On Squeezed Limits From Single Field Inflation

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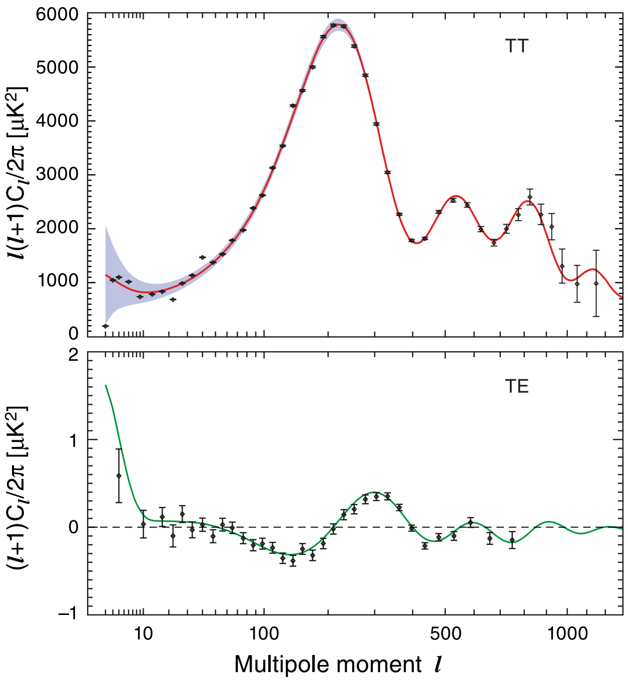
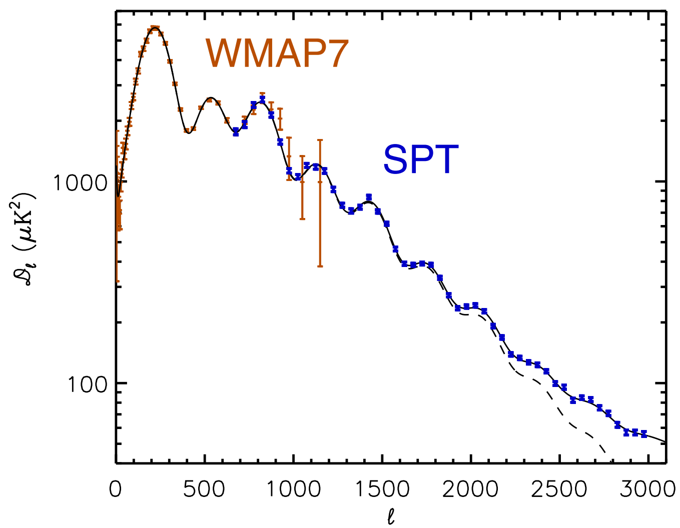
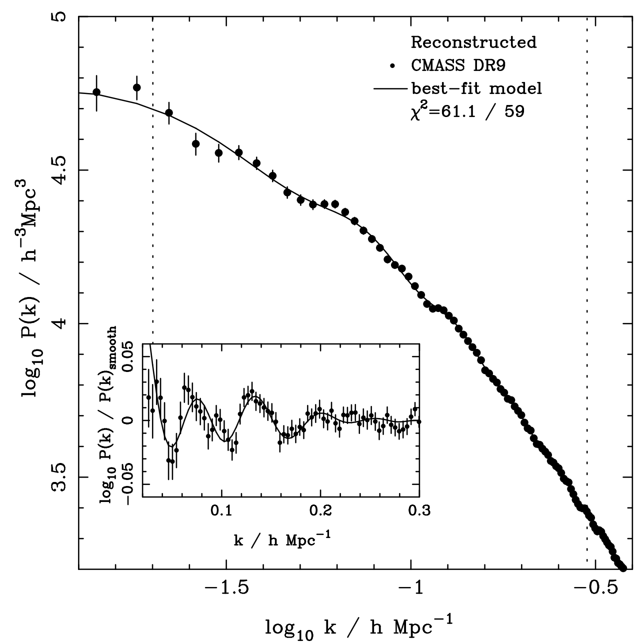
Introduction







Data is consistent with an early inflationary phase.

To what extent can learn about its detailed properties?

Introduction

Was there a single light degree of freedom or many?

For models with a single clock in the Bunch-Davies state the squeezed limit of 3-pt function of curvature perturbations obeys the consistency relation

$$\lim_{k_L \to 0} \langle \zeta_{k_L} \zeta_{k_S} \zeta_{k_S} \rangle' = -P_\zeta(k_L) P_\zeta(k_S) \left[(n_s - 1) + \mathcal{O}\left(k_L^2/k_S^2\right) \right]$$

"The local physics is independent of (frozen) long wavelength modes."

Looks promising to distinguish between single and multi-field models, but can be violated for $X < k_L/k_S \ll 1$.

Introduction

Experimentally we must be sensitive to triangles with $k_L/k_S < X$ to rule out single field inflation.

The goal will be to show that for a large class of single field models

$$X \sim \Delta_{\mathcal{R}}^{1/2} \sim 10^{-2}$$

So a detection for $k_L/k_S < \Delta_{\mathcal{R}}^{1/2}$ would rule out a large class of single field models.

What limits the range is backreaction and weak coupling.

Outline

- Why does the consistency condition hold?
- What does it take to violate it for intermediate momenta?
- What is X in simple single field models?
- Generalizations
- Forecasts

The consistency condition

Why does the consistency condition hold?

- In single field models, modes freeze after horizon crossing. (once the attractor is reached)
- Frozen superhorizon modes correspond to a rescaling of coordinates and are unobservable.
- If no correlations between short and long modes are generated before the long modes exit, the consistency condition holds.

The consistency condition

What does it take to violate it for intermediate scales?

- Since correlations must be generated before the long modes exits, we can use flat space intuition to understand the physics better.

Why are the correlations in the Bunch-Davies state small and generated near horizon crossing?

Consider a massless scalar field in flat space with cubic self-interaction

$$\langle \varphi_{\vec{k}_1}\varphi_{\vec{k}_2}\varphi_{\vec{k}_3}(t)\rangle' = -i\frac{1}{8k_1k_2k_3}\int_{-\infty(1-i\epsilon)}^{t} dt'\,\mu e^{i(k_1+k_2+k_3)(t'-t)} - \text{c.c.}$$

$$= -\mu\frac{1}{4k_1k_2k_3(k_1+k_2+k_3)}$$

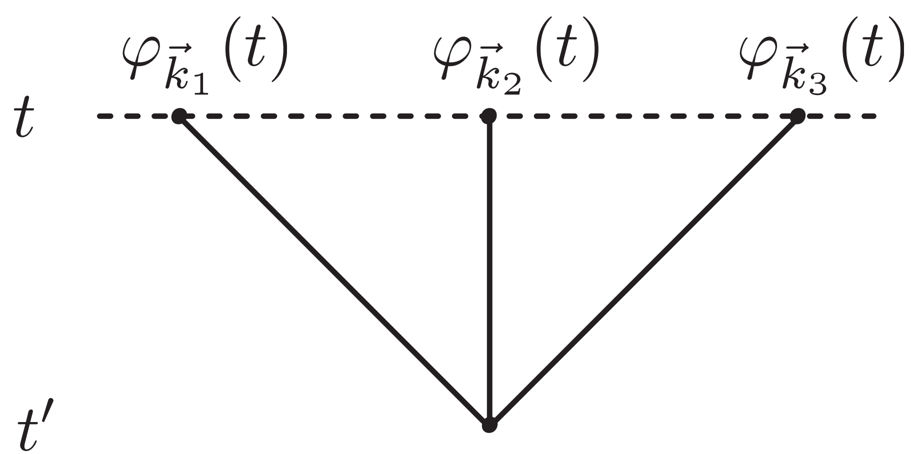
The consistency condition

What does it take to violate it for intermediate scales?

Key properties

- Generated $\Delta t \sim 1/(k_1 + k_2 + k_3)$ before t
- proportional to $1/k_t$ (no interesting squeezed limit)

Can be understood intuitively from



The consistency condition

What does it take to violate it for intermediate scales?

Crucial assumptions

- Time translation invariance
- The system is in the ground state

We can break these with time-dependent interactions that excite the state dynamically or by an excited initial state.

Either way we need energy which will affect the dynamics of the system.

The consistency condition

What does it take to violate it for intermediate scales?

In both scenarios

$$\frac{k_S}{a} \lesssim \frac{k_\star}{a} \lesssim \dot{\phi}^{1/2}$$

$$k_L \sim a(t) H$$

$$\frac{k_L}{k_S} \gtrsim \frac{H}{\dot{\phi}^{1/2}} \sim \Delta_{\mathcal{R}}^{1/2}$$

Non-trivial correlations exist only between modes that are separated in wave number by less than ~100

The bound

Non-adiabatic evolution

Consider the concrete example

$$V = V_0 + \Lambda^4 \cos\left(\frac{\phi}{f}\right)$$

Then (with $\omega = \dot{\phi}/f$)

$$\langle \zeta(\mathbf{k_1}, t)\zeta(\mathbf{k_2}, t)\zeta(\mathbf{k_3}, t)\rangle' = (2\pi)^4 \Delta_{\mathcal{R}}^4 \frac{1}{k_1^2 k_2^2 k_3^2}$$

$$\times f^{\rm res}\left[\sin\left(\frac{\omega}{H_0}\ln k_t/k_0\right) + \frac{H_0}{\omega}\sum_{i\neq j}\frac{k_i}{k_j}\cos\left(\frac{\omega}{H_0}\ln k_t/k_0\right) + \dots\right]$$

violates the consistency condition for $\dfrac{H_0}{\omega} < \dfrac{k_L}{k_S} < 1$

The bound

Non-adiabatic evolution

To make the range $\frac{H_0}{\omega} < \frac{k_L}{k_S} < 1$ as large as possible, should maximize ω / H_0 .

The theory becomes strongly coupled at energies $\sim f$

$$\omega < f \implies \omega < \dot{\phi}^{1/2}$$

So violations of the consistency condition are limited to

$$\frac{k_L}{k_S} \gtrsim \frac{H}{\dot{\phi}^{1/2}} \sim \Delta_{\mathcal{R}}^{1/2}$$

The bound

Excited initial state

If we allow an arbitrary state, essentially anything goes.

We will make the assumption that only modes between

$$k_L = a_I H_I \quad \text{and} \quad k_S = k_\star$$

are significantly excited.

Then the consistency condition holds for

$$k_L/k_S < a_I H_I/k_\star$$

(but $k_\star$ a priori is arbitrary.)

The bound

Excited initial state

The power spectrum is

$$\langle\Psi|\zeta_H(\mathbf{k_1},t)\zeta_H(\mathbf{k_2},t)|\Psi\rangle' = \langle\Psi|\zeta_I(\mathbf{k_1},t)\zeta_I(\mathbf{k_2},t)|\Psi\rangle' + \ldots$$
$$= \frac{H^4}{2\dot{\phi}^2 k^3}(1+2N_k+\ldots)+\ldots$$

These dots are negligible only if $\langle\Psi|\delta\dot{\varphi}^2|\Psi\rangle \ll \dot{\phi}^2$

But

$$\langle\Psi|\delta\dot{\varphi}^2|\Psi\rangle \sim \frac{1}{a^4}\int\frac{d^3k}{(2\pi)^3}k\,N_k \sim \frac{k_\star^4}{8\pi^2 a^4}N_{k_\star}$$

The bound

Excited initial state

So $\langle\Psi|\delta\dot{\varphi}^2|\Psi\rangle \ll \dot{\phi}^2$ implies

$$\frac{k_L}{k_S} > \frac{\Delta_{\mathcal{R}}^{1/2} N_{k_\star}^{1/4}}{(1+2N_{k_\star}+\dots)^{1/4}}$$

Large occupation numbers ($\gtrsim 0.1$) are thus ruled out by the power spectrum because it is scale invariant over more than 2 orders of magnitude.

Small occupation numbers require more attention.

The bound

Excited initial state

What if $\langle\Psi|\delta\dot{\varphi}^2|\Psi\rangle \gtrsim \dot{\phi}^2$?

The energy density stored in the excitations is large enough to affect the background evolution.

In particular there is a phase with $\delta = \frac{\ddot{H}}{2\dot{H}H} \approx -2$

The power spectrum for modes that exit during this phase is generically scale-dependent.

Canceling the scale dependence by hand leads to highly non-scale invariant higher n-point functions.

Forecasts

Can excited states with small occupation numbers and

$$\frac{k_L}{k_S} > \Delta_{\mathcal{R}}^{1/2} N_{k_\star}^{1/4}$$

lead to halo bias detectable say with Euclid?

15,000 sq.deg.
z=0.5...2.1(12 bins)

Compute

$$P(k) = (b_E^{(g)} + \Delta b)^2 P_m(k)$$

and then

$$\Delta\chi^2 = \sum_i \frac{V(z_i)}{(2\pi)^2} \int dk k^2 \left(1 - \frac{1}{n_g(z_i)P(k)}\right)^2 \left(\frac{\Delta P(k, z_i)}{P(k, z_i)}\right)^2$$

Forecasts

In practice

$$\Delta b(k,M) = \frac{1}{\mathcal{M}_M(k)}\left(\frac{(b_E^{(g)}-1)\delta_c}{D(z)}\mathcal{F}(k,M) + \frac{d\mathcal{F}(k,M)}{d\ln\sigma_M}\right)$$

with

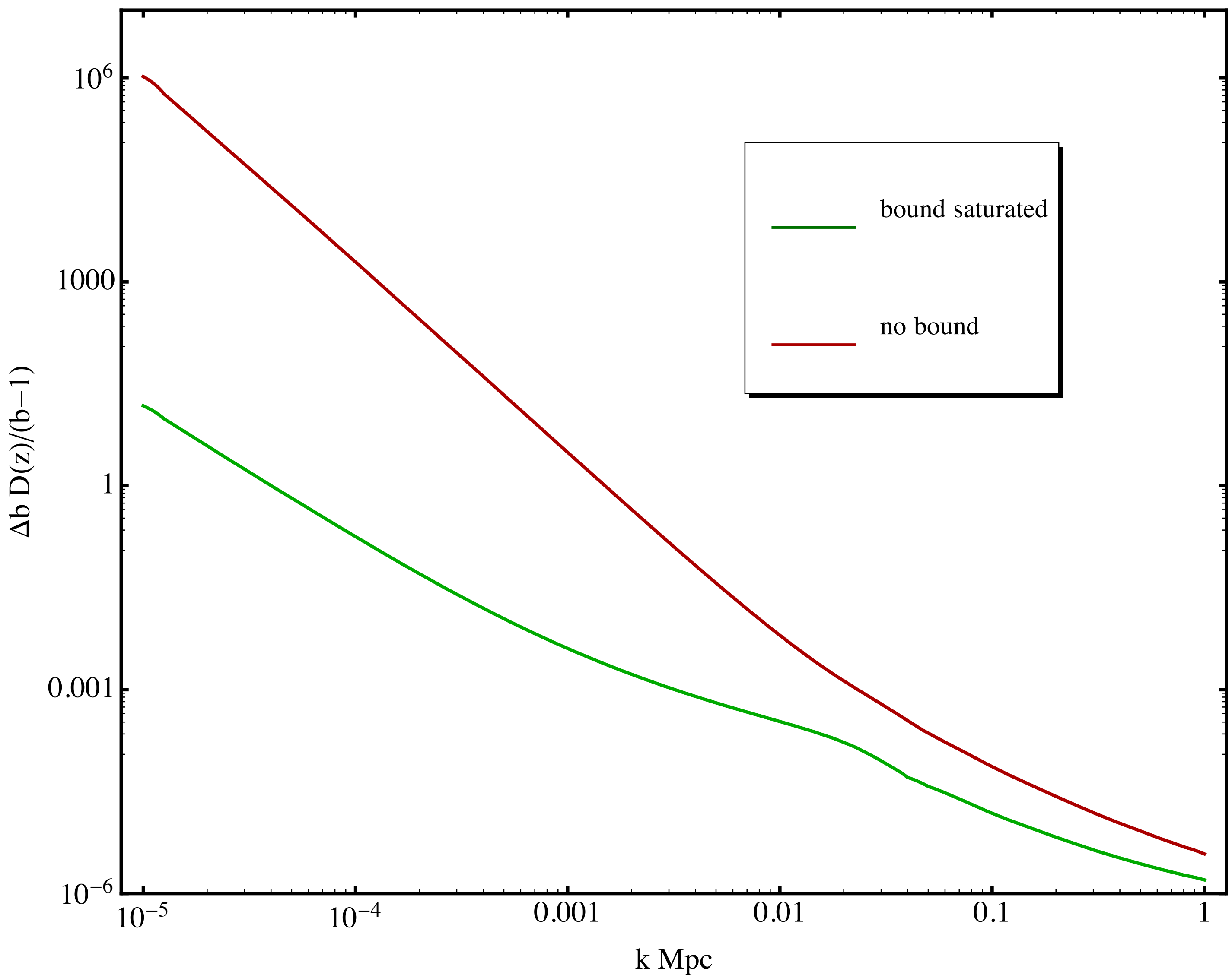
$$\mathcal{M}_M(k) = k^2 T(k) W(kR(M))$$

$$\mathcal{F}(k,M) = \frac{1}{8\pi^2\sigma_M^2 P_\zeta(k)}\int dk' k'^2 \mathcal{M}_M(k')\int_{-1}^{1} d\mu \mathcal{M}_M(|\vec{k}+\vec{k}'|) B_\zeta(k,k',|\vec{k}+\vec{k}'|)$$

We use slow roll inflation with Bogoliubov state for B_ζ excited from $k_L = 1/\tau_0$ to k_S saturating the bound

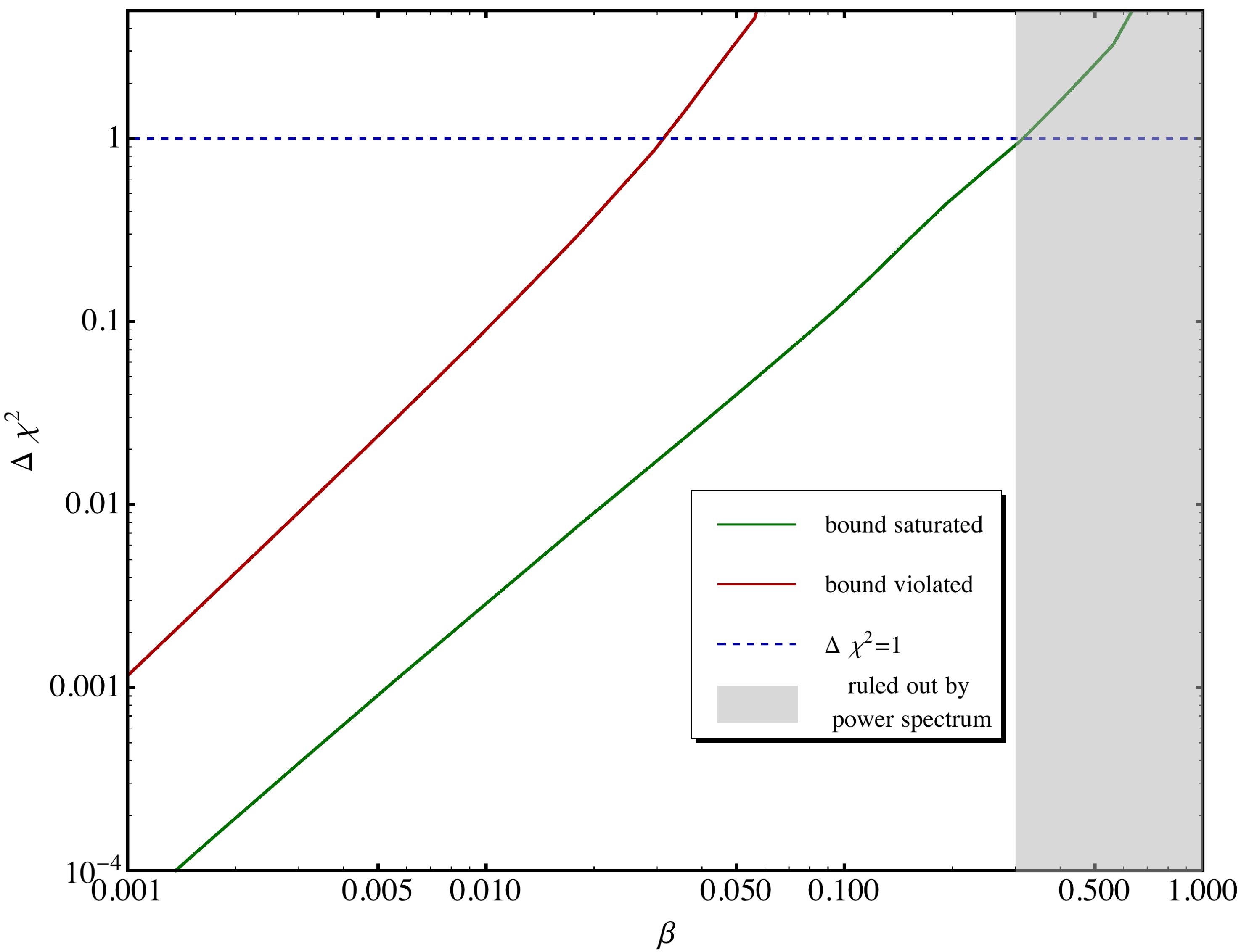
and a model for the Gaussian bias, number of galaxies, etc., from Orsi, A., Baugh, C., Lacey, C., Cimatti, A., Wang, Y., et al. 2010

Forecasts



bound saturated
no bound
10^6
1000
1
0.001
10^{-6}
$\Delta b\ D(z)/(b-1)$
10^{-5}
10^{-4}
0.001
0.01
0.1
1
k Mpc

Forecasts



1
0.1
0.01
0.001
10^{-4}
$\Delta\,\chi^2$
0.001
0.005
0.010
0.050
0.100
0.500
1.000
β
bound saturated
bound violated
$\Delta\,\chi^2=1$
ruled out by power spectrum

Conclusions

- This can be generalized conveniently with the framework of the effective field theory for fluctuations during inflation to more general models
- Generically, single field models violate the consistency relation and lead to interesting behavior in the squeezed limit only for $$\frac{k_L}{k_S} \gtrsim \frac{H}{\dot{\phi}^{1/2}} \sim \Delta_{\mathcal{R}}^{1/2}$$
- Special states can circumvent this and extend the range to $$\frac{k_L}{k_S} \gtrsim \Delta_{\mathcal{R}} f_{\rm NL}^{1/2}$$

Thank you!