Statistics of CMB Anisotropies, and (some) WMAP Results

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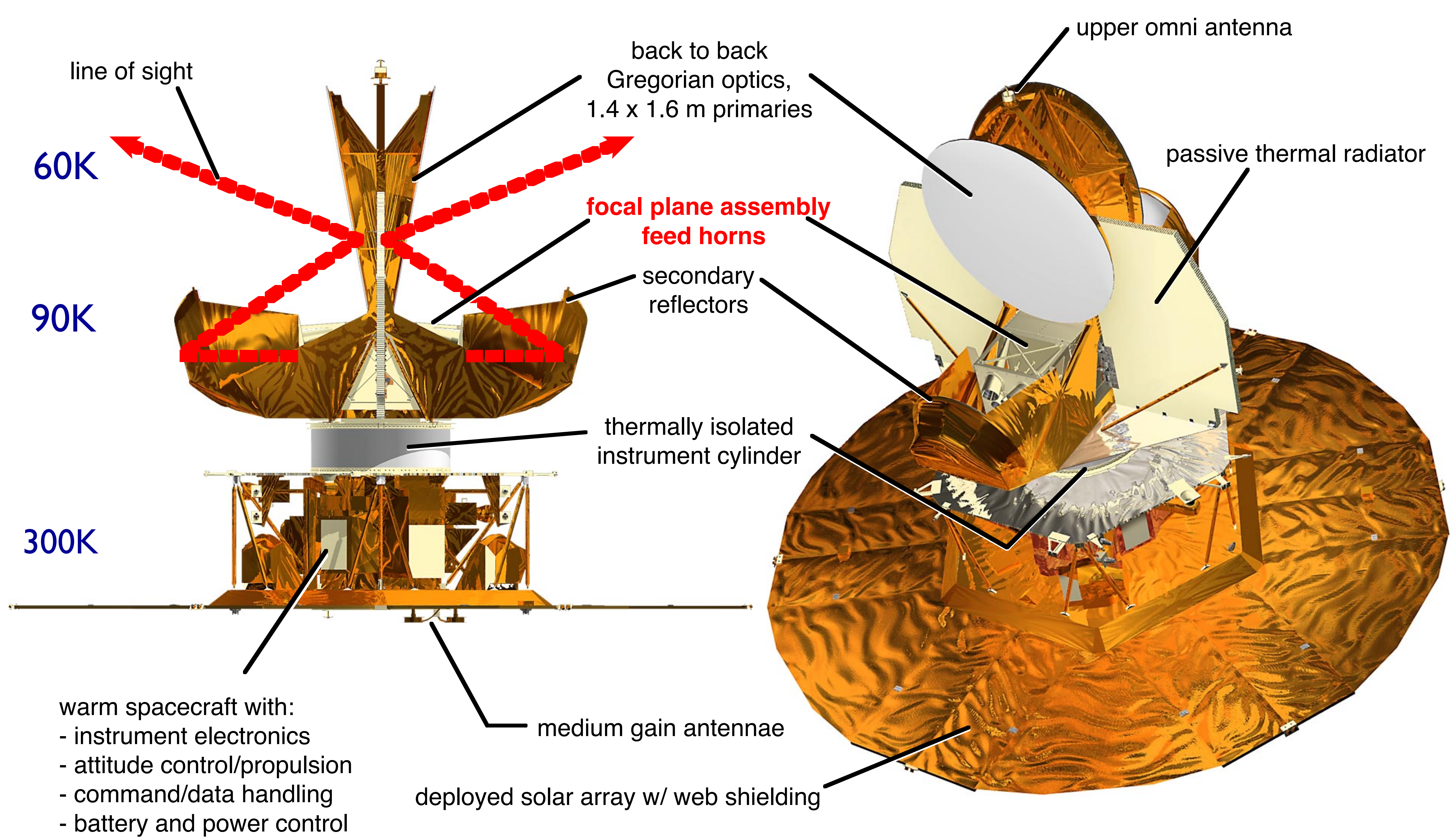
The purpose of this talk

- I would like to walk you through some key steps and assumptions that we make when we analyze the CMB data (WMAP data in particular).
- It would be best if you could listen to my talk while thinking, "how does this apply to ***my*** data?"

The Problem

- The WMAP satellite records the temperature *difference* between two locations in the sky (141 degrees apart)



The Problem

- The WMAP satellite records the temperature *difference* between two locations in the sky (141 degrees apart)
  - We further difference between two polarization states to measure linear polarization ("double differencing")
- We create a sky map from the time series: $T=Ad+n$
- We measure the power spectrum and bispectrum from this map: $\langle TT \rangle$ and $\langle TTT \rangle$
- We then turn the measurements into parameters

Once you are given maps

- We decompose maps into components. Maps in microwave bands (centimeters to millimeters) contain:
  - Primary CMB
  - Secondary CMB (caused by the intervening stuff affecting CMB photons)
  - Galactic foreground (emission from our own Milky Way)
  - Extra-galactic foreground (point and extended sources)
  - Noise

Have a PDF!!

- A powerful lesson I have learned from 12 years of dealing with CMB data:
- **Write down a PDF of your data before you start doing anything on the data**
- Is it a Gaussian? Poisson? Non-Gaussian but only weakly non-Gaussian? Strongly non-Gaussian but with known distribution (log-normal)? Strongly non-Gaussian without any clue?

Decent Working Hypothesis

- PDFs of Primary CMB and noise are Gaussian, and they are uncorrelated
- Then, we describe the data as

$$-2\ln(\mathrm{PDF}) = ([\mathrm{data}]_i - [\mathrm{stuff}]_i)^T (C^{-1})_{ij} ([\mathrm{data}]_j - [\mathrm{stuff}]_j) + |C|$$

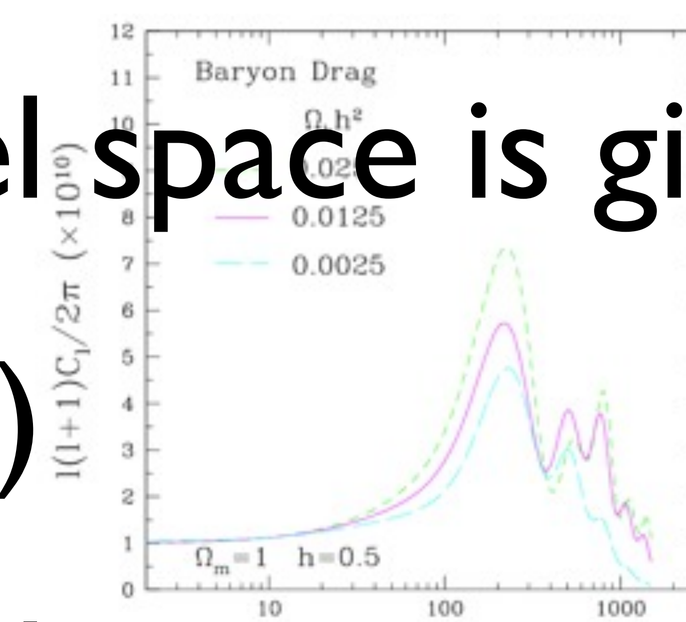
where "C_{ij}" describes the two-point correlation of CMB and noise in either pixel or Fourier (harmonics) space

Why decent? Because it is (i) expected theoretically (inflation); and (ii) verified a posteriori by data

The problem is well-defined, in principle

$-2\ln(\mathrm{PDF}) = ([\mathrm{data}]_i - [\mathrm{stuff}]_i)^T (C^{-1})_{ij} ([\mathrm{data}]_j - [\mathrm{stuff}]_j) + |C|$

- $C_{ij} = [\mathrm{signal}]_{ij} + [\mathrm{noise}]_{ij}$
- For the WMAP case, we understand $[\mathrm{noise}]_{ij}$ quite well
- The signal covariance matrix in pixel space is given by
  - $[\mathrm{signal}]_{ij} = (4\pi)^{-1} \sum (2l+1) S_l P_l(\cos\theta_{ij})$



- The signal covariance matrix in pixel space is given by
  - $[\mathrm{signal}]_{lm,l'm'} = S_l \delta_{ll'} \delta_{mm'}$

Caveat

- $[\text{signal}]_{ij} = (4\pi)^{-1}\sum(2l+1)S_l P_l(\cos\theta_{ij})$ in pixel space; or
- $[\text{signal}]_{lm,l'm'} = S_l \delta_{ll'} \delta_{mm'}$ in harmonic space;
- are the consequences of spatial translation and rotation invariance of PDF.
  - If either of the two is broken, then we must specify the full covariance matrix, $[\text{signal}]_{lm,l'm'}$.
  - *You have just seen an example of broken rotation invariance, presented by Jaiseung Kim.*

Case I: no noise

$-2\ln(\mathrm{PDF}) = ([\mathrm{data}]_i - [\mathrm{stuff}]_i)^T (C^{-1})_{ij} ([\mathrm{data}]_j - [\mathrm{stuff}]_j) + |C|$

- $(C^{-1})_{ij} = [\mathrm{signal}]^{-1}{}_{ij} = (4\pi)^{-1}\sum(2l+1)(1/S_l)P_l(\cos\theta_{ij})$
- This is trivial to evaluate.
- So, we can perform, in pixel space (or harmonic space), simultaneous fits to S_l at each multipole and models of $[\mathrm{stuff}]_i$.
- Or, we can write S_l as a function of cosmological parameters, and fit the parameters instead.

Case II: with noise

$$-2\ln(\mathrm{PDF}) = ([\mathrm{data}]_i - [\mathrm{stuff}]_i)^T (C^{-1})_{ij} ([\mathrm{data}]_j - [\mathrm{stuff}]_j) + |C|$$

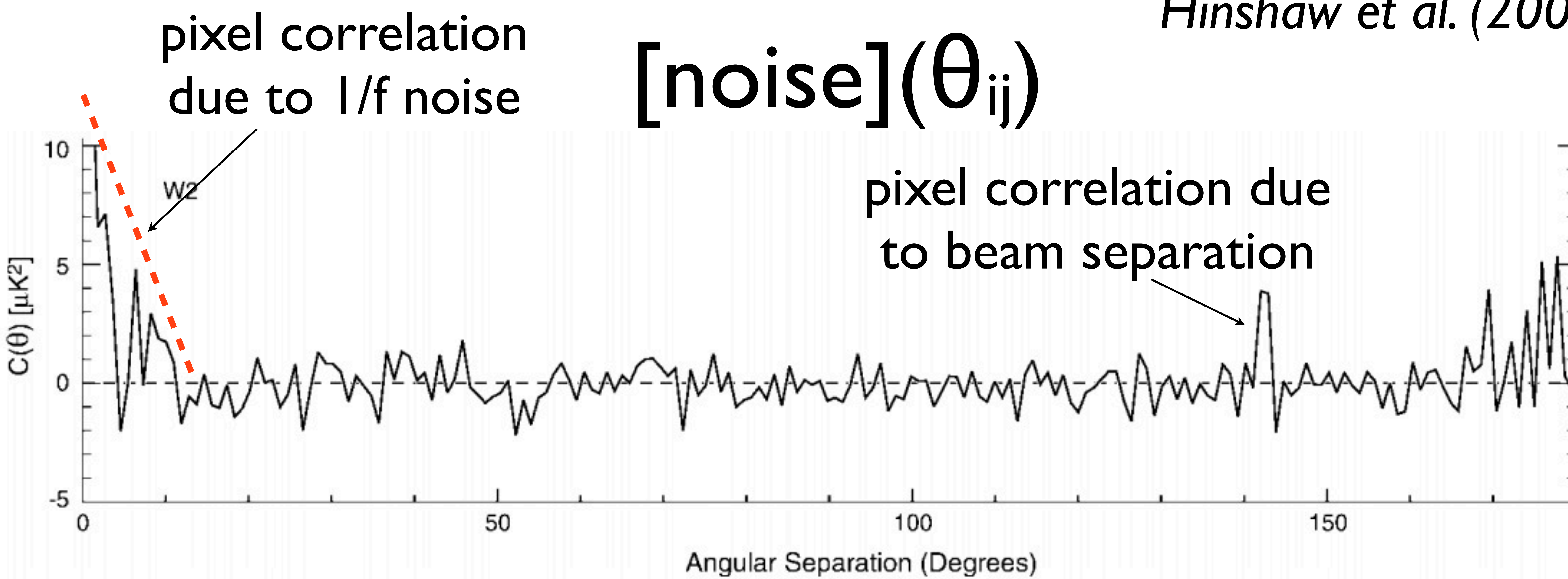
- $(C^{-1})_{ij} = ([\mathrm{signal}] + [\mathrm{noise}])^{-1}{}_{ij}$
- This is trivial to evaluate, **if** PDF of noise is also invariant under spatial translation and rotation. Then, $(C^{-1})_{ij} = [\mathrm{signal}]^{-1}{}_{ij} = (4\pi)^{-1}\sum(2l+1)[1/(S_l+\mathbf{N_l})]\, P_l(\cos\theta_{ij})$
- However, WMAP noise is not invariant under translation because the r.m.s. of noise per pixel is not uniform

Noise properties of WMAP

1. WMAP's noise map is spatially correlated
   - This is because WMAP is a differential experiment, taking difference between temperature values at two locations in the sky, separated by 141 degrees
   - Also, detector's 1/f noise correlates noise in pixels
2. WMAP's noise map is spatially inhomogeneous
   - This is because WMAP's scan pattern is such that it avoids the Sun. Then it does not observe the ecliptic plane as much as the poles

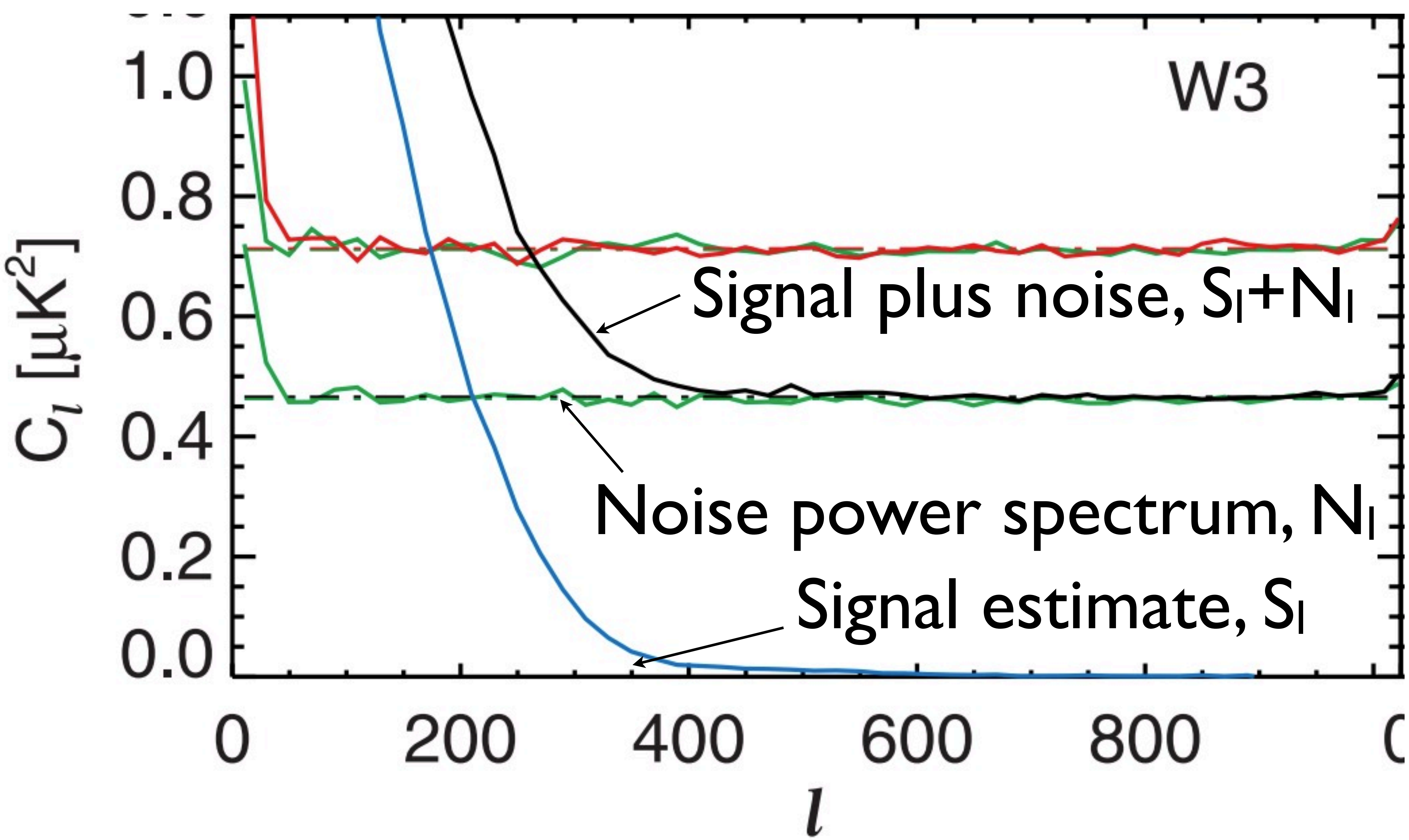
[noise](θ_{ij})

Hinshaw et al. (2003)

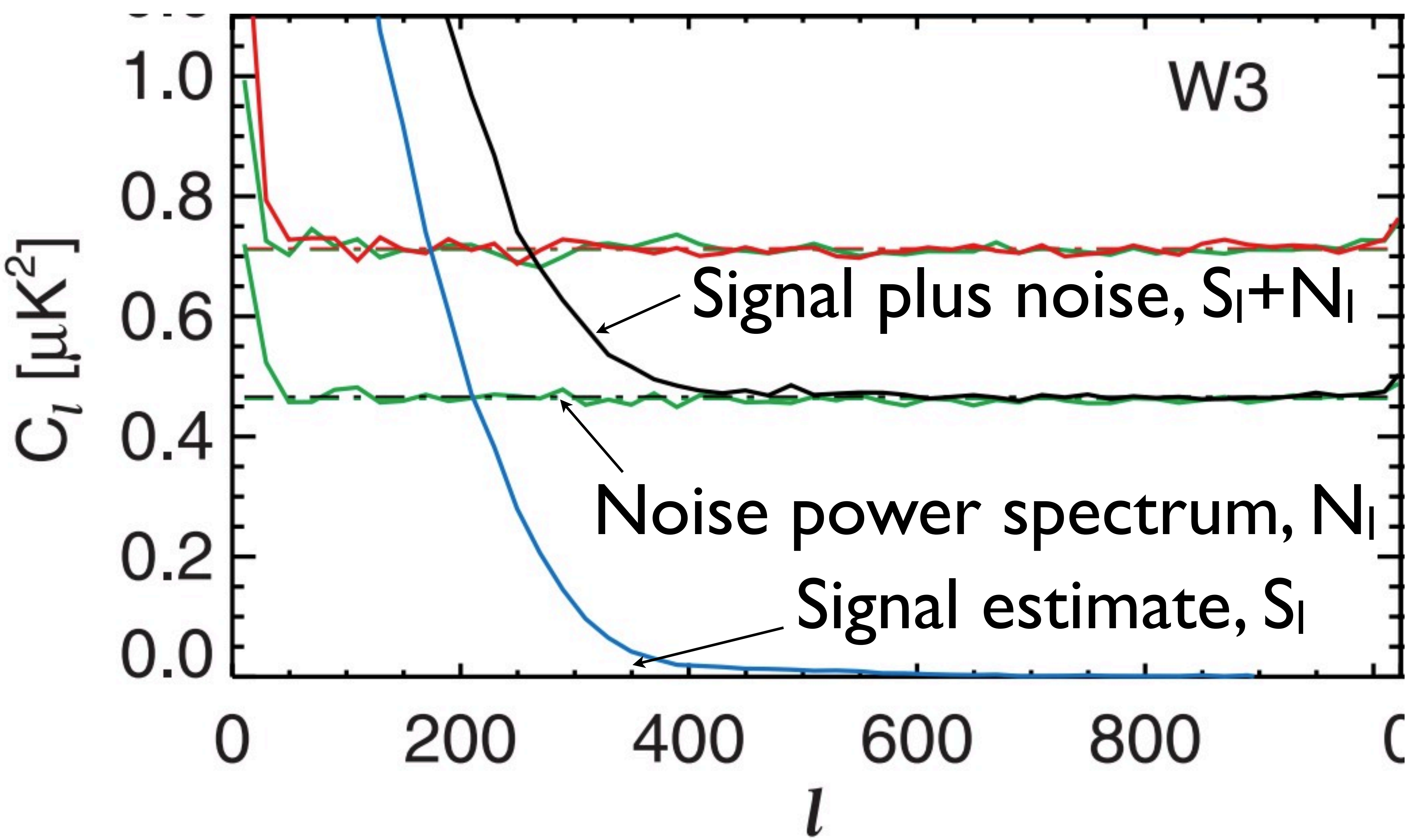


- [noise](0) is the average noise variance
- [noise](θ) does not decline toward larger θ as fast as it would for uncorrelated noise: a signature of 1/f noise
- There is a spike in the noise correlation at the beam separation at 141 degrees

Hinshaw et al. (2003)



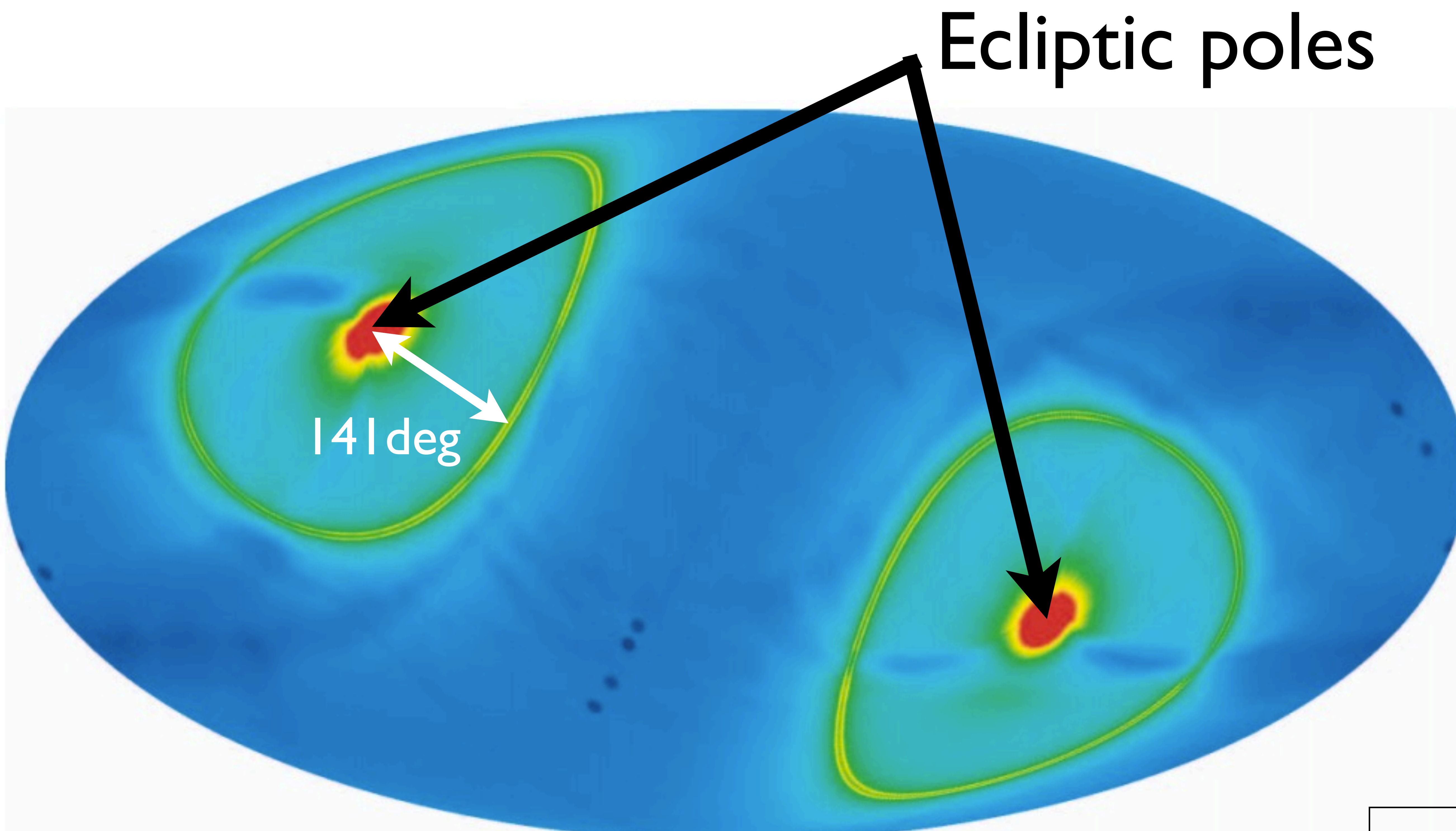
- The signal power spectrum grows rapidly toward low multipoles, $S_l \sim l^{-2}$
- The noise power spectrum is roughly constant, except for low multipoles.



- Assuming that noise is approximately uncorrelated in the noise-dominated region (i.e., higher multipoles) is a good approximation!!

Uncorrelated, but inhomogeneous noise

Bennett et al. (2003)



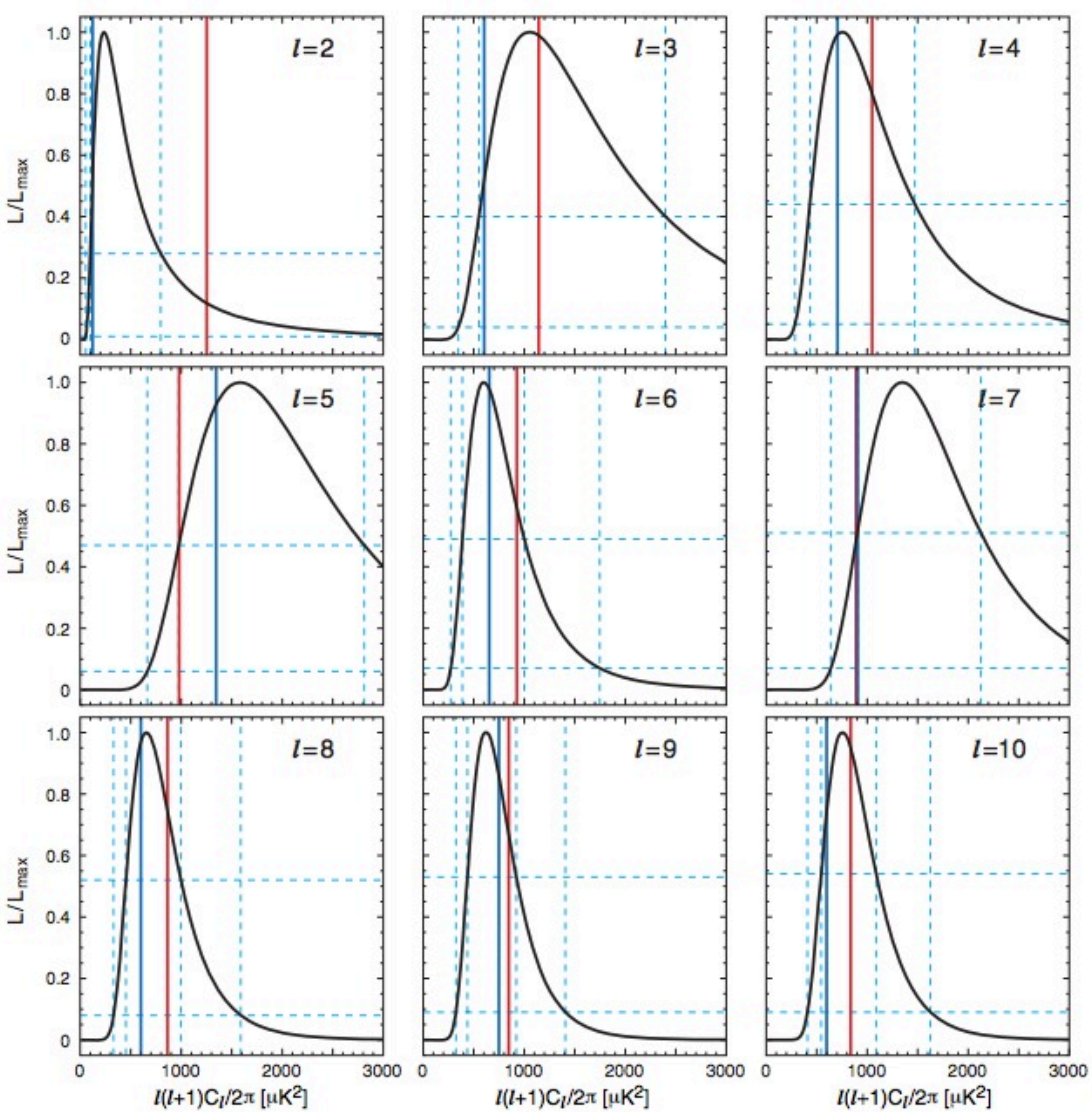
Map of the number of "hits" (observations), $N_{obs,i}$

- Noise matrix is diagonal in pixel space:
- $[noise]_{ij}=\delta_{ij}\ \sigma_0/N_{obs,i}$
- σ_0: noise per hit
- $N_{obs,i}$: # of hits

Broken translation invariance…
$[noise]_{ij}$ is no longer diagonal in harmonic space

Now, a challenge

- We need to invert the signal-plus-noise covariance matrix, to compute $(C^{-1})_{ij}=([\text{signal}]+[\text{noise}])^{-1}{}_{ij}$
- Unless BOTH signal and noise are diagonal in some space, inversion requires N_{pix}^3 operations, where $N_{pix}\sim10^6$ is the number of WMAP pixels
- The issue: the signal is diagonal in harmonic space; while noise is (approximately) diagonal in pixel space
- This means that we cannot evaluate PDF directly at all scales - but only at low multipoles where the signal dominates the data and the number of (low-resolution) pixels is small enough (say, $N_{pix}\sim10^3$)



Hinshaw et al. (2007)

Full PDF at l=2–10

- Solid lines: posterior PDF of C_l estimated from the three-year data
- Red lines: best-fit ΛCDM model
- Blue lines: "pseudo-C_l" estimator (sub-optimal)

A (tentative) solution: estimator

- First of all, do not forget the principle: given data set, we always have distribution of answers, i.e., PDF
- However, when estimating PDF is not possible due to, e.g., large computational costs, an alternative is to use an estimator for the mean (just a number for the best guess of a quantity you wish to measure) and the variance (second-order moment of the PDF)

Typical CMB Analysis

- Ideally: evaluate PDF directly, varying all the parameters characterizing primary CMB and [stuff]=(secondary CMB, foregrounds), **simultaneously**, using

$$-2\ln(\mathrm{PDF}) = ([\mathrm{data}]_i - [\mathrm{stuff}]_i)^T (C^{-1})_{ij} ([\mathrm{data}]_j - [\mathrm{stuff}]_j) + |C|$$

- In reality:

1. Estimate and remove [stuff] from the data
2. Estimate the power spectrum, C_l, and its covariance, $\langle C_l C_{l'} \rangle$
3. Construct an approximate PDF for C_l from C_l and $\langle C_l C_{l'} \rangle$ and use it to estimate the cosmo. parameters

How to remove [stuff]?

- We use templates. There are three emission processes which are important in the WMAP frequencies:

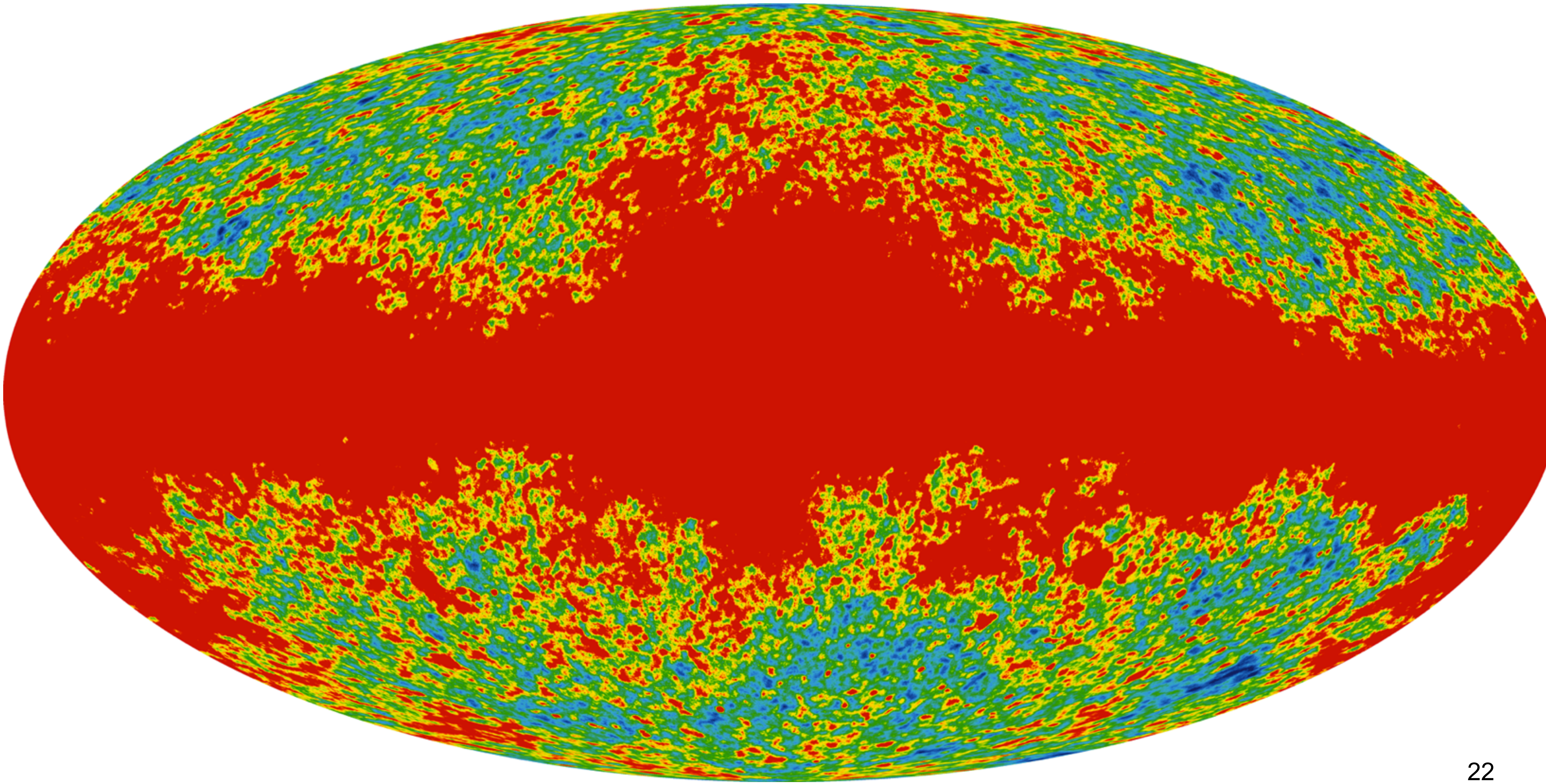
internal 1. ***Synchrotron emission***, traced by the difference between 23 and 33 GHz maps

external 2. ***Free-free emission***, traced by a map of Hα emission

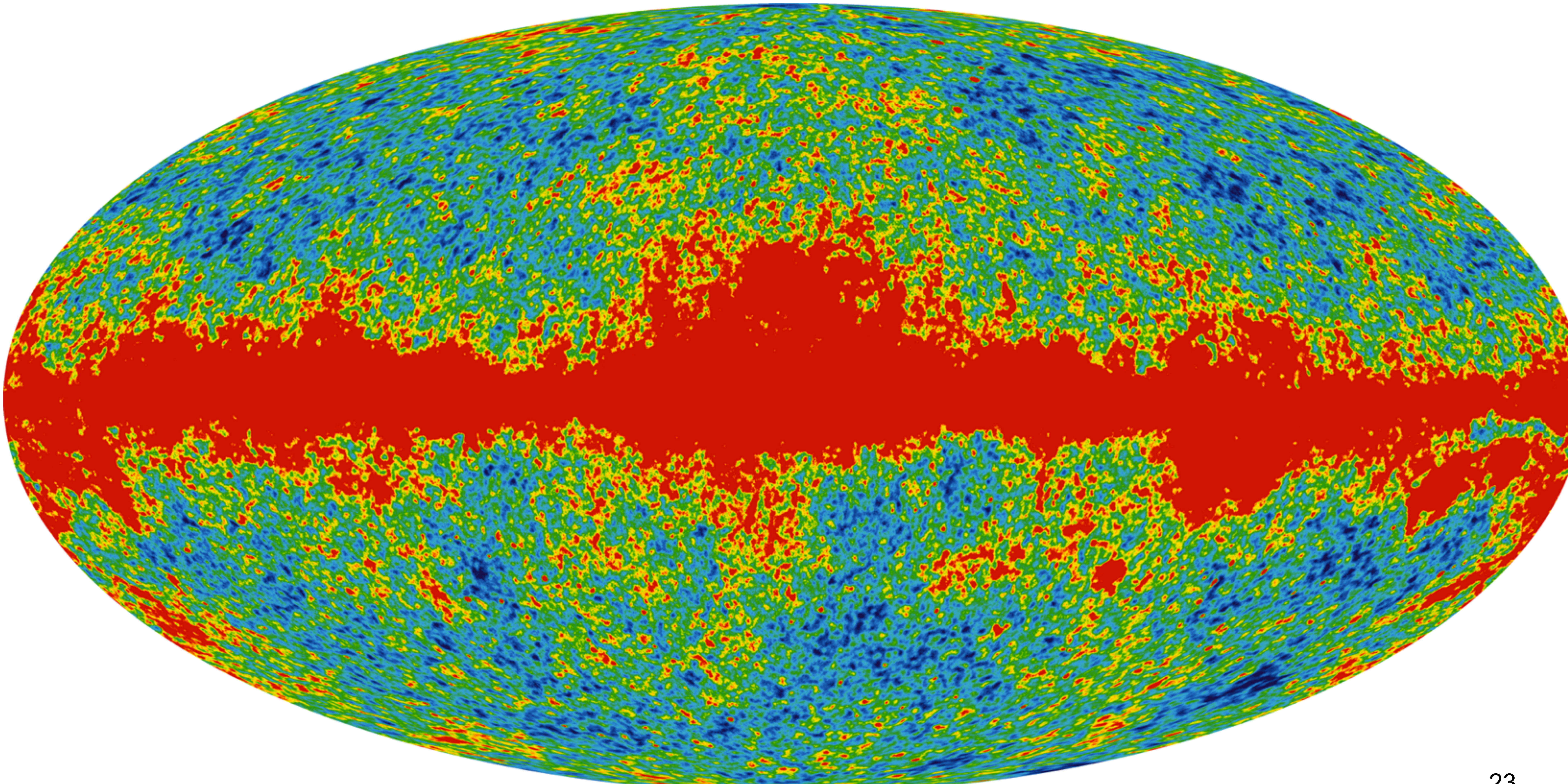
external 3. ***Thermal dust emission***, traced by infrared data (Finkbeiner, Davis & Schlegel dust map)

- We then smooth these maps to one-degree FWHM beam, fit them to and remove them from 41, 61, and 94 GHz maps, **yielding [data]-[stuff] maps**

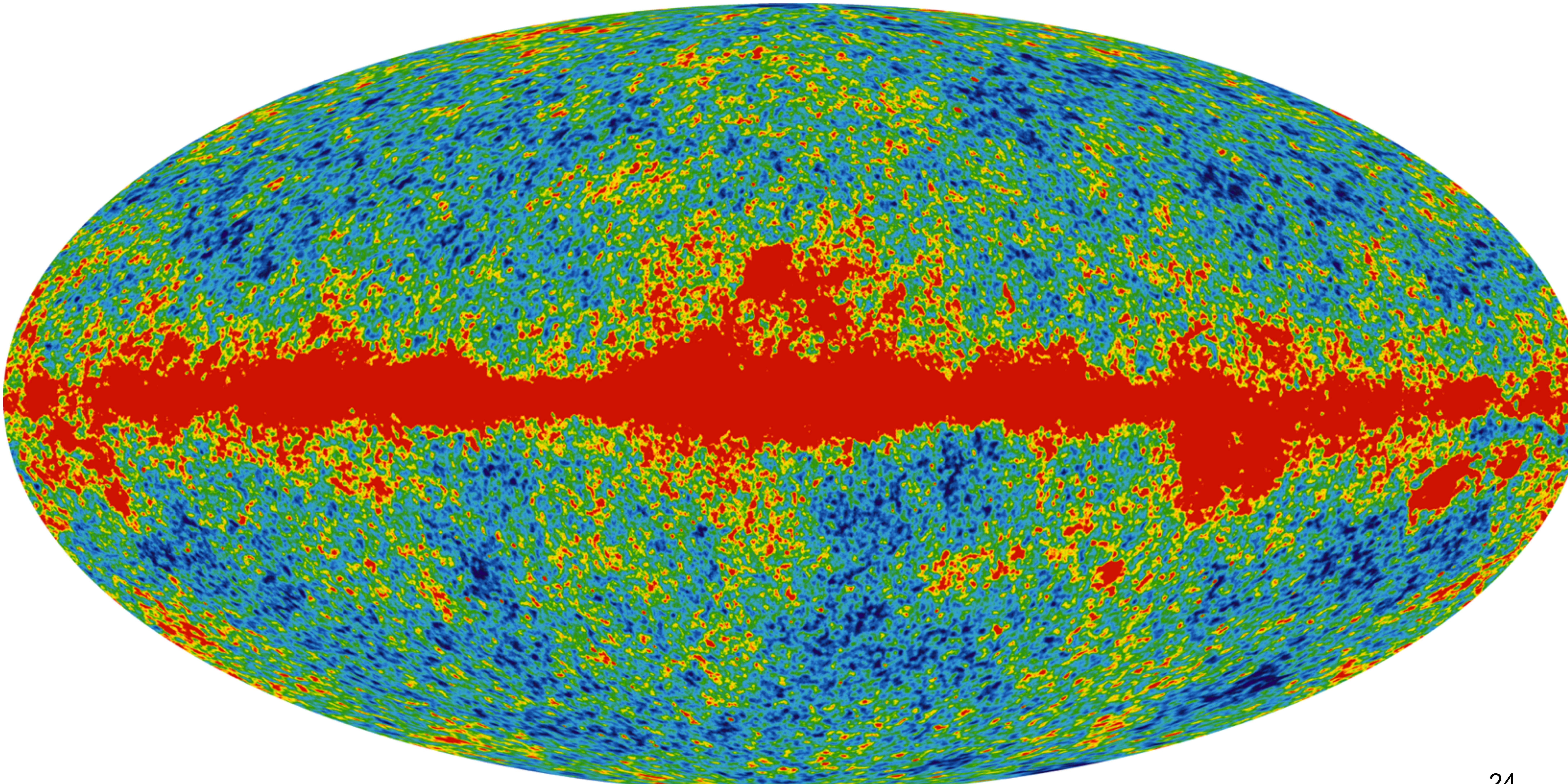
23 GHz [unpolarized]



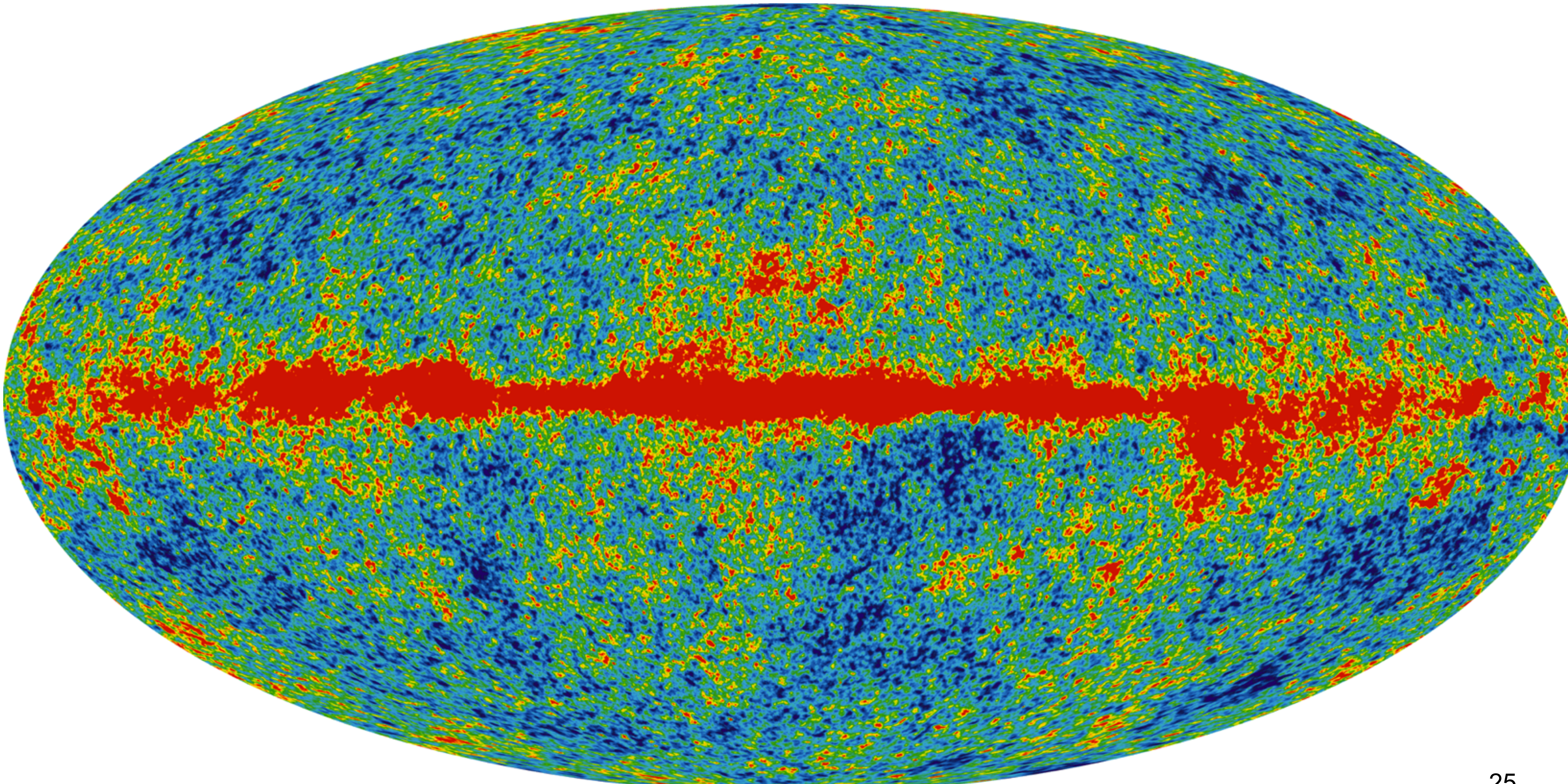
33 GHz [unpolarized]



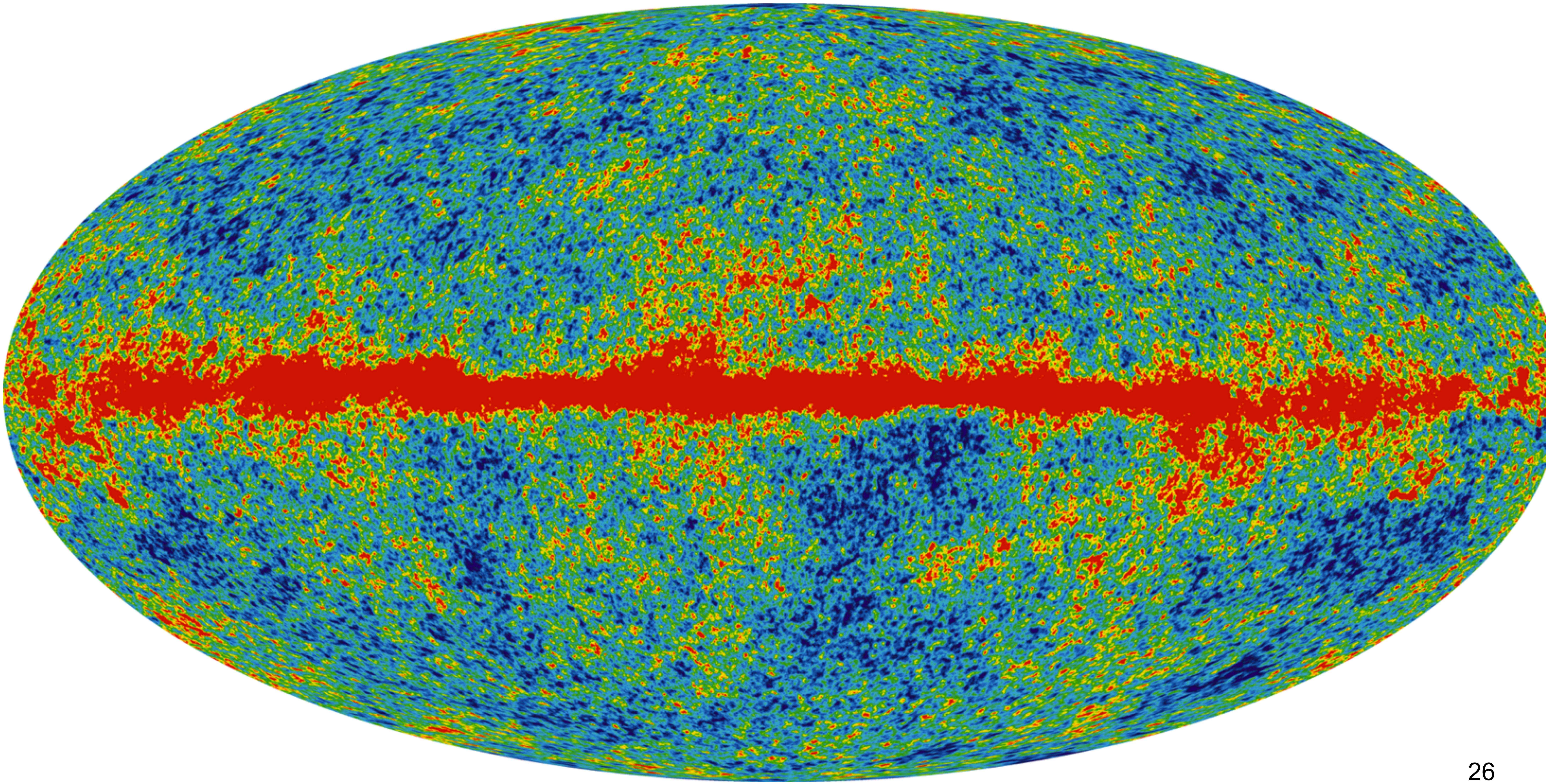
41 GHz [unpolarized]



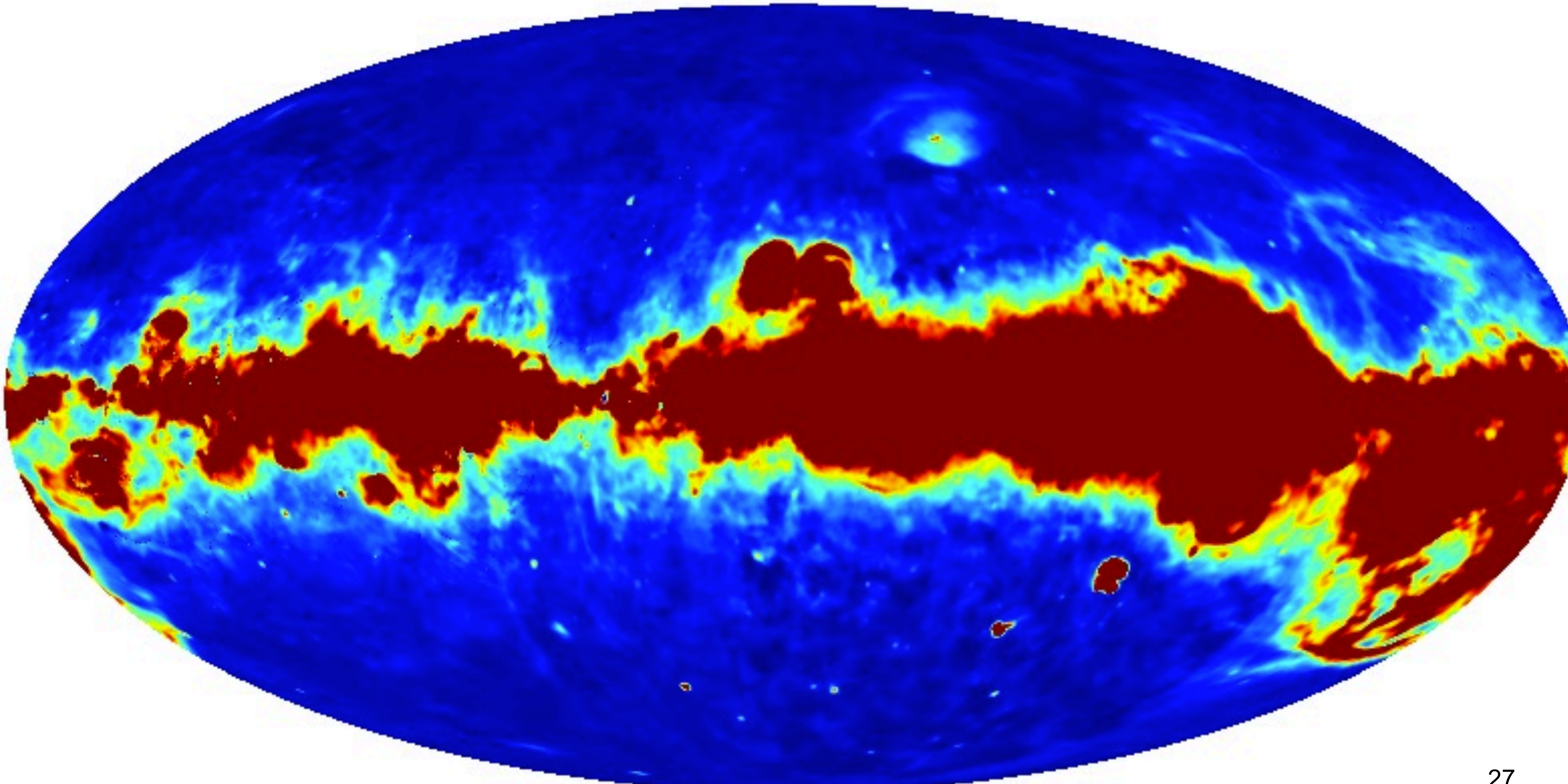
61 GHz [unpolarized]



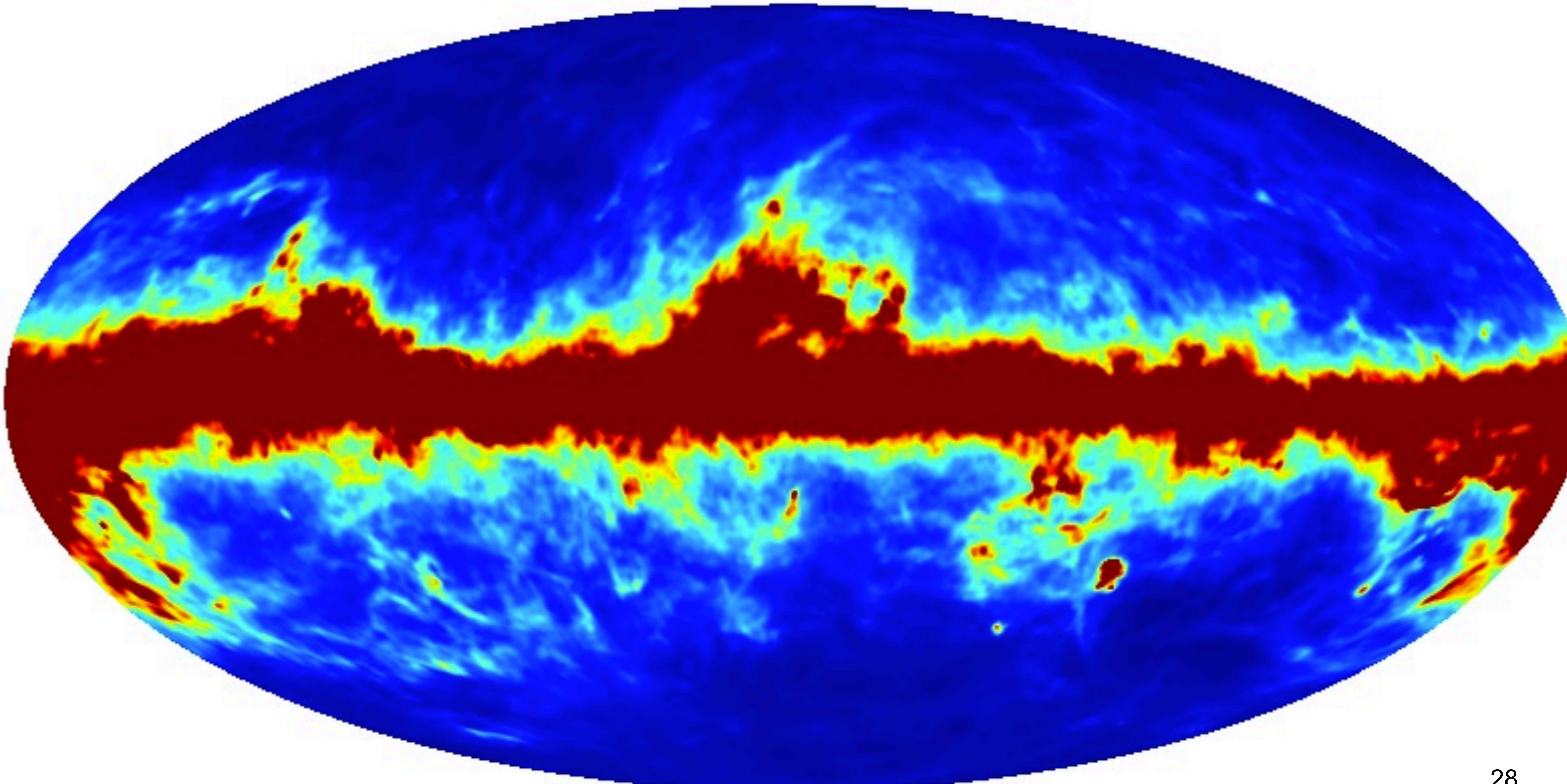
94 GHz [unpolarized]



Free-free template



Dust template



How to estimate C_l?

$$-2\ln(\mathrm{PDF}) = ([\mathrm{data}]_i - [\mathrm{stuff}]_i)^T (C^{-1})_{ij} ([\mathrm{data}]_j - [\mathrm{stuff}]_j) + |C|$$

- An estimator for C_l (i.e., the best-fit value of C_l) is given by maximizing PDF with respect to C_l:
  - **$d\ln(\mathrm{PDF})/dC_l = 0$**, yielding

$$\boxed{C_l = ([\mathrm{data}]_i - [\mathrm{stuff}]_i)^T (C_{\mathrm{fid}}^{-1})_{ij} (2l+1) P_l(\cos\theta_{jk}) (C_{\mathrm{fid}}^{-1})_{kp} ([\mathrm{data}]_p - [\mathrm{stuff}]_p),}$$

up to a normalization

$$C_l = ([\text{data}]_i-[\text{stuff}]_i)^T(C_{fid}^{-1})_{ij}(2l+1)P_l(\cos\theta_{jk})(C_{fid}^{-1})_{kp}([\text{data}]_p-[\text{stuff}]_p),$$

up to a normalization

- Now, notice that this expression has two problems:
  1. We need to assume a fiducial power spectrum, C_{fid}, to estimate the power spectrum
     - Solution: we can iterate until the answer converges
  2. We still need to evaluate $(C_{fid}^{-1})_{kp}([\text{data}]_p-[\text{stuff}]_p)$
     - Solution: conjugate gradient method (which works if we need to do this evaluation only a few times)

How to estimate $<C_l C_{l'}>$?

$$-2\ln(\text{PDF}) = ([\text{data}]_i - [\text{stuff}]_i)^T (C^{-1})_{ij} ([\text{data}]_j - [\text{stuff}]_j) + |C|$$

- We estimate the covariance of C_l from curvature of the PDF near the maximum:
  - $<d^2\ln(\text{PDF})/(dC_l dC_{l'})> = [\text{Cov}(C_l,C_{l'})]^{-1}$

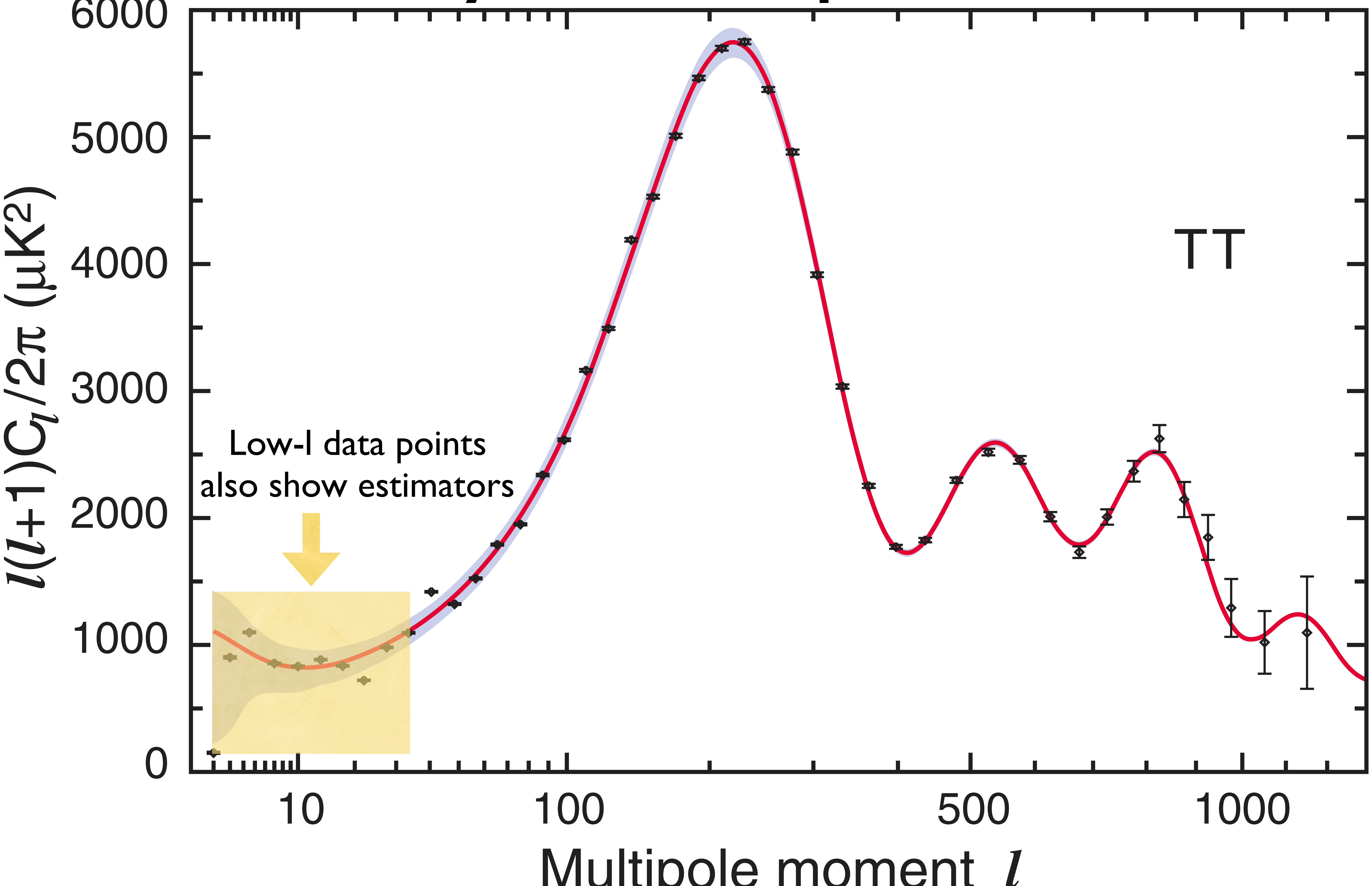
$$\boxed{\begin{aligned}&[\text{Cov}(C_l,C_{l'})]^{-1}\\ &= (1/2)(C^{-1})_{ij}(2l+1)P_l(\cos\theta_{jk})(C^{-1})_{kp}(2l'+1)P_{l'}(\cos\theta_{pi})]\end{aligned}}$$

WMAP Nine-year Approach

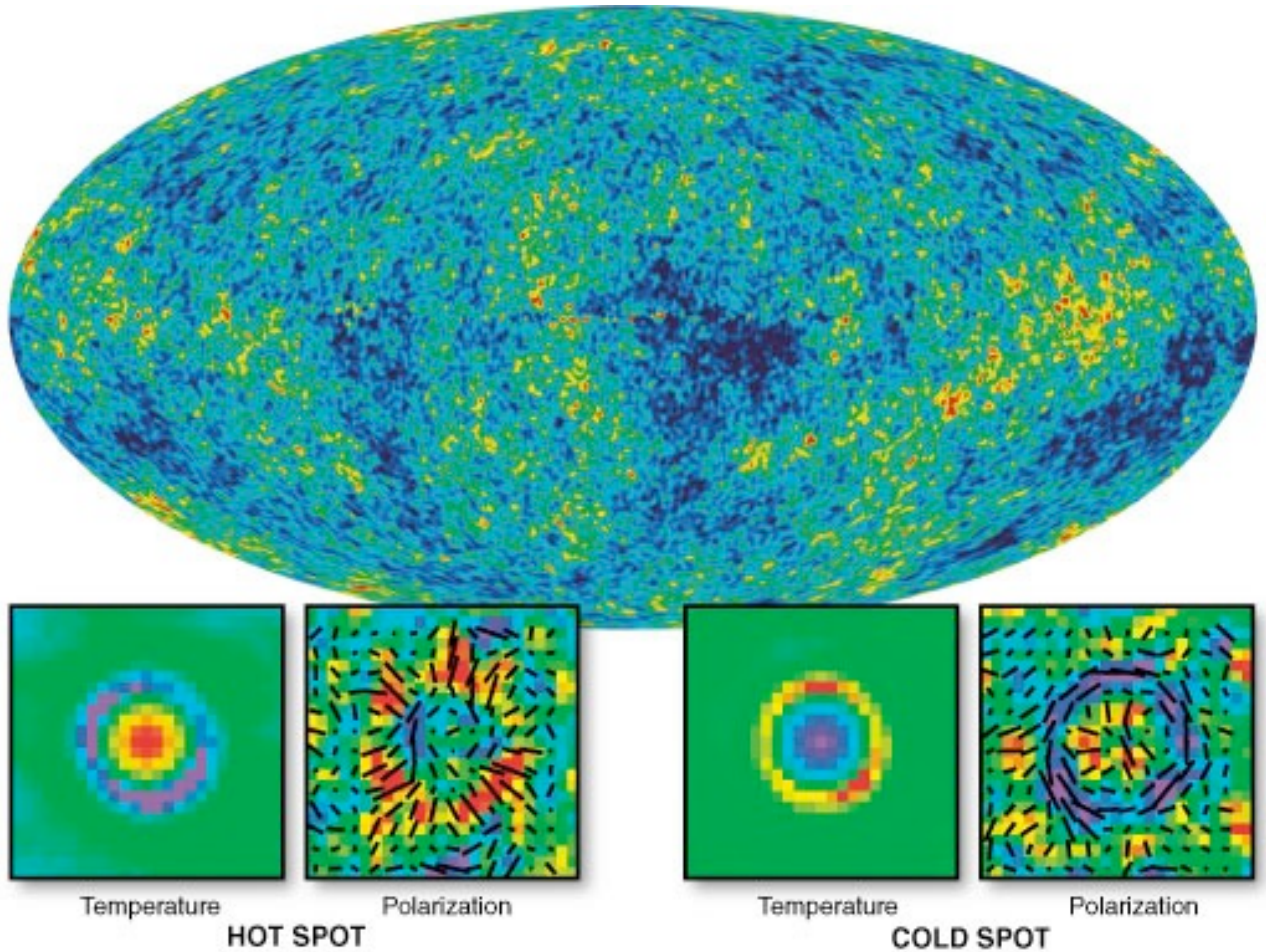
- We use a hybrid approach:
  - Use the exact PDF of [data]-[stuff] at low multipoles, $l<=32$. We do this not only because we can do it, but also because the PDF is highly non-Gaussian
  - Use the estimator at high multipoles, $32<l<=1200$, and construct an approximate, nearly-Gaussian PDF of C_l from $\mathrm{Cov}[C_l,C_{l'}]$

Bennett et al. (2013)

Nine-year Temperature C_l

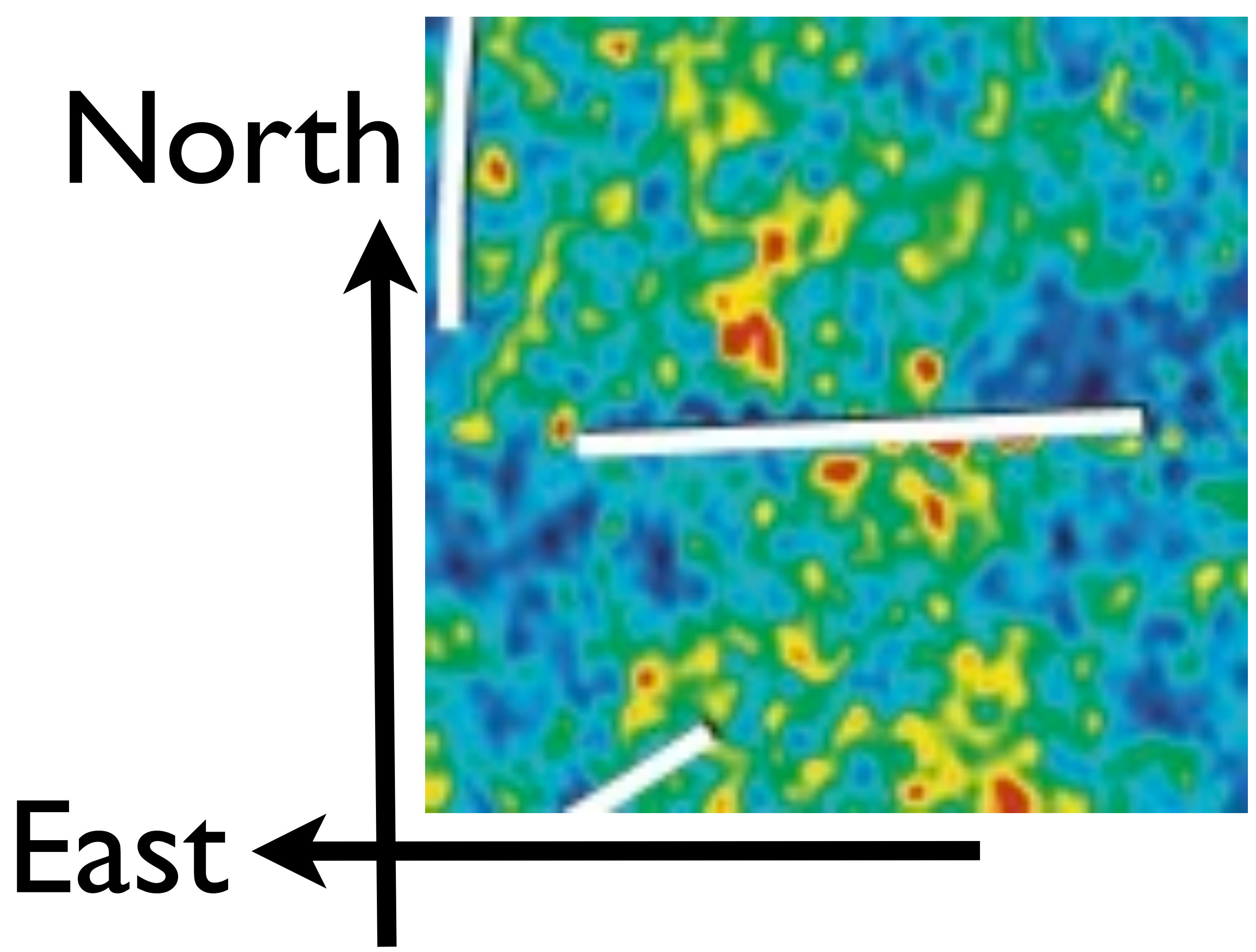


CMB Polarization

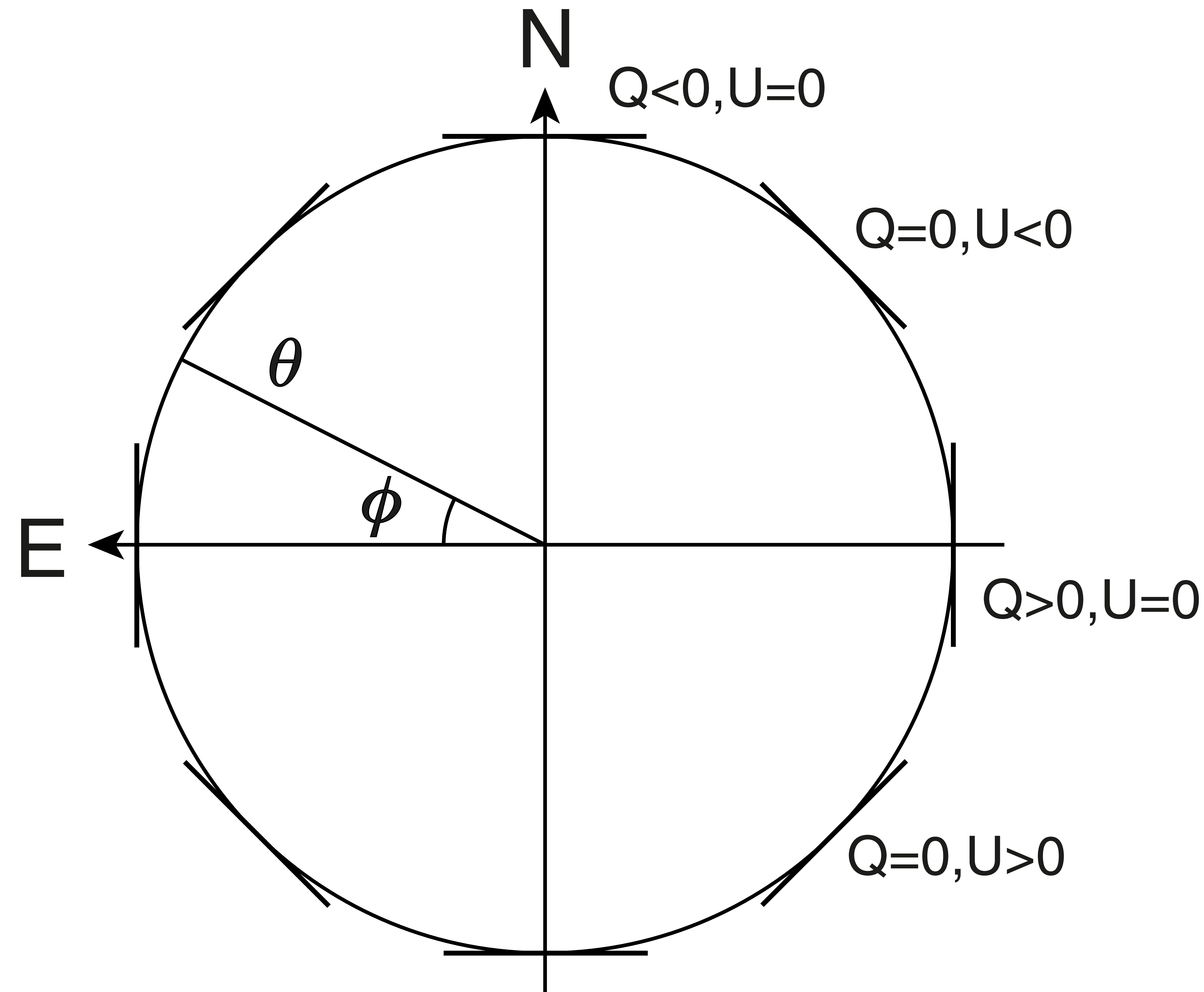


- *CMB is (very weakly) polarized!*

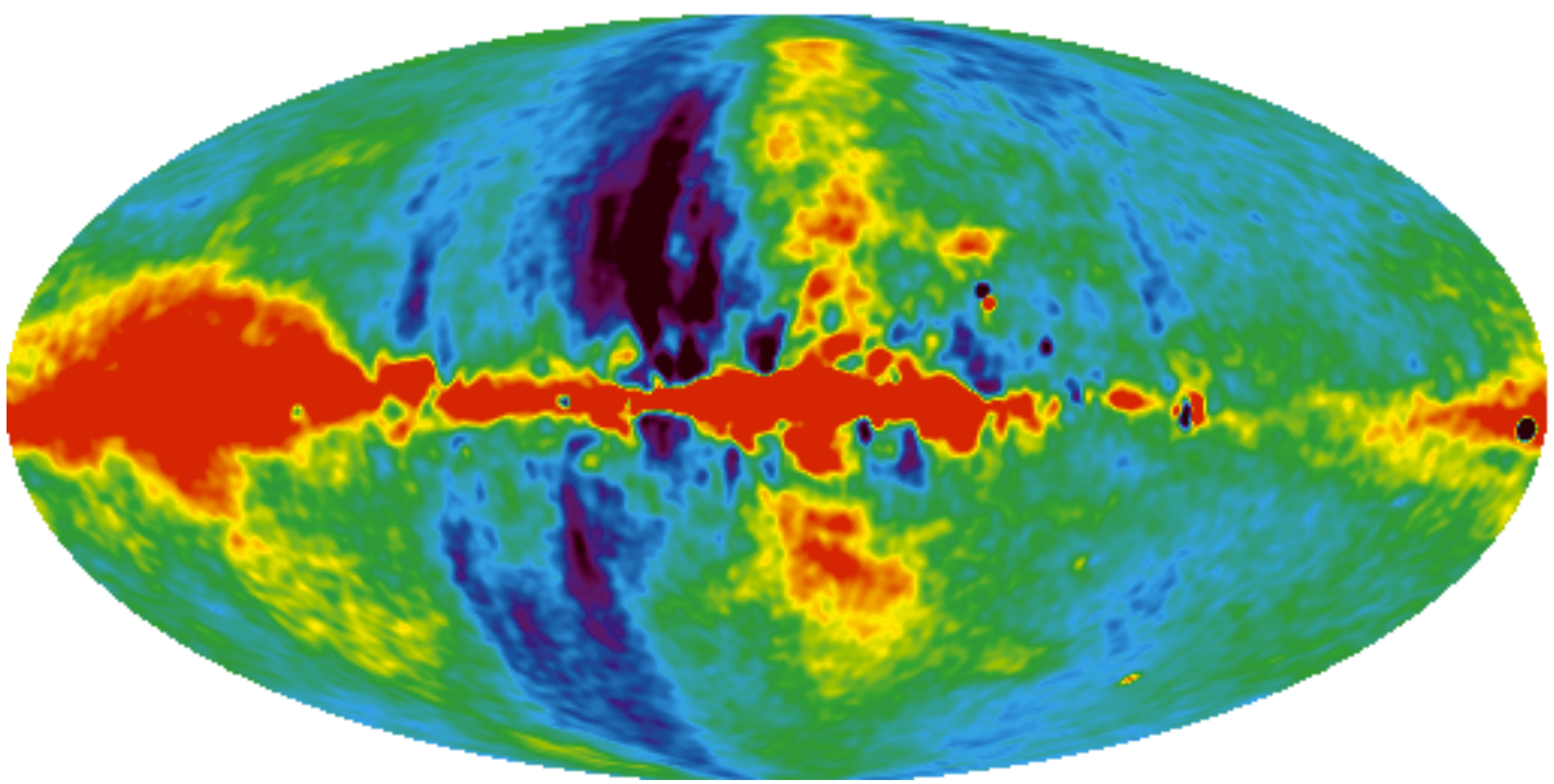
"Stokes Parameters"



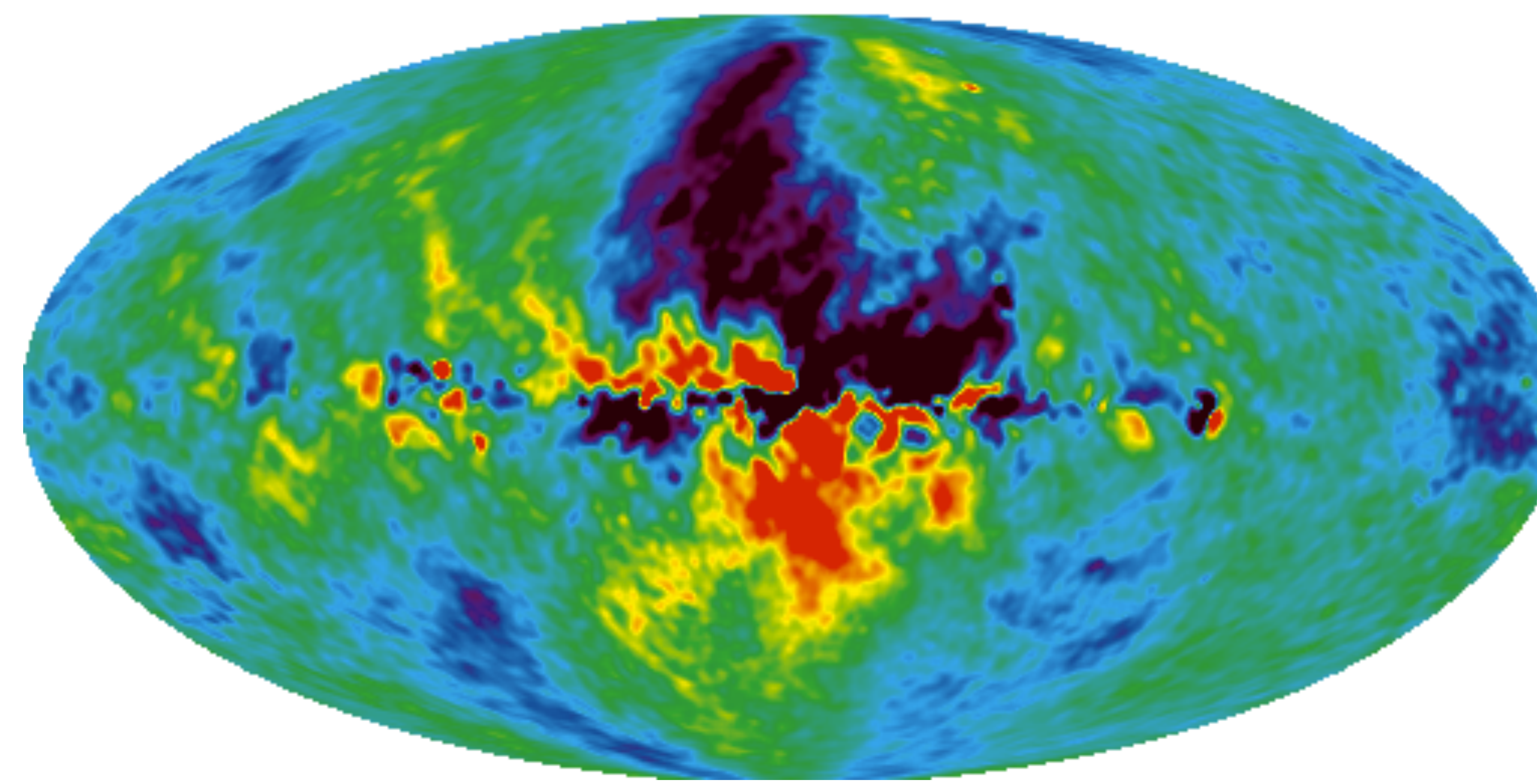
Q<0; U=0



23 GHz [polarized]

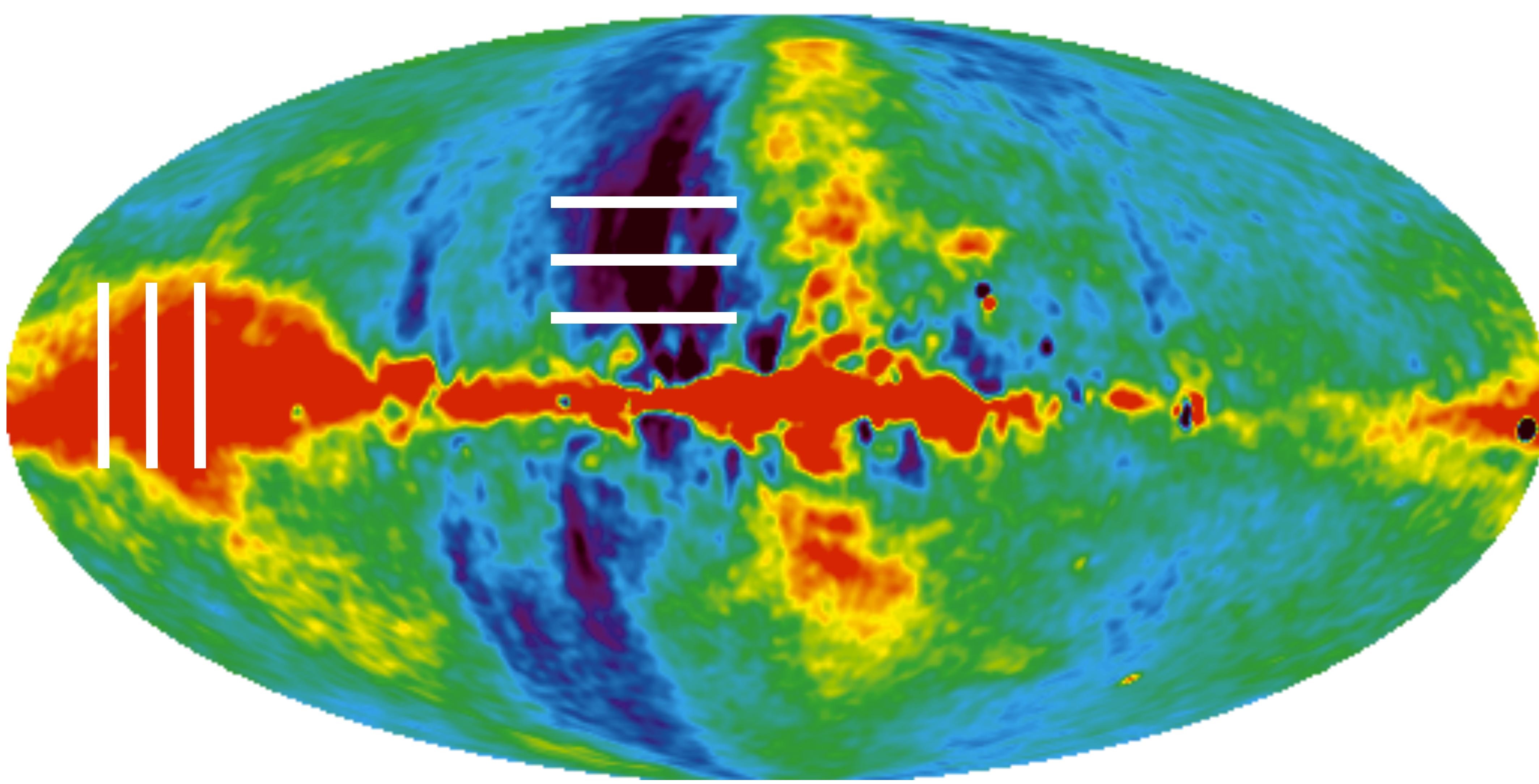


Stokes Q

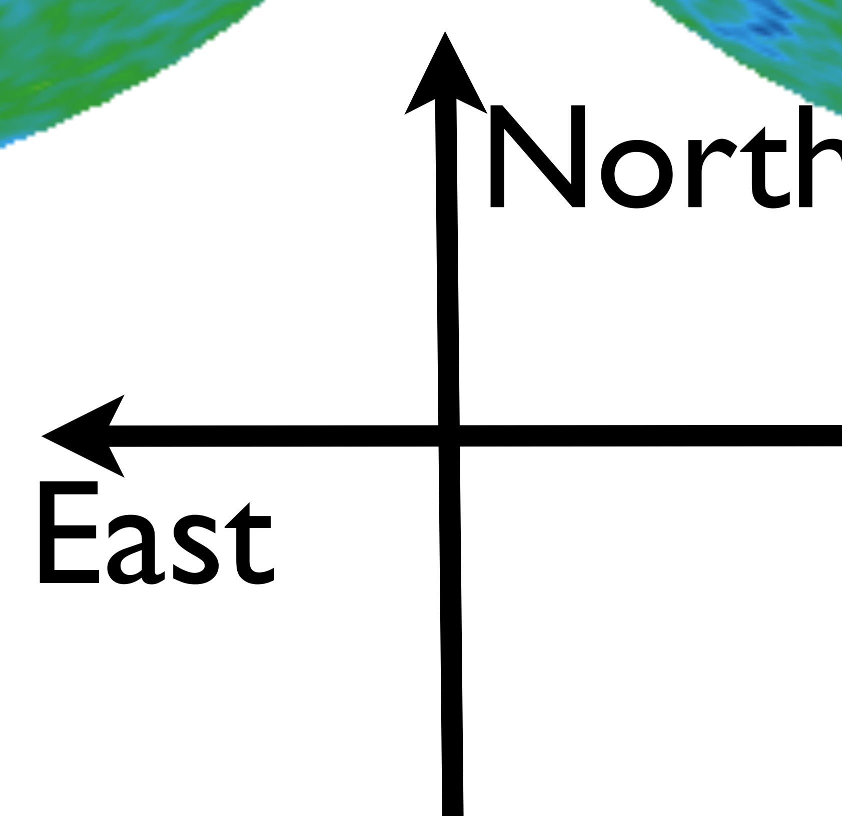


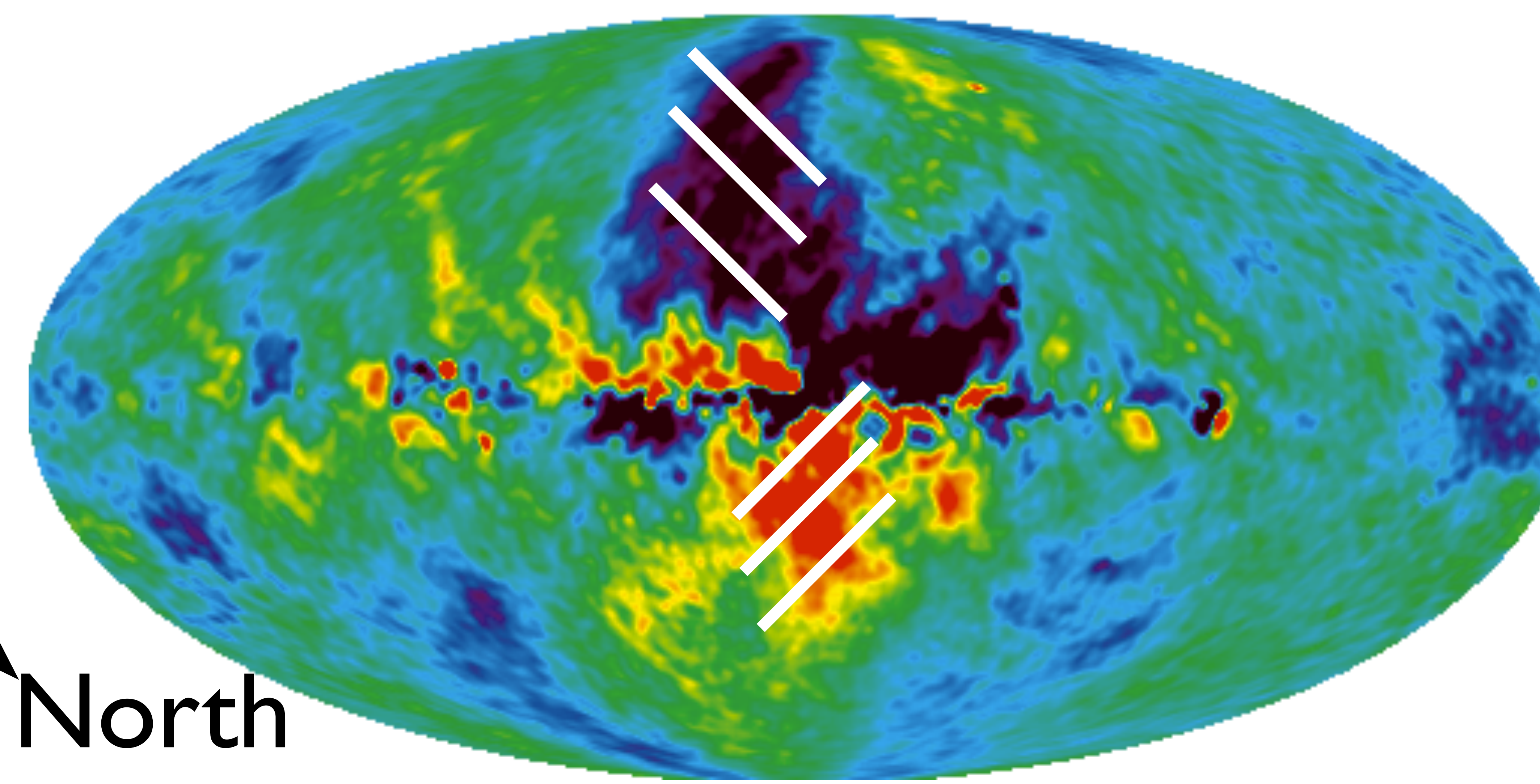
Stokes U

23 GHz [polarized]



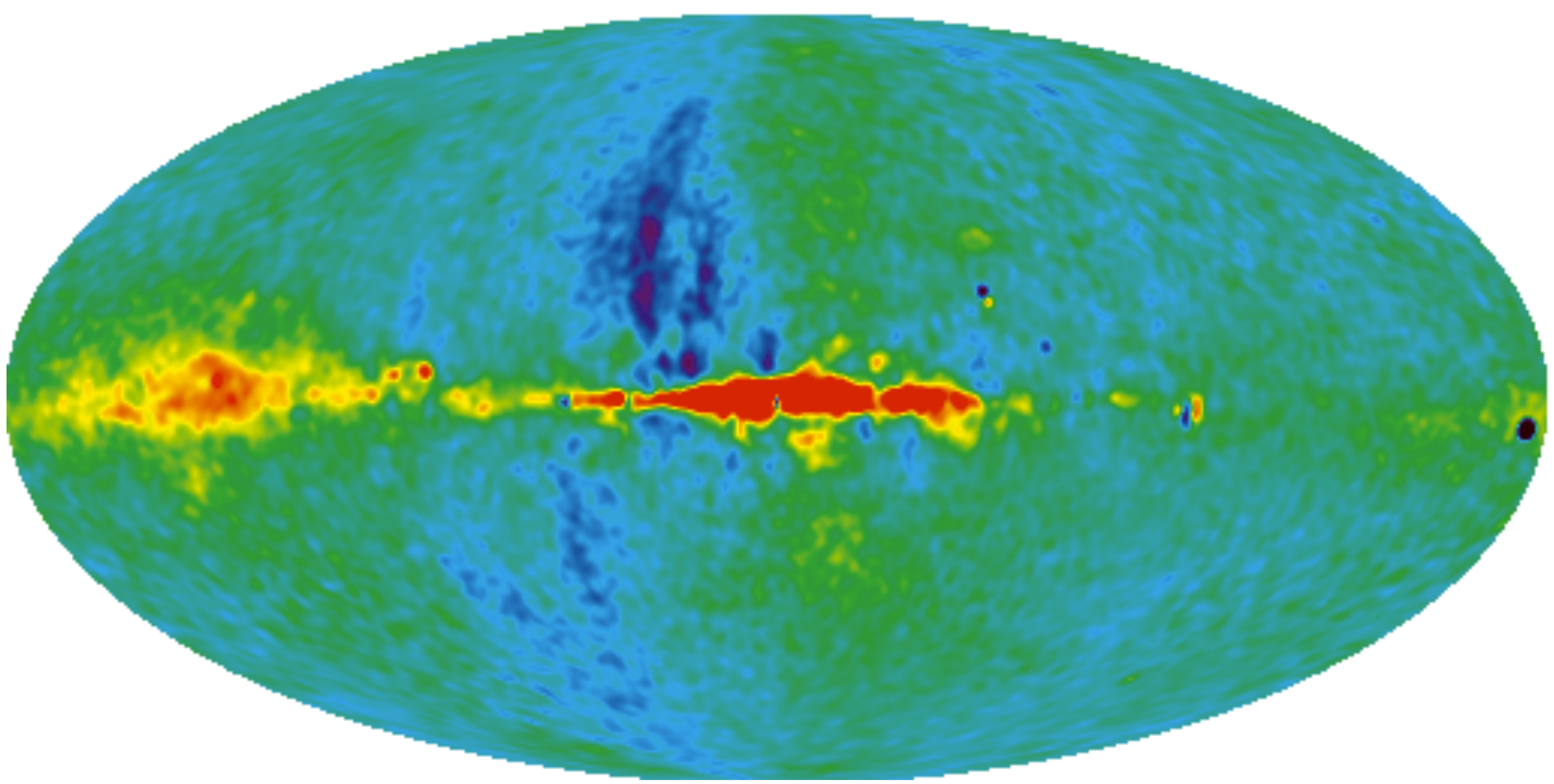
Stokes Q



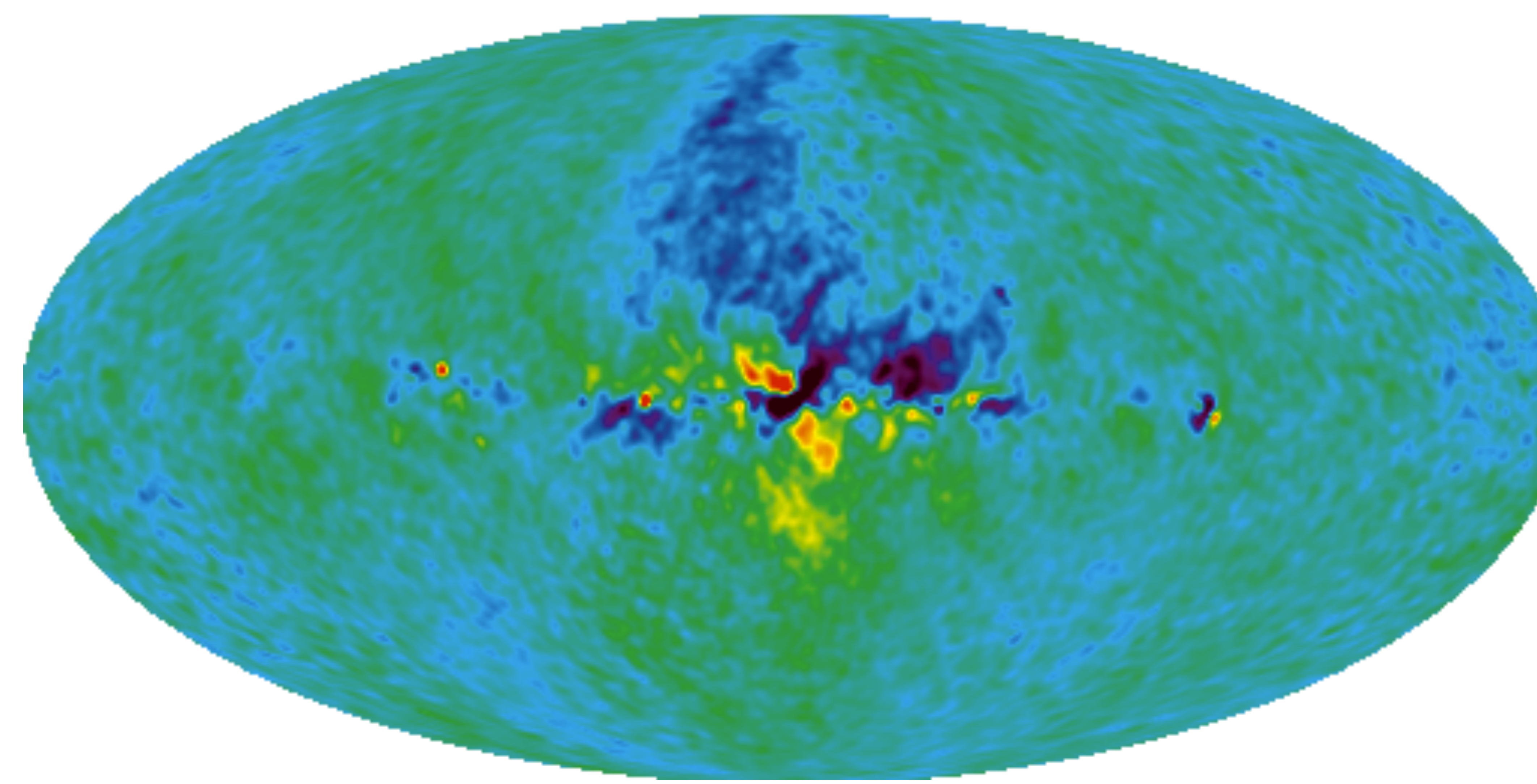


Stokes U

33 GHz [polarized]

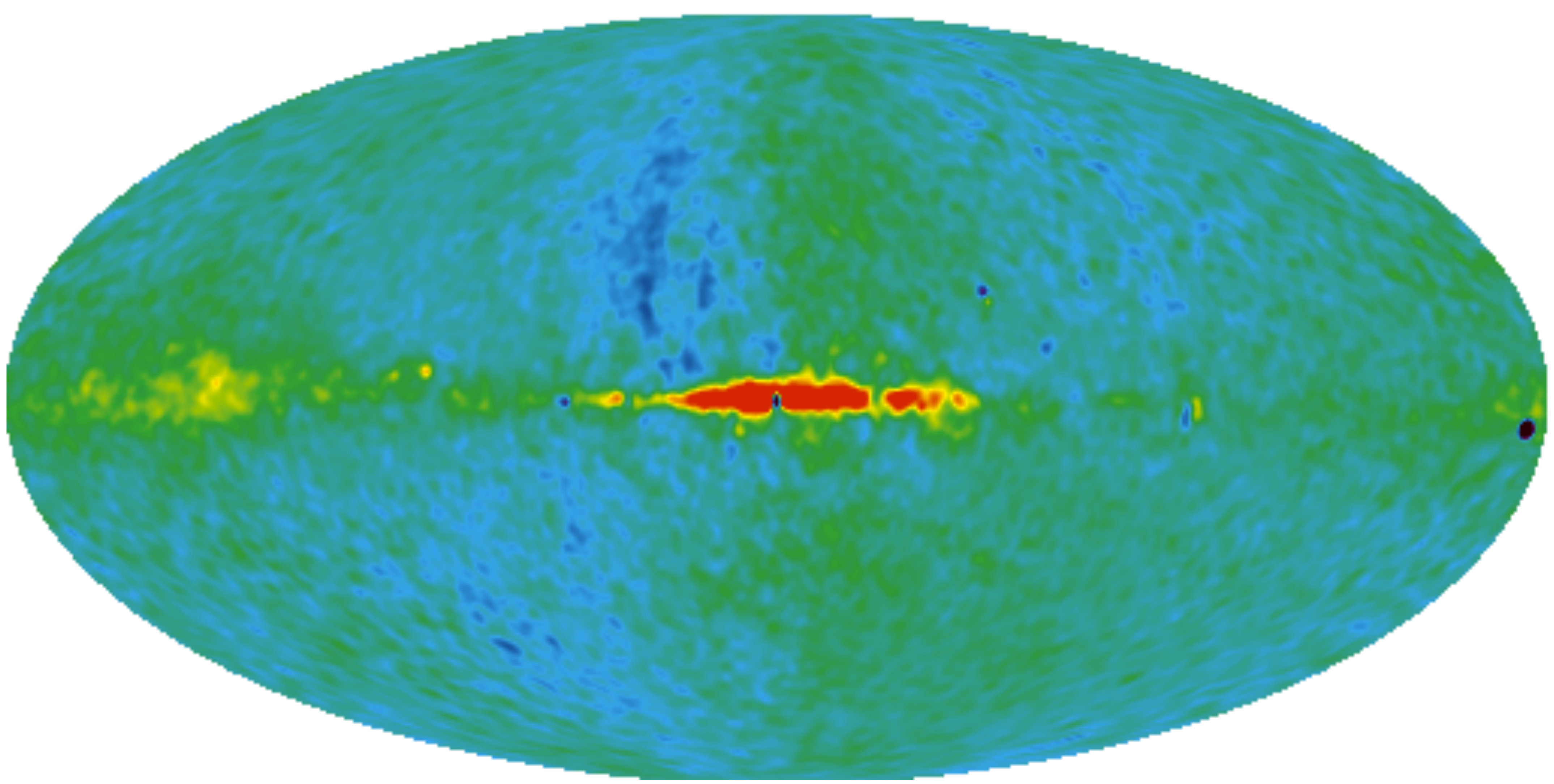


Stokes Q

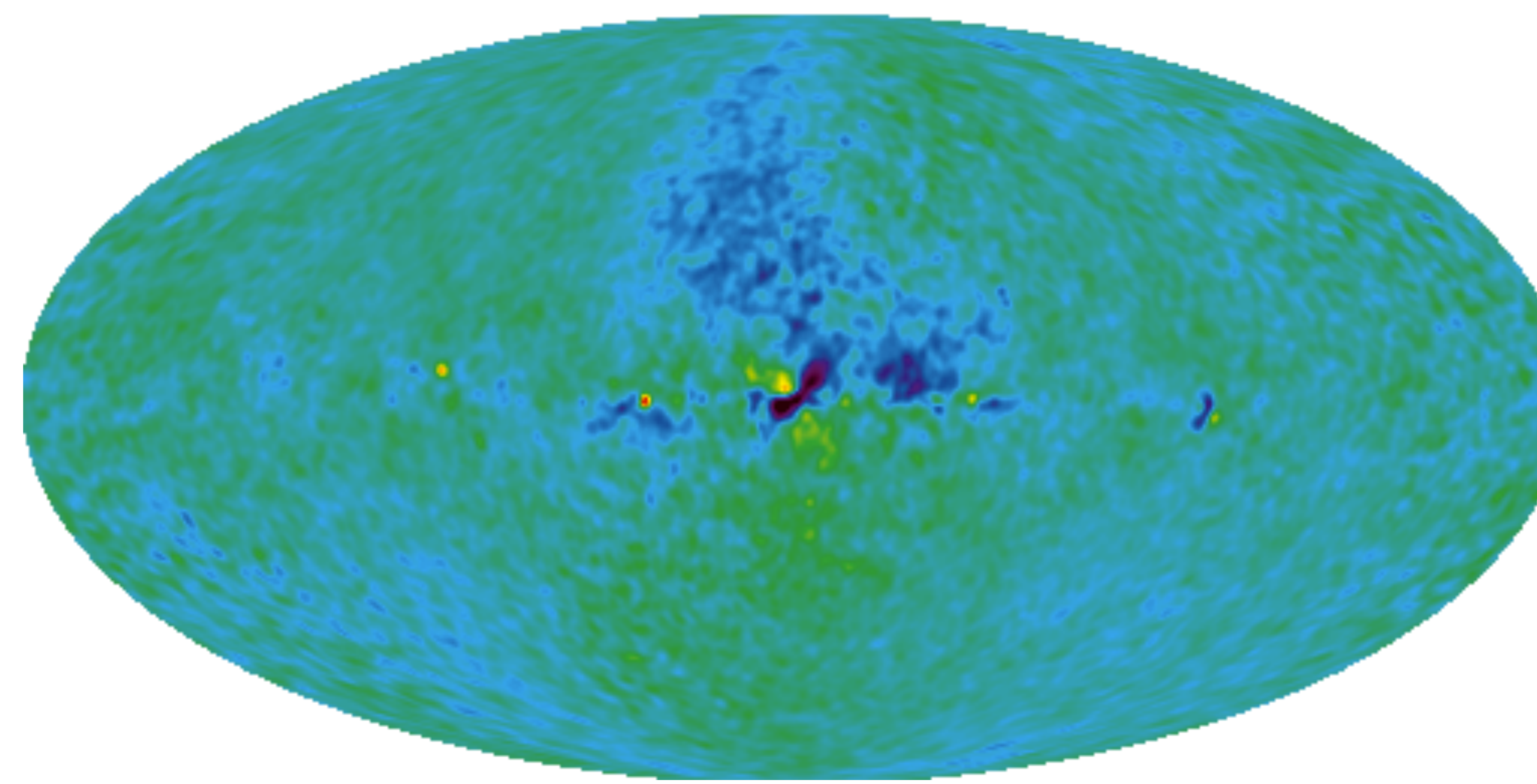


Stokes U

41 GHz [polarized]

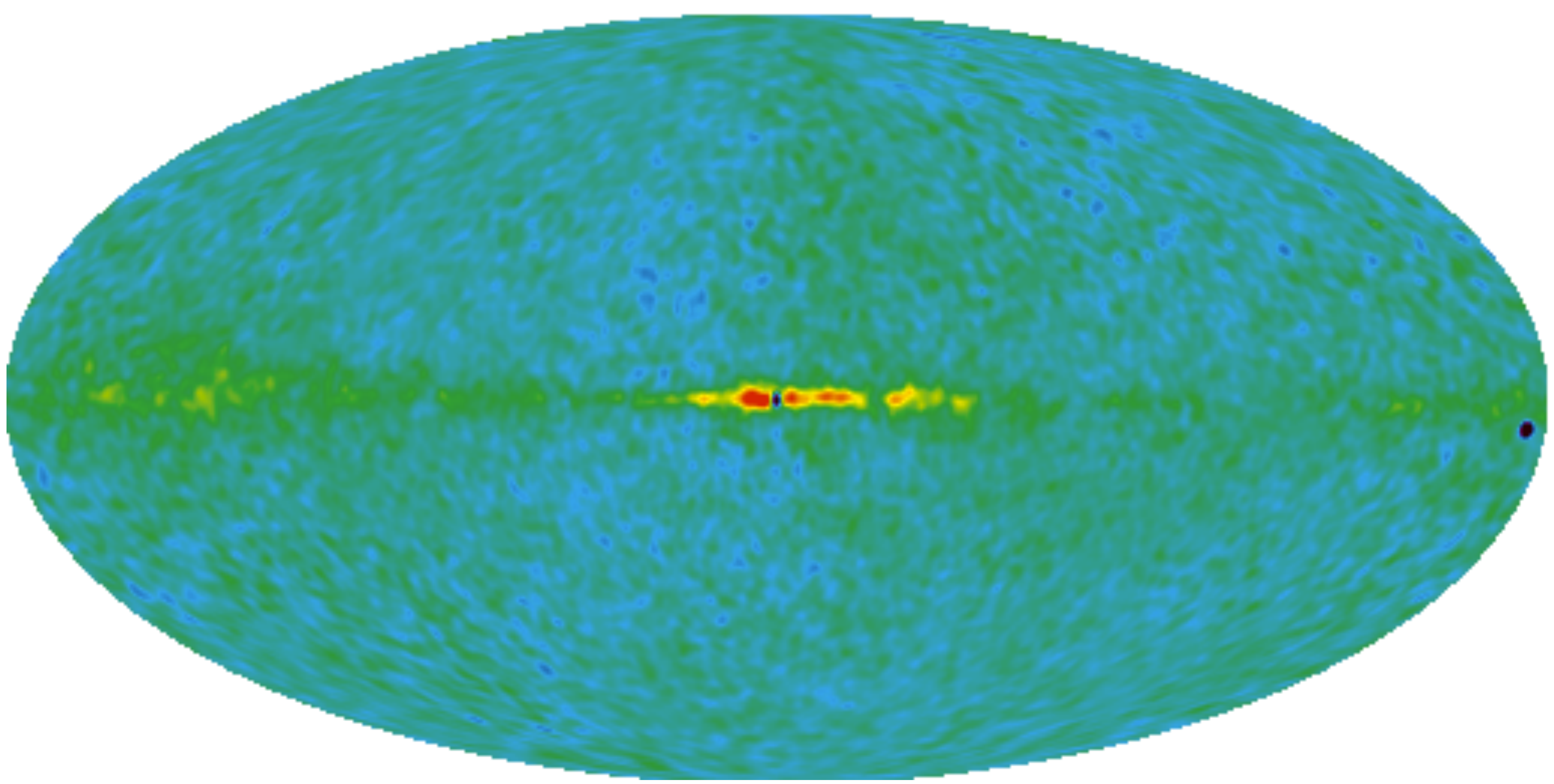


Stokes Q

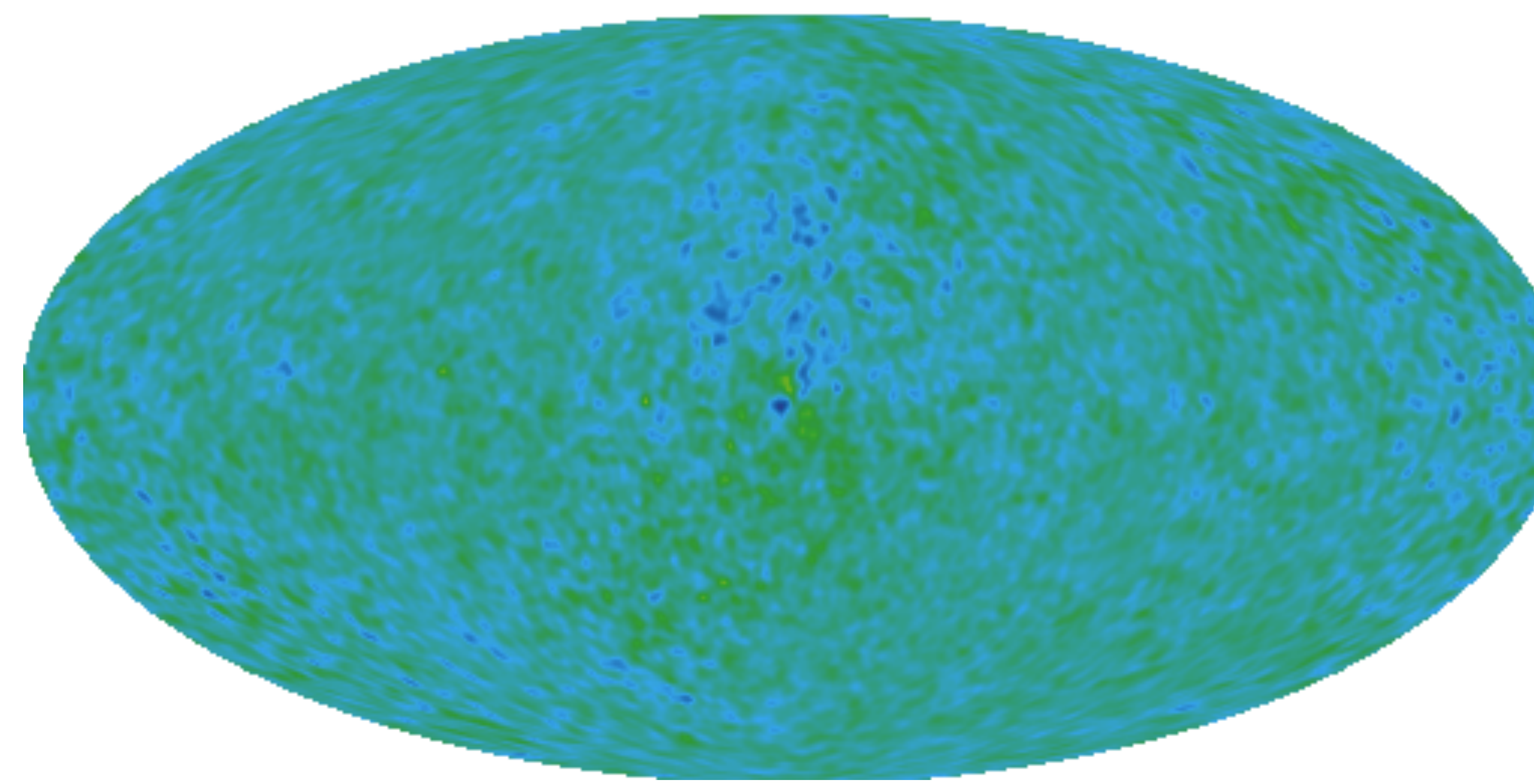


Stokes U

61 GHz [polarized]

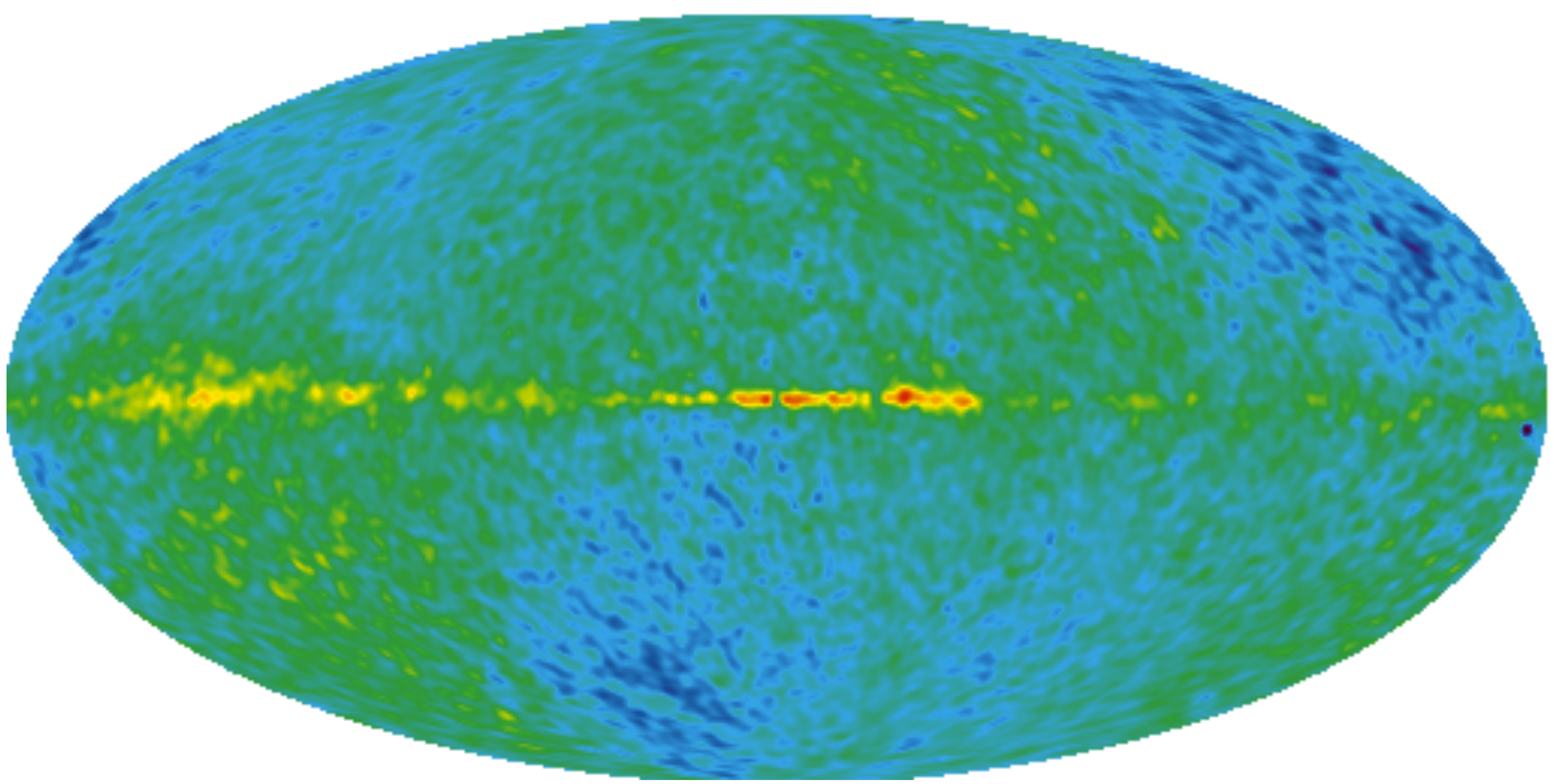


Stokes Q

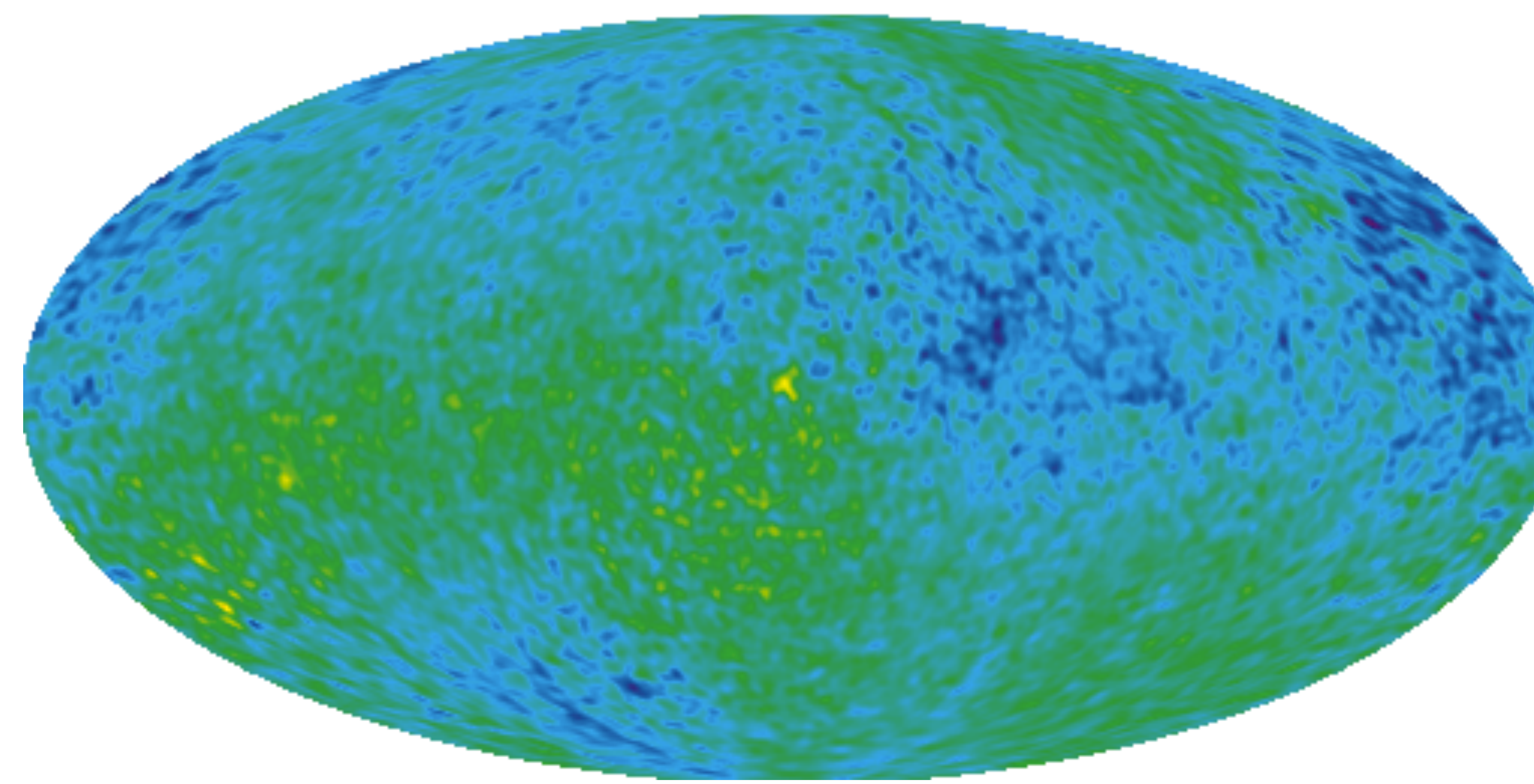


Stokes U

94 GHz [polarized]



Stokes Q



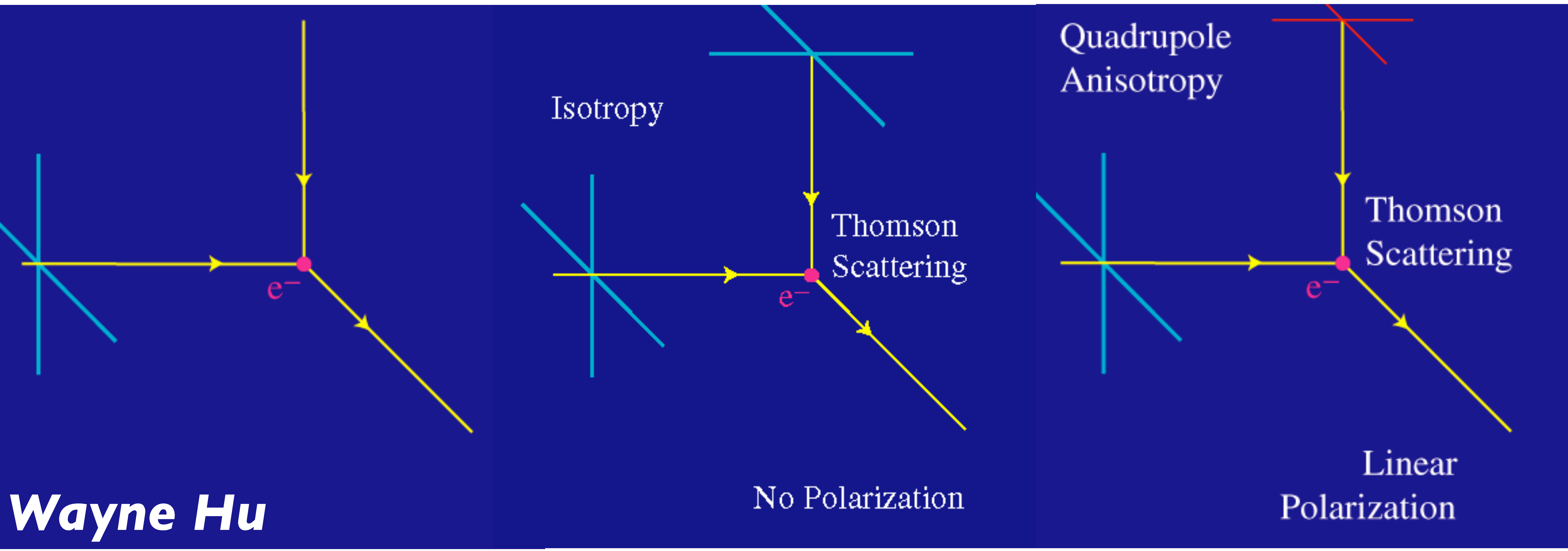
Stokes U

How many components?

1. **CMB**: $T_\nu \sim \nu^0$
2. **Synchrotron** (electrons going around magnetic fields): $T_\nu \sim \nu^{-3}$
3. ~~**Free-free** (electrons colliding with protons): $T_\nu \sim \nu^{-2}$~~
4. **Dust** (heated dust emitting thermal emission): $T_\nu \sim \nu^2$
5. ~~**Spinning dust** (rapidly rotating tiny dust grains): T_ν~complicated~~

*You need at least **THREE** frequencies to separate them!*

Physics of CMB Polarization

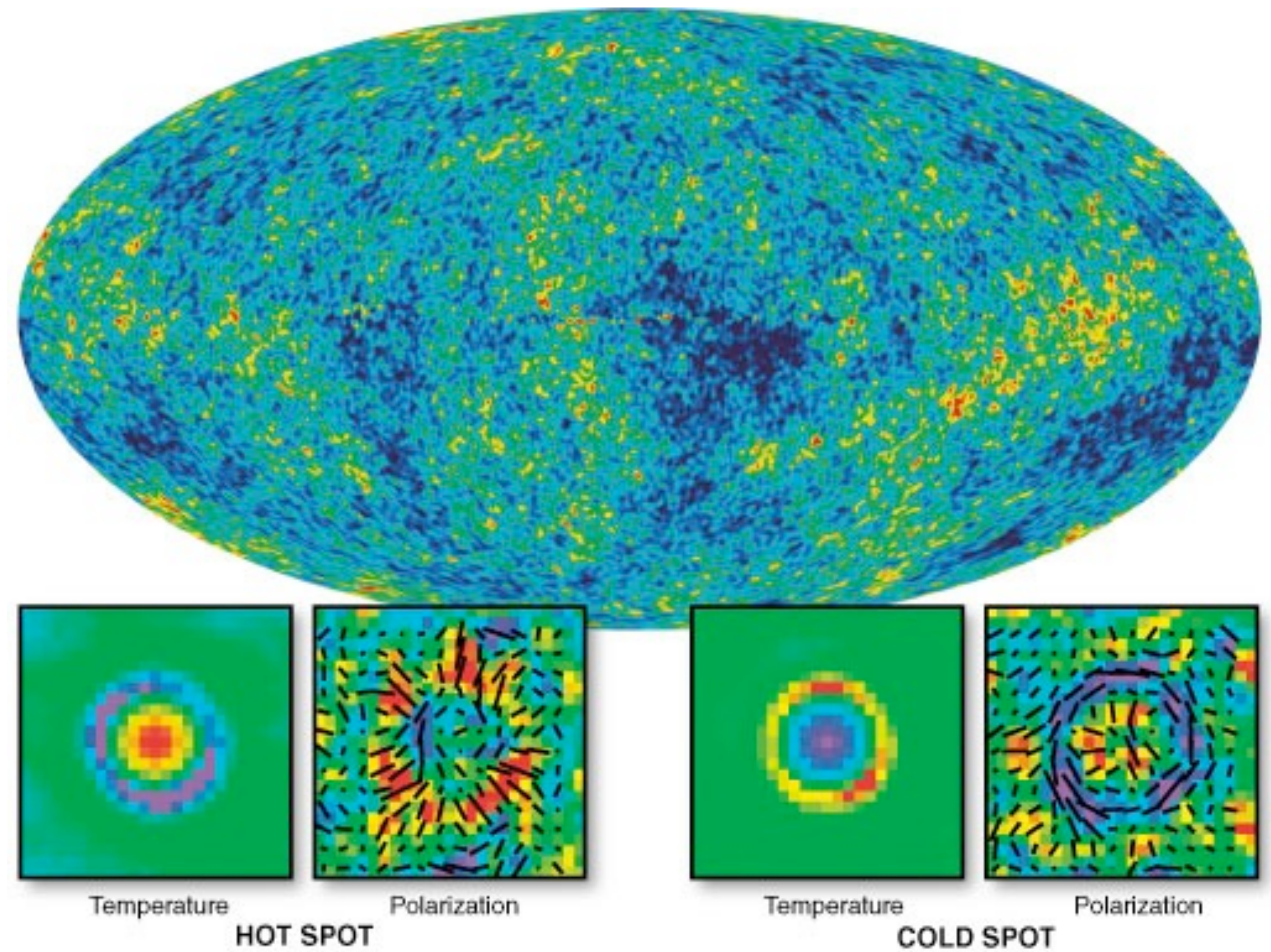


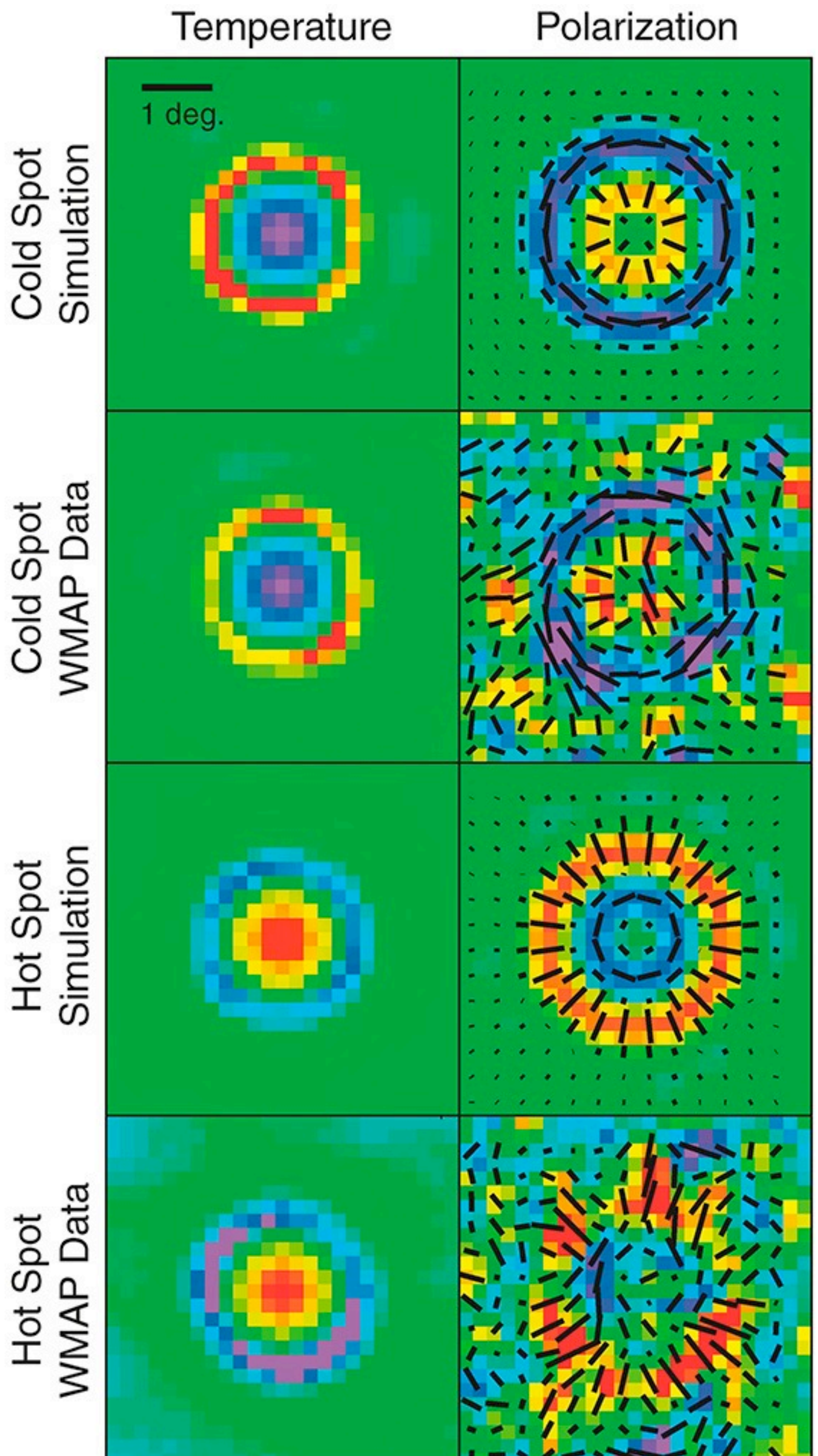
- CMB Polarization is created by a local temperature **quadrupole** anisotropy.

Stacking Analysis

- Stack polarization images around temperature hot and cold spots.

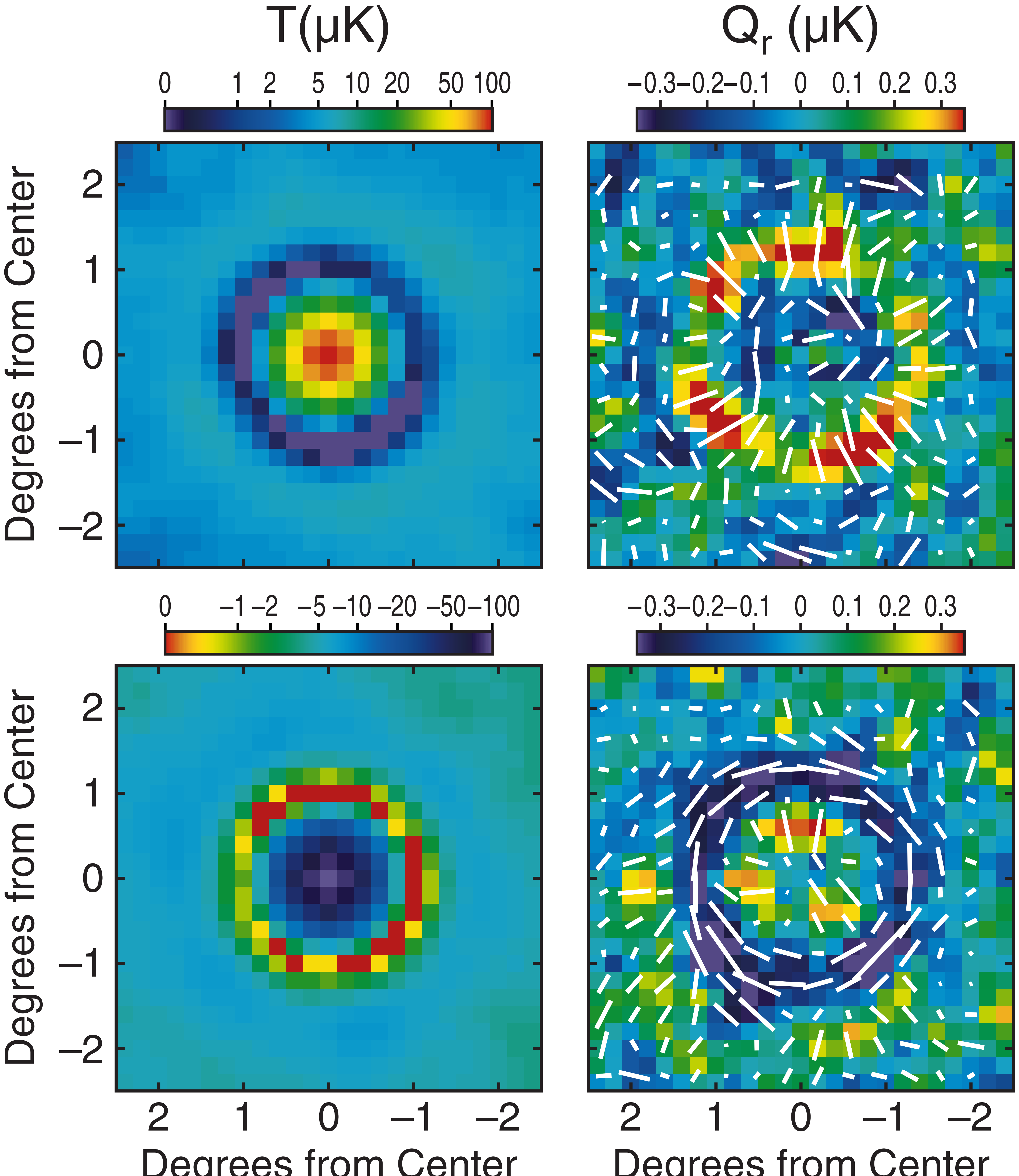
- Outside of the Galaxy mask (not shown), there are **11536 hot spots** and **11752 cold spots**.





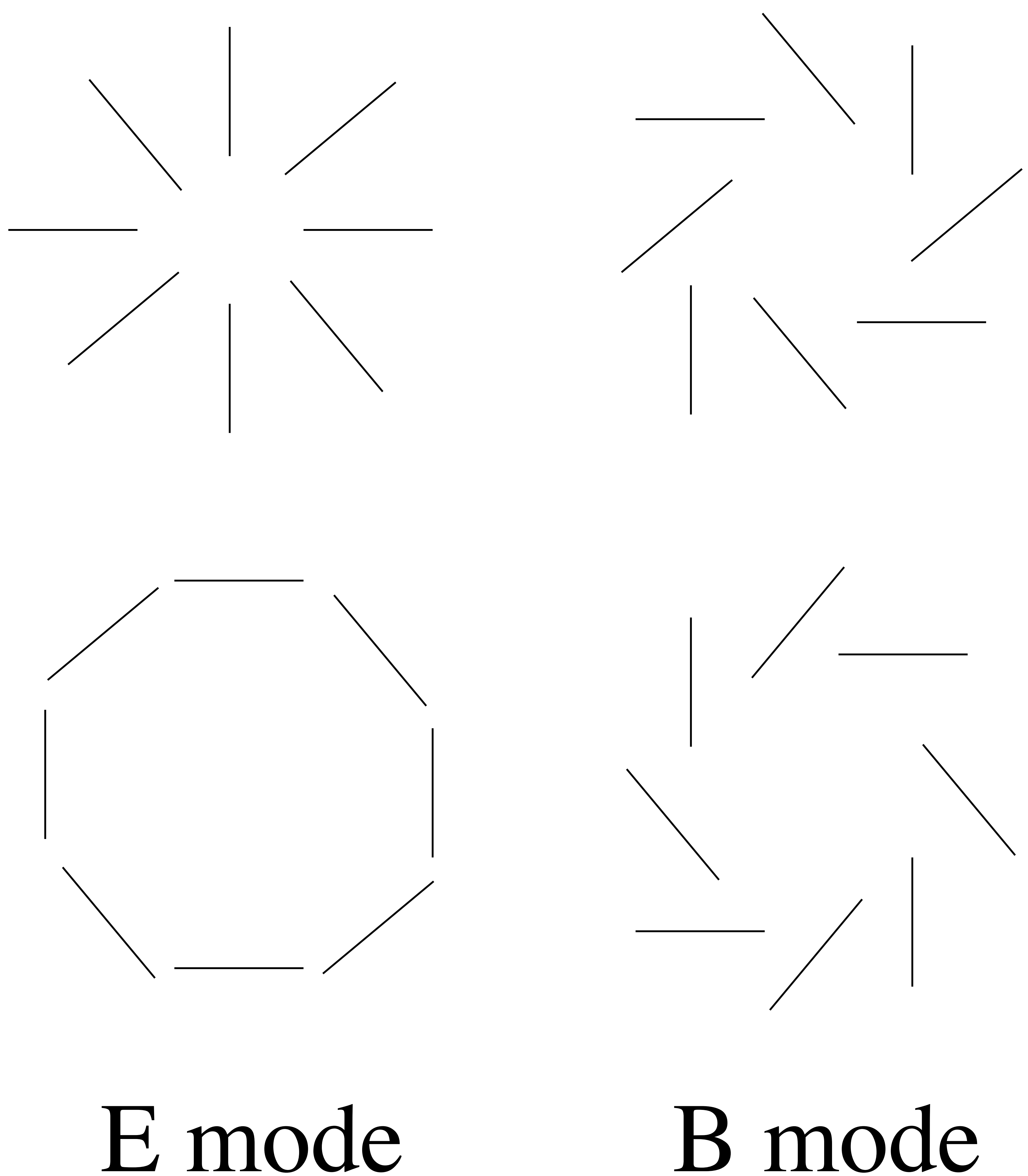
Radial and Tangential Polarization Patterns around Temp. Spots

- All hot and cold spots are stacked
- "Compression phase" at θ=1.2 deg and "slow-down phase" at θ=0.6 deg are predicted to be there and we observe them!
- The 7-year overall significance level: 8σ



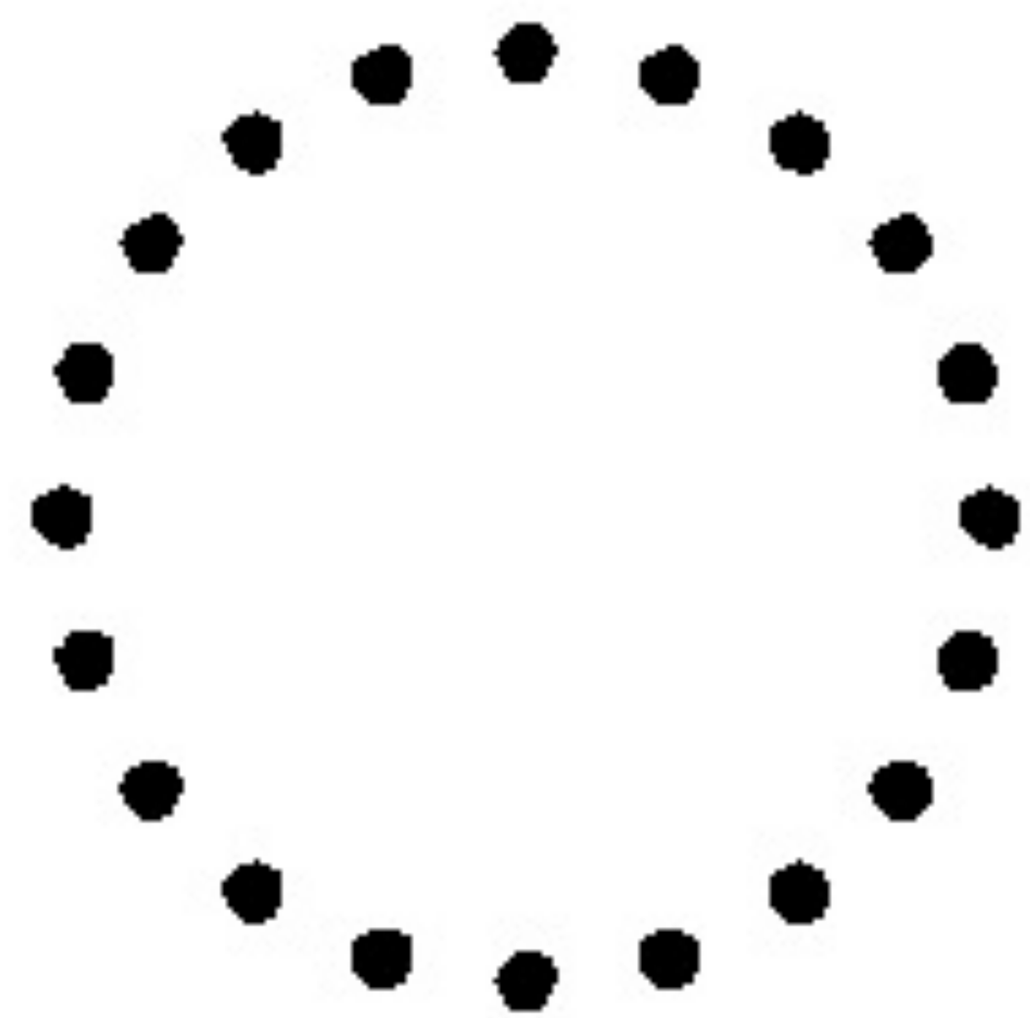
- The 9-year overall significance level: 10σ

E-mode and B-mode



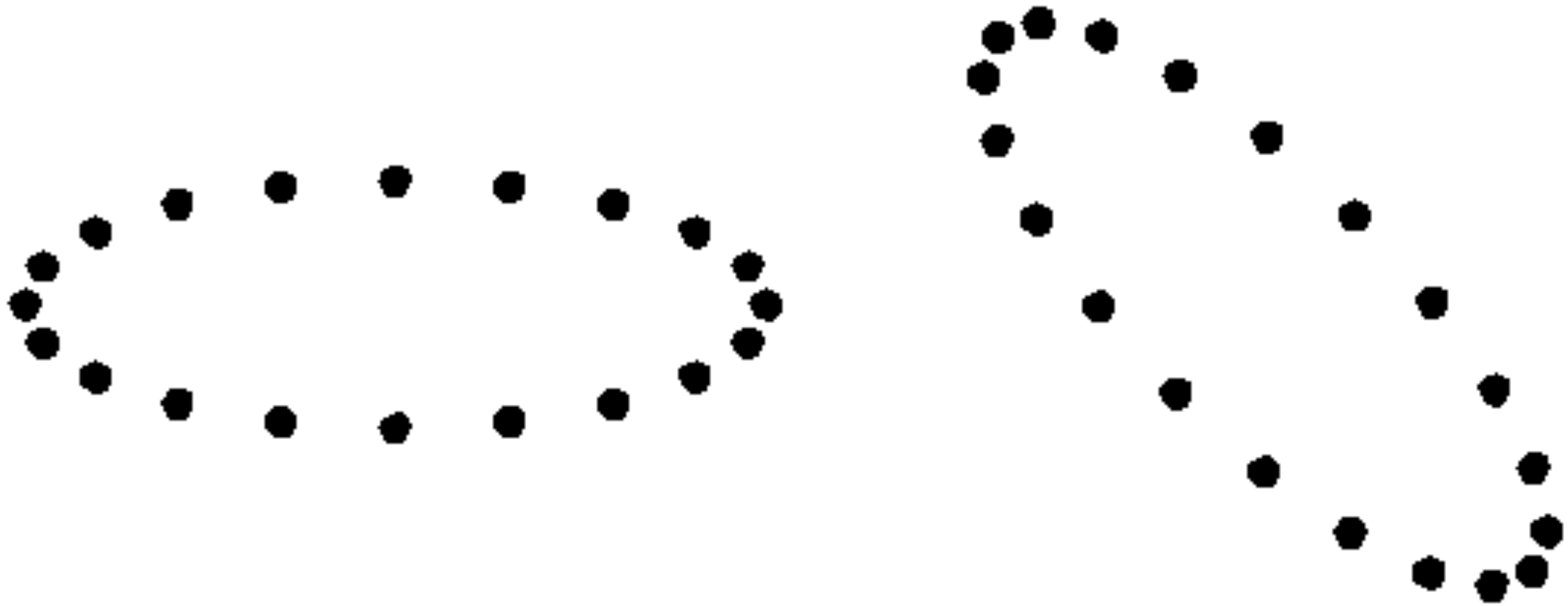
- Gravitational potential can generate the E-mode polarization, but not B-modes.
- **Gravitational waves** can generate both E- and B-modes!

Gravitational waves are coming toward you... What do you do?



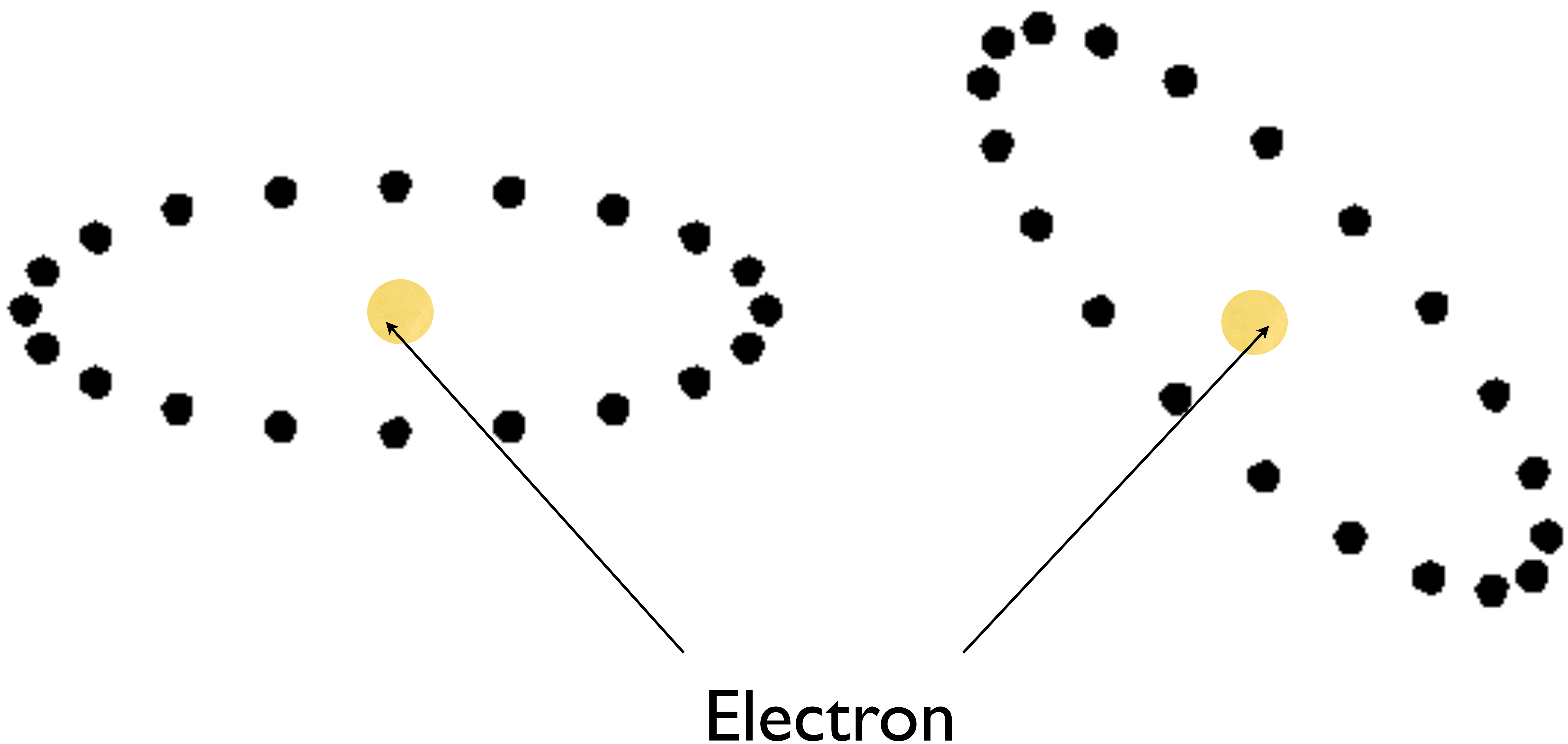
- Gravitational waves stretch space, causing particles to move.

Two Polarization States of GW

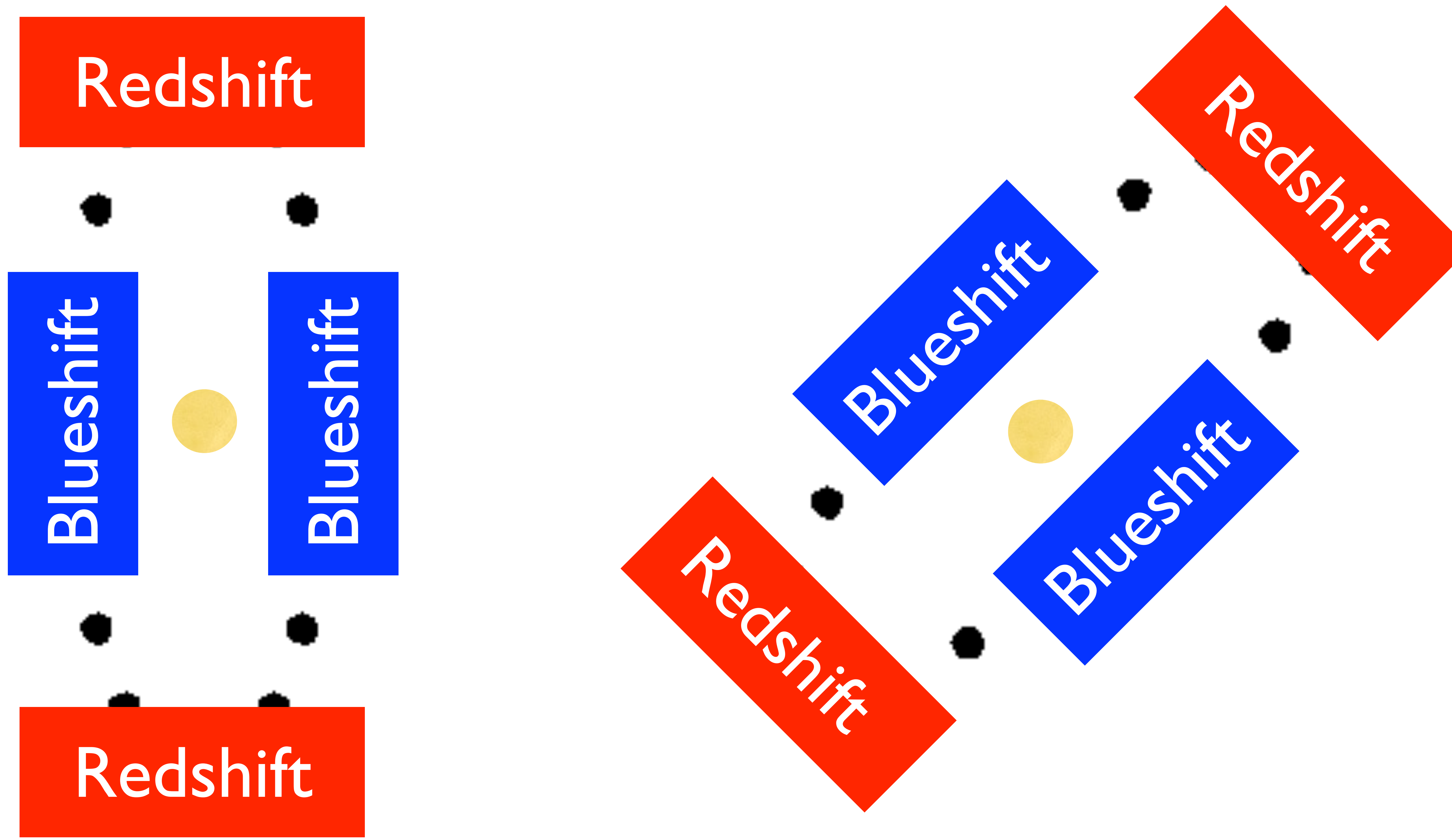


- This is great - this will automatically generate quadrupolar anisotropy around electrons!

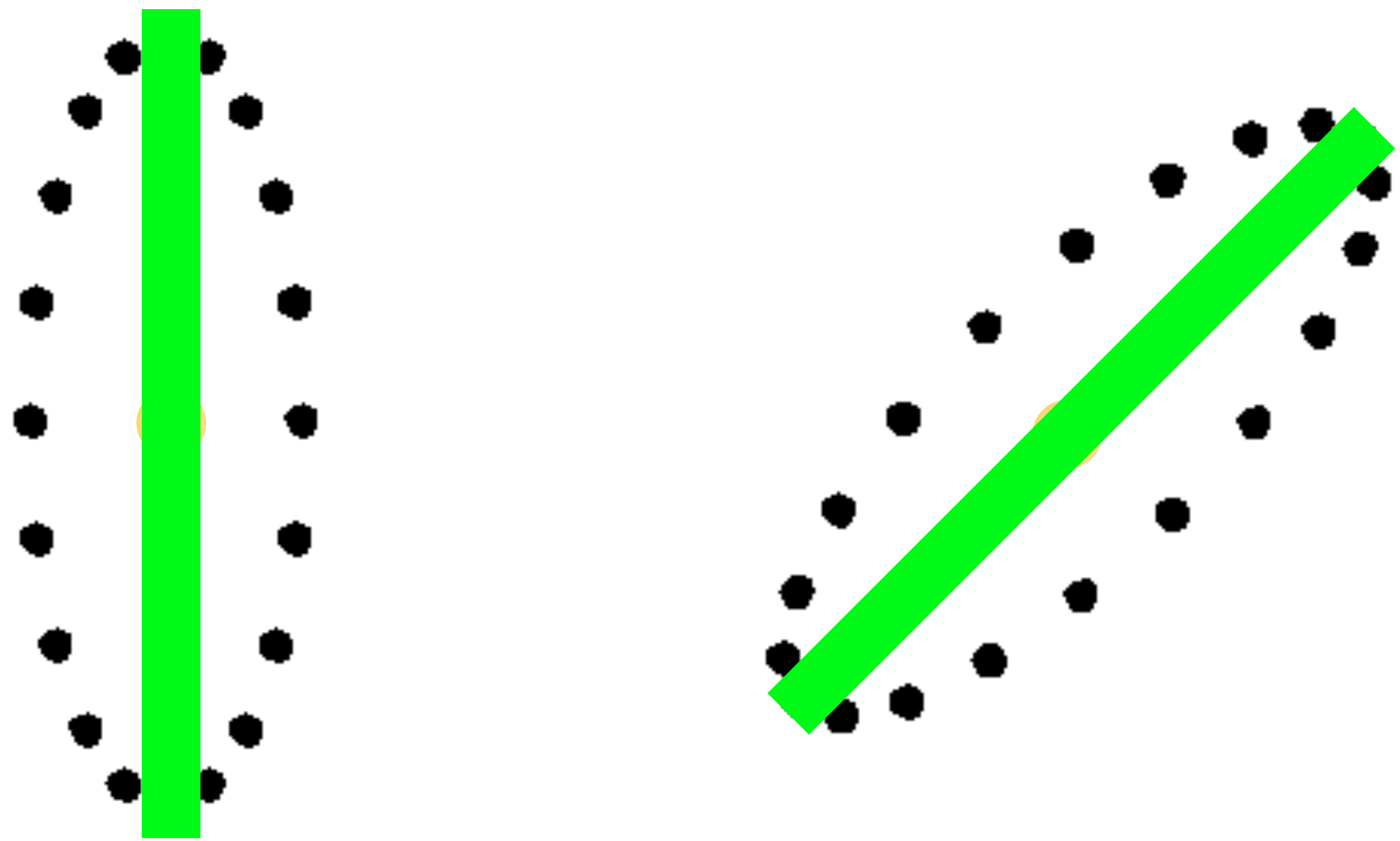
From GW to CMB Polarization



From GW to CMB Polarization



From GW to CMB Polarization



Gravitational waves can produce
both E- and B-mode polarization

Polarization Analysis

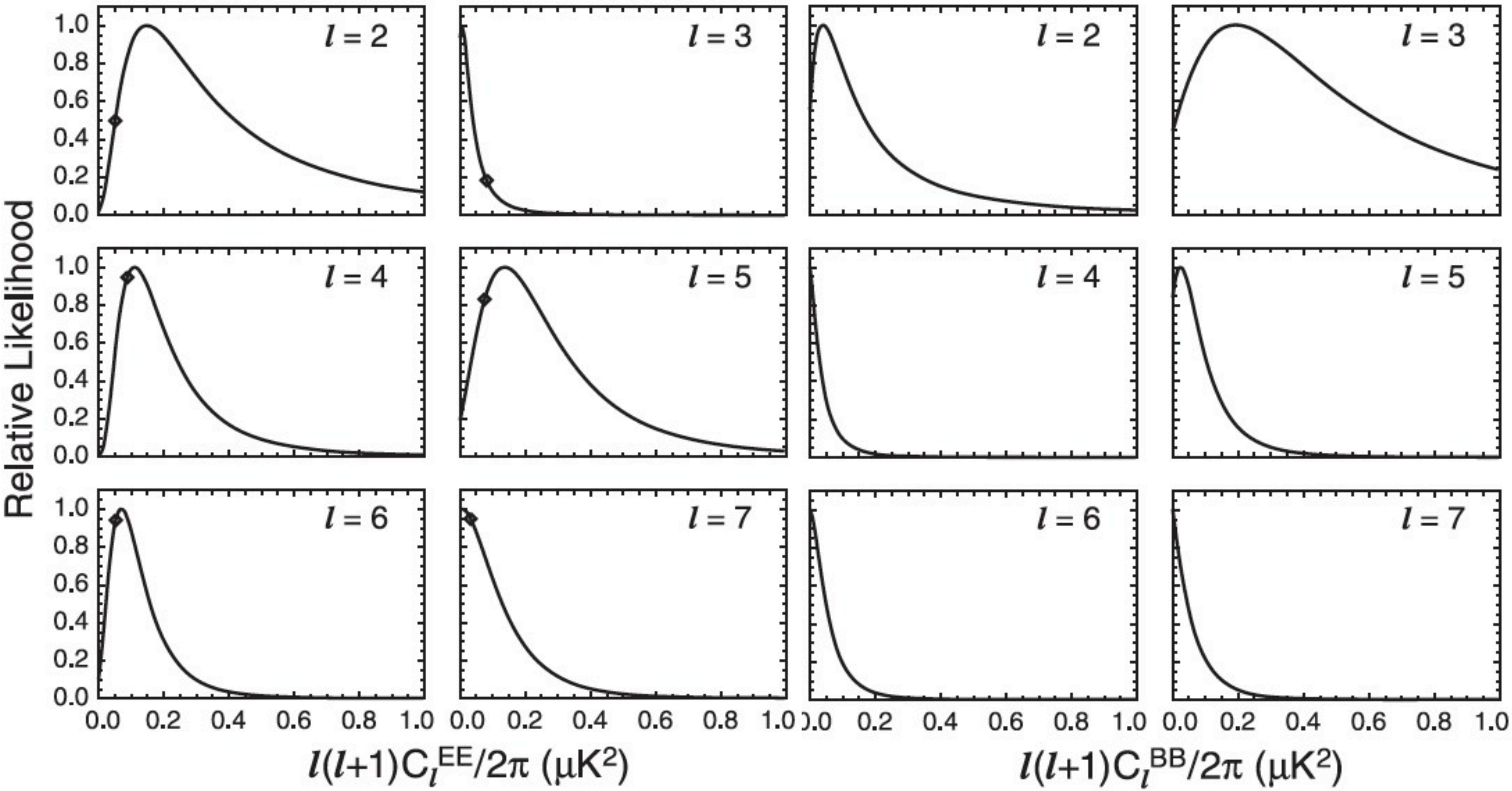
$$-2\ln(\mathrm{PDF}) = ([\mathrm{data}]_i - [\mathrm{stuff}]_i)^T (C^{-1})_{ij} ([\mathrm{data}]_j - [\mathrm{stuff}]_j) + |C|$$

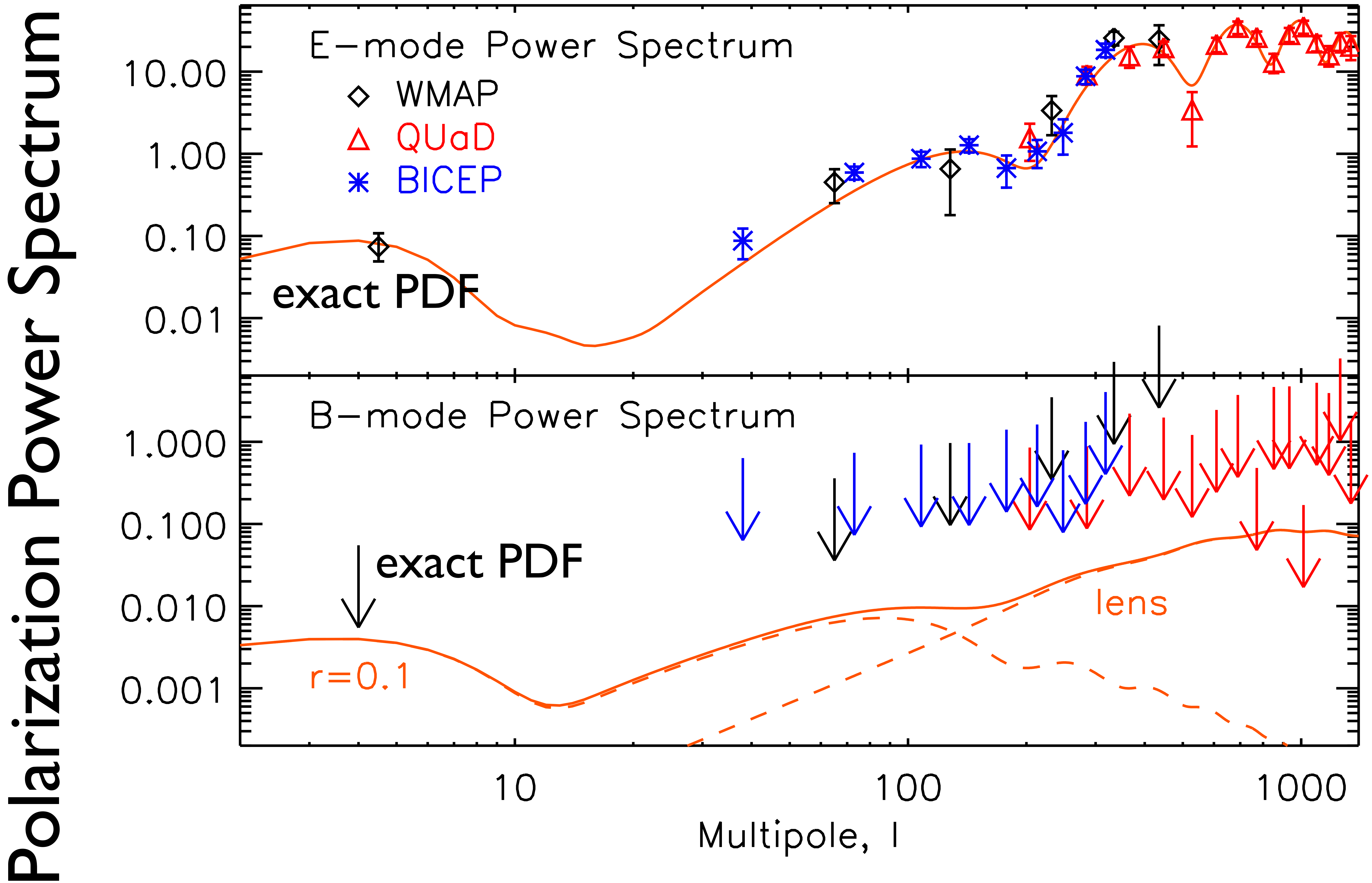
- The polarization data at l>~10 are totally dominated by noise, and thus we evaluate the exact PDF of [data]-[stuff] of polarization at low multipoles
- We again use internal/external template maps to remove [stuff]:

internal
- ***Synchrotron emission***: Use 23 GHz map

external
- ***Thermal dust emission***: Use a map of polarized star light

E-mode PDF *B-mode PDF*





- No detection of B-mode polarization yet. **B-mode is the next holy grail!**

Beyond Power Spectrum

- What if your PDF is not Gaussian?

“Taylor-expanding PDF”

- When non-Gaussianity is expected to be weak, we can “Taylor-expand” PDF around a Gaussian distribution
  - E.g., Gram-Charlier expansion

Gram-Charlier Expansion

- Let $G(x)$ be a 1-d Gaussian, $G(x)=\exp(-x^2/2)/\mathrm{sqrt}(2\pi)$
- Then, a PDF, $P(x)$, would be expanded as
  - $P(x)=\sum_n c_n\, d^nG/dx^n$

    $= G(x)[c_0+(-1)c_1x+c_2(x^2-1)+(-1)c_3(x^3-3x)+\ldots]$

    $= G(x)\sum_n(-1)^n c_n\, He_n(x)$
- Inverting this, we get the coefficients in terms of P:
  - $n!c_n=(-1)^n\int dx\, P(x)He_n(x)$

Gram-Charlier Expansion

- $P(x) = G(x)[c_0+(-1)c_1x+c_2(x^2-1)+(-1)c_3(x^3-3x)+\ldots]$

  where $c_0=1$; $c_1=0$; $c_2=0$; $c_3=-(1/6)\langle x^3\rangle=-(1/6)\kappa_3$; ...

- Thus:

$$\boxed{P(x) = G(x)[1 + (1/6)(x^3-3x)\kappa_3 + \ldots]}$$

This is your PDF!!

Estimating skewness, κ_3

$$\boxed{P(x) = G(x)[1 + (1/6)(x^3-3x)\kappa_3 + \ldots]}$$

- Taking $\langle d\ln P(x)/d\kappa_3 \rangle = 0$, we find
  - $\kappa_3 = \langle x^3 \rangle - 3\sigma^2 \langle x \rangle$ [σ^2: variance]
  - The first term is perhaps obvious, while the second term is not! **The Power of Knowing PDF**
- When the variance depends on pixels [broken translation invariance], we get

$$\kappa_3 = \frac{\frac{1}{6}\sum_i \left(\frac{x_i^3}{\sigma_i^6} - 3\frac{x_i}{\sigma_i^4} \right)}{\frac{1}{6}\sum_i \frac{1}{\sigma_i^6}}$$

[where "i" refers to an "ith measurement"]

Generalization to CMB

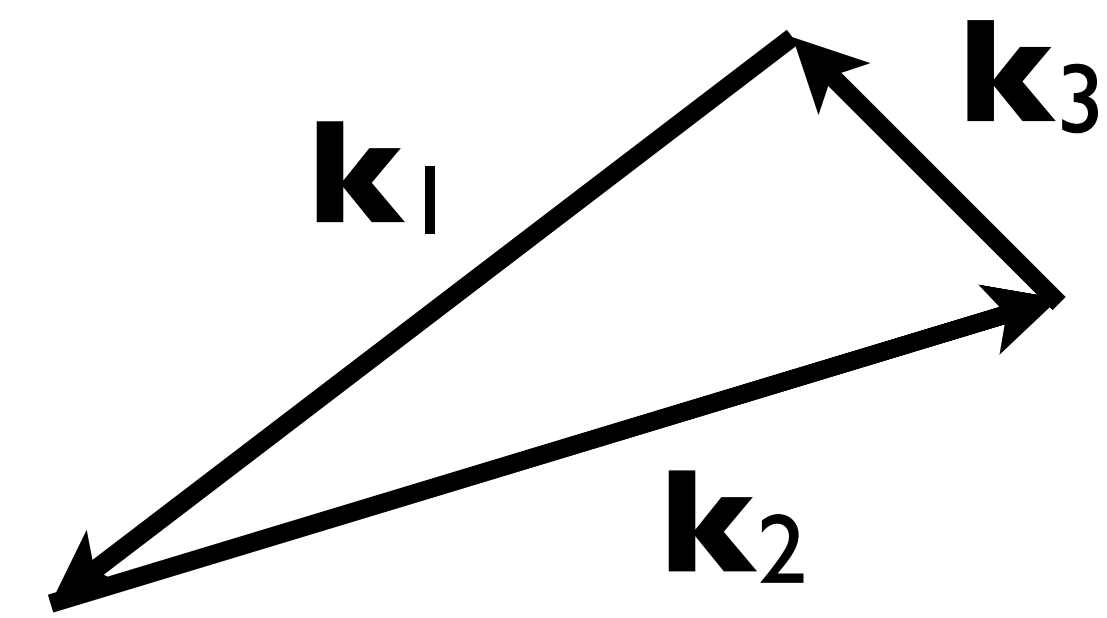
- Gaussian PDF for CMB:

$$P(a) = \frac{1}{(2\pi)^{N_{\rm harm}/2}|C|^{1/2}} \exp\left[-\frac{1}{2}\sum_{lm}\sum_{l'm'} a^*_{lm}(C^{-1})_{lm,l'm'}a_{l'm'}\right]$$

- To a non-Gaussian PDF! [truncated at the 3rd-order]

$$P(a) = \left[1 - \frac{1}{6}\sum_{\text{all } l_i m_j} \underbrace{\langle a_{l_1m_1}a_{l_2m_2}a_{l_3m_3}\rangle}_{\text{“bispectrum”}} \frac{\partial}{\partial a_{l_1m_1}}\frac{\partial}{\partial a_{l_2m_2}}\frac{\partial}{\partial a_{l_3m_3}}\right] \frac{e^{-\frac{1}{2}\sum_{lm}\sum_{l'm'} a^*_{lm}(C^{-1})_{lm,l'm'}a_{l'm'}}}{(2\pi)^{N_{\rm harm}/2}|C|^{1/2}}$$

Bispectrum



- Three-point function!
- $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$
  $= \langle\zeta_{\mathbf{k}1}\zeta_{\mathbf{k}2}\zeta_{\mathbf{k}3}\rangle = (\text{amplitude}) \times (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)F(k_1,k_2,k_3)$

model-dependent function

Primordial fluctuation
[giving CMB anisotropy via dT/T=–ζ/5 on large scales]

parameterized by “f_{NL}”

Estimating "f_{NL}"

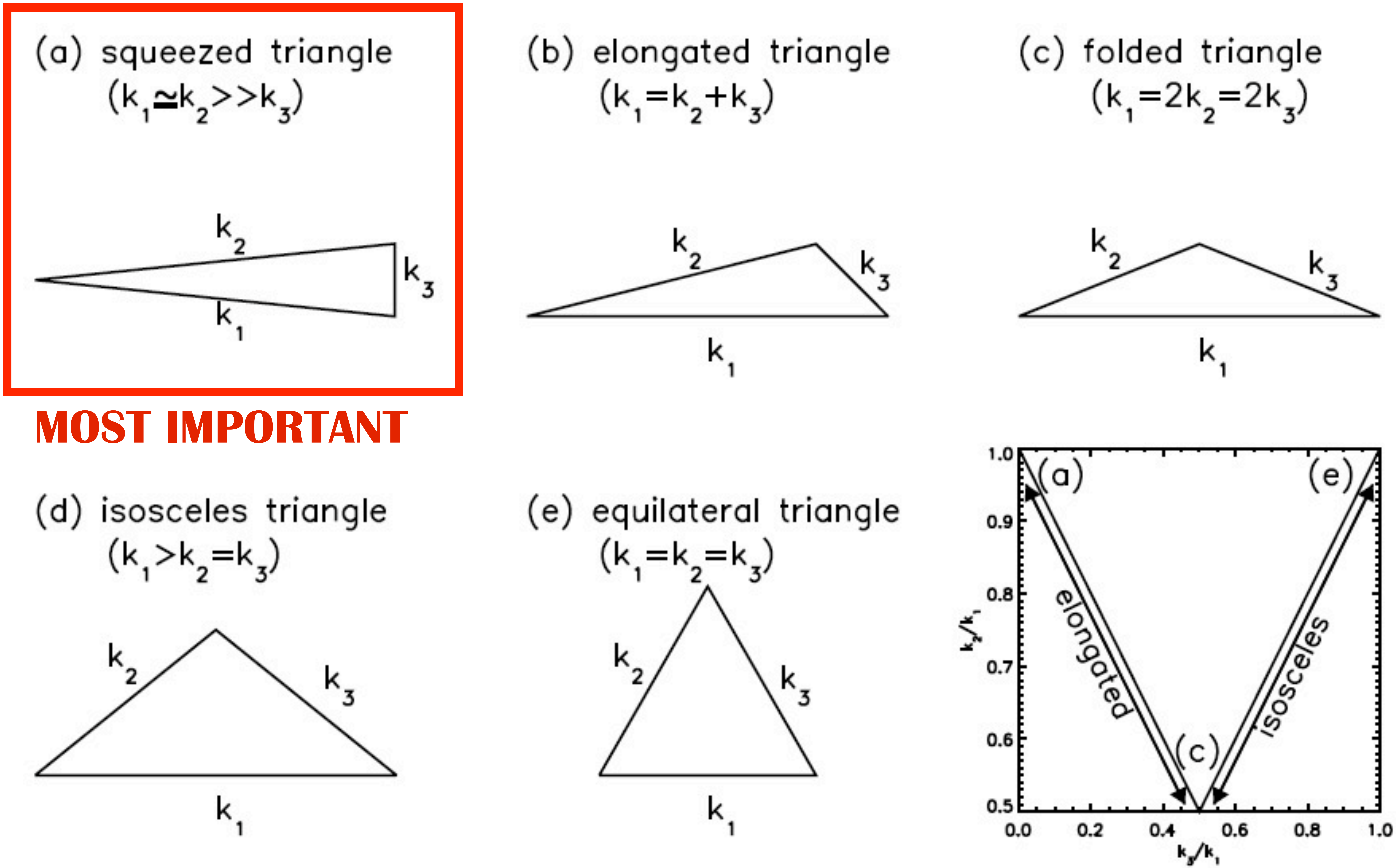
- To a non-Gaussian PDF! [truncated at the 3rd-order]

$$P(a) = \left[1 - \frac{1}{6}\sum_{\text{all } l_i m_j} \underbrace{\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle}_{\text{"bispectrum"}} \frac{\partial}{\partial a_{l_1 m_1}} \frac{\partial}{\partial a_{l_2 m_2}} \frac{\partial}{\partial a_{l_3 m_3}}\right] \frac{e^{-\frac{1}{2}\sum_{lm}\sum_{l'm'} a^*_{lm}(C^{-1})_{lm,l'm'} a_{l'm'}}}{(2\pi)^{N_{\text{harm}}/2}|C|^{1/2}}$$

"*bispectrum*"
has a known shape (model)
with an unknown coefficient "f_{NL}"

The same procedure: $\langle d\ln P/df_{NL} \rangle = 0$ gives an estimator

The error bar can be computed from
Monte-Carlo simulations [frequentist approach]



(a) squeezed triangle
($k_1 \simeq k_2 >> k_3$)
k_2
k_3
k_1
MOST IMPORTANT
(b) elongated triangle
($k_1 = k_2 + k_3$)
k_2
k_3
k_1
(c) folded triangle
($k_1 = 2k_2 = 2k_3$)
k_2
k_3
k_1
(d) isosceles triangle
($k_1 > k_2 = k_3$)
k_2
k_3
k_1
(e) equilateral triangle
($k_1 = k_2 = k_3$)
k_2
k_3
k_1
1.0
0.9
0.8
0.7
0.6
0.5
k_2/k_1
0.0
0.2
0.4
0.6
0.8
1.0
k_3/k_1
(a)
(e)
(c)
elongated
isosceles

Probing Inflation (3-point Function)

- Inflation models predict that primordial fluctuations are very close to Gaussian.
  - In fact, ALL SINGLE-FIELD models predict a particular form of 3-point function to have the amplitude of $f_{NL}=0.02$.
  - Detection of $f_{NL}>1$ would rule out ALL single-field models!
- No detection of 3-point functions of primordial curvature perturbations. The 68% CL limit is:
  - **$f_{NL} = 37 \pm 20$ (1σ)**
  - The WMAP data are consistent with the prediction of **simple single-field inflation** models: $1-n_s \approx r \approx f_{NL}$

PHYSICAL REVIEW D, VOLUME 63, 063002 *Komatsu&Spergel (2001)*

Acoustic signatures in the primary microwave background bispectrum

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(Received 25 October 2000; published 13 February 2001)

If the primordial fluctuations are non-Gaussian, then this non-Gaussianity will be apparent in the cosmic microwave background (CMB) sky. With their sensitive all-sky observation, MAP and Planck satellites should be able to detect weak non-Gaussianity in the CMB sky. On a large angular scale, there is a simple relationship between the CMB temperature and the primordial curvature perturbation: $\Delta T/T = -\Phi/3$. On smaller scales, however, the radiation transfer function becomes more complex. In this paper, we present the angular bispectrum of the primary CMB anisotropy that uses the full transfer function. We find that the bispectrum has a series of acoustic peaks that change a sign and a period of acoustic oscillations is twice as long as that of the angular power spectrum. Using a single non-linear coupling parameter to characterize the amplitude of the bispectrum, we estimate the expected signal-to-noise ratio for COBE, MAP, and Planck experiments. In order to detect the primary CMB bispectrum by each experiment, we find that the coupling parameter should be larger than 600, 20, and 5 for COBE, MAP, and Planck experiments, respectively. Even for the ideal noise-free and infinitesimal thin-beam experiment, the parameter should be larger than 3. We have included effects from the cosmic variance, detector noise, and foreground sources in the signal-to-noise estimation. Since the simple inflationary scenarios predict that the parameter is an order of 0.01, the detection of the primary bispectrum by any kind of experiments should be problematic for those scenarios. We compare the sensitivity of the primary bispectrum to the primary skewness and conclude that, when we can compute the predicted form of the bispectrum, it becomes a ‘‘matched filter’’ for detecting the non-Gaussianity in the data and a much more powerful tool than the skewness. For example, we need the coupling parameter of larger than 800, 80, 70, and 60 for each relevant experiment in order to detect the primary skewness. We also show that MAP and Planck can separate the primary bispectrum from various secondary bispectra on the basis of the shape difference. The primary CMB bispectrum is a test of the inflationary scenario and also a probe of the non-linear physics in the very early universe.

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Planck Result: f_{NL} = 2.7 ± 5.8 (68%CL)

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Summary

- If you have not done so yet, write down your PDF!
  - E.g., cosmic-rays & gamma-rays: Poisson distribution
- Even if it is not a Gaussian, you can obtain an approximate PDF if non-Gaussianity is weak
- Even if it is strongly non-Gaussian, perhaps you can transform it into a Gaussian shape [e.g., log-normal]; and then expand it
- Neither works? Well, we could talk!