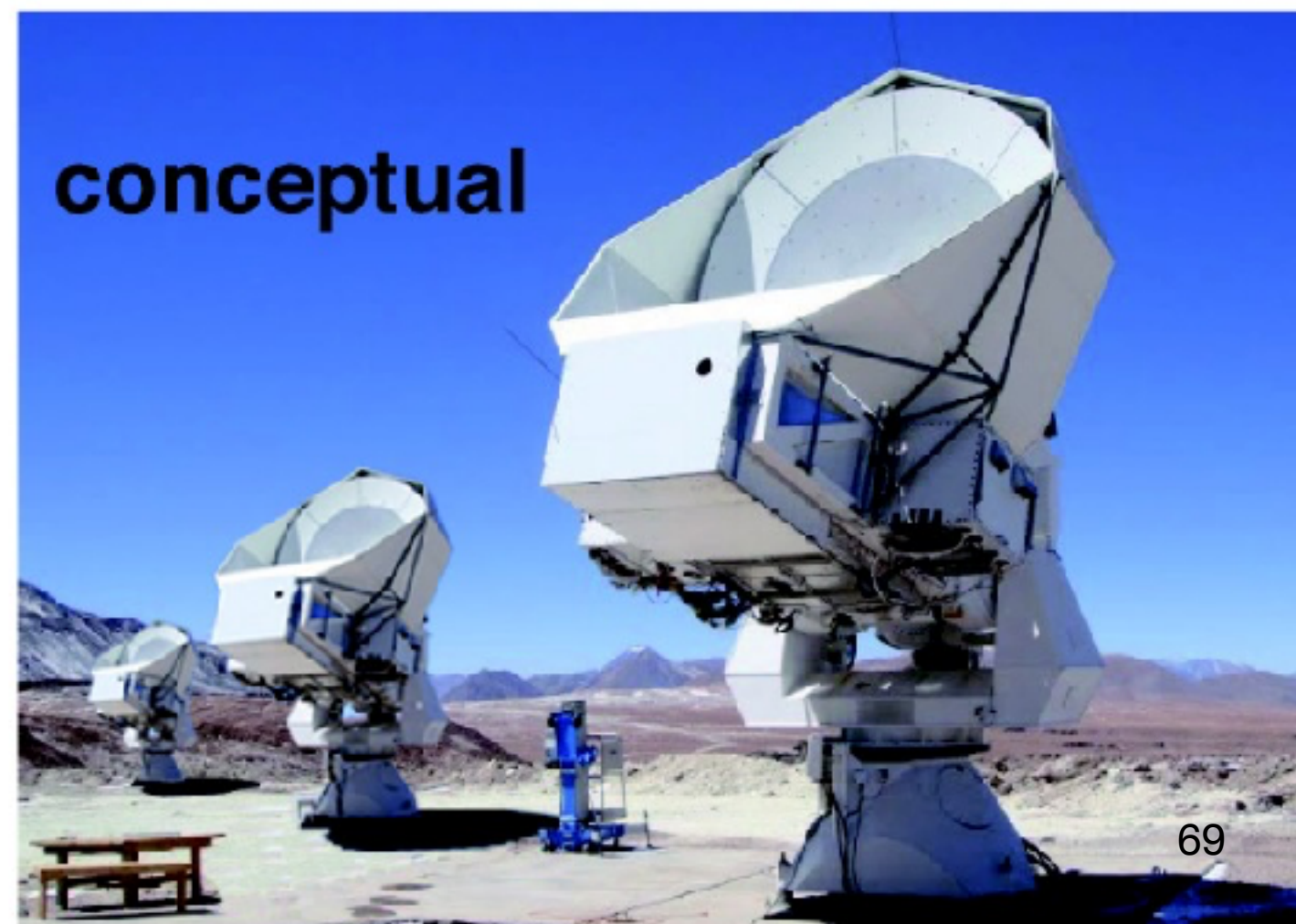


**South Pole Telescope "3G"**



# On-going Ground-based Experiments

**The Simons Array**



**BICEP/Keck Array**



**CLASS**





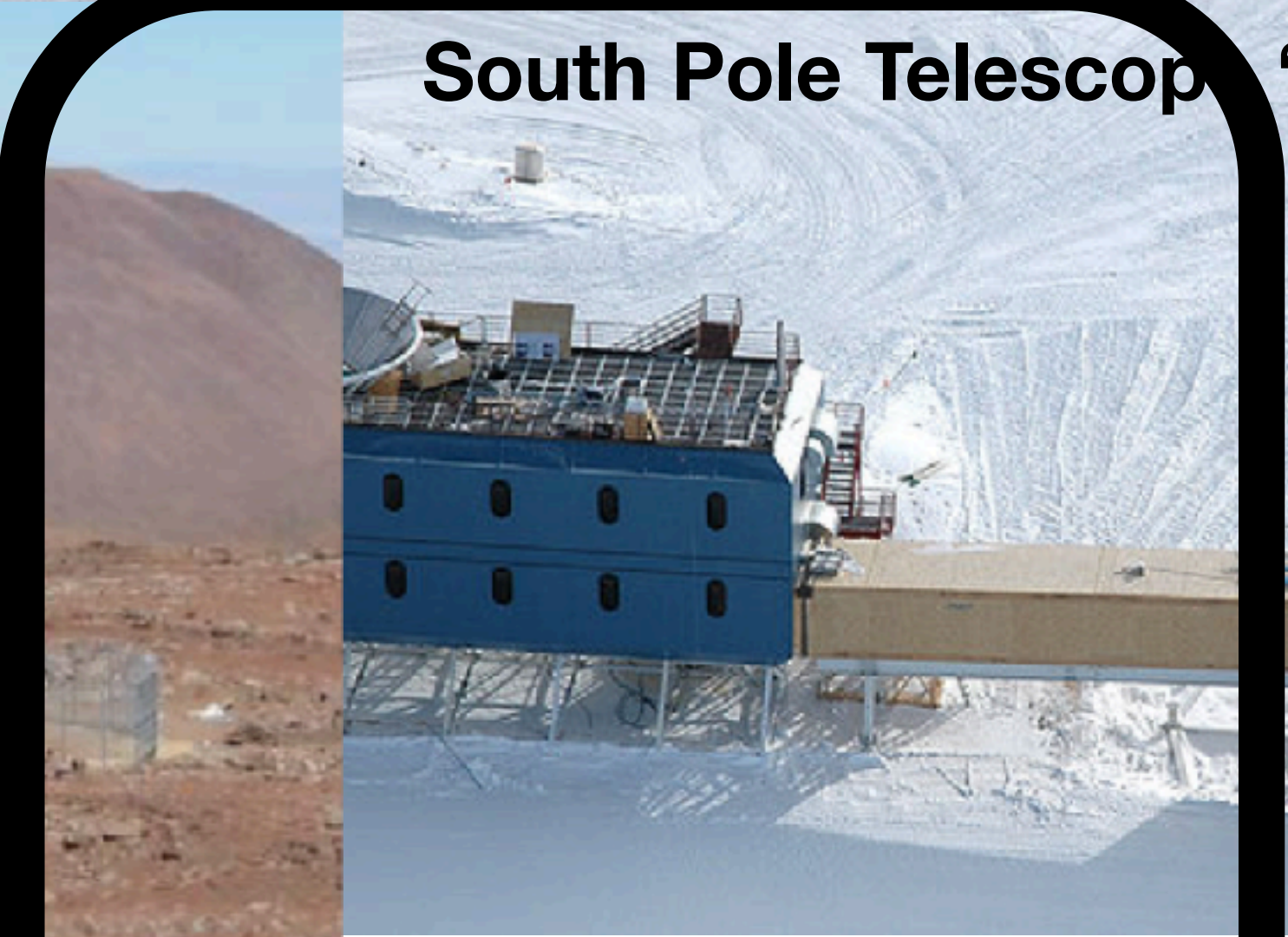
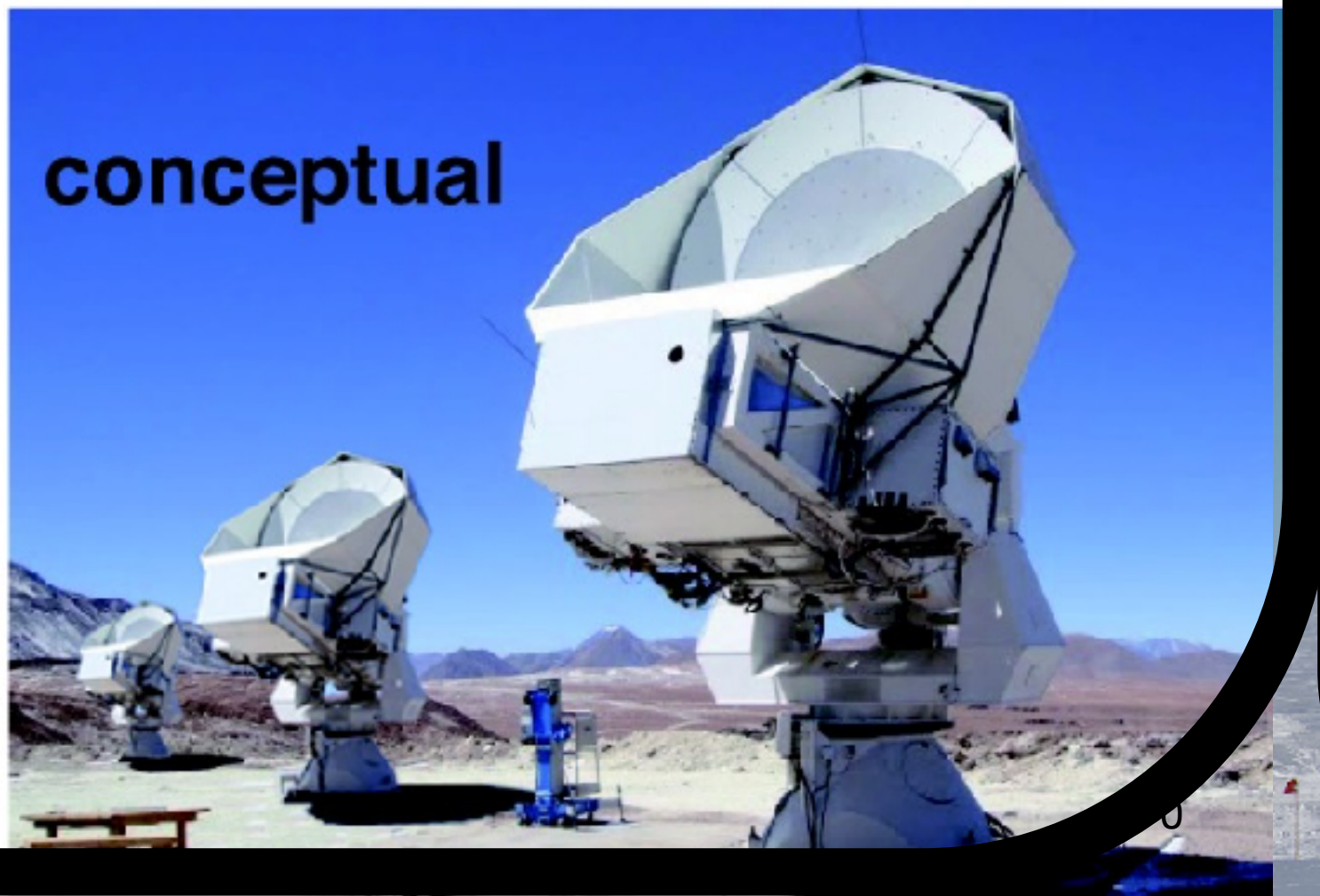



**Early 2020s**  
~\$100M



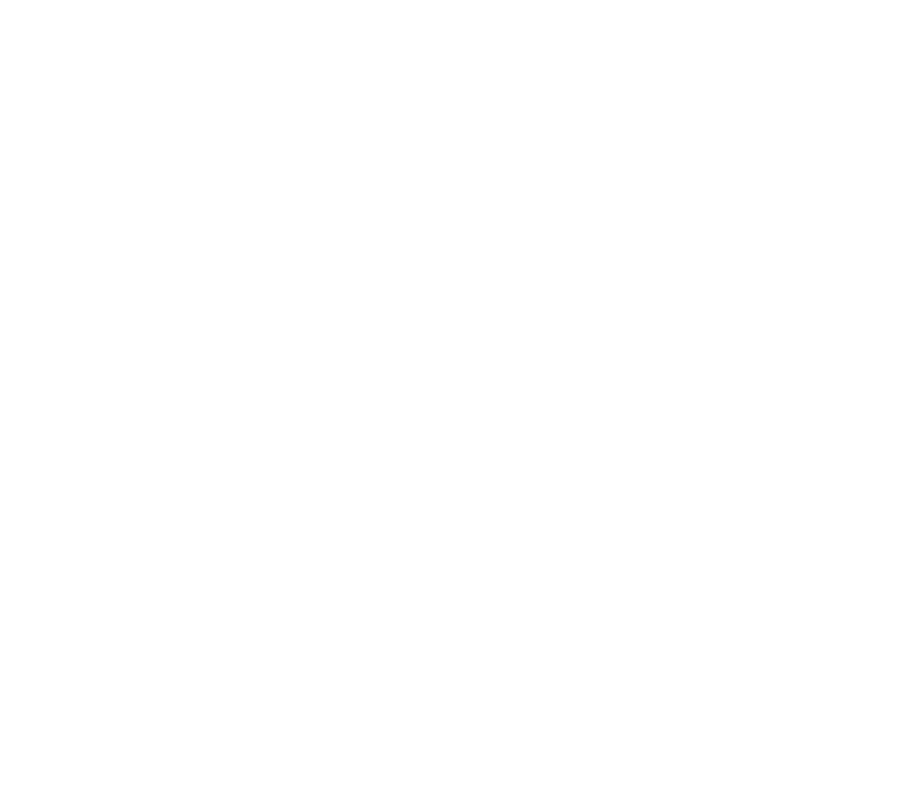
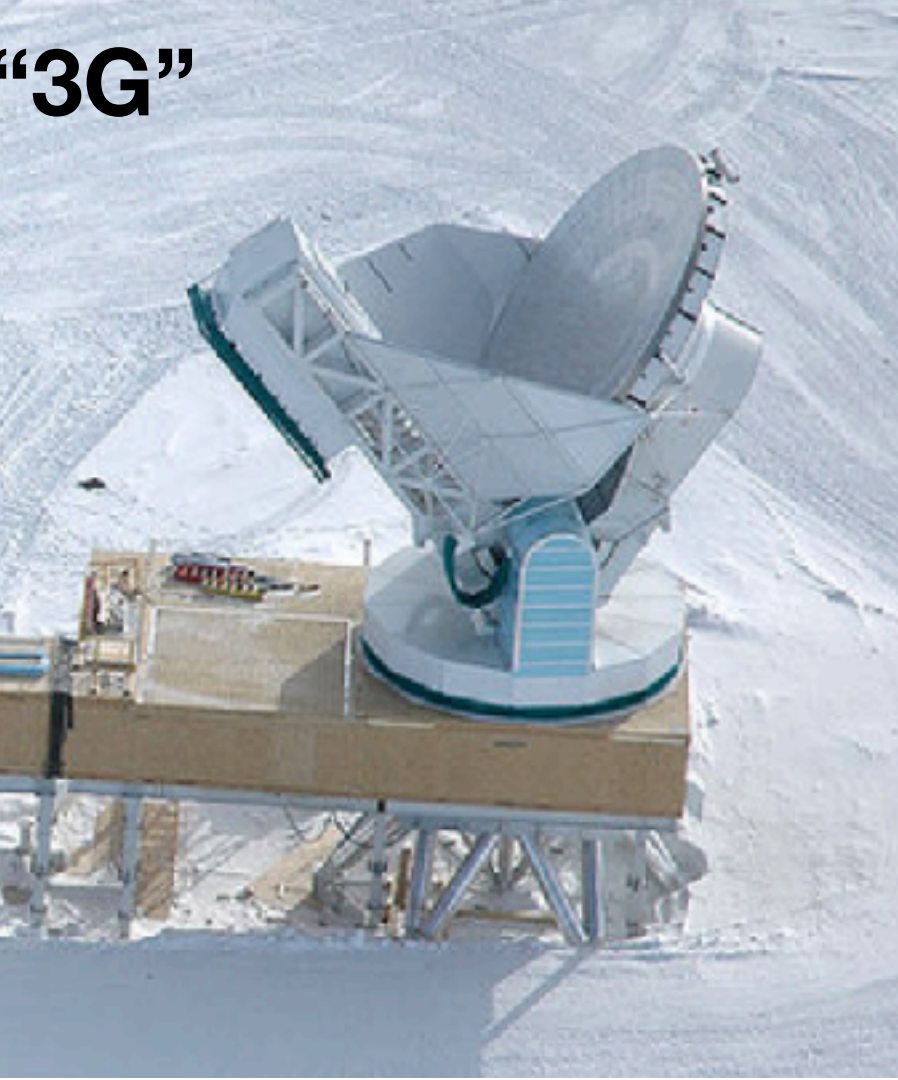
**= +**

**The Simons Array**

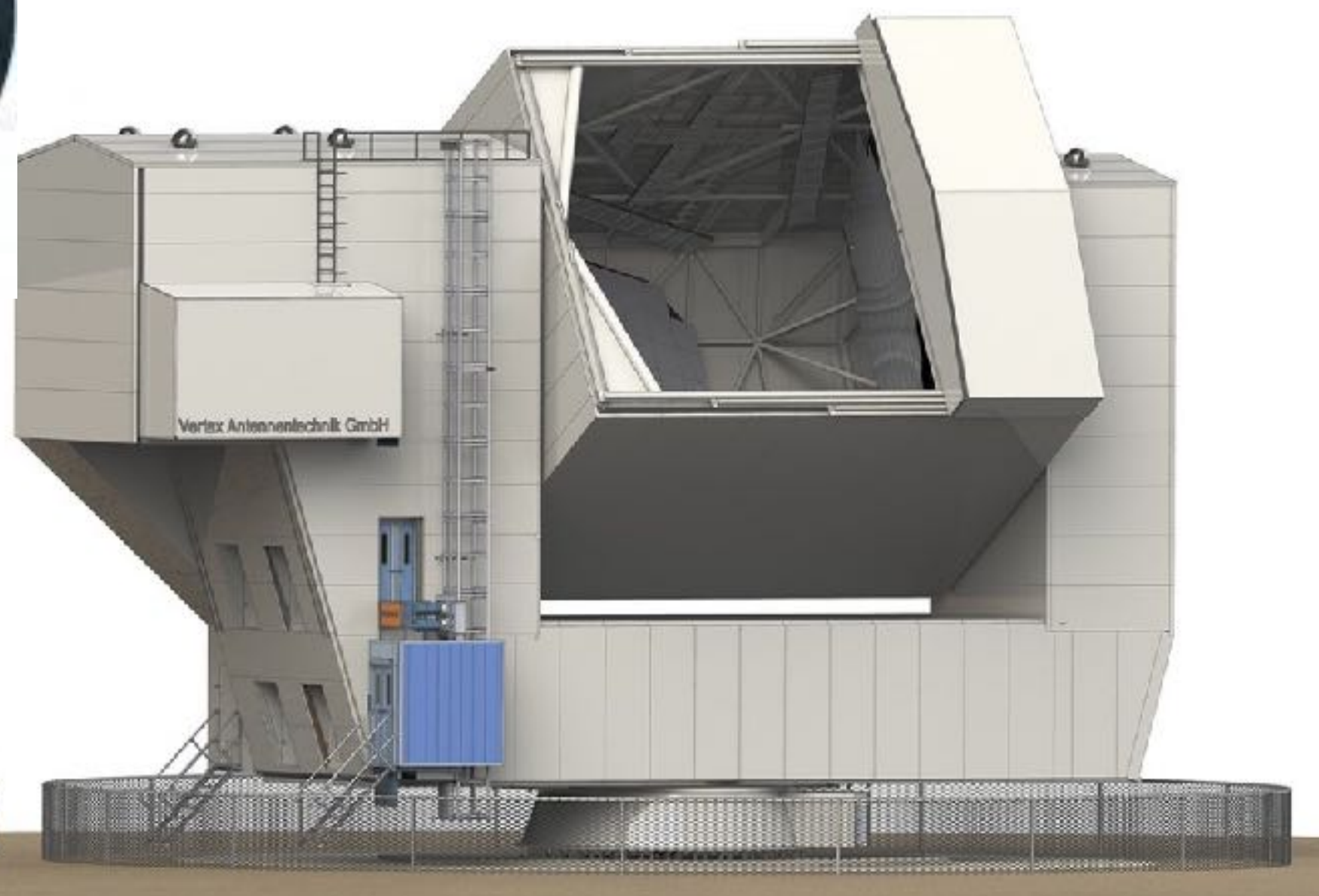
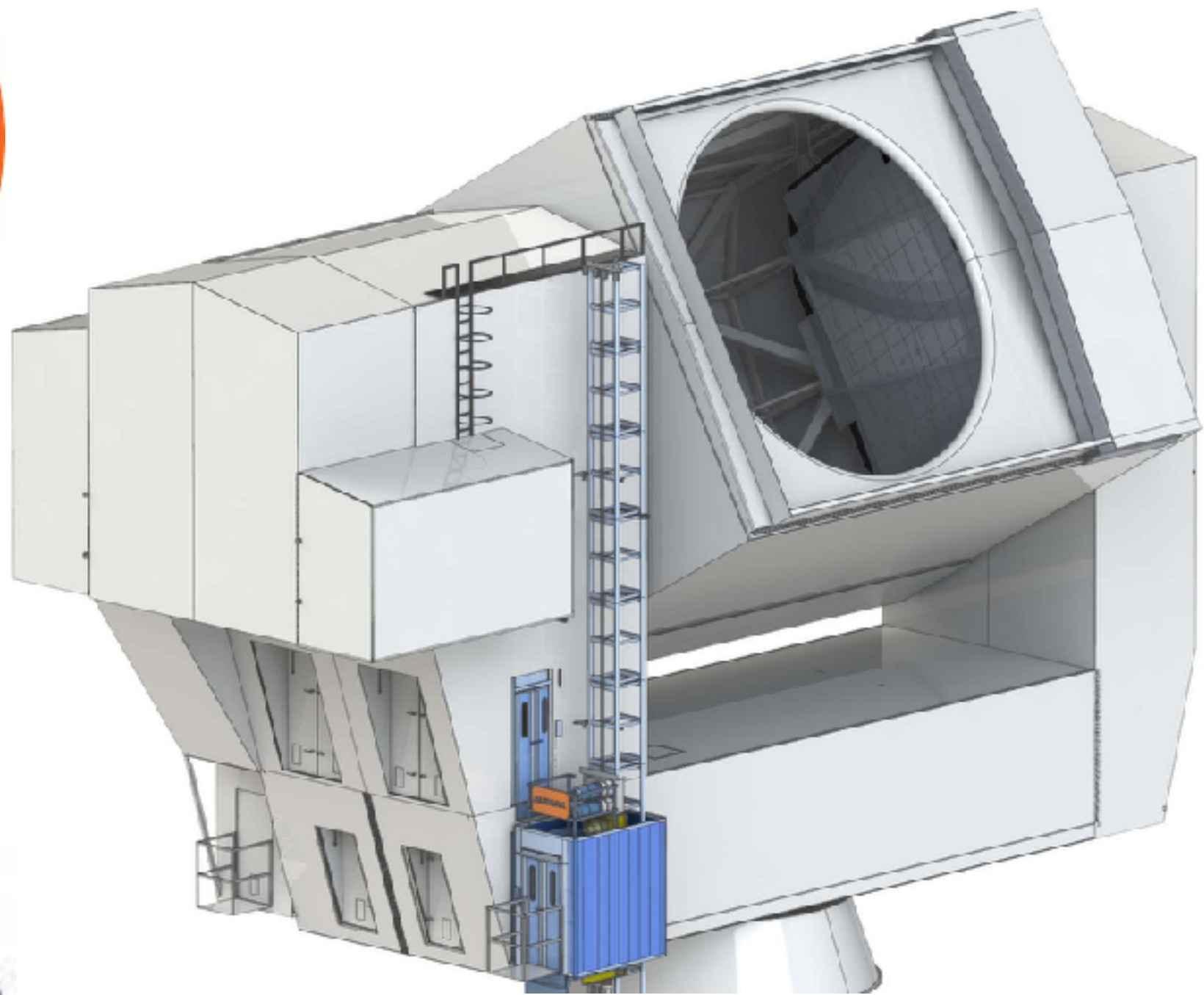


**+ =**

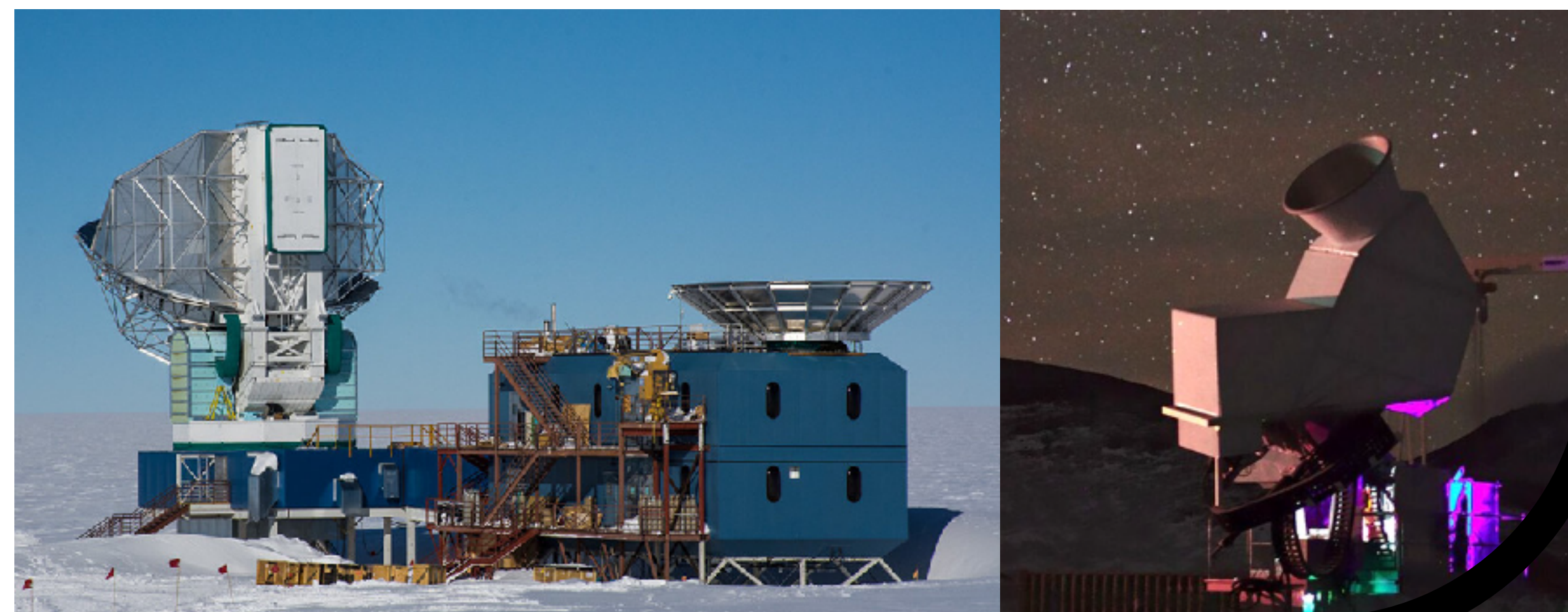
**The South Pole Observatory**







Bringing all together:  
US-led CMB Stage IV  
Late 2020s (~\$600M)



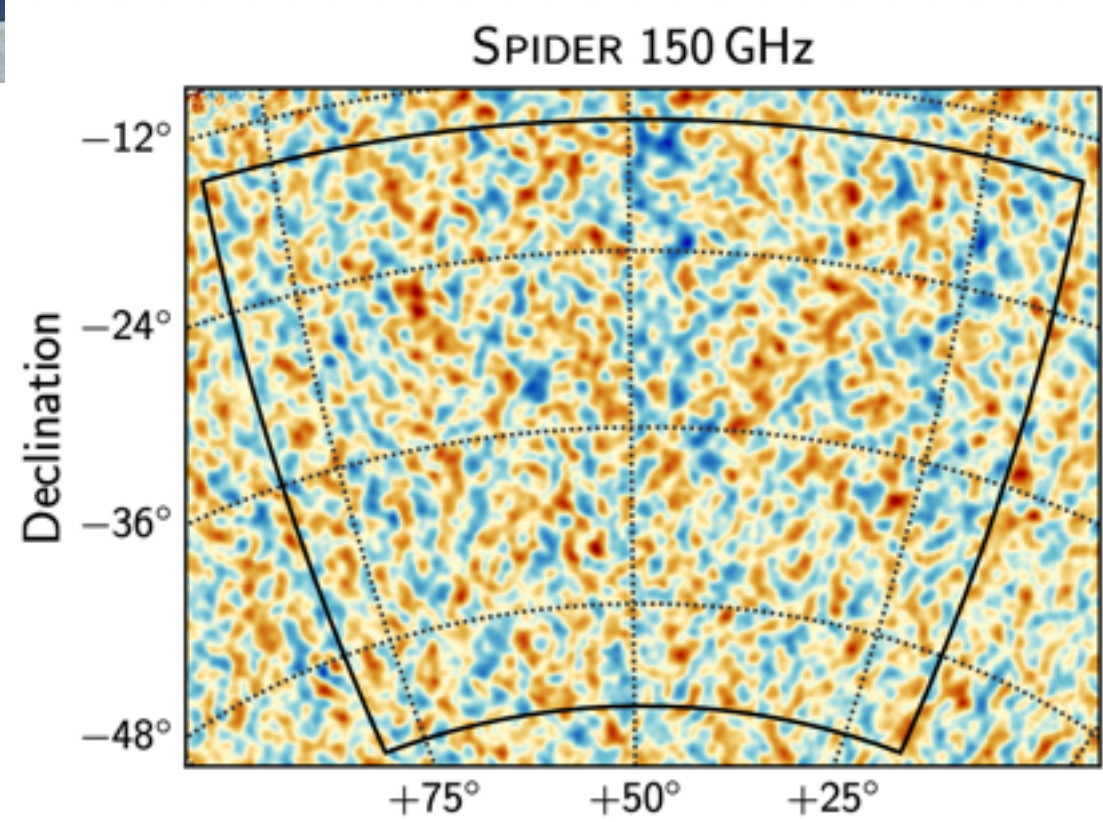


# Balloons!

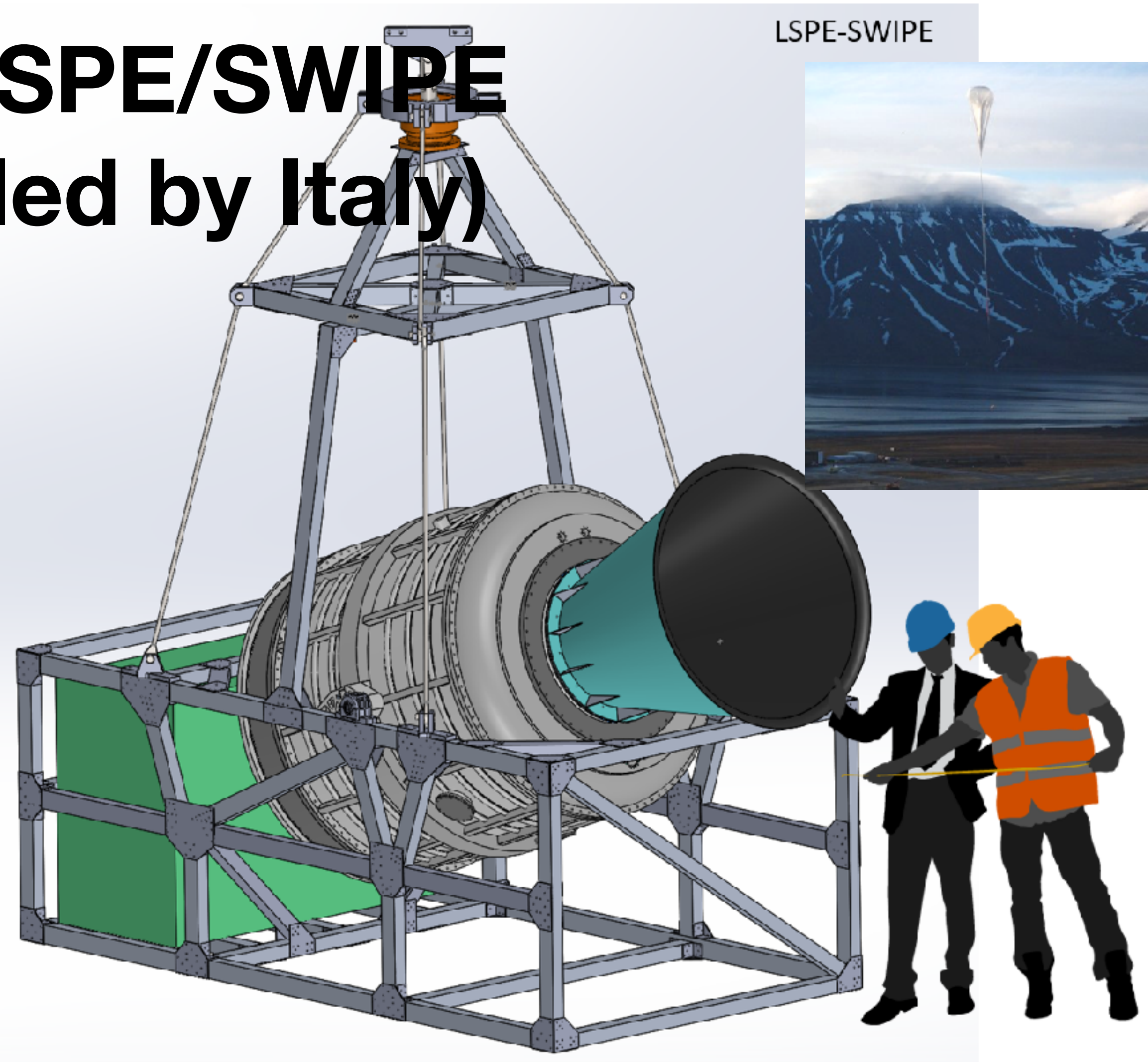
“Almost space”

## SPIDER (led by USA)

*First B-mode result:  
arXiv:2103.13334*



## LSPE/SWIPE (led by Italy)





# 2029- LiteBIRD



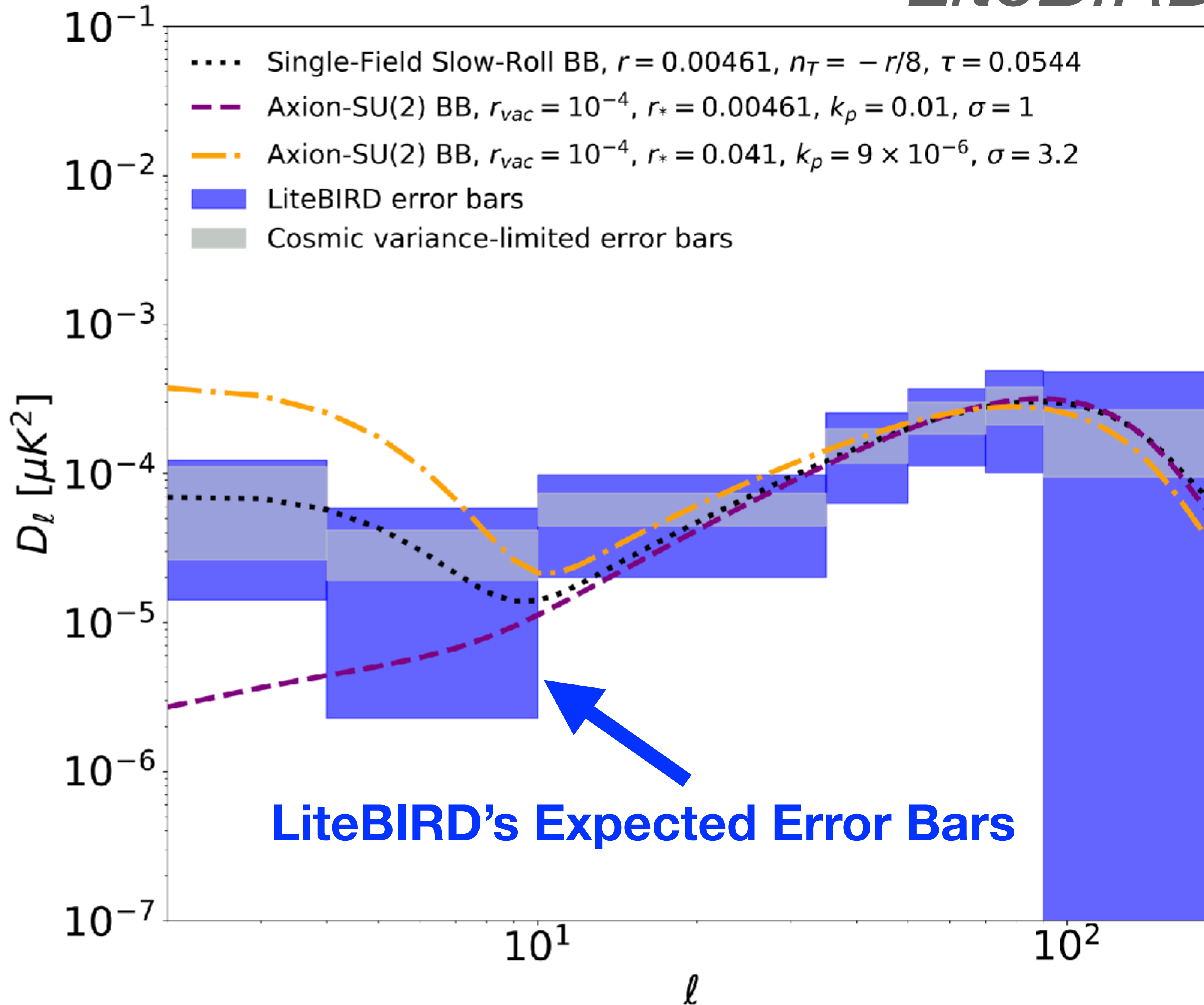
JAXA  
+ NASA  
+ CSA  
+ Europe

**A few thousand super-conducting  
microwave sensors in space.  
Selected by JAXA to fly to L2!**



# B-mode Power Spectrum

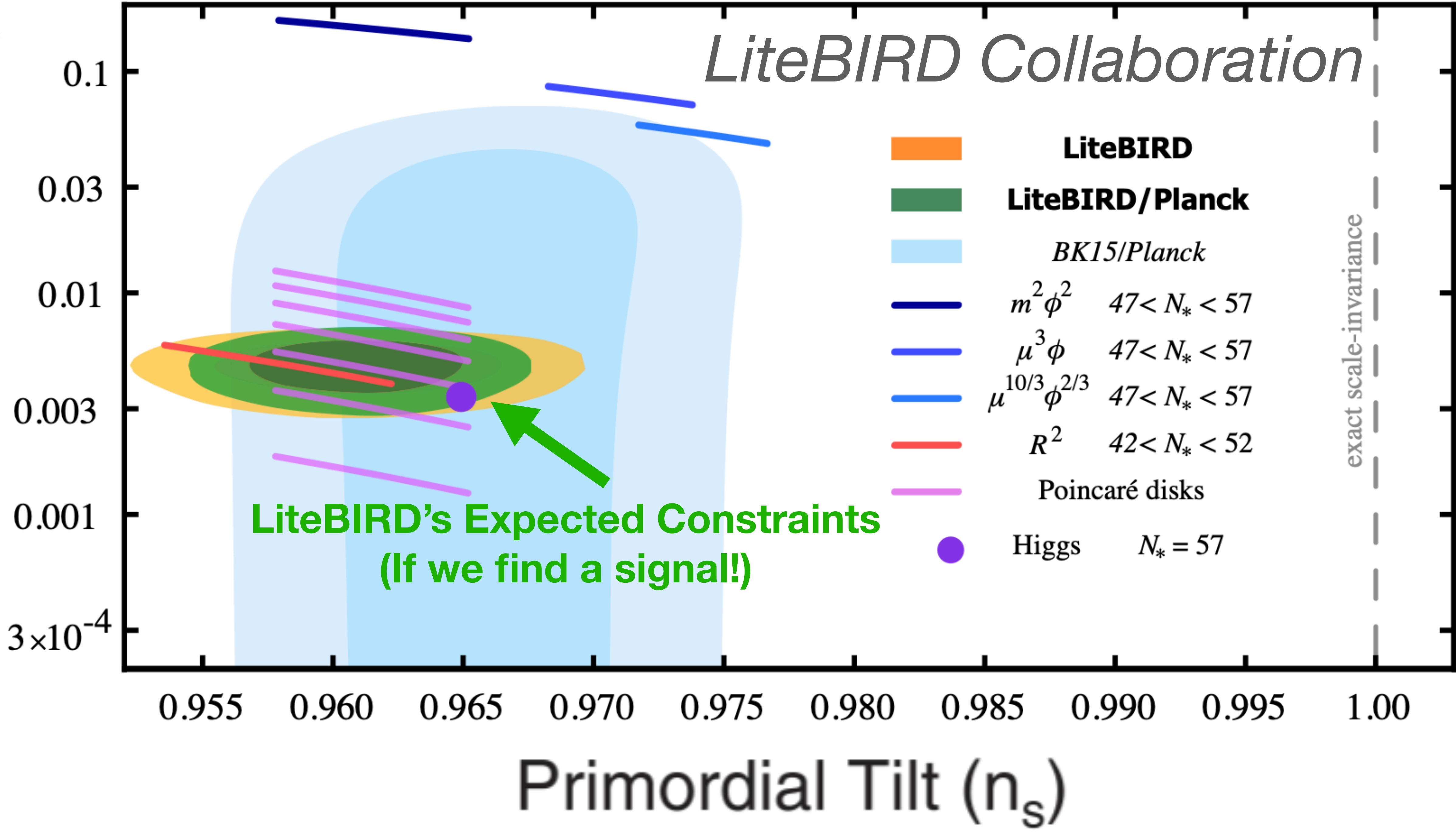
LiteBIRD Collaboration





Tensor-to-Scalar Ratio ( $r$ )

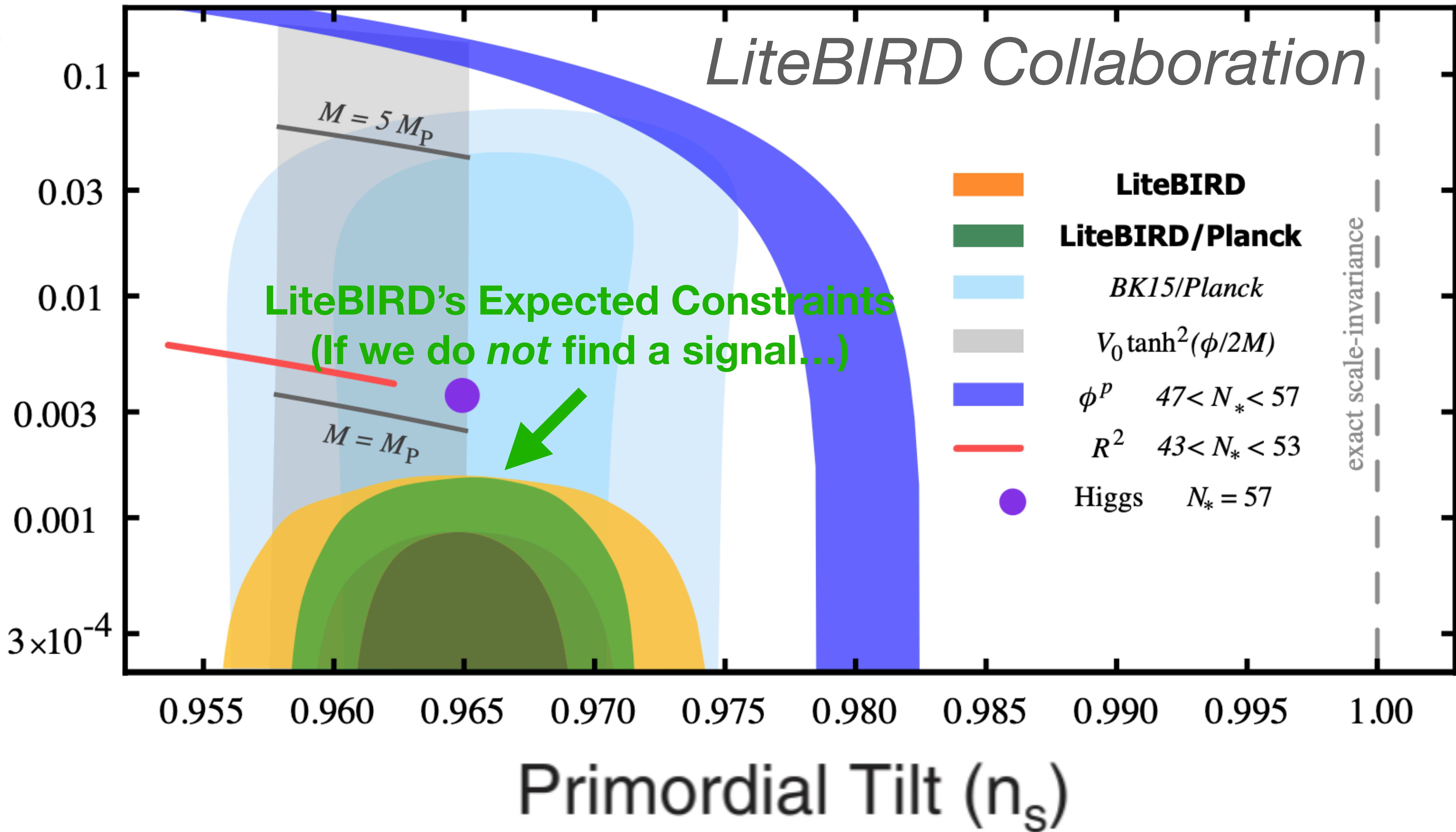
*LiteBIRD Collaboration*





Tensor-to-Scalar Ratio ( $r$ )

*LiteBIRD Collaboration*

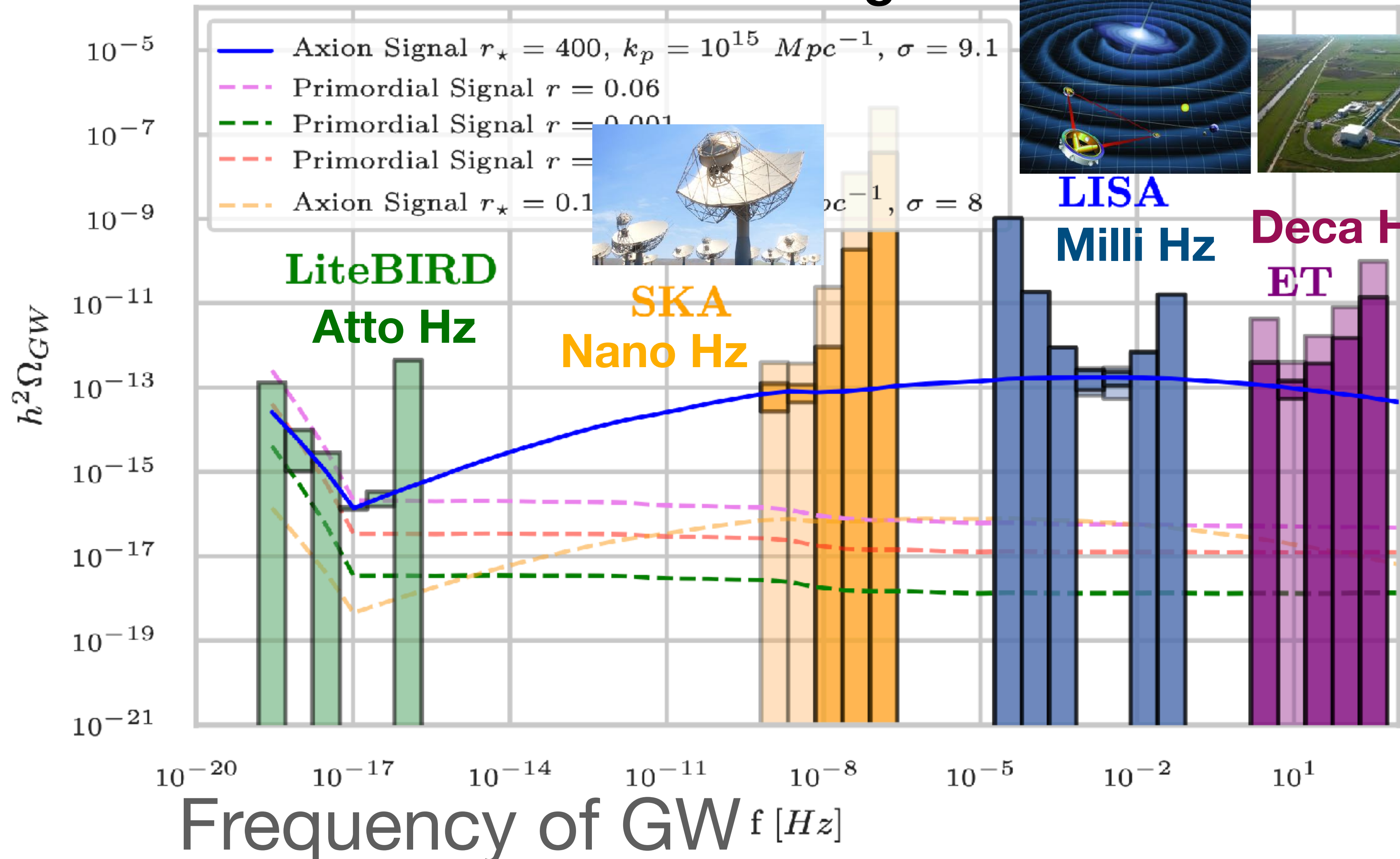




# But let's recall again: not just CMB!

We can measure it across 21 orders of magnitude in the GW frequency

Energy Density of GW  
today





# Summary

## Towards finding our origins

- **The Quest So Far:**

- There is very good evidence that we all came from the quantum fluctuation in the early Universe, generated during the period of **cosmic inflation**.

- **The New Quest:**

- Discovery of the primordial gravitational wave with the wavelength of billions of light years gives **definitive evidence for inflation**.
- Hoping to find the first evidence from ground-based and balloon-borne experiments within the next 10 years.
- Then, the definitive measurement will come from **LiteBIRD** in early 2030s.