NON-GAUSSIANITY IN THE CMB: ANALYSIS ISSUES

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PLAN

Data processing
Error calculation
Estimators
Blind checks
Signal identification

DATA PROCESSING

Part 1: Raw data to TOI
Part 2: TOI processing
Part 3: TOI to Map
Part 4: Clean Map

DATA PROCESSING

Part 1: Raw data to TOI

-detector

-pointing

-orbit

-timing

Combines & compresses data into usable form

DATA PROCESSING

Part 2: TOI processing

- -De-modulation (Remove AC carrier wave)
- -De-glitch (Remove cosmic ray strikes)
- -Volts to Temp (Correct for non-linear gain and gain variation)
- -Thermal decorrelation (Remove temp fluctuation using dark bolometers)
- -Remove cooler systematics (EM interference, Micro-phonics)
- -Deconvolve bolometer time constant (Correct time response)

DATA PROCESSING

Part 3: TOI to Map
-De-stripe to create rings (Remove low frequency correlated noise)
-Add rings (correcting with offsets from de-striping algorithm)

Part 4: Clean Map
-Beam deconvolution
-Foreground removal (Dust, etc..)
-Point source

SIMULATION

Part 1: $$a_{lm} = b_l \sqrt{C_l} \times \mathrm{RNG}$$

Part 2: $$N(\hat{\mathbf{n}}) = \frac{A_N}{\sqrt{\mathrm{HC}(\hat{\mathbf{n}})}} \times \mathrm{RNG}$$

Part 3: $$M(\hat{\mathbf{n}}) = \sum_{lm} a_{lm} Y_{lm}(\hat{\mathbf{n}}) + N(\hat{\mathbf{n}})$$

Error is statistical, and around 0. So 30 +/- 20 is really saying:
30 is consistent with 0 +/- 20
A detection of fnl would need more

ESTIMATION

Suppose we have a bispectrum we wish to constrain

$$(B^{l_1 l_2 l_3}_{m_1 m_2 m_3})^{Theory}$$

Which has some amplitude parameter f_{NL}

The maximum likelihood estimator for f_{NL} is

$$\mathcal{E} = \frac{\sum_{l_i m_i} (B^{l_1 l_2 l_3}_{m_1 m_2 m_3})^{Theory}_{f_{NL}=1} C^{-1}_{l_1 m_1 l'_1 m'_1} C^{-1}_{l_2 m_2 l'_2 m'_2} C^{-1}_{l_3 m_3 l'_3 m'_3} a_{l'_1 m'_1} a_{l'_2 m'_2} a_{l'_3 m'_3}}{\sum_{l_i m_i} (B^{l_1 l_2 l_3}_{m_1 m_2 m_3})^{Theory}_{f_{NL}=1} C^{-1}_{l_1 m_1 l'_1 m'_1} C^{-1}_{l_2 m_2 l'_2 m'_2} C^{-1}_{l_3 m_3 l'_3 m'_3} (B^{l'_1 l'_2 l'_3}_{m'_1 m'_2 m'_3})^{Theory}_{f_{NL}=1}}$$

where

$$C_{l_1 m_1 l_2 m_2} = \langle a_{l_1 m_1} a_{l_2 m_2} \rangle$$

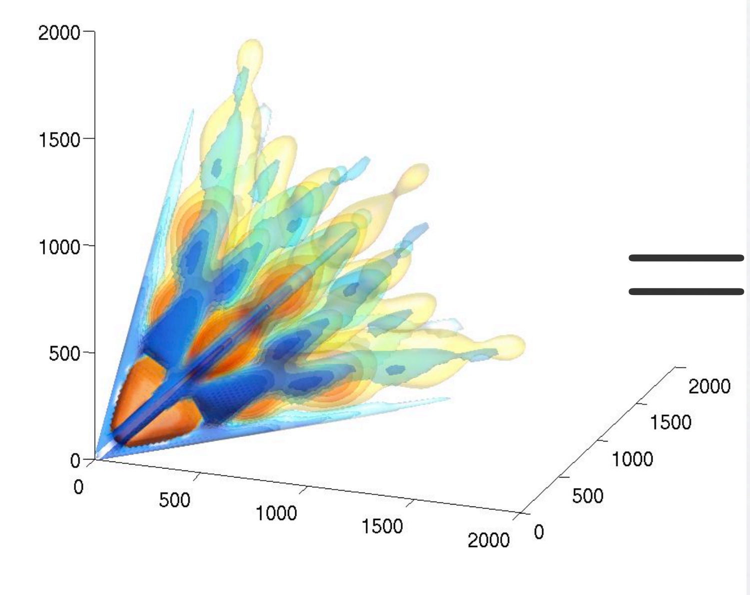
PROJECTION

So how do we calculate it?
First we start with the primordial bispectrum.

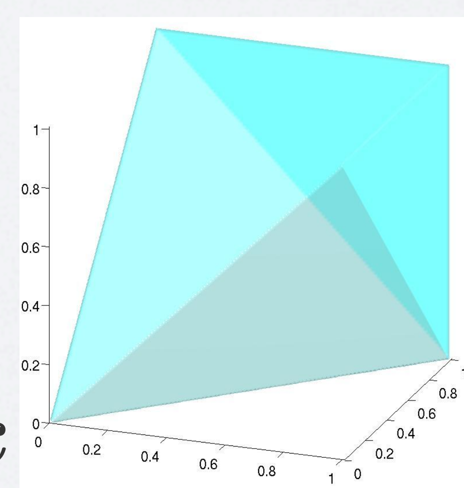
$$\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3)\rangle = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B(k_1, k_2, k_3)$$

Then project it forward with transfer functions....

$$B^{l_1 l_2 l_3}_{m_1 m_2 m_3} = \int B(k_1, k_2, k_3)\, \Delta^{l_1 l_2 l_3}_{m_1 m_2 m_3}(k_1, k_2, k_3)$$



$$= \int_{\mathcal{V}_k}$$



$$\times \Delta$$

PROBLEM

But...

$$\Delta^{l_1 l_2 l_3}_{m_1 m_2 m_3}(k_1, k_2, k_3) = \int d^3x \tilde{\Delta}_{l_1 m_1}(k_1, \mathbf{x}) \tilde{\Delta}_{l_2 m_2}(k_2, \mathbf{x}) \tilde{\Delta}_{l_3 m_3}(k_3, \mathbf{x})$$

$$\tilde{\Delta}_{l_1 m_1}(k_1, \mathbf{x}) = j_{l_1}(k_1 x) Y_{l_1 m_1}(\hat{x}) \Delta_{l_1}(k_1)$$

To be solvable we need separability

$$B(k_1, k_2, k_3) = X(k_1) X(k_2) X(k_3)$$

SOLUTION?

The problem is that in general

$$B(k_1, k_2, k_3) \neq X(k_1)X(k_2)X(k_3)$$

We need to find a representation of B which is separable

$$B(k_1, k_2, k_3) = \sum_n \alpha_n R_n(k_1, k_2, k_3)$$

$$R_n(k_1, k_2, k_3) = r_i(k_1)r_j(k_2)r_k(k_3) + 5\,\text{permutations}$$

$$\langle R_n R_m \rangle = \delta_{nm}$$

ORTHONORMAL BASIS

- Now how to construct our R?

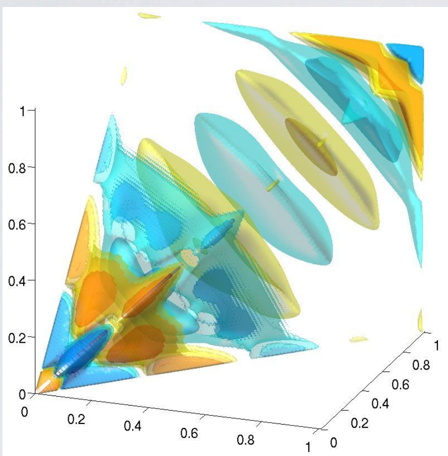
$$R_n(k_1, k_2, k_3) = \sum_m \lambda_{nm} Q_m(k_1, k_2, k_3)$$

$$Q_m(k_1, k_2, k_3) = \frac{1}{6}\left(q_i(k_1)q_j(k_2)q_k(k_3) + 5\,(permutations)\right)$$

Where the q are arbitrary functions and λ_{nm} is the product of some orthogonalisation procedure. We must also chose an ordering

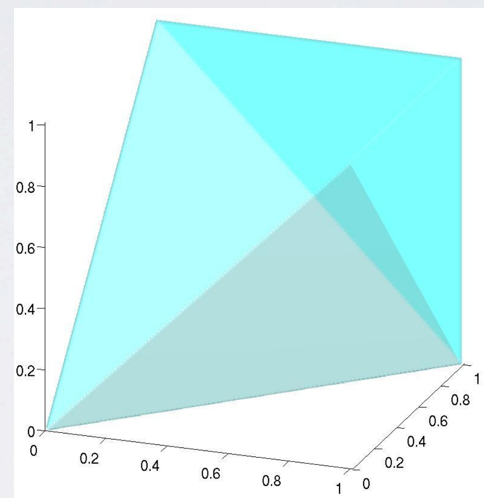
$\underline{0 \to 000}$	$4 \to 111$	$8 \to 022$	$12 \to 113$
$\underline{1 \to 001}$	$5 \to 012$	$9 \to 013$	$13 \to 023$
$2 \to 011$	$\underline{6 \to 003}$	$\underline{10 \to 004}$	$14 \to 014$
$\underline{3 \to 002}$	$7 \to 112$	$11 \to 122$	$\underline{15 \to 005}$ $\cdots$

ORTHONORMAL BASIS



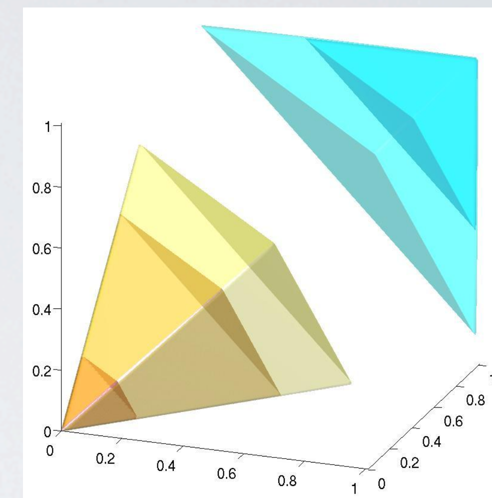
$B(k_1, k_2, k_3)$

$= \alpha_0$



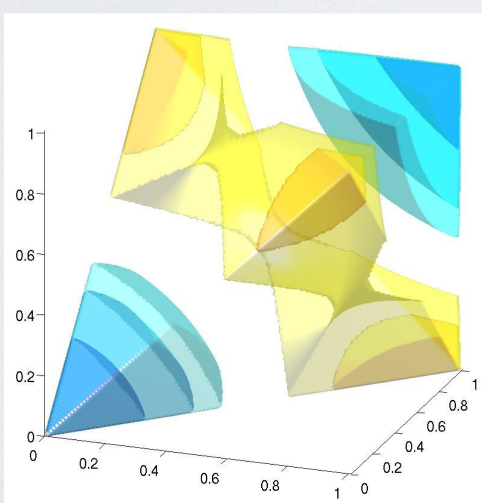
$000 \rightarrow 1$

$+\alpha_1$



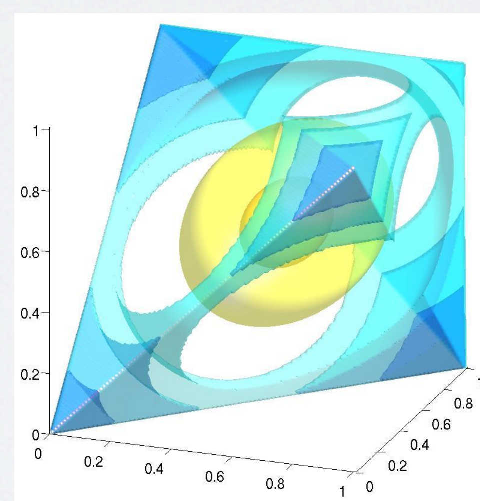
$001 \rightarrow k_1 + k_2 + k_3$

$+\alpha_2$



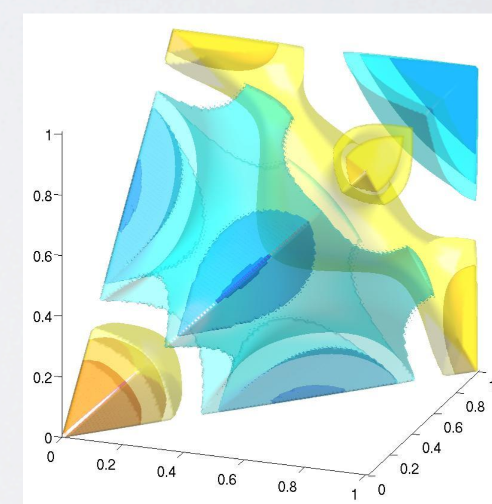
$011 \rightarrow k_1 k_2 + k_2 k_3 + k_3 k_1$

$+\alpha_3$



$002 \rightarrow k_1^2 + k_2^2 + k_3^2$

$+\alpha_4$



$111 \rightarrow k_1 k_2 k_3$

$\cdots$

ORTHONORMAL BASIS

We can now use this method to calculate the estimator

$$\mathcal{E} = \frac{1}{N}\sum_n \alpha_n \beta_n$$

$$\beta_n^Q = \int d^3x M_i(\mathbf{x}) M_j(\mathbf{x}) M_k(\mathbf{x})$$

$$M_i(\mathbf{x}) = \sum_{lm} \tilde{q}^i_{lm}(\mathbf{x}) C^{-1}_{lml'm'} a_{l'm'}$$

$$\tilde{Q}_n = \int x^2 dx \tilde{q}^{\{i}_{l_1 m_1}(x) \tilde{q}^{j}_{l_2 m_2}(x) \tilde{q}^{k\}}_{l_3 m_3}(x)$$

$$\tilde{q}^i_{lm}(\mathbf{x}) = \int dk\, q_i(k) \Delta_l(k) j_l(xk) Y_{lm}(\hat{\mathbf{x}})$$

KSW EXAMPLE

If we consider the three models constrained by KSW we find they can be represented by the following choices of monomials for the q and an ordering which only includes scale invariant combinations.

$$q_0(k) = k^{-1} \qquad 0 \rightarrow 003 \qquad \alpha^Q_{local} = \{2,\ 0,\ 0\}$$

$$q_1(k) = 1 \qquad 1 \rightarrow 012 \qquad \alpha^Q_{equi} = \{-1,\ 1,\ -2\}$$

$$q_2(k) = k \qquad 2 \rightarrow 111 \qquad \alpha^Q_{ortho} = \{-3,\ 3,\ -8\}$$

$$q_3(k) = k^2$$

The only difference is they never use orthonormality as they can read off the coefficients directly from their templates

KSW EXAMPLE

$$4\mathrm{E} - \mathrm{L} \propto \mathrm{O}$$

$$\frac{\frac{4}{6}0.19\sigma - 1.52\sigma}{\frac{4}{6}^2 - 2 \times 0.4 + 1^2} = -2.16\sigma$$

$$\frac{\langle EL \rangle}{\sqrt{\langle EE \rangle \langle LL \rangle}} \approx 0.4 \qquad \sqrt{\frac{\langle EE \rangle}{\langle LL \rangle}} \approx 6$$

LATE TIME ESTIMATION

We will now go one step further by defining the weighted vectors (and matrix)

$$\mathcal{A}_\wp = \frac{\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle}{\sqrt{C_{l_1} C_{l_2} C_{l_3}}}\,, \qquad \mathcal{B}_\wp = \frac{a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} - 3 C_{l_1 m_1 l_2 m_2} a_{l_3 m_3}}{\sqrt{C_{l_1} C_{l_2} C_{l_3}}}\,, \qquad \mathcal{C}_{\wp\wp'} = \frac{C_{l_1 m_1 l'_1 m'_1} \ldots C_{l_3 m_3 l'_3 m'_3}}{\sqrt{C_{l_1} C_{l'_1} \ldots C_{l_3} C_{l'_3}}}\,,$$

And we can then write the estimator in matrix form as

$$\bar{\mathcal{E}} = \frac{\mathcal{A}^T \mathcal{C}^{-1} \mathcal{B}}{\mathcal{A}^T \mathcal{C}^{-1} \mathcal{A}}$$

CMB BASIS

We can perform the same modal decomposition on the data to obtain the estimator

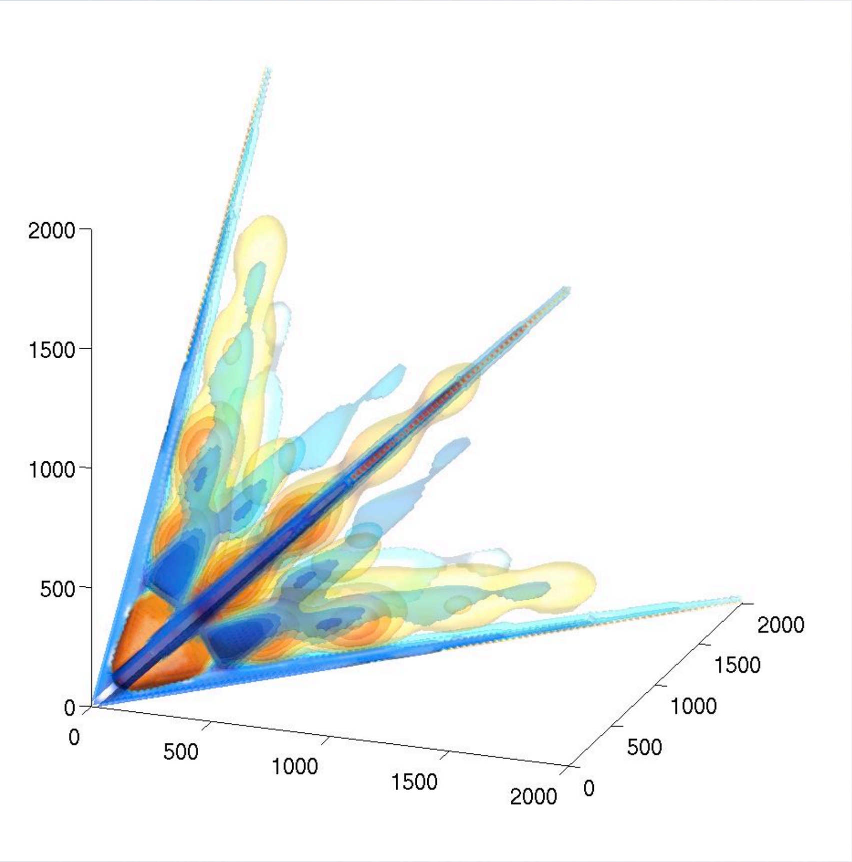
$$\bar{\alpha} = \bar{\mathcal{R}}\mathcal{A} \rightarrow \mathcal{A} = \bar{\mathcal{R}}^T \bar{\alpha}$$

$$\bar{\beta} = \bar{\mathcal{R}}\mathcal{B} \rightarrow \mathcal{P}\mathcal{B} = \bar{\mathcal{R}}^T \bar{\beta} \qquad \langle \bar{\beta} \rangle = \bar{\alpha}$$

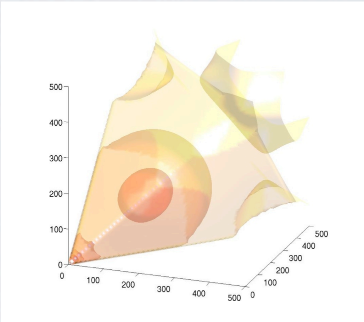
$$\zeta = 6\, \bar{\mathcal{R}}\mathcal{C}\bar{\mathcal{R}}^T = \left\langle \bar{\beta}\bar{\beta}^T \right\rangle$$

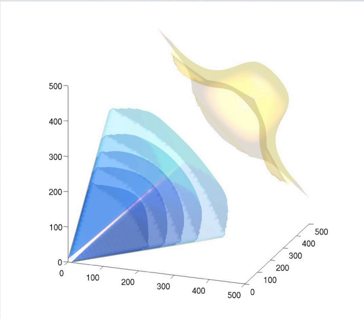
$$\mathcal{E} = \frac{\bar{\alpha}^T \zeta^{-1} \bar{\beta}}{\bar{\alpha}^T \zeta^{-1} \bar{\alpha}}$$

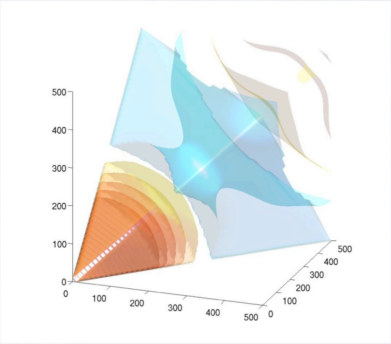
CMB BASIS

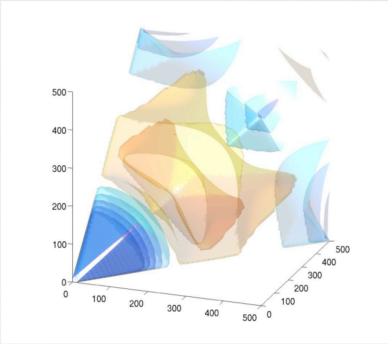


$=$

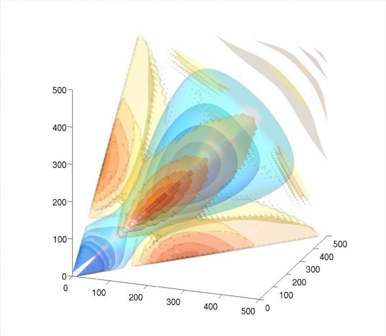
α_0 

α_1 

α_2 

α_3 

$+$

α_4 

$\ldots\ldots$

ORTHONORMAL BASIS

If we also calculate the decomposition of the primordial basis modes projected forward

$$\bar{\mathcal{R}}\tilde{\mathcal{R}}^T = \Gamma \qquad \left(\tilde{\mathcal{R}}_l = \int_{\mathcal{V}_k} \mathcal{R}(k) \times \Delta\right)$$

Then we can transform between the primordial and CMB expansions

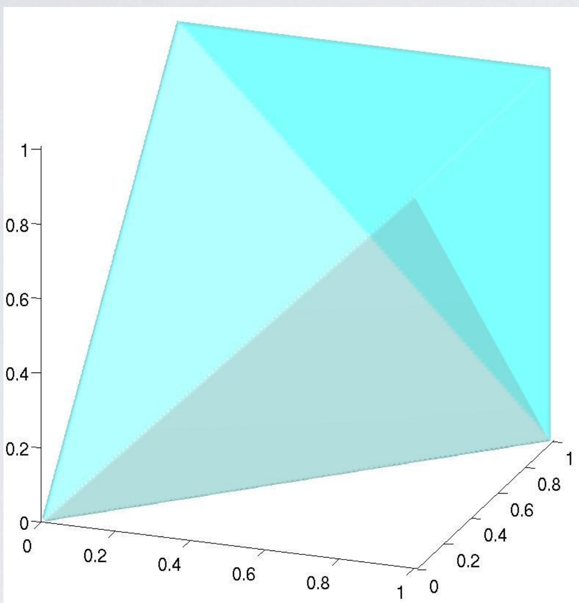
$$\bar{\alpha}^{\mathcal{R}} = \Gamma\alpha^{\mathcal{R}}$$

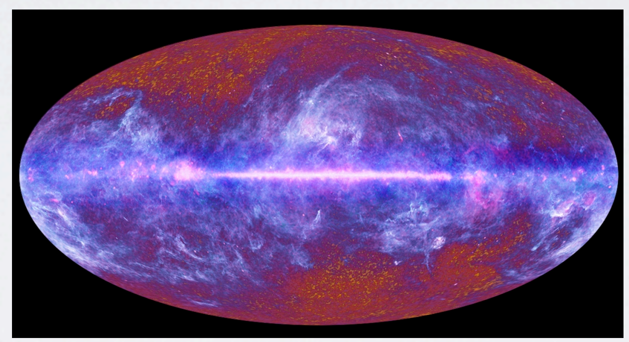
$$\left(\bar{\alpha}^{\mathcal{Q}} = \bar{\lambda}\Gamma{\lambda^{-1}}^T\alpha^{\mathcal{Q}}\right)$$

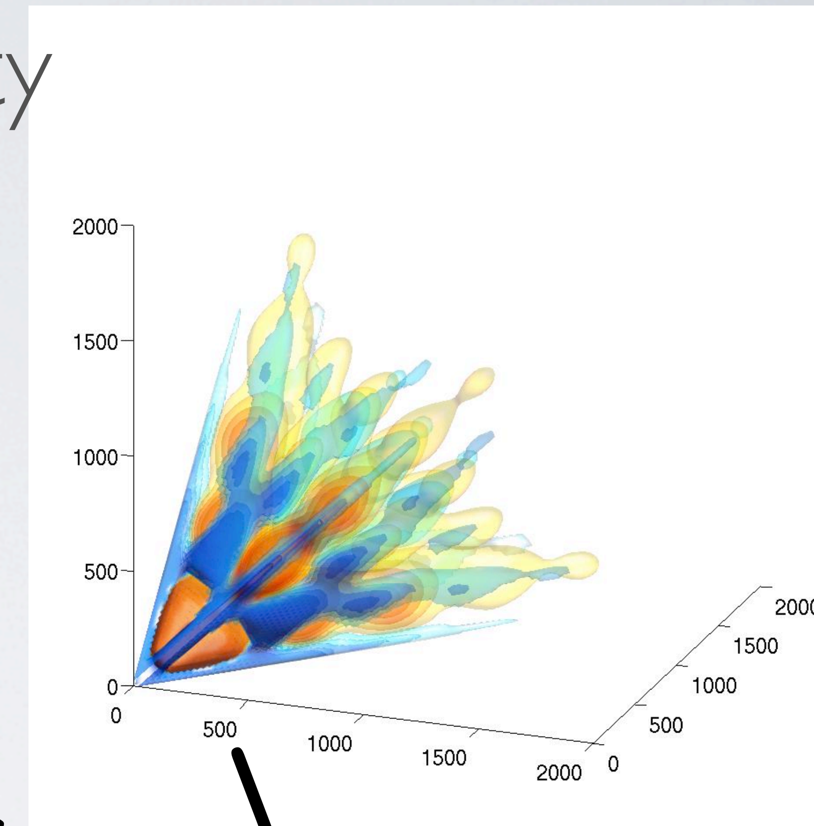
RECAP

Separability = Tractability

Basis = Good







$$\mathcal{E} = \frac{1}{N}\sum_n \alpha_n \beta_n \longleftarrow \Gamma = \bar{\mathcal{R}}\tilde{\mathcal{R}}^T \longrightarrow \mathcal{E} = \frac{\bar{\alpha}^T \zeta^{-1} \bar{\beta}}{\bar{\alpha}^T \zeta^{-1} \bar{\alpha}}$$

ArXiv:1006.1642

LATE TIME EXAMPLES

$$R_n(l_1, l_2, l_3) = \sum_m \lambda_{nm} Q_m(l_1, l_2, l_3)$$

$$Q_m(l_1, l_2, l_3) = \frac{1}{6}\left(q_i(l_1) q_j(l_2) q_k(l_3) + 5\,(permutations)\right)$$

Now:

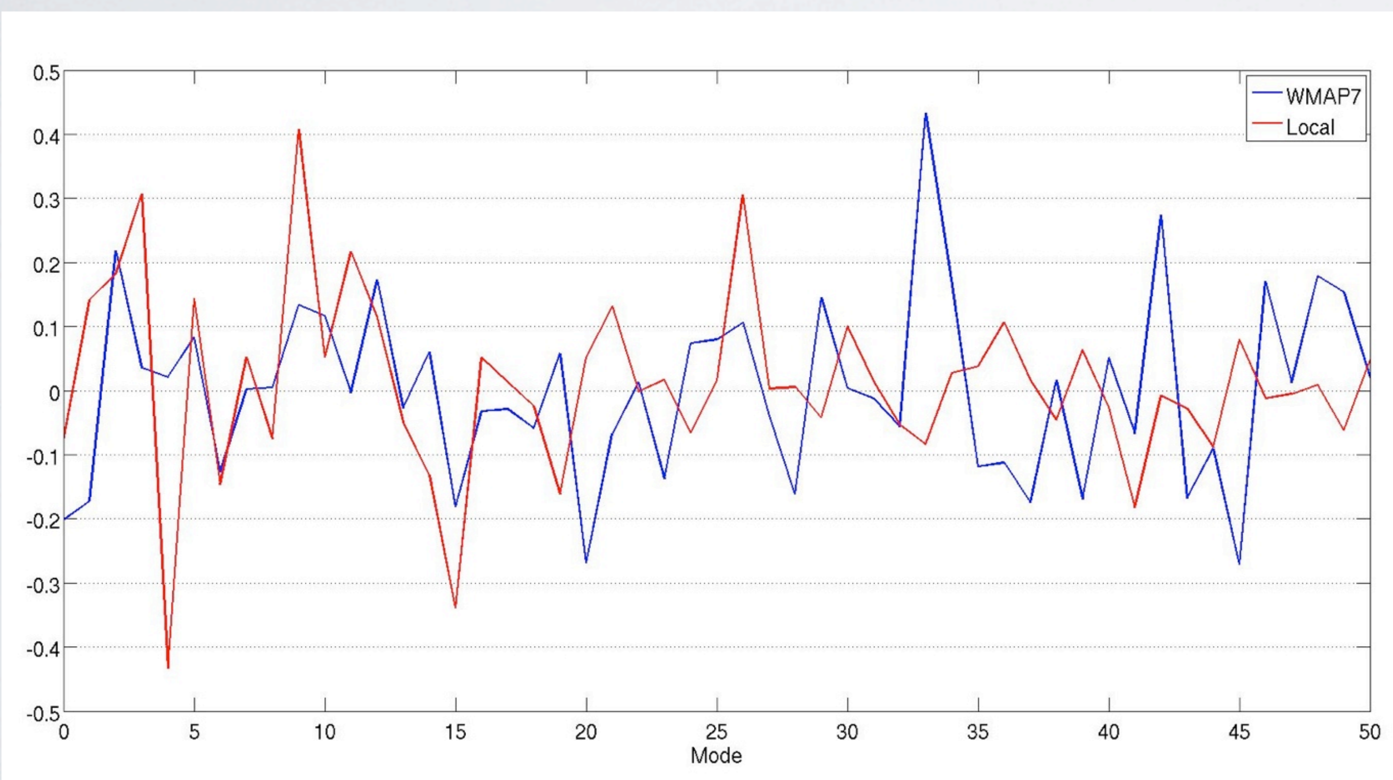
q = Harmonic transform of SMHW for wavelet estimators

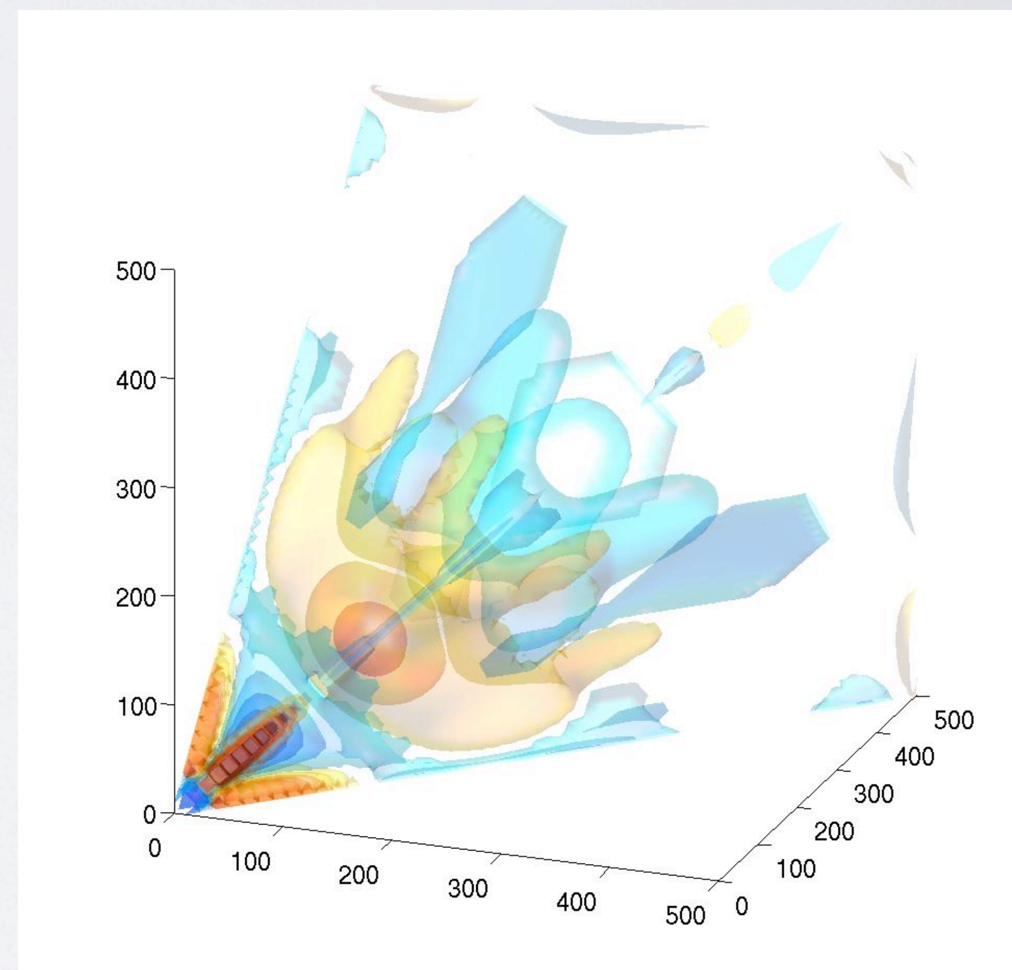
q = Top hat functions for binned estimators

q = Continuous functions for Modal estimators (eg polynomials, trigonometric functions...)

RECONSTRUCTION

We have $\langle\beta\rangle = \alpha$ so can reconstruct the best fit bispectrum to the data by using the β as our α. If we have constructed a primordial basis as well then we can use Γ to find the best fit primordial bispectrum





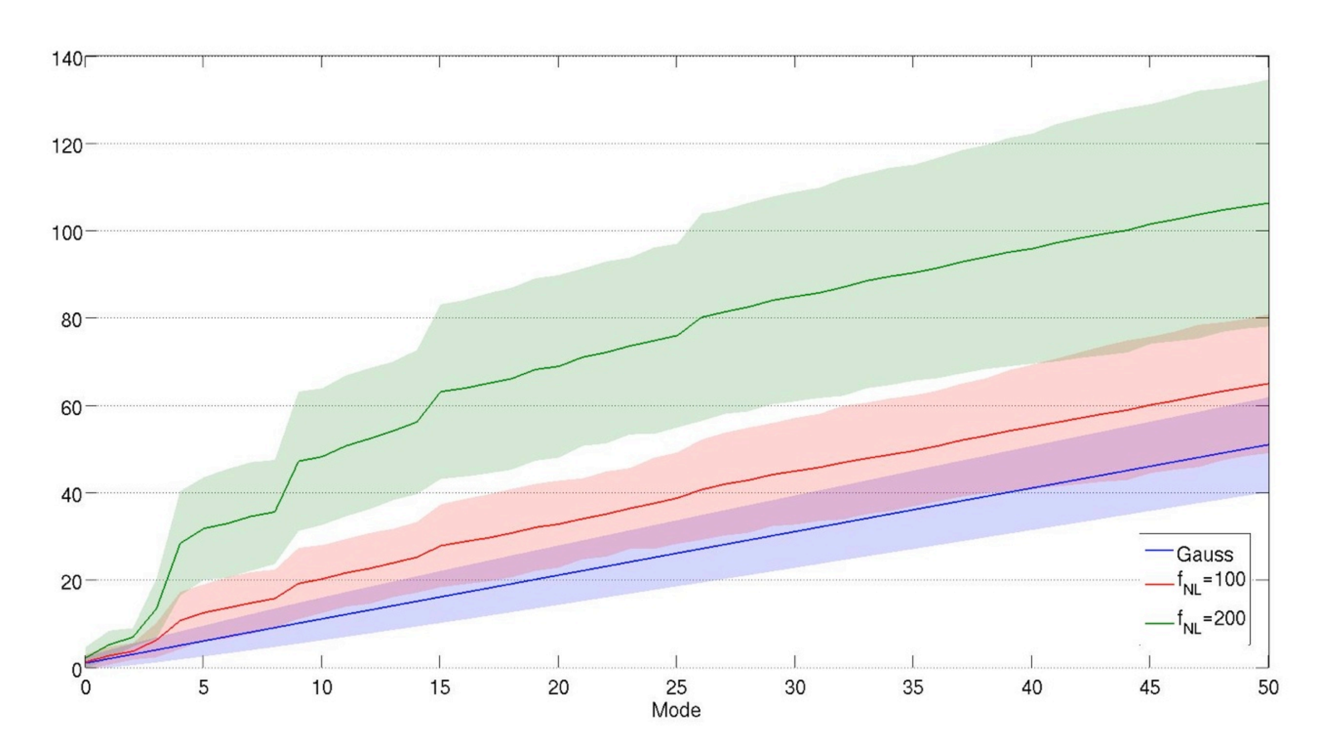
RECONSTRUCTION

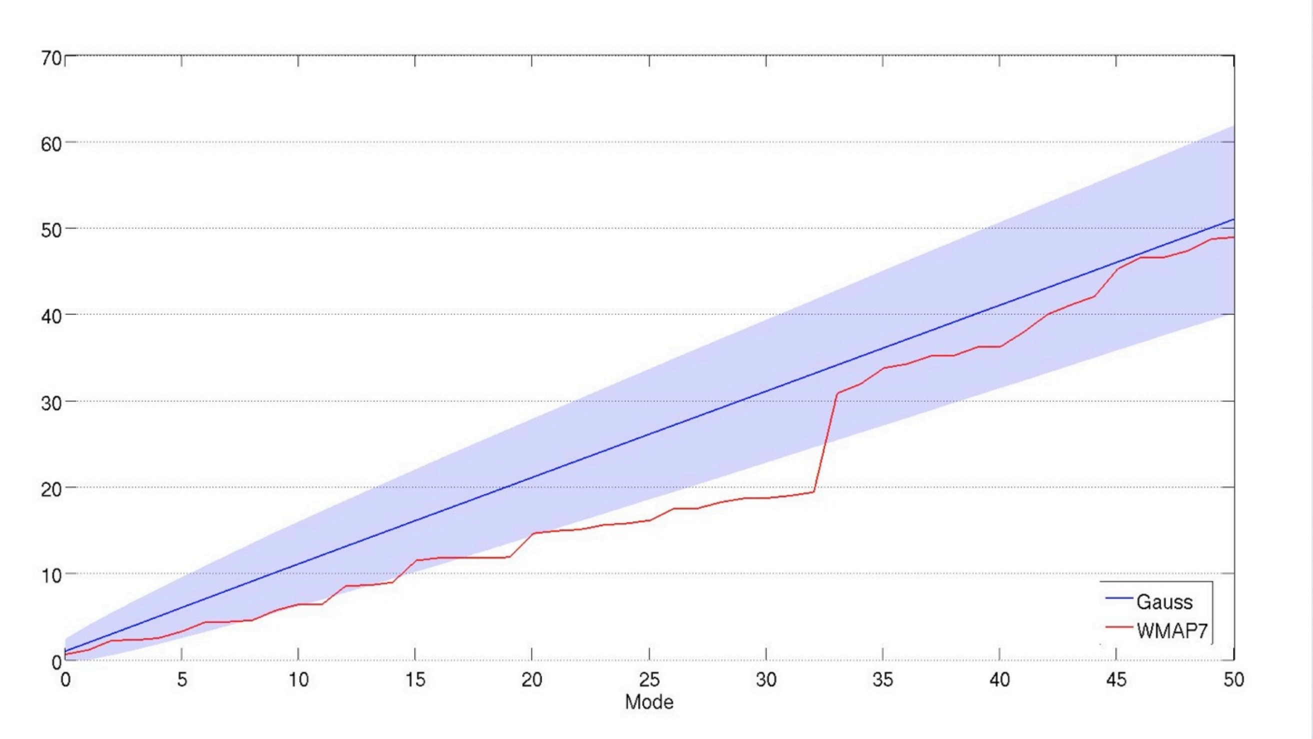
We can perform a blind search for excess variance

$$F^2_{NL}(N) = \sum_{n,n'=0}^{N} \beta_n \zeta^{-1}_{n,n'} \beta_{n'}$$

$$\langle F^2_{NL} \rangle = N \qquad \text{(Gaussian)}$$

$$\delta F^2_{NL} = \sqrt{2N} \; \text{(Gaussian)}$$





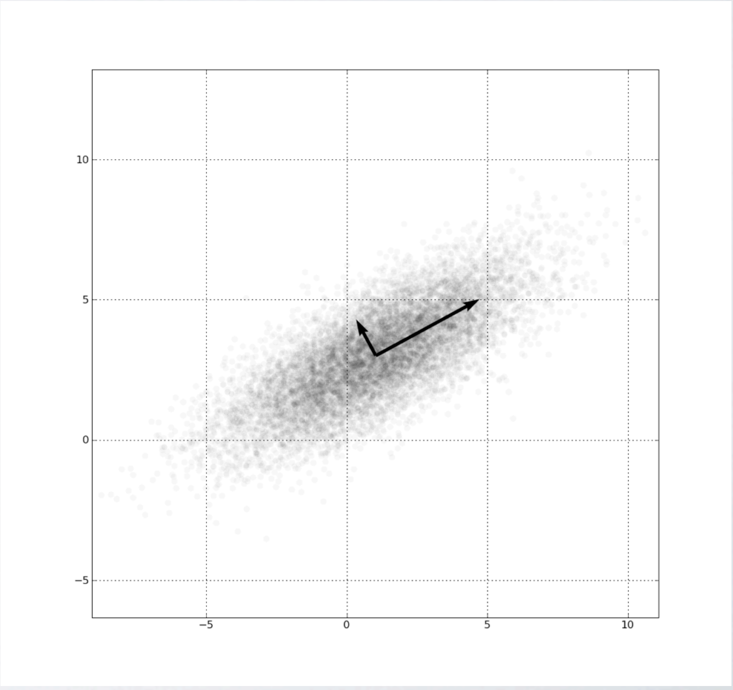
CONTAMINANTS

As we expect the covariance matrix to be the identity we can use principle component analysis to identify the shape of contaminants.

We first calculate the covariance matrix for beta from simulations

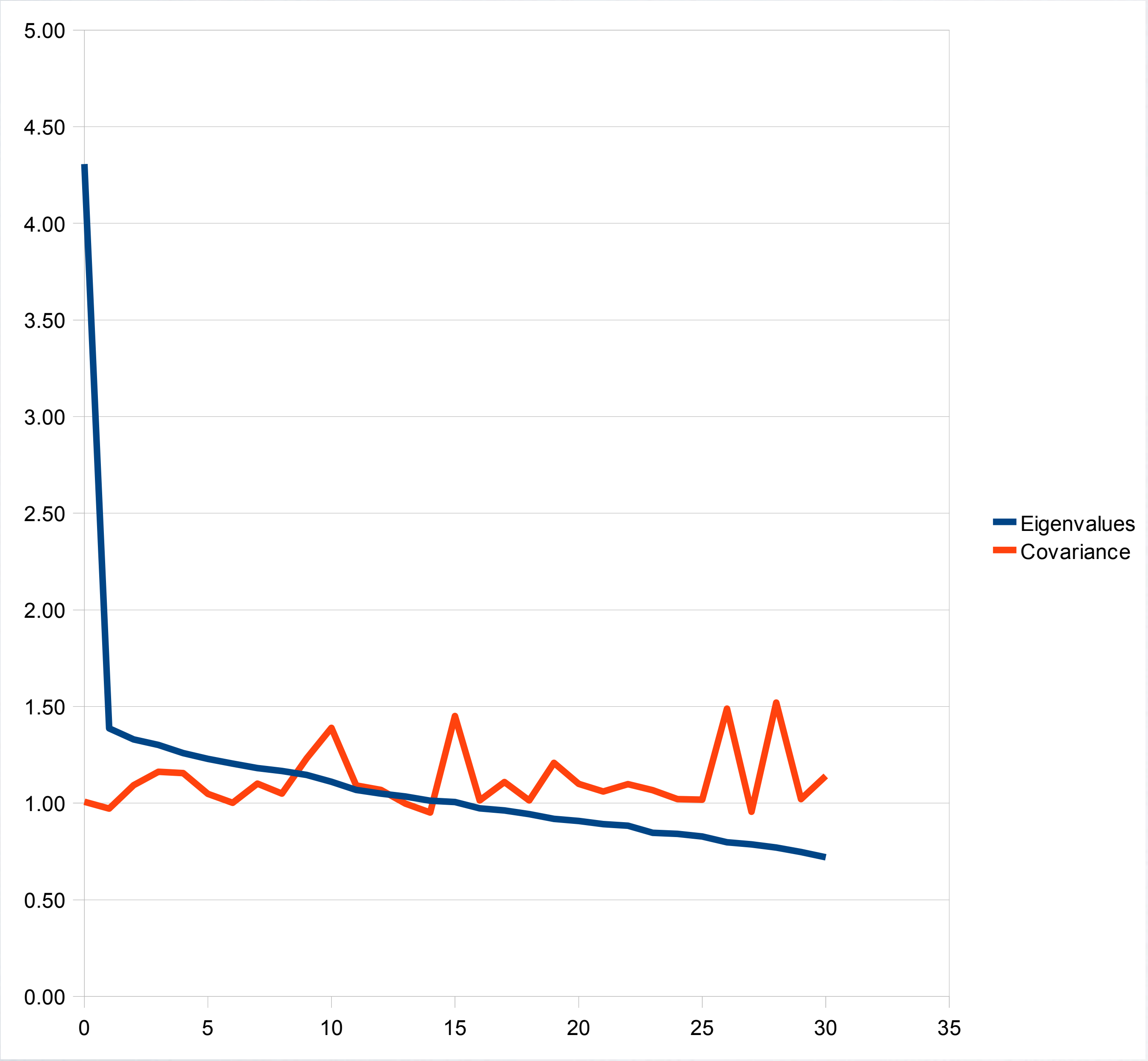
$$V \zeta V^T = D$$

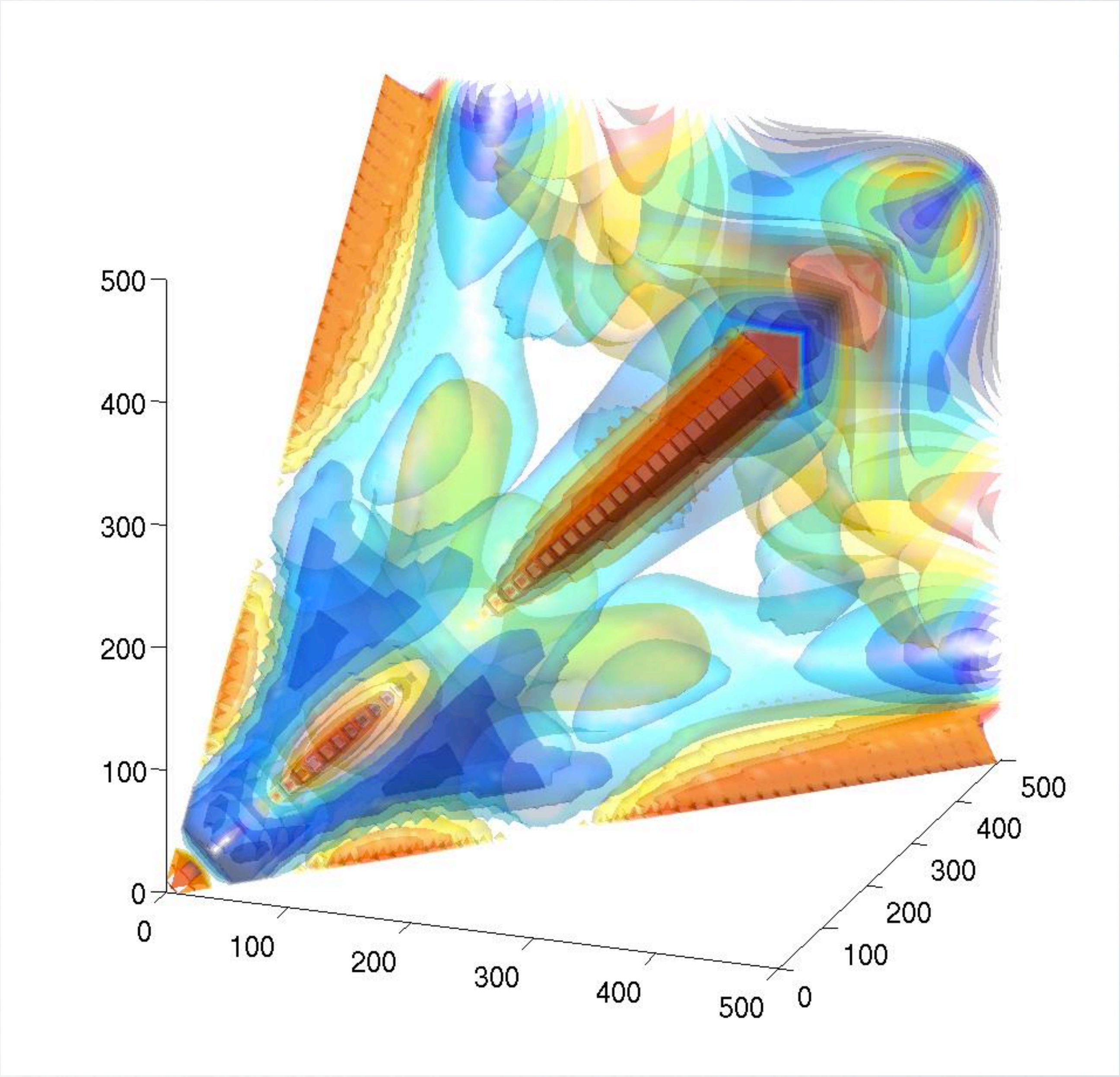
And then find the rotation which diagonalises it. This is equivalent to performing an eigen decomposition. The result is that you obtain a new orthonormal basis but now your modes are uncorrelated and ordered from greatest to least variance.



CONTAMINANTS

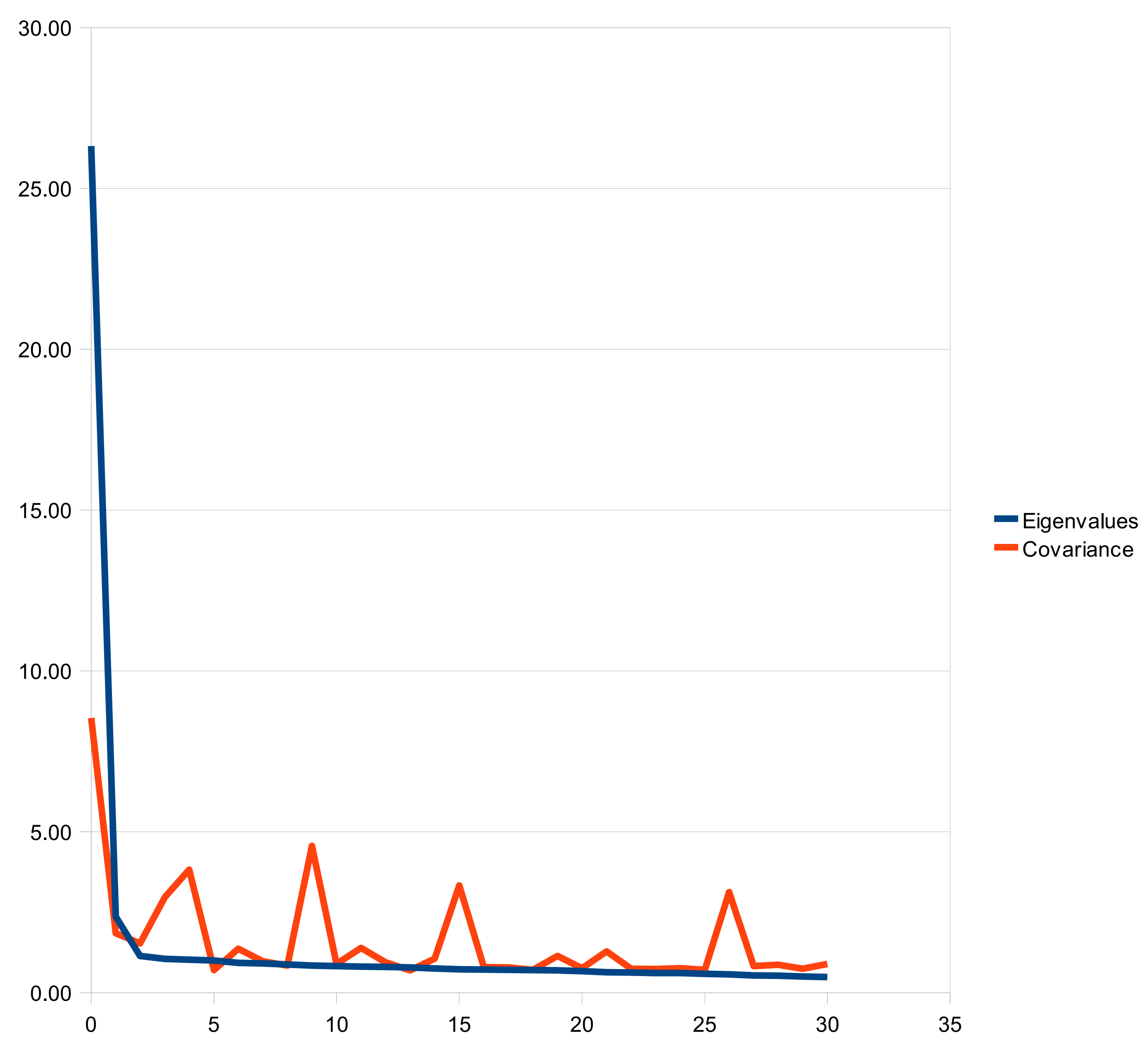
WMAP inhomogeneous noise

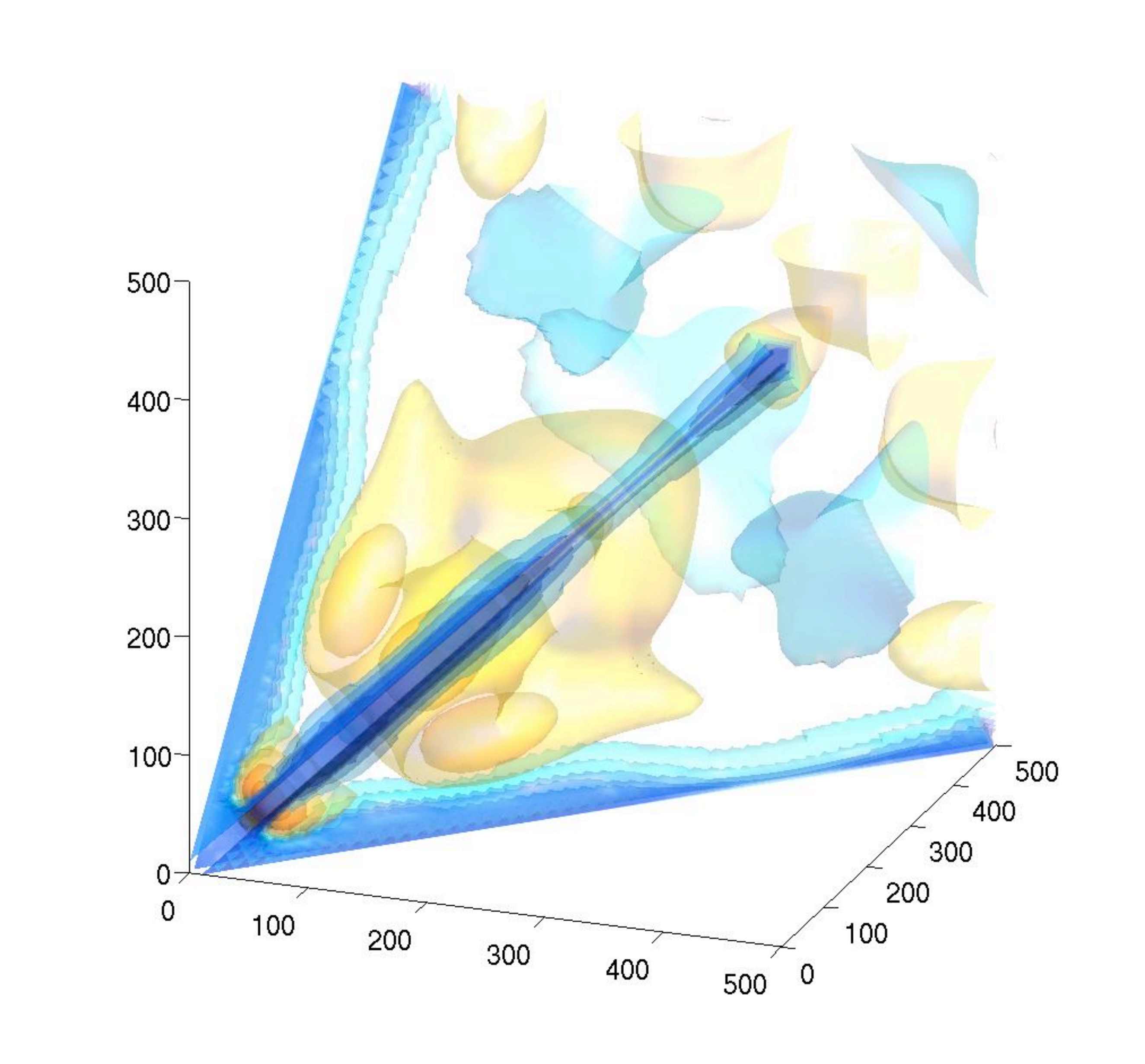




CONTAMINANTS

WMAP Mask





CONTAMINANTS

Point sources