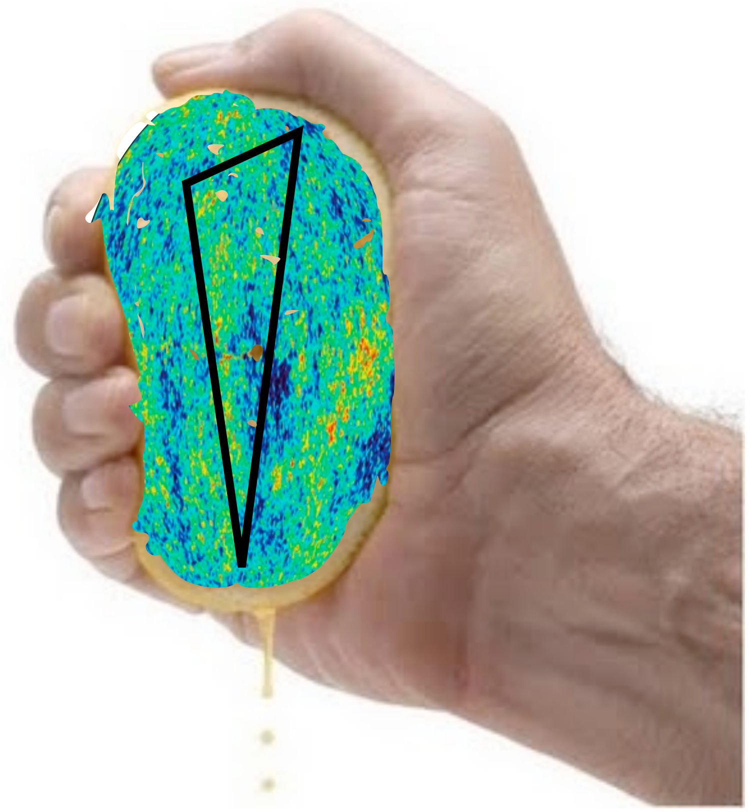
Squeezing the CMB bispectrum



Filippo Vernizzi - IPhT, CEA Saclay

With Boubekeur, Creminelli, D'Amico, Noreña,
(arXiv:0806.1016, arXiv:0906.0980),
Creminelli, Pitrou (arXiv:1109.1822),
and Zhiqi Huang (work in progress)

November 8, 2012 : MPI, Garching, Germany

Preparing for the best...

Monday discussion by Xingang Chen

$$f_{\rm NL}^{\rm loc} \sim 32$$

Preparing for the worst...

$$f_{\mathrm{NL}}^{\mathrm{loc}} \ll 1$$

...or the not-so bad

$$f_{\mathrm{NL}}^{\mathrm{loc}} \sim \mathrm{few}$$

CMB non-Gaussianity

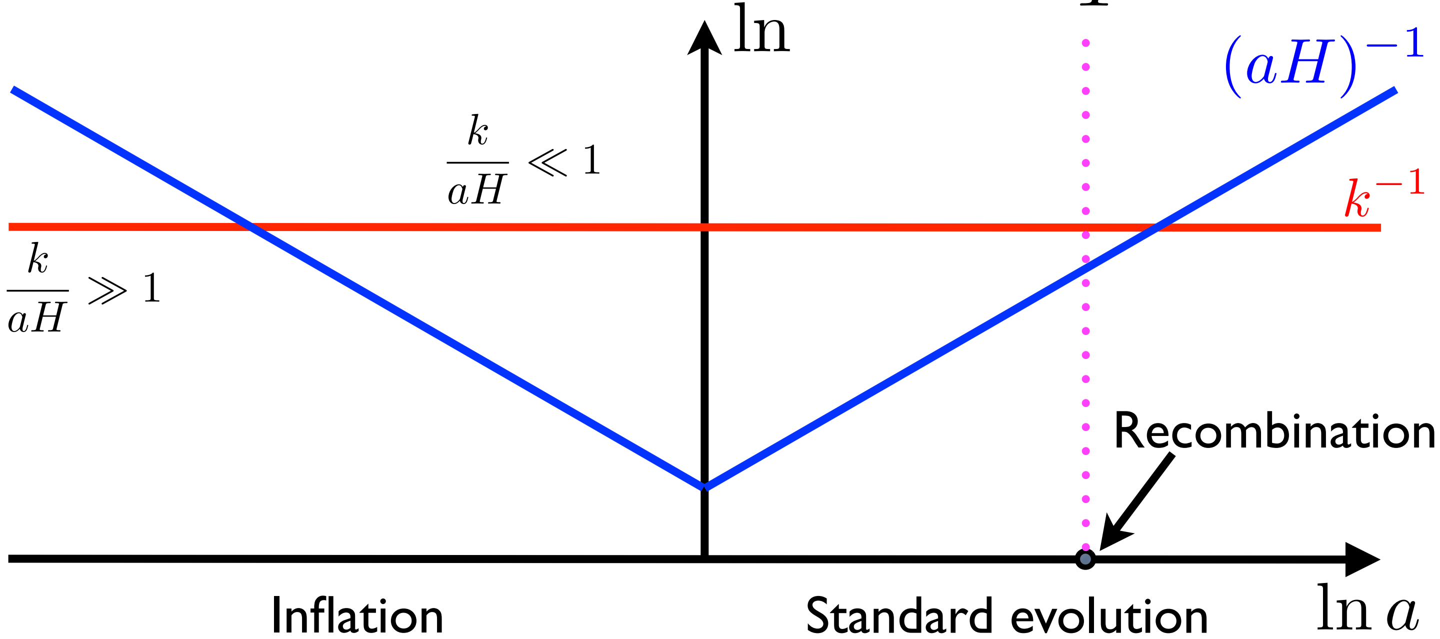
Even in the absence of primordial non-Gaussianity, $\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = 0$, the CMB is non-Gaussian!

2nd-order effects induce NG:

- late time: ISW-lensing (previous talk);
- at recombination: 2nd-order perturbations in the fluid + GR nonlinearities.

Nonlinear relation

$$\zeta \quad \Rightarrow \quad \frac{\delta T}{T}$$

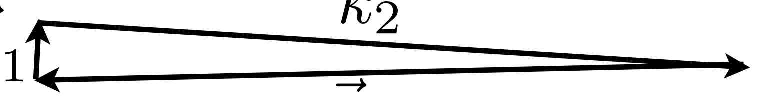


$$\delta = \delta^{(1)} + \delta^{(2)} \quad \Rightarrow \quad \begin{array}{l} D[\delta^{(1)}] = 0 \\ D[\delta^{(2)}] = S[\delta^{(1)2}] \end{array} \quad \Rightarrow \quad f_{\rm NL} \sim \frac{\langle \delta^{(2)} \delta^{(1)} \delta^{(1)} \rangle}{\langle \delta^{(1)} \delta^{(1)} \rangle^2} \sim \text{few}$$

All these effects are order ~ may be important to interpret Planck data!

Squeezed limit

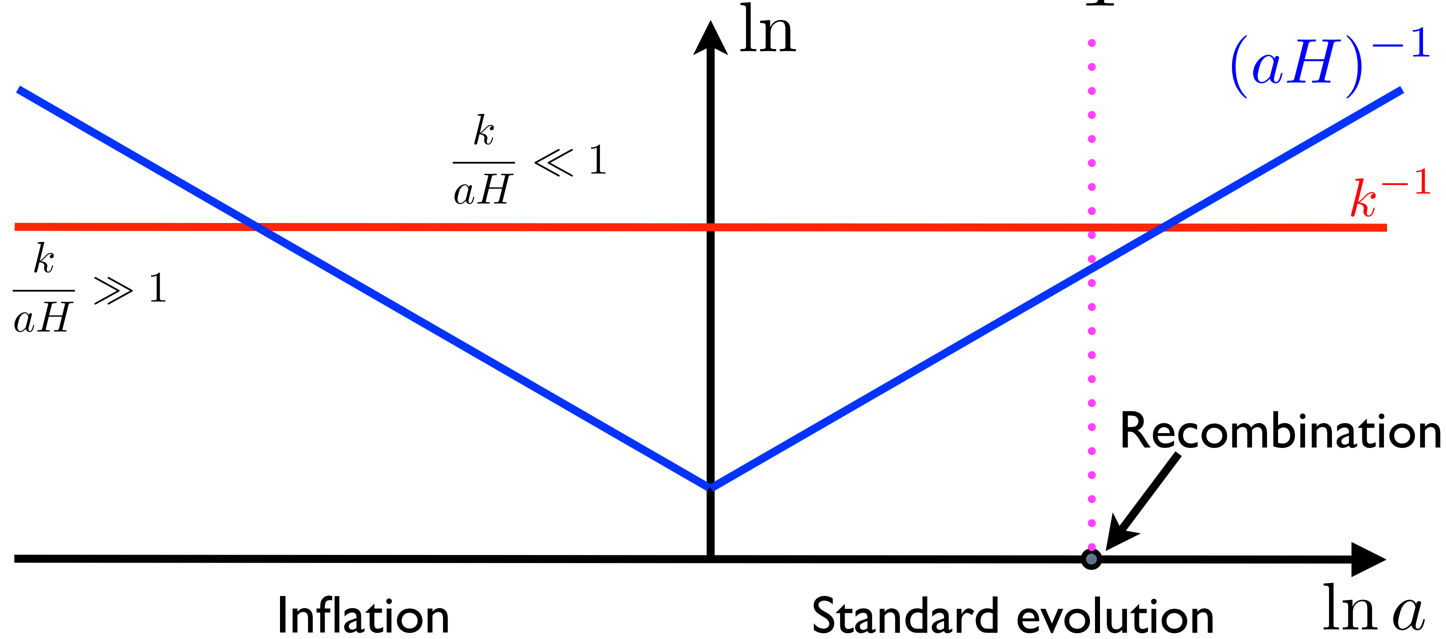
$$k_L \equiv k_1 \ll k_S \equiv k_2 \sim k_3$$



$$\langle \zeta_{\vec{k}_L} \zeta_{\vec{k}_S} \zeta_{-\vec{k}_S} \rangle = -P_{k_L} P_{k_S} \frac{d \ln(k_S^3 P_{k_S})}{d \ln k_S}$$

Maldacena '02, Creminelli & Zaldarriaga '04, Cheung et al. '07

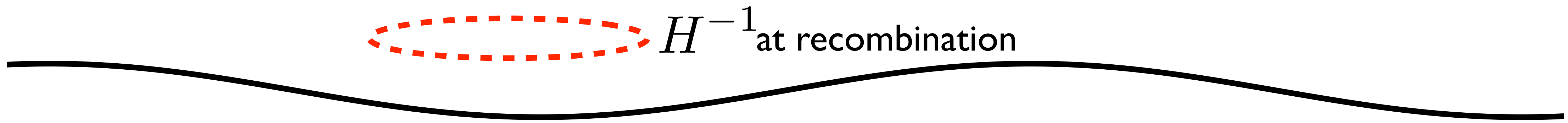
$$\zeta \quad \Rightarrow \quad \frac{\delta T}{T}$$

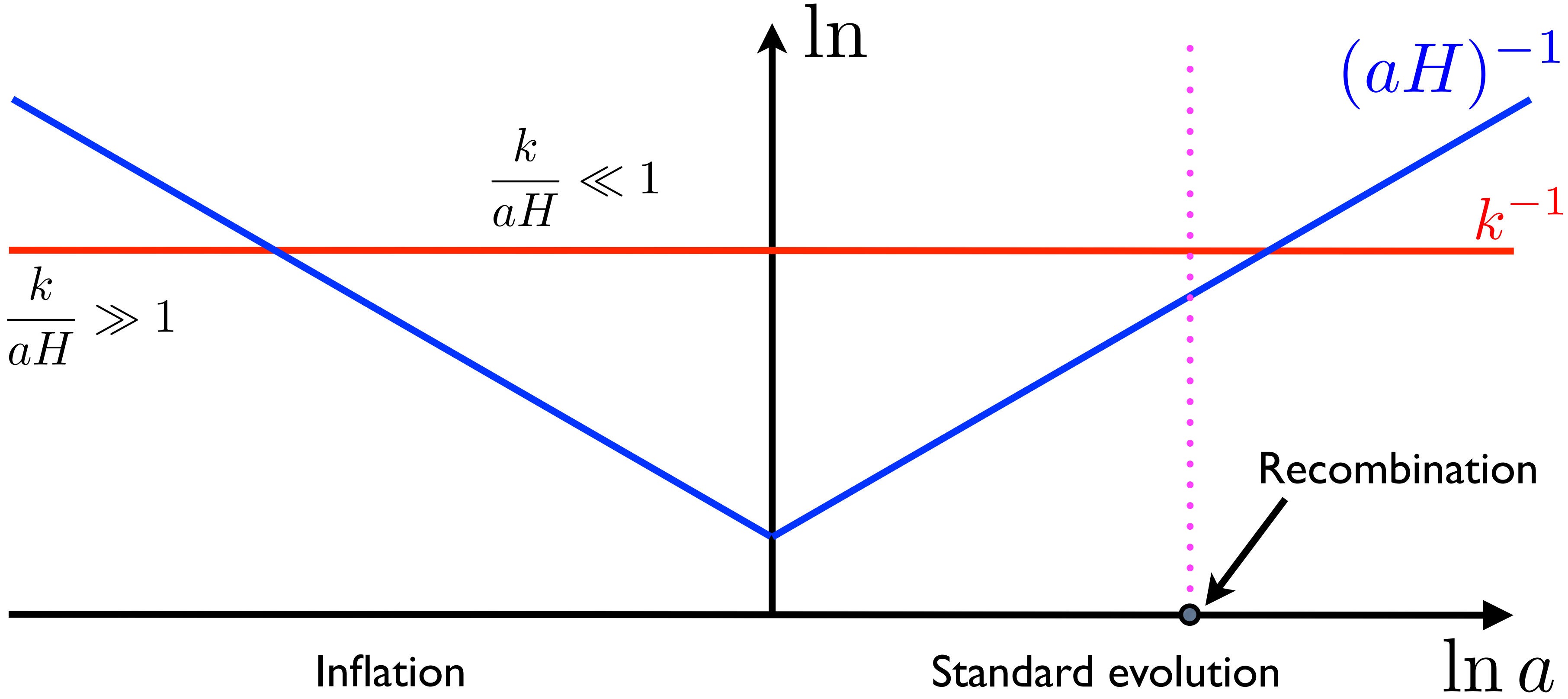


- Directly addresses the local shape (important to rule out single-field models)
- Eventually we would like to have a 2nd-order Boltzmann code: squeezed limit can be used as a consistency check

Particular squeezed limit

One of the angles must subtend a scale longer than Hubble radius at recombination (but smaller than Hubble radius today):





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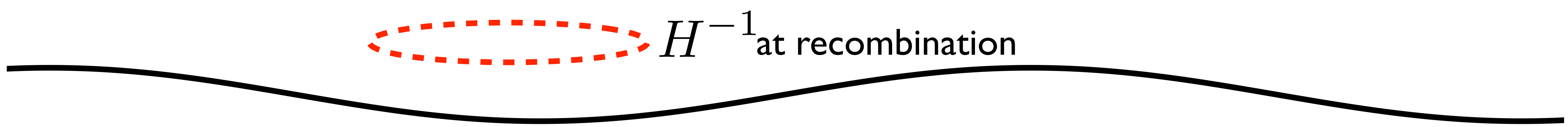
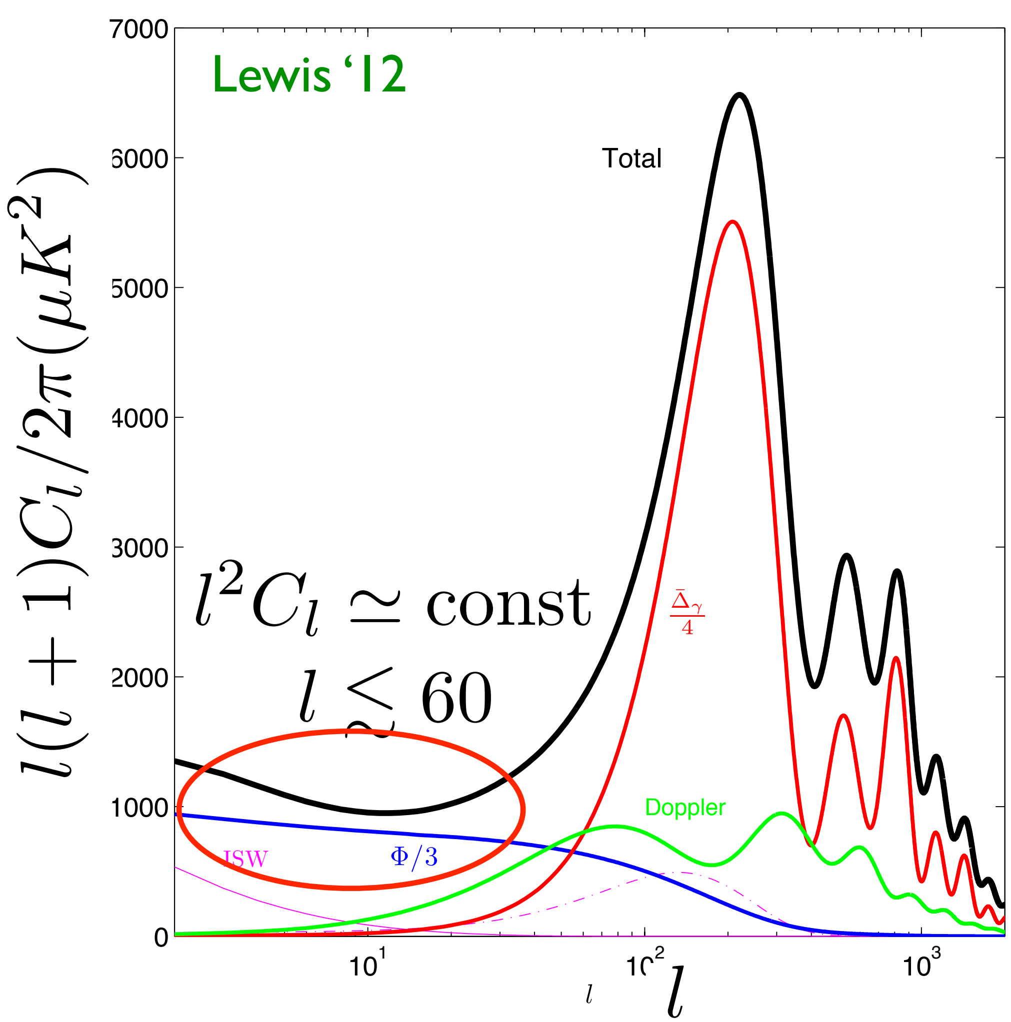


For the bispectrum: $B_{l_1 l_2 l_3}\ , \quad l_1 \ll l_2 \simeq l_3 \quad \& \quad l_1 \ll 200$



Sachs-Wolfe effect:

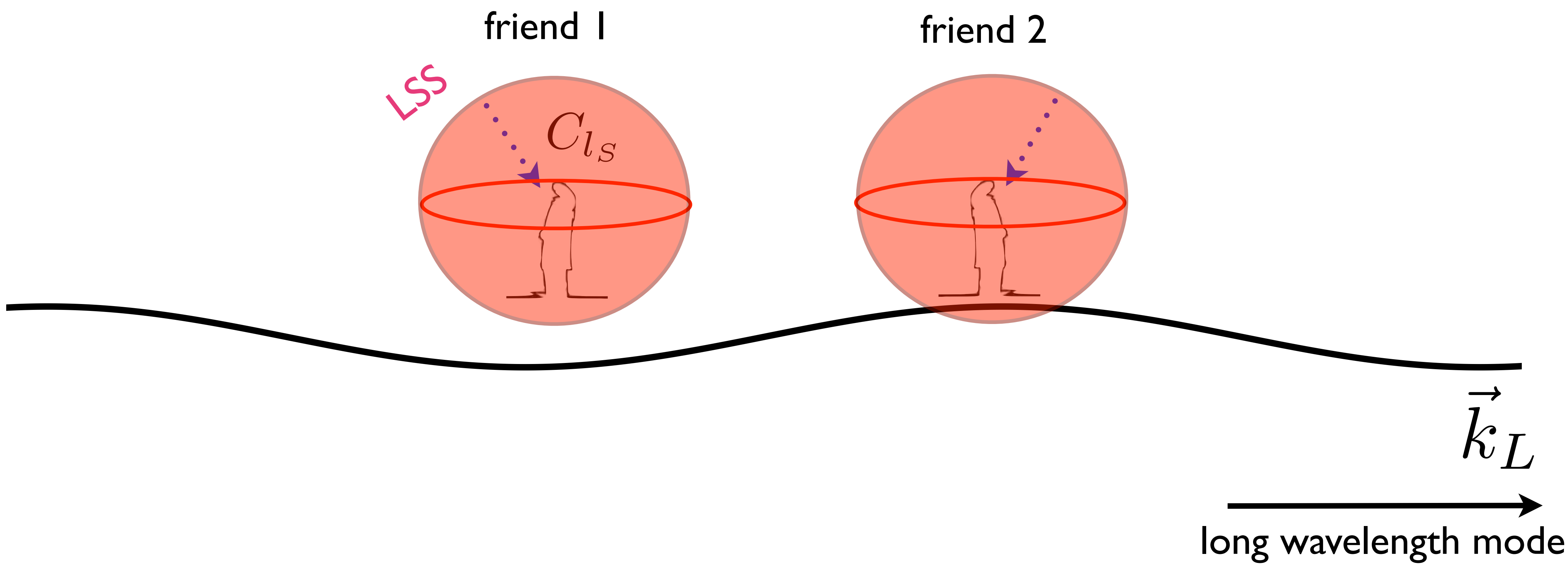
$$\frac{\delta T}{T} = \frac{1}{3}\Phi(\vec{x}_{\rm rec}) = -\frac{1}{5}\zeta(\vec{x}_{\rm rec}) \quad \Rightarrow$$

$$C_l \simeq \frac{A_T}{l^2}\left(\frac{l}{l_*}\right)^{n_s-1}$$

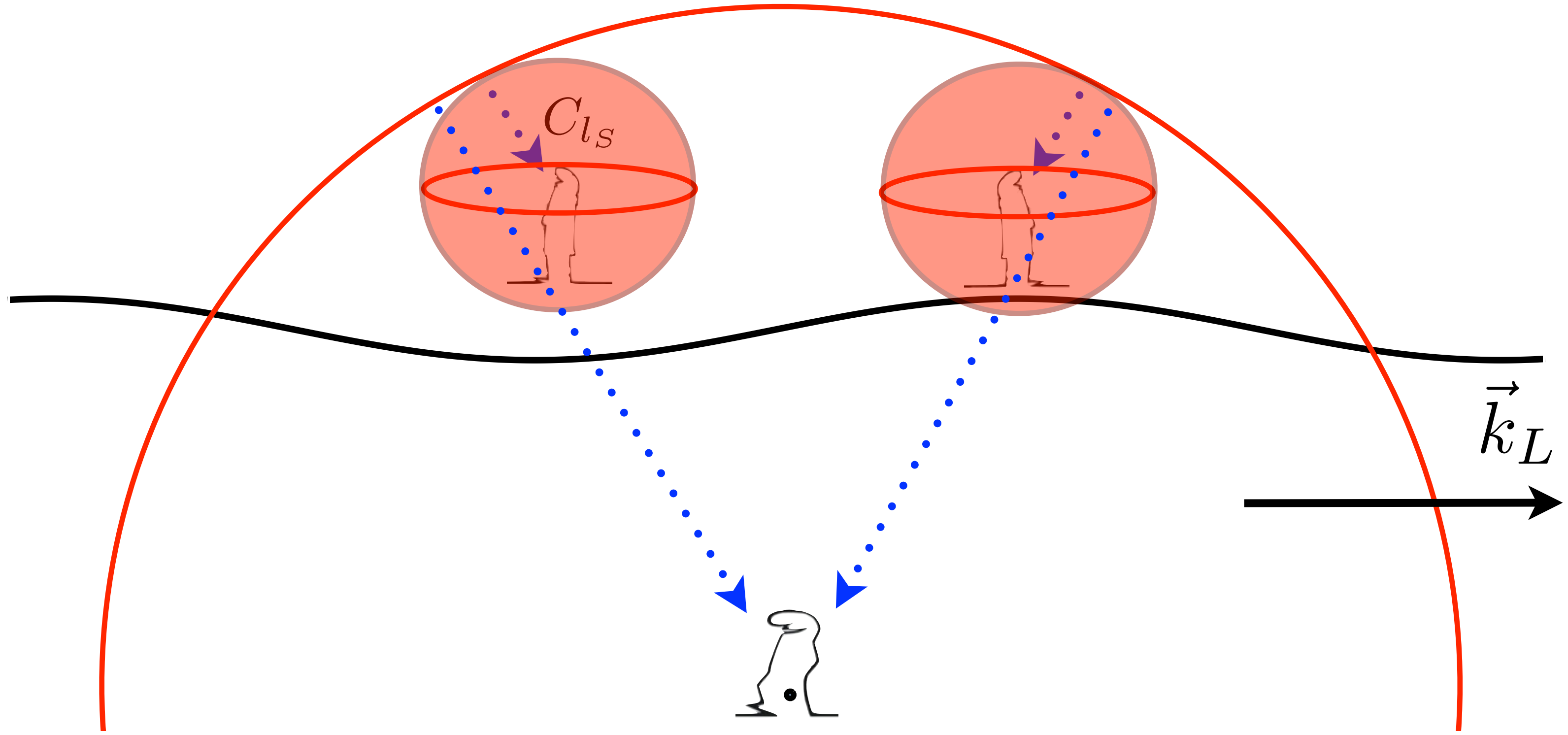
Physical argument

Creminelli, Zaldarriaga, '04

Single-field inflation: 1 clock, e.g. everything is determined by T.



Local physics is identical in Hubble patches that differ only by super-horizon modes: two observers in different places on LSS will see exactly the same CMB anisotropies (at given T).



The long mode is inside the horizon and I can compare different patches. Will see a modulation of the 2-point function due large scale T:

$$C_l \to C_l + \Theta_L \frac{d}{d\Theta_L} C_l \qquad \Theta \equiv \Delta T/T \qquad (\Theta_L = -\frac{1}{5}\zeta)$$

• Long mode changes the local average temperature: $T \to (1+\Theta_L)T$

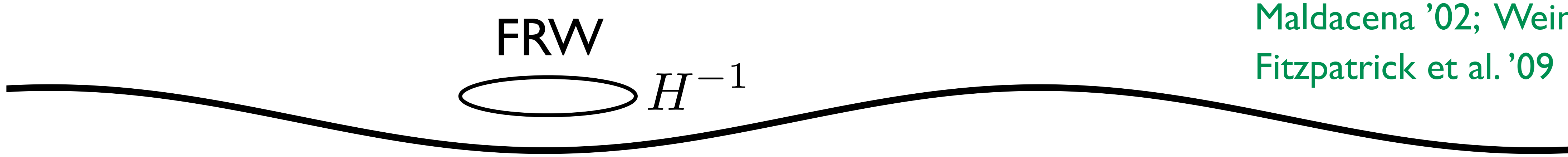
$$C_l \to C_l + 2\Theta_L C_l \quad \Rightarrow \quad B_{l_L l_S l_S} = 2 C_{l_L} C_{l_S} \qquad (f_{\rm NL}^{\rm loc} = -\frac{1}{6})$$

• Transverse rescaling of spatial coords ⇒ rescaling of angles:

$$C_l \to C_l - 5\Theta_L(\hat{n} \cdot \nabla_{\hat{n}} C_l) \quad \Rightarrow \quad B_{l_L l_S l_S} = 5 C_{l_L} C_{l_S} \frac{d \ln(l_S^2 C_{l_S})}{d \ln l_S}$$

• Lensing close to last scattering displaces the 2-p function

2^{nd}-order evolution as a coord change



Maldacena '02; Weinberg '03; Fitzpatrick et al. '09

Locally, possible to rewrite a perturbed FRW metric as an unperturbed one by reabsorbing the long mode with a coordinate transformation. Ex, in matter dominance:

$$ds^2 = a^2(\eta)\left[-(1+2\Phi_L)d\eta^2 + (1-2\Phi_L)dx^2\right] \quad \Rightarrow \quad ds^2 = a^2(\tilde{\eta})\left[-d\tilde{\eta}^2 + d\tilde{x}^2\right]$$

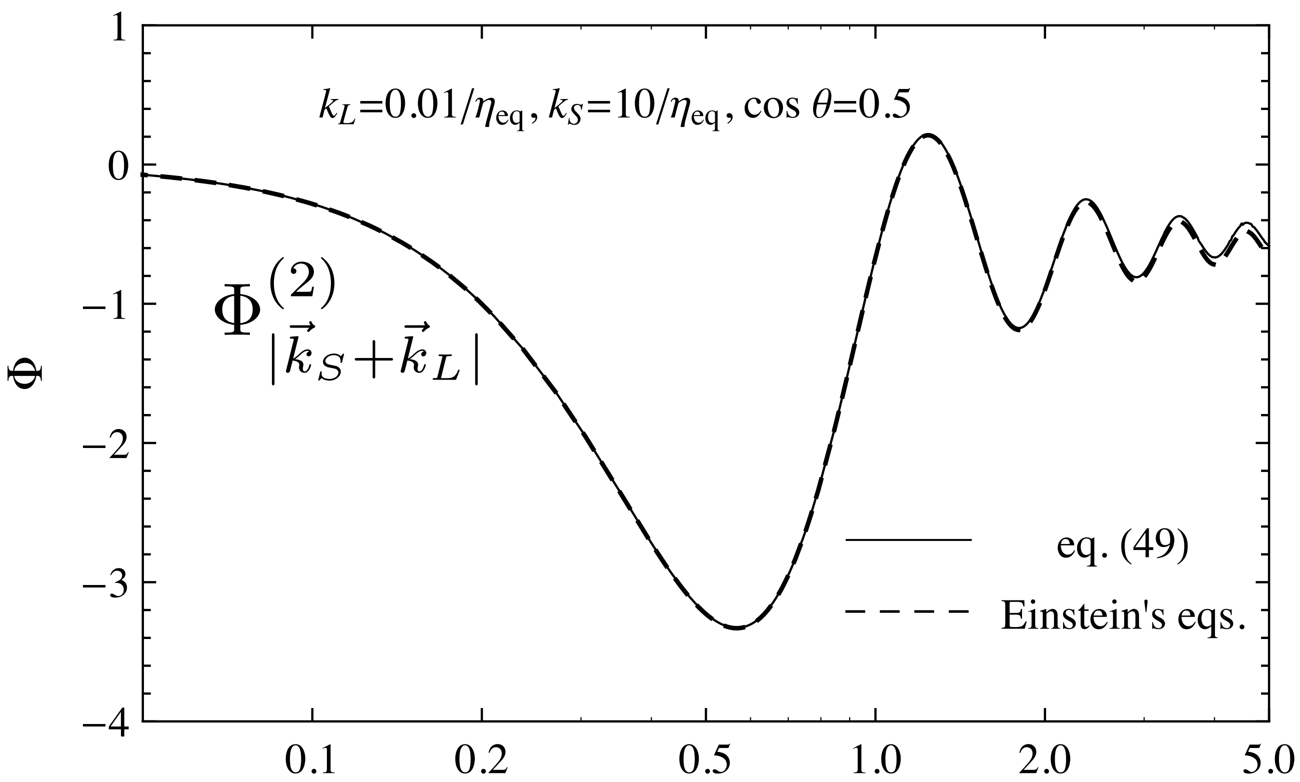
$$\tilde{\eta} = \eta(1+\Phi_L/3)$$
$$\tilde{x}^i = x^i(1-5\Phi_L/3) \qquad (\zeta = -\frac{5}{3}\Phi_L)$$

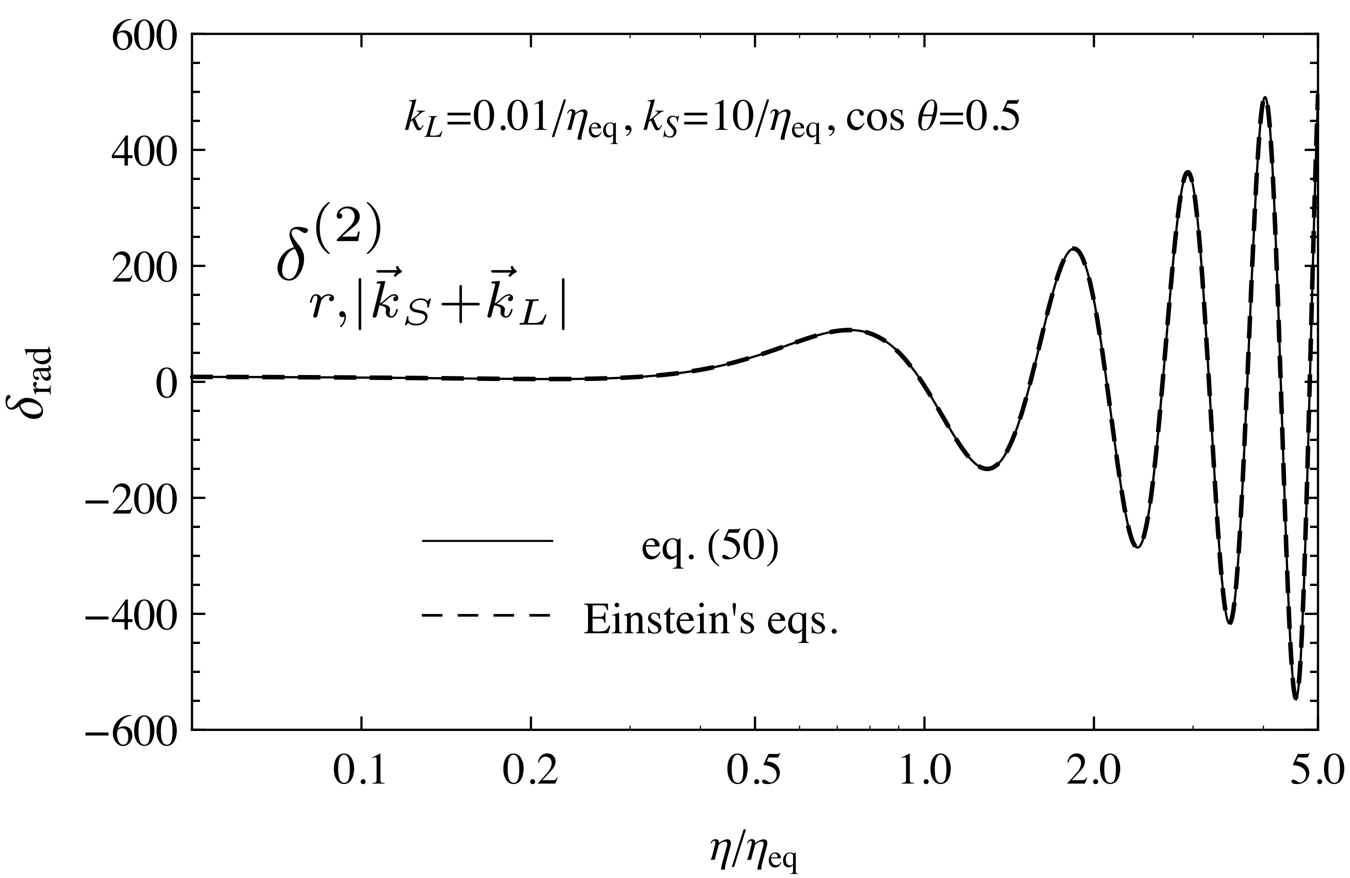
Conversely, start from a perturbed metric at 1^{st}-order and "generate" 2^{nd}-order couplings between short and long modes by the inverse coordinate transformation:

$$ds^2 = a^2(\tilde{\eta})\left[-(1+2\tilde{\Phi}_S)d\tilde{\eta}^2 + (1-2\tilde{\Psi}_S)d\tilde{x}^2\right] \Rightarrow ds^2 = a^2(\eta)\left[-e^{2\Phi}d\eta^2 + e^{2\Psi}dx^2\right]$$

$$\Phi = \tilde{\Phi}_S + \Phi_L + \frac{1}{3}\Phi_L\frac{\partial\tilde{\Phi}_S}{\partial\ln\eta} - \frac{5}{3}\Phi_L x^i\frac{\partial\tilde{\Phi}_S}{\partial x^i}$$

2nd-order evolution as a coord change





Example: radiation-to-matter transition:

- - - - - - Solution of Einstein's eqs. at 2nd order

——— Coordinate transformation:

$$\Phi_{k_S} \to \Phi_{k_S} - \frac{5}{3}\Phi_{k_L}\left(f(\eta)\frac{\partial\Phi_{k_S}}{\partial\ln\eta} - \frac{\partial\Phi_{k_S}}{\partial\ln k}\right)$$

$$f(\eta) = -\frac{20+15\alpha\eta+3\alpha^2\eta^2}{5(2+\alpha\eta)}$$

$$\alpha = (\sqrt{2}-1)/\eta_{eq}$$

$$\delta_{k_S} \to \delta_{k_S} - \frac{5}{3}\Phi_{k_L}\left(f(\eta)\frac{\partial\delta_{k_S}}{\partial\ln\eta} - \frac{\partial\delta_{k_S}}{\partial\ln k}\right)$$

Coordinate transformation on the CMB

Creminelli, Pitrou, FV '11

Apply this coordinate transformation to the observed CMB: $$\Theta_{\rm obs}(\hat{n}) = \frac{T_{\rm obs}(\hat{n}) - \langle T_{\rm obs}\rangle}{\langle T_{\rm obs}\rangle}$$

$$\Theta_{\rm obs} = [\Theta + \Phi - \hat{n}\cdot\vec{v}](\eta_{\rm rec}, \vec{x}_{\rm rec})$$

In pure matter dominance and instantaneous recombination

Coordinate transformation on the CMB

Creminelli, Pitrou, FV '11

Apply this coordinate transformation to the observed CMB: $\Theta_{\rm obs}(\hat n) = \frac{T_{\rm obs}(\hat n) - \langle T_{\rm obs}\rangle}{\langle T_{\rm obs}\rangle}$

$$\Theta_{\rm obs} = \Theta_{{\rm obs},S} + \Theta_{{\rm obs},L} + \Theta_{{\rm obs},L}\left(1 + \frac{\partial}{\partial \ln \eta_{\rm rec}} - 5\hat n\cdot\nabla_{\hat n}\right)\Theta_{{\rm obs},S}$$

$$\Theta_{{\rm obs},S} = [\Theta_S + \Phi_S - \hat n \cdot \vec v_S](\eta_{\rm rec}, \vec x_{\rm rec}) \qquad \Theta_{{\rm obs},L} = \frac{1}{3}\Phi_L(\eta_{\rm rec}, \vec x_{\rm rec})$$

Holds also when including radiation/matter transition (early Sachs-Wolfe) and finite recombination.

Check: a mode out of Hubble radius today is unobservable. Cancels out from this expression.

Time derivative is geometrically suppressed as $\sim \frac{\eta_{\rm rec}}{\eta_{\rm obs}}$

Bispectrum: $$B_{l_L l_S l_S} = C_{l_L} C_{l_S}\left(2 + 5\frac{d\ln(l_S^2 C_{l_S})}{d\ln l_S}\right)$$

See also Bartolo Matarrese Riotto '11; Lewis '12

Extension of the Maldacena relation. Cf: $B_{k_L k_S k_S} = -P_{k_L} P_{k_S}\frac{d\ln(k_S^3 P_{k_S})}{d\ln k_S}$

Final result

Coordinate and average temperature redefinition

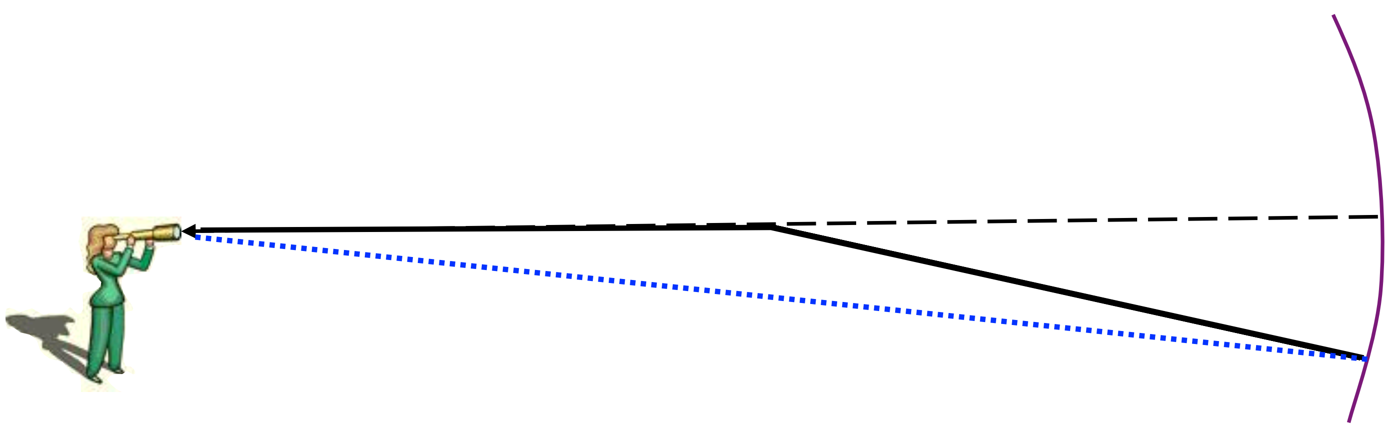
Lensing

$$B_{l_L l_S l_S} = C_{l_L} C_{l_S} \left(2 + 5 \frac{d \ln (l_S^2 C_{l_S})}{d \ln l_S} \right) + 6 C_{l_L} C_{l_S} \left[2 \cos 2\theta - (1 + \cos 2\theta) \frac{d \ln (l_S^2 C_{l_S})}{d \ln l_S} \right]$$

Boubekeur et al. '09

$$(\cos \theta = \hat{l}_L \cdot \hat{l}_S)$$

Lensing due to correlation between temperature at recombination and transverse displacement:



$$\delta \vec{x}^{\perp} = -2 \int_{\eta_{\rm rec}}^{\eta_0} (1 - \frac{\eta_{\rm rec}}{\eta}) \vec{\nabla}_{\hat{n}} \Phi(\vec{x}) d\eta$$

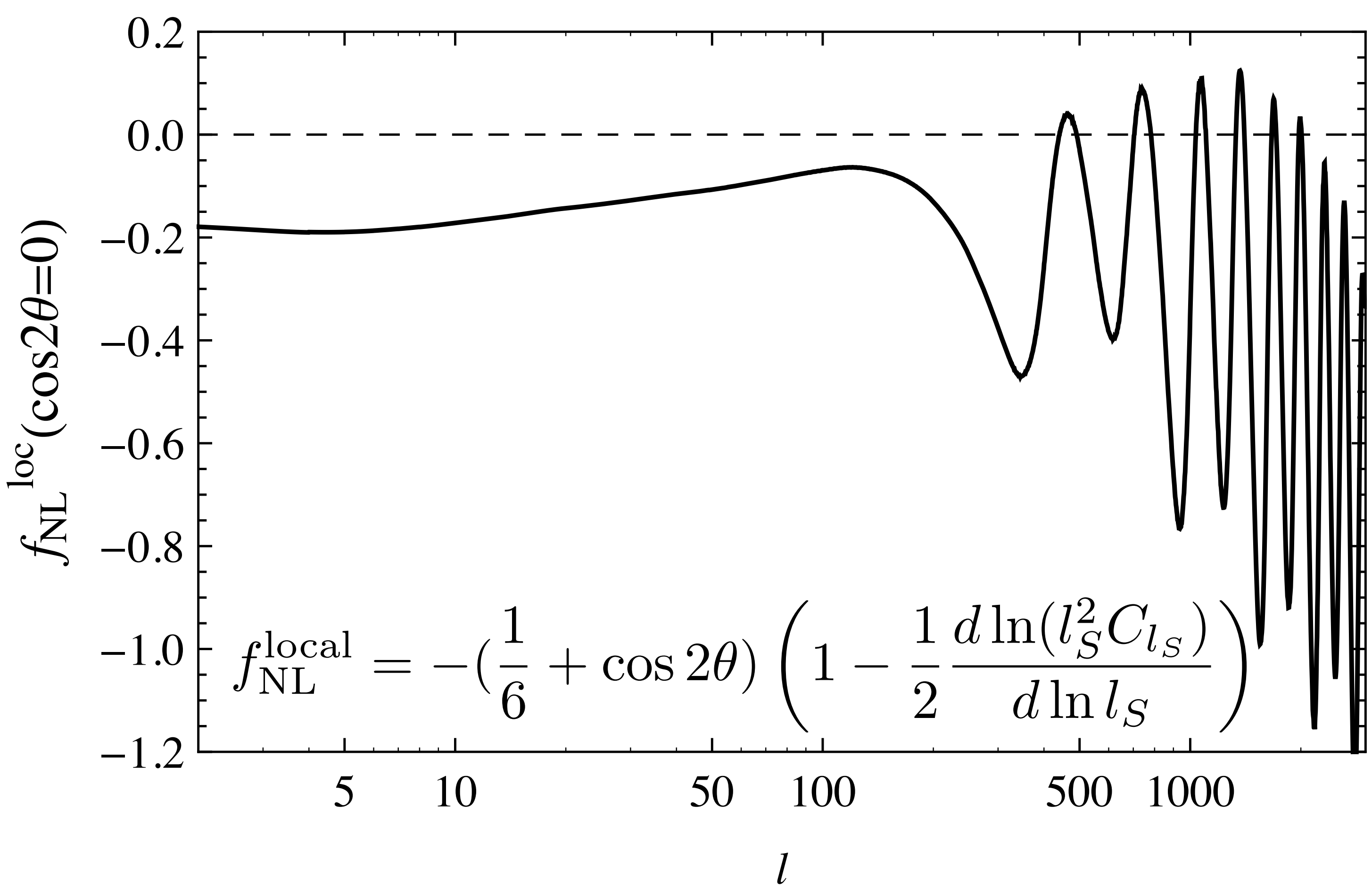
$$\Theta_{\rm obs}^{\rm lensed} = \Theta_{{\rm obs},S}(\eta_{\rm rec}, \vec{x}_* + \delta \vec{x}^{\perp}) = \Theta_{{\rm obs},S}(\eta_*, \vec{x}_*) + \delta \vec{x}^{\perp} \cdot \vec{\nabla} \Theta_{{\rm obs},S}(\eta_*, \vec{x}_*)$$

Partial cancellation between (isotropic) lensing convergence and space redefinition. Overdense regions (negative potential) give positive convergence, moving the spectrum towards larger angles, while coordinate redefinition shrinks it.

Contamination

Creminelli, Pitrou, FV '11

$$B_{l_L l_S l_S} = C_{l_L} C_{l_S} (1 + 6\cos 2\theta)\left(2 - \frac{d\ln(l_S^2 C_{l_S})}{d\ln l_S}\right)$$



Integrating only in the squeezed limit $l_L \lesssim 60$:

$f_{NL}^{loc} = -0.39\ , \qquad l_{max} = 2000$ See also Bartolo, Riotto '11

Other effects at $l_L \gtrsim 200$ Khatri, Wandelt '08; Senatore, Tassev, Zaldarriaga '09

Seemingly negligible contamination to f_{NL}^{loc}

From Lewis '12	
Data used	$\sigma_{f_{NL}}$
T	4.3
Planck T	5.9
T ($l_1 < 60$)	4.6
Planck T ($l_1 < 60$)	6.2
T+E	2.1
Planck T+E	5.2

Boltzmann code: CMBquick

This relation can be used as consistency check of Boltzmann codes based on a physical limit

There have been several contributions to development of Boltzmann numerical code at 2nd order.

Bartolo, Matarrese, Riotto '06; Bernardeau, Pitrou, Uzan '08; Pitrou '08; Bartolo, Riotto '08; Khatri, Wandelt '08; Senatore, Tassev, Zaldarriaga '09; Nitta et al. '09, Beneke and Fidler '10

One of the most complete code is Pitrou's CMBquick (see also next talk):

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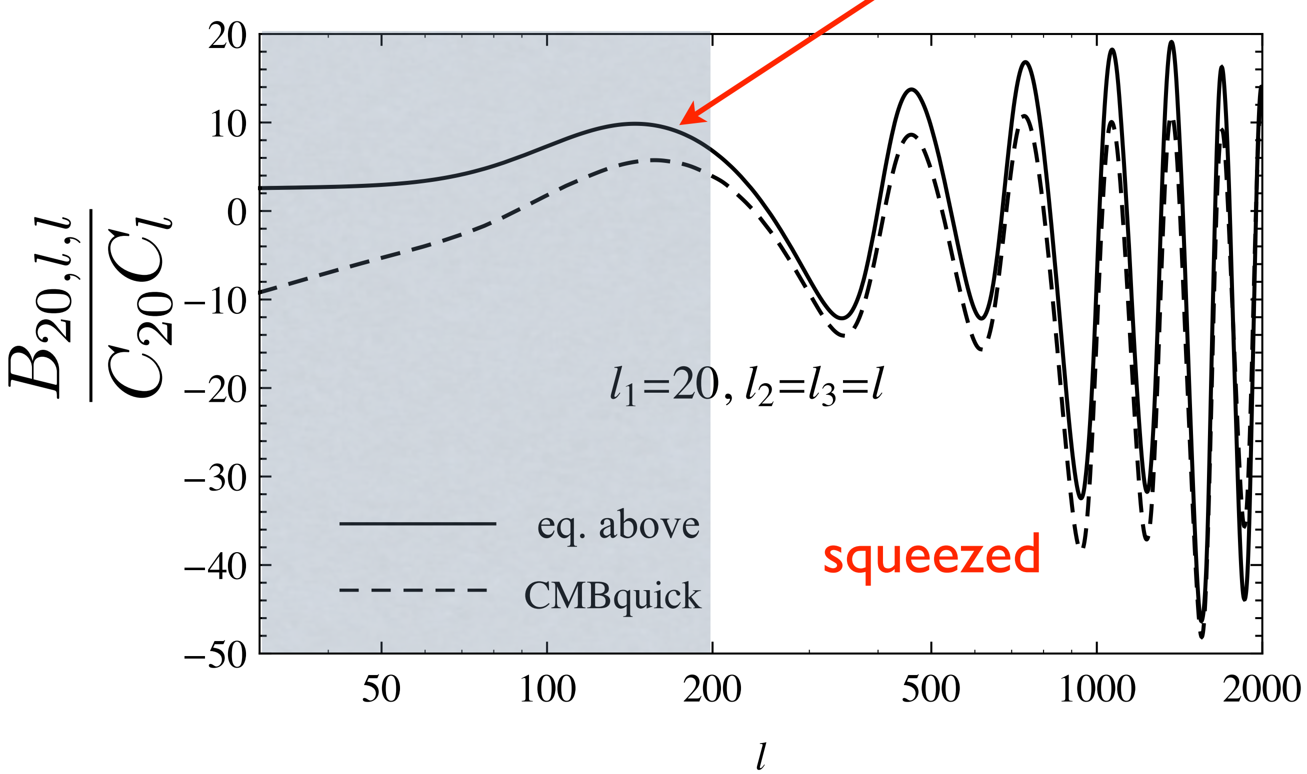
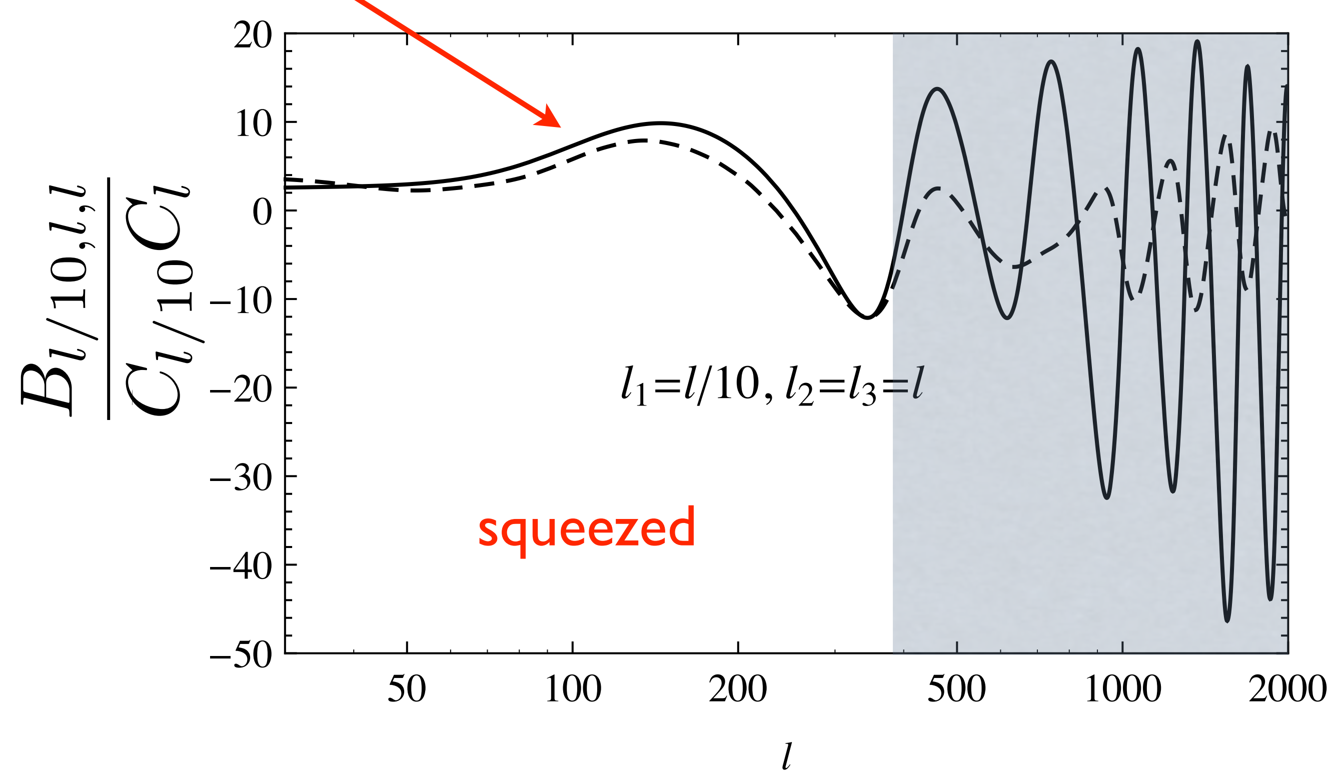
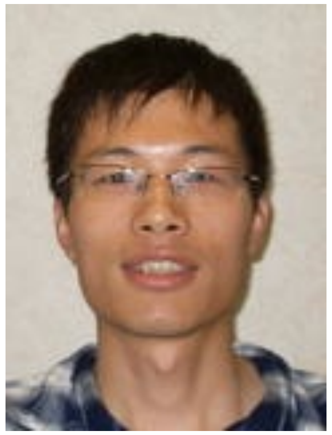
$$B_{l_L l_S l_S} = C_{l_L} C_{l_S} \left(2 + 5 \frac{d \ln (l_S^2 C_{l_S})}{d \ln l_S} \right)$$





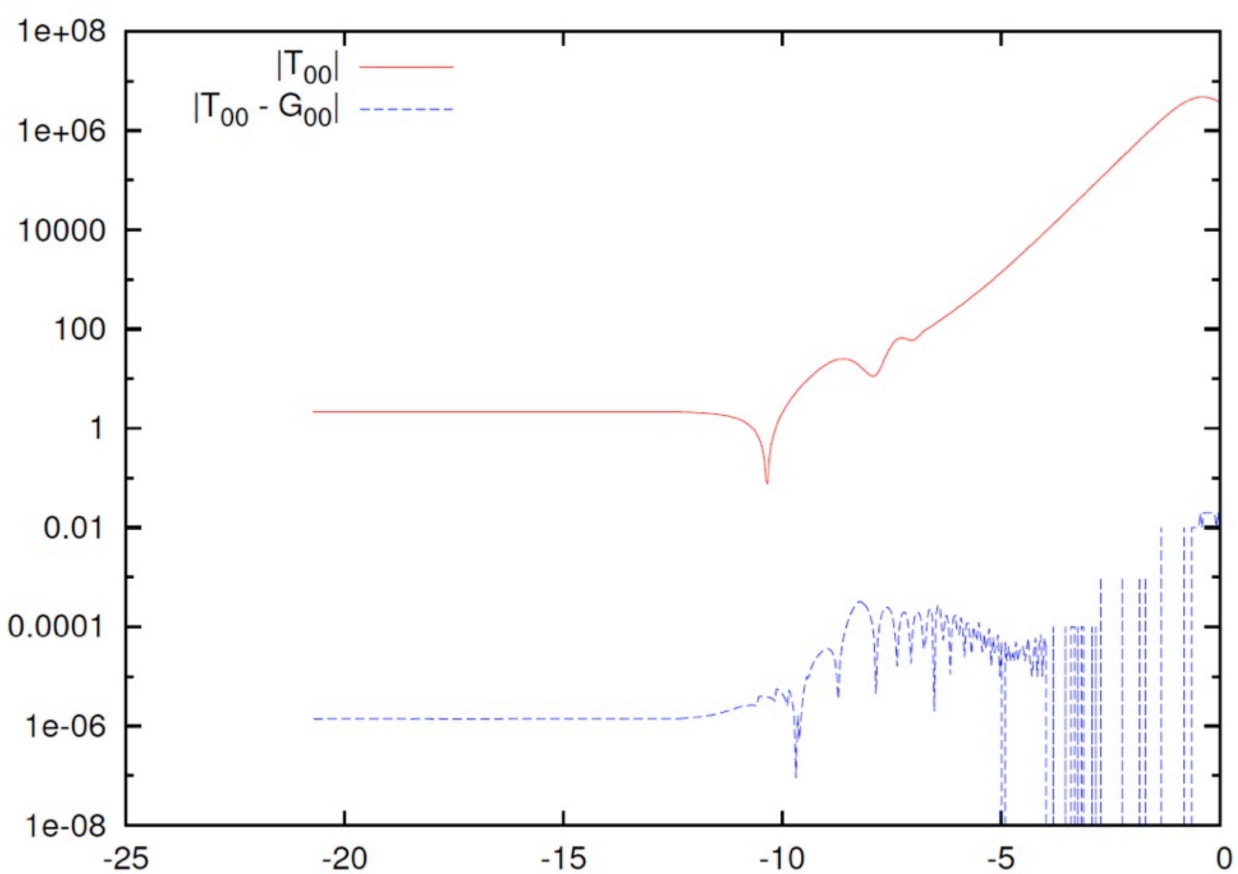
The check is nontrivial! Even though analytically the squeezed limit is easy, in the code all 2nd-order effects must conspire to reproduce the simple analytical formula.

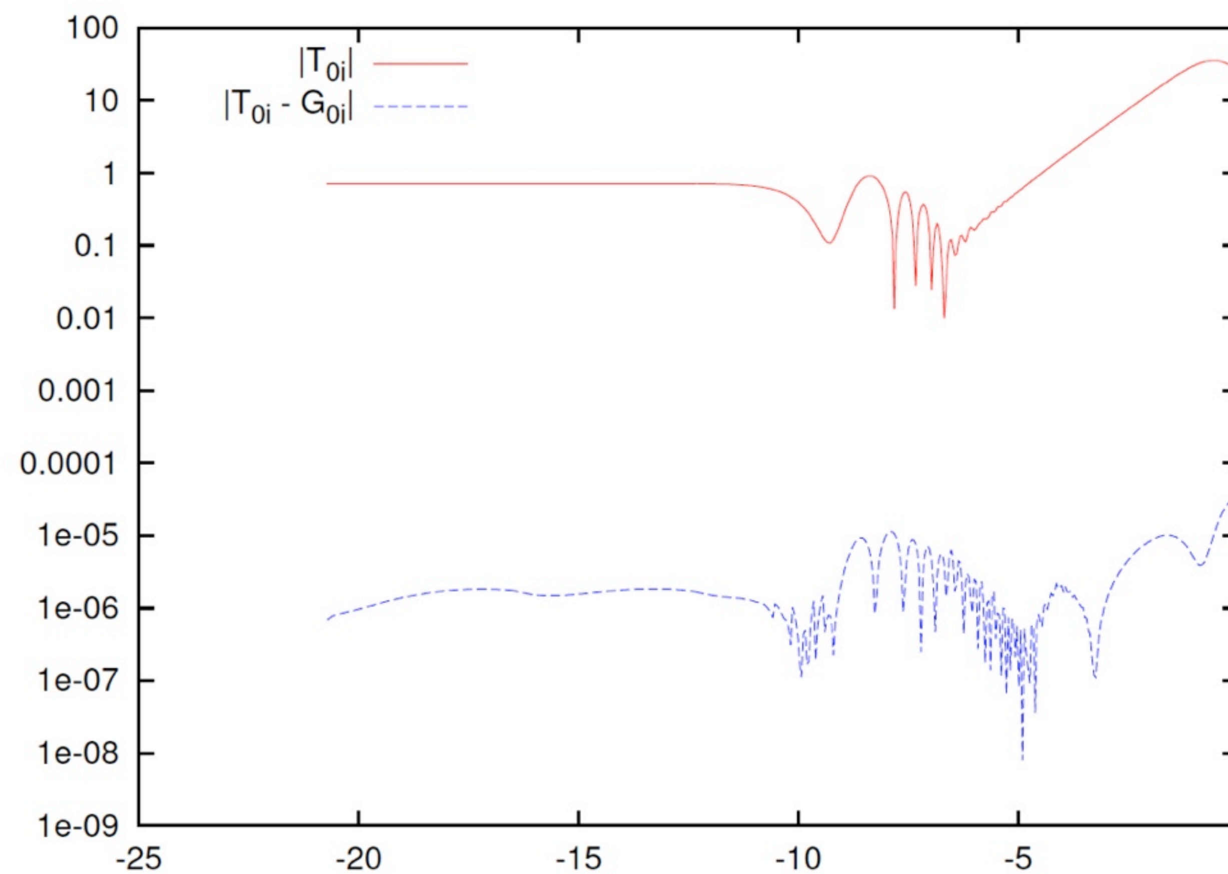
New Boltzmann code: CosmoLib++

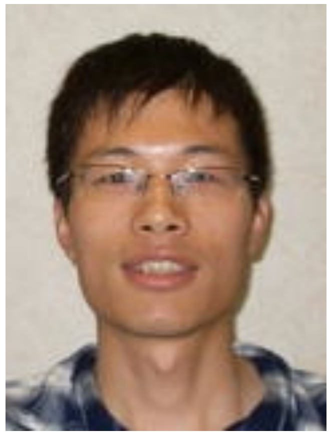


by Zhiqi Huang

	CMBquick	CosmoLib++
language	Mathematica	Fortran (mixed with C)
multipoles treatment	flat-sky approximation	full-sky exact
Boltzmann code squeezed limit check	✓	✓
Einstein constraints check	relative error < 0.1	relative error < 10^{-6}







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Results will appear soon!

Conclusion

In the squeezed limit (one mode longer than horizon at recombination), it is possible to compute the CMB bispectrum exactly.

Valid for adiabatic (single clock) perturbations. Already takes into account NG from single-field models. It is a consistency relation on the observable (CMB temperature) in the squeezed limit.

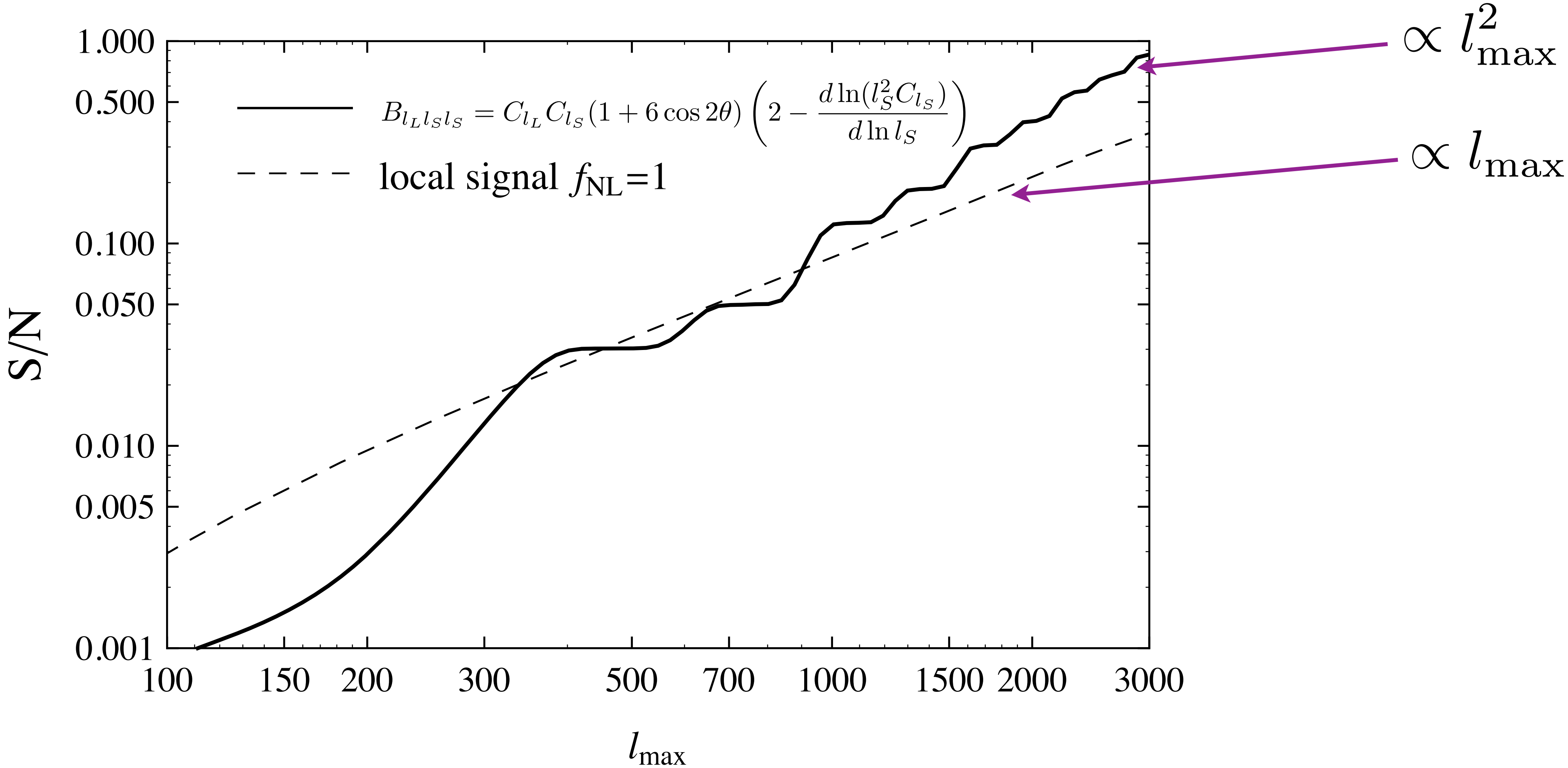
- Planck will (most likely) not be biased by 2^{nd}-order effects at recombination (may be detectable).

- Test Boltzmann codes at 2^{nd} order. Reasonable agreement with Pitrou's CMBquick code. Need of better codes: CosmoLib++ is including 2^{nd}-order perturbations and bispectrum computation.

Observability

Can we observe this signal? Signal-to-noise ratio is:

$$\left(\frac{S}{N}\right)^2 = \frac{1}{\pi}\int \frac{d^2l_2 d^2l_3}{(2\pi)^2}\frac{[B(l_1,l_2,l_3)]^2}{6C_{l_1}C_{l_2}C_{l_3}}$$



We are integrating only in the squeezed limit and we do not include polarization. Boltzmann code would give a better estimate. Possibly measurable effect.