Simple foreground cleaning algorithm for detecting primordial B-mode polarization of the CMB

Eiichiro Komatsu (Max-Planck-Institut für Astrophysik)

Big3 Workshop, APC, Paris, September 20, 2013

This presentation is based on:

- Review part: WMAP 7-year papers
- Main part: *Katayama & Komatsu, ApJ, 737, 78 (2011)*

I agree with Lloyd Knox

- ***Simplicity*** *can be a useful guiding principle!*
  - I have only ~10 years of experience of analyzing the CMB data, but my limited experience has shown that:
  - "If a simple method does not work *at all* for some problem, then it is usually a good indication that the problem is unsolvable."

Our Problem

- Can we reduce the polarized Galactic foreground emission down to the level that is sufficient to allow us to detect a signature of primordial gravitational waves from inflation at the level of 0.1% of gravitational potential? (It means $r=10^{-3}$ for cosmologists.)

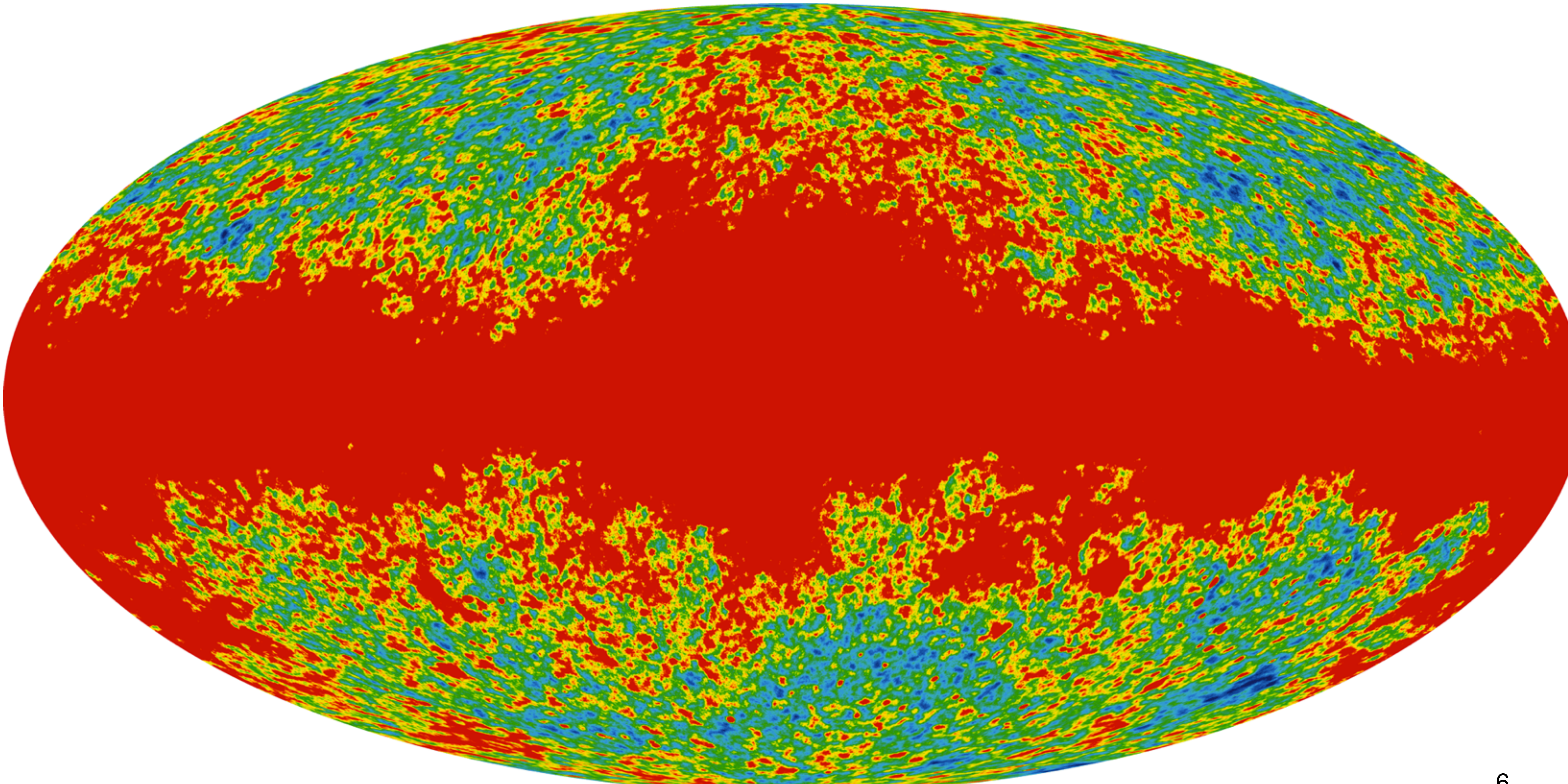
- If a simple method does not get us anywhere near $r \sim 10^{-3}$, then perhaps we should just give up reaching such a low level. ***Good News: a simple method does get you to $r \sim 10^{-3}$!***

Let me emphasize:

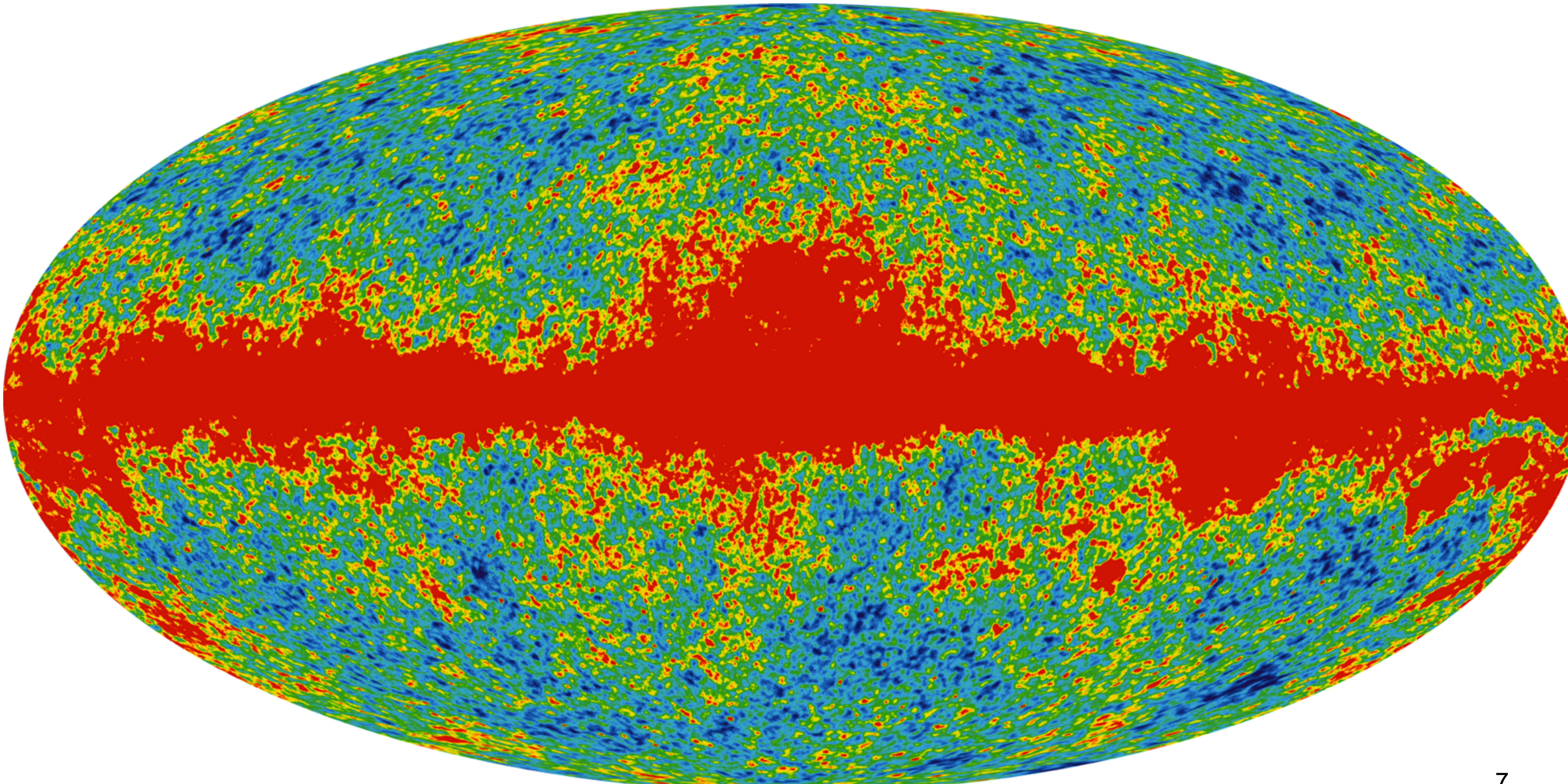
- However, a simple method that I am going to present here will not give you the final word.

- Rather, our results show that, as the simple method gets us to $r=O(10^{-3})$, it is worth going beyond the simple method and refining the algorithm to reduce the remaining bias in the gravitational wave amplitude (i.e., r) by a factor of order unity (rather than a factor of >100).

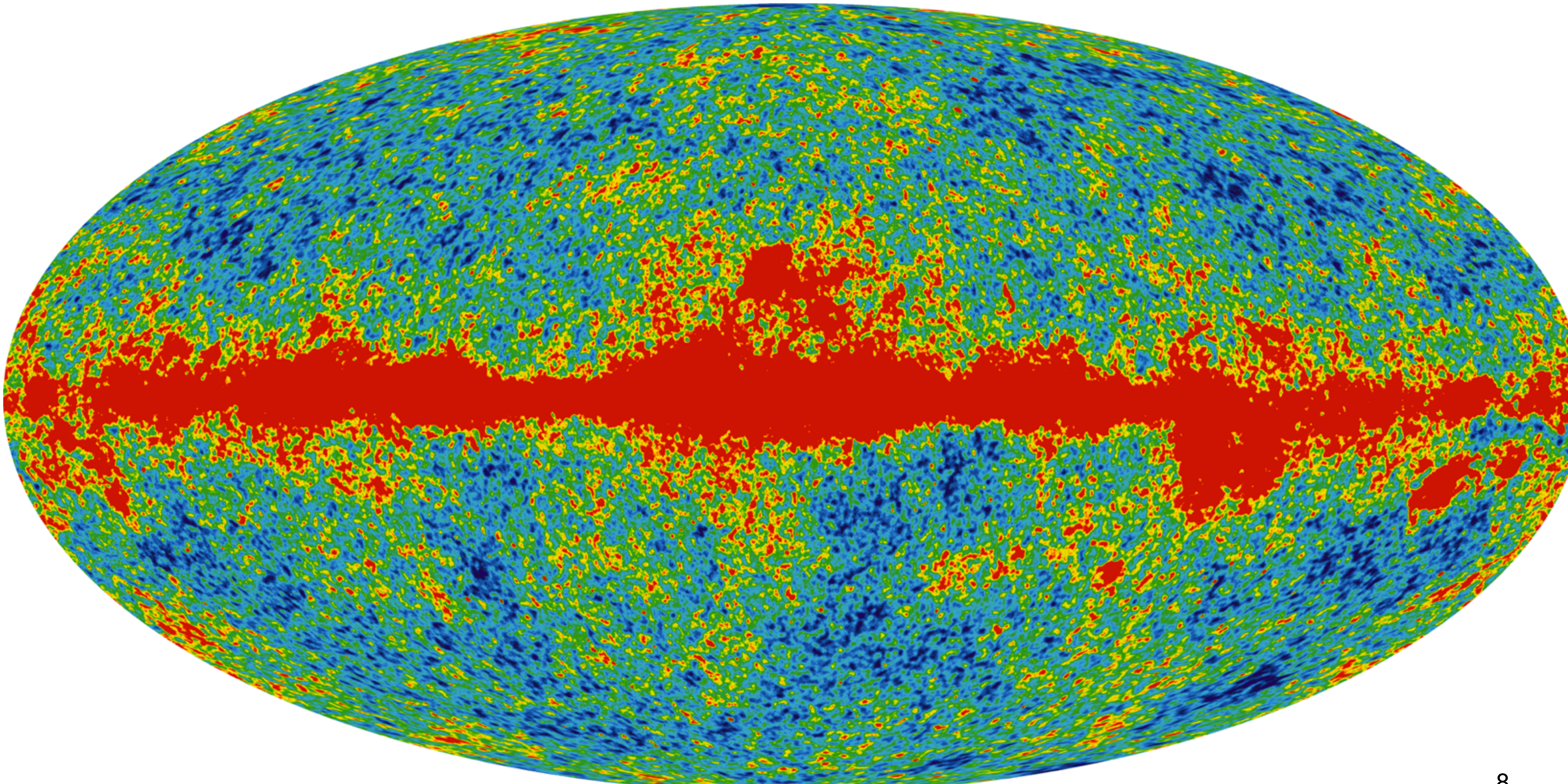
23 GHz [unpolarized]



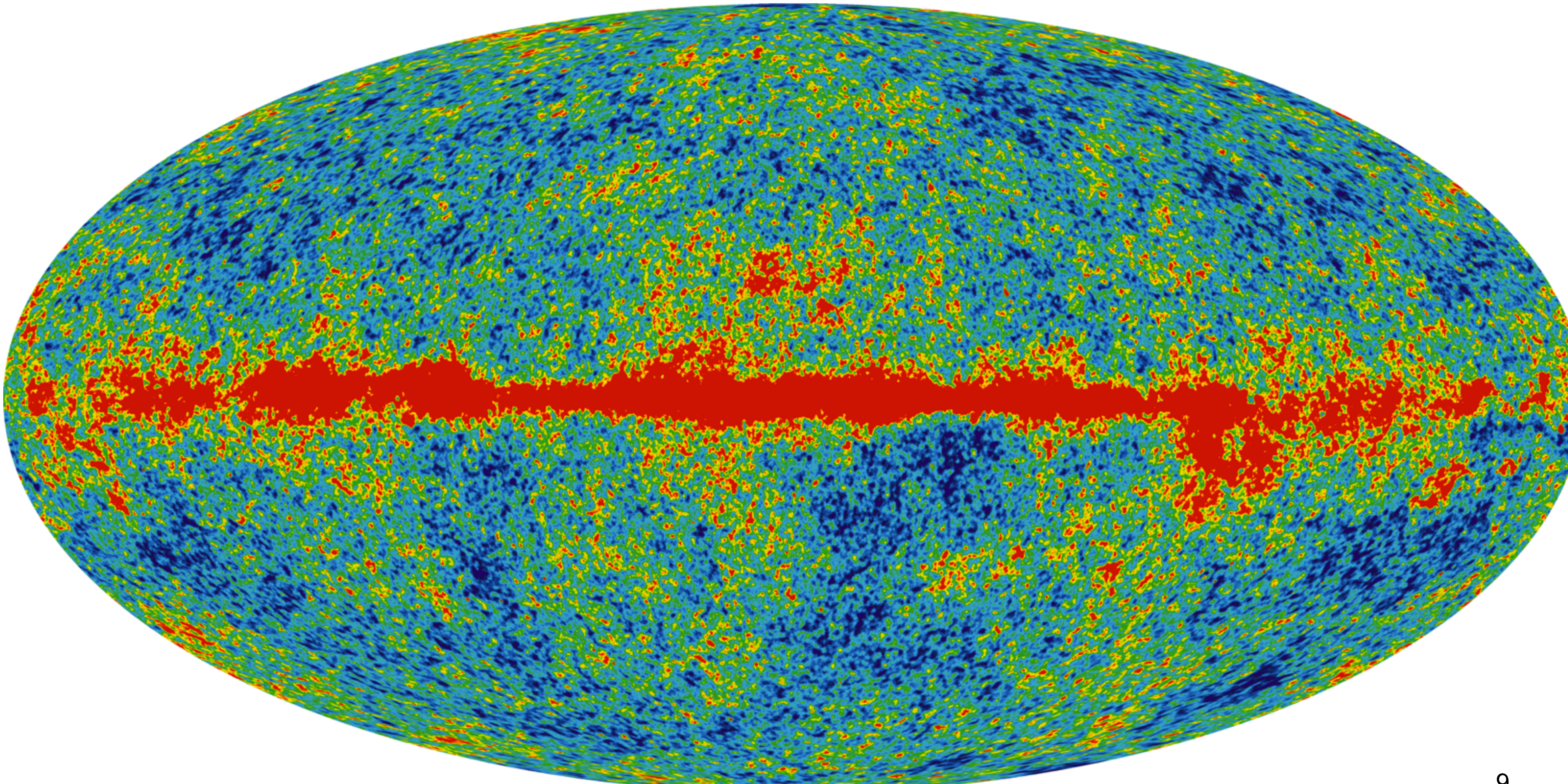
33 GHz [unpolarized]



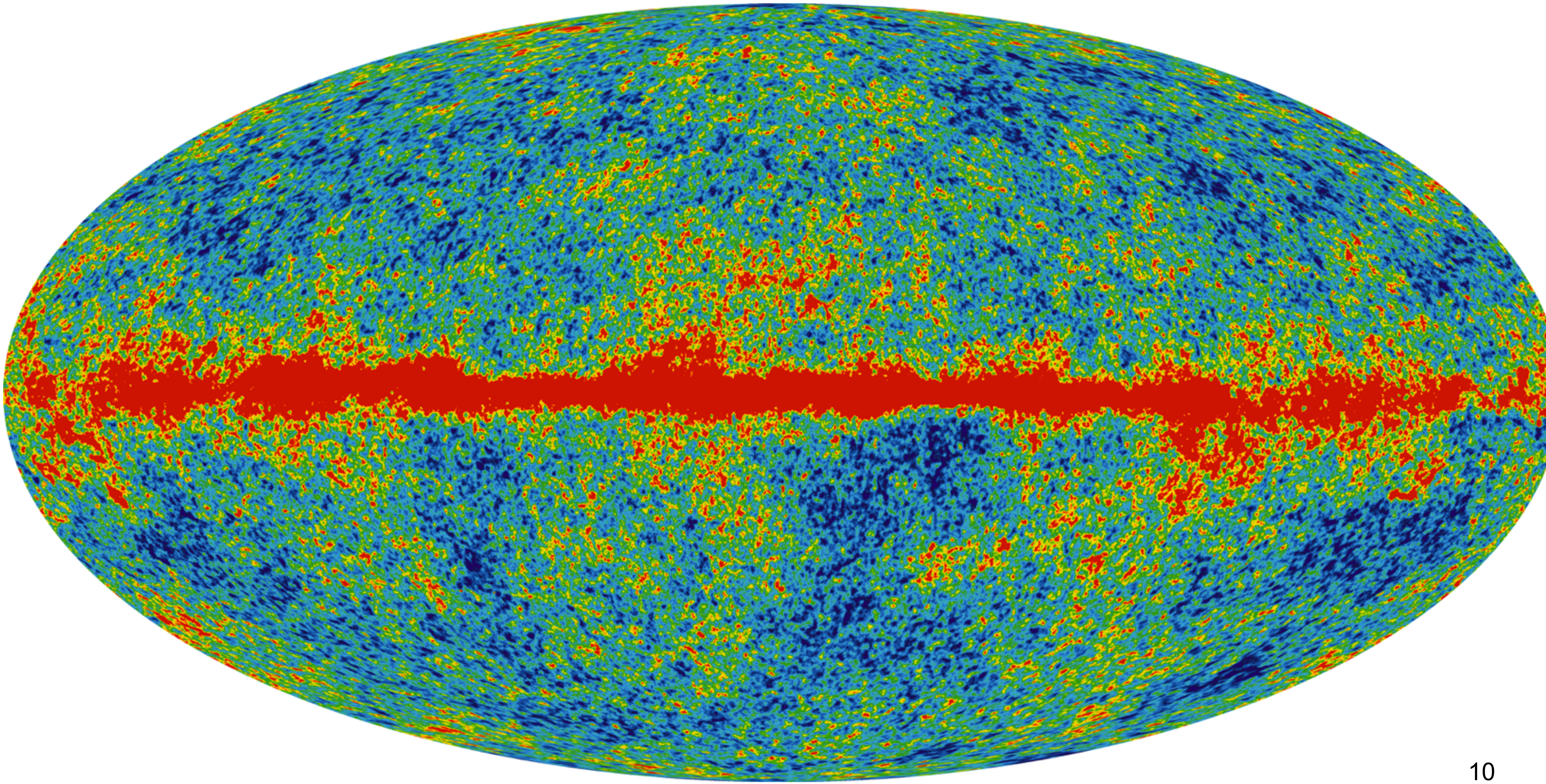
41 GHz [unpolarized]



61 GHz [unpolarized]



94 GHz [unpolarized]

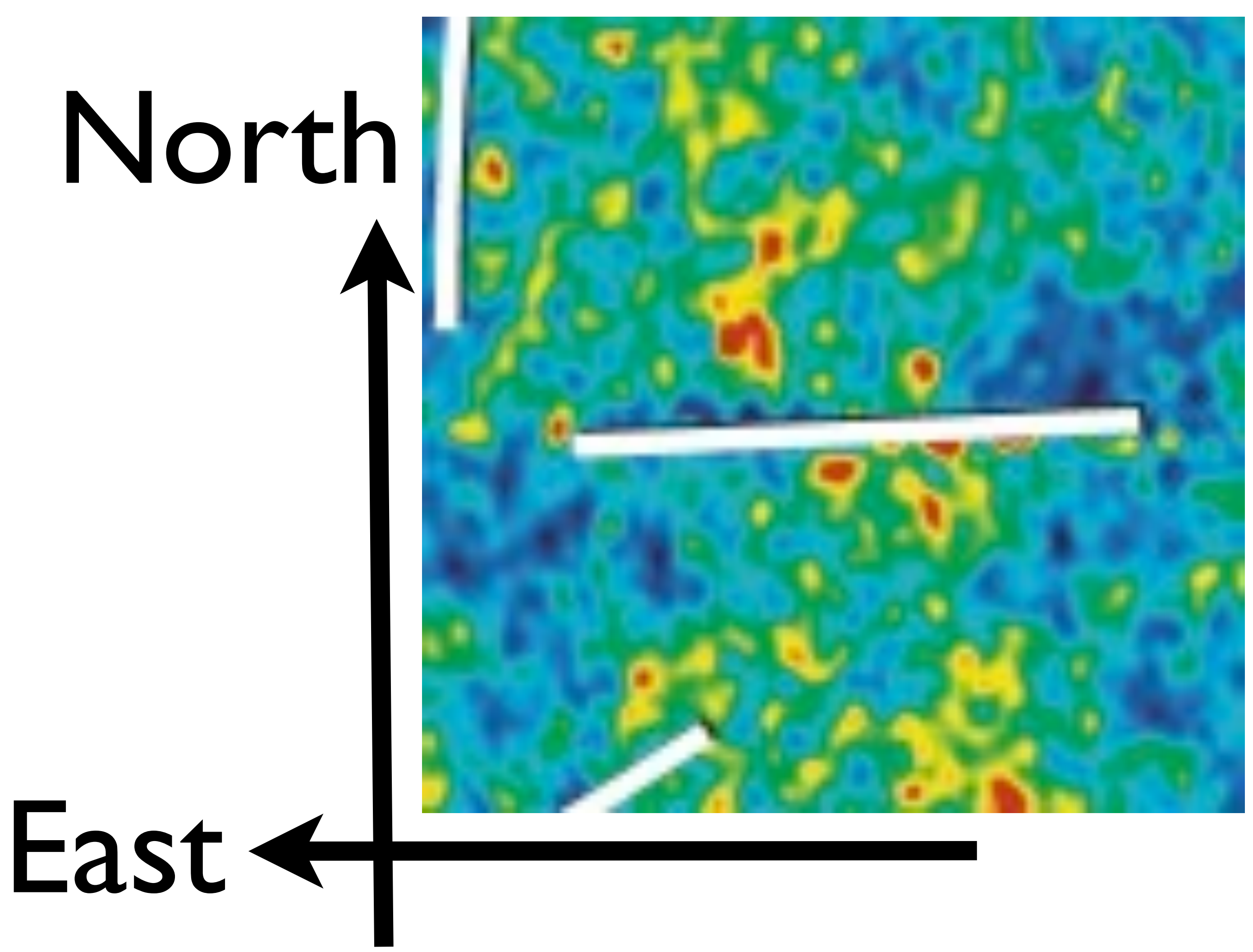


How many components?

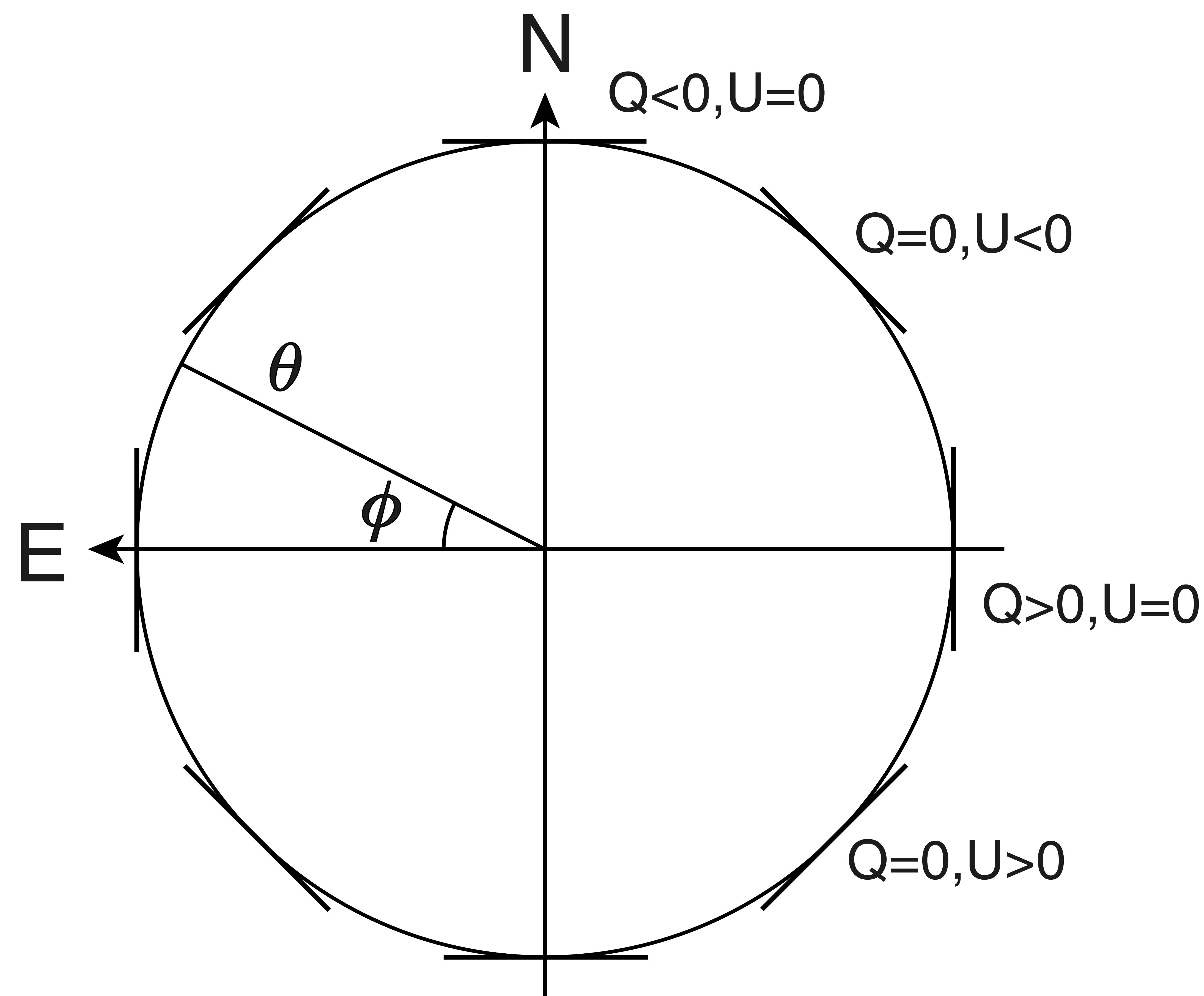
1. **CMB**: $T_\nu \sim \nu^0$
2. **Synchrotron** (electrons going around magnetic fields): $T_\nu \sim \nu^{-3}$
3. **Free-free** (electrons colliding with protons): $T_\nu \sim \nu^{-2}$
4. **Dust** (heated dust emitting thermal emission): $T_\nu \sim \nu^2$
5. **Spinning dust** (rapidly rotating tiny dust grains): $T_\nu \sim$complicated

You need at least five frequencies to separate them!

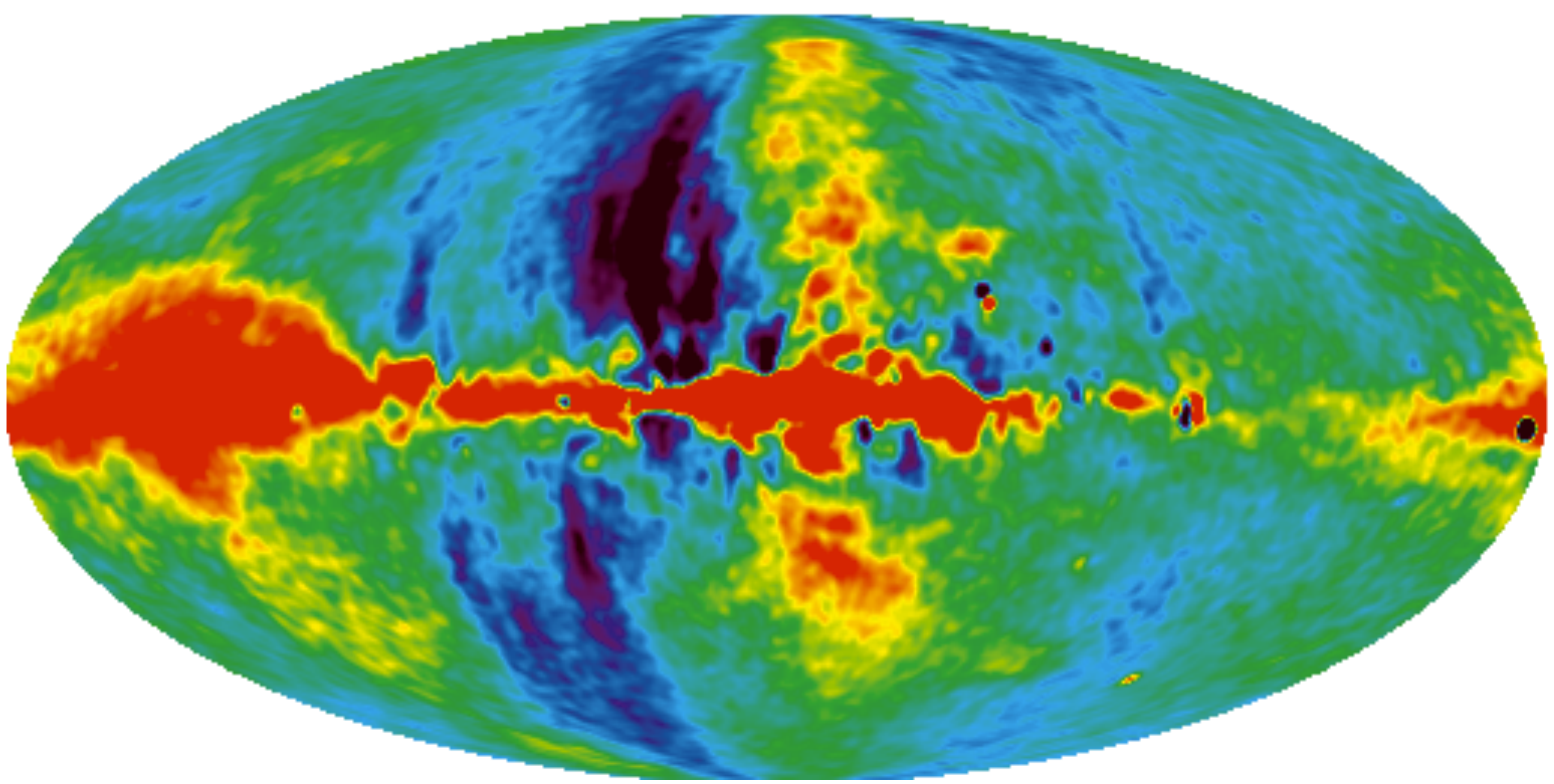
“Stokes Parameters”



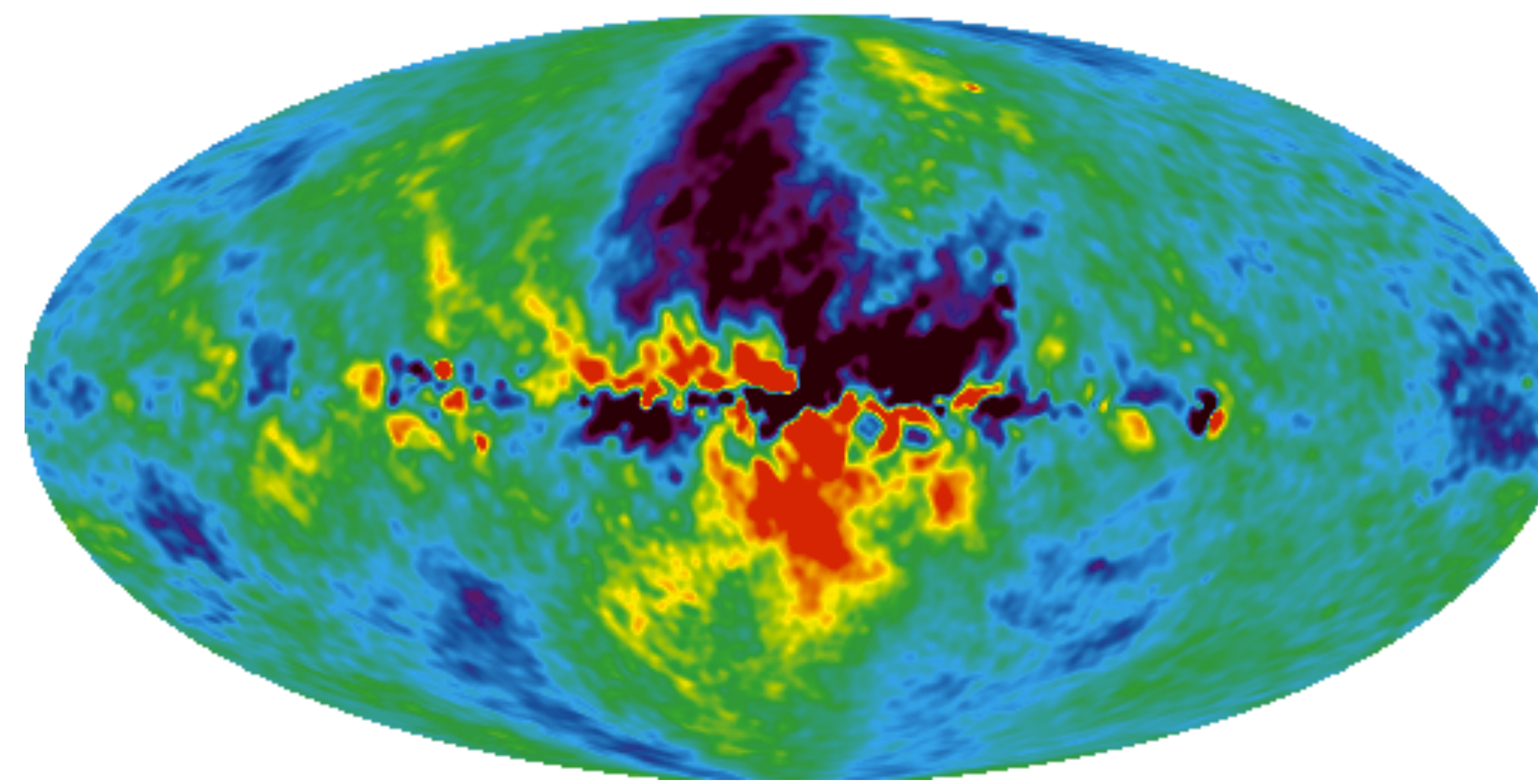
Q<0; U=0



23 GHz [polarized]

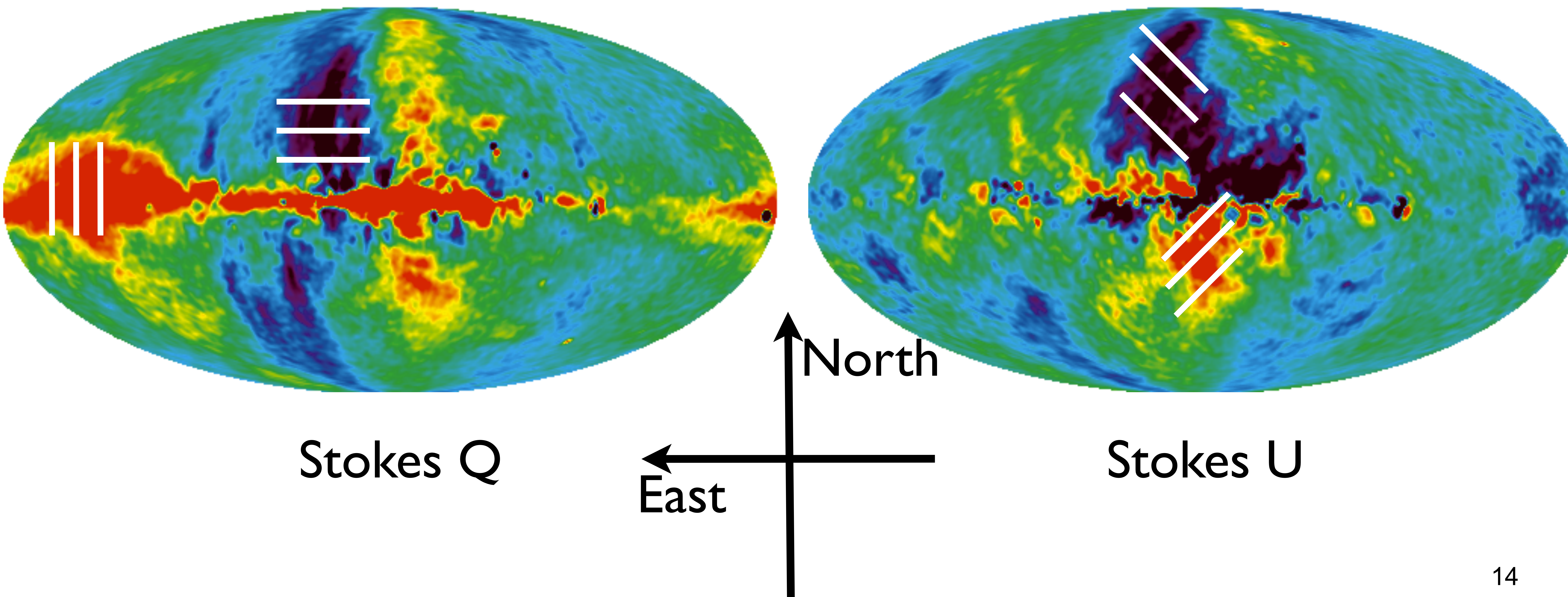


Stokes Q



Stokes U

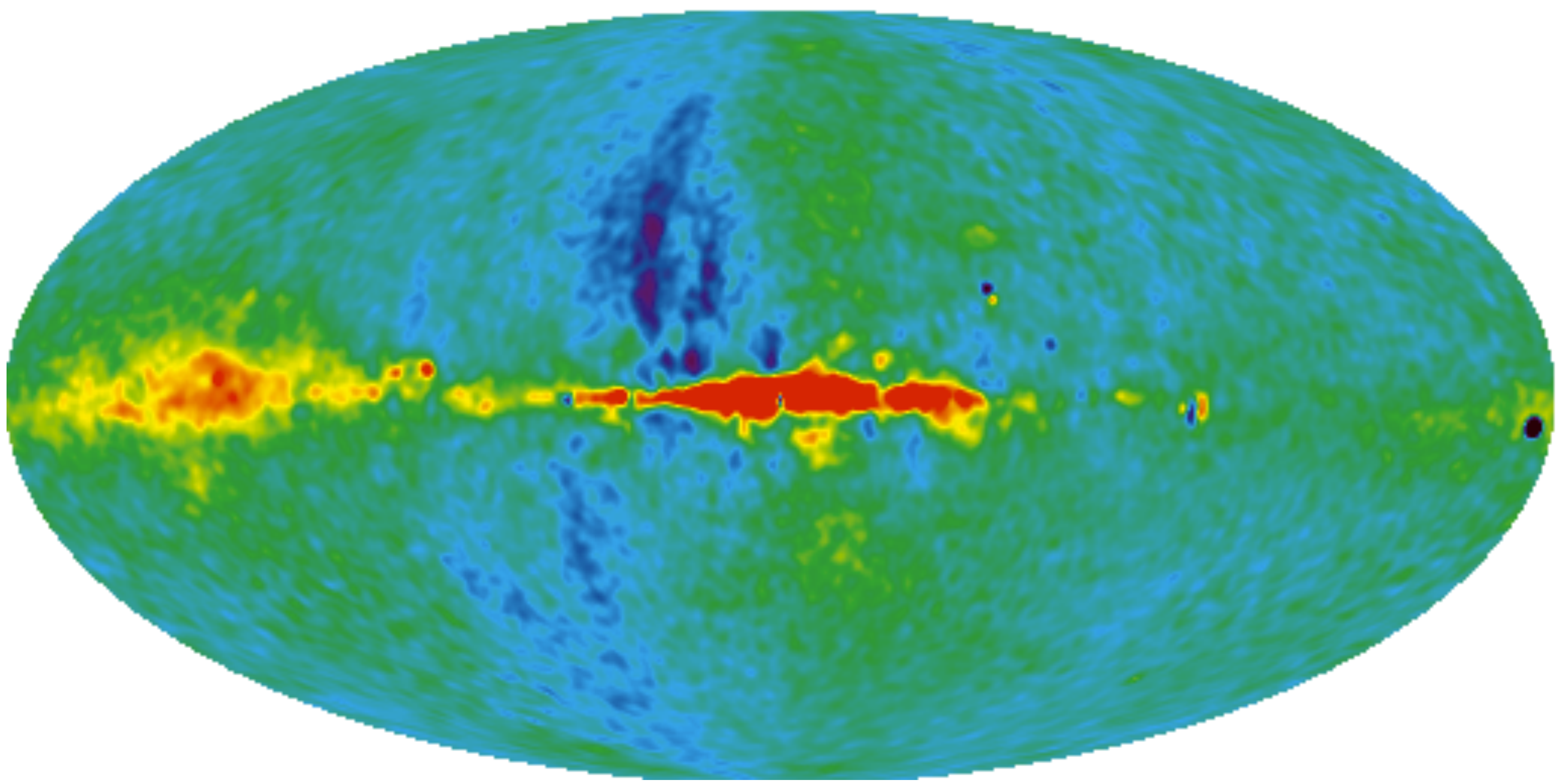
23 GHz [polarized]



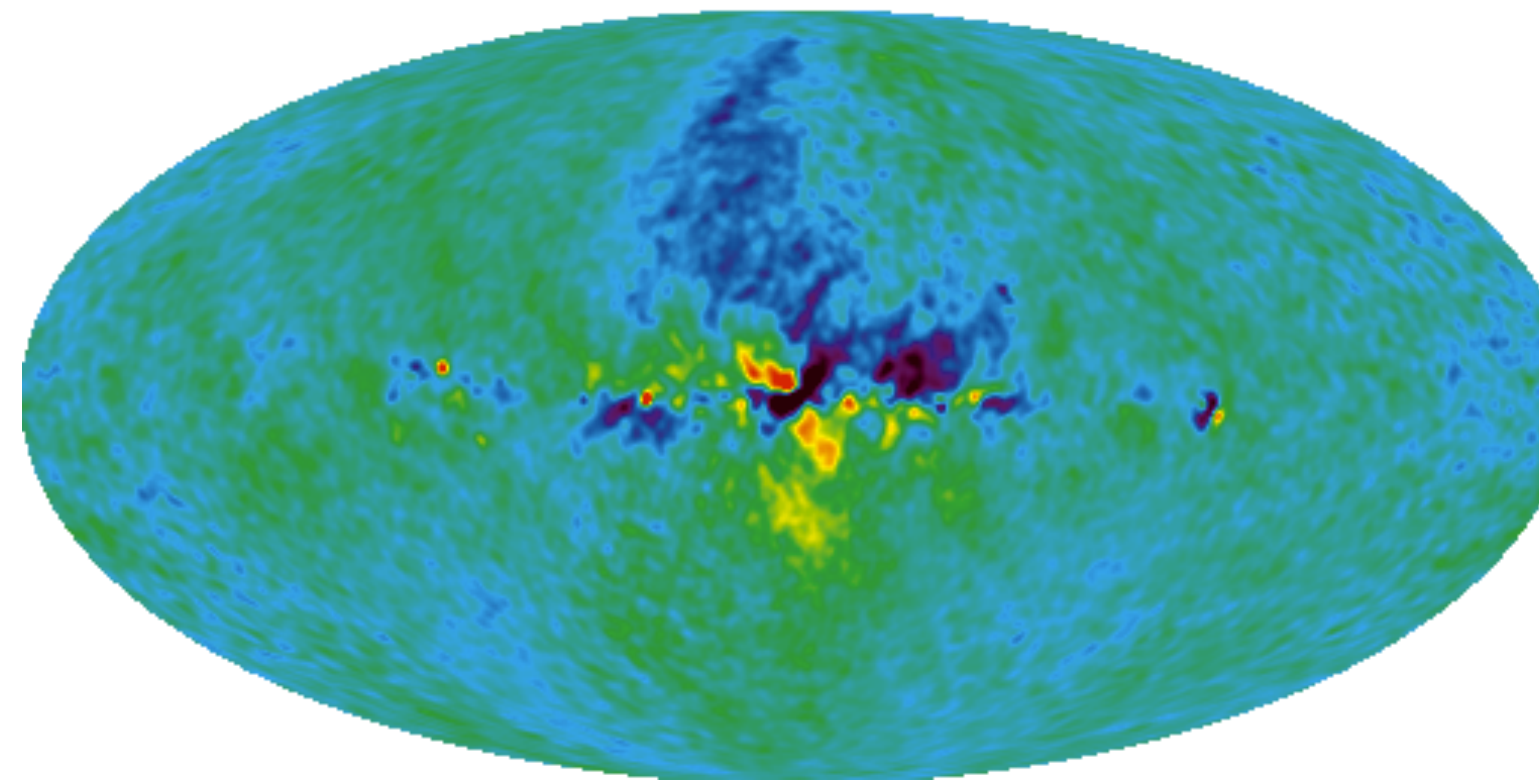
Stokes Q

Stokes U

33 GHz [polarized]



Stokes Q



Stokes U

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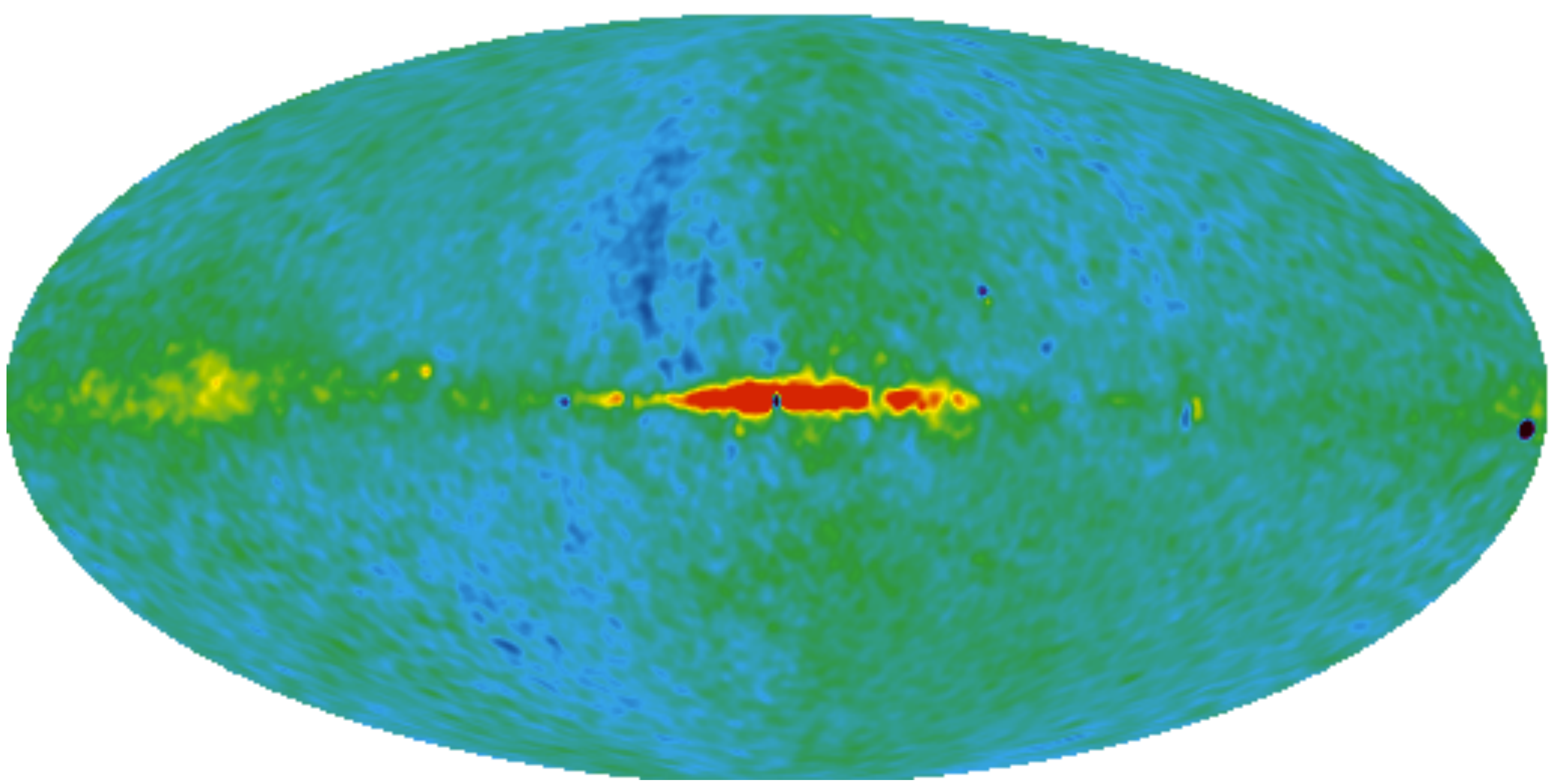
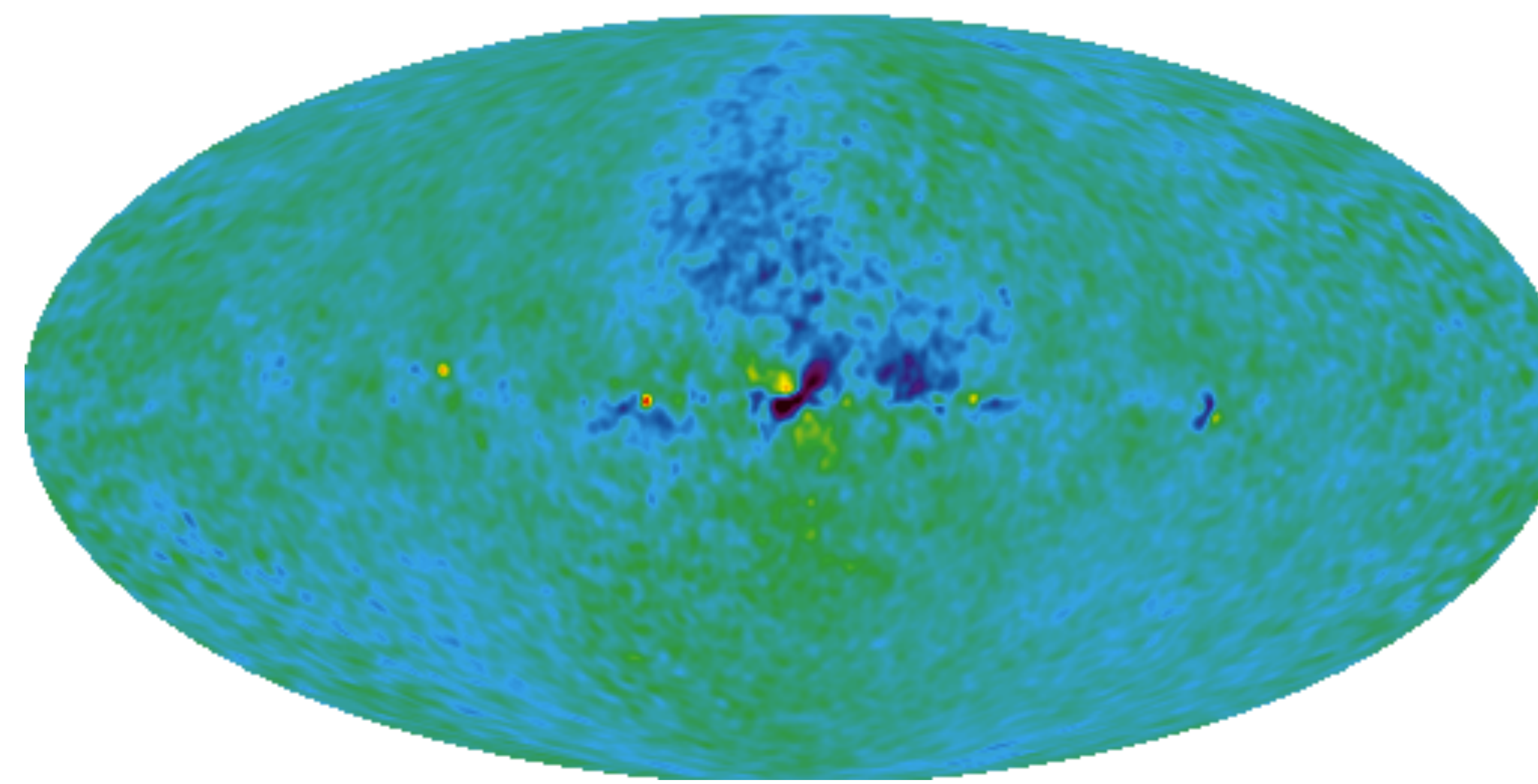
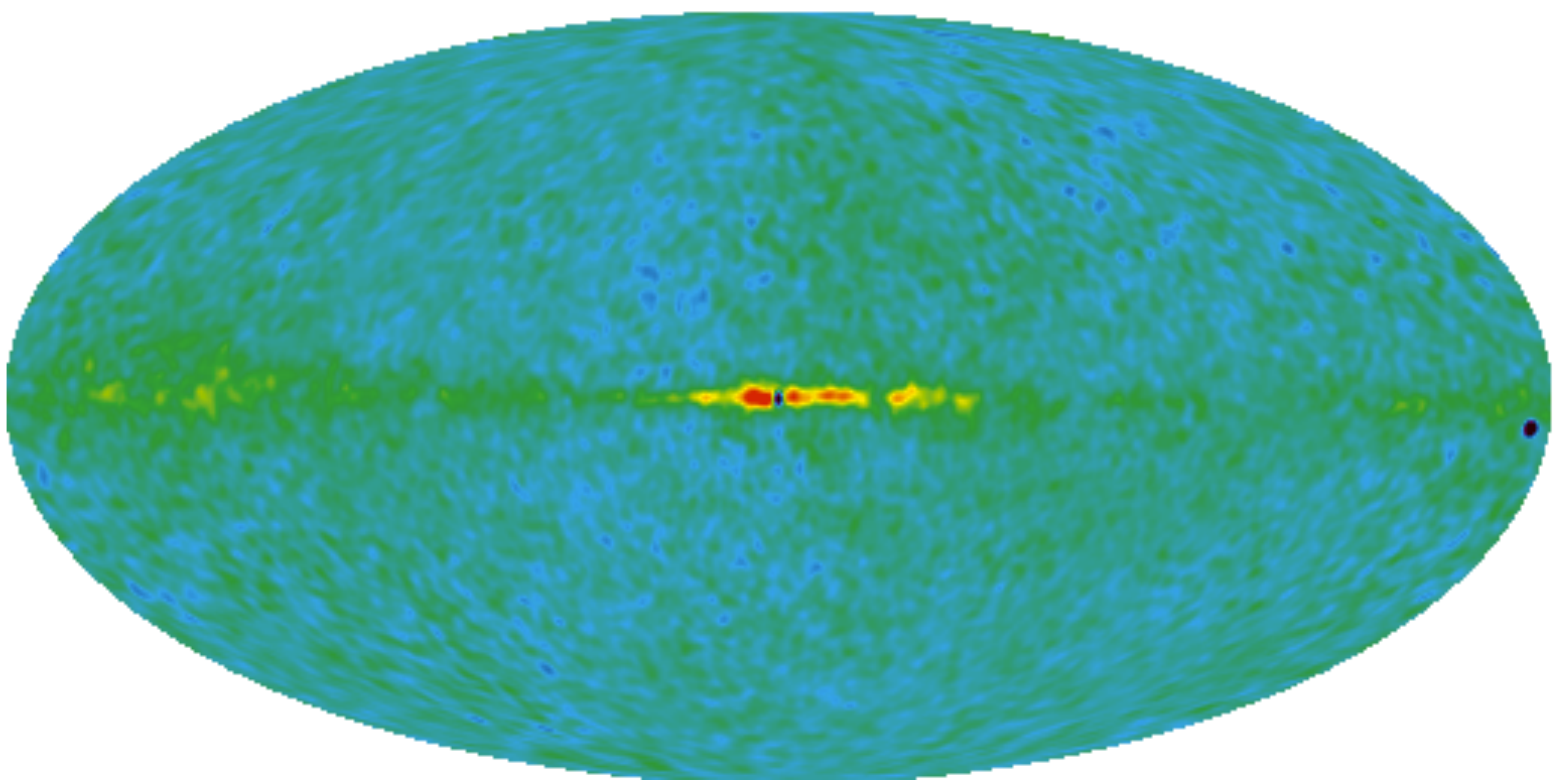


Stokes Q

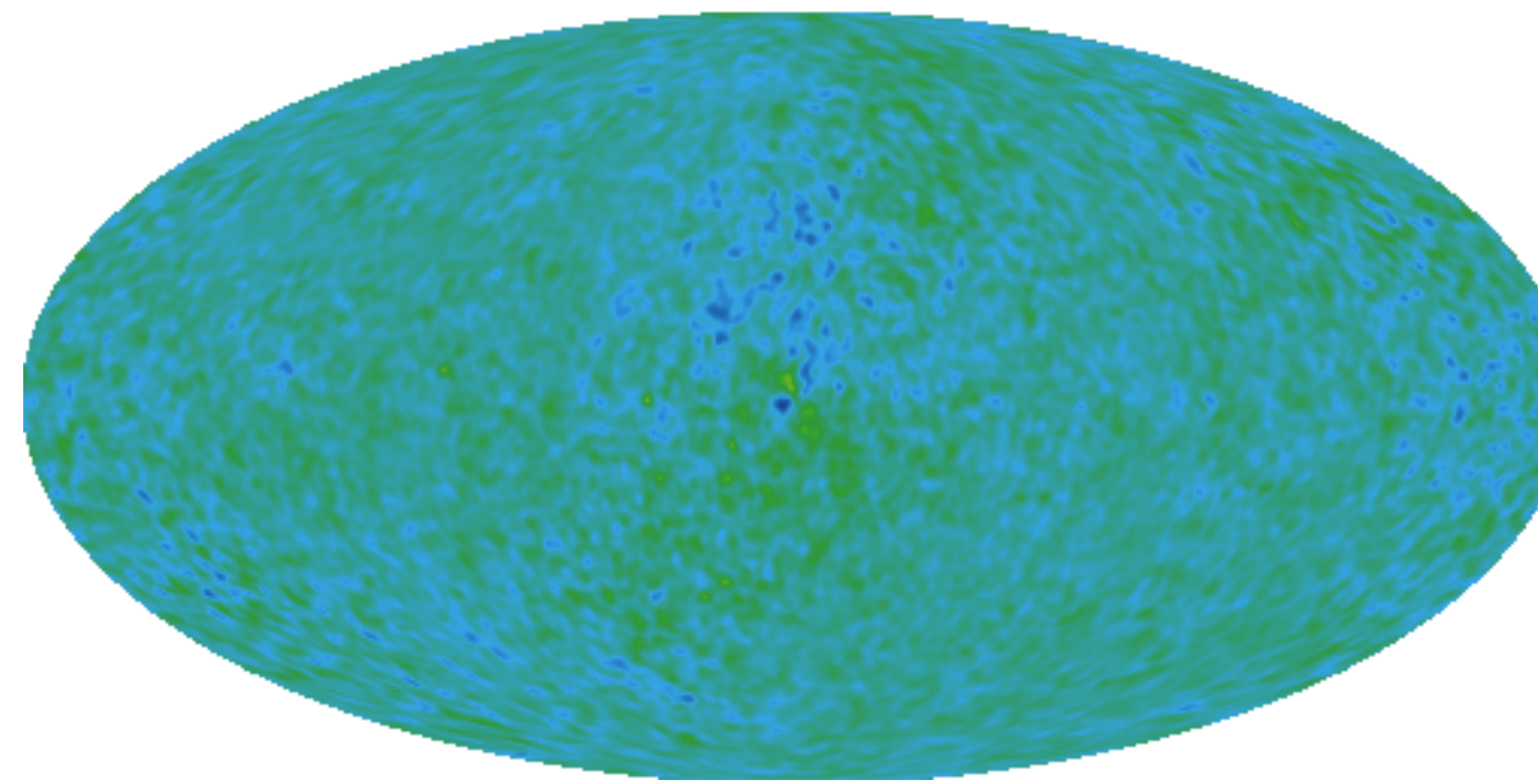


Stokes U

61 GHz [polarized]

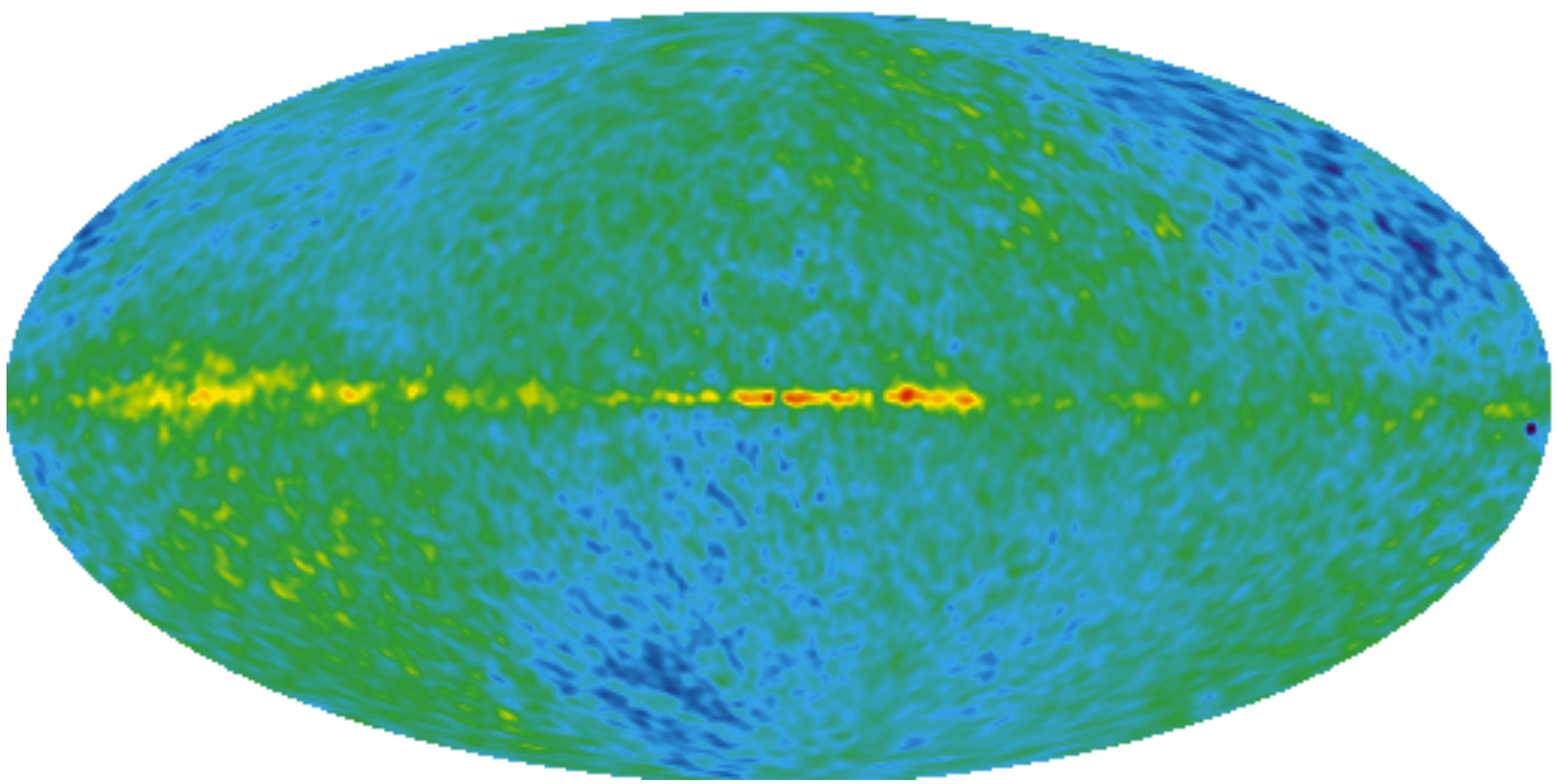


Stokes Q

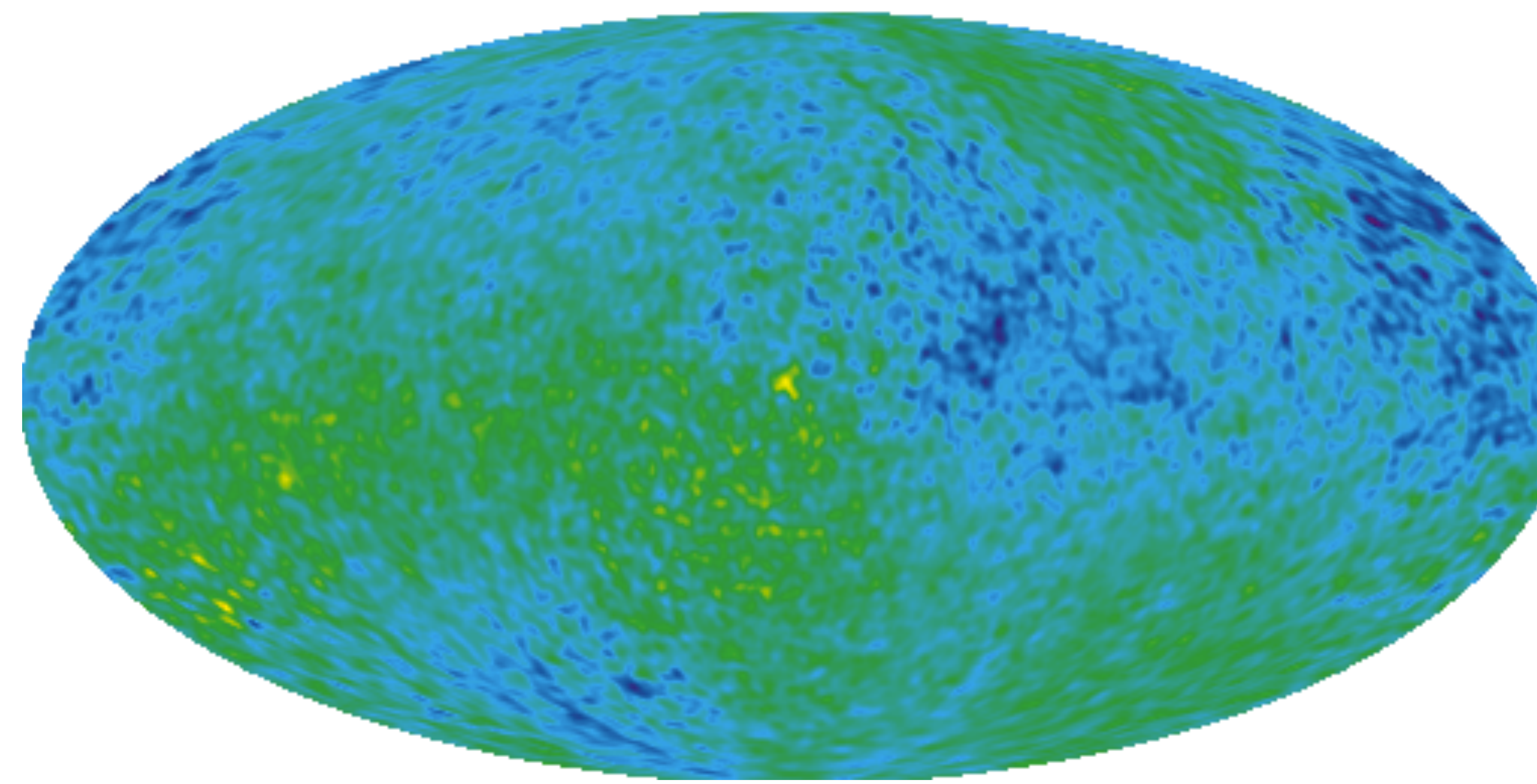


Stokes U

94 GHz [polarized]



Stokes Q



Stokes U

How many components?

1. **CMB**: $T_\nu \sim \nu^0$
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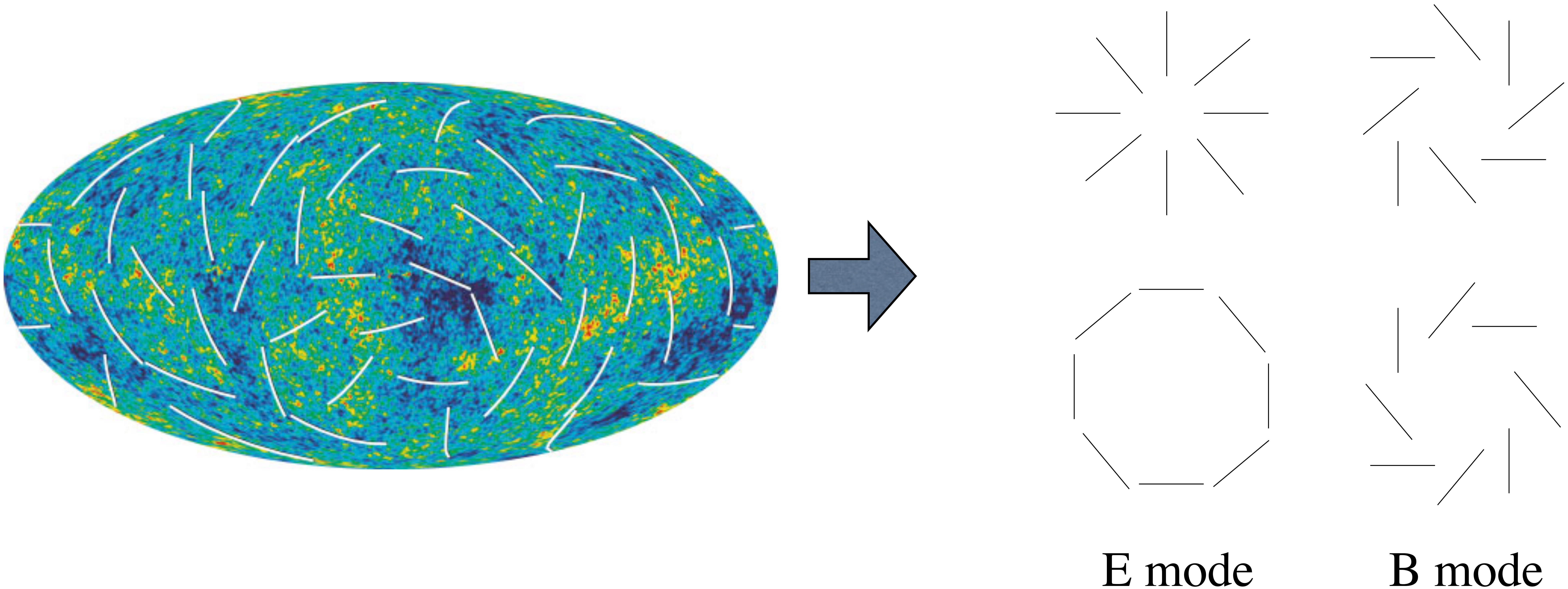
3. ~~**Free-free** (electrons colliding with protons): $T_\nu \sim \nu^{-2}$~~
4. **Dust** (heated dust emitting thermal emission): $T_\nu \sim \nu^2$
5. ~~**Spinning dust** (rapidly rotating tiny dust grains): $T_\nu \sim$ complicated~~

*You need at least **THREE** frequencies to separate them!*

A simple question

- *How well can we reduce the polarized foreground using* ***only three*** *frequencies?*
- An example configuration:
  - 100 GHz for CMB "science channel"
  - 60 GHz for synchrotron "foreground channel"
  - 240 GHz for dust "foreground channel"

Decomposing Polarization

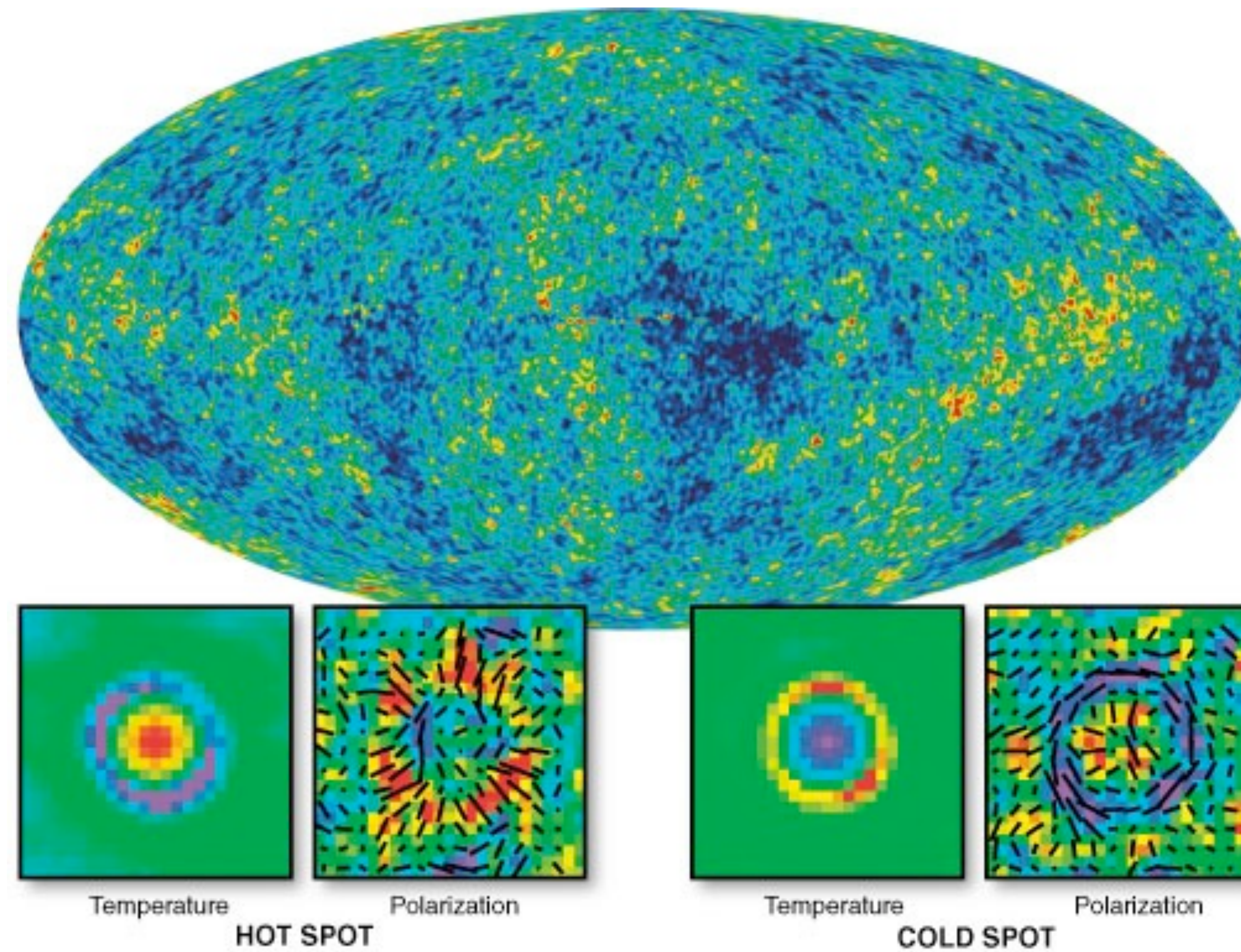


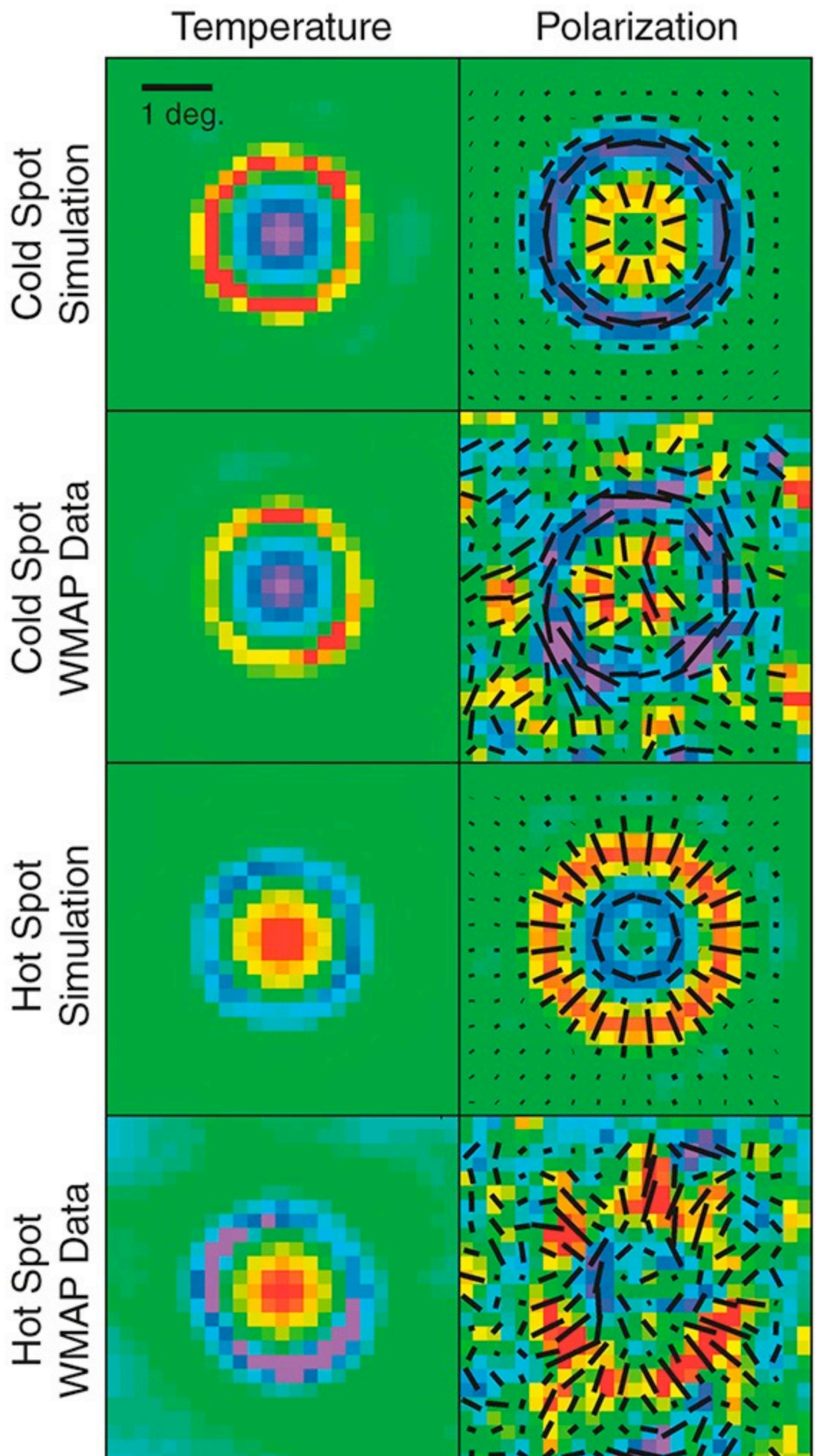
- Q&U decomposition depends on coordinates.
- A rotationally-invariant decomposition: E&B

E-mode Detected (by "stacking")

- Co-add polarization images around temperature hot and cold spots.

- Outside of the Galaxy mask (not shown), there are **12387 hot spots** and **12628 cold spots**.

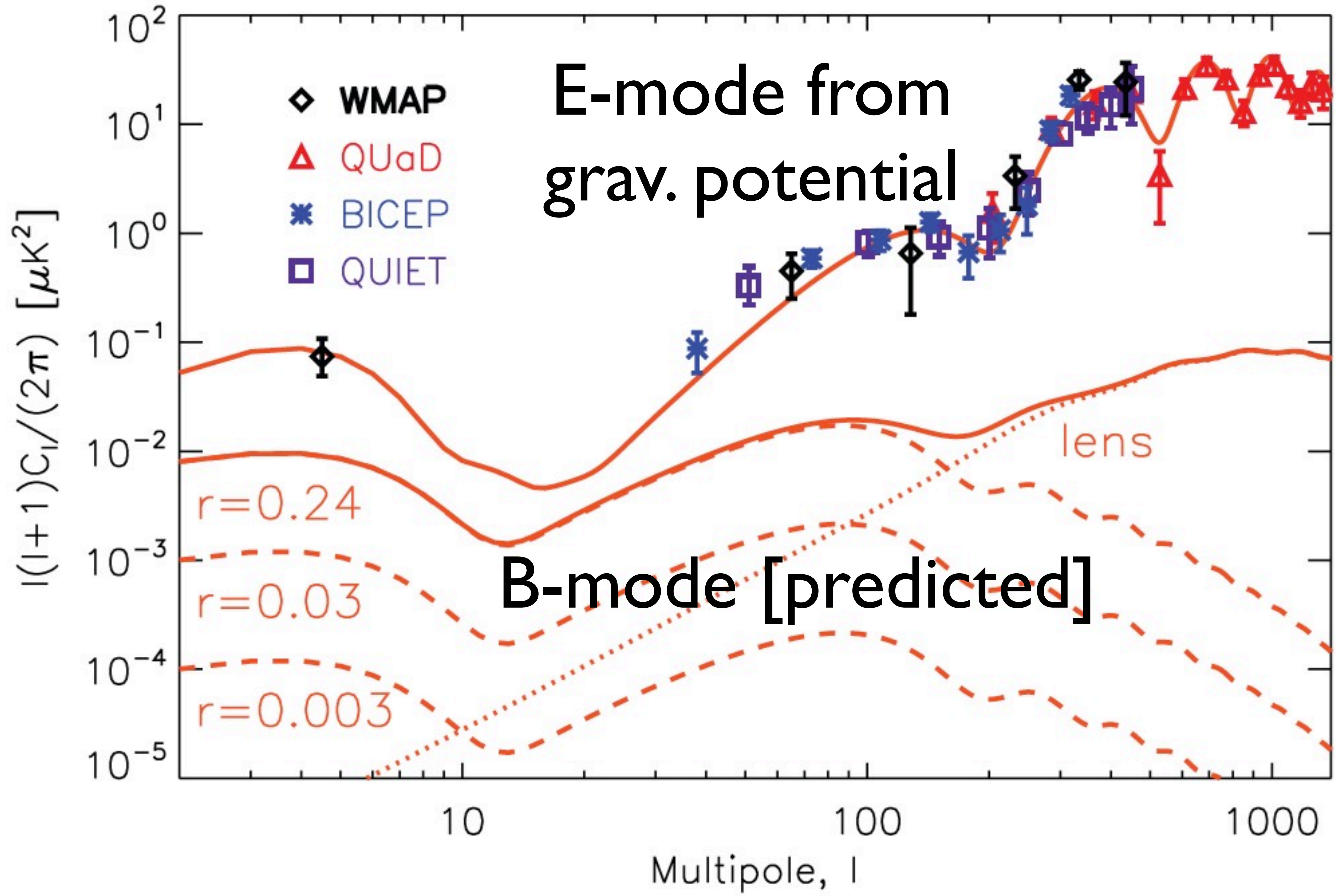




E-mode Detected

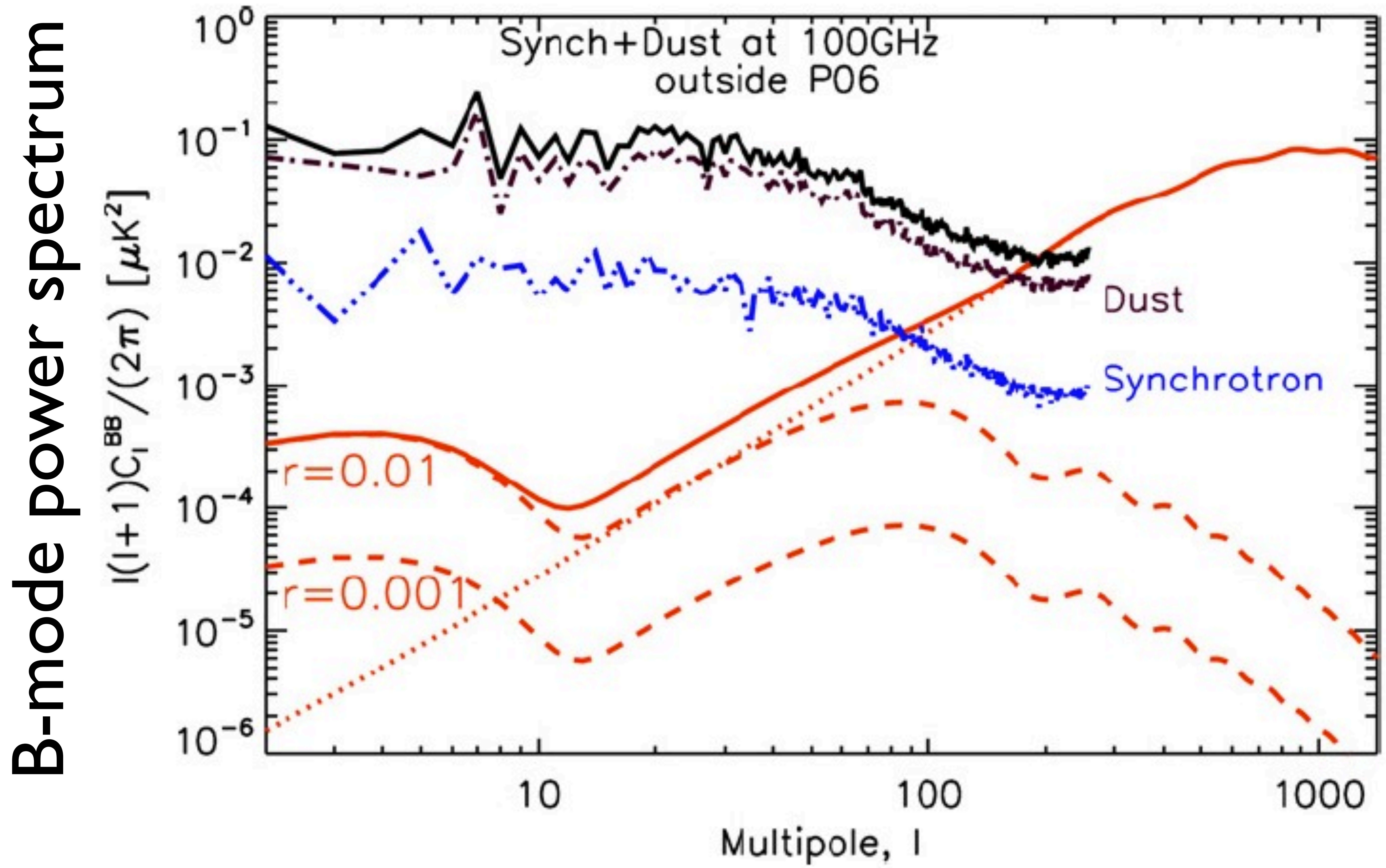
- All hot and cold spots are stacked
- "Compression phase" at θ=1.2 deg and "slow-down phase" at θ=0.6 deg are predicted to be there and we observe them!
  - The overall significance level: 8σ
- Physics: a hot spot corresponds to a potential well, and matter is flowing into it. **Gravitational potential can create only E-mode!**

Polarization Power Spectrum



- Detection of B-modes is the next holy grail in cosmology!

It's not going to be easy



- Even in the science channel (100GHz), foreground is a few orders of magnitude bigger in power at l<~30

Gauss will help you

- Don't be scared too much: the power spectrum captures only a fraction of information.
- Yes, CMB is very close to a Gaussian distribution. But, foreground is highly non-Gaussian.
- CMB scientist's best friend is this equation:

$$\boxed{-2\ln L = ([\text{data}]_i - [\text{stuff}]_i)^T (C^{-1})_{ij} ([\text{data}]_j - [\text{stuff}]_j)}$$

where "C_{ij}" describes the two-point correlation of CMB and noise

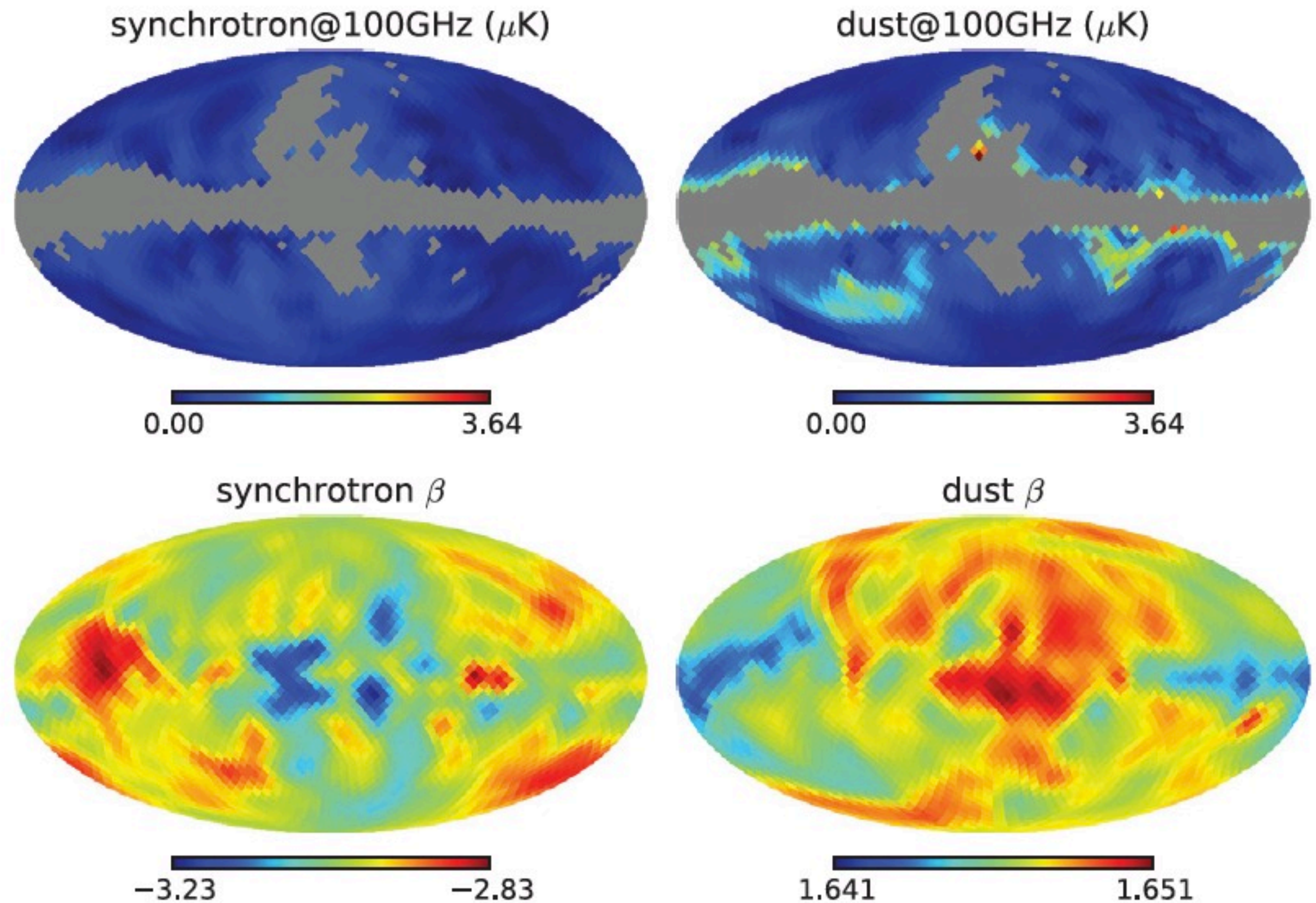
WMAP's Simple Approach

$$[\text{data}]= [Q', U'](\nu) = \frac{[Q, U](\nu) - \alpha_S(\nu)[Q, U](\nu = 23\,\text{GHz})}{1 - \alpha_S(\nu)}$$

- Use the 23 GHz map as a tracer of synchrotron.
- Fit the 23 GHz map to a map at another frequency (with a single amplitude α_S), and subtract.
- After correcting for "CMB bias," this method removes foreground completely, provided that:
  - Spectral index ("β" of $T_\nu \sim \nu^\beta$; e.g., $\beta \sim -3$ for synchrotron) does not vary across the sky.

Limitation of the simplest approach

Planck Sky Model (ver 1.6.2)



- The index β <u>does</u> vary at lot for synchrotron!
- We don’t really know what β does for dust (just yet)

Nevertheless...

- Let’s try and see how far we can go with the simplest approach. The biggest limitation of this method is a position-dependent index.

- And, obvious improvements are possible anyway:
  - Fit multiple coefficients to different locations in the sky
  - Use more frequencies to constrain the index

We describe the data (=CMB+noise+PSMv1.6.2) by

- Amplitude of the B-mode polarization: **r** [this is what we want to measure at the level of $r \sim 10^{-3}$]
- Amplitude of the E-mode polarization from gravitational potential: **s** [which we wish to marginalize over]
- Amplitude of synchrotron: α_{Synch} [which we wish to marginalize over]
- Amplitude of dust: α_{Dust} [which we wish to marginalize over]

We shall maximize the following likelihood function for estimating r, s, and α_i:

$$\mathcal{L}(r, s, \alpha_i) \propto \frac{\exp\left[-\frac{1}{2}\boldsymbol{x}'(\alpha_i)^T \boldsymbol{C}^{-1}(r, s, \alpha_i)\boldsymbol{x}'(\alpha_i)\right]}{\sqrt{|\boldsymbol{C}(r, s, \alpha_i)|}}, \tag{9}$$

where

$$\boldsymbol{x}' = \frac{[Q, U](\nu) - \sum_i \alpha_i(\nu)[Q, U]\left(\nu_i^{\mathrm{template}}\right)}{1 - \sum_i \alpha_i(\nu)} \tag{10}$$

is a template-cleaned map. This is a generalization of Equation (6) for a multi-component case. In this paper, i takes on "S" and "D" for synchrotron and dust, respectively, unless noted otherwise. For definiteness, we shall choose

$$\begin{aligned} \nu &= 100\,\mathrm{GHz}, \\ \nu_{\mathrm{S}}^{\mathrm{template}} &= 60\,\mathrm{GHz}, \\ \nu_{\mathrm{D}}^{\mathrm{template}} &= 240\,\mathrm{GHz}. \end{aligned}$$

$$L \propto \frac{\exp\left[-\frac{1}{2}\boldsymbol{x}'(\alpha_i)^T \boldsymbol{C}^{-1}(r, s, \alpha_i)\boldsymbol{x}'(\alpha_i)\right]}{\sqrt{|\boldsymbol{C}(r, s, \alpha_i)|}}$$

$$\boldsymbol{C}(r, s, \alpha_i) = \underbrace{r\boldsymbol{c}^{\text{tensor}} + s\boldsymbol{c}^{\text{scalar}}}_{\text{signal part}} + \underbrace{\frac{\boldsymbol{N}_1 + \boldsymbol{N}_2}{(1 - \sum_i \alpha_i)^2}}_{\text{noise part}}$$

noise part
(after correcting for CMB bias)

SIGNAL COVARIANCE MATRIX

Given power spectra, c_ℓ^{BB} and c_ℓ^{EE}, the components of the signal covariance matrix for Q and U can be computed analytically. We have

$$\boldsymbol{c}(\hat{\boldsymbol{n}}, \hat{\boldsymbol{n}}') = \begin{pmatrix} c_{QQ}(\hat{\boldsymbol{n}}, \hat{\boldsymbol{n}}') & c_{QU}(\hat{\boldsymbol{n}}, \hat{\boldsymbol{n}}') \\ c_{UQ}(\hat{\boldsymbol{n}}, \hat{\boldsymbol{n}}') & c_{UU}(\hat{\boldsymbol{n}}, \hat{\boldsymbol{n}}') \end{pmatrix},$$

where

$$\begin{aligned}
c_{QQ}(\hat{\boldsymbol{n}}, \hat{\boldsymbol{n}}') &= \sum_l c_l^{EE} w_l^2 \sum_m W_{lm}(\hat{\boldsymbol{n}}) W_{lm}^*(\hat{\boldsymbol{n}}') \\
&\quad + \sum_l c_l^{BB} w_l^2 \sum_m X_{lm}(\hat{\boldsymbol{n}}) X_{lm}^*(\hat{\boldsymbol{n}}') \\
c_{QU}(\hat{\boldsymbol{n}}, \hat{\boldsymbol{n}}') &= \sum_l c_l^{EE} w_l^2 \sum_m [-W_{lm}(\hat{\boldsymbol{n}}) X_{lm}^*(\hat{\boldsymbol{n}}')] \\
&\quad + \sum_l c_l^{BB} w_l^2 \sum_m X_{lm}(\hat{\boldsymbol{n}}) W_{lm}^*(\hat{\boldsymbol{n}}') \\
c_{UQ}(\hat{\boldsymbol{n}}, \hat{\boldsymbol{n}}') &= \sum_l c_l^{EE} w_l^2 \sum_m [-X_{lm}(\hat{\boldsymbol{n}}) W_{lm}^*(\hat{\boldsymbol{n}}')] \\
&\quad + \sum_l c_l^{BB} w_l^2 \sum_m W_{lm}(\hat{\boldsymbol{n}}) X_{lm}^*(\hat{\boldsymbol{n}}') \\
c_{UU}(\hat{\boldsymbol{n}}, \hat{\boldsymbol{n}}') &= \sum_l c_l^{EE} w_l^2 \sum_m X_{lm}(\hat{\boldsymbol{n}}) X_{lm}^*(\hat{\boldsymbol{n}}') \\
&\quad + \sum_l c_l^{BB} w_l^2 \sum_m W_{lm}(\hat{\boldsymbol{n}}) W_{lm}^*(\hat{\boldsymbol{n}}')
\end{aligned}$$

and

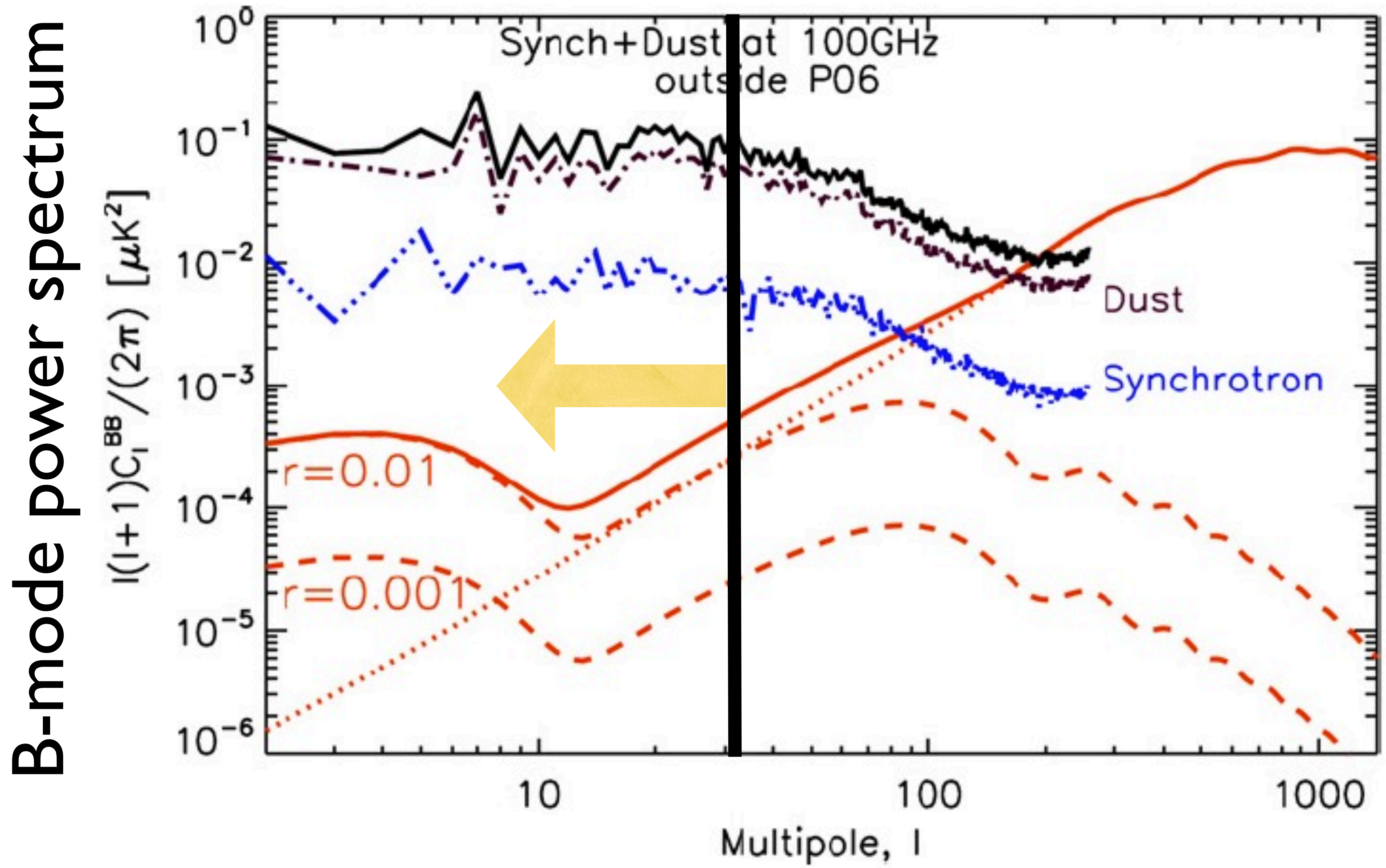
$$\begin{aligned}
W_{lm}(\hat{\boldsymbol{n}}) &\equiv (-1)[{}_2Y_{lm}(\hat{\boldsymbol{n}}) + {}_{-2}Y_{lm}(\hat{\boldsymbol{n}})]/2, \\
X_{lm}(\hat{\boldsymbol{n}}) &\equiv (-i)[{}_2Y_{lm}(\hat{\boldsymbol{n}}) - {}_{-2}Y_{lm}(\hat{\boldsymbol{n}})]/2.
\end{aligned}$$

Here goes O(N^3)

$$\mathcal{L}(r, s, \alpha_i) \propto \frac{\exp\left[-\frac{1}{2}\boldsymbol{x}'(\alpha_i)^T \boldsymbol{C}^{-1}(r, s, \alpha_i)\boldsymbol{x}'(\alpha_i)\right]}{\sqrt{|\boldsymbol{C}(r, s, \alpha_i)|}}$$

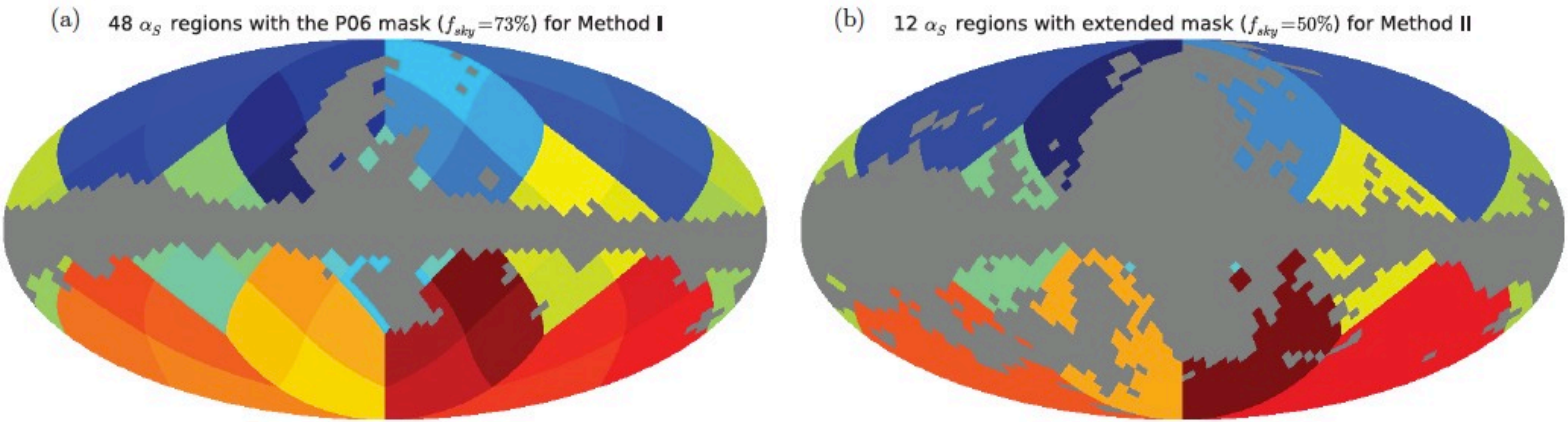
- A numerical challenge: for each set of r, s, α_{Synch} and α_{Dust}, we need to invert the covariance matrix.
- For this study, we use low-resolution Q&U maps with 3072 pixels per map (giving a 6144x6144 matrix).

We target the low-l bump



- This is a semi-realistic configuration for a future satellite mission targeting the B-modes from inflation.

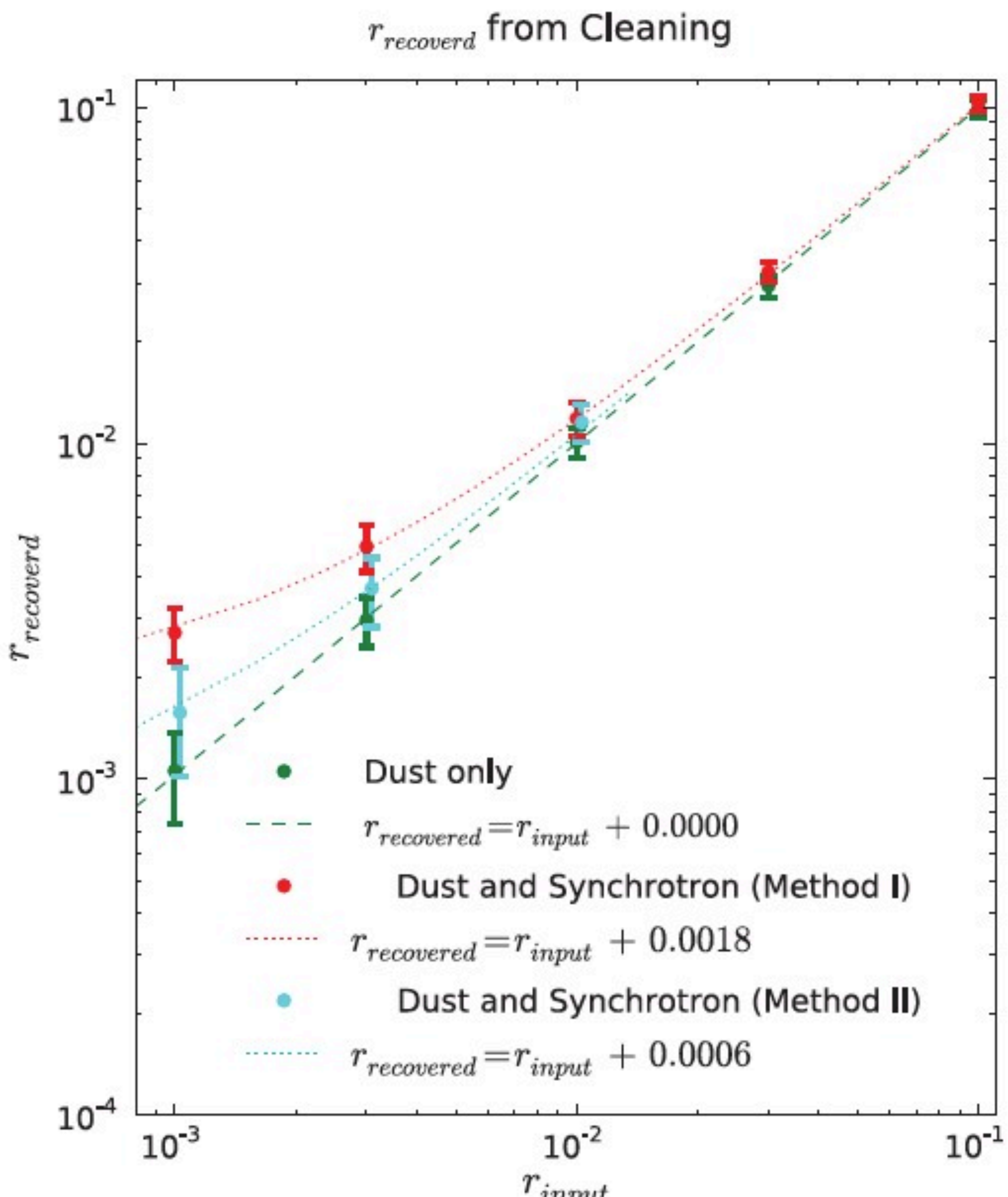
Two Masks and Choice of Regions for Synch Index



"Method I" "Method II"

Katayama & Komatsu, ApJ, 737, 78 (2011)

Results (3 frequency bands: 60, 100, 240 GHz)



- It works quite well!
- For dust-only case (for which the index does not vary much): we observe ***no bias*** in the B-mode amplitude, as expected.
- For Method I (synch+dust), the bias is $\Delta r = 2\times10^{-3}$
- For Method II (synch+dust), the bias is $\Delta r = 0.6\times10^{-3}$

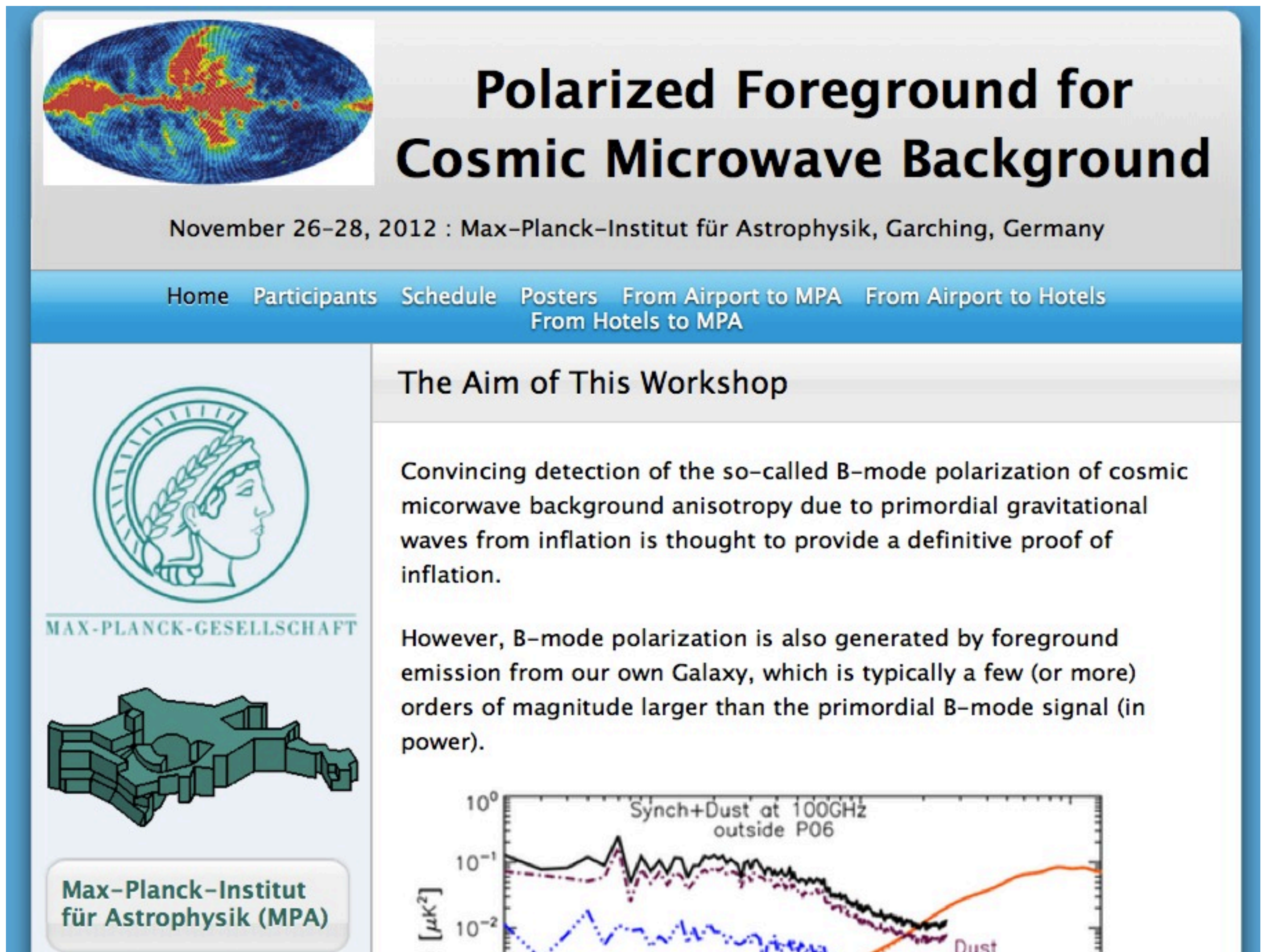
Conclusion

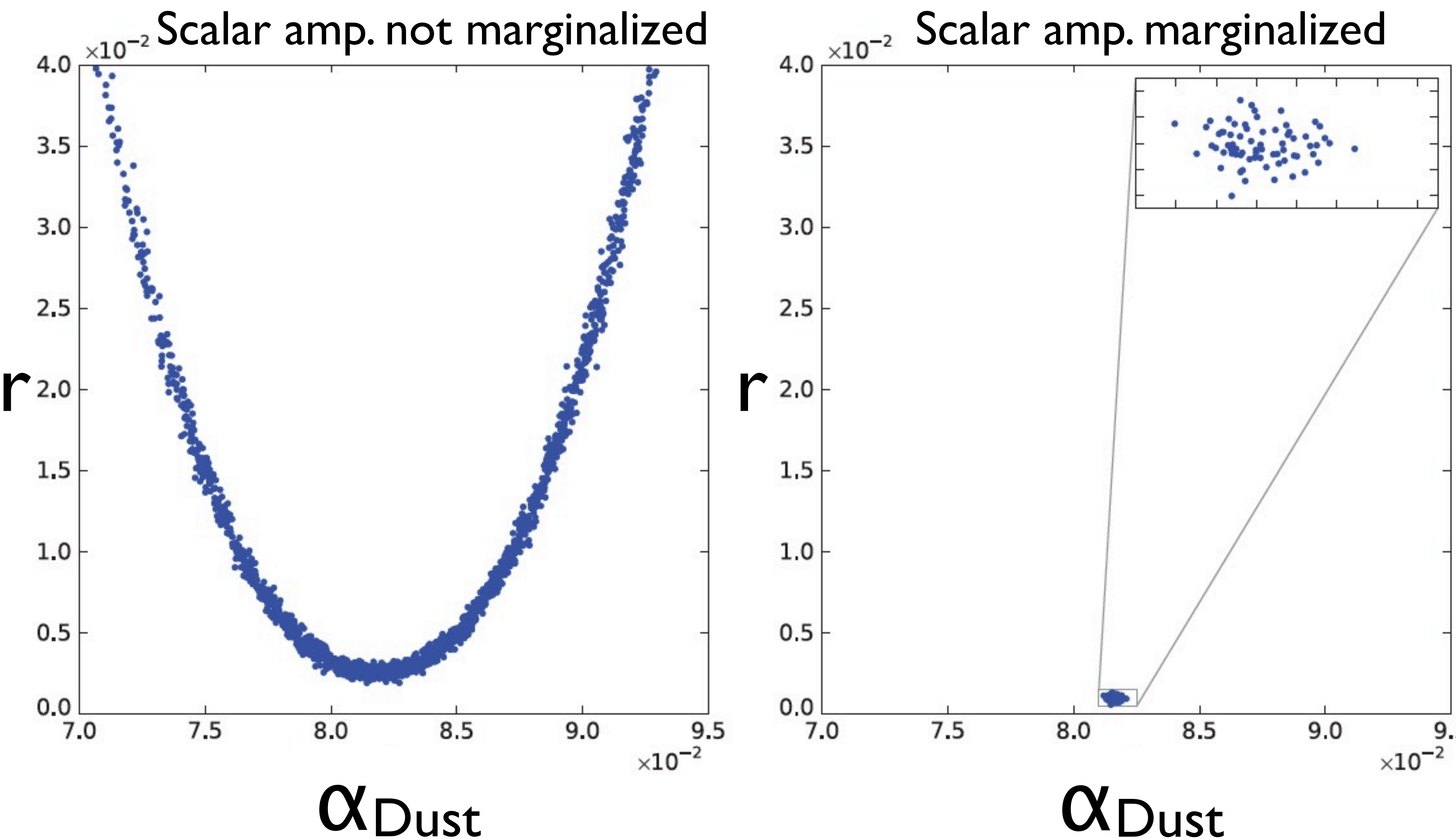
- The simplest approach is already quite promising
  - *Using just 3 frequencies gets the bias down to $\Delta r < 10^{-3}$*
- The bias is totally dominated by the spatial variation of the synchrotron index
  - How to improve further? We can use 4 frequencies: two frequencies for synchrotron to constrain the index
  - The biggest worry: we do not know much about the dust index variation (yet; until March 2013). Perhaps we should have two frequencies for the dust index as well
- **The minimum number of frequencies = 5**

Really? Is it really that easy?

- Let's discuss that in Munich from **November 26–28**:

http://www.mpa-garching.mpg.de/~komatsu/meetings/fg2012/





Scalar amp. not marginalized
Scalar amp. marginalized
r
$\times 10^{-2}$
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
7.0
7.5
8.0
8.5
9.0
9.5
$\times 10^{-2}$
α_{Dust}