Modulated Reheating Mechanism
and
Spectral Index of Powerspectrum

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$c = \hbar = M_G^2 = 1/(8\pi G) = 1$

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  What is the modulated reheating mechanism

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Introduction

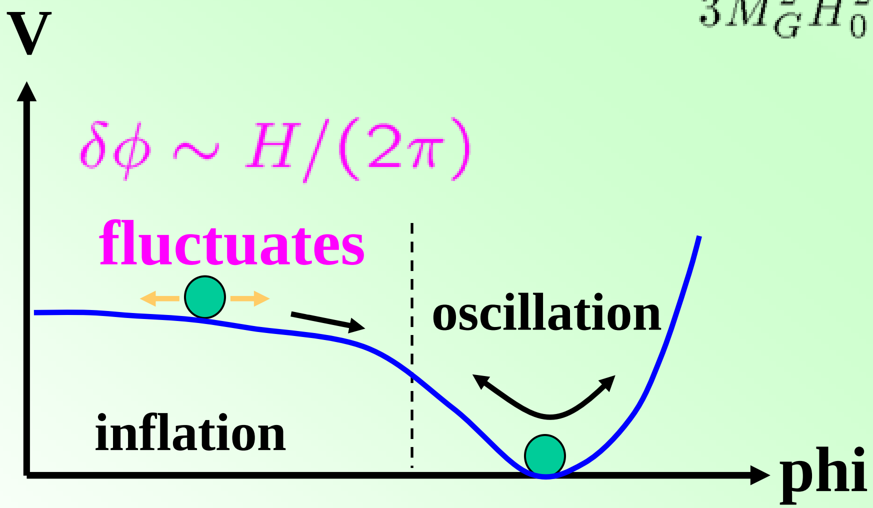
Inflation

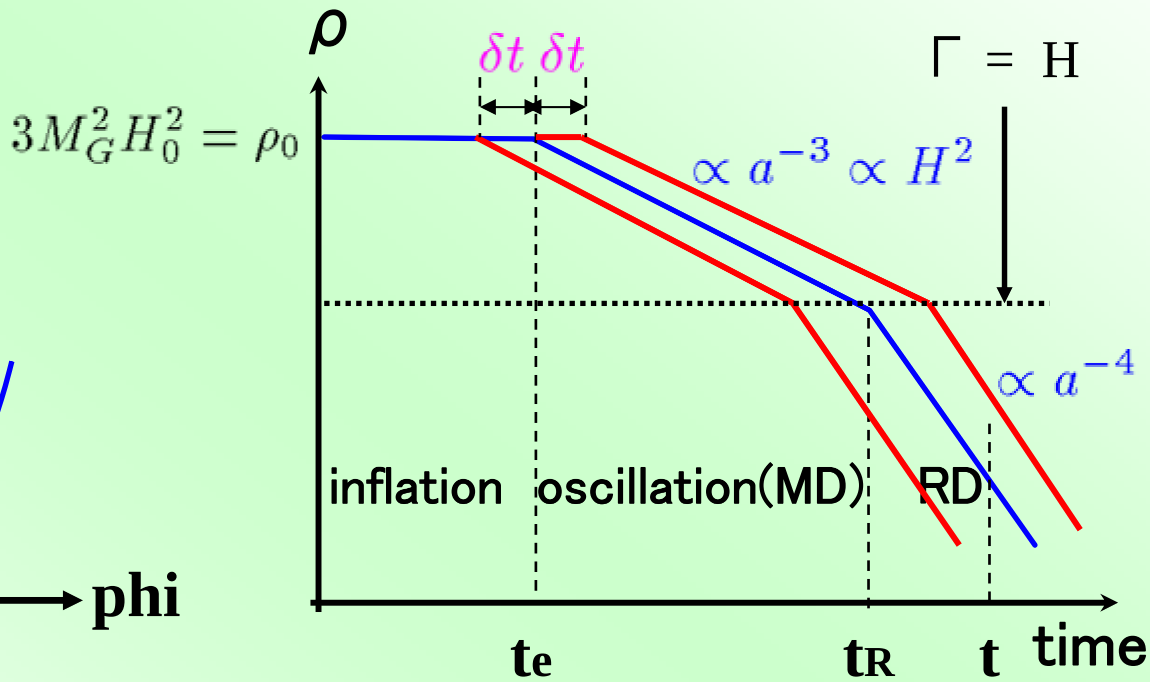
Inflation can naturally solve the problems of the standard big bang cosmology.

- **The horizon problem**
- **The flatness problem**
- **The origin of density fluctuations**
- **The monopole problem**
- …

After inflation, inflaton decays into the standard particles to (re)heat the Universe.

Primordial density fluctuations originating from inflaton





Other sources for primordial density fluctuations

- However, inflaton is not necessarily only the candidate responsible for the curvature perturbations.
- Other light fields, which acquire quantum fluctuations during inflation, can contribute to them.
- In fact, there can be light fields like moduli in string theory, which determine coupling constants.

Modulated reheating mechanism

Modulated reheating mechanism

Dvali, Gruzinov, Zaldarriaga
Kofman

The decay rate of the inflaton depends on a light scalar field σ .

$$\Gamma = \Gamma(\sigma) \Leftarrow \quad \text{e.g.} \quad \mathcal{L} = h(\sigma)\phi\bar{\psi}\psi$$

(Inflaton must couple to a light scalar field (modulus) and SM particles.)

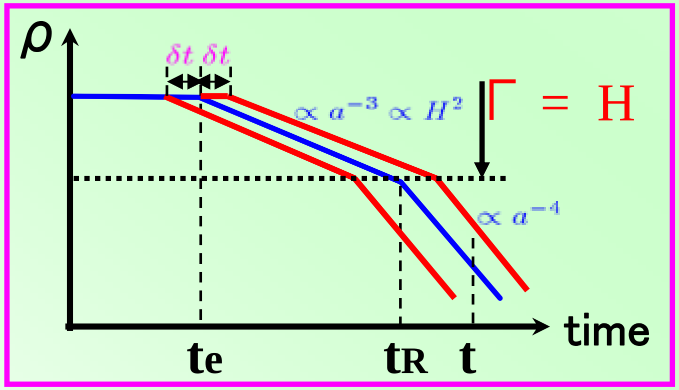
$$\sigma = \langle\sigma\rangle + \delta\sigma, \quad \delta\sigma \simeq \frac{H}{2\pi}$$ during inflation

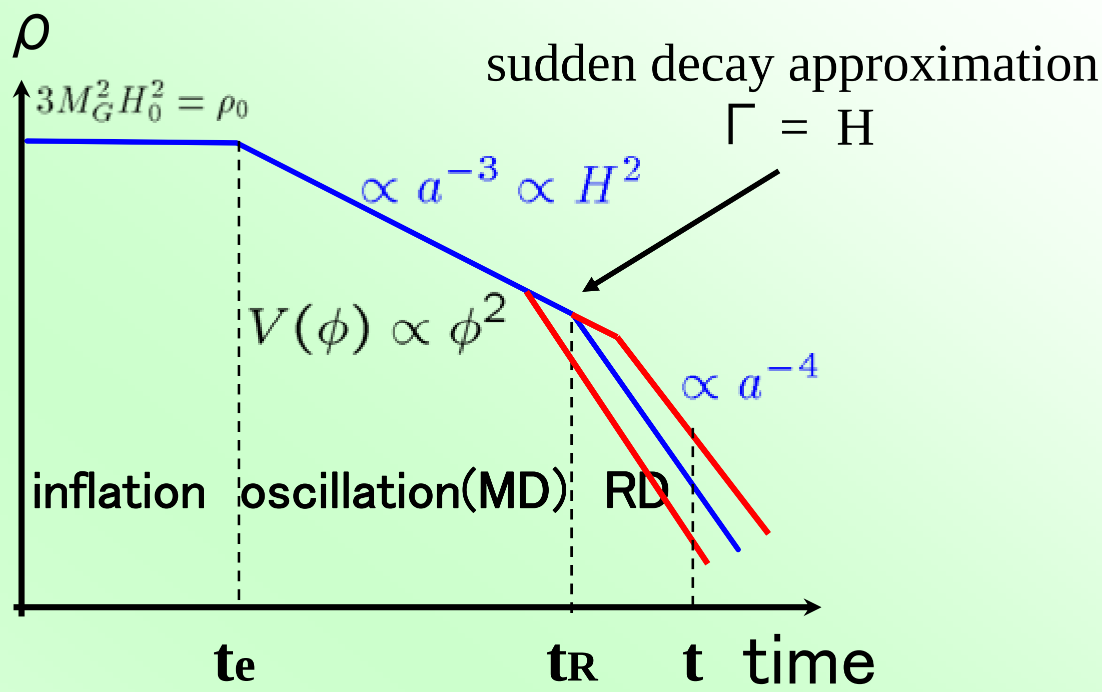
$$\Rightarrow \quad \Gamma(\sigma) = \Gamma(\langle\sigma\rangle) + \delta\Gamma$$

$$T_R \propto \Gamma^{1/2} \quad \Rightarrow \quad \frac{\delta T_R}{T_R} = \frac{1}{2}\frac{\delta\Gamma}{\Gamma}$$

How does this fluctuation lead to the curvature perturbation ?

Curvature perturbation in the modulated reheating mechanism





$$\rho(t) = \rho_0 \left(\frac{a_R}{a_e}\right)^{-3} \left(\frac{a(t)}{a_R}\right)^{-4}$$

$$= \rho_0 \left(\frac{a_e}{a(t)}\right)^{4} \left(\frac{a_R}{a_e}\right) \quad \left(\frac{a_R}{a_e} = \left(\frac{\Gamma}{H_0}\right)^{-2/3}\right)$$

$$\frac{\delta\rho(t)}{\rho(t)} = \frac{\delta a_R}{a_R} = -\frac{2}{3}\frac{\delta\Gamma}{\Gamma}$$ (on flat slicings)

$$\zeta = \frac{1}{4}\frac{\delta\rho(t)}{\rho(t)} = -\frac{1}{6}\frac{\delta\Gamma}{\Gamma} = -\frac{1}{3}\frac{\delta T_R}{T_R}$$

(The minus sign reflects that higher decay rate leads to lower energy.)

More general discussions

Decay channels and potentials of inflaton

Ichikawa, Suyama, Takahashi, MY

Inflaton starts the oscillation around its minimum of the potential after inflation.

$$V(\phi) \propto \phi^{2n}.$$

$$\Rightarrow \rho_\phi \propto a^{-\frac{6n}{(n+1)}} \propto \left(a^{-3} \text{ for } n=1,\ a^{-4} \text{ for } n=2,\ a^{-\frac{9}{2}} \text{ for } n=3\right).$$

decrease slower **radiation** **decrease faster**

Decay channels: $\mathcal{L}_{\text{int}} \supset -\sum_a y_a(\sigma)\phi\bar{\psi}_a\psi_a - \sum_a M_a(\sigma)\phi\chi_a^2 - \sum_a h_a(\sigma)\phi^2\chi_a^2.$

$$\Rightarrow \Gamma_\phi^{(n)}(\sigma) = \sum_a A_n \frac{y_a^2(\sigma)}{8\pi} m_\phi^{\text{eff}} + \sum_a B_n \frac{M_a^2(\sigma)}{8\pi m_\phi^{\text{eff}}} + \sum_a C_n \frac{h_a^2(\sigma)}{8\pi (m_\phi^{\text{eff}})^3}\rho_\phi.$$

$$(m_\phi^{\text{eff}})^2 = V_{\phi\phi}|_{\phi=\bar{\phi}}$$

	n=1	n=2	n=3
An	1	0.676	0.546
Bn	1	2.693	4.362
Cn	0.5	8.86	37.26

e-folding number

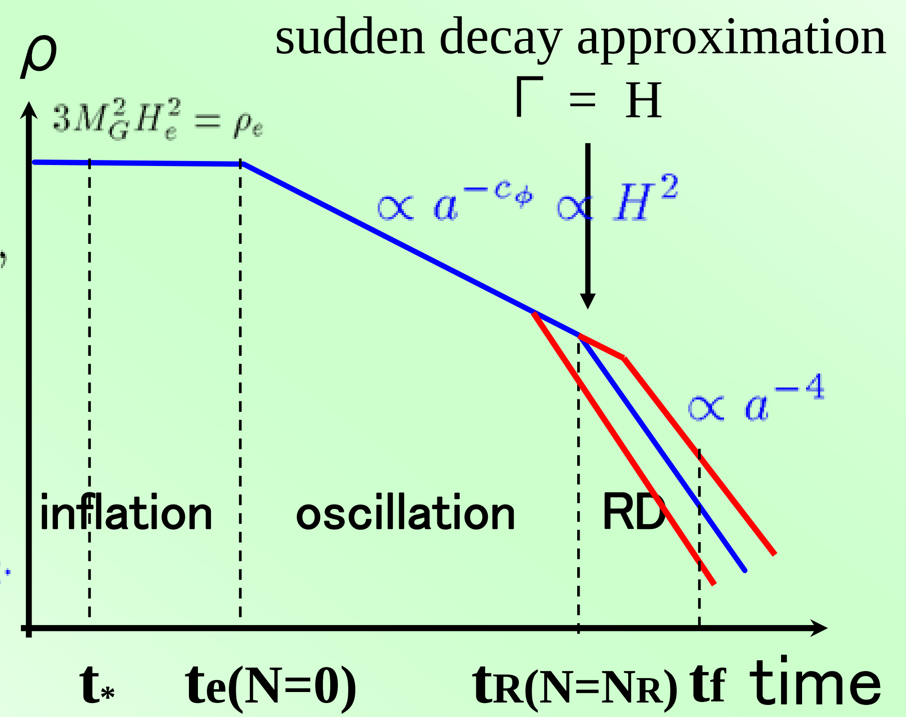
$$N(t_f, t_*, \phi_*, \sigma_*) = N(t_e, t_*, \phi_*) + N(t_f, t_e, \sigma_*)$$

final time, horion exit **negligible(assumption)** **inflation end**

$$N(t_f, t_e, \sigma_*) = \frac{1}{4}\log\frac{\rho_e}{\rho_f} + Q\left[\frac{\Gamma_\phi(\sigma_*, t_e)}{H_e}\right].$$

$$\begin{cases} \dfrac{d\rho_r}{dN} + 4\rho_r = \dfrac{\Gamma_\phi}{H}\rho_\phi\delta(N - N_R), \\ \dfrac{d\rho_\phi}{dN} + c_\phi\rho_\phi = -\dfrac{\Gamma_\phi}{H}\rho_\phi\delta(N - N_R), \\ H^2 = \dfrac{1}{3M_{\rm pl}^2}(\rho_\phi + \rho_r). \end{cases}$$

$$N(t_f, t_e, \sigma_*) = \frac{1}{4}\log\frac{\rho_e}{\rho_f} + \frac{1}{2}\log\frac{\Gamma_R}{H_e} + N_R.$$



e-folding number II

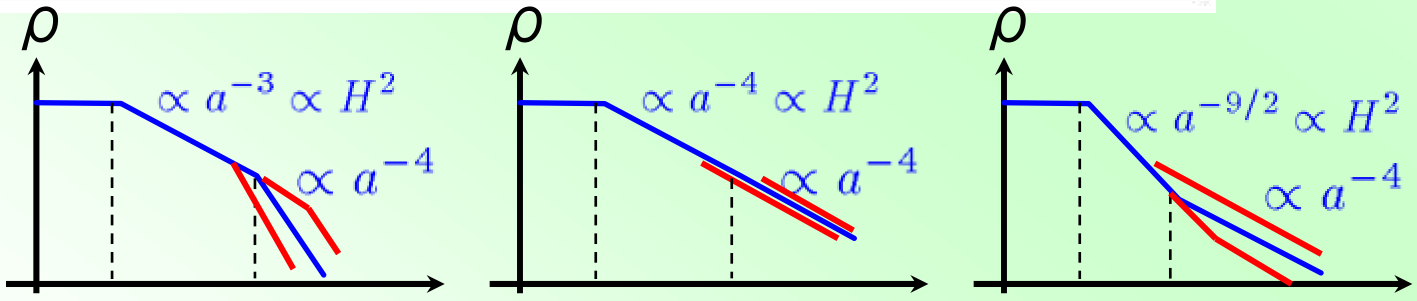
$$N(t_f, t_e, \sigma_*) = \frac{1}{4}\log\frac{\rho_e}{\rho_f} + Q\left[\frac{\Gamma_\phi(\sigma_*, t_e)}{H_e}\right].$$

$\left(\Gamma \propto a^{-c_\Gamma}\right)$

$$\Rightarrow Q\left[\frac{\Gamma_\phi(\sigma_*, t_e)}{H_e}\right] = \frac{1}{2}\log\frac{\Gamma_R}{H_e} + N_R = \frac{4-c_\phi}{2(2c_\Gamma - c_\phi)}\log\frac{\Gamma_\phi(\sigma_*, t_e)}{H_e}.$$

$$\Rightarrow Q(x) = a_0 \log x, \quad x = \frac{\Gamma_\phi(\sigma_*, t_e)}{H_e}, \quad a_0 = \frac{4-c_\phi}{2(2c_\Gamma - c_\phi)} = \frac{2-n}{2[(c_\Gamma - 3)n + c_\Gamma]}.$$

$\mathcal{L}_{\rm int}$	$n=1$ **(c_Φ=3)**	$n=2$ **(c_Φ=4)**	$n=3$ **(c_Φ=9/2)**
$-y\phi\bar{\psi}\psi$	$-\frac{1}{6}$ **(c_Γ=0)**	0	$\frac{1}{6}$ **(c_Γ=3/2)**
$-M\phi\chi\chi$	$-\frac{1}{6}$ **(c_Γ=0)**	0	$\frac{1}{30}$ **(c_Γ=-3/2)**
$-h\phi^2\chi^2$	$\cdots$ **(cannot decay)**	0	$\frac{1}{18}$ **(c_Γ=0)**



Non-gaussianity from δN

Zaldarriaga,
Bartolo, Matarrese, Riotto,
Vernizzi, Suyama & MY

$$\zeta(t) = \delta N = N^*_\sigma \delta\sigma_* + \frac{1}{2} N^*_{\sigma\sigma} \delta\sigma_*^2 + \frac{1}{6} N^*_{\sigma\sigma\sigma} \delta\sigma_*^3 + \cdots .$$

$$N^*_\sigma = \left.\frac{\partial N}{\partial \sigma}\right|_{\sigma=\sigma(t^*)} \cdots$$, $\delta\sigma_*$: Gaussian

$$N(t_f, t_e, \sigma_*) = \frac{1}{4}\log\frac{\rho_e}{\rho_f} + Q(x) = \frac{1}{4}\log\frac{\rho_e}{\rho_f} + a_0 \log x, \quad x = \frac{\Gamma(\sigma_*)}{H_e}.$$

$$\begin{cases} N_\sigma = xQ'(x)\dfrac{\Gamma_\sigma}{\Gamma} = A(x)\dfrac{\Gamma_\sigma}{\Gamma}, \\ N_{\sigma\sigma} = xQ'(x)\dfrac{\Gamma_{\sigma\sigma}}{\Gamma} + x^2 Q''(x)\dfrac{\Gamma_\sigma^2}{\Gamma^2} = A(x)\dfrac{\Gamma_{\sigma\sigma}}{\Gamma} + B(x)\dfrac{\Gamma_\sigma^2}{\Gamma^2}, \\ N_{\sigma\sigma\sigma} = xQ'(x)\dfrac{\Gamma_{\sigma\sigma\sigma}}{\Gamma} + 3x^2 Q''(x)\dfrac{\Gamma_\sigma\Gamma_{\sigma\sigma}}{\Gamma^2} + x^3 Q'''(x)\dfrac{\Gamma_\sigma^3}{\Gamma^3} = A(x)\dfrac{\Gamma_{\sigma\sigma\sigma}}{\Gamma} + 3B(x)\dfrac{\Gamma_\sigma\Gamma_{\sigma\sigma}}{\Gamma^2} + C(x)\dfrac{\Gamma_\sigma^3}{\Gamma^3}. \end{cases}$$

$$(A(x) = a_0, \;\; B(x) = -a_0, \;\; C(x) = 2a_0)$$

$$\begin{cases} \dfrac{6}{5} f_{NL} = \dfrac{N^*_{\sigma\sigma}}{N^{*2}_\sigma} = \dfrac{1}{a_0}\left(-1 + \dfrac{\Gamma\Gamma_{\sigma\sigma}}{\Gamma_\sigma^2}\right), \qquad \tau_{NL} = \dfrac{36}{25} f_{NL}^2 \\ \dfrac{54}{25} g_{NL} = \dfrac{N^*_{\sigma\sigma\sigma}}{N^{*3}_\sigma} = \dfrac{1}{a_0^2}\left(2 - 3\dfrac{\Gamma\Gamma_{\sigma\sigma}}{\Gamma_\sigma^2} + \dfrac{\Gamma^2\Gamma_{\sigma\sigma\sigma}}{\Gamma_\sigma^3}\right). \end{cases}$$

Non-gaussianity in the modulated reheating scenario

$$\frac{6}{5} f_{NL} = \frac{1}{a_0}\left(-1 + \frac{\Gamma\Gamma_{\sigma\sigma}}{\gamma_\sigma^2}\right), \qquad \frac{54}{25} g_{NL} = \frac{1}{a_0^2}\left(2 - 3\frac{\Gamma\Gamma_{\sigma\sigma}}{\gamma_\sigma^2} + \frac{\Gamma^2\Gamma_{\sigma\sigma\sigma}}{\gamma_\sigma^3}\right).$$

Non-linearlity between ζ (= δN) and $\delta\Gamma$.

a_0:

$\mathcal{L}_{\rm int}$	$n=1$	$n=2$	$n=3$
$-y\phi\bar{\psi}\psi$	$-\frac{1}{6}$	0	$\frac{1}{6}$
$-M\phi\chi\chi$	$-\frac{1}{6}$	0	$\frac{1}{30}$
$-h\phi^2\chi^2$	$\cdots$	0	$\frac{1}{18}$

Inflaton potential for the oscillation:

$$V(\phi) \propto \phi^{2n}$$

- **n=1: Non-linearlity between ζ & $\delta\Gamma$ generates f_{NL} = 5, but non-linearlity Γ & σ can generate large (either sign) f_{NL}.**
- **n=2: No perturbation is generated.**
- **n=3: Non-linearlity between ζ & $\delta\Gamma$ as well as Γ & σ can generate large (minus) f_{NL},**

Large (local type) non-Gaussianity can be easily generated in the modulated reheating scenario.

Relation between bispectrum & trispectrum

Suyama, Takahashi, MY, Yokoyama

$$\frac{6}{5}f_{NL} = \frac{1}{a_0}\left(-1 + \frac{\Gamma\Gamma_{\sigma\sigma}}{\gamma_\sigma^2}\right), \qquad \frac{54}{25}g_{NL} = \frac{1}{a_0^2}\left(2 - 3\frac{\Gamma\Gamma_{\sigma\sigma}}{\gamma_\sigma^2} + \frac{\Gamma^2\Gamma_{\sigma\sigma\sigma}}{\gamma_\sigma^3}\right).$$

$(\Gamma_{\sigma\sigma\sigma} = 0)$ ⇒

$$g_{NL} = -\frac{5}{3a_0}f_{NL} - \frac{25}{54a_0^2}.$$

(n=1, a0 = -1/6) ⇒

$$g_{NL} = 10 f_{NL} - \frac{50}{3}.$$

(f_{NL} & g_{NL} have the same sign,
while the curvaton predicts opposite in general.)

&

$$\tau_{NL} = \frac{36}{25}f_{NL}^2.$$

Spectral index of a light field model

Spectral index in a light field model

Kobayashi, Takahashi, Takahashi, MY

$$n_s - 1 \equiv \frac{d \ln P_\zeta}{d \ln k} \simeq \frac{2}{3}\frac{V''(\sigma_*)}{H_*^2} + 2\frac{\dot{H}_*}{H_*^2}.$$

WMAP7 : ns ~ 0.967, SPT : ns ~ 0.955 …

Generally speaking, a light field mass is really light and ε is small. But,

① **ε ~ 0.016 can reproduce ns~0.967, which requires large field inflation.**

$$\epsilon = -\frac{\dot{H}}{H^2} = -\frac{1}{2M_{\rm pl}^2}\left(\frac{d\phi}{dN}\right)^2 \implies \Delta\phi \simeq \sqrt{1-n_s} N M_{\rm pl} \gg M_{\rm pl} \text{ for } N \gtrsim 60.$$

e.g. $V(\phi) \propto \phi^n \implies 2\epsilon \simeq \frac{n^2}{\phi_N^2} \simeq \frac{n}{2N}.$

$$n_s \simeq 0.967 \iff n \sim 2(1-n_s)N \sim 4 \text{ for } N \sim 60.$$

The quartic potential is favored in comparison to the conventional wisdom.

② **V'' ~ - 0.05 H^2, that is, a hill-top type potential is also good.** (Kawasaki et al. 2011)

e.g. $V(\sigma) \simeq V_0 - \frac{1}{2}m^2(\sigma - \sigma_0)^2$ $\implies$ $f_{\rm NL} \simeq \frac{5(4+3f)}{18f}\frac{\sigma_{\rm osc}}{\sigma_0 - \sigma_{\rm osc}}, \quad f = \frac{\rho_\sigma}{\rho_r}.$

Even if the curvaton dominates at its decay, the sizable f$_{\rm NL}$ can be obtained.

Spectral index in a light field model II

Extensions of minimal cosmological assumptions may allow an extremely scale invariant spectral index $n_s \simeq 1$ because they provide powers only for large scales or reduce small scale powers.

① **Addition of isocurvature perturbations:**
residual isocurvature modes in curvaton (Lyth & Wands)
intrinsic isocurvature modes of axion …

② **Additional relativistic degree of freesom (dark radiation):**
inflaton must decay into a modulus in modulated reheating, which can behave like dark radiation. (Kobayashi, Takahashi, Takahashi, MY)

③ **Tensor perturbations:**
multibrid model (Naruko & Sasaki) …

Addition of isocurvature perturbations

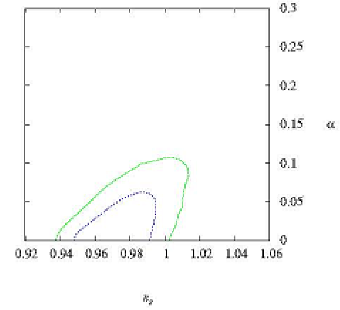


Figure 5: 1σ and 2σ limits in the n_s–α_0 plane for uncorrelated CDM isocurvature perturbations (WMAP+BAO+H0).

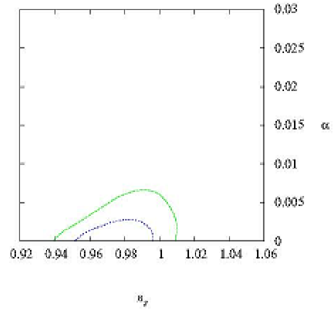


Figure 6: 1σ and 2σ limits in the n_s–α_1 plane for positively-correlated CDM isocurvature perturbations (WMAP+BAO+H0).

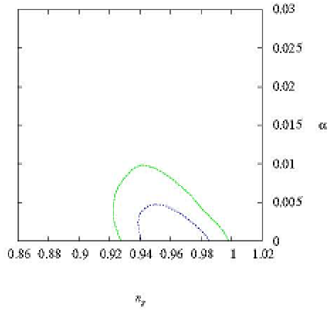


Figure 7: 1σ and 2σ limits in the n_s–α_1 plane for negatively-correlated CDM isocurvature perturbations (WMAP+BAO+H0).

Addition of uncorrelated or positively correlated CDM isocurvature perturbations can still accommodate an extremely scale invariant spectral index ns ~ 1.

Spectral index in a light field model II

Extensions of minimal cosmological assumptions may allow an extremely scale invariant spectral index $n_s \simeq 1$ because they provide powers only for large scales or reduce small scale powers.

① Addition of isocurvature perturbations:
residual isocurvature modes in curvaton (Lyth & Wands)
intrinsic isocurvature modes of axion …

② Additional relativistic degree of freesom (dark radiation):
inflaton must decay into a modulus in modulated reheating, which can behave like dark radiation. (Kobayashi, Takahashi, Takahashi, MY)

③ Tensor perturbations:
multibrid model (Naruko & Sasaki) …

Additional relativistic degree of freesom

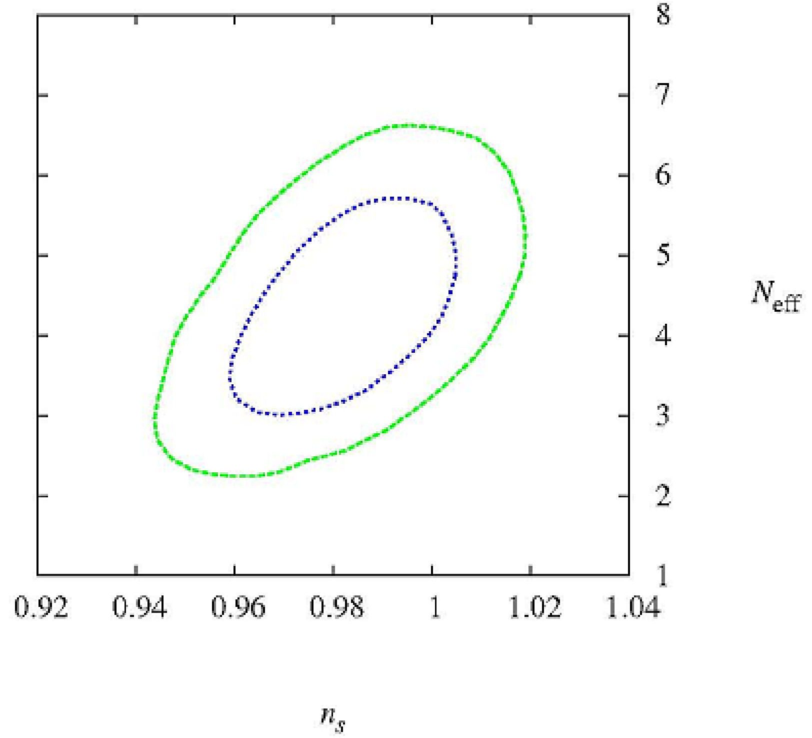


Figure 20: 1σ and 2σ limits in the n_s–$N_{\rm eff}$ plane (WMAP+BAO+H0).

Addition of extra relativisitic degree of freedom (dark radiation)
can still accommodate an extremely scale invariant spectral index ns ~ 1.

Spectral index in a light field model II

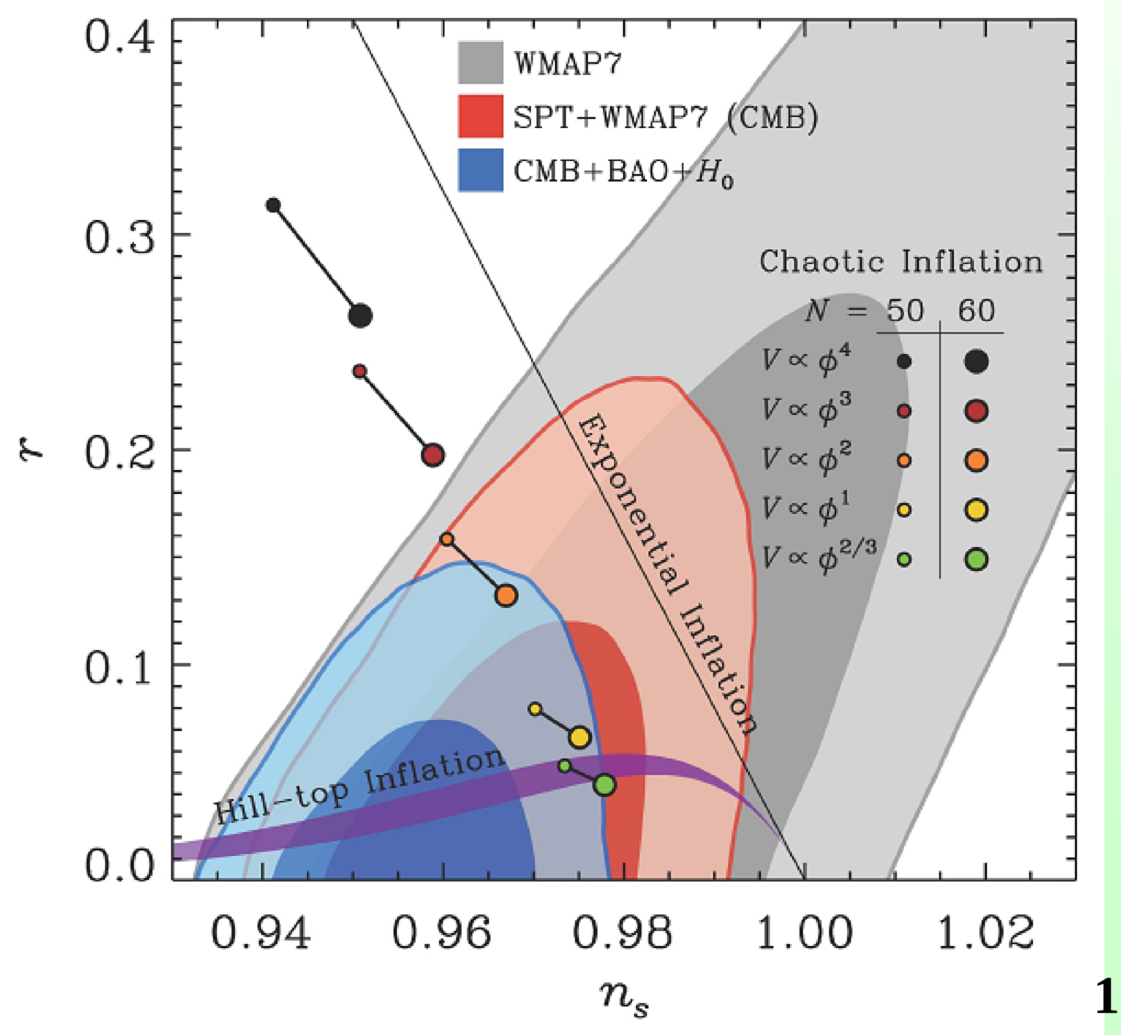
Extensions of minimal cosmological assumptions may allow an extremely scale invariant spectral index $n_s \simeq 1$ because they provide powers only for large scales or reduce small scale powers.

① **Addition of isocurvature perturbations:**
  residual isocurvature modes in curvaton **(Lyth & Wands)**
  intrinsic isocurvature modes of axion …

② **Additional relativistic degree of freesom (dark radiation):**
  inflaton must decay into a modulus in modulated reheating, which can behave like dark radiation. **(Kobayashi, Takahashi, Takahashi, MY)**

③ **Tensor perturbations:**
  multibrid model **(Naruko & Sasaki) …**

Tensor contributions



1210.7231 SPT

Though n_s ∼ 1 is marginally consistent for CMB, additions of BAO and H rule out it even if we take into account tensor contributions.

Let’s look forward to the Planck results !!

Summary

- **The modulated reheating mechanism can provide large non-Gaussianities.**
- **The relation between f_{NL} & g_{NL} can be useful to discriminate the modulated reheating mechanism from other models.**
- **The spectral index may be a serious problem for light field models generating large local type non-Gaussianities.**