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Single-field consistency relations

10 Years !!

with

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and M. Zaldarriaga

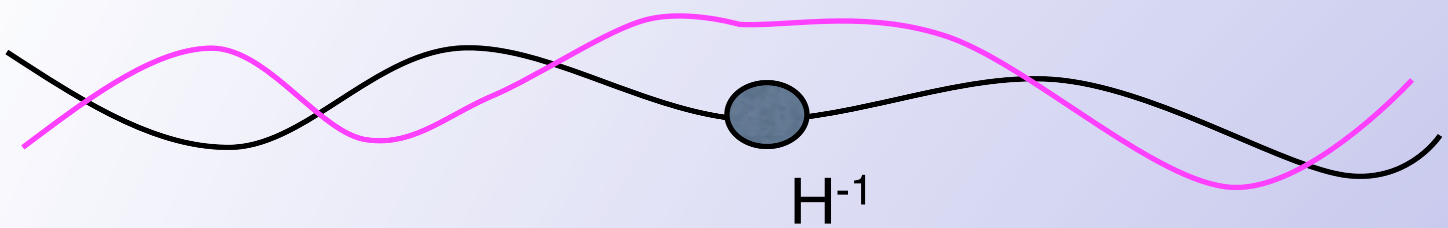
Outline

- Derivation of **Conformal Consistency Relations**
- Consequences
- **Assumptions** and **Counterexamples**
- Full conformal invariance

Non-linearly adiabatic

A long adiabatic mode is … nothing

VS

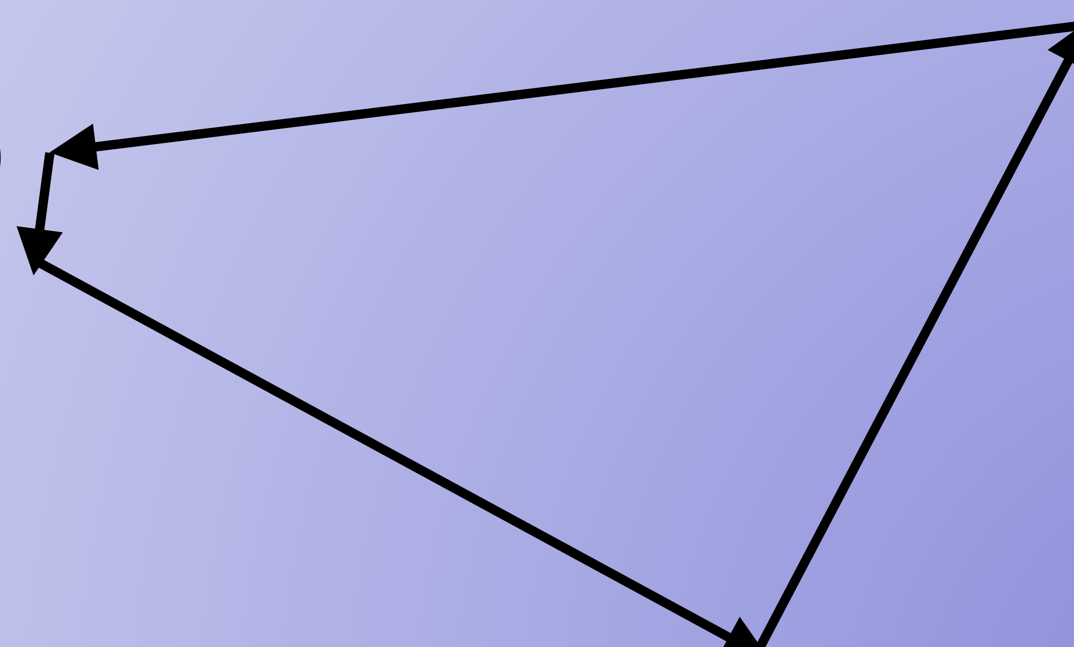


Adiabaticity:

$$\frac{\delta\rho_m}{\rho_m} = \frac{3}{4}\frac{\delta\rho_\gamma}{\rho_\gamma}$$

Long adiabatic modes only induce (locally unobservable) coordinate redefinitions of the short modes

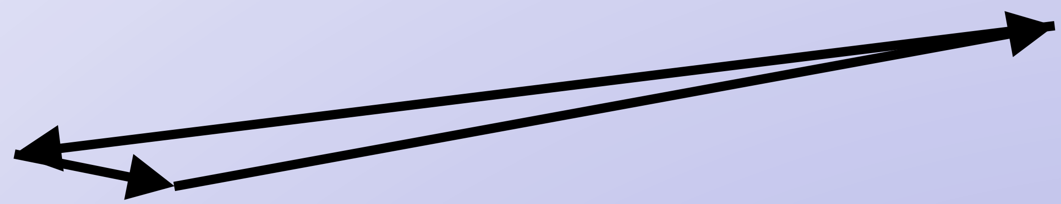
$k_1 \to 0$



3pf consistency relation

Maldacena 02
PC, Zaldarriaga 04
Cheung etal 07

Squeezed limit of the 3-point function in single-field models



$k_1 \to 0$

Single field !

$$\phi(t,\vec{x}) = \phi_0(t) \qquad h_{ij} = e^{2\zeta(t,\vec{x})}\delta_{ij}$$

ζ goes to a constant: attractor!

The long mode is already classical when the other freeze and acts just as a rescaling of the coordinates

$$\langle\zeta(\vec{x}_2)\zeta(\vec{x}_3)\rangle|_{\bar{\zeta}(x)} \simeq \xi(\vec{x}_3-\vec{x}_2) + \bar{\zeta}(\vec{x}_+)[(\vec{x}_3-\vec{x}_2)\cdot\vec{\nabla}\xi(|\vec{x}_3-\vec{x}_2|)]$$

Bunch-Davies!

3pf consistency relation

$$\langle\bar\zeta(\vec x_1)\zeta(\vec x_2)\zeta(\vec x_3)\rangle \simeq \langle\bar\zeta(\vec x_1)\bar\zeta(\vec x_+)\rangle[(\vec x_3-\vec x_2)\cdot\vec\nabla\xi(|\vec x_3-\vec x_2|)]$$

$$\simeq \int\frac{\mathrm{d}^3k_L}{(2\pi)^3}\int\frac{\mathrm{d}^3k_S}{(2\pi)^3}\,e^{i\vec k_L\cdot(\vec x_1-\vec x_+)}\,P(k_L)P(k_S)\left[\vec k_S\cdot\frac{\partial}{\partial\vec k_S}\right]e^{i\vec k_S\cdot\vec x_-}$$

$$= -\int\frac{\mathrm{d}^3k_1\,\mathrm{d}^3k_L\,\mathrm{d}^3k_S}{(2\pi)^9}\,e^{-i\vec k_1\cdot\vec x_1-i\vec k_L\cdot\vec x_++i\vec k_S\cdot\vec x_-}\left[(2\pi)^3\delta(\vec k_1+\vec k_L)P(k_1)P(k_S)\frac{\mathrm{d}\ln k_S^3P(k_S)}{\mathrm{d}\ln k_S}\right]$$

$$\downarrow$$

$$\langle\zeta(\vec k_1)\zeta(\vec k_2)\zeta(\vec k_3)\rangle \underset{k_1\to 0}{\simeq} -(2\pi)^3\delta(\vec k_1+\vec k_2+\vec k_3)P(k_1)P(k_S)\frac{\mathrm{d}\ln(k_S^3P(k_S))}{\mathrm{d}\ln k_S}$$

$$\vec k_S = (\vec k_2-\vec k_3)/2$$

- No slow-roll approximation
- It holds also in non-inflationary models (if assumptions hold)
- It can be proven wrong

Adiabatic mode including gradients

Adiabatic modes can be constructed from unfixed gauge transformations (k=0)

In ζ gauge:

Weinberg 03

$$\phi(t,\vec{x}) = \phi_0(t) \qquad h_{ij} = e^{2\zeta(t,\vec{x})}\delta_{ij}$$

- Cannot touch t
- Conformal transformation of the spatial coordinates:

$$x^i \to x^i - b^i \vec{x}^2 + 2x^i(\vec{b}\cdot\vec{x})$$

$$\zeta = 2\vec{b}(t)\cdot\vec{x} + \lambda(t)$$

- Impose it is the k → 0 limit of a physical solution

$$\partial_j(H\delta N - \dot{\zeta}) = 0 \qquad (3H^2 + \dot{H})\delta N + H\partial_i N^i = -\frac{\nabla^2}{a^2}\zeta + 3H\dot{\zeta}$$

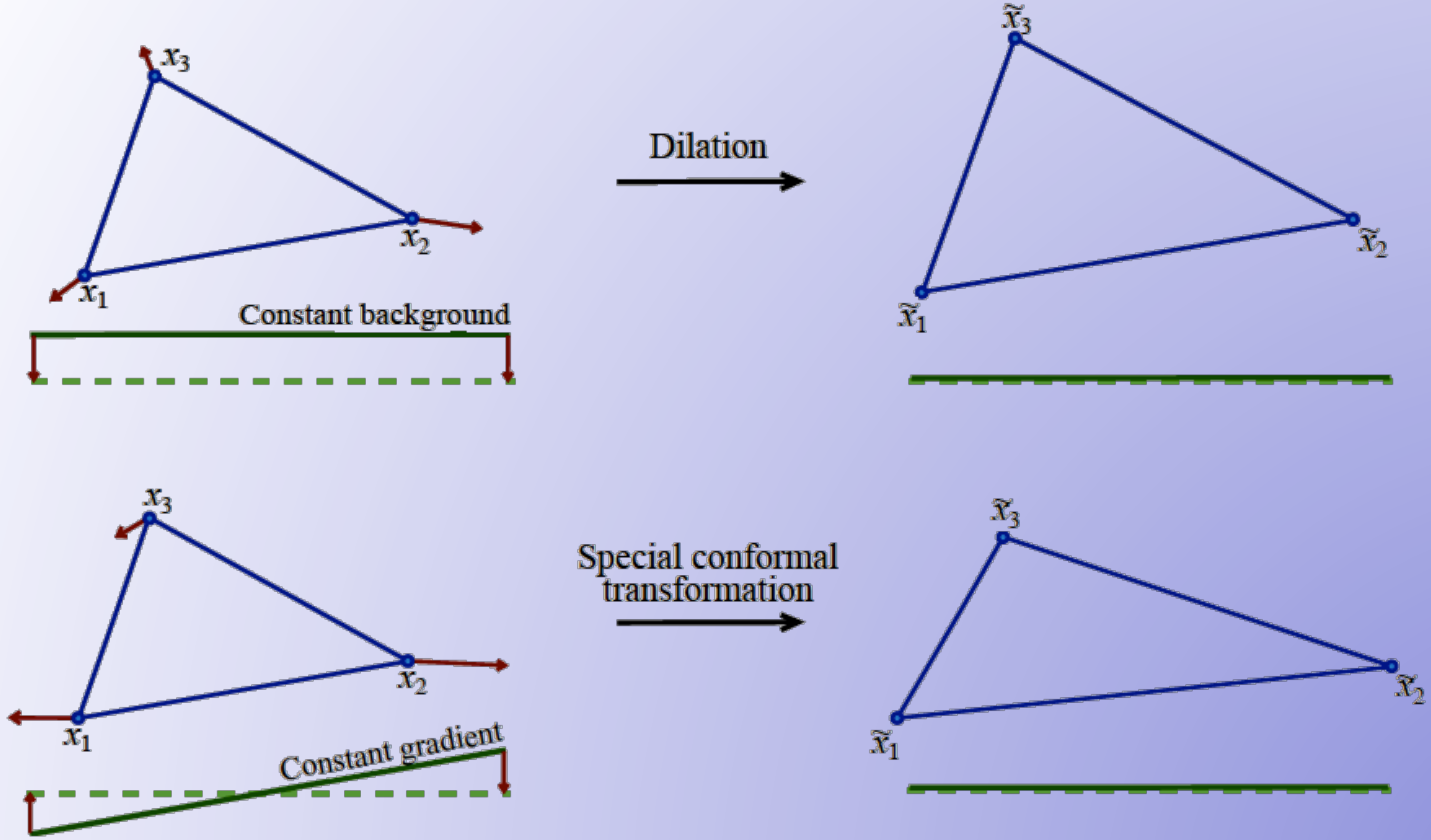
- b and λ are time-independent + need a time-dep translation to induce the N^i

Long wavelength approx of an adiabatic mode up to $O(k^2)$

Extension to the full SO(4,1)

A special conformal transformation induces a conformal factor linear in x

$$\zeta = 2\vec{b}\cdot\vec{x} + \lambda$$



Conformal consistency relations

PC, Noreña, Simonović 12

$$\langle \zeta_{\vec{q}} \zeta_{\vec{k}_1} \ldots \zeta_{\vec{k}_n} \rangle' \stackrel{q \to 0}{=} - P(q) \left[3(n-1) + \sum_a \vec{k}_a \cdot \vec{\partial}_{k_a} + \frac{1}{2} q^i D_i \right] \langle \zeta_{\vec{k}_1} \ldots \zeta_{\vec{k}_n} \rangle' + \mathcal{O}(q/k)^2$$

$$\text{with} \quad q^i D_i \equiv \sum_{a=1}^{n} \left[6 \vec{q} \cdot \vec{\partial}_{k_a} - \vec{q} \cdot \vec{k}_a \vec{\partial}^2_{k_a} + 2 \vec{k}_a \cdot \vec{\partial}_{k_a} (\vec{q} \cdot \vec{\partial}_{k_a}) \right] .$$

2- and 3-pf only depend on moduli and $q^i D_i$ reduces to: $\sum_{a=1}^{n} \vec{q} \cdot \vec{k}_a \left[\frac{4}{k_a} \frac{\partial}{\partial k_a} + \frac{\partial^2}{\partial k_a^2} \right]$

The variation of the 2-point function is zero: no linear term in the 3pf

PC, D'Amico, Musso and Noreña 11

Consistency relations as Ward identities and with OPE methods:
see Baumann's and Khoury's talks

3pf - 4pf in slow-roll inflation

Maldacena 02

$$\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle = (2\pi)^3\delta^3(\sum \vec{k}_i)\frac{\dot\rho_*^4}{\dot\phi_*^4}\frac{H_*^4}{M_{pl}^4}\frac{1}{\prod_i(2k_i^3)}\mathcal{A}_*$$

$$\mathcal{A} = 2\frac{\ddot\phi_*}{\dot\phi_*\dot\rho_*}\sum_i k_i^3 + \frac{\dot\phi_*^2}{\dot\rho_*^2}\left[\frac{1}{2}\sum_i k_i^3 + \frac{1}{2}\sum_{i\neq j} k_i k_j^2 + 4\frac{\sum_{i>j} k_i^2 k_j^2}{k_t}\right]$$

$$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\zeta_{\mathbf{k}_4}\rangle^{\mathrm{CI}} = (2\pi)^3\delta(\sum_a \mathbf{k}_a)\frac{H_*^6}{4\epsilon^2\prod_a(2k_a^3)}\sum_{\mathbf{perms}}\mathcal{M}_4(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)$$

$$\mathcal{M}_4(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) = -2\frac{k_1^2k_3^2}{k_{12}^2k_{34}^2}\frac{W_{24}}{k_t}\left(\frac{\mathbf{Z}_{12}\cdot\mathbf{Z}_{34}}{k_{34}^2} + 2\mathbf{k}_2\cdot\mathbf{Z}_{34} + \frac{3}{4}\sigma_{12}\sigma_{34}\right) - \frac{1}{2}\frac{k_3^2}{k_{34}^2}\sigma_{34}\left(\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_t}W_{124} + 2\frac{k_1^2k_2^2}{k_t^3} + 6\frac{k_1^2k_2^2k_4}{k_t^4}\right),$$

$$\sigma_{ab} = \mathbf{k}_a\cdot\mathbf{k}_b + k_b^2\,,$$

$$\mathbf{Z}_{ab} = \sigma_{ab}\mathbf{k}_a - \sigma_{ba}\mathbf{k}_b\,,$$

$$W_{ab} = 1 + \frac{k_a+k_b}{k_t} + \frac{2k_ak_b}{k_t^2}\,,$$

$$W_{abc} = 1 + \frac{k_a+k_b+k_c}{k_t} + \frac{2(k_ak_b+k_bk_c+k_ak_c)}{k_t^2} + \frac{6k_ak_bk_c}{k_t^3}$$

Lidsey, Seery, Sloth 06
Seery, Sloth, Vernizzi 09

Small speed of sound: large NG

$$S = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left(M_{\rm Pl}^2 R + 2P(X,\phi)\right) \qquad X = -\tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

$$c_s^2 \equiv \frac{P_{,X}}{P_{,X} + 2XP_{,XX}}$$

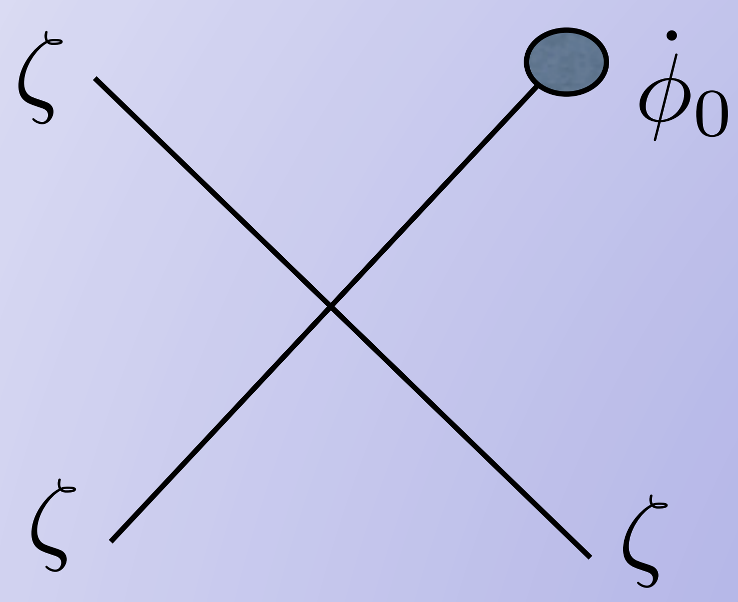
E.g. Chen etal 09

$$\lambda = X^2 P_{,XX} + \frac{2}{3}X^3 P_{,XXX}$$

$$\Sigma = X^2 P_{,X} + 2X^2 P_{,XX}\,.$$

$$P_\zeta = \frac{1}{2M_{\rm Pl}^2}\frac{H^2}{c_s\epsilon}$$

$$\mu = \frac{1}{2}X^2 P_{,XX} + 2X^3 P_{,XXX} + \frac{2}{3}X^4 P_{,XXXX}$$



CCR encode at the level of observables the
non-linear relation among operators in the Lagrangian

Small speed of sound

$$\mathcal{M}^{(3)} = \left(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma}\right)\frac{3k_1^2k_2^2k_3^2}{2k_t^3} + \left(\frac{1}{c_s^2} - 1\right)\left(-\frac{1}{k_t}\sum_{i>j}k_i^2k_j^2 + \frac{1}{2k_t^2}\sum_{i\neq j}k_i^2k_j^3 + \frac{1}{8}\sum_i k_i^3\right)$$

$$\mathcal{M}^{(4)}_{cont} = \left[\frac{3}{2}\left(\frac{\mu}{\Sigma} - \frac{9\lambda^2}{\Sigma^2}\right)\frac{\prod_{i=1}^4 k_i^2}{k_t^5} - \frac{1}{8}\left(\frac{3\lambda}{\Sigma} - \frac{1}{c_s^2} + 1\right)\frac{k_1^2k_2^2(\vec{k}_3\cdot\vec{k}_4)}{k_t^3}\left(1 + \frac{3(k_3+k_4)}{k_t} + \frac{12k_3k_4}{k_t^2}\right)\right.$$
$$\left. + \frac{1}{32}\left(\frac{1}{c_s^2} - 1\right)\frac{(\vec{k}_1\cdot\vec{k}_2)(\vec{k}_3\cdot\vec{k}_4)}{k_t}\left(1 + \frac{\sum_{i<j}k_ik_j}{k_t^2} + \frac{3k_1k_2k_3k_4}{k_t^3}\sum_{i=1}^4\frac{1}{k_i} + \frac{12k_1k_2k_3k_4}{k_t^4}\right)\right] + 23 \text{ perm.}$$

- λ corresponds to $(g^{00}+1)^3$: relation between contribution to 3pf and 4pf
- μ is $(g^{00}+1)^4$ and it does not have a squeezed limit 3pf
- Squeezed limit is $1/c_s^2$ while the full 4pf is $1/c_s^4$

$$\langle\zeta_{\vec{q}}\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle \simeq -\frac{1}{2}P(q)q^iD_i\langle\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle \propto \frac{1}{c_s^2}$$

Generalizations

- **Graviton correlation functions**:

$$x_i \to x_i + A_{ij}x_j + B_{ijk}x_jx_k$$

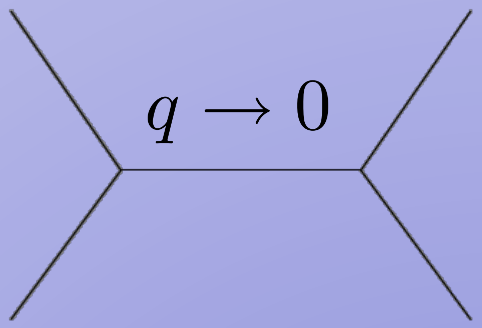
Induce long graviton with

$$A_{ij} = \frac{1}{2}\gamma_{ij}\ , \quad B_{ijk} = \frac{1}{4}(\partial_k\gamma_{ij} - \partial_i\gamma_{jk} + \partial_j\gamma_{ik})$$

$$\langle\gamma^s_{\vec q}\zeta_{\vec k_1}\ldots\zeta_{\vec k_n}\rangle'_{q\to 0} = -\frac{1}{2}P_\gamma(q)\sum_a \epsilon^s_{ij}(\vec q)k_{ai}\partial_{k_{aj}}\langle\zeta_{\vec k_1}\ldots\zeta_{\vec k_n}\rangle'$$
$$-\frac{1}{4}P_\gamma(q)\sum_a \epsilon^s_{ij}(\vec q)\left(2k_{ai}(\vec q\cdot\vec\partial_{k_a}) - (\vec q\cdot\vec{k_a})\partial_{k_{ai}}\right)\partial_{k_{aj}}\langle\zeta_{\vec k_1}\ldots\zeta_{\vec k_n}\rangle'$$

Not more than one…

- **Soft internal lines**



$$\langle\zeta_{\vec k_1}\cdots\zeta_{\vec k_n}\rangle'_{\vec q\to 0} = P_\zeta(q)\langle\zeta_{-\vec q}\zeta_{\vec k_1}\cdots\zeta_{\vec k_m}\rangle^*_{\vec q\to 0}\langle\zeta_{\vec q}\zeta_{\vec k_{m+1}}\cdots\zeta_{\vec k_n}\rangle^*_{\vec q\to 0}+$$
$$+P_\gamma(q)\sum_s\langle\gamma^s_{-\vec q}\zeta_{\vec k_1}\cdots\zeta_{\vec k_m}\rangle^*_{\vec q\to 0}\langle\gamma^s_{\vec q}\zeta_{\vec k_{m+1}}\cdots\zeta_{\vec k_n}\rangle^*_{\vec q\to 0}$$

- **More than one q going to zero together**

Assumptions

(No-go) theorems have a way of relying on apparently technical assumptions that later turn out to have exceptions of great physical interest (S. Weinberg)

1. Quasi-single field (see Chen's talk). Solid inflation. Endlich, Nicolis, Wang 12

2. Bunch-Davies vacuum (see Shandera's and Flauger's talks)

3. The solution is an attractor

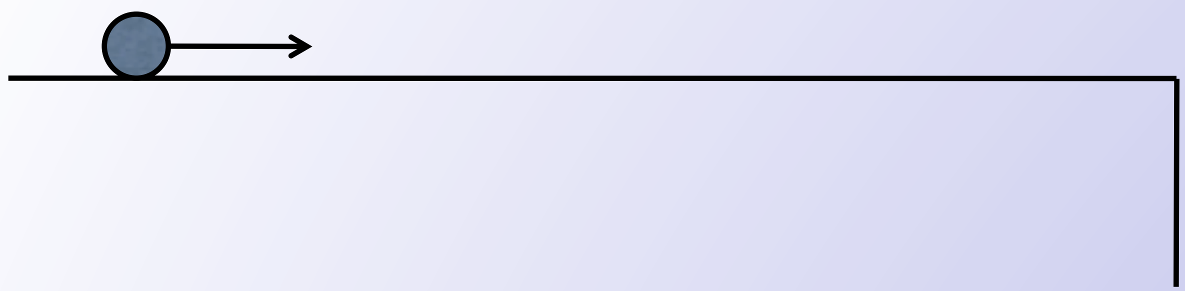
4. Time-dependence of modes decays as a^{-2}

Not an attractor

E.g. Namjoo, Firouzjahi, Sasaki 12

Khoury and Piazza 11

$$\ddot{\phi} + 3H\dot{\phi} = 0\,, \quad 3{M_{\rm Pl}}^2 H^2 = \frac{1}{2}\dot{\phi}^2 + V_0 \simeq V_0$$



$$\epsilon \propto a^{-6}\,, \quad \eta \simeq -6$$

Scale invariant: $\delta\phi$ is a test scalar in dS and $\zeta = -\frac{H\delta\phi}{\dot{\phi}}\Big|_{\rm end}$

$$\phi_k^g \propto \cos k\tau + k\tau \sin k\tau \simeq 1\,,$$
$$\phi_k^d \propto k\tau \cos k\tau - \sin k\tau \simeq \frac{1}{3}(-k\tau)^3$$

← The growing mode describes a perturbation of the position: different velocity at the same point

Time translations

Inflation takes place in ~ dS $$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2$$

Scale invariance of correlation functions due to dilation isometry $$t \to t - H^{-1} \log \lambda \quad \vec{x} \to \lambda \vec{x}$$

+ invariance under time translation of inflaton dynamics: $t \to \tilde{t} = t + \mathrm{const}$

Or equivalently: $\phi \to \phi + c$

$$\varphi_{\vec{k}} \to \lambda^3 \varphi_{\vec{k}/\lambda} \qquad \langle \varphi_{\vec{k}_1} \varphi_{\vec{k}_2} \rangle = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2) \frac{1}{k_1^3} F(k_1 \eta)$$

$$\langle \varphi_{\vec{k}_1} \dots \varphi_{\vec{k}_n} \rangle = (2\pi)^3 \delta(\sum_i \vec{k}_i) F(\vec{k}_1, \dots, \vec{k}_n)$$

Approximately valid in a given interval → Reheating

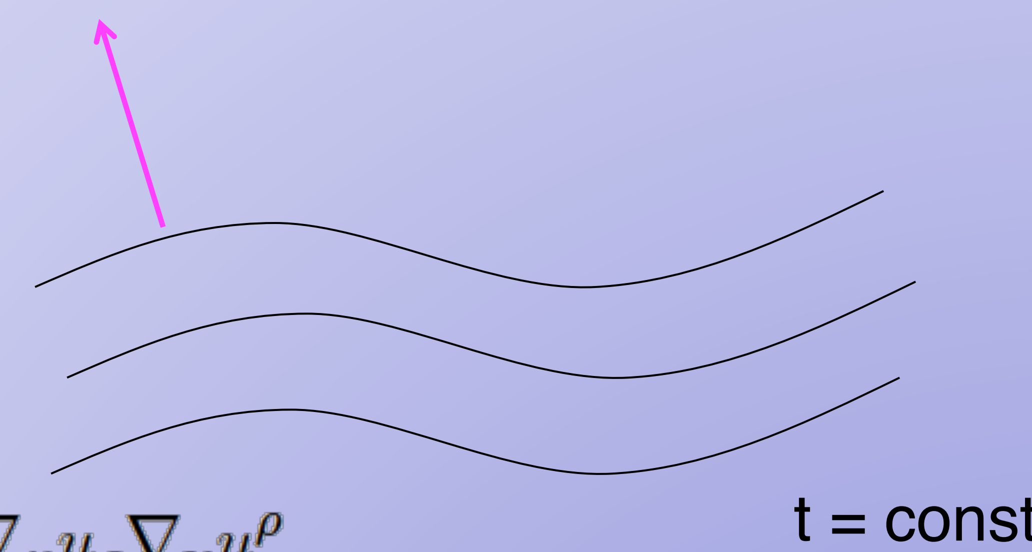
Time independence

What happens if the symmetry: $t \to \tilde{t} = t + \mathrm{const}$ is promoted to $t \to \tilde{t}(t)$?

This is the same symmetry discussed in the healthy Horava gravity

Blas, Pujolas and Sibiryakov 10

$$u_\mu = \frac{\partial_\mu \phi}{\sqrt{-g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi}}$$



$$(\nabla_\mu u^\mu)^2\,; \quad \nabla_\mu u^\nu \nabla_\nu u^\mu; \quad \nabla_\mu u^\nu \nabla^\mu u_\nu; \quad u^\mu u^\nu \nabla_\mu u_\rho \nabla_\nu u^\rho$$

By parts:

Frobenius theorem: $\nabla_\mu u^\nu \nabla_\nu u^\mu = \nabla_\mu u^\nu \nabla^\mu u_\nu + u^\mu u^\nu \nabla_\mu u_\rho \nabla_\nu u^\rho$

Only two operators + higher derivative corrections

Khronon inflation

$$S=\frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left(M_{Pl}^2R-2\Lambda-\underline{M_\lambda^2}\left(\nabla_\mu u^\mu-3H\right)^2+\underline{M_\alpha^2}u^\mu u^\nu\nabla_\mu u_\rho\nabla_\nu u^\rho\right)$$

Geometrically: $x_i\to\tilde{x}_i\left(\mathbf{x},t\right);\quad t\to\tilde{t}(t)$ $\qquad \cancel{g^{00}}$

$$S=\frac{M_P^2}{2}\int \mathrm{d}^3x\,\mathrm{d}t\sqrt{h}N\left(R^{(3)}+K_{ij}K^{ij}-\underline{\lambda(K}-3H)^2+\underline{\alpha a_i a^i}\right)$$

$$K_{ij}=\frac{1}{2N}\left(\dot{h}_{ij}-\nabla_iN_j-\nabla_jN_i\right)$$

$$ds^2=-N^2dt^2+h_{ij}(dx^i+N^idt)(dx^j+N^jdt)$$

$$a_i\equiv N^{-1}\partial_iN$$

All correlation functions depend on c_S only

Power spectrum

$$S_2 = \int \mathrm{d}^3x\, \mathrm{d}\eta \left(\frac{M_\alpha^2}{2}(\partial\pi')^2 - \frac{M_\lambda^2}{2}(\partial^2\pi)^2 \right)$$

No dependence on a!

$$\pi_k(\eta) = \frac{1}{\sqrt{2k^3}} \frac{1}{\sqrt{M_\alpha M_\lambda}} e^{\pm i \frac{M_\lambda}{M_\alpha} k\eta}$$

~ Minkowski

$$c_s^2 \equiv \frac{M_\lambda^2}{M_\alpha^2} \qquad \zeta = -H\pi$$

$$\langle \zeta_{\vec{k}} \zeta_{\vec{k}'} \rangle = (2\pi)^3 \delta(\vec{k} + \vec{k}') \frac{1}{2k^3} \frac{H^2}{M_\alpha M_\lambda}$$

- $\omega = k/a \ll H$: the decaying mode decays, but only as 1/a (not 1/a^3)
- ζ is conserved out of H, as its derivative is small
- Small breaking terms will become relevant

$$S = \int \mathrm{d}^3x\, \mathrm{d}\eta \left[\frac{M_\alpha^2}{2}(\partial\pi')^2 - \frac{M_\lambda^2}{2}(\partial^2\pi)^2 + \beta a^2 H^2 \left(\frac{M_\alpha^2}{2}\pi'^2 - \frac{M_\lambda^2}{2}(\partial_i\pi)^2 \right) \right]$$

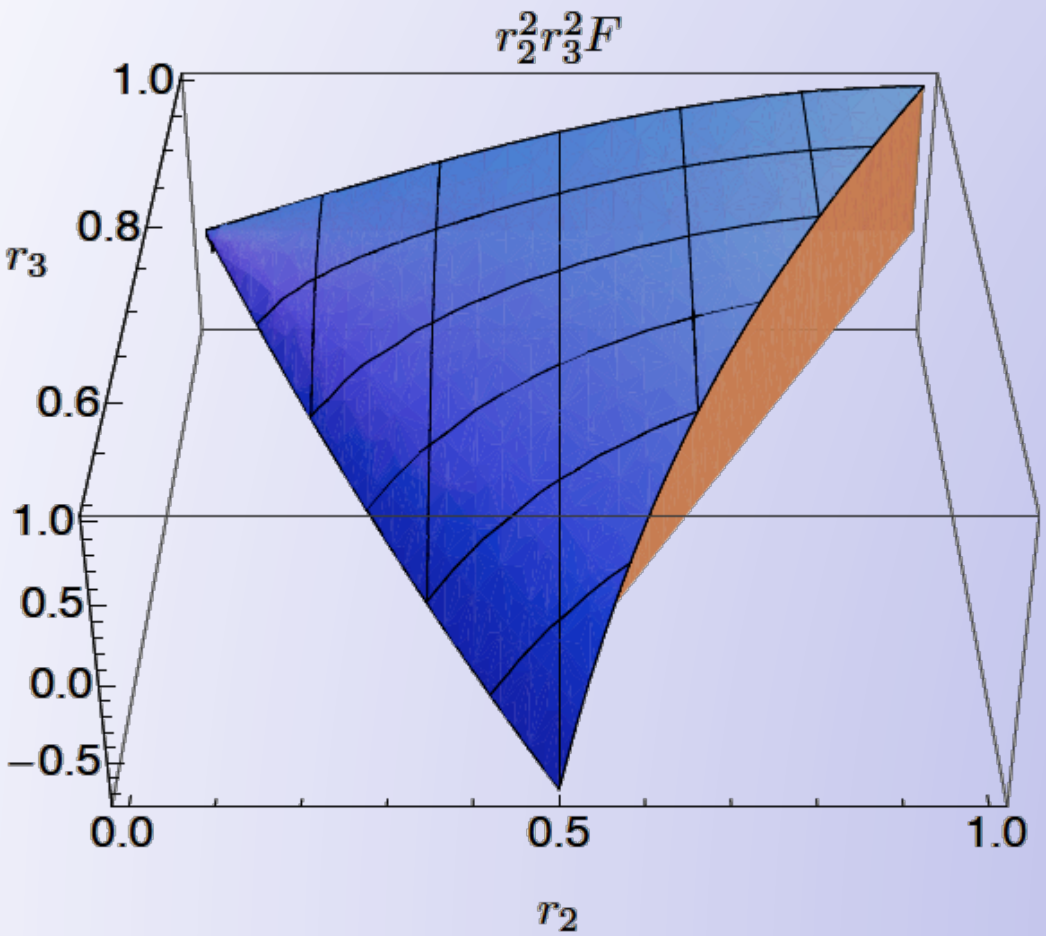
3-point function

$$S_3 = \int \mathrm{d}^3x\, \mathrm{d}\eta\, \frac{1}{a}\left[M_\lambda^2\left(2\partial_i\pi'\partial_i\pi\partial^2\pi + \pi'\partial_i\partial_j\pi\partial_i\partial_j\pi\right) + M_\alpha^2\left(\pi'\partial_i\pi''\partial_i\pi - \underline{\partial_i\pi'\partial_j\pi\partial_i\partial_j\pi}\right)\right]$$

$$\langle\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle \equiv (2\pi)^3\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)F_\zeta(k_1,k_2,k_3)$$

$$F_\zeta(k_1,k_2,k_3) = \frac{1}{\prod k_i^3}P_\zeta^2\left[-\frac{k_1}{k_t^2}(k_3^2\vec{k}_1\cdot\vec{k}_2 + k_2^2\vec{k}_1\cdot\vec{k}_3) - \frac{k_1^2}{k_t}\vec{k}_2\cdot\vec{k}_3 - \underline{\frac{M_\alpha^2}{M_\lambda^2}\frac{k_1^3}{k_t^2}\vec{k}_2\cdot\vec{k}_3}\right]$$

$$+ \text{ cyclic perms.}$$



$$\propto \frac{1}{c_s^2}$$

Violation of CCR

$$S_3 = \int \mathrm{d}^3x\,\mathrm{d}\eta\,\frac{1}{a}\Big[M_\lambda^2\big(2\partial_i\pi'\partial_i\pi\partial^2\pi + \pi'\partial_i\partial_j\pi\partial_i\partial_j\pi\big) + M_\alpha^2\big(\pi'\partial_i\pi''\partial_i\pi - \partial_i\pi'\partial_j\pi\partial_i\partial_j\pi\big)\Big]$$

• Linear correction to 3pf, violating CCR

• In the limit of exact symmetry they cancel. A homogeneuos time-dependent mode can be set to zero using the symmetry

$$t+\pi \to F(t+\pi) \simeq t+\pi+\epsilon(t+\pi)+\ldots = t+\pi+\epsilon(t)+\dot\epsilon(t)\pi+\frac{1}{2}\ddot\epsilon(t)\pi^2+\ldots$$

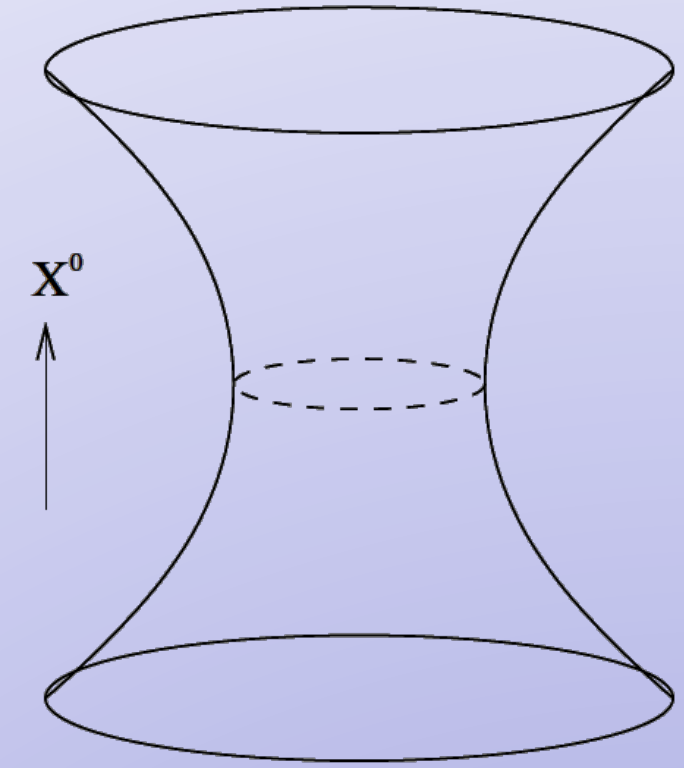
• Violation of CCR suppressed by the small breaking of the field-redefinition symmetry. Totally unobservable.

$$S \supset \int \mathrm{d}^3x\,\mathrm{d}t\sqrt{h}N\,(g^{00}-1)(K-3H)^2$$

• Related to spatial non locality $\quad [\hat{\dot\zeta}_{\boldsymbol k}(\tau), \hat\zeta_{\boldsymbol k'}(\tau)] = \frac{i}{k^2}\delta(\boldsymbol k+\boldsymbol k')$

Assassi, Baumann, Green 12

de Sitter: SO(4,1)



Inflation takes place in ~ dS

$$ds^2 = \frac{1}{H^2\eta^2}(-d\eta^2 + d\vec{x}^2)$$

- **Translations, rotations**: ok
- **Dilations** (if slow-roll) $\eta \to \lambda\eta,\ \vec{x} \to \lambda\vec{x}$

→ scale-invariance

$$\varphi_{\vec{k}} \to \lambda^3 \varphi_{\vec{k}/\lambda} \qquad \langle \varphi_{\vec{k}_1}\varphi_{\vec{k}_2}\rangle = (2\pi)^3\delta(\vec{k}_1 + \vec{k}_2)\frac{1}{k_1^3}F(k_1\eta)$$

In general:

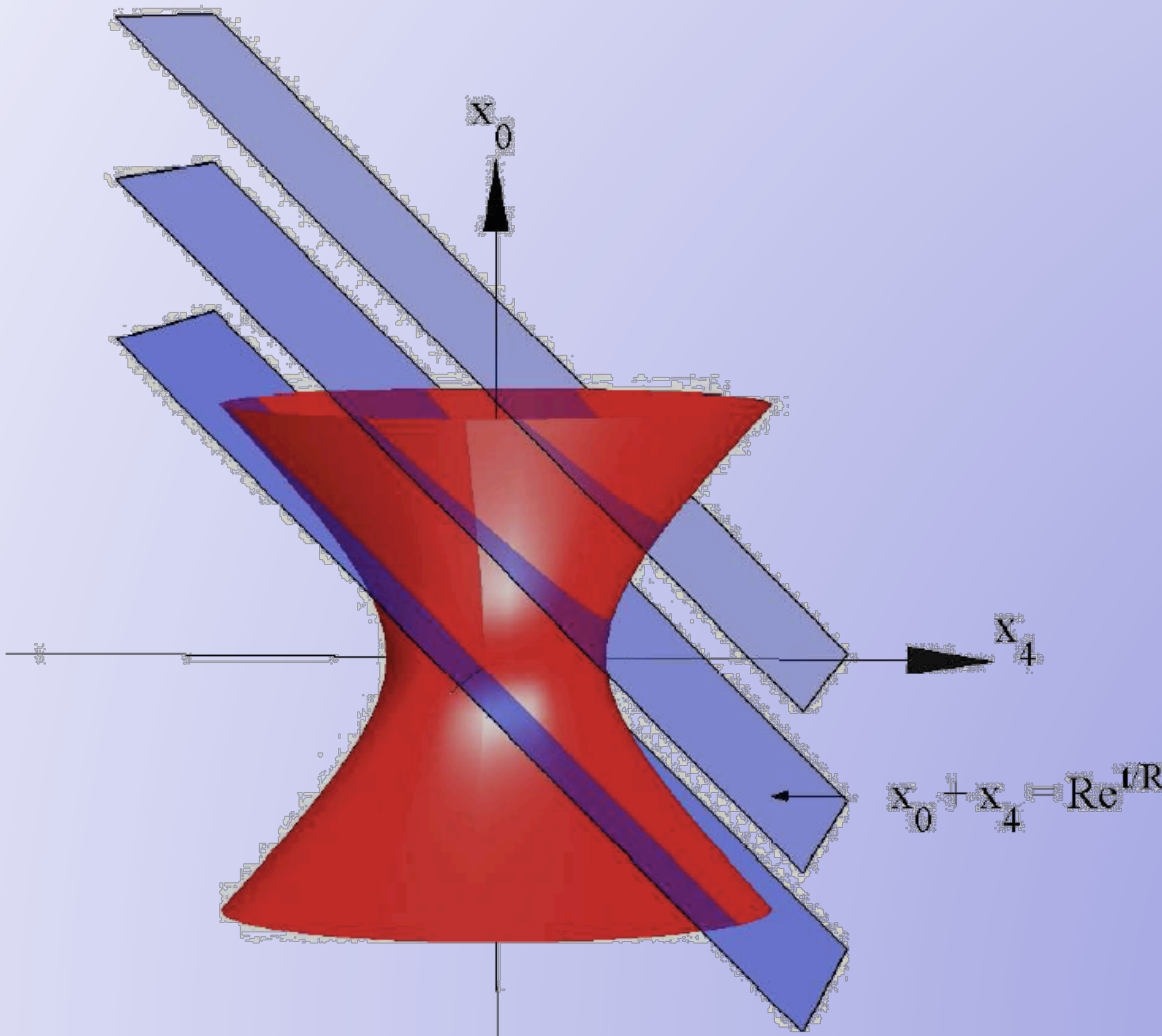
$$\langle \varphi_{\vec{k}_1}\ldots\varphi_{\vec{k}_n}\rangle = (2\pi)^3\delta(\sum_i \vec{k}_i)F(\vec{k}_1,\ldots,\vec{k}_n)$$

with F homogeneous of degree -3(n-1)

Special conformal

$$\eta \to \eta - 2\eta(\vec{b}\cdot\vec{x})\ , \quad x^i \to x^i + b^i(-\eta^2 + \vec{x}^2) - 2x^i(\vec{b}\cdot\vec{x})$$

The inflaton background breaks these symmetries



Scale → Conformal invariance

Antoniadis, Mazur and Mottola, 11
Maldacena and Pimental, 11

PC 11

Curvaton, modulated reheating…

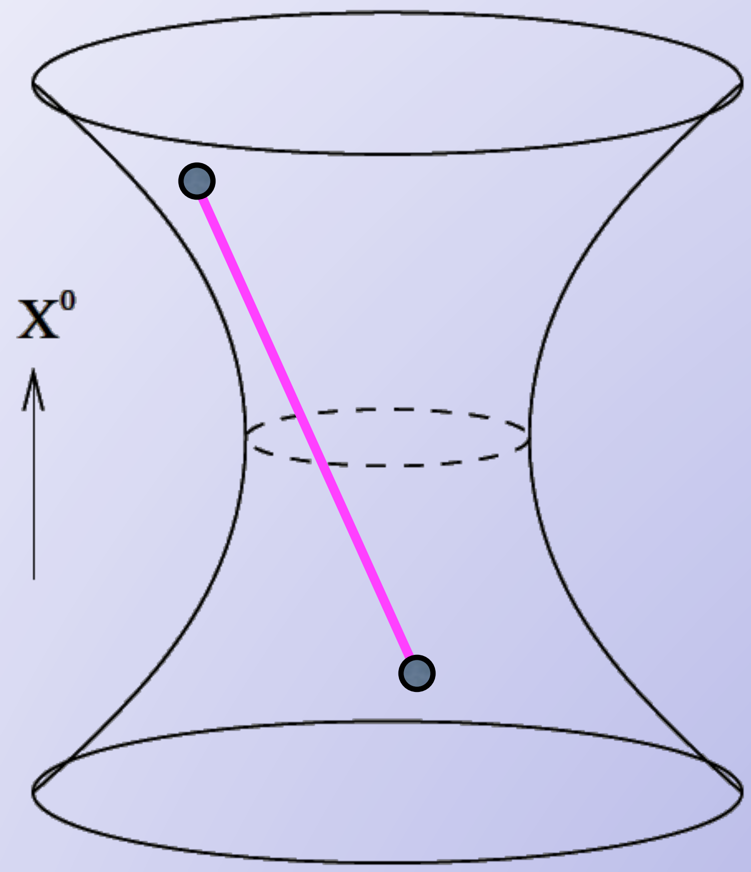
If perturbations are created by a sector with negligible interactions with the inflaton, correlation functions have the full SO(4,1) symmetry

They are conformal invariant

Independently of any details about this sector, even at strong coupling

Same as AdS/CFT

dS-invariant distance



$$\frac{|\vec{x}_i - \vec{x}_j|^2}{\eta_i \eta_j} - \left(\frac{\eta_i}{\eta_j} + \frac{\eta_j}{\eta_i} \right)$$

Scale → Conformal invariance

We are interested in correlators at late times

$$x^i \to x^i + b^i \vec{x}^2 - 2x^i(\vec{b}\cdot\vec{x}) \qquad\qquad \eta \to \eta - 2\eta(\vec{b}\cdot\vec{x})$$

$$\varphi \sim \eta^\Delta\,, \quad \Delta = \frac{3}{2}\left(1 - \sqrt{1 - \frac{4m^2}{9H^2}}\right) \ll 1$$

This is the transformation of the a primary of conformal dim Δ

Example: $m = \sqrt{2}H \qquad \Delta = 1$

$$\int d^4x\sqrt{-g}\frac{M}{6}\varphi^3 \quad \longrightarrow \quad \langle \varphi_{\vec{k}_1}\varphi_{\vec{k}_2}\varphi_{\vec{k}_3}\rangle = (2\pi)^3\delta(\sum_i \vec{k}_i)\frac{\pi}{16}MH^2\eta_*^3 \cdot \frac{1}{k_1k_2k_3}$$

$$\langle \varphi(\vec{x}_1)\varphi(\vec{x}_2)\varphi(\vec{x}_3)\rangle = \frac{MH^2\eta_*^3}{128\pi^2} \cdot \frac{1}{|\vec{x}_1 - \vec{x}_2||\vec{x}_1 - \vec{x}_3||\vec{x}_2 - \vec{x}_3|}$$

Massless scalars

$$\langle \varphi_{\vec{k}_1}\varphi_{\vec{k}_2}\varphi_{\vec{k}_3}\rangle = (2\pi)^3\delta(\sum_i \vec{k}_i)\frac{H^2}{\prod_i 2k_i^3}\frac{2M}{3}\left[\sum_i k_i^3(-1+\gamma+\log(-k_t\eta_*)) + k_1k_2k_3 - \sum_{i\neq j} k_i^2 k_j\right]$$

$$k_t \equiv \sum_i k_i$$

Zaldarriaga 03
Seery, Malik,Lyth 08

$$\langle \varphi_1(\vec{x}_1)\varphi_2(\vec{x}_2)\varphi_3(\vec{x}_3)\rangle = \frac{MH^2}{48\pi^2}\log\frac{|\vec{x}_1-\vec{x}_2|}{A\eta_*}\log\frac{|\vec{x}_1-\vec{x}_3|}{A\eta_*}\log\frac{|\vec{x}_2-\vec{x}_3|}{A\eta_*}$$

Everything determined up to two constants

Independently of the interactions!

$$\frac{1}{M}\int d^4x\sqrt{-g}\nabla_\mu\varphi_1\nabla^\mu\varphi_2\varphi_3 \longrightarrow \frac{1}{M}\int d^4x\sqrt{-g}\frac{1}{2}(\Box\varphi_3\varphi_1\varphi_2 - \Box\varphi_1\varphi_2\varphi_3 - \Box\varphi_2\varphi_1\varphi_3)$$

The conversion to ζ will add a local contribution: $\zeta(\vec{x}) = A_I\varphi^I(\vec{x}) + B_{IJ}\varphi^I(\vec{x})\varphi^J(\vec{x})$

4-point function

$$\int d^4x \frac{1}{8M^4}(\partial_\mu\varphi)^2(\partial_\nu\varphi)^2$$

$$\langle\varphi_{\vec{k}_1}\varphi_{\vec{k}_2}\varphi_{\vec{k}_3}\varphi_{\vec{k}_4}\rangle = (2\pi)^3\delta(\sum_i \vec{k}_i)\frac{1}{M^4}\frac{H^8}{\prod_i 2k_i^3}\Bigg[-\frac{144k_1^2k_2^2k_3^2k_4^2}{k_t^5} - 4\left(\frac{12k_1k_2k_3k_4}{k_t^5} + \frac{3\prod_{i<j<l}k_ik_jk_l}{k_t^4} + \frac{\prod_{i<j}k_ik_j}{k_t^3} + \frac{1}{k_t}\right)$$
$$((\vec{k}_1\cdot\vec{k}_2)(\vec{k}_3\cdot\vec{k}_4)+(\vec{k}_1\cdot\vec{k}_3)(\vec{k}_2\cdot\vec{k}_4)+(\vec{k}_1\cdot\vec{k}_4)(\vec{k}_2\cdot\vec{k}_3))+(\vec{k}_1\cdot\vec{k}_2)\left(\frac{4k_3^2k_4^2}{k_t^3} + \frac{12(k_1+k_2)k_3^2k_4^2}{k_t^4} + \frac{48k_1k_2k_3^2k_4^2}{k_t^5}\right)+6\text{perm.}\Bigg],$$

Not so obvious it is conformal invariant…

I can check it in **Fourier space**

Maldacena and Pimental, 11

$$\sum_{a=1,2,3,4}\left[6\vec{b}\cdot\vec{\partial}_{k_a} - \vec{b}\cdot\vec{k}_a\vec{\partial}_{k_a}^2 + 2\vec{k}_i\cdot\vec{\partial}_{k_a}(\vec{b}\cdot\vec{\partial}_{k_a})\right]\langle\varphi_{\vec{k}_1}\varphi_{\vec{k}_2}\varphi_{\vec{k}_3}\varphi_{\vec{k}_4}\rangle' = 0$$

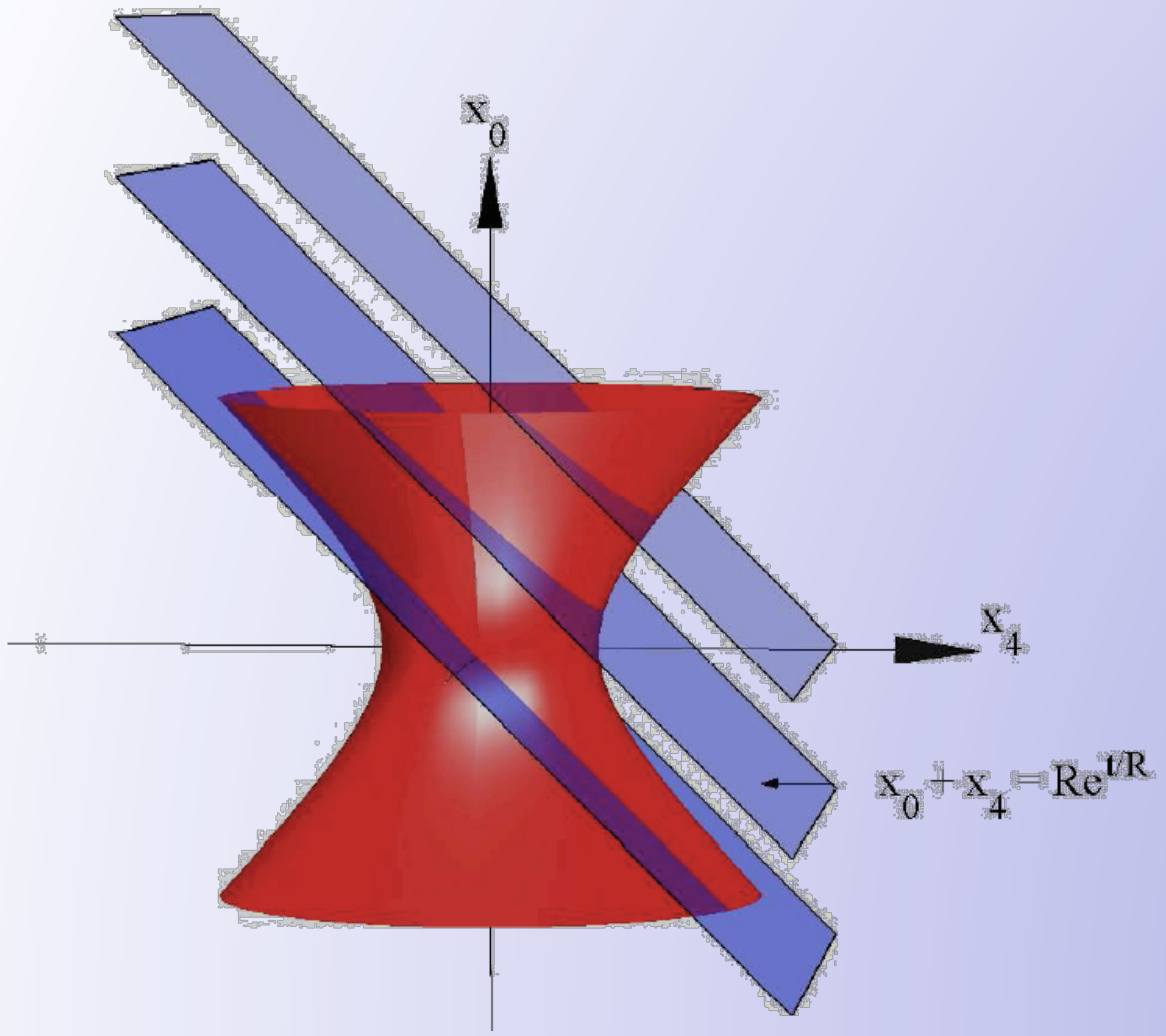
In general: $F\left(\frac{r_{13}r_{24}}{r_{12}r_{34}}, \frac{r_{23}r_{41}}{r_{12}r_{34}}\right)\prod_{i<j} r_{ij}^{-2\Delta/3}$ 2 parameters instead of 5

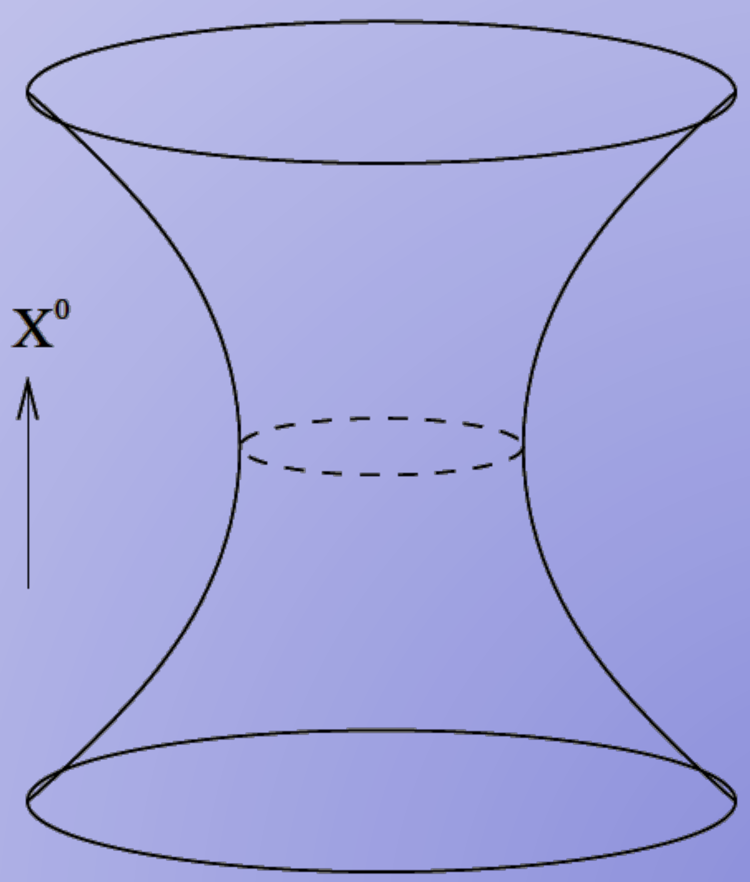
Therefore

If we see something beyond the spectrum

- Something not conformal would be a probe of a "sliced" de Sitter

- Something conformal would be a probe of pure de Sitter





Conclusions

- CCR

$$\langle \zeta_{\vec{q}} \zeta_{\vec{k}_1} \ldots \zeta_{\vec{k}_n} \rangle' \overset{q \to 0}{=} - P(q) \left[3(n-1) + \sum_a \vec{k}_a \cdot \vec{\partial}_{k_a} + \frac{1}{2} q^i D_i \right] \langle \zeta_{\vec{k}_1} \ldots \zeta_{\vec{k}_n} \rangle' + \mathcal{O}(q/k)^2$$

$$\text{with} \quad q^i D_i \equiv \sum_{a=1}^{n} \left[6\vec{q} \cdot \vec{\partial}_{k_a} - \vec{q} \cdot \vec{k}_a \vec{\partial}^2_{k_a} + 2\vec{k}_a \cdot \vec{\partial}_{k_a} (\vec{q} \cdot \vec{\partial}_{k_a}) \right] .$$

- Assumptions:
  - Attractor
  - Decay of time derivative. Khronon inflation.
- Conformal correlation functions for mechanisms decoupled from inflaton