Beyond BAO

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MPE Seminar, August 7, 2008

Papers To Talk About

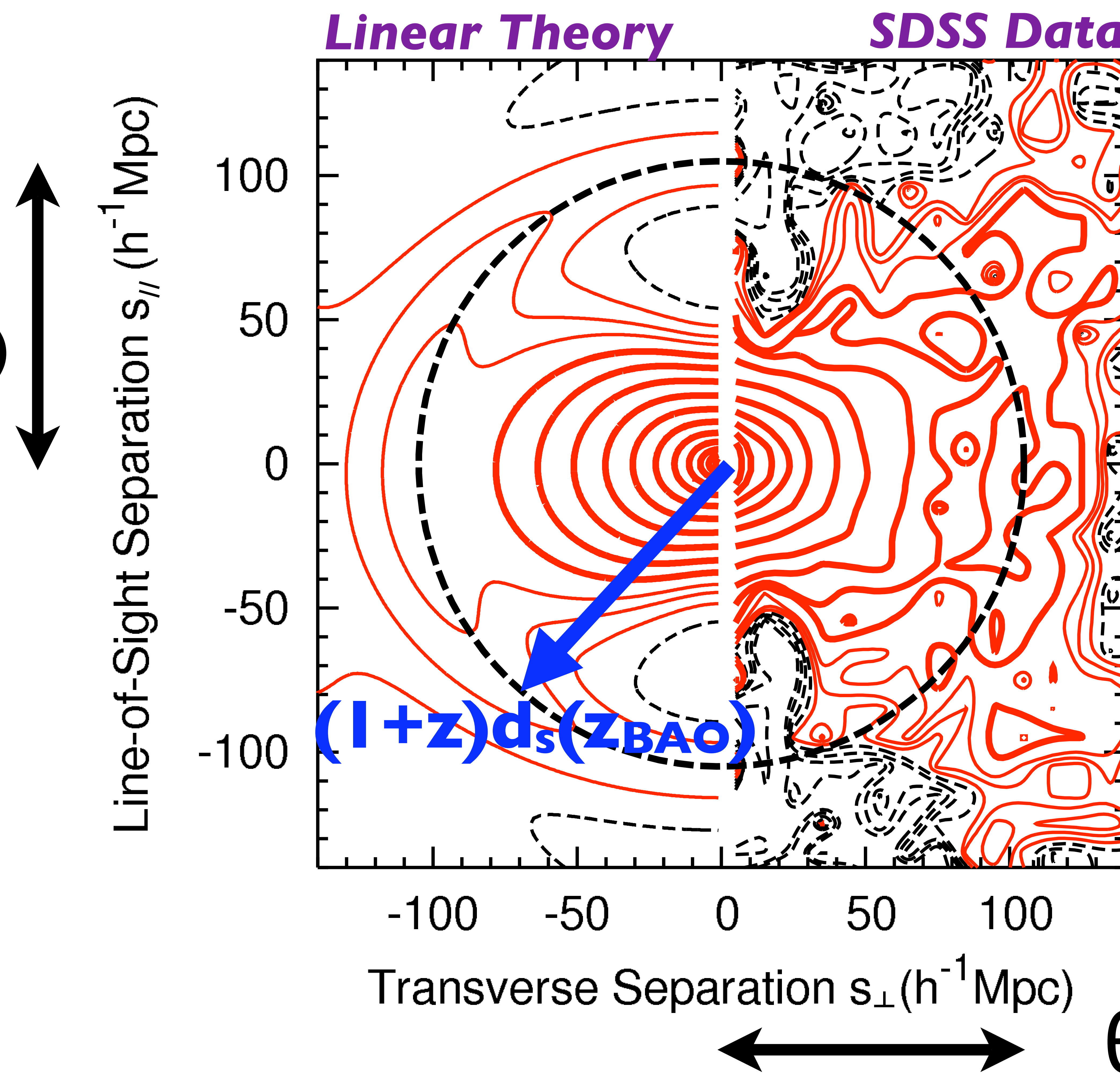
- Donghui Jeong & EK, ApJ, 651, 619 (2006)
- Donghui Jeong & EK, arXiv:0805.2632
- Masatoshi Shoji, Donghui Jeong & EK, arXiv:0805.4238
- Jeong, Sefusatti & Komatsu (in preparation)

Why BAO? In 5 Minutes

- We can measure:
  - Angular Diameter Distances, $D_A(z)$
  - Hubble Expansion Rates, $H(z)$
- $D_A(z)$ & $H(z)$. These are fundamental quantities to measure in cosmology!

Transverse=$D_A(z)$; Radial=$H(z)$

$c\Delta z/(1+z)$
$= d_s(z_{BAO})\mathbf{H(z)}$



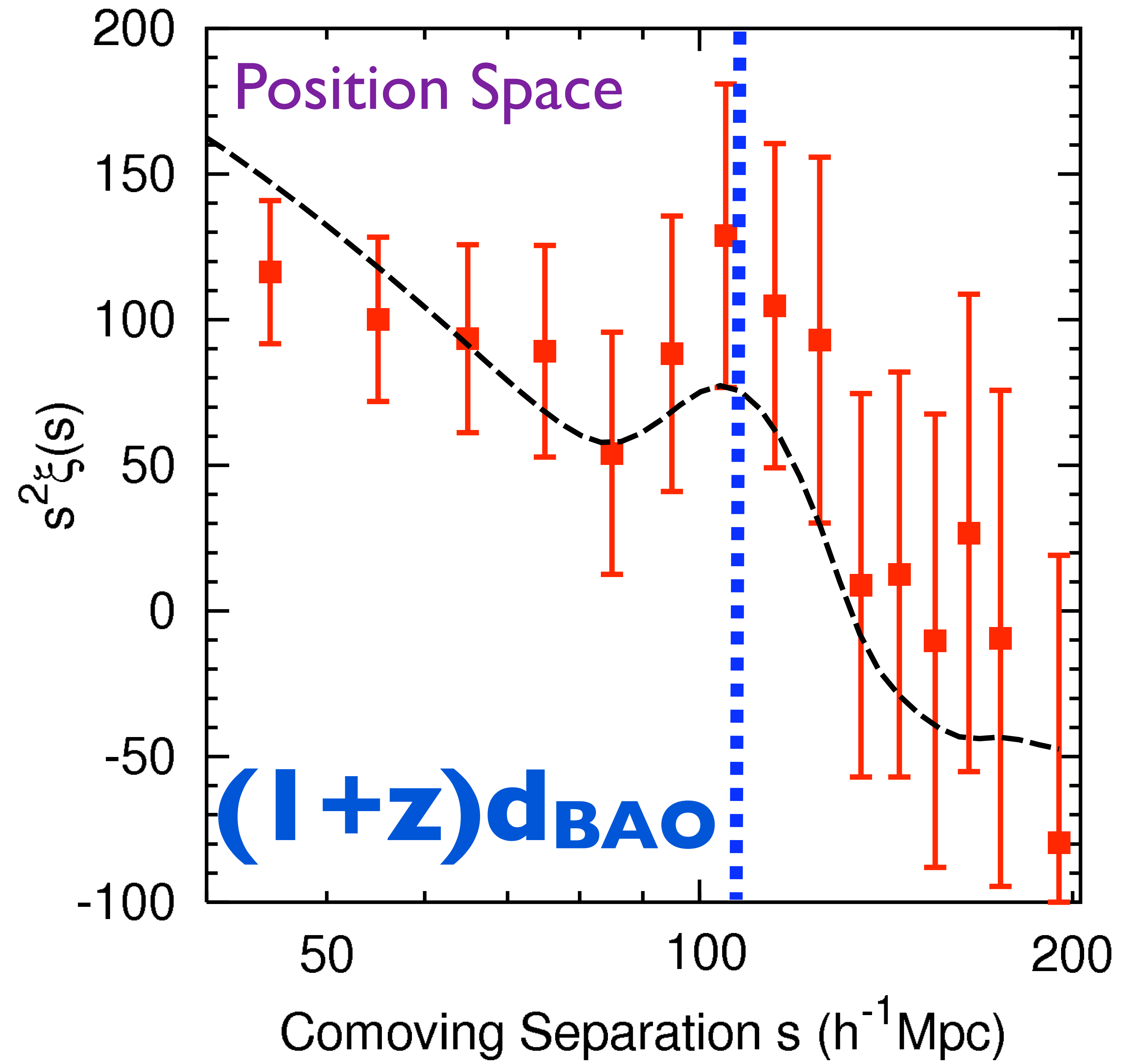
Two-point correlation function measured from the SDSS Luminous Red Galaxies

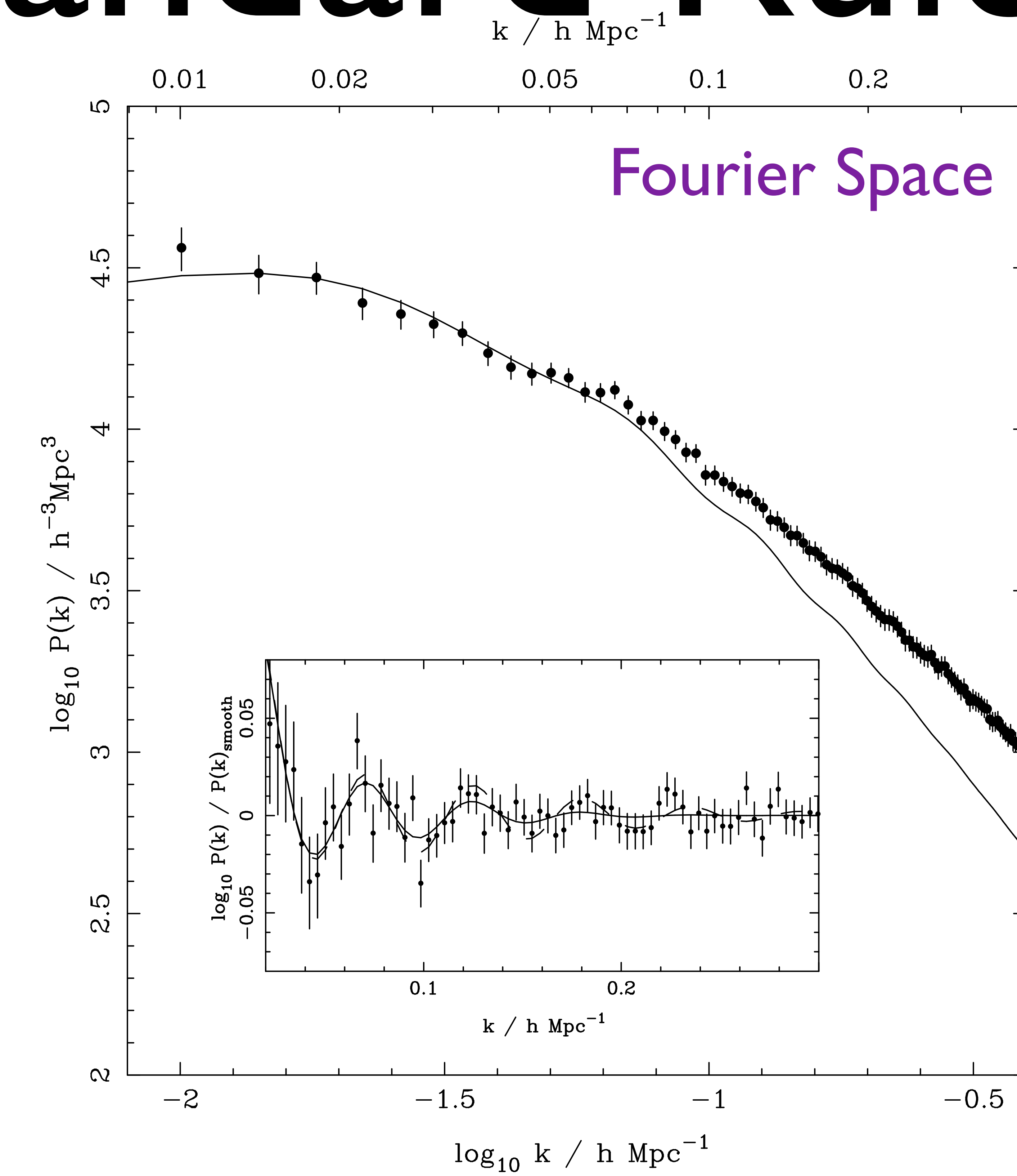
(Okumura et al. 2007)

$\theta = d_s(z_{BAO})/\mathbf{D_A(z)}$

BAO as a Standard Ruler

Okumura et al. (2007)



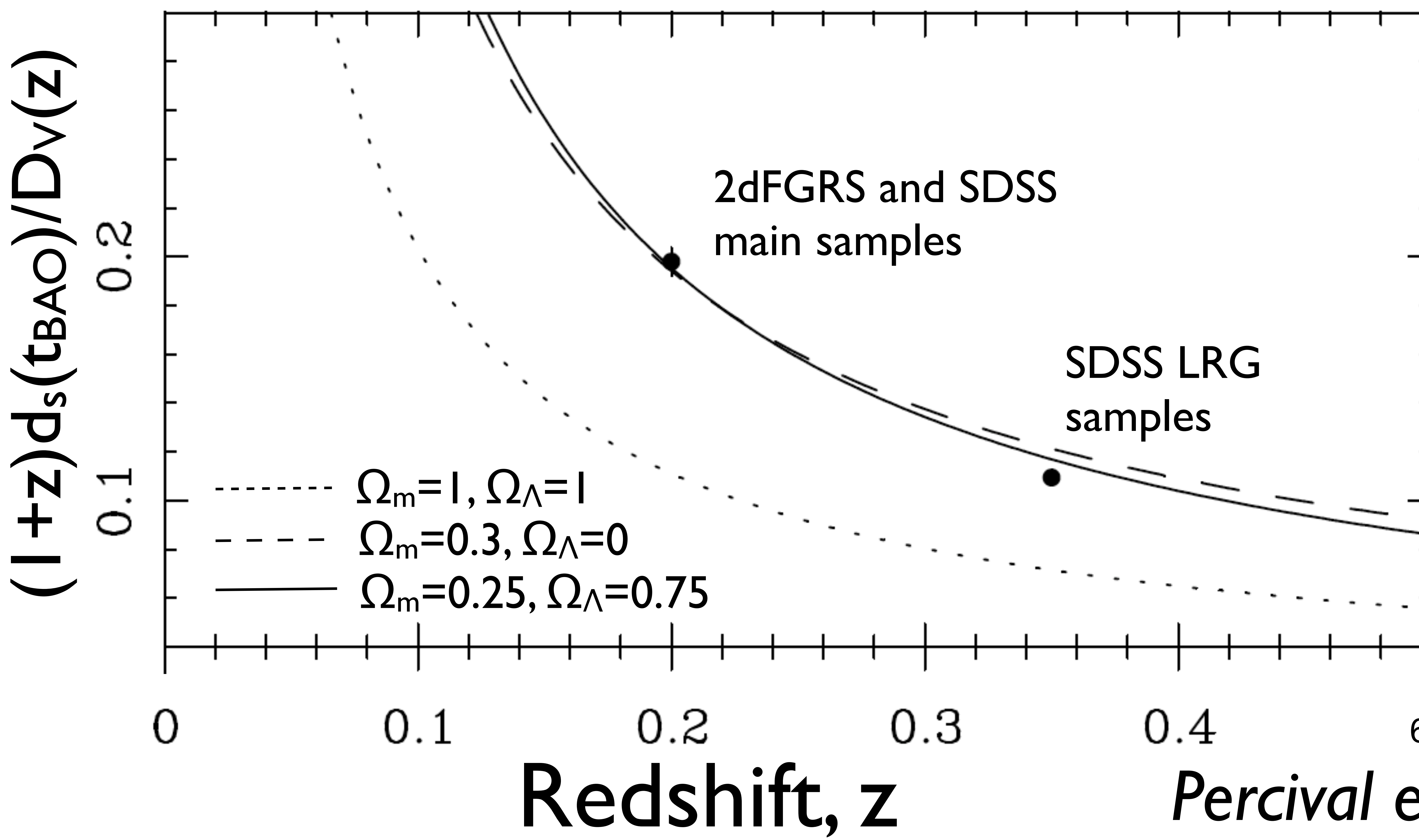


Percival et al. (2006)

- The existence of a localized clustering scale in the 2-point function yields oscillations in Fourier space.

$$D_V(z) = \{(1+z)^2 D_A^2(z)[cz/H(z)]\}^{1/3}$$

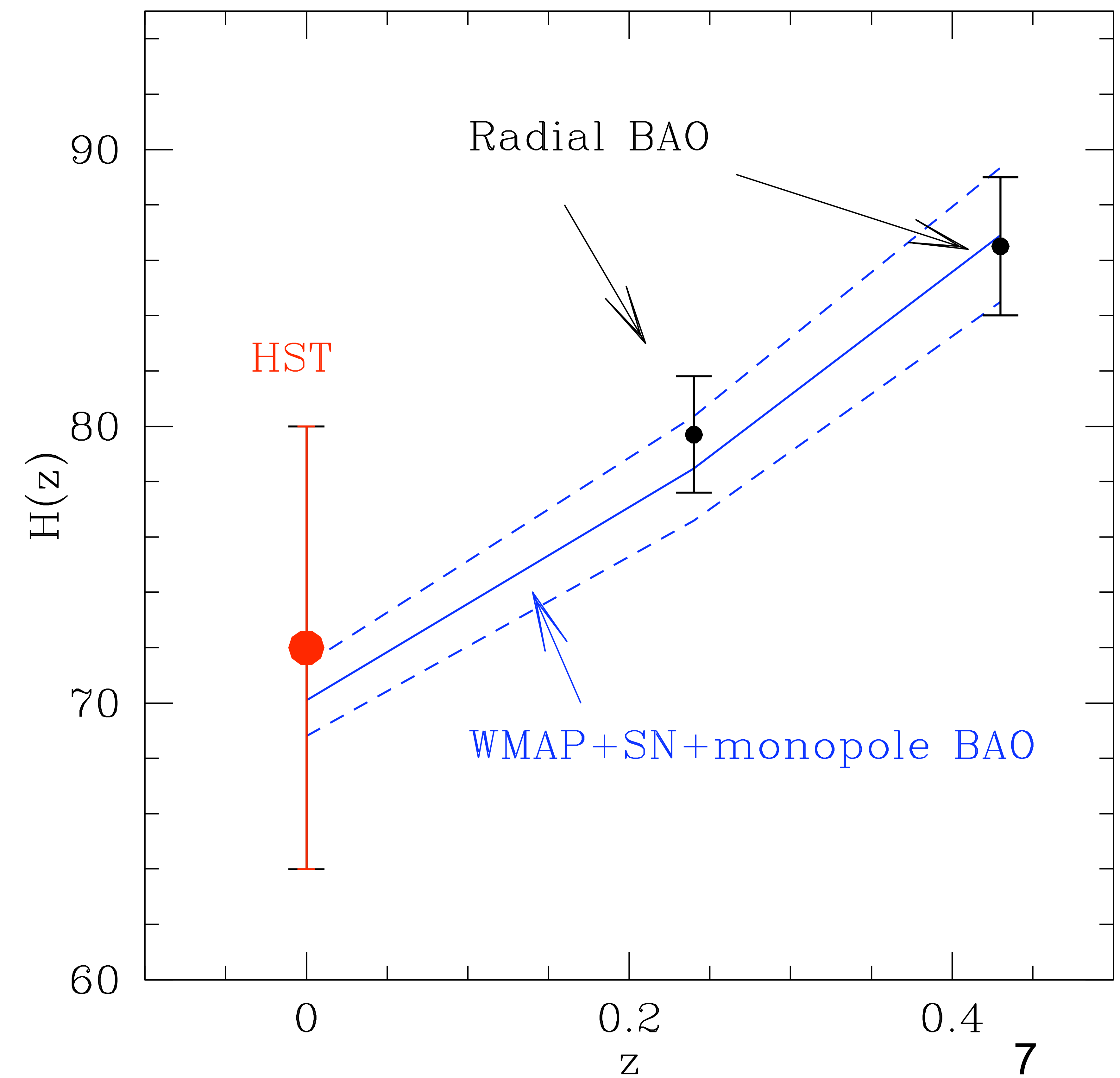
Once spherically averaged, $D_A(z)$ and $H(z)$ are mixed.
A combination distance, **$D_V(z)$**, has been constrained.



Percival et al. (2007)

H(z) also determined recently!

- SDSS DR6 data are now good enough to constrain H(z) from the 2-dimension correlation function *without spherical averaging*.
- Excellent agreement with ΛCDM model.

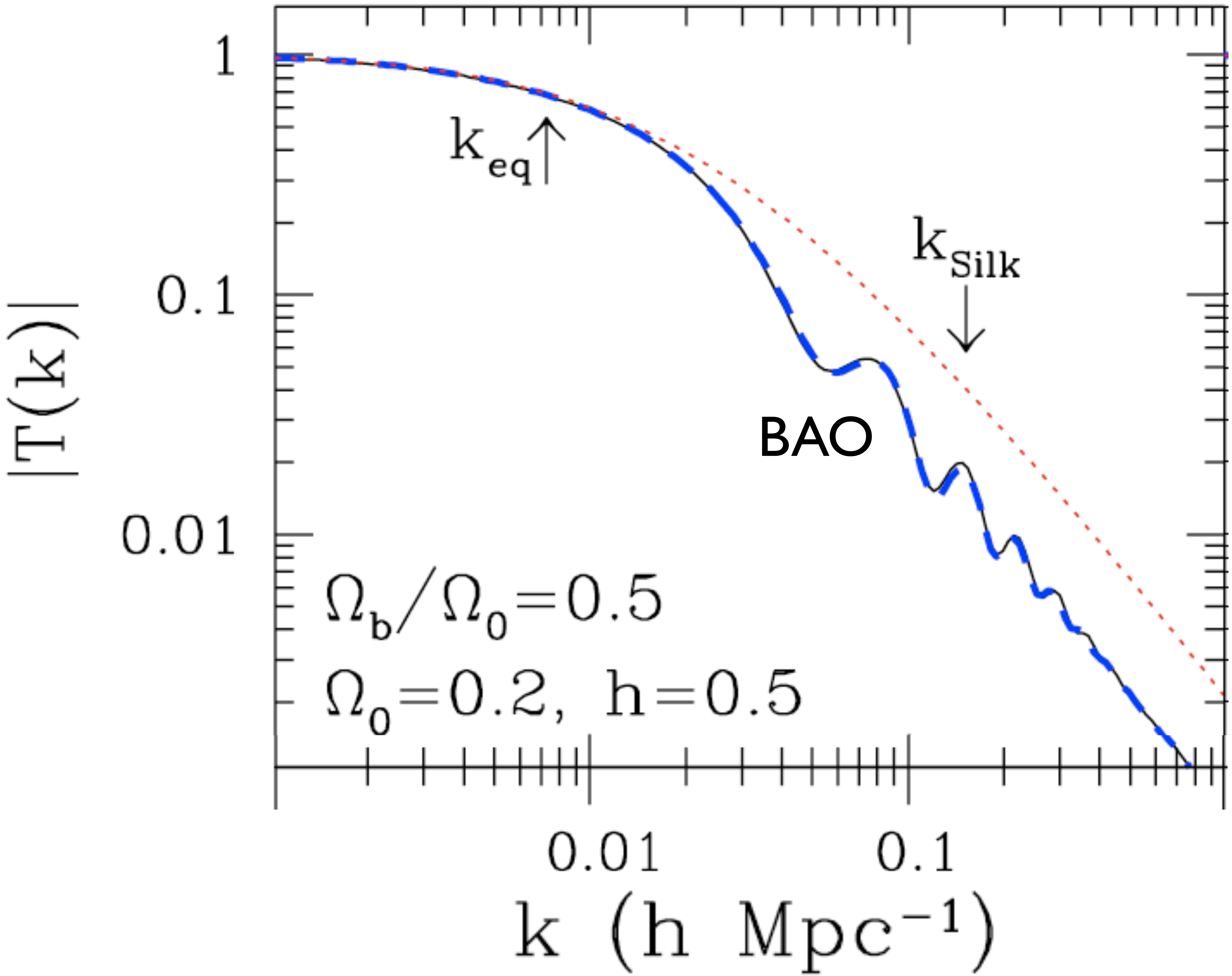


Gaztanaga, Cabre & Hui (2008)

Why Go Beyond BAO?

- BAOs capture only a **fraction** of the information contained in the galaxy power spectrum!
- BAOs use the sound horizon size at z~1020 as the standard ruler.
- However, there are other standard rulers:
  - Horizon size at the matter-radiation equality epoch (z~3200)
  - Silk damping scale

Eisenstein & Hu (1998)



...and, these are all well known

- Cosmologists have been measuring k_{eq} over the last three decades.
- This was usually called the "Shape Parameter," denoted as Γ.
- Γ is proportional to k_{eq}/h, and:
  - The effect of the Silk damping is contained in the constant of proportionality.
  - Easier to measure than BAOs: the signal is much stronger.

WMAP & Standard Ruler

- **With WMAP 5-year data only,** the scales of the standard rulers have been determined accurately (Komatsu et al. 2008). Even when $w \neq -1$, $\Omega_k \neq 0$,

1.3% • $d_s(z_{BAO}) = 153.4^{+1.9}_{-2.0}$ Mpc ($z_{BAO}=1019.8 \pm 1.5$)

4.6% • $k_{eq}=(0.975^{+0.044}_{-0.045})\times10^{-2}$ Mpc^{-1} ($z_{eq}=3198^{+145}_{-146}$)

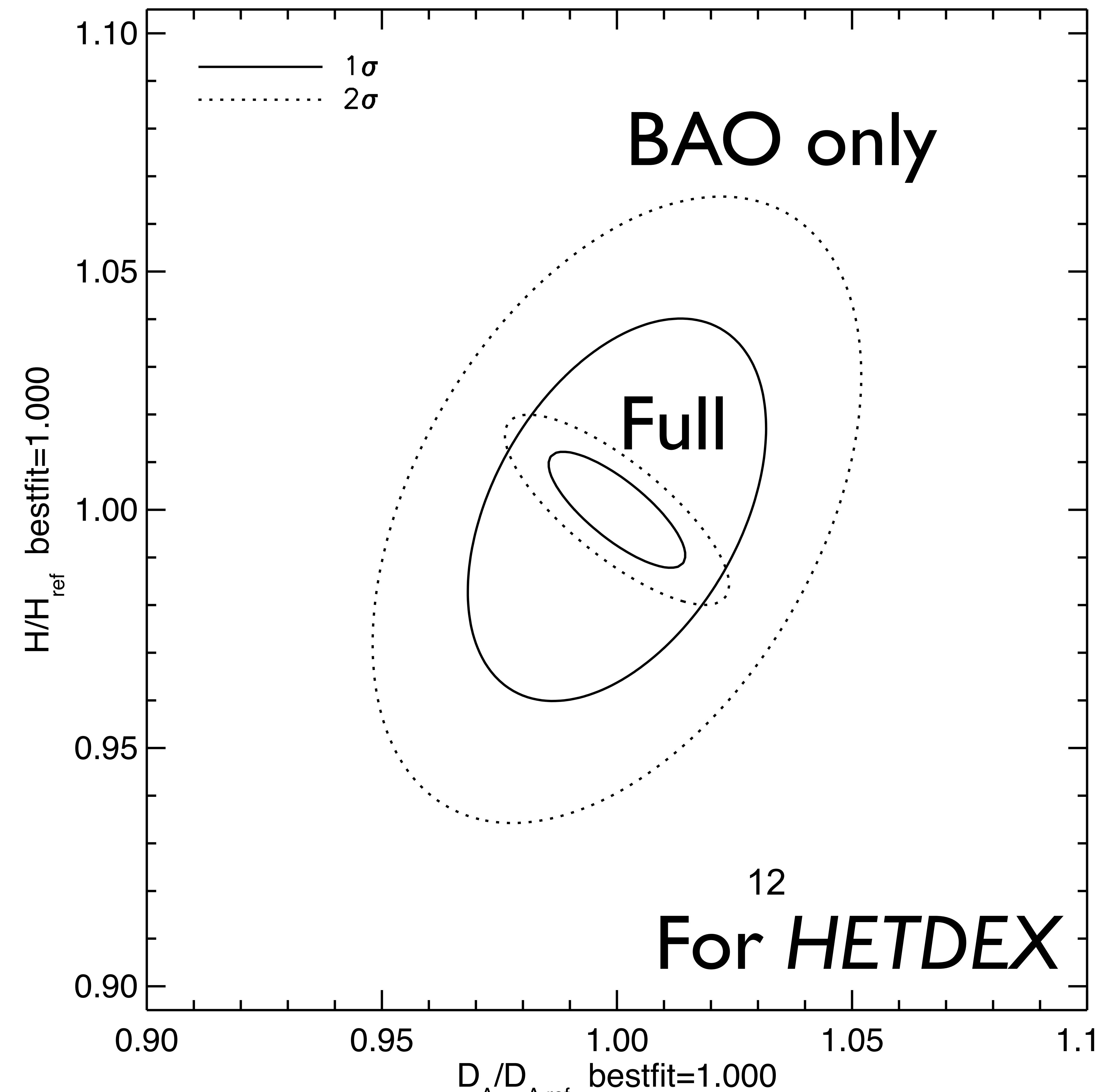
2.3% • $k_{silk}=(8.83 \pm 0.20)\times10^{-2}$ Mpc^{-1}

With Planck, they will be determined to higher precision.

Shoji, Jeong & Komatsu (2008)

BAO vs Full Modeling

- Full modeling improves upon the determinations of D_A & H by more than a factor of two.
- On the D_A-H plane, the size of the ellipse shrinks by more than a factor of four.



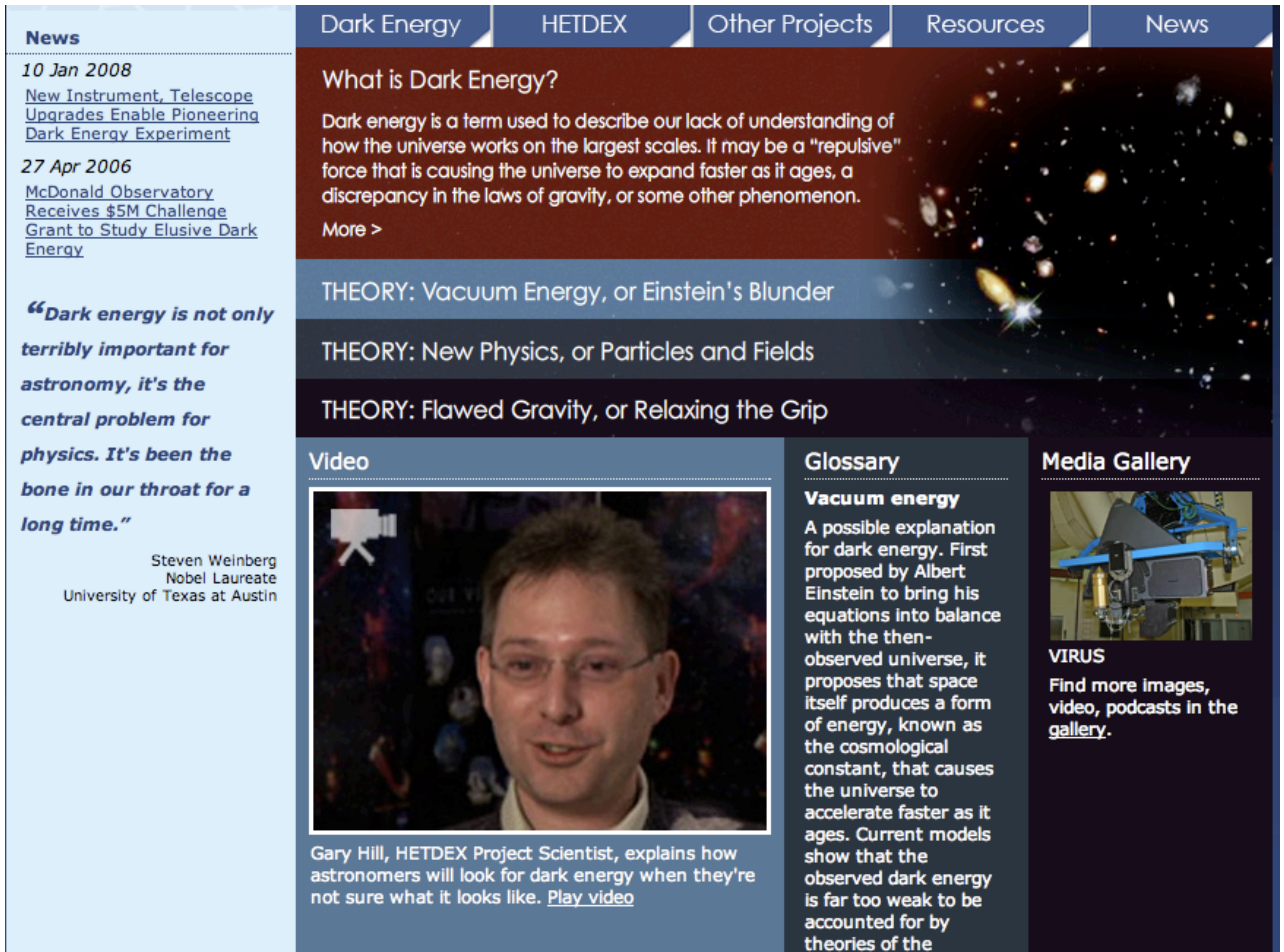
Shoji, Jeong & Komatsu (2008)

For the analysis of HETDEX

- BAO only
  - D_A: 2.1%, H: 2.6%
  - Correlation coefficient: 0.43
- Full Modeling
  - D_A: 0.96%, H: 0.80%
  - Correlation coefficient: -0.79

HETDEX ?

- www.hetdex.org



Effective Use of Resources

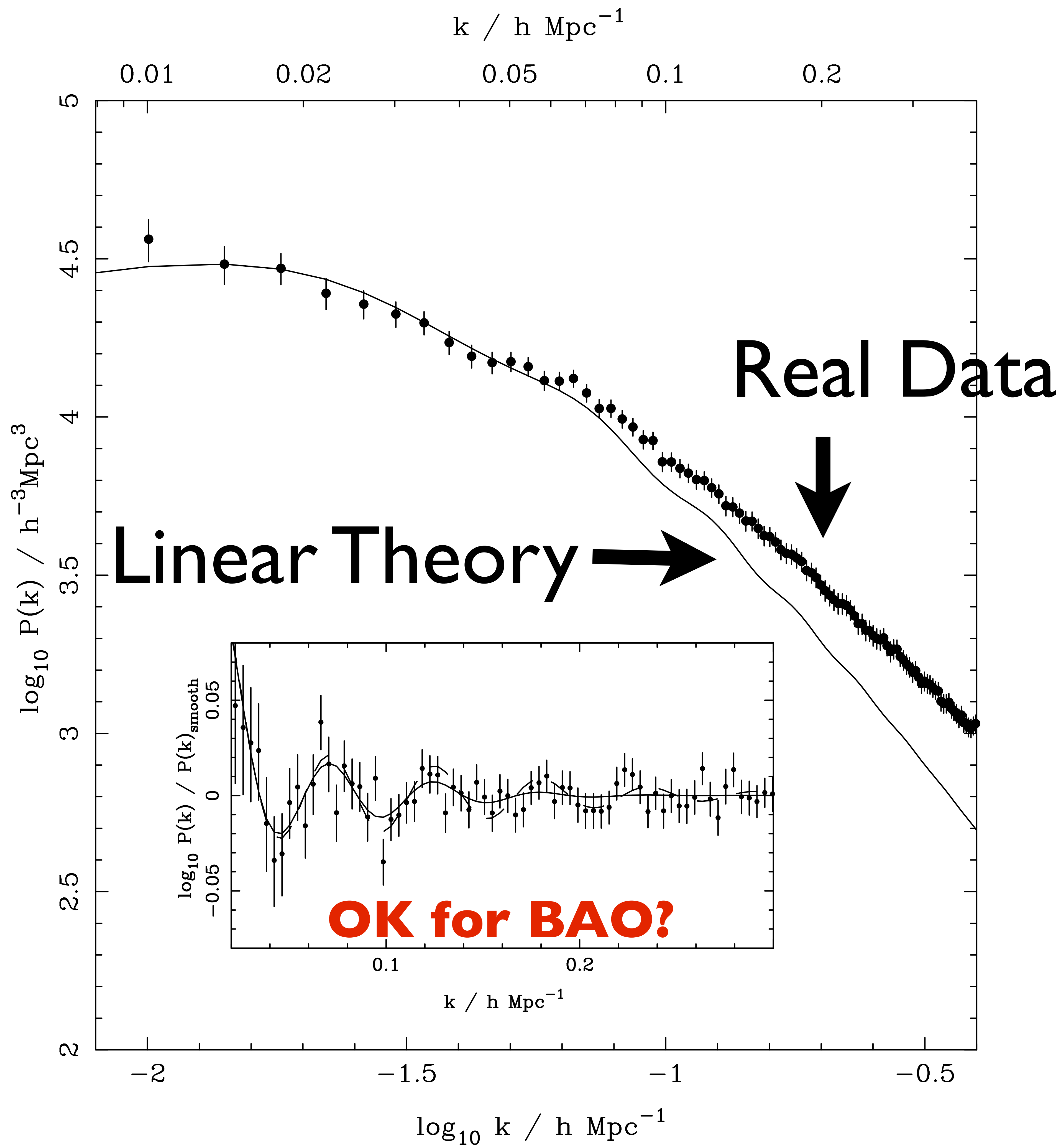
- Using the full information is equivalent to having four times as much volume as you would have with the BAO-only analysis.
- Save the integration time by a factor of four!

Still, BAO.

- If what I am saying is correct, why would people talk only about the BAOs these days, and tend to ignore the full information?

- **NON-LINEARITY**

Non-linear Effects

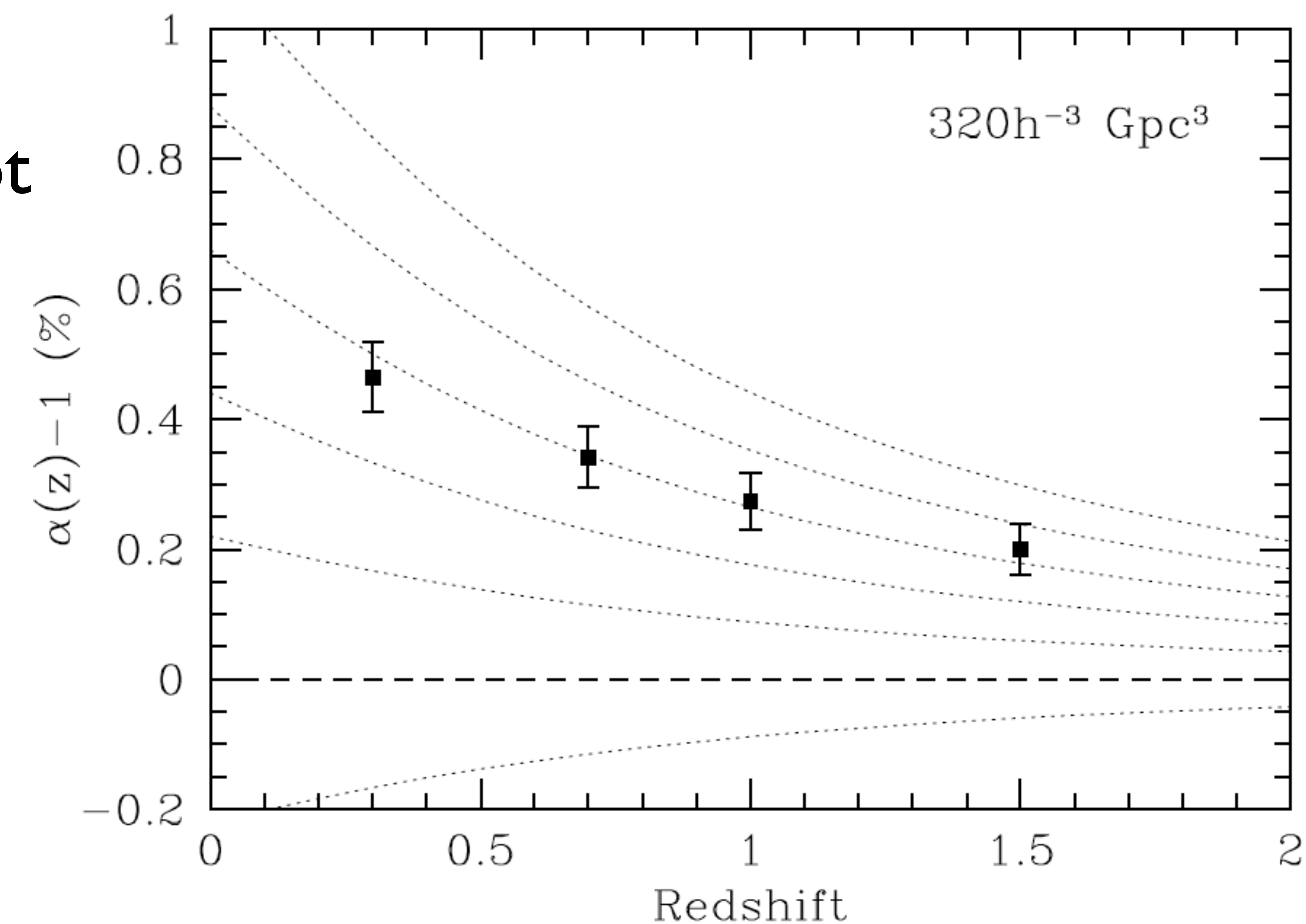


- Three non-linearities
  1. Non-linear matter clustering
  2. Non-linear galaxy bias
  3. Non-linear peculiar velocity
- The effects of k_{eq} and k_{silk} can be affected by these non-linear effects much more strongly than the effects of BAOs.

Seo, Siegel, Eisenstein & White (2008)

According to Dan Eisenstein:

- The phases of BAOs are not affected by the non-linear evolution very much.
- The effects are correctable.
  - z=0.3: 0.54%
  - z=1.5: 0.25%

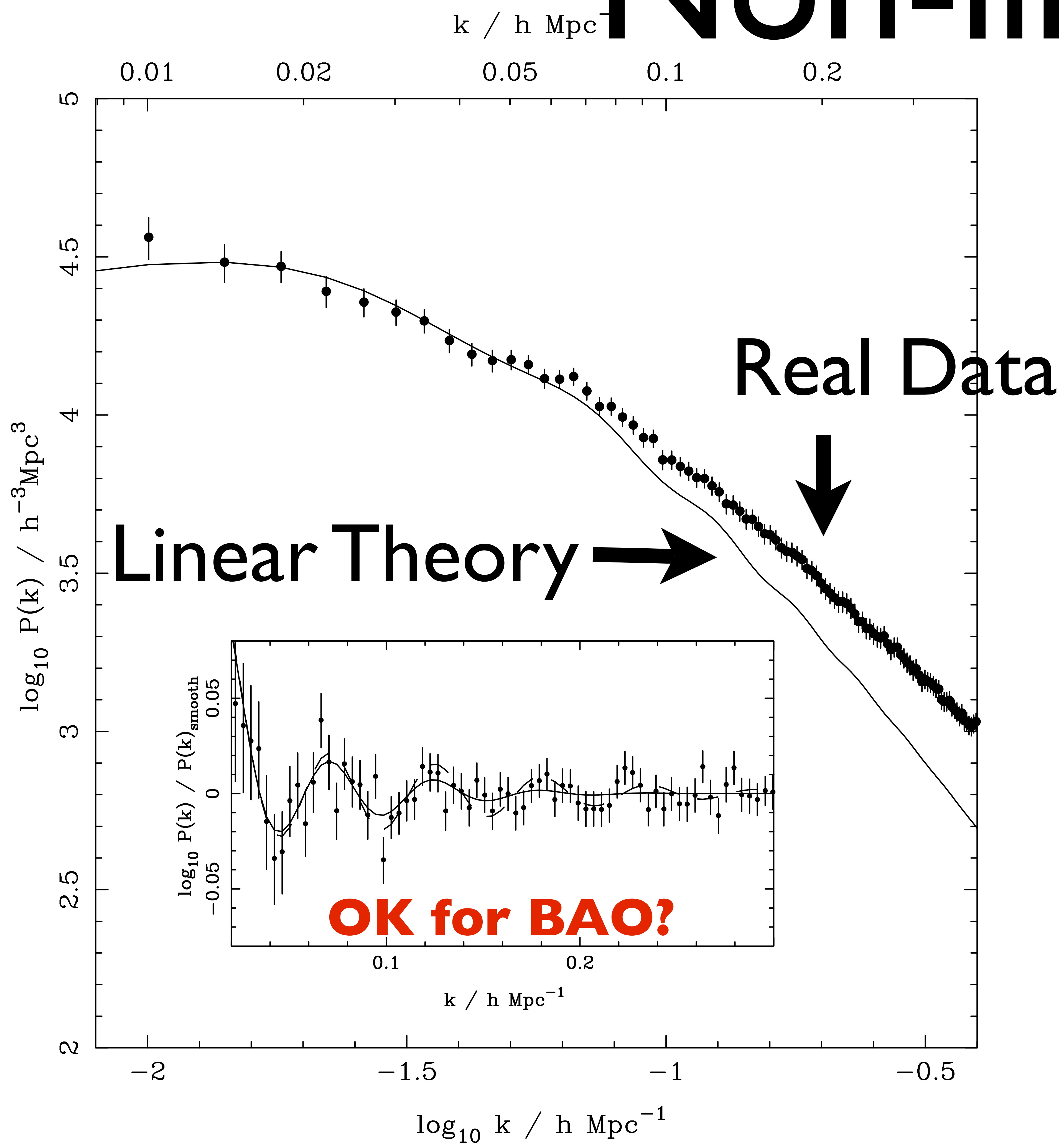


Takada, Komatsu & Futamase (2006)

Why Full Information? Reason II

- Not only do we improve upon the determinations of D_A & H, but also:
    - We can constrain inflationary models, and
    - We can measure the neutrino masses and the number of massive neutrino species.
- Therefore, just using the BAOs is such a waste of information!

Toward Understanding Non-linearities



- Three non-linearities
  1. Non-linear matter clustering
  2. Non-linear galaxy bias
  3. Non-linear peculiar velocity
- Solid theoretical framework is necessary for avoiding any empirical, calibration factors

Toward Understanding Non-linearities

- Solid framework: **Perturbation Theory (PT)**
  - Validity of the cosmological ***linear*** perturbation theory has been verified *observationally* (Remember WMAP!)
  - So, we just go beyond the linear theory, and calculate higher order terms in perturbations.
  - **3rd-order perturbation theory (3PT)**

Is 3PT New?

- No. It is more than 25 years old.
- Active investigations in 1990's
    - Most popular in European and Asian countries, but was not very popular in USA for some reason
- 3PT has never been applied to the real data so far. Why?
    - Non-linearity is too strong to model by PT at z~0

Why Perturbation Theory Now?

- The time has changed.
- **High-redshift (z>1) galaxy redshift surveys are now possible.**
- And now, such surveys are needed for Dark Energy studies
- **Non-linearities are weaker at z>1, making it possible to use the cosmological perturbation theory!**

Just Three Equations to Solve

- Consider large scales, where the baryon pressure is negligible, i.e., the scales larger than the Jeans scale
- Ignore the shell-crossing, i.e., the velocity field of particles has zero curl: rot*V*=0.
- Equations to solve are:

$$\dot{\delta} + \nabla \cdot [(1+\delta)\boldsymbol{v}] = 0$$

$$\dot{\boldsymbol{v}} + (\boldsymbol{v} \cdot \nabla)\,\boldsymbol{v} = -\frac{\dot{a}}{a}\boldsymbol{v} - \nabla\phi$$

$$\nabla^2\phi = 4\pi G a^2 \bar{\rho}\delta$$

Fourier Transform...

$$\dot{\delta}(\boldsymbol{k},\tau)+\theta(\boldsymbol{k},\tau)$$

$$= -\int \frac{d^3k_1}{(2\pi)^3}\int d^3k_2\delta_D(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k})\frac{\boldsymbol{k}\cdot\boldsymbol{k}_1}{k_1^2}\delta(\boldsymbol{k}_2,\tau)\theta(\boldsymbol{k}_1,\tau),$$

$$\dot{\theta}(\boldsymbol{k},\tau)+\frac{\dot{a}}{a}\theta(\boldsymbol{k},\tau)+\frac{3\dot{a}^2}{2a^2}\Omega_{\rm m}(\tau)\delta(\boldsymbol{k},\tau)$$

$$= -\int \frac{d^3k_1}{(2\pi)^3}\int d^3k_2\delta_D(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k})\frac{k^2(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)}{2k_1^2k_2^2}\theta(\boldsymbol{k}_1,\tau)\theta(\boldsymbol{k}_2,\tau)$$

- Here, $\theta=\nabla\cdot\boldsymbol{v}$ is the velocity divergence.

Taylor-expand in δ_1

- δ_1 is the linear perturbation

$$\delta(\boldsymbol{k},\tau) = \sum_{n=1}^{\infty} a^n(\tau) \int \frac{d^3q_1}{(2\pi)^3} \cdots \frac{d^3q_{n-1}}{(2\pi)^3} \int d^3q_n \delta_D(\sum_{i=1}^{n} \boldsymbol{q}_i - \boldsymbol{k}) F_n(\boldsymbol{q}_1, \boldsymbol{q}_2, \cdots, \boldsymbol{q}_n) \delta_1(\boldsymbol{q}_1) \cdots \delta_1(\boldsymbol{q}_n),$$

$$\theta(\boldsymbol{k},\tau) = -\sum_{n=1}^{\infty} \dot{a}(\tau) a^{n-1}(\tau) \int \frac{d^3q_1}{(2\pi)^3} \cdots \frac{d^3q_{n-1}}{(2\pi)^3} \int d^3q_n \delta_D(\sum_{i=1}^{n} \boldsymbol{q}_i - \boldsymbol{k}) G_n(\boldsymbol{q}_1, \boldsymbol{q}_2, \cdots, \boldsymbol{q}_n) \delta_1(\boldsymbol{q}_1) \cdots \delta_1(\boldsymbol{q}_n)$$

Keep terms up to 3rd order

- $\delta=\delta_1+\delta_2+\delta_3$, where $\delta_2=O(\delta_1{}^2)$, $\delta_3=O(\delta_1{}^3)$.
- Power spectrum, $P(k)=$**$P_L(k)$**$+$**$P_{22}(k)$**$+2$**$P_{13}(k)$**, may be written, order-by-order, as

$$
\begin{aligned}
&(2\pi)^3 P(k)\delta_D(\boldsymbol{k}+\boldsymbol{k}') \\
&\equiv \langle \delta(\boldsymbol{k},\tau)\delta(\boldsymbol{k}',\tau)\rangle \\
&= \langle \delta_1(\boldsymbol{k},\tau)\delta_1(\boldsymbol{k}',\tau)\rangle + \langle \delta_2(\boldsymbol{k},\tau)\delta_1(\boldsymbol{k}',\tau) + \delta_1(\boldsymbol{k},\tau)\delta_2(\boldsymbol{k}',\tau)\rangle \\
&\quad + \langle \delta_1(\boldsymbol{k},\tau)\delta_3(\boldsymbol{k}',\tau) + \delta_2(\boldsymbol{k},\tau)\delta_2(\boldsymbol{k}',\tau) + \delta_3(\boldsymbol{k},\tau)\delta_1(\boldsymbol{k}',\tau)\rangle \\
&\quad + \mathcal{O}(\delta_1^6)
\end{aligned}
$$

Odd powers in δ_1 vanish (Gaussianity)

P_L, **P_{13}**, **P_{22}**, **P_{13}**

Vishniac (1983); Fry (1984); Goroff et al. (1986); Suto&Sasaki (1991); Makino et al. (1992); Jain&Bertschinger (1994); Scoccimarro&Frieman (1996)

P(k): 3rd-order Solution

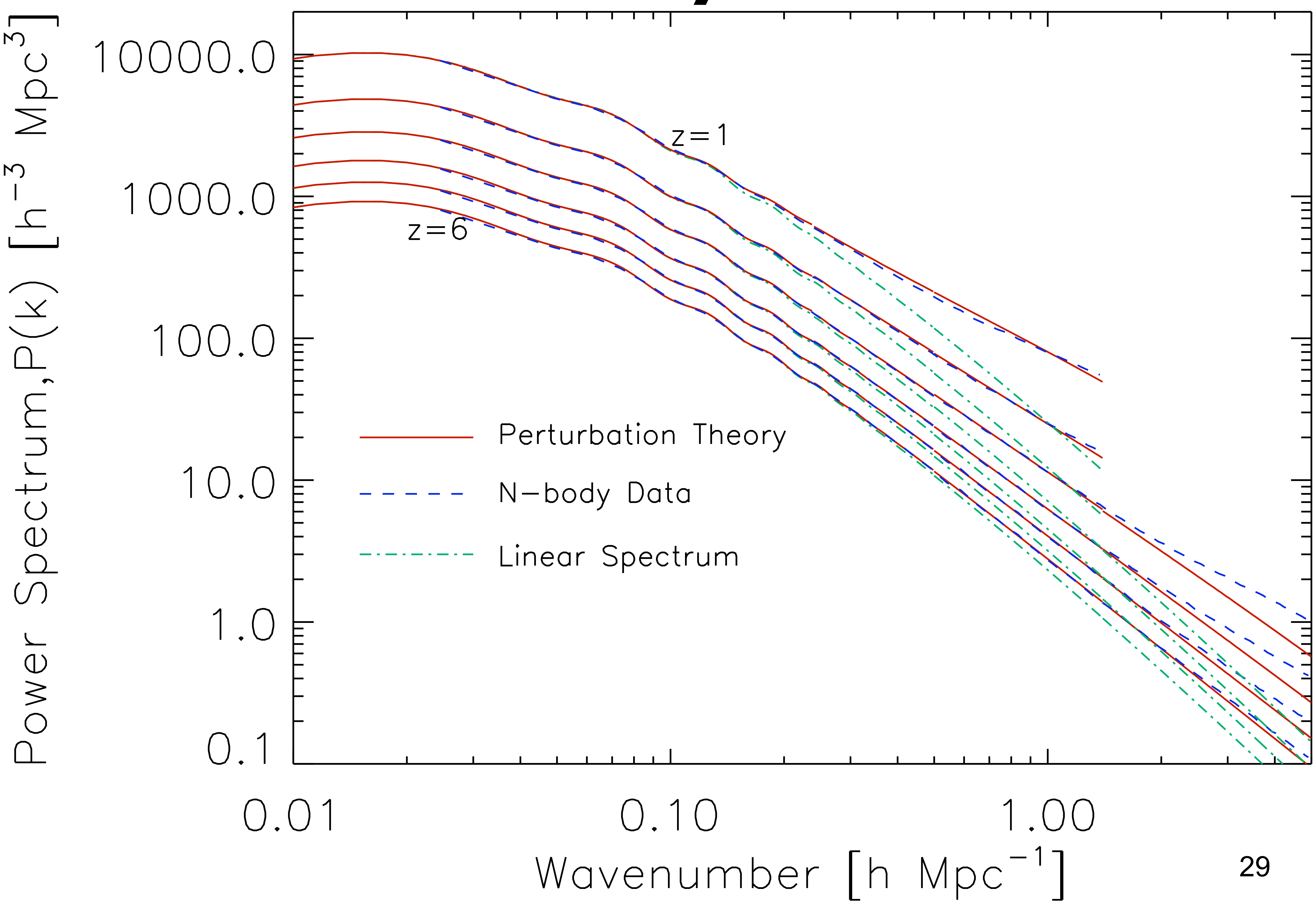
$$P_{22}(k) = 2\int \frac{d^3q}{(2\pi)^3} P_L(q) P_L(|\boldsymbol{k}-\boldsymbol{q}|) \left[F_2^{(s)}(\boldsymbol{q}, \boldsymbol{k}-\boldsymbol{q})\right]^2$$

$$\begin{aligned} 2P_{13}(k) &= \frac{2\pi k^2}{252} P_L(k) \int_0^\infty \frac{dq}{(2\pi)^3} P_L(q) \\ &\times \left[100\frac{q^2}{k^2} - 158 + 12\frac{k^2}{q^2} - 42\frac{q^4}{k^4} \right. \\ &+ \left. \frac{3}{k^5 q^3}(q^2-k^2)^3(2k^2+7q^2)\ln\left(\frac{k+q}{|k-q|}\right)\right] \end{aligned}$$

- $F_2^{(s)}$ is a known mathematical function (Goroff et al. 1986)

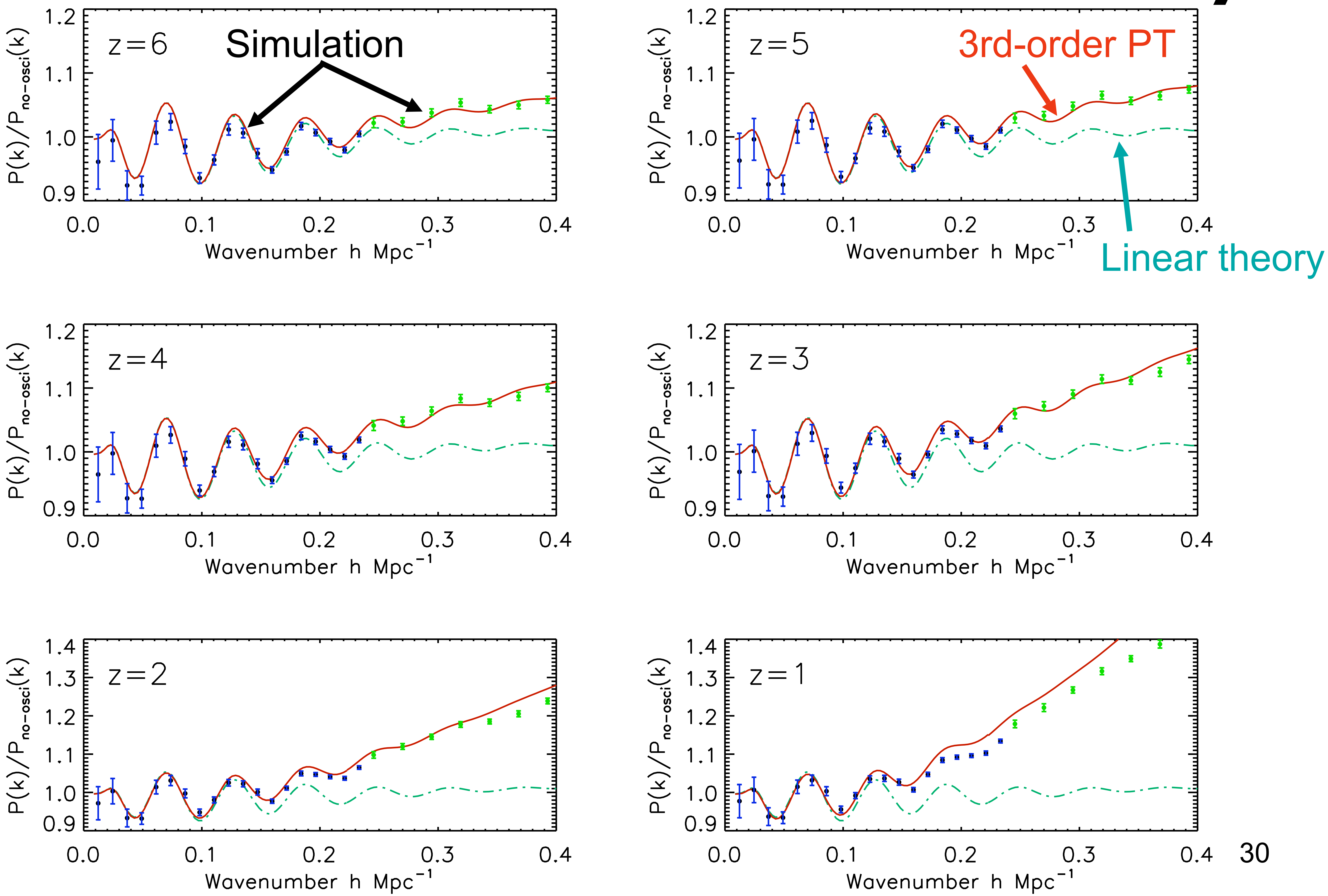
Jeong & Komatsu (2006)

3PT vs N-body Simulations



Jeong & Komatsu (2006)

BAO: Matter Non-linearity



What About Galaxies?

- We measure the **galaxy** power spectrum.
  - Who cares about the *matter* power spectrum?
  - How can we use 3PT for galaxies?

Local Bias Assumption

- The distribution of galaxies is not the same as the distribution of matter fluctuations
- Usually, this fact is modeled by the so-called "linear bias," meaning $P_g(k)=b_1^2\,P(k)$, where b_1 a scale-independent (but time-dependent) factor.
- How do we extend this to the non-linear form? We have to assume something about the galaxy formation
- Assumption: galaxy formation is a local process, at least on the scales that cosmologists care about.

Gaztanaga & Fry (1993); McDonald (2006)

Taylor-expand δ_g in δ

$$\delta_g(x) = c_1\delta(x) + c_2\delta^2(x) + c_3\delta^3(x) + O(\delta^4) + \varepsilon(x)$$

Here, δ is the non-linear matter perturbation, ε is stocastic "noise," uncorrelated with δ, i.e., $<\delta(x)\varepsilon(x)>=0$.

- Both sides of this equation are evaluated at the same spatial location, x, hence the term "local."
- We know that the local assumption breaks down at some small scales. That's where we must stop using PT.

McDonald (2006)

3PT Galaxy Power Spectrum

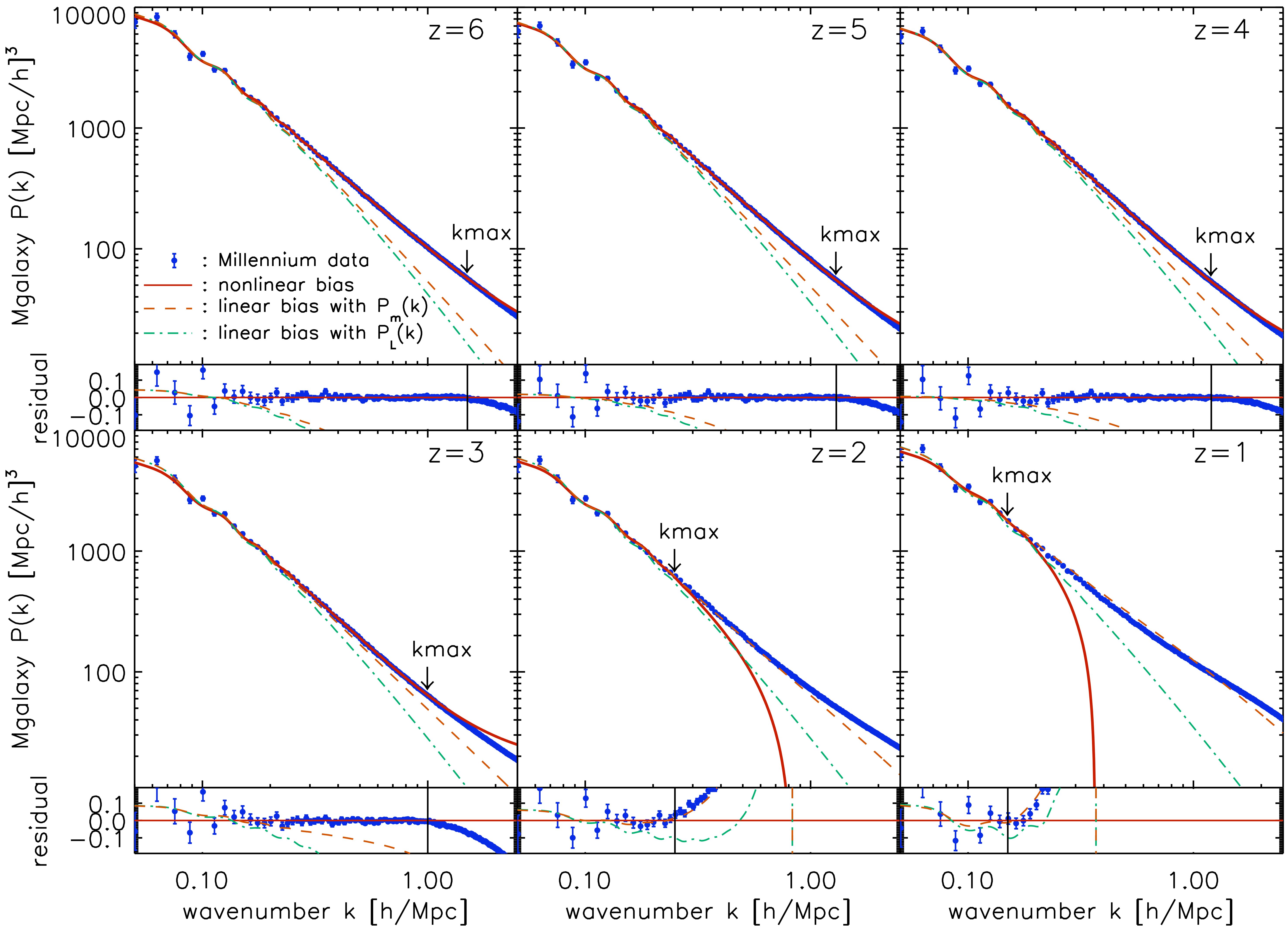
$$P_g(k) = N + b_1^2\Bigg[P(k) + \frac{b_2^2}{2}\int \frac{d^3\boldsymbol{q}}{(2\pi)^3}P(q)\Big[P(|\boldsymbol{k}-\boldsymbol{q}|) - P(q)\Big] + 2b_2\int \frac{d^3\boldsymbol{q}}{(2\pi)^3}P(q)P(|\boldsymbol{k}-\boldsymbol{q}|)F_2^{(s)}(\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q})\Bigg]$$

- 3 bias parameters (b_1, b_2, N) are linearly related to the coefficients of the Taylor expansion (c_1, c_2, c_3, ε)
- These parameters contain the information of the physics of galaxy formation; however, we shall marginalize over them because we are not interested in them. (b_1, b_2, N are *<u>nuisance parameters</u>*)

Millennium "Galaxy" Catalogue

- Let's compare 3PT with galaxy simulations...
- The best simulation available today: Millennium Simulation (Springel et al. 2005).
- Millinnium Simulation is a N-body simulation. How did they create galaxies? Semi-analytical galaxy formation recipe.
    - MPA code: De Lucia & Blaizot (2007)
    - Durham code: Croton et al. (2006)

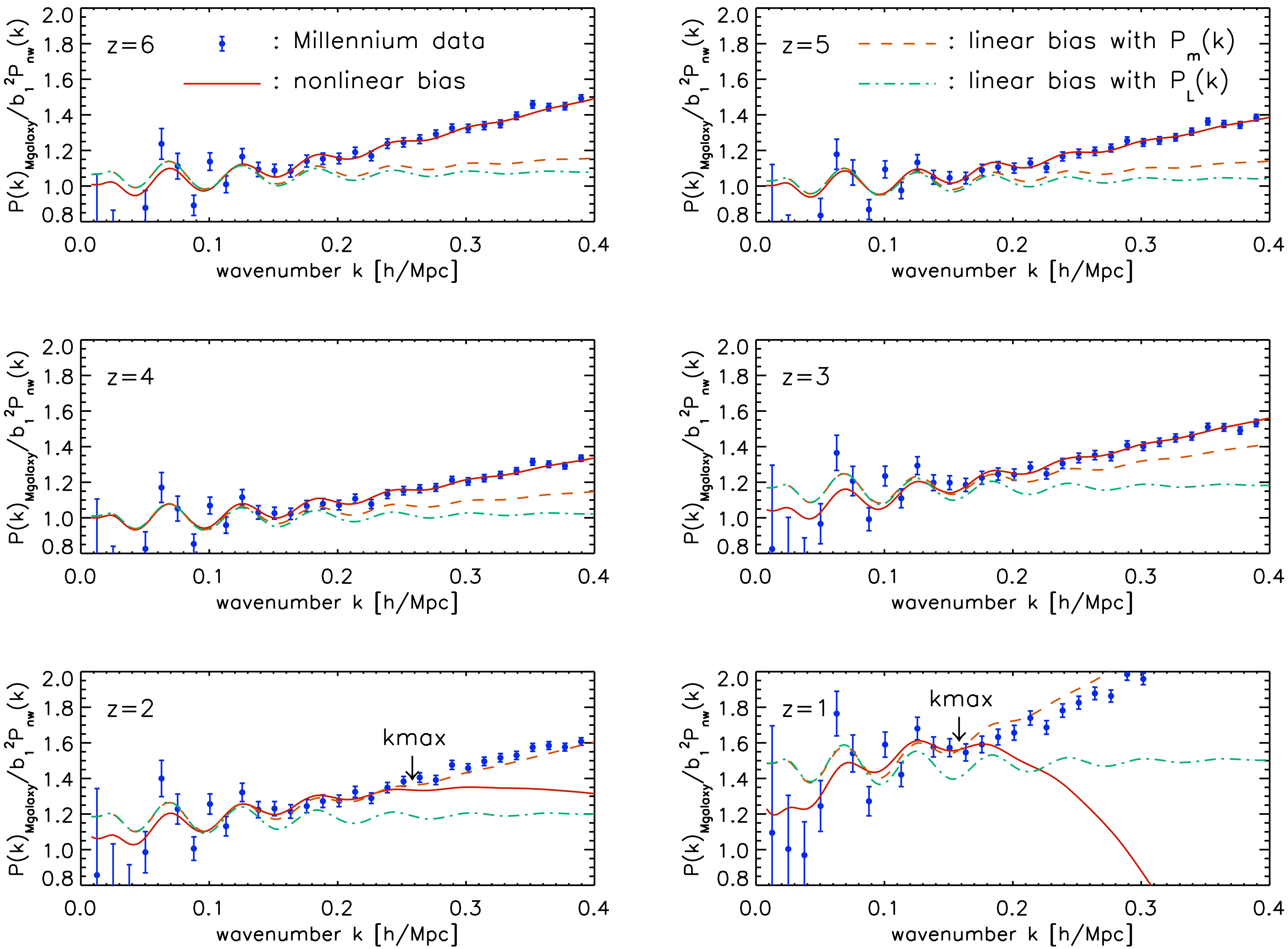
3PT vs MPA galaxies



- k_{max} is where 3PT deviates from the matter P(k) at 1%.
- So, we must stop using 3PT for galaxies at k_{max} also.
- 3PT with local bias assumption fits the Millennium Simulation very well.

Jeong & Komatsu (2008)

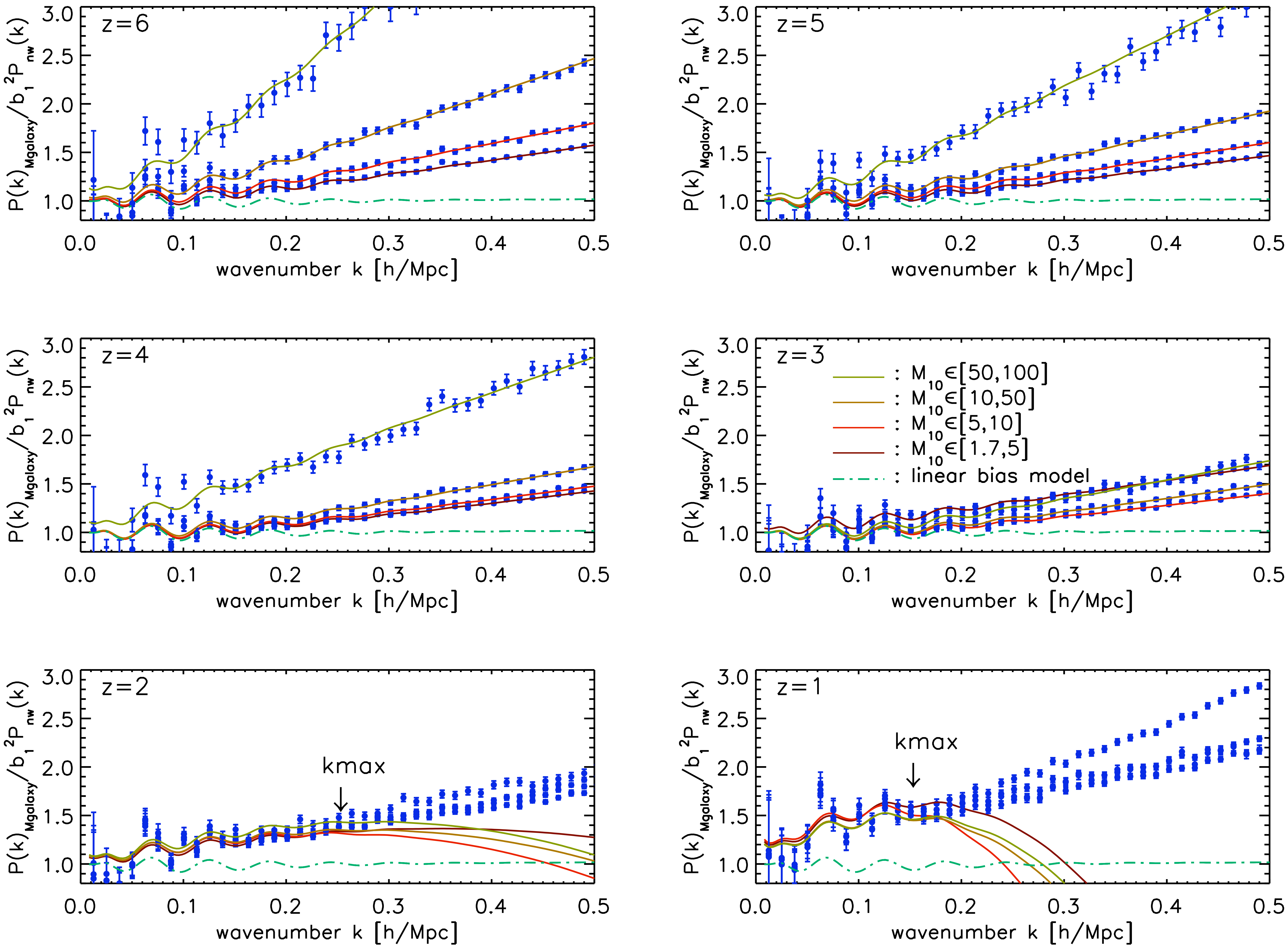
BAO: Non-linear Bias



- It is obvious that non-linear bias is going to be important for BAOs
- But, we now know how to model the effect!

Jeong & Komatsu (2008)

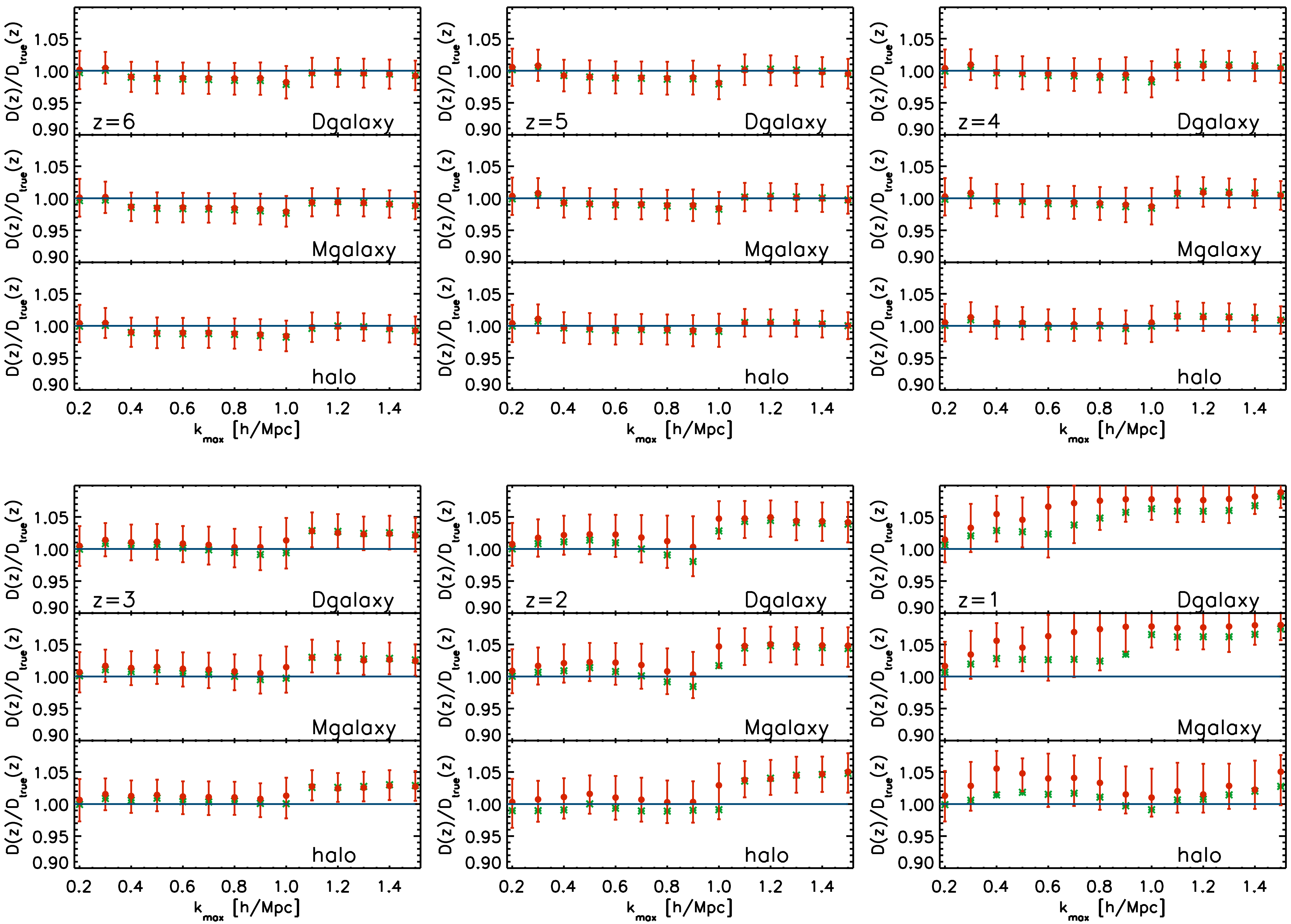
Galaxy Mass Dependence



- Massive galaxies are more strongly biased with greater non-linearities
- This is a well-known fact, by the way.
- 3PT works just fine for ***any*** masses, as long as we apply it only up to k_{max} that is given by the matter power spectrum

Jeong & Komatsu (2008)

$D_A(z)$ From $P_g(k)$



- **Result**

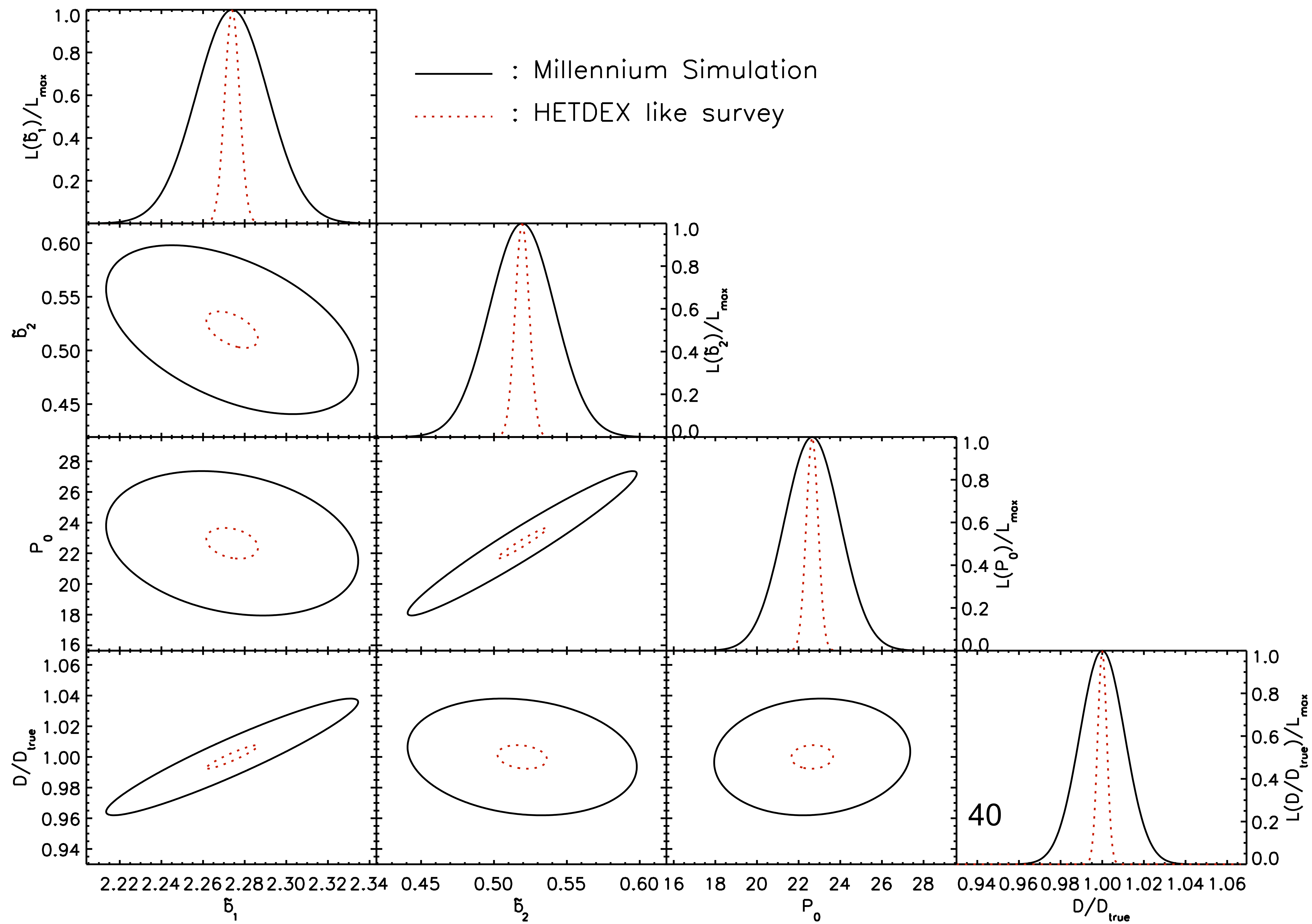
  With 3PT, we succeeded in measuring the correct $D_A(z)$ from the "observed" galaxy power spectra in the Millennium Simualtion at z>2

- However, z=1 still seems challenging
  - Better PT is needed, e.g., Renormalized PT

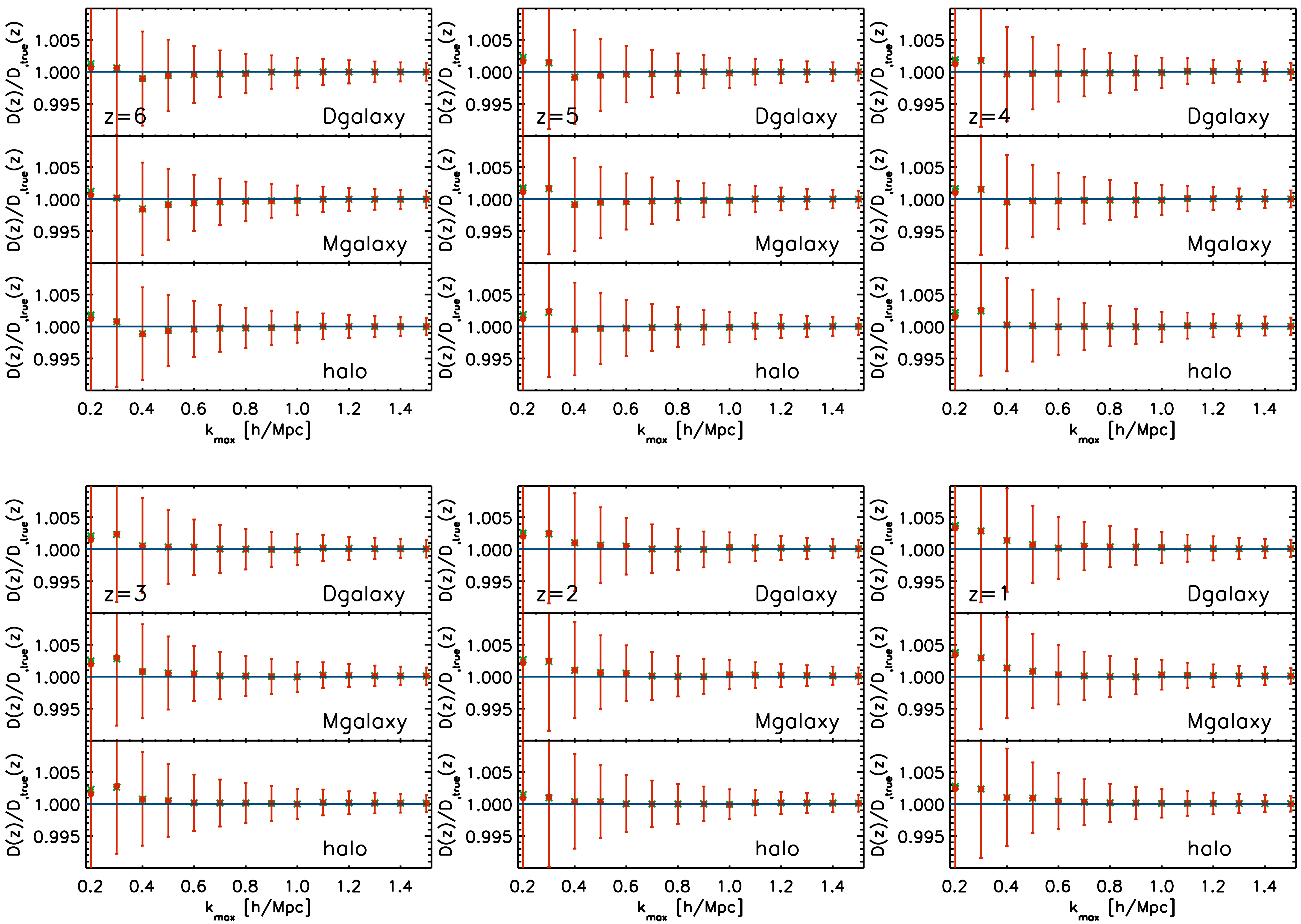
Jeong & Komatsu (2008)

So Much Degeneracies...

- Bias parameters and the distance are strongly degenerate, if we use the power spectrum information only.
- Solution?
  - Use the bispectrum!



Let's say, we determine b_1 and b_2 from the galaxy bispectra...



- **Result**

  The errors in the distance determinations are reduced substantially.

  WE MUST USE THE BISPECTRUM

Bispectrum?

- Bispectrum (3-point correlation) depends on b_1 and b_2 as:

$$Q_g(k_1,k_2,k_3)=(1/\mathbf{b_1})[Q_m(k_1,k_2,k_3)+\mathbf{b_2}]$$

Q_m is the matter bispectrum, given by PT.

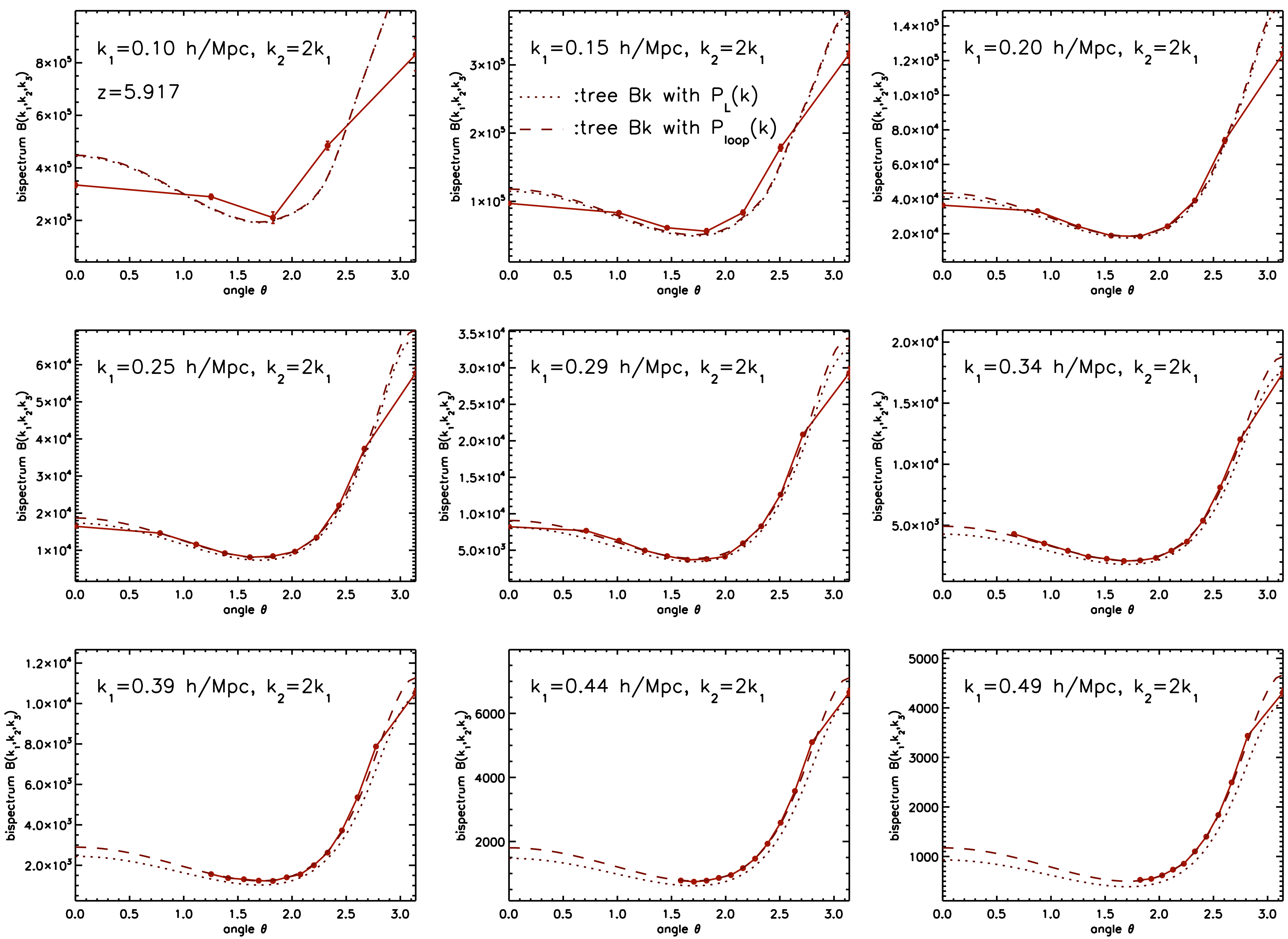
- This method has been applied to the real data (2dFGRS): b_1=1.04±0.11; b_2=-0.054±0.08 at z=0.17 (Verde et al. 2002)

- At higher redshifts, we expect x10 better results (Sefusatti & Komatsu 2007)

- The bispectrum is an indispensable tool for measuring the bias parameters.

Jeong, Sefusatti & Komatsu (in preparation)

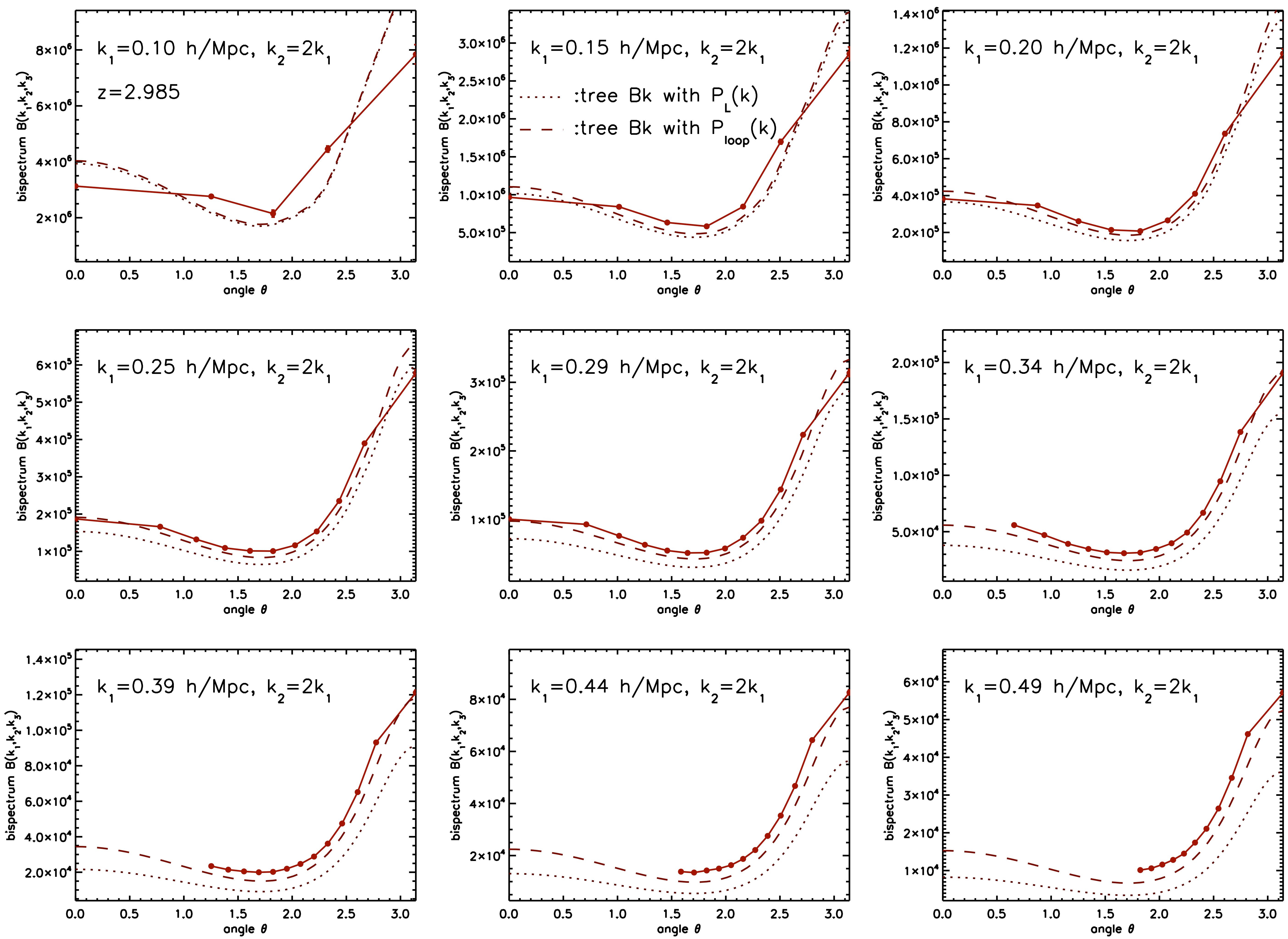
PT vs Bispectrum (z=6)



- 2nd-order PT
  - Good agreement at z=6
- Preliminary!

Jeong, Sefusatti & Komatsu (in preparation)

PT vs Bispectrum (z=3)



- 2nd-order PT
  - Agreement is not satisfactory, even at z=3
- 4th-order PT is necessary?
- Preliminary!

Results So Far

- We understood the effects of matter non-linearity on P(k) at z>2, using cosmological perturbation theory.
- Galaxy bias is also understood, at least on large scales where 3PT is valid.
- Bispectrum must be used: we are now developing a joint analysis pipeline using the power spectrum and bispectrum.

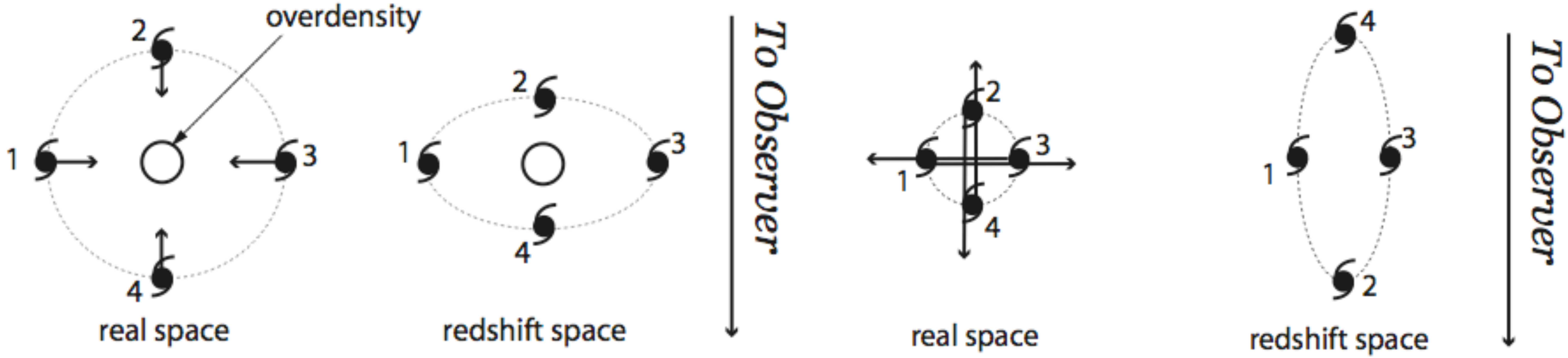
Biggest Limitation

These results are all in real space. We still need to go to redshift space...

Most Difficult Problem

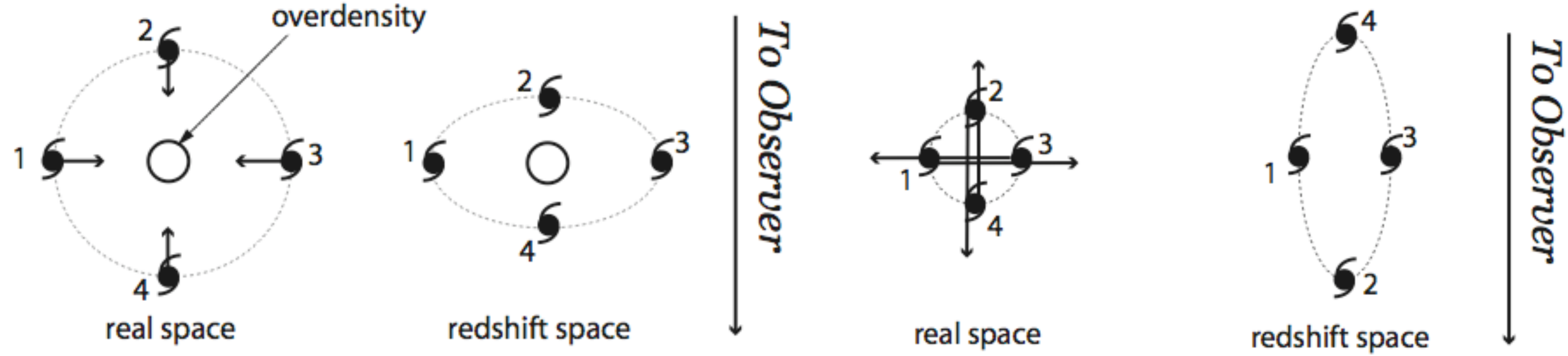
- The most difficult (and unsolved) problem in modeling $P_g(k)$ is the "redshift space distortion" arising from the peculiar velocity of galaxies
- Understanding this effect is crucial for getting H(z) out of the observed galaxy power spectrum
- Why so difficult?
  - Perturbation theory calculation breaks down, even at z~3

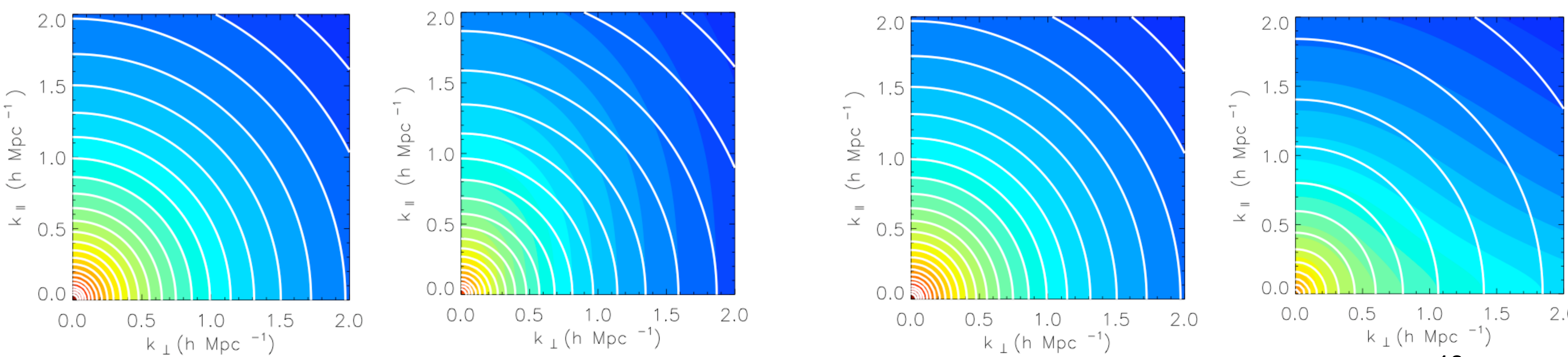
Redshift Space Distortion



- (Left) Coherent velocity field => Clustering **enhanced** along the line of sight
  - "Kaiser" effect
- (Right) Virial-like random motion => Clustering **diminished** along the line of sight
  - "Finger-of-God" effect

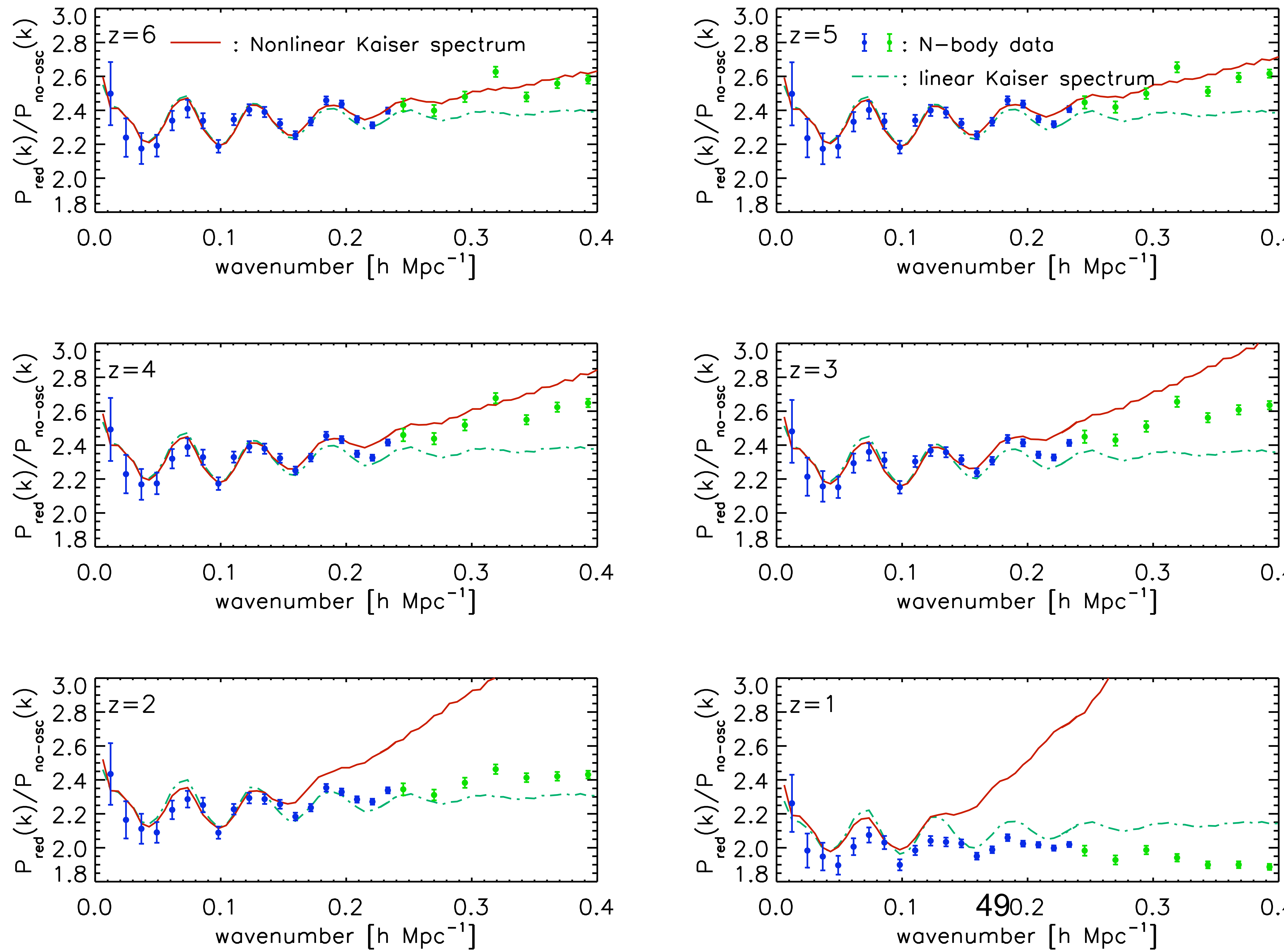
Redshift Space Distortion





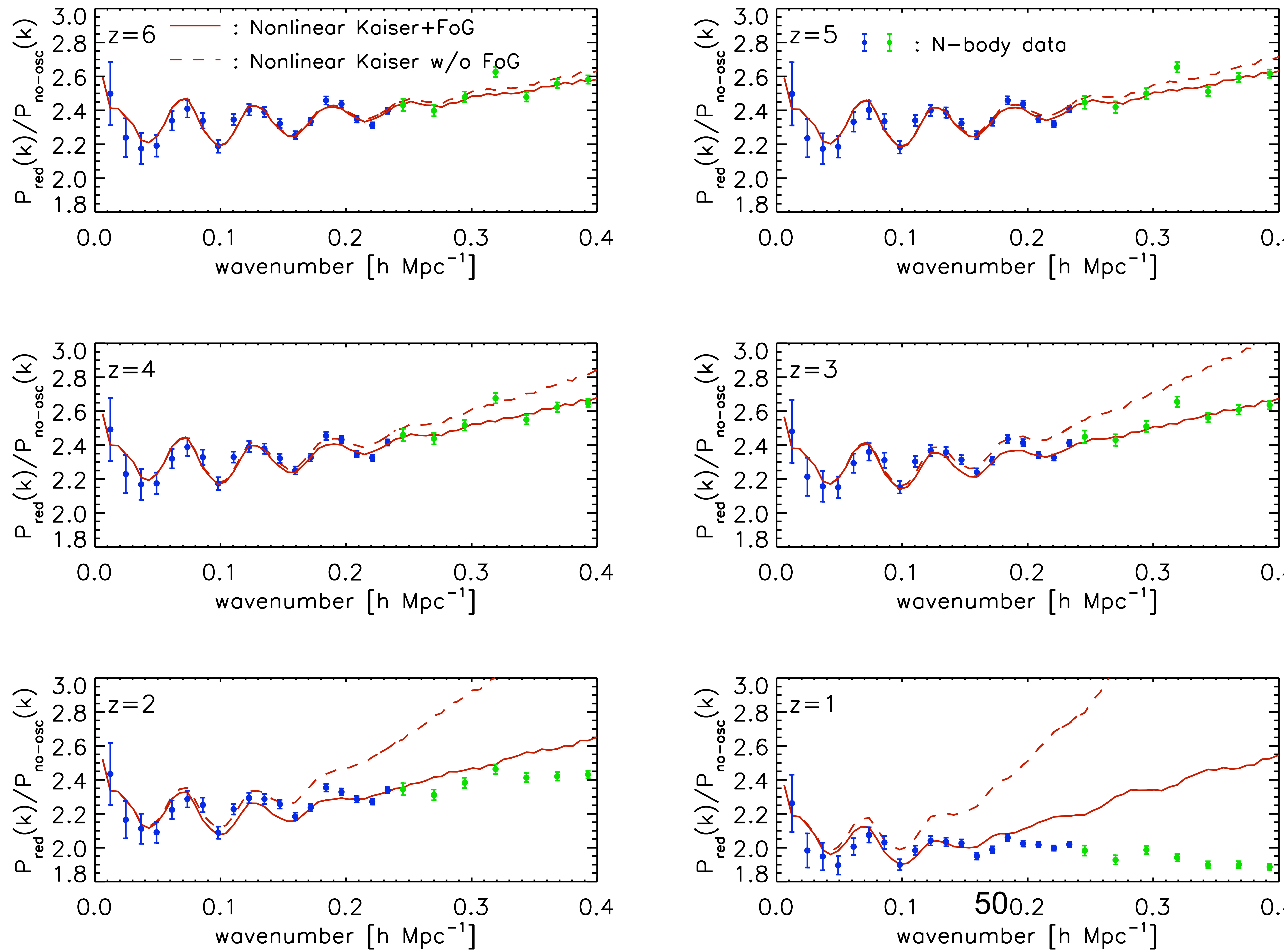
PT in Redshift Space

- Non-linear Kaiser effect can be calculated by PT
- But, PT overestimates power at z<3...
- This is caused by the Finger-of-God effect, which is **non-perturbative** and is absent in the existing PT calculations



PT in Redshift Space

- Empirical (and historical) modeling of FoG
  - $P_g(k)/(1+k_{para}^2\sigma^2)$
- Agreement is a sort of OK, but this is a wrong approach.
  - We need to remove any room for empirical calibrations.
- Work to do. (There are some ideas.)



Even Worse

- Bispectrum needs to be computed in redshift space also!
- Seems like a long way to go, but serious investigations have already begun. E.g.,
  - "An Analytic Model for the Bispectrum of Galaxies in Redshift Space" by Smith, Sheth & Scoccimarro, PRD, in press (0712.0017)

Summary

- With perturbation theory, we think we can model
  - non-linear matter clustering, and
  - non-linear and stochastic galaxy bias
- Redshift space distortion requires more work. It is likely that we need to give up perturbative descriptions of FoG.
  - Need for a hybrid approach: PT for P(k) in real space, convolved with the velocity distribution function computed in some other way
- HETDEX starting in 2011: we still have 3 years...