On the amplitude of non-Gaussian halo bias

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Critical test of inflation using non-Gaussianity, MPA Garching, Nov 7 2012

- Since Dalal et al (2008):

high peaks

Matarrese & Verde (2008)

Verde & Matarrese (2009)

...

Peak-background split

Slosar et al (2008)

Schmidt & Kamionkowski (2010)

...

local bias

Taruya, Koyama & Matsubara (2008)

Sefusatti (2009)

...

Multivariate bias

McDonald (2008)

Giannantonio & Porciani (2010)

...

- Since Dalal et al (2008):

high-threshold regions
Matarrese & Verde (2008)
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Peak-background split
Slosar et al (2008)
Schmidt & Kamionkowski (2010)
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local bias
Taruya, Koyama & Matsubara (2008)
Sefusatti (2009)
...

Multivariate bias
McDonald (2008)
Giannantonio & Porciani (2010)
...

Discrete density peaks

- Peak-background split (PBS) with Gaussian ICs:

$$\delta(\mathbf{x}) = \delta_l(\mathbf{x}) + \delta_s(\mathbf{x})$$

$$\delta_g(\mathbf{x}) \equiv \frac{n_g(\mathbf{x})}{\bar{n}_g} - 1 = \frac{\bar{n}_g\left[\left(\delta_c - \delta_l(\mathbf{x})\right)/\sigma_s\right]}{\bar{n}_g(\delta_c/\sigma_s)} - 1 = \left(-\frac{1}{\bar{n}_g}\frac{d\bar{n}_g}{d\delta_c}\right)\delta_l(\mathbf{x}) + \ldots$$

$$\bar{n}_g = \bar{n}_g\left(\frac{\delta_c}{\sigma_s}\right)$$

Universal mass function

$$\equiv b_1$$

linear bias

(Kaiser 1984, Bardeen et al 1986, Cole & Kaiser 1989, Mo & White 1996, Sheth & Tormen 1999, ...)

- PBS with local quadratic PNG: $\Phi(\mathbf{x}) = \phi(\mathbf{x}) + f_{\rm NL}\phi(\mathbf{x})^2$

$$\phi(\mathbf{x}) = \phi_l(\mathbf{x}) + \phi_s(\mathbf{x})$$

$$\Phi = \left(\phi_l + f_{\rm NL}\phi_l^2\right) + \phi_s\left(1 + 2f_{\rm NL}\phi_l\right) + f_{\rm NL}\phi_s^2$$

$$\sigma_s \;\to\; \sigma_s\Big(1 + 2f_{\rm NL}\phi_l(\mathbf{x})\Big)$$

Slosar et al (2008)

$$\delta_{\rm g}(\mathbf{x}) = \left(-\frac{1}{\bar{n}_{\rm g}}\frac{d\bar{n}_{\rm g}}{d\delta_c}\right)\delta_l(\mathbf{x}) + \left(\frac{1}{\bar{n}_{\rm g}}\frac{d\bar{n}_{\rm g}}{d\sigma_s}\right)2f_{\rm NL}\sigma_s\phi_l(\mathbf{x}) + \cdots$$

$$= b_1\delta_l(\mathbf{x}) + \left(-\frac{1}{\bar{n}_{\rm g}}\frac{d\bar{n}_{\rm g}}{d\delta_c}\right)2f_{\rm NL}\delta_c\phi_l(\mathbf{x}) + \cdots \longleftarrow \bar{n}_{\rm g} \equiv \bar{n}_{\rm g}\left(\frac{\delta_c}{\sigma_s}\right)$$

$$= b_1\delta_l(\mathbf{x}) + 2f_{\rm NL}\delta_c b_1\phi_l(\mathbf{x}) + \cdots$$

$$\delta_l(\mathbf{k}) = \mathcal{M}(k)\phi_l(\mathbf{k})$$

$$\delta_g(\mathbf{k}) = \Big(b_1 + 2f_{\rm NL}\delta_c b_1\mathcal{M}^{-1}(k)\Big)\delta_l(\mathbf{k}) + \dots$$

- Generalize to arbitrary PNG with PBS formulated in terms of i) split or ii) conditional mass function

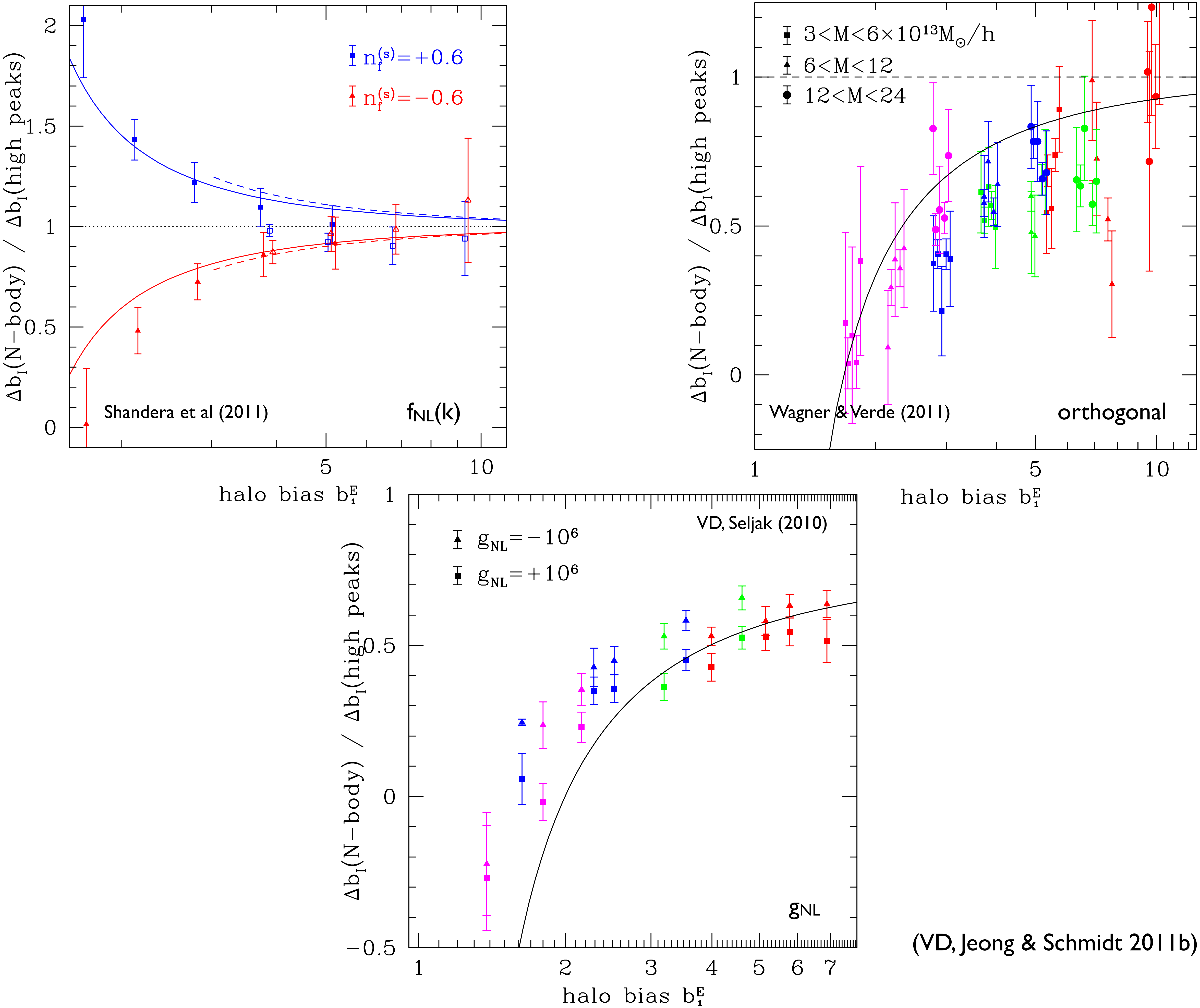
(VD, Jeong & Schmidt 2011a)

- Non-Gaussian bias corrections from peak-background split

$$\Delta b_1(k) = \frac{4}{(N-1)!}\left\{b_{N-2}\delta_c + b_{N-3}\left[3 - N + \frac{\partial \ln \mathcal{F}_s^{(N)}(k)}{\partial \ln \sigma_s}\right]\right\}\mathcal{F}_s^{(N)}(k)\mathcal{M}_s(k)^{-1}$$

negligible only for constant-$f_{\rm NL}\phi^2$ model

(VD, Jeong & Schmidt 2011a)



(VD, Jeong & Schmidt 2011b)

Relax the assumption of a universal mass function using, e.g., the excursion set formalism:

$$\bar{n}(M) = \frac{\bar{\rho}}{M} \left| \frac{d\sigma_0^2}{dM} \right| \mathcal{F}_0(\delta_c, \sigma_0^2)$$

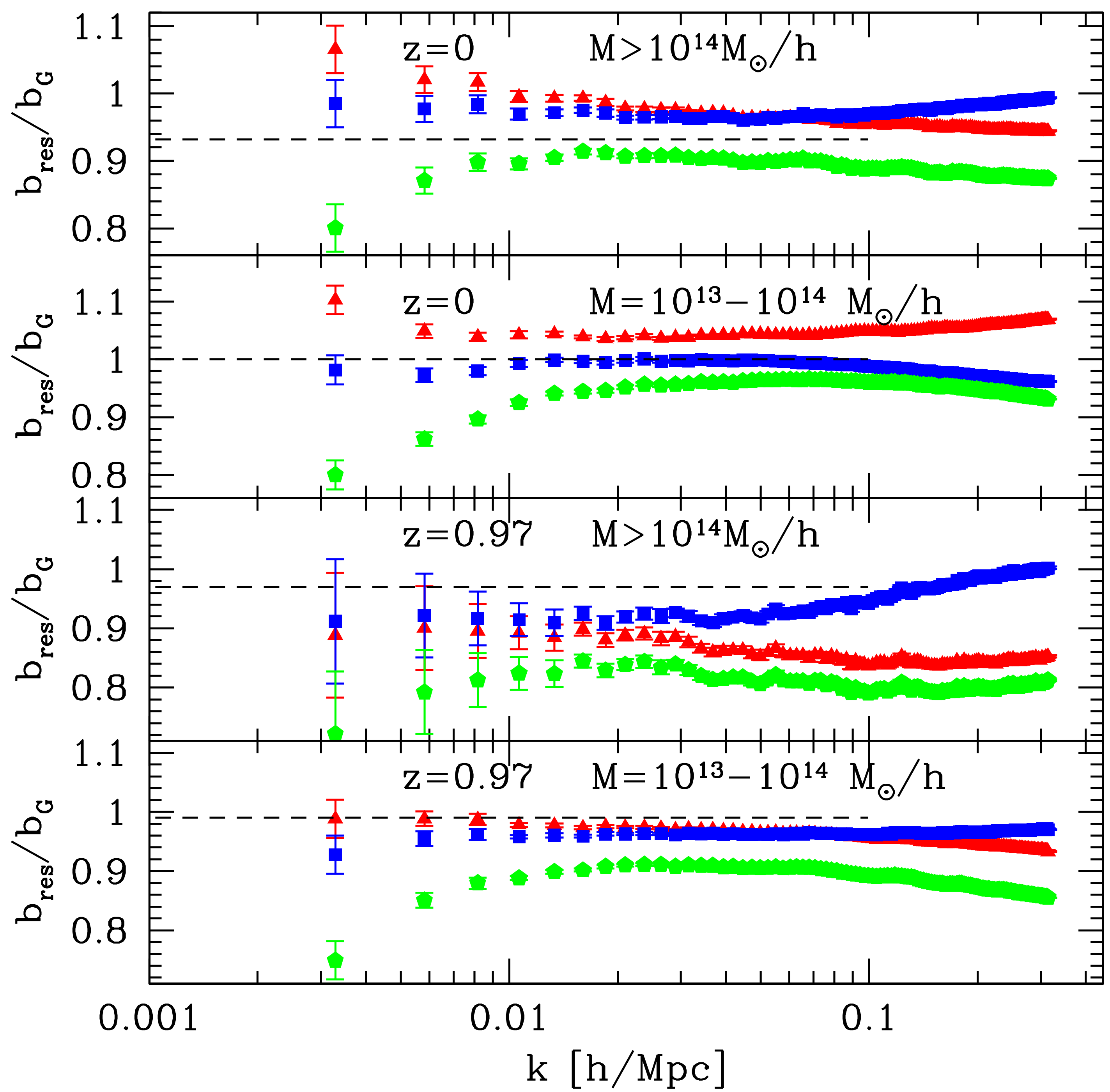
The non-Gaussian bias becomes:

$$\Delta b_1(k) = \frac{\partial_{\sigma_0^2} \left[I_{21} \mathcal{F}_0 \right]}{\mathcal{M}(k) \mathcal{F}_0} = \frac{\partial_M \left[I_{21}(k, M) \left(\frac{dn}{d \ln M} \right) \left(\frac{d\sigma_0^2}{dM} \right)^{-1} \right]}{\mathcal{M}(k) \left(\frac{dn}{d \ln M} \right)}$$

$$I_{21}^{\rm loc}(k \to 0, M) = 4 f_{\rm NL} \sigma_0^2 + \mathcal{O}(k^2)$$

(Scoccimarro et al 2012)

Orthogonal shape



(Scoccimarro et al 2012)

Use path-integral formalism to compute additional corrections to non-Gaussian bias:

$$\mathcal{F}_0(\delta_c, \sigma_0^2|\delta_l) = -\frac{\partial}{\partial \sigma_0^2} \int_{-\infty}^{\delta_c} d\delta\, \Pi_0(\delta, \sigma_0^2|\delta_l)$$

$$\Pi_0(\delta_s, \sigma_0^2|\delta_l) = \int_{-\infty}^{\delta_c} d\delta_1 \ldots \int_{-\infty}^{\delta_c} d\delta_{n-1}\, W(\delta_l; \delta_1, \ldots, \delta_{n-1}, \delta_s; \sigma_0^2)$$

$$W(\delta_l, \ldots, \delta_s; \sigma_0^2) = \int \mathcal{D}\lambda \, \exp\left(i \sum_{j=1}^{n} \lambda_j \delta_j - \frac{1}{2} \sum_{j,k=1}^{n} \lambda_j \lambda_k \left\langle \delta_j \delta_k \right\rangle_c \right)$$
$$\times \exp\left(\frac{(-i)^3}{6} \sum_{j,k,l=1}^{n} \lambda_j \lambda_k \lambda_l \left\langle \delta_j \delta_k \delta_l \right\rangle_c \right)$$
$$\times \ldots$$

(Maggiore & Riotto 2010; D'Aloisio et al 2012; Ashead et al 2012)

- Discrete density peaks (a la BBKS):

$$\nu(\mathbf{x}) = \frac{1}{\sigma_0}\delta_s(\mathbf{x}), \quad \eta_i(\mathbf{x}) = \frac{1}{\sigma_1}\partial_i\delta_s(\mathbf{x}), \quad \zeta_{ij}(\mathbf{x}) = \frac{1}{\sigma_2}\partial_i\partial_j\delta_s(\mathbf{x})$$

- Kac-Rice formula:

$$n_{\rm pk}(\nu', \mathbf{x}) = \sum_{\rm pk} \delta_D(\mathbf{x} - \mathbf{x}_{\rm pk}) = \frac{3^{3/2}}{R_\star^3} \, |{\rm det}\zeta(\mathbf{x})| \; \delta_D[\eta(\mathbf{x})] \; \theta(\lambda_3) \, \delta_D[\nu' - \nu(\mathbf{x})]$$

- Calculate ensembles averages

$$\langle n_{\rm pk}(\nu, \mathbf{x}_1) \times \cdots \times n_{\rm pk}(\nu, \mathbf{x}_n) \rangle$$

$$R_\star = \sqrt{3}\frac{\sigma_1}{\sigma_2}$$

$$\gamma = \frac{\sigma_1^2}{\sigma_0 \sigma_2}$$

$$\sigma_n^2 = \frac{1}{2\pi^2} \int_0^\infty dk\, k^{2(n+1)}\, P_s(k)$$

- For Gaussian initial conditions, the average peak number density is

$$\langle n_{\rm pk}(\nu, \mathbf{x})\rangle \equiv \bar{n}_{\rm pk}(\nu, R_s) = \frac{1}{(2\pi)^2 R_\star^3}\, G_0(\gamma, \gamma\nu) e^{-\nu^2/2}$$

$$G_n(\gamma, \omega) = \int_0^\infty du\, u^n\, f(u) \frac{e^{-(u-\omega)^2/2(1-\gamma^2)}}{\sqrt{2\pi(1-\gamma^2)}}$$

$$\propto (\gamma u)^3 \quad \text{for} \quad \nu \gg 1$$

$$u = -\sum_i \zeta_{ii} = \text{peak curvature}$$

(Bardeen et al 1986)

Non-Gaussian bias from discrete density peaks

VD, Jinn-Ouk Gong & Toni Riotto, in preparation

- 2-point correlation of discrete peaks with non-Gaussian ICs:

$$\langle n_{\rm pk}(\nu, \mathbf{x}_1) n_{\rm pk}(\nu, \mathbf{x}_2)\rangle = \bar{n}^2 \left[1 + \xi_{\rm pk}(\nu, |\mathbf{x}_1 - \mathbf{x}_1|)\right]$$

$$P(\mathbf{y}_1, \mathbf{y}_2; r) = P_G(\mathbf{y}_1, \mathbf{y}_2; r) \times \text{Edgeworth series}$$

- Contributions from 3-point correlators:

$$\left\langle \nu^2(\mathbf{x}_1)\nu(\mathbf{x}_2)\right\rangle \qquad \left\langle \nu^2(\mathbf{x}_1)\zeta_{ij}(\mathbf{x}_2)\right\rangle$$

$$\langle \nu(\mathbf{x}_1)\zeta_{ij}(\mathbf{x}_1)\nu(\mathbf{x}_2)\rangle \qquad \langle \nu(\mathbf{x}_1)\zeta_{ij}(\mathbf{x}_1)\zeta_{kl}(\mathbf{x}_2)\rangle$$

$$\langle \eta_i(\mathbf{x}_1)\eta_j(\mathbf{x}_1)\nu(\mathbf{x}_2)\rangle \qquad \langle \eta_i(\mathbf{x}_1)\eta_j(\mathbf{x}_1)\zeta_{kl}(\mathbf{x}_2)\rangle$$

$$\langle \zeta_{ij}(\mathbf{x}_1)\zeta_{kl}(\mathbf{x}_1)\nu(\mathbf{x}_2)\rangle \qquad \langle \zeta_{ij}(\mathbf{x}_1)\zeta_{kl}(\mathbf{x}_1)\zeta_{mn}(\mathbf{x}_2)\rangle$$

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- Summing up all the contributions, we find (specialized to local quadratic PNG)

$$\Delta b_1(k) = 2f_{\rm NL}\left[\left(\sum_{j=0}^{2}\frac{\partial\ln n_{\rm pk}}{\partial\ln\sigma_j}\right)-1\right]\mathcal{M}^{-1}(k)$$

- To make connection with dark matter halos, we should ensure that first-crossing occurs on the scale R_s:

$$\bar{n}_{\rm pk}(\nu, R_s) \;\rightarrow\; f_{\rm ESP}(\nu, R_s) = \frac{e^{-\nu^2/2}}{\sqrt{2\pi}}\left(\frac{V}{V_\star}\right)\frac{G_1(\gamma, \gamma\nu)}{\gamma\nu}$$

(Paranjape & Sheth 2012; Paranjape, Sheth & VD 2012)

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(Paranjape & Sheth 2012; Paranjape, Sheth & VD 2012)

- The halo mass function is

$$\bar{n}(M) = \frac{\bar{\rho}}{M} f_{\rm ESP}(\nu)\frac{d\nu}{dM} = \frac{\bar{\rho}}{M^2}\nu f_{\rm ESP}(\nu)\frac{d\,{\rm ln}\nu}{d\,{\rm ln}M}$$

- The non-Gaussian halo bias thus is

$$\Delta b_1(k) = 2 f_{\rm NL}\left(\sum_{j=0}^{2}\frac{\partial\ln\bar{n}}{\partial\ln\sigma_j}\right)\mathcal{M}^{-1}(k)$$

- PBS not assumed but recovered from the calculation

$$\sigma_j \to (1+\alpha)\,\sigma_j$$

so that $R_\star$ and γ remain unchanged

- The *k*-dependence is in the correlators $\langle \nu^2(\mathbf{x}_1)\nu(\mathbf{x}_2)\rangle$ etc.

- First-crossing of strongly correlated walks so no significant corrections from correlators of fields at scales different than R_s

- No contribution from zero-lag moments

Summary

$\Delta P_{halo}(k)$ is in good shape but ...