

# ***Non-Gaussianity*** as a Probe of the Physics of the Primordial Universe

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Cook's Branch, April 15, 2010



# Motivation

- Non-Gaussianity (3- and 4-point functions of fluctuations) can be used to rule out **(almost) all** inflation models!
- That's the slide#42. Please stay awake...

# How Do We Test Inflation?

- How can we answer a simple question like this:
  - “*How were primordial fluctuations generated?*”

# Power Spectrum

- A very successful explanation (Mukhanov & Chibisov; Guth & Pi; Hawking; Starobinsky; Bardeen, Steinhardt & Turner) is:
- *Primordial fluctuations were generated by quantum fluctuations of the scalar field that drove inflation.*
- The prediction: a nearly scale-invariant power spectrum in the curvature perturbation,  $\zeta$ :
  - **$P_{\zeta}(\mathbf{k}) = A/k^{4-n_s} \sim A/k^3$**
  - where  $n_s \sim 1$  and  $A$  is a normalization.

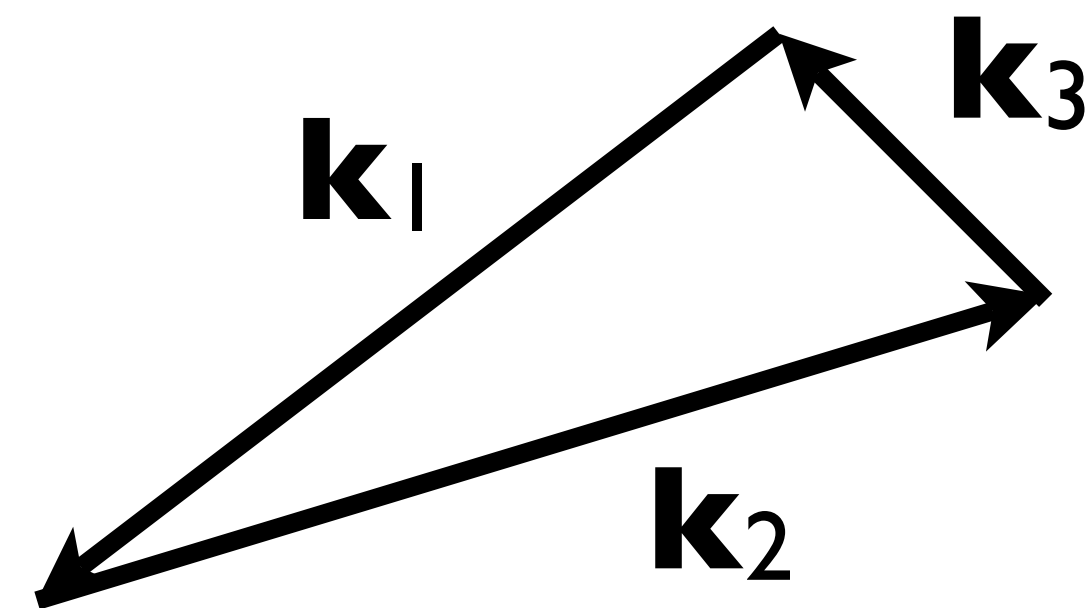
# $n_s < 1$ Observed (at $>3\sigma$ )

- The latest results from the WMAP 7-year data:
  - **$n_s = 0.963 \pm 0.012$**  (68%CL; for tensor modes = zero)
- $n_s \neq 1$ : another line of evidence for inflation
- Detection of non-zero tensor modes is a next important step

# Beyond Power Spectrum

- These are based upon fitting the observed power spectrum (of scalar and tensor perturbations).
- Is there any more information one can obtain, beyond the power spectrum?

# Bispectrum



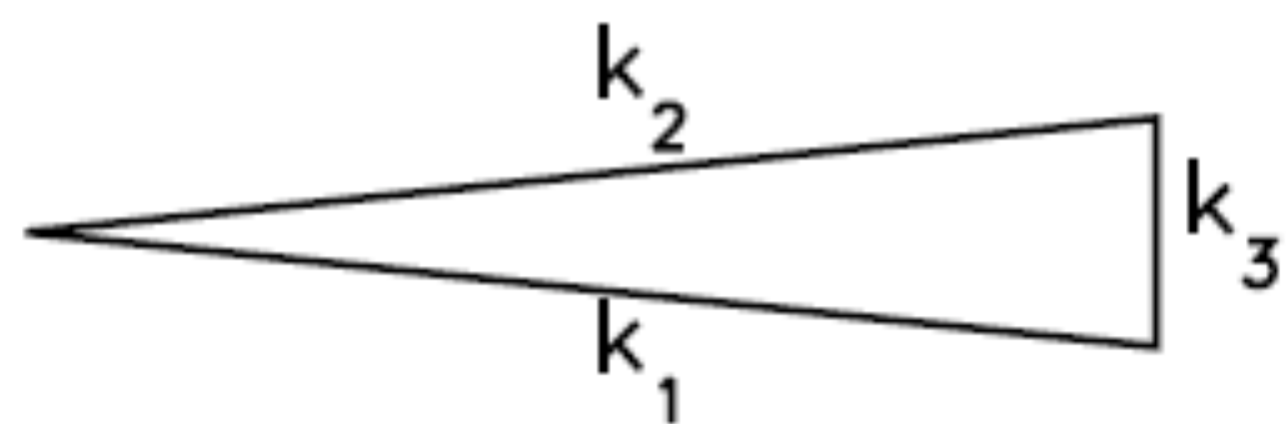
- Three-point function!

- $B_\zeta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$

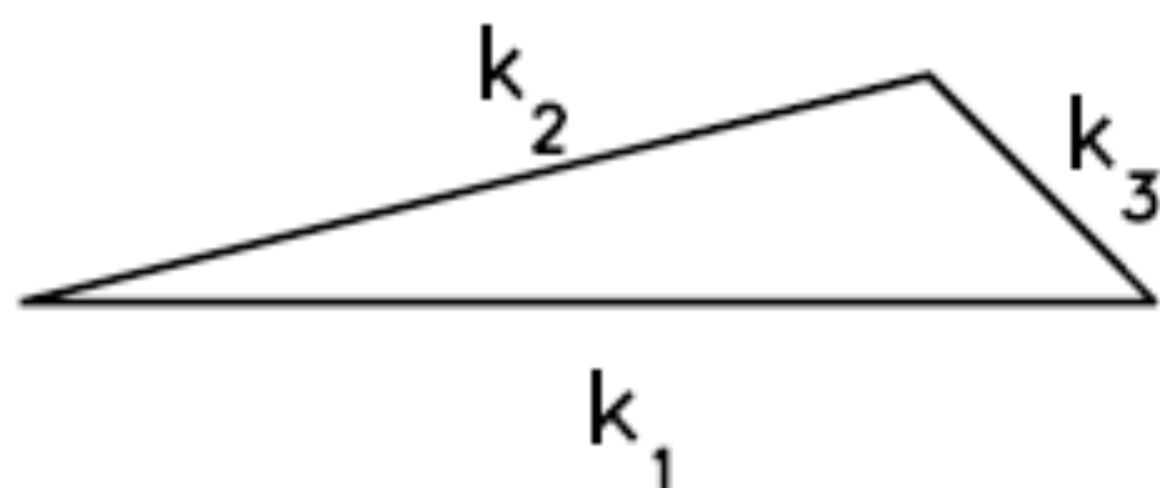
$$= \langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = (\text{amplitude}) \times (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) F(k_1, k_2, k_3)$$

model-dependent function

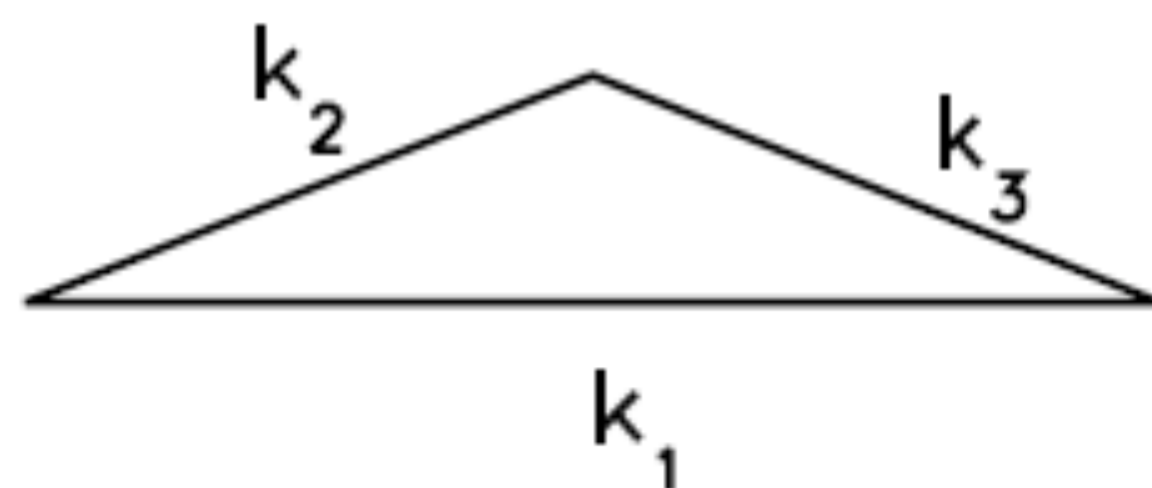
(a) squeezed triangle  
( $k_1 \approx k_2 \gg k_3$ )



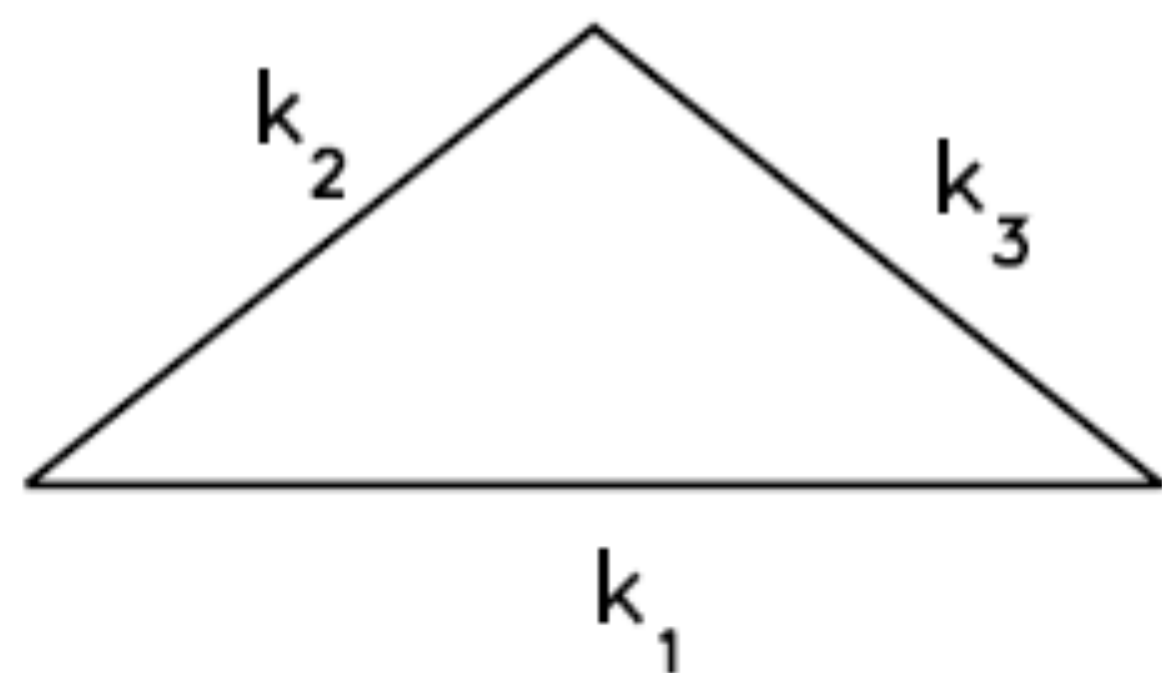
(b) elongated triangle  
( $k_1 = k_2 + k_3$ )



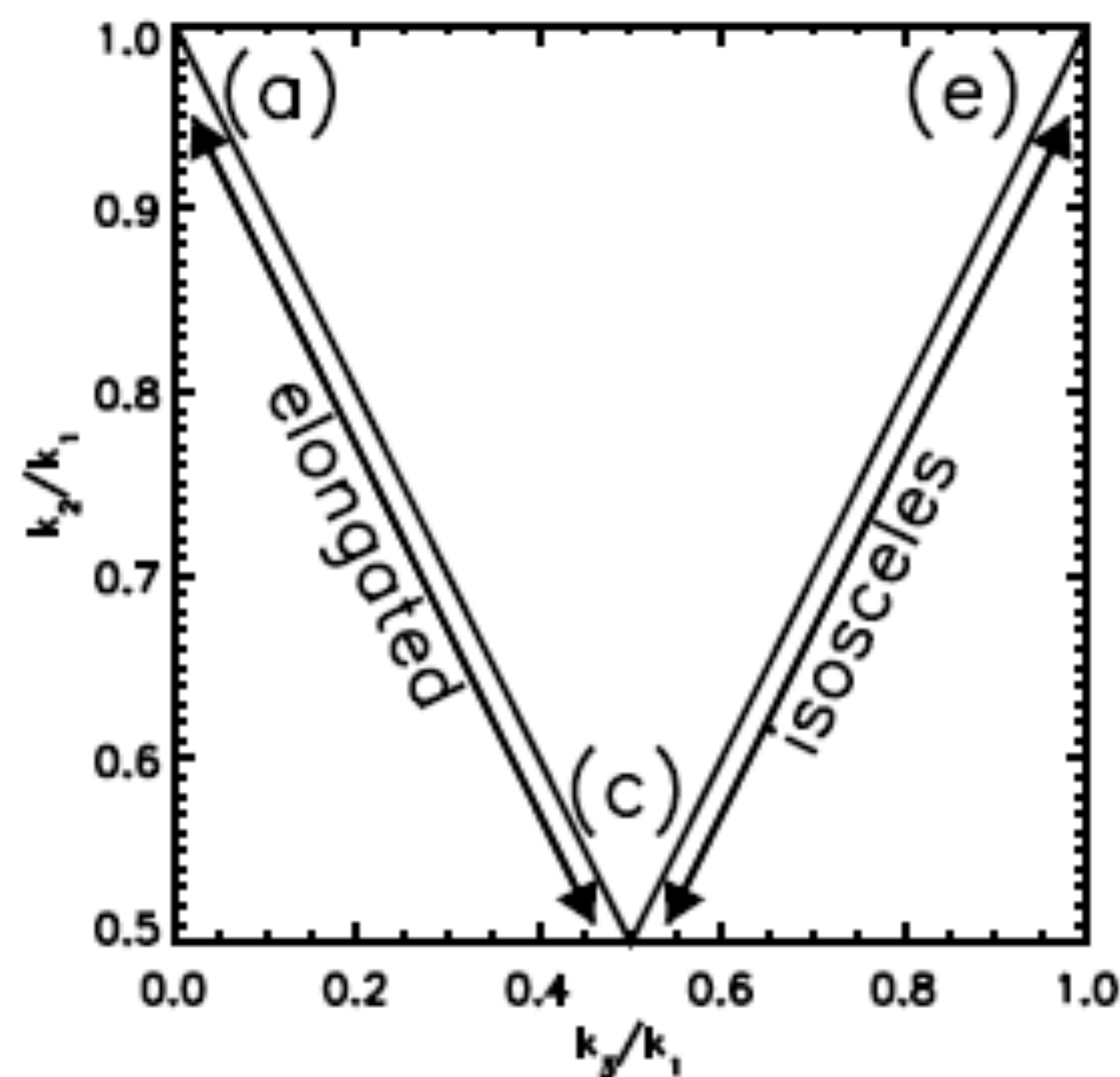
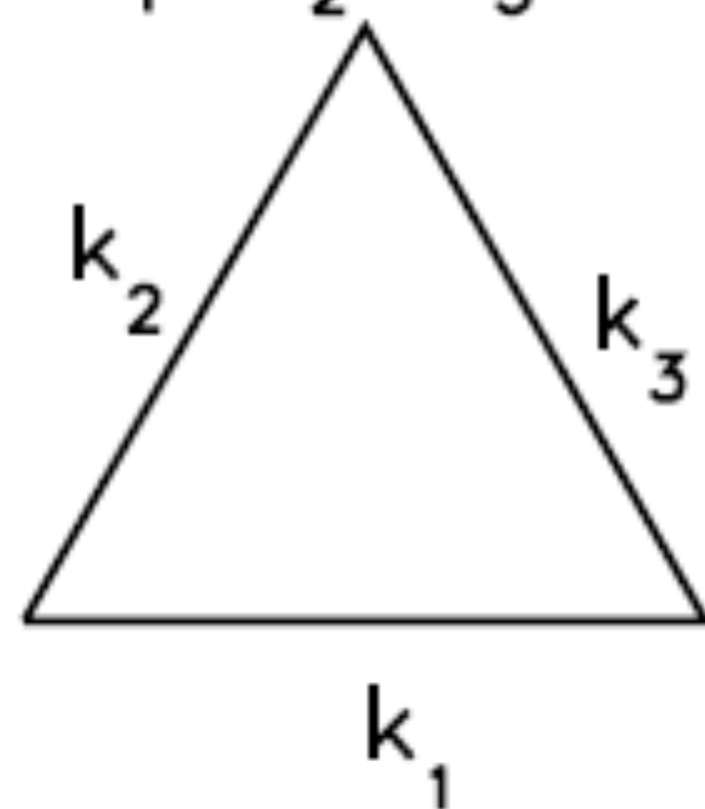
(c) folded triangle  
( $k_1 = 2k_2 = 2k_3$ )



(d) isosceles triangle  
( $k_1 > k_2 = k_3$ )



(e) equilateral triangle  
( $k_1 = k_2 = k_3$ )



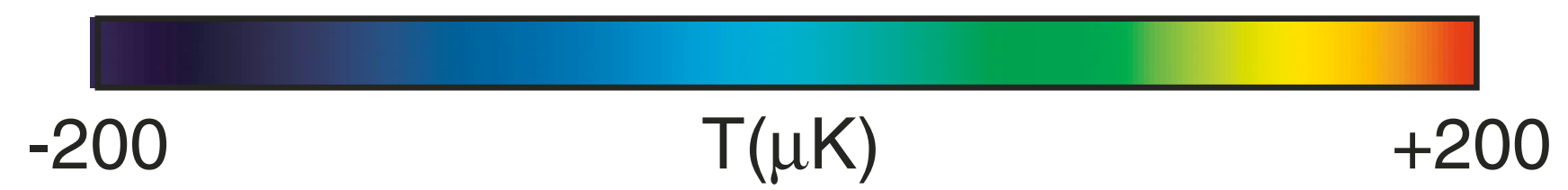
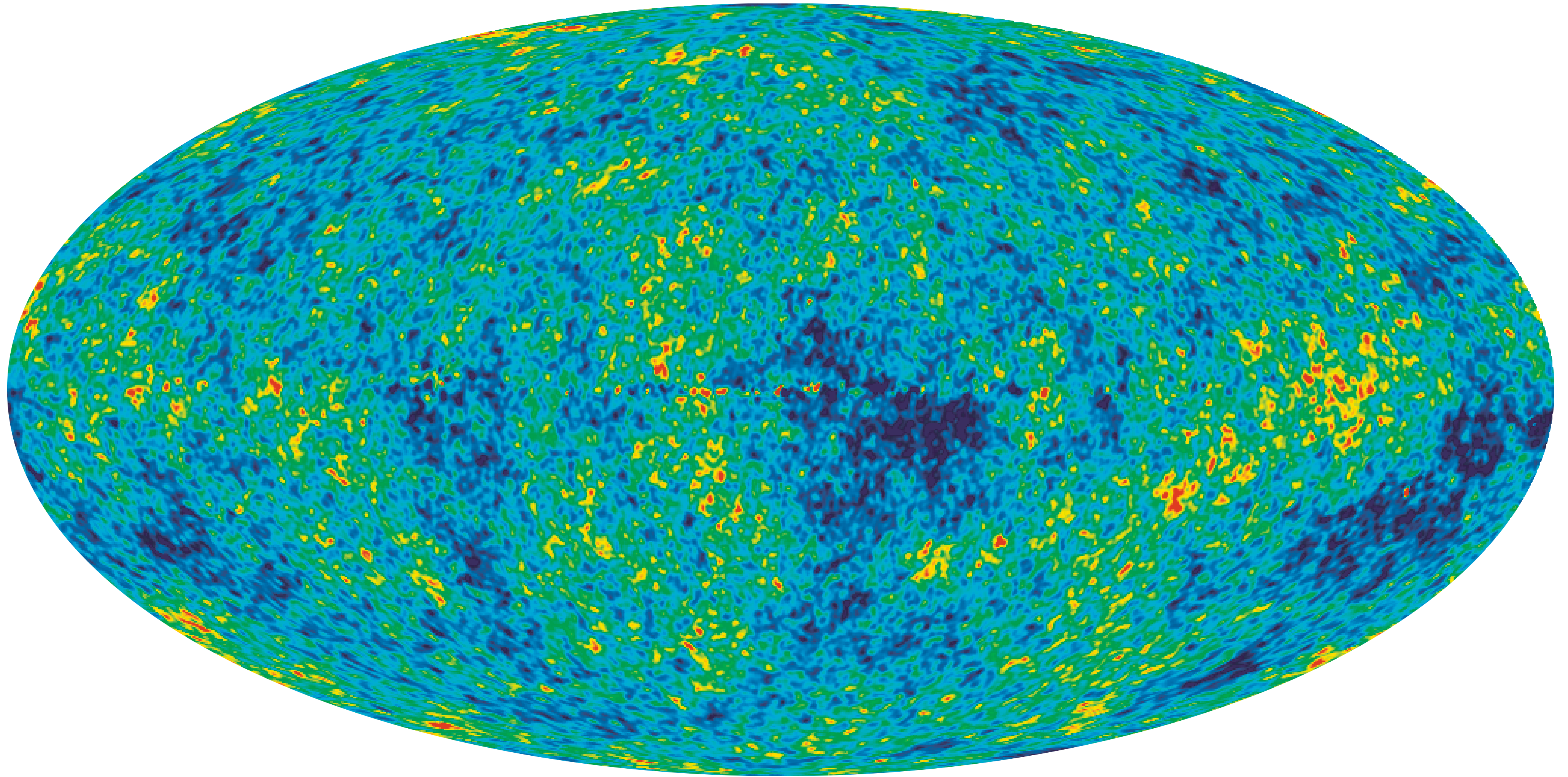


# Why Study Bispectrum?

- It probes the interactions of fields - new piece of information that cannot be probed by the power spectrum
- But, above all, it provides us with a **critical test** of the simplest models of inflation: “***are primordial fluctuations Gaussian, or non-Gaussian?***”
- Bispectrum vanishes for Gaussian fluctuations.
- Detection of the bispectrum = detection of non-Gaussian fluctuations

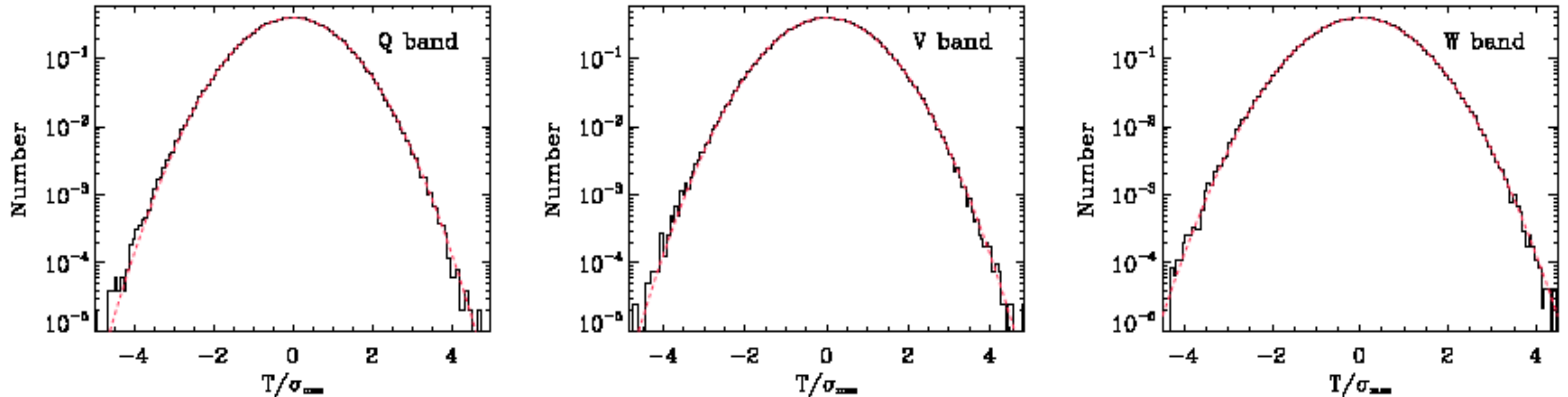
# Gaussian?

WMAP5



WMAP 5-year

# Take One-point Distribution Function



- The one-point distribution of WMAP map looks pretty Gaussian.
  - Left to right: Q (41GHz), V (61GHz), W (94GHz).
- Deviation from Gaussianity is small, if any.

# Inflation Likes This Result

- According to inflation (Mukhanov & Chibisov; Guth & Yi; Hawking; Starobinsky; Bardeen, Steinhardt & Turner), CMB anisotropy was created from **quantum fluctuations of a scalar field in Bunch-Davies vacuum** during inflation
- Successful inflation (with the expansion factor more than  $e^{60}$ ) *demands* the scalar field be almost interaction-free
- The wave function of free fields in the ground state is a Gaussian!

# But, Not Exactly Gaussian

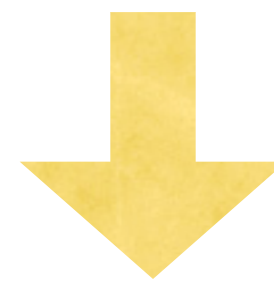
- Of course, there are always corrections to the simplest statement like this.
- For one, inflaton field **does** have interactions. They are simply weak – they are suppressed by the so-called slow-roll parameter,  $\epsilon \sim \mathcal{O}(0.01)$ , relative to the free-field action.

# A Non-linear Correction to Temperature Anisotropy

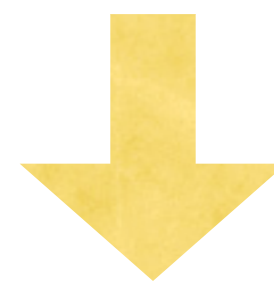
- The CMB temperature anisotropy,  $\Delta T/T$ , is given by the curvature perturbation in the matter-dominated era,  $\Phi$ .
- On large scales (the Sachs-Wolfe limit),  $\Delta T/T = -\Phi/3$ .  
For the Schwarzschild metric,  $\Phi = +GM/R$ .
- Add a non-linear correction to  $\Phi$ :
  - $\Phi(\mathbf{x}) = \Phi_g(\mathbf{x}) + f_{\text{NL}}[\Phi_g(\mathbf{x})]^2$  (Komatsu & Spergel 2001)
  - $f_{\text{NL}}$  was predicted to be small ( $\sim 0.01$ ) for slow-roll models (Salopek & Bond 1990; Gangui et al. 1994)

# $f_{\text{NL}}$ : Form of $B_{\zeta}$

- $\Phi$  is related to the primordial curvature perturbation,  $\zeta$ , as  $\Phi = (3/5)\zeta$ .



- $\zeta(\mathbf{x}) = \zeta_g(\mathbf{x}) + (3/5)f_{\text{NL}}[\zeta_g(\mathbf{x})]^2$



- $B_{\zeta}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = (6/5)f_{\text{NL}} \times (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \times [P_{\zeta}(k_1)P_{\zeta}(k_2) + P_{\zeta}(k_2)P_{\zeta}(k_3) + P_{\zeta}(k_3)P_{\zeta}(k_1)]$

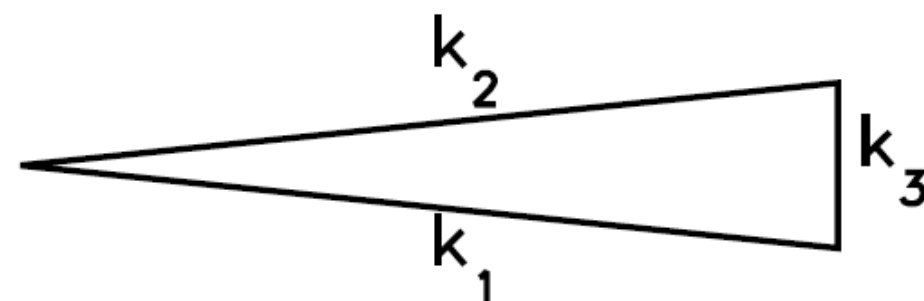
# $f_{NL}$ : Shape of Triangle

- For a scale-invariant spectrum,  $P_\zeta(k)=A/k^3$ ,
  - $B_\zeta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = (6A^2/5)f_{NL} \times (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \times [1/(k_1 k_2)^3 + 1/(k_2 k_3)^3 + 1/(k_3 k_1)^3]$
- Let's order  $k_i$  such that  $k_3 \leq k_2 \leq k_1$ . For a given  $k_1$ , one finds the largest bispectrum when the smallest  $k$ , i.e.,  $k_3$ , is very small.

- $B_\zeta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$  peaks when  $k_3 \ll k_2 \sim k_1$

- Therefore, the shape of  $f_{NL}$  bispectrum is the squeezed triangle!

(Babich et al. 2004)





# $B_\zeta$ in the Squeezed Limit

- In the squeezed limit, the  $f_{\text{NL}}$  bispectrum becomes:

$$B_\zeta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \approx (12/5)f_{\text{NL}} \times (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \times P_\zeta(k_1)P_\zeta(k_3)$$

# Single-field Theorem (Consistency Relation)

- For **ANY** single-field models\*, the bispectrum in the squeezed limit is given by
- $B_{\zeta}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \approx (1-n_s) \times (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \times P_{\zeta}(k_1) P_{\zeta}(k_3)$
- Therefore, all single-field models predict  $f_{NL} \approx (5/12)(1-n_s)$ .
- With the current limit  $n_s=0.963$ ,  $f_{NL}$  is predicted to be 0.015.

\* for which the single field is solely responsible for driving inflation and generating observed fluctuations.

# Understanding the Theorem

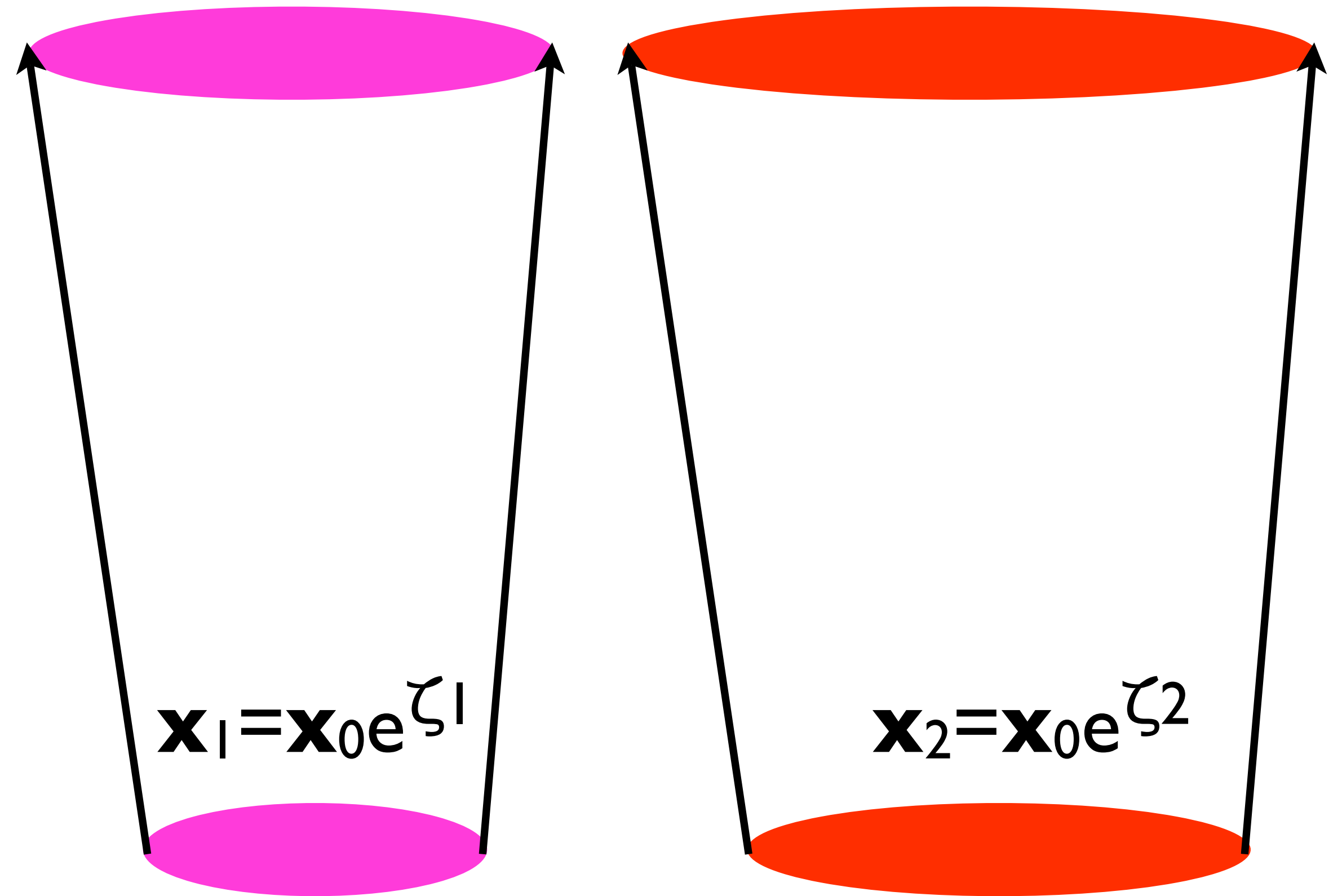
- First, the squeezed triangle correlates one very long-wavelength mode,  $k_L (=k_3)$ , to two shorter wavelength modes,  $k_S (=k_1 \approx k_2)$ :
  - $\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle \approx \langle (\zeta_{k_S})^2 \zeta_{k_L} \rangle$
- Then, the question is: “why should  $(\zeta_{k_S})^2$  ever care about  $\zeta_{k_L}$ ?”
  - The theorem says, “it doesn’t care, if  $\zeta_{k_S}$  is exactly scale invariant.”

# $\zeta_{\mathbf{k}L}$ rescales coordinates

- The long-wavelength curvature perturbation rescales the spatial coordinates (or changes the expansion factor) within a given Hubble patch:

- $ds^2 = -dt^2 + [a(t)]^2 e^{2\zeta} (d\mathbf{x})^2$

Separated by more than  $H^{-1}$

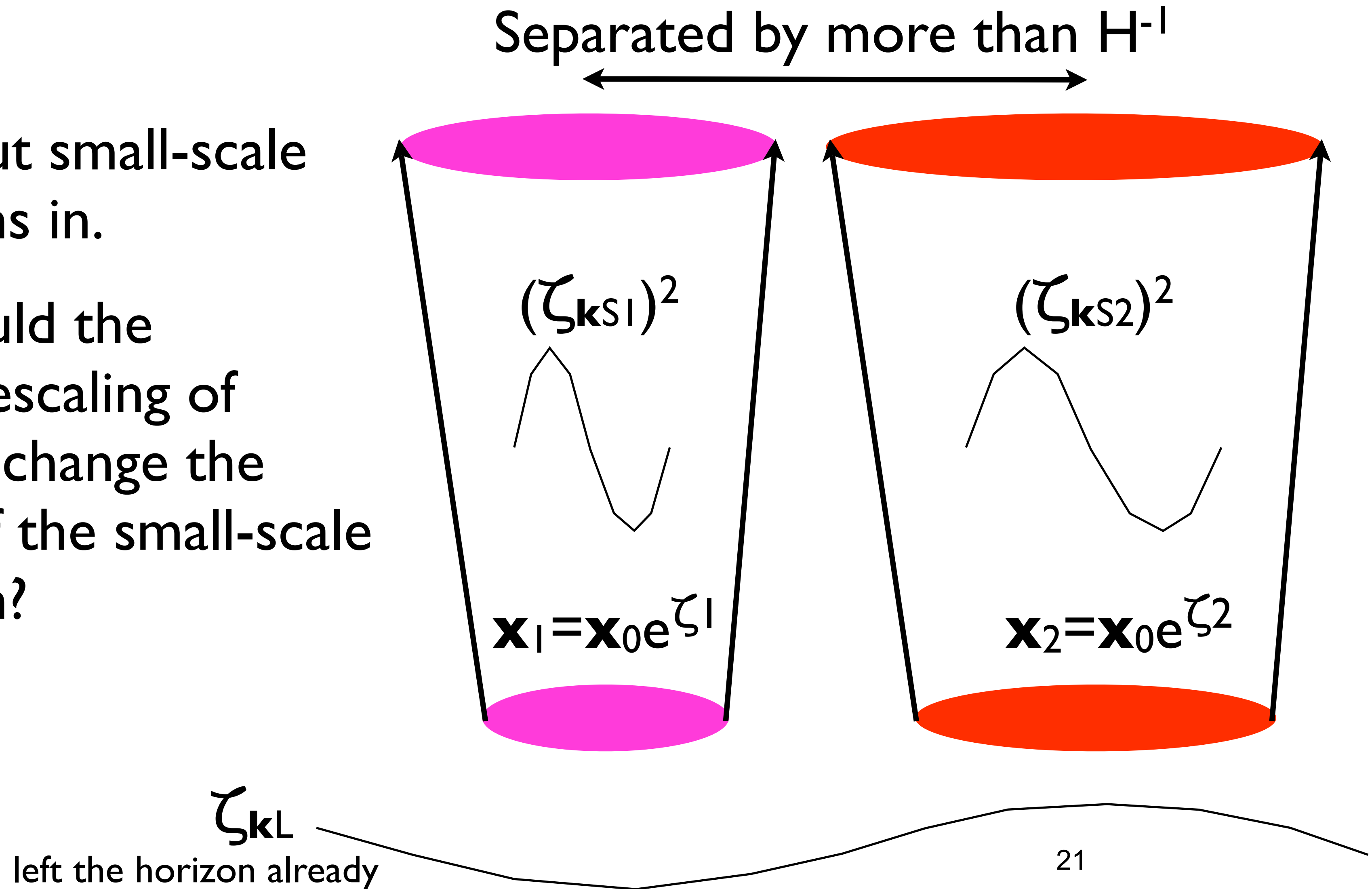


$\zeta_{\mathbf{k}L}$

left the horizon already

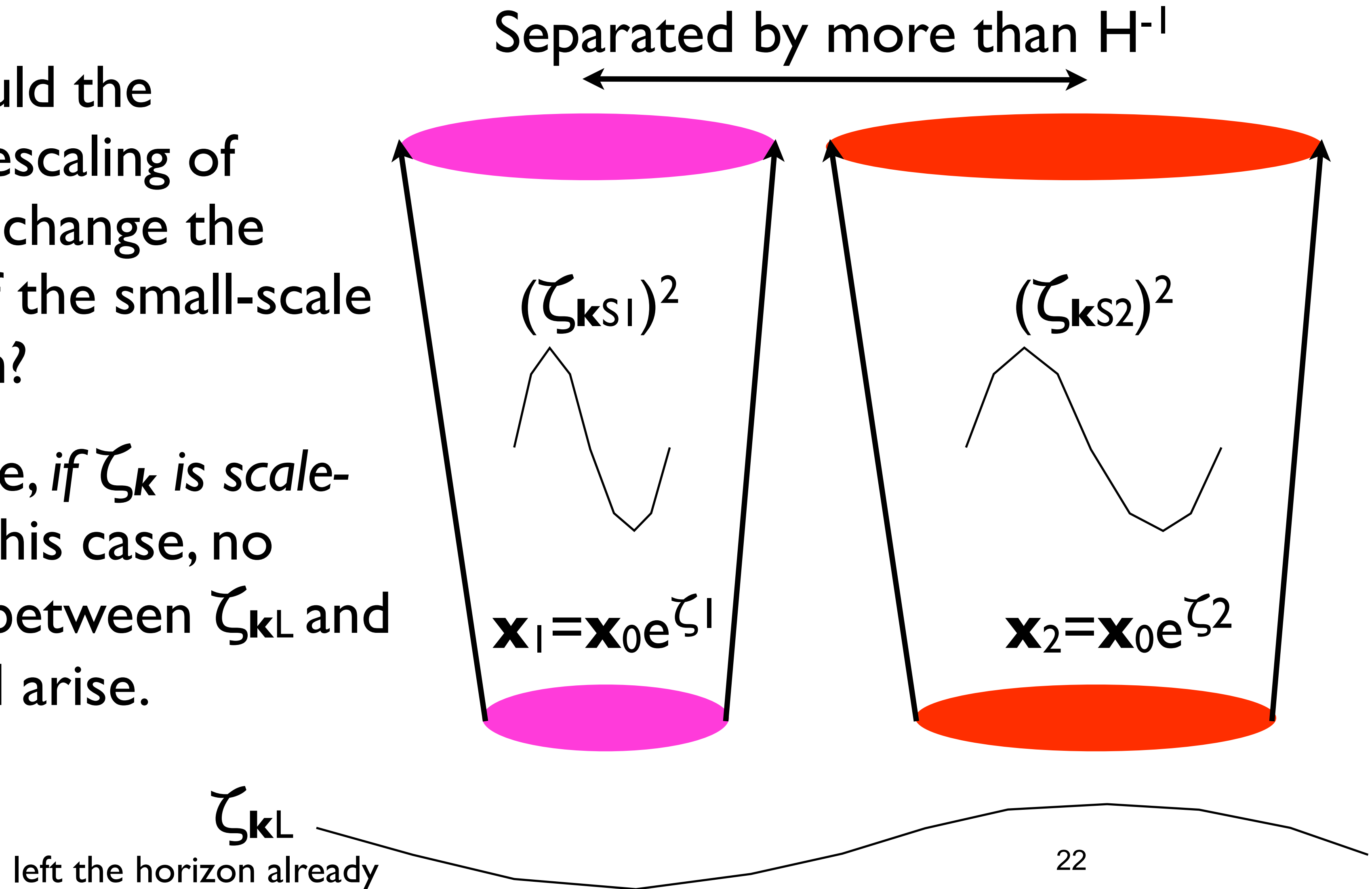
# $\zeta_{kL}$ rescales coordinates

- Now, let's put small-scale perturbations in.
- Q. How would the conformal rescaling of coordinates change the amplitude of the small-scale perturbation?



# $\zeta_{\mathbf{k}L}$ rescales coordinates

- Q. How would the conformal rescaling of coordinates change the amplitude of the small-scale perturbation?
- A. No change, if  $\zeta_{\mathbf{k}}$  is scale-invariant. In this case, no correlation between  $\zeta_{\mathbf{k}L}$  and  $(\zeta_{\mathbf{k}S})^2$  would arise.



# Real-space Proof

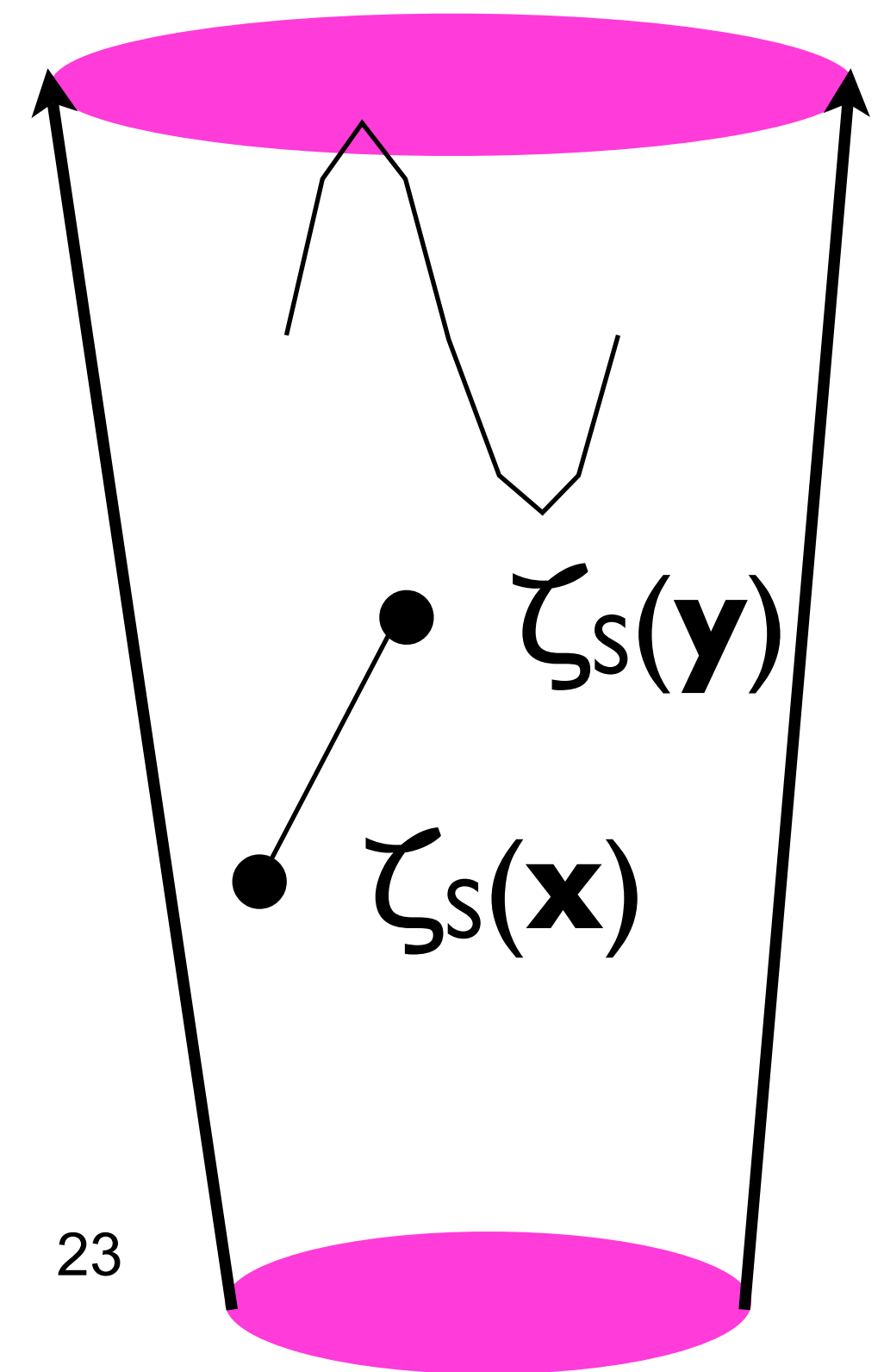
- The 2-point correlation function of short-wavelength modes,  $\xi = \langle \zeta_s(\mathbf{x}) \zeta_s(\mathbf{y}) \rangle$ , within a given Hubble patch can be written in terms of its vacuum expectation value (in the absence of  $\zeta_L$ ),  $\xi_0$ , as:

- $\xi_{\zeta_L} \approx \xi_0(|\mathbf{x}-\mathbf{y}|) + \zeta_L [d\xi_0(|\mathbf{x}-\mathbf{y}|)/d\zeta_L]$

- $\xi_{\zeta_L} \approx \xi_0(|\mathbf{x}-\mathbf{y}|) + \zeta_L [d\xi_0(|\mathbf{x}-\mathbf{y}|)/d\ln|\mathbf{x}-\mathbf{y}|]$

- $\xi_{\zeta_L} \approx \xi_0(|\mathbf{x}-\mathbf{y}|) + \zeta_L (1-n_s)\xi_0(|\mathbf{x}-\mathbf{y}|)$

$$\begin{aligned} \text{3-pt func.} &= \langle (\zeta_s)^2 \zeta_L \rangle = \langle \xi_{\zeta_L} \zeta_L \rangle \\ &= (1-n_s) \xi_0(|\mathbf{x}-\mathbf{y}|) \langle \zeta_L^2 \rangle \end{aligned}$$



# Where was “Single-field”?

- Where did we assume “single-field” in the proof?
- For this proof to work, it is crucial that there is only one dynamical degree of freedom, i.e., it is only  $\zeta_L$  that modifies the amplitude of short-wavelength modes, and nothing else can modify it.
- Also,  $\zeta$  must be constant outside of the horizon (otherwise anything can happen afterwards). This is also the case for single-field inflation models.



# Therefore...

- A convincing detection of  $f_{\text{NL}} > 1$  would rule out ***all*** of the single-field inflation models, regardless of:
  - the form of potential
  - the form of kinetic term (or sound speed)
  - the initial vacuum state
- A convincing detection of  $f_{\text{NL}}$  would be a breakthrough.

Side  
Note:

# Large Non-Gaussianity from Single-field Inflation

But not in the squeezed limit

- $S = (1/2) \int d^4x \sqrt{-g} [R - (\partial_\mu \varphi)^2 - 2V(\varphi)]$
- 2nd-order (which gives  $P_\zeta$ )
  - $S_2 = \int d^4x \epsilon [a^3 (\partial_t \zeta)^2 - a (\partial_i \zeta)^2]$
- 3rd-order (which gives  $B_\zeta$ )
  - $S_3 = \int d^4x \epsilon^2 [\dots a^3 (\partial_t \zeta)^2 \zeta + \dots a (\partial_i \zeta)^2 \zeta + \dots a^3 (\partial_t \zeta)^3] + O(\epsilon^3)$

Cubic-order interactions are suppressed by an additional factor of  $\epsilon$ .  
(Maldacena 2003)

Side  
Note:

# Large Non-Gaussianity from Single-field Inflation

But not in the squeezed limit

- $S = (1/2) \int d^4x \sqrt{-g} \{R - 2P[(\partial_\mu \varphi)^2, \varphi]\}$  [general kinetic term]
- 2nd-order
  - $S_2 = \int d^4x \varepsilon [a^3 (\partial_t \zeta)^2 / c_s^2 - a (\partial_i \zeta)^2]$ 

“Speed of sound”  
 $c_s^2 = P_{,x} / (P_{,x} + 2XP_{,xx})$
- 3rd-order
  - $S_3 = \int d^4x \varepsilon^2 [\dots a^3 (\partial_t \zeta)^2 \zeta / c_s^2 + \dots a (\partial_i \zeta)^2 \zeta + \dots a^3 (\partial_t \zeta)^3 / c_s^2] + O(\varepsilon^3)$

**Some interactions are enhanced for  $c_s^2 < 1$ .**

(Seery & Lidsey 2005; Chen et al. 2007)

Side  
Note:

# Large Non-Gaussianity from Single-field Inflation

But not in the squeezed limit

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- 2nd-order

- $S_2 = \int d^4x \epsilon [a^3 (\partial_t \zeta)^2 / c_s^2 - a (\partial_i \zeta)^2]$

“Speed of sound”  
 $c_s^2 = P_{,X} / (P_{,X} + 2XP_{,XX})$

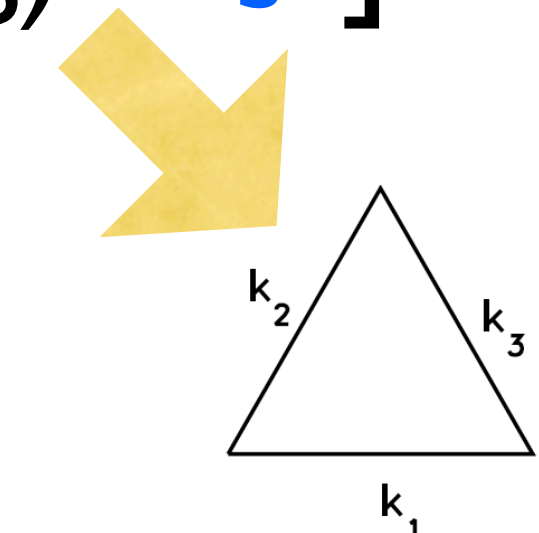
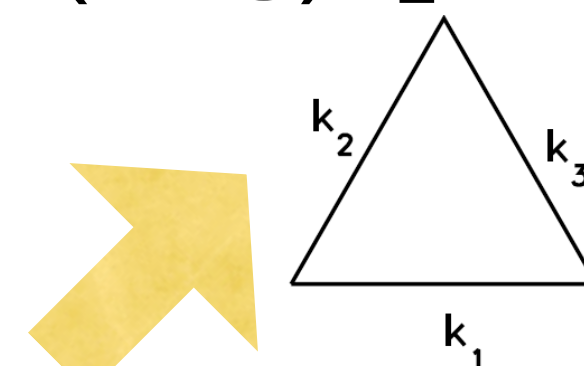
- 3rd-order

- $S_3 = \int d^4x \epsilon^2 [\dots a^3 (\partial_t \zeta)^2 \zeta / c_s^2 + \dots a (\partial_i \zeta)^2 \zeta + \dots a^3 (\partial_t \zeta)^3 / c_s^2] + O(\epsilon^3)$

**Some interactions are enhanced for  $c_s^2 < 1$ .**

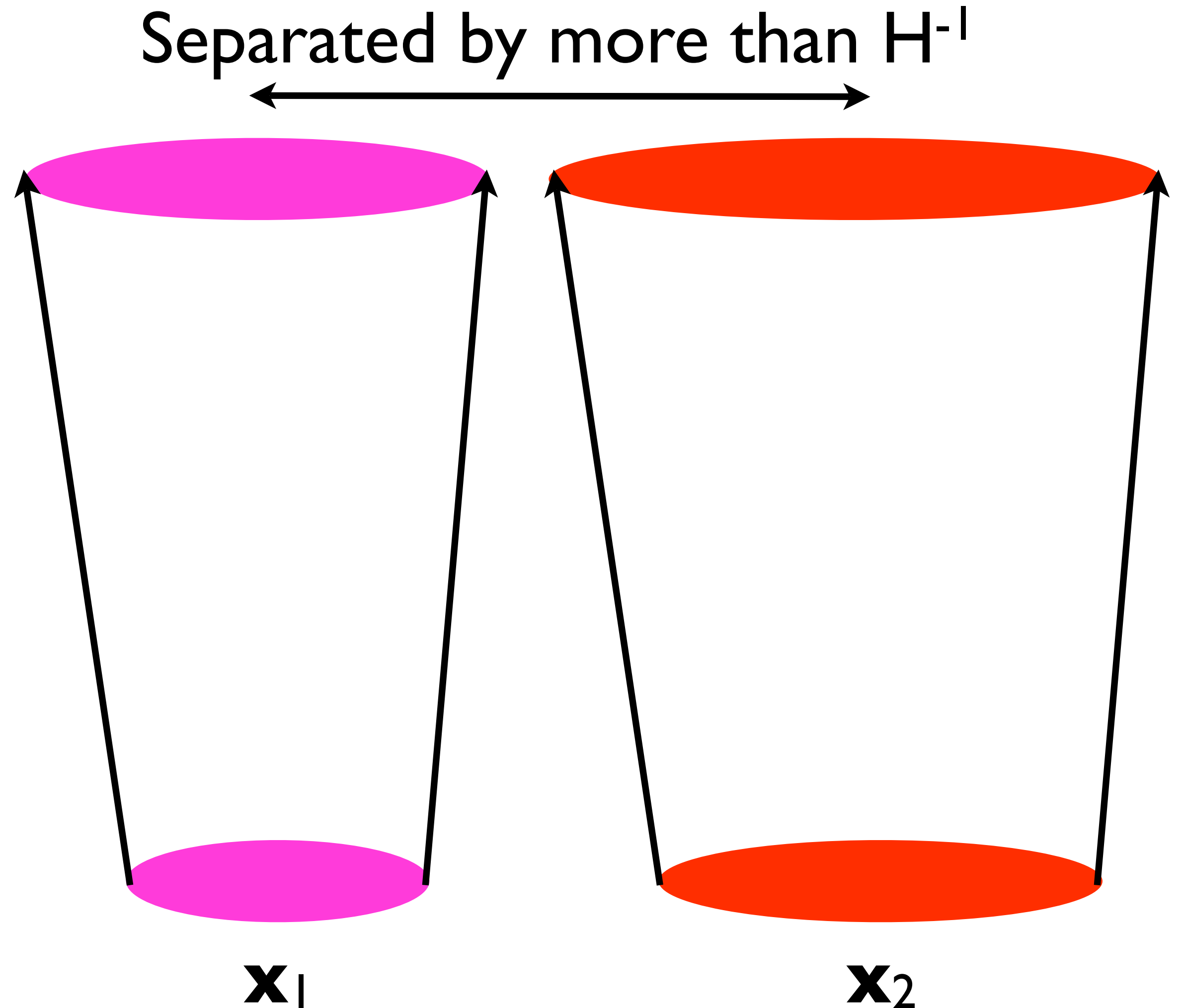
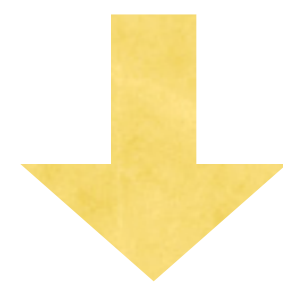
(Seery & Lidsey 2005; Chen et al. 2007)

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# Another Motivation For $f_{\text{NL}}$

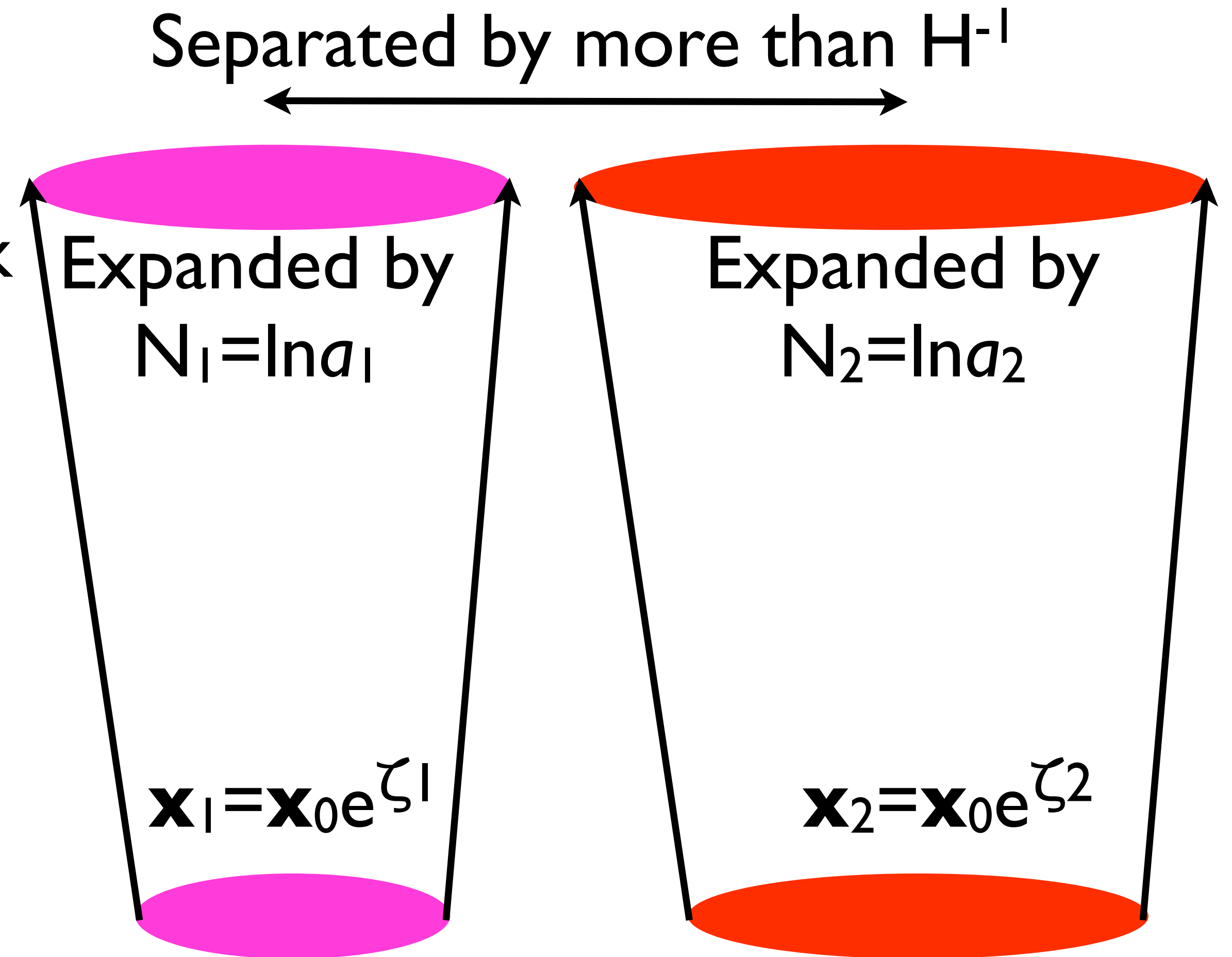
- In multi-field inflation models,  $\zeta_{\mathbf{k}}$  can evolve outside the horizon.
- This evolution can give rise to non-Gaussianity; however, causality demands that the form of non-Gaussianity must be local!



$$\zeta(\mathbf{x}) = \zeta_{\text{g}}(\mathbf{x}) + (3/5)f_{\text{NL}}[\zeta_{\text{g}}(\mathbf{x})]^2 + A\chi_{\text{g}}(\mathbf{x}) + B[\chi_{\text{g}}(\mathbf{x})]^2 + \dots$$

# The $\delta N$ Formalism

- The  $\delta N$  formalism (Starobinsky 1982; Salopek & Bond 1990; Sasaki & Stewart 1996) states that the curvature perturbation is equal to the difference in  $N = \ln a$ .
- $\zeta = \delta N = N_2 - N_1$
- where  $N = \int H dt$



# Getting the familiar result

- Single-field example at the linear order:
- $\zeta = \delta\{\int H dt\} = \delta\{\int (H/\varphi') d\varphi\} \approx (H/\varphi') \delta\varphi$
- Mukhanov & Chibisov; Guth & Pi; Hawking; Starobinsky; Bardeen, Steinhardt & Turner

# Extending to non-linear, multi-field cases

$$\zeta = \sum_I \frac{\partial N}{\partial \phi_I} \delta\phi_I + \frac{1}{2} \sum_{IJ} \frac{\partial^2 N}{\partial \phi_I \partial \phi_J} \delta\phi_I \delta\phi_J + \dots$$

(Lyth & Rodriguez 2005)

- Calculating the bispectrum is then straightforward. Schematically:

- $\langle \zeta^3 \rangle = \langle (\text{1st}) \times (\text{1st}) \times (\text{2nd}) \rangle \sim \langle \delta\varphi^4 \rangle \neq 0$

- $f_{\text{NL}} \sim \langle \zeta^3 \rangle / \langle \zeta^2 \rangle^2$

$$\frac{6}{5} f_{\text{NL}}^{\text{local}} = \frac{\sum_{IJ} N_{,IJ} N_{,I} N_{,J}}{[\sum_I (N_{,I})^2]^2}$$

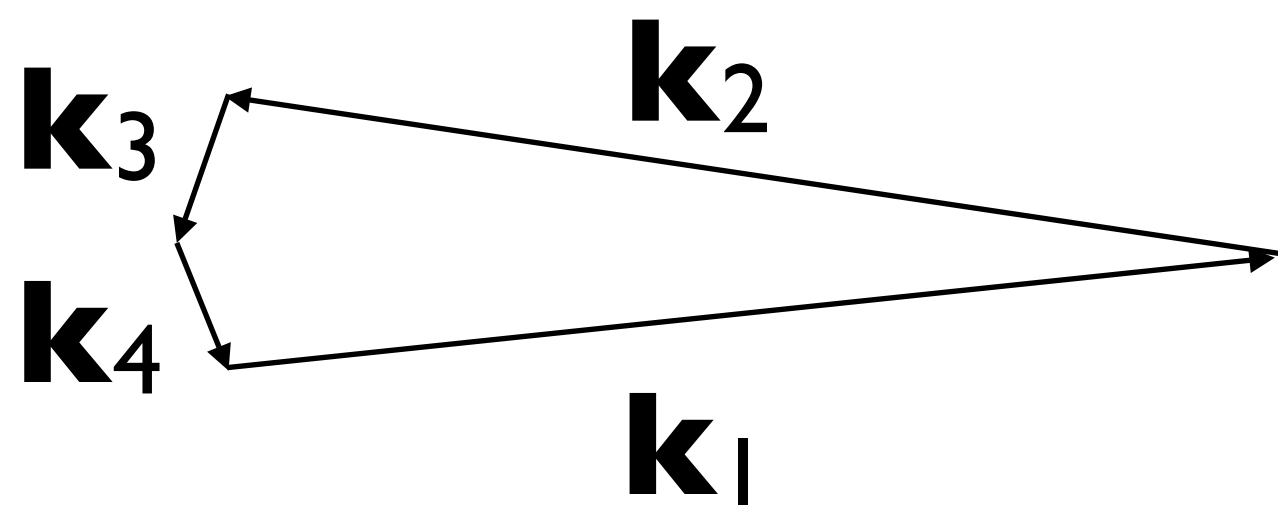


# Trispectrum: Next Frontier

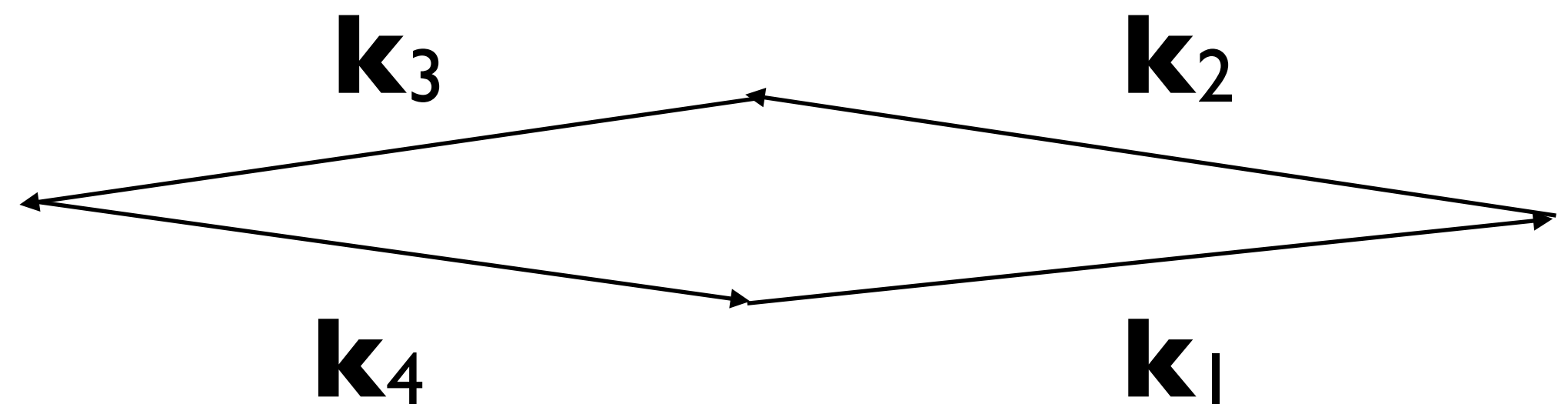
- The local form bispectrum,  
$$B_{\zeta}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) f_{\text{NL}} [(6/5) P_{\zeta}(k_1) P_{\zeta}(k_2) + \text{cyc.}]$$
- is equivalent to having the curvature perturbation in position space, in the form of:
  - $\zeta(\mathbf{x}) = \zeta_g(\mathbf{x}) + (3/5) f_{\text{NL}} [\zeta_g(\mathbf{x})]^2$
- This can be extended to higher-order:
  - $\zeta(\mathbf{x}) = \zeta_g(\mathbf{x}) + (3/5) f_{\text{NL}} [\zeta_g(\mathbf{x})]^2 + (9/25) g_{\text{NL}} [\zeta_g(\mathbf{x})]^3$

# Local Form Trispectrum

- For  $\zeta(\mathbf{x}) = \zeta_g(\mathbf{x}) + (3/5)f_{\text{NL}}[\zeta_g(\mathbf{x})]^2 + (9/25)g_{\text{NL}}[\zeta_g(\mathbf{x})]^3$ , we obtain the trispectrum:
  - $T_\zeta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4) \{ g_{\text{NL}}[(54/25)P_\zeta(k_1)P_\zeta(k_2)P_\zeta(k_3) + \text{cyc.}] + (f_{\text{NL}})^2[(18/25)P_\zeta(k_1)P_\zeta(k_2)(P_\zeta(|\mathbf{k}_1 + \mathbf{k}_3|) + P_\zeta(|\mathbf{k}_1 + \mathbf{k}_4|)) + \text{cyc.}] \}$



$g_{\text{NL}}$

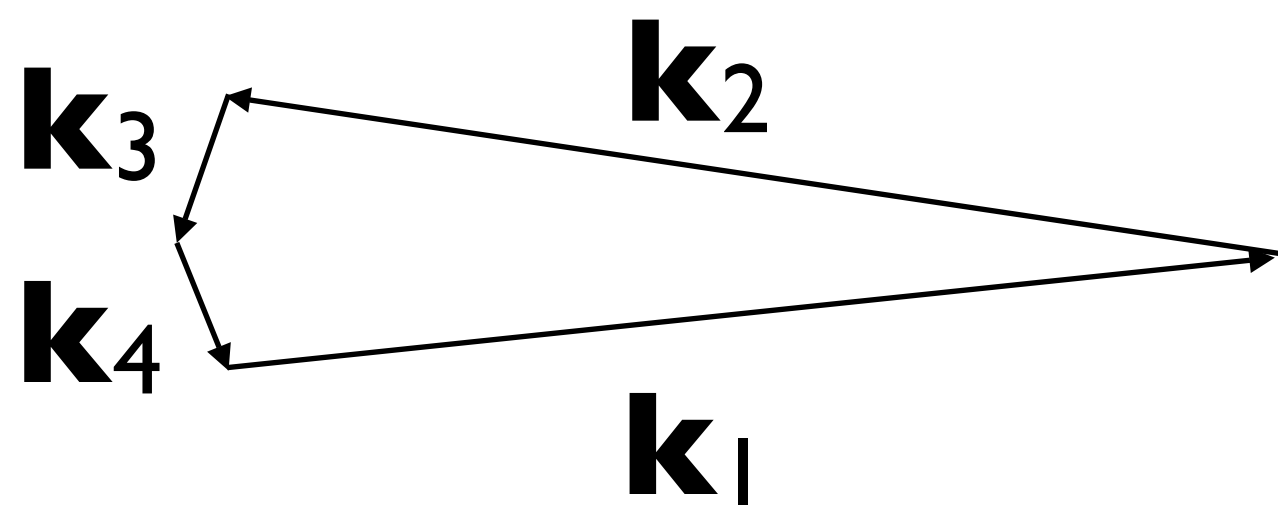


$f_{\text{NL}}^2$

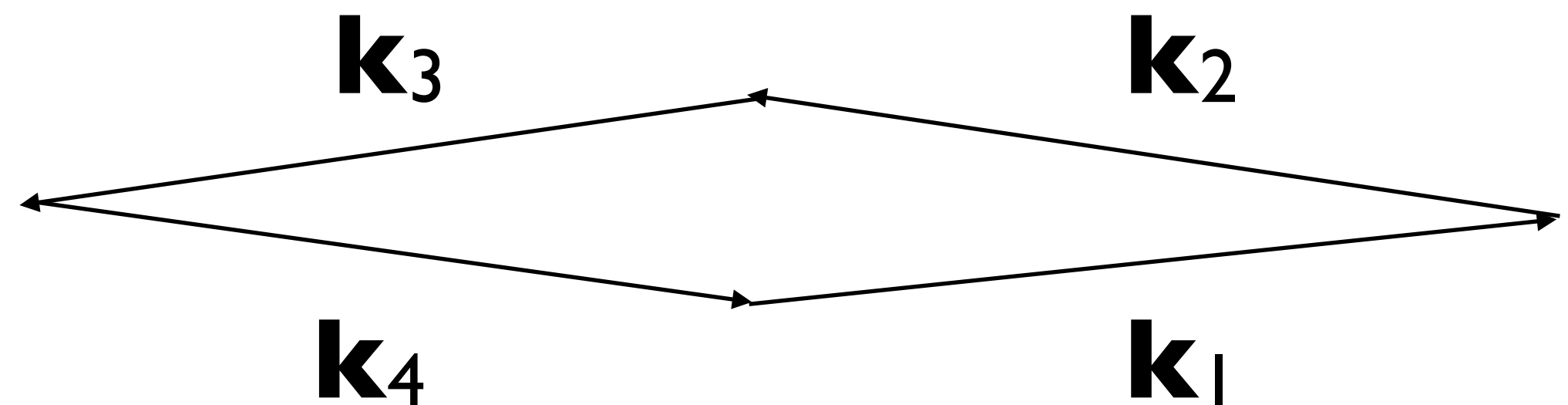
# (Slightly) Generalized Trispectrum

- $T_{\zeta}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4)$   
 $\{ \mathbf{g}_{NL} [(54/25) P_{\zeta}(k_1) P_{\zeta}(k_2) P_{\zeta}(k_3) + \text{cyc.}]$   
 $+ \mathbf{T}_{NL} [P_{\zeta}(k_1) P_{\zeta}(k_2) (P_{\zeta}(|\mathbf{k}_1 + \mathbf{k}_3|) + P_{\zeta}(|\mathbf{k}_1 + \mathbf{k}_4|)) + \text{cyc.}] \}$

*The local form consistency relation,  
 $T_{NL} = (6/5)(f_{NL})^2$ , may not be respected –  
 additional test of multi-field inflation!*



$\mathbf{g}_{NL}$



$\mathbf{T}_{NL}$

# Coming back to $\delta N$ ...

$$\zeta = \sum_I \frac{\partial N}{\partial \phi_I} \delta \phi_I + \frac{1}{2} \sum_{IJ} \frac{\partial^2 N}{\partial \phi_I \partial \phi_J} \delta \phi_I \delta \phi_J + \dots$$

(Lyth & Rodriguez 2005)

- Calculating the trispectrum is also straightforward. Schematically:

- $\langle \zeta^4 \rangle = \langle (\text{1st})^2 (\text{2nd})^2 \rangle \sim \langle \delta \varphi^6 \rangle \neq 0$

- $f_{\text{NL}} \sim \langle \zeta^4 \rangle / \langle \zeta^2 \rangle^3$

$$\tau_{\text{NL}} = \frac{\sum_{IJK} N_{,IJ} N_{,J} N_{,IK} N_{,K}}{[\sum_I (N_{,I})^2]^3} = \frac{\sum_I (\sum_J N_{,IJ} N_{,J})^2}{[\sum_I (N_{,I})^2]^3}$$

# Now, stare at these.

$$\frac{6}{5} f_{\text{NL}}^{\text{local}} = \frac{\sum_{IJ} N_{,IJ} N_{,I} N_{,J}}{[\sum_I (N_{,I})^2]^2},$$

$$\tau_{\text{NL}} = \frac{\sum_{IJK} N_{,IJ} N_{,J} N_{,IK} N_{,K}}{[\sum_I (N_{,I})^2]^3} = \frac{\sum_I (\sum_J N_{,IJ} N_{,J})^2}{[\sum_I (N_{,I})^2]^3}$$

# Change the variable...

$$\frac{6}{5} f_{\text{NL}}^{\text{local}} = \frac{\sum_{IJ} N_{,IJ} N_{,I} N_{,J}}{[\sum_I (N_{,I})^2]^2},$$

$$\tau_{\text{NL}} = \frac{\sum_{IJK} N_{,IJ} N_{,J} N_{,IK} N_{,K}}{[\sum_I (N_{,I})^2]^3} = \frac{\sum_I (\sum_J N_{,IJ} N_{,J})^2}{[\sum_I (N_{,I})^2]^3}$$

$$a_I = \frac{\sum_J N_{,IJ} N_{,J}}{[\sum_J (N_{,J})^2]^{3/2}}$$

$$b_I = \frac{N_{,I}}{[\sum_J (N_{,J})^2]^{1/2}}$$

$$(6/5) f_{\text{NL}} = \sum_I a_I b_I$$

$$\tau_{\text{NL}} = (\sum_I a_I)^2 (\sum_I b_I)^2$$

# Then apply the Cauchy-Schwarz Inequality

$$\left(\sum_I a_I^2\right) \left(\sum_J b_J^2\right) \geq \left(\sum_I a_I b_I\right)^2$$

- Implies (Suyama & Yamaguchi 2008)

$$\tau_{\text{NL}} \geq \left(\frac{6 f_{\text{NL}}^{\text{local}}}{5}\right)^2$$

This holds for almost all (if not **all** - left unproven) for  
multi-field models!

# Be careful when $0=0$

- The Suyama-Yamaguchi inequality does not always hold because the Cauchy-Schwarz inequality can be  $0=0$ . For example:

$$\zeta = \frac{\partial N}{\partial \phi_1} \delta \phi_1 + \frac{1}{2} \frac{\partial^2 N}{\partial \phi_2^2} \delta \phi_2^2$$

In this harmless two-field case, the Cauchy-Schwarz inequality becomes  $0=0$  (both  $f_{\text{NL}}$  and  $\tau_{\text{NL}}$  result from the second term).

In this case,

$$\tau_{\text{NL}} \sim 10^3 (f_{\text{NL}}^{\text{local}})^{4/3}$$

(Suyama & Takahashi 2008) 40



# But, even in this case...

$$\tau_{\text{NL}} \sim 10^3 (f_{\text{NL}}^{\text{local}})^{4/3}$$

still satisfies

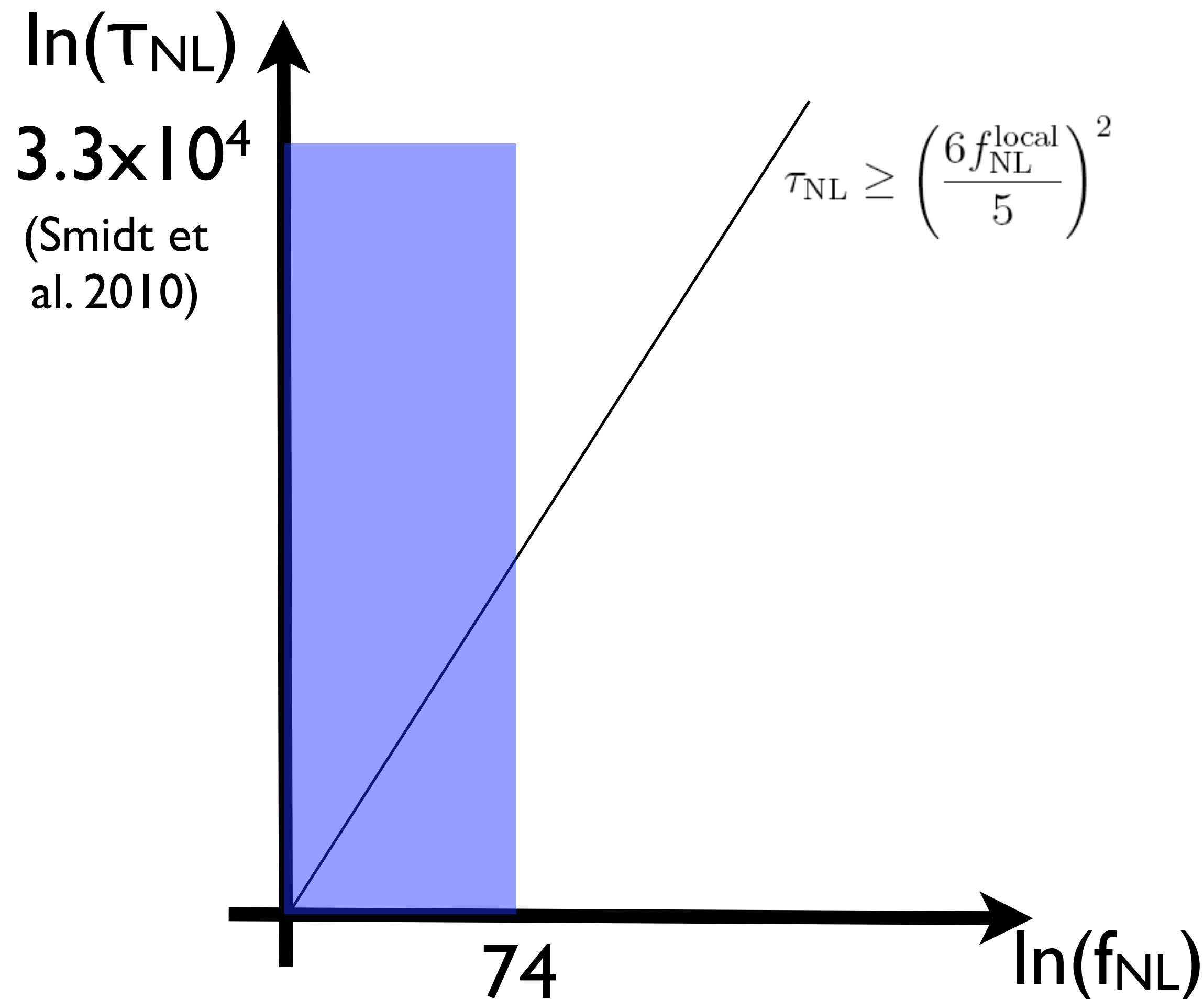
$$\tau_{\text{NL}} \geq \left( \frac{6 f_{\text{NL}}^{\text{local}}}{5} \right)^2$$

as long as  $f_{\text{NL}} < 18000$ . Current limit?

$$f_{\text{NL}}^{\text{local}} = 32 \pm 21 \text{ (68\% CL)}$$

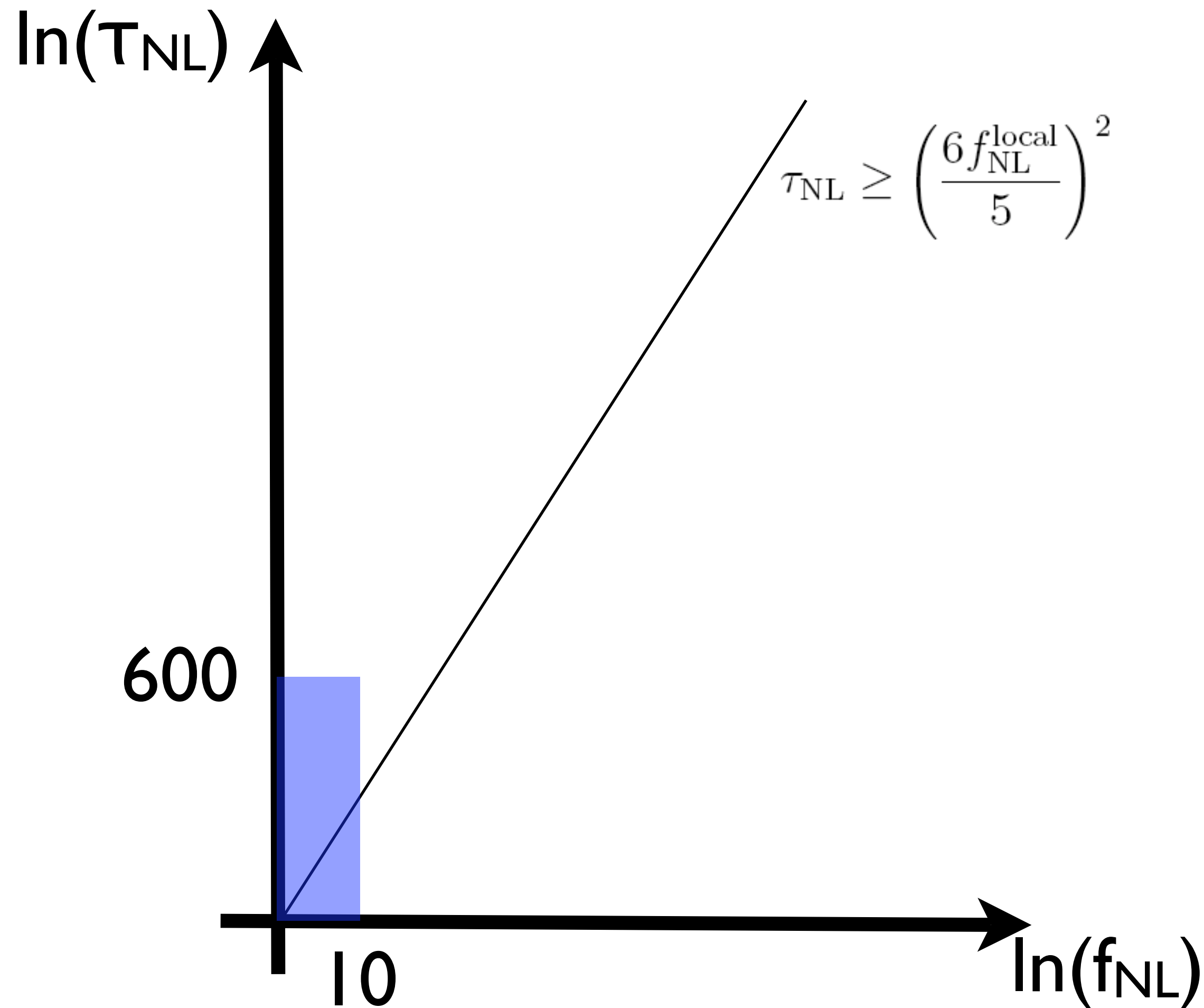
(Komatsu et al. 2010)

# The diagram that you should take away from this talk.



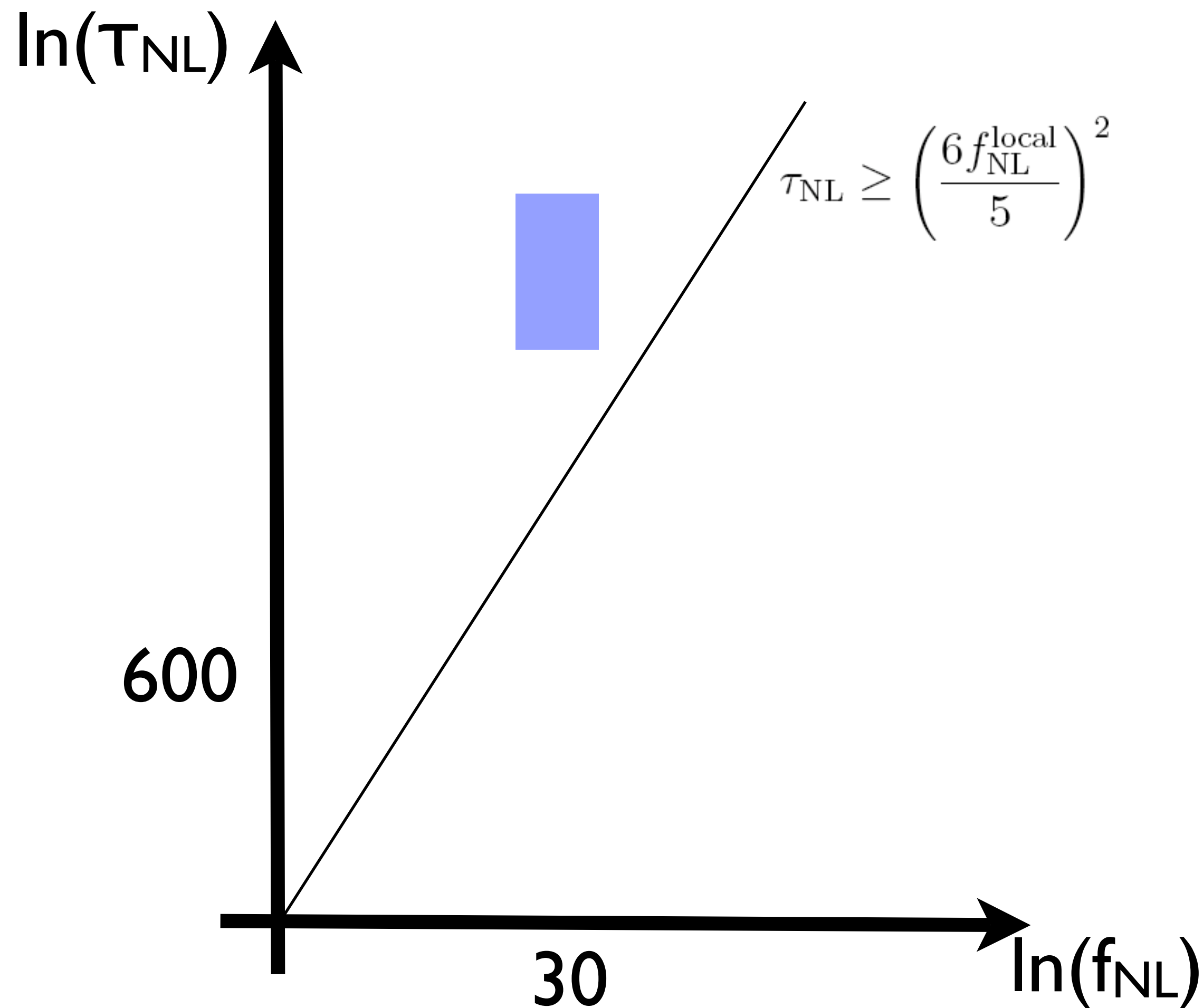
- The current limits from WMAP 7-year are consistent with single-field or multi-field models.
- So, let's play around with the future.

# Case A: Single-field Happiness



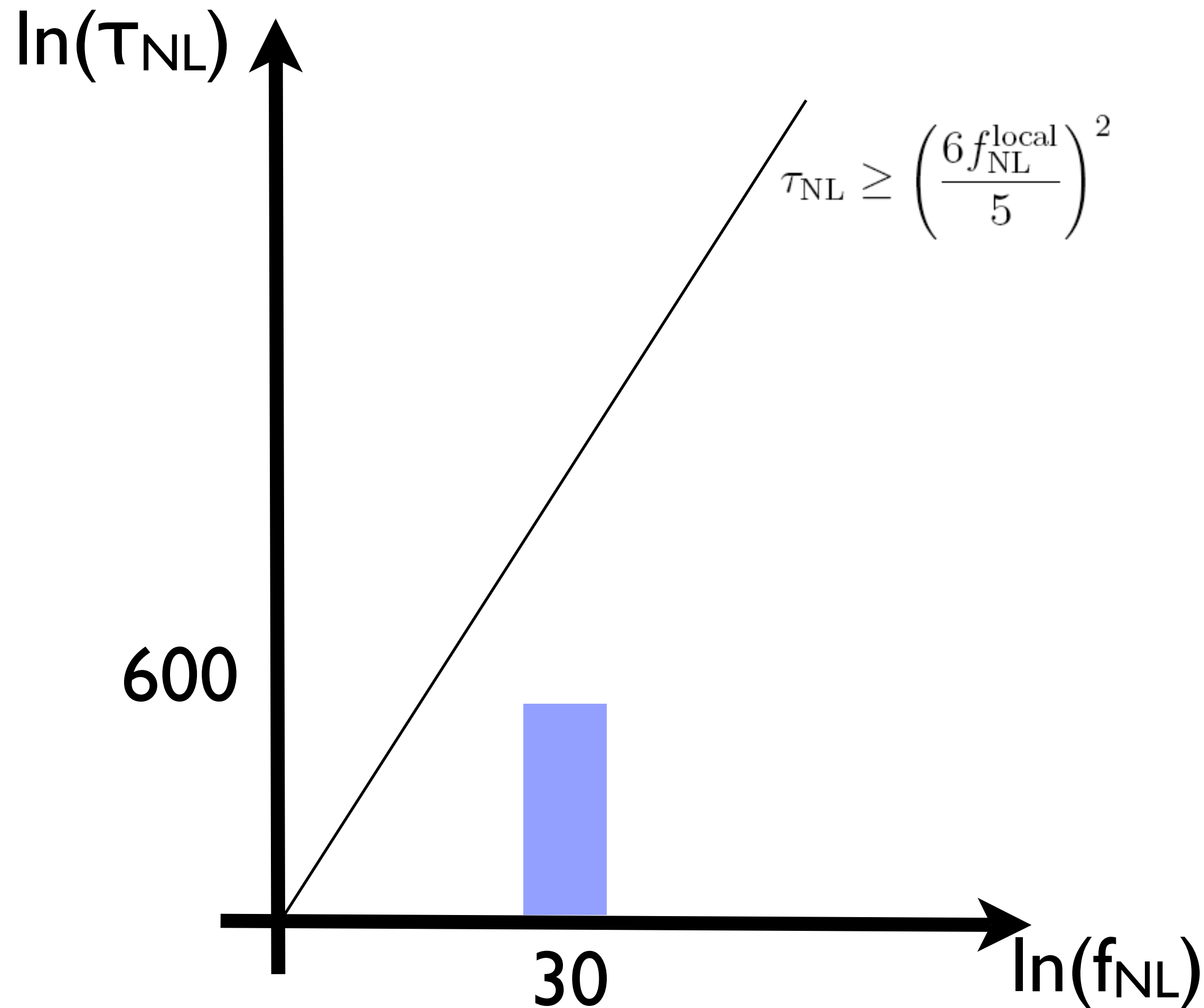
- No detection of anything after Planck. Single-field survived the test (for the moment: the future galaxy surveys can improve the limits by a factor of ten).

# Case B: Multi-field Happiness



- $f_{\text{NL}}$  is detected. Single-field is dead.
- But,  $\tau_{\text{NL}}$  is also detected, in accordance with the Suyama-Yamaguchi inequality, as expected from most (if not all - left unproven) of multi-field models.

# Case C: Madness



- $f_{\text{NL}}$  is detected. Single-field is dead.
- But,  $\tau_{\text{NL}}$  is **not** detected, inconsistent with the Suyama-Yamaguchi inequality.
- (With the caveat that this may not be completely general) BOTH the single-field and multi-field are gone.

# An exciting field

## **Non-Gaussianity as a Probe of the Physics of the Primordial Universe and the Astrophysics of the Low Redshift Universe**

E.Komatsu, N.Afshordi, N.Bartolo, D.Baumann, J.R.Bond, E.I.Buchbinder, C.T.Byrnes, X.Chen, D.J.H.Chung, A.Cooray, P.Creminelli, N.Dalal, O.Dore, R.Easter, A.V.Frolov, K.M.Gorski, M.G. Jackson, J.Khoury, W.H.Kinney, L.Kofman, K.Koyama, L.Leblond, J.-L.Lehners, J.E.Lidsey, M.Liguori, E.A.Lim, A.Linde, D.H.Lyth, J.Maldacena, S.Matarrese, L.McAllister, P.McDonald, S.Mukohyama, B.Ovrut, H.V.Peiris, C.Raeth, A.Riotto, Y.Rodriguez, M.Sasaki, R.Scoccimarro, D.Seery, E.Sefusatti, U.Seljak, L.Senatore, S.Shandera, E.P.S.Shellard, E.Silverstein, A.Slosar, K.M.Smith, A.A.Starobinsky, P.J.Steinhardt, F.Takahashi, M.Tegmark, A.J.Tolley, L.Verde, B.D.Wandelt, D.Wands, S.Weinberg, M.Wyman, A.P.S.Yadav, M.Zaldarriaga

Science White Paper submitted to the Cosmology and Fundamental Physics (CFP) Science Frontier Panel of the Astro 2010 Decadal Survey

# Summary

- Non-Gaussianity provides the only means (so far) to rule out single-field inflation models altogether.
- Non-Gaussianity provides the only, possible means (because it has not been proven completely yet) to rule out multi-field inflation models altogether.
- As a result, non-Gaussianity can be used to rule out inflation models altogether - something that was not conceived to be possible before.

# Summary

- *Planck* is well-positioned to achieve this.
- If not, inflation still needs to pass more stringent tests from (near; ~5 years) future data, reaching  $f_{\text{NL}} \sim 1$  and  $\tau_{\text{NL}} \sim 10$ .