

# The halo bispectrum in N-body simulations with non-Gaussian initial conditions

Critical Tests of Inflation Using Non-Gaussianity  
@ MPA, Munich - November 7th, 2012

Emiliano SEFUSATTI - 

*in collaboration with Martin Crocce, Vincent Desjacques (ArXiv:1003.0007, ArXiv: 1111.6966)  
+ Dani Figueroa, Toni Riotto & Filippo Vernizzi (ArXiv: 1205.2015)*

# The **Galaxy Bispectrum**: *NG and nonlinear bias*

The galaxy bispectrum at large scales

$$B_g(k_1, k_2, k_3) = b_1^3 B(k_1, k_2, k_3) + b_1^2 b_2 P(k_1) P(k_2) + 2 \text{ perm.} + \dots$$

# The Galaxy Bispectrum: NG and nonlinear bias

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$$B = B_0 + B_G^{tree}[P_0] + B_G^{loop}[P_0] + B_{NG}^{loop}[P_0, B_0]$$

primordial component  
(large scales)

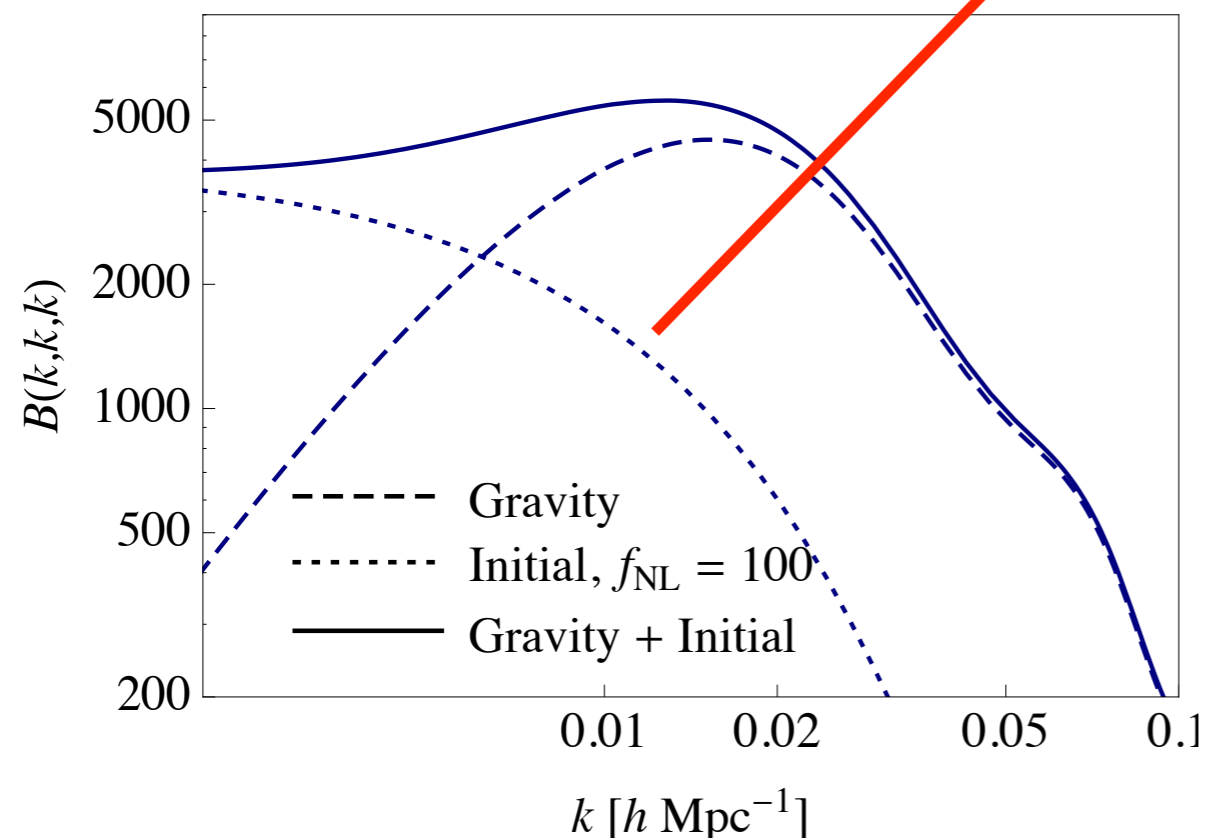
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primordial component  
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$$\frac{B_0(k, k, k)}{B_G^{tree}(k, k, k)} \underset{k \rightarrow 0}{\sim} \frac{f_{NL}}{D(z)k^2}$$

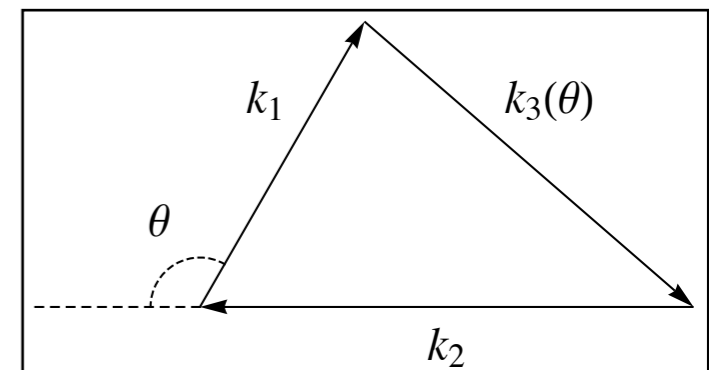
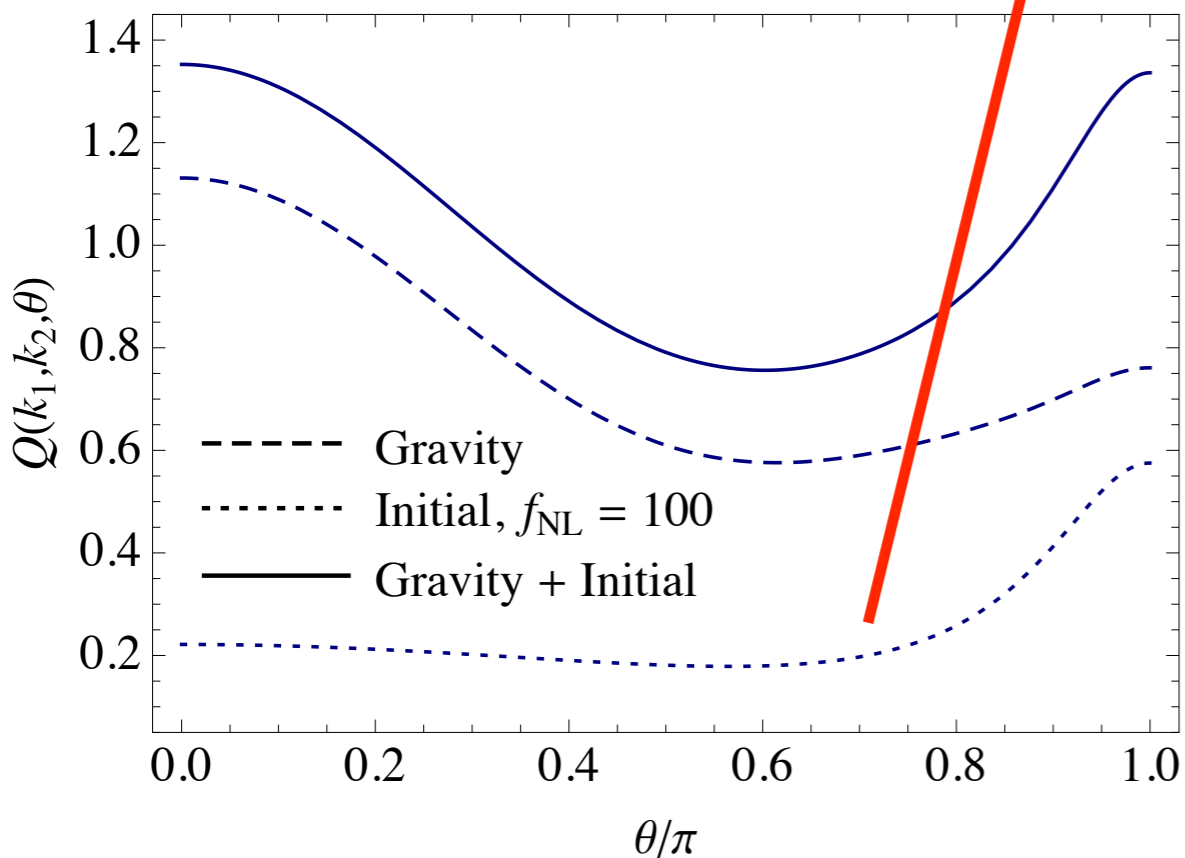
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primordial component  
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$$Q(k_1, k_2, k_3) = \frac{B(k_1, k_2, k_3)}{P(k_1)P(k_2) + P(k_1)P(k_3) + P(k_2)P(k_3)}$$

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$$B_g(k_1, k_2, k_3) = b_1^3 B(k_1, k_2, k_3) + b_1^2 b_2 P(k_1) P(k_2) + 2 \text{ perm.} + \dots$$


$$B = B_0 + B_G^{tree}[P_0] + B_G^{loop}[P_0] + B_{NG}^{loop}[P_0, B_0]$$

primordial component  
(large scales)

If  $B_0$  was the *only effect* of NG initial conditions on the LSS then future, large volume surveys ( $\sim 100 \text{ Gpc}^3$ ) could provide:

$$\Delta f_{\text{NL}}^{\text{local}} < 5 \text{ and } \Delta f_{\text{NL}}^{\text{eq}} < 10$$

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The galaxy bispectrum at large scales

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primordial component  
(large scales)

effect on nonlinear  
evolution (small scales)

# The Galaxy Bispectrum: $NG$ and nonlinear bias

The galaxy bispectrum at large scales

$$B_g(k_1, k_2, k_3) = b_{1,G}^3 B(k_1, k_2, k_3) + b_{1,G}^2 b_{2,G} P(k_1) P(k_2) + 2 \text{ perm.} + \dots$$

$b_{1,G} + \Delta b_{1,NG}(f_{NL}, k)$        $b_{2,G} + \Delta b_{2,NG}(f_{NL}, \vec{k}_1, \vec{k}_2)$       bias corrections

$P = P_0 + P_G^{loop}[P_0] + P_{NG}^{loop}[P_0, B_0]$

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primordial component  
(large scales)

effect on nonlinear  
evolution (small scales)

$$\Delta b_{1,NG}(f_{NL}, \vec{k}) = \Delta b_{1,si}(f_{NL}) + \Delta b_{1,sd}(f_{NL}, b_{1,G}, \vec{k})$$

$$\Delta b_{2,NG}(f_{NL}, \vec{k}_1, \vec{k}_2) = \Delta b_{2,si}(f_{NL}) + \Delta b_{2,sd}(f_{NL}, b_{1,G}, b_{2,G}, \vec{k}_1, \vec{k}_2)$$

Giannantonio & Porciani (2010)  
Baldauf, Seljak & Senatore (2010)

# The Galaxy Bispectrum: NG and nonlinear bias

The galaxy bispectrum at large scales

$$B_g(k_1, k_2, k_3) = \overset{b_{1,G} + \Delta b_{1,NG}(f_{NL}, k)}{\uparrow} b_1^3 B(k_1, k_2, k_3) + \overset{b_{2,G} + \Delta b_{2,NG}(f_{NL}, \vec{k}_1, \vec{k}_2)}{\uparrow} b_1^2 b_2 P(k_1) P(k_2) + 2 \text{ perm.} + \dots$$

**bias corrections**

$$P = P_0 + P_G^{loop}[P_0] + P_{NG}^{loop}[P_0, B_0]$$

$$B = B_0 + B_G^{tree}[P_0] + B_G^{loop}[P_0] + B_{NG}^{loop}[P_0, B_0]$$

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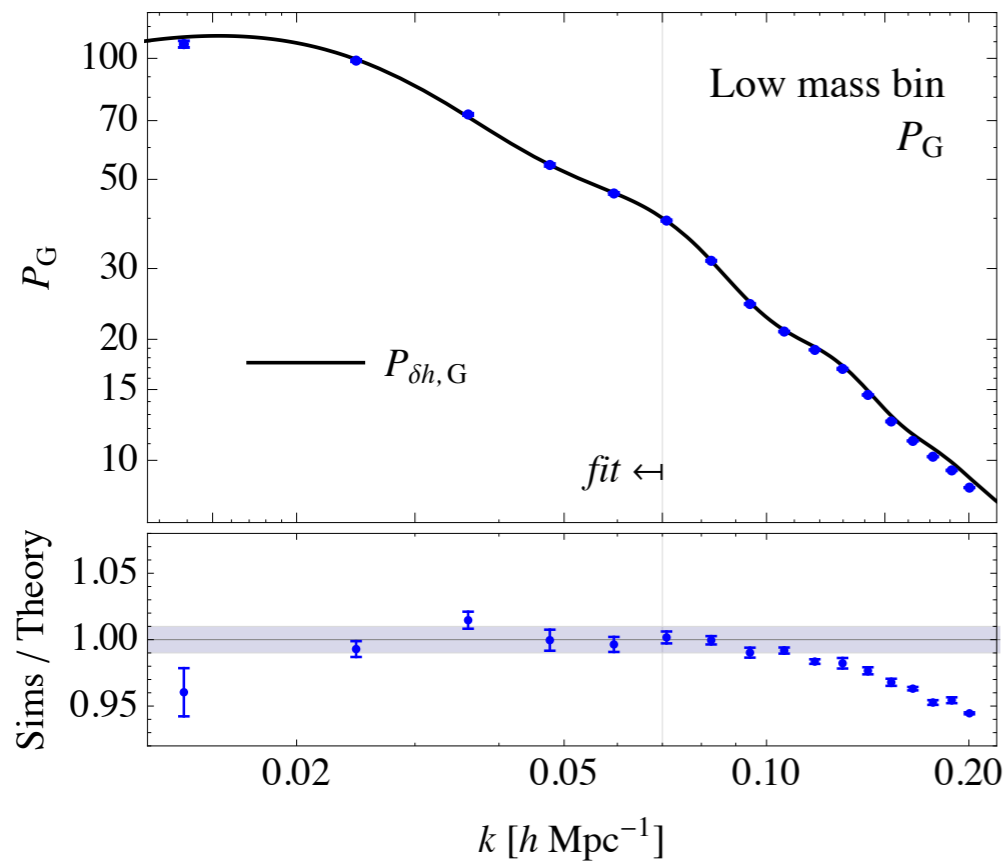
$$\Delta b_{2,sd,b}(k_1, k_2, f_{NL}) = 2 f_{NL} \delta_c \left[ b_{2,G} + \left( \frac{13}{21} - \frac{1}{\delta_c} \right) (b_{1,G} - 1) \right] \left[ \frac{1}{M(k_1, z)} + \frac{1}{M(k_2, z)} \right]$$

We test this model in N-body simulations with  
local NG initial conditions

$$\langle \delta \delta \delta_h \rangle = \delta_D(\vec{k}_{123}) B_{mmh}$$

$$\langle \delta_h \delta_h \delta_h \rangle = \delta_D(\vec{k}_{123}) B_h$$

# The Halo Bispectrum: *theory vs. simulations*



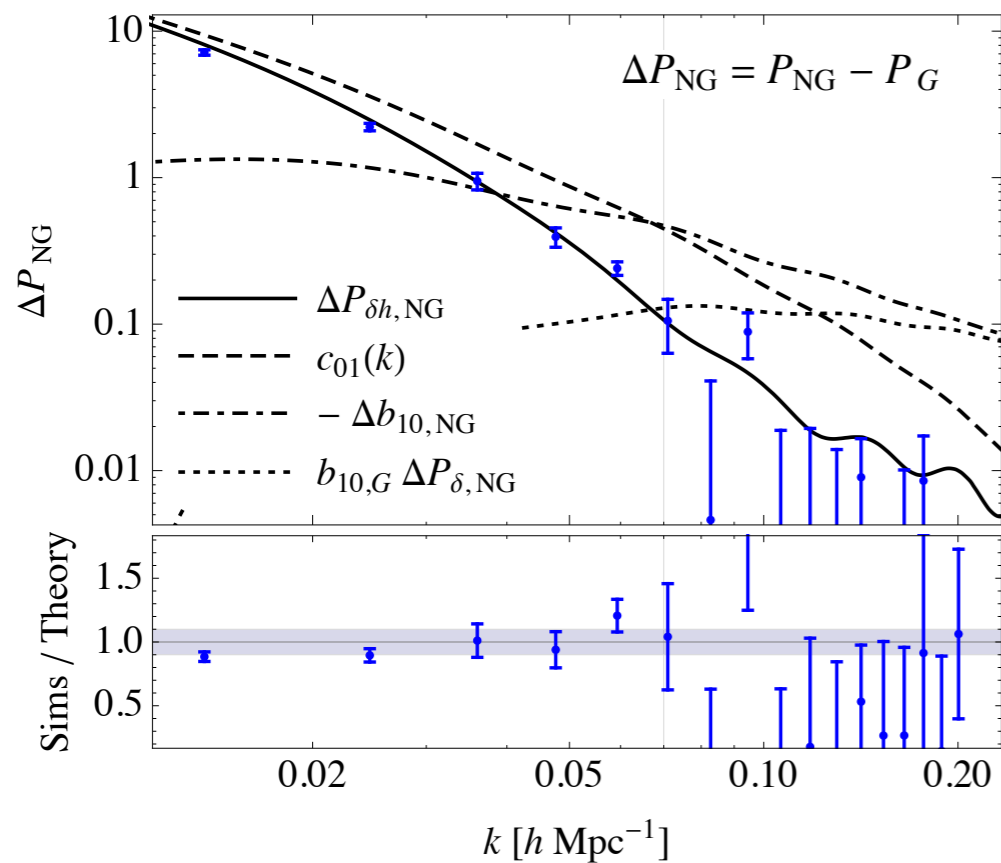
$$B_{mmh} = b_1 B + b_2 P P + \text{perm.}$$

$$b_1 = b_{1,G} + \Delta b_{1,si} + \Delta b_{1,sd}(b_{1,G}, \vec{k})$$

$$b_2 = b_{2,G} + \Delta b_{2,si} + \Delta b_{2,sd}(b_{1,G}, b_{2,G}, \vec{k}_1, \vec{k}_2)$$

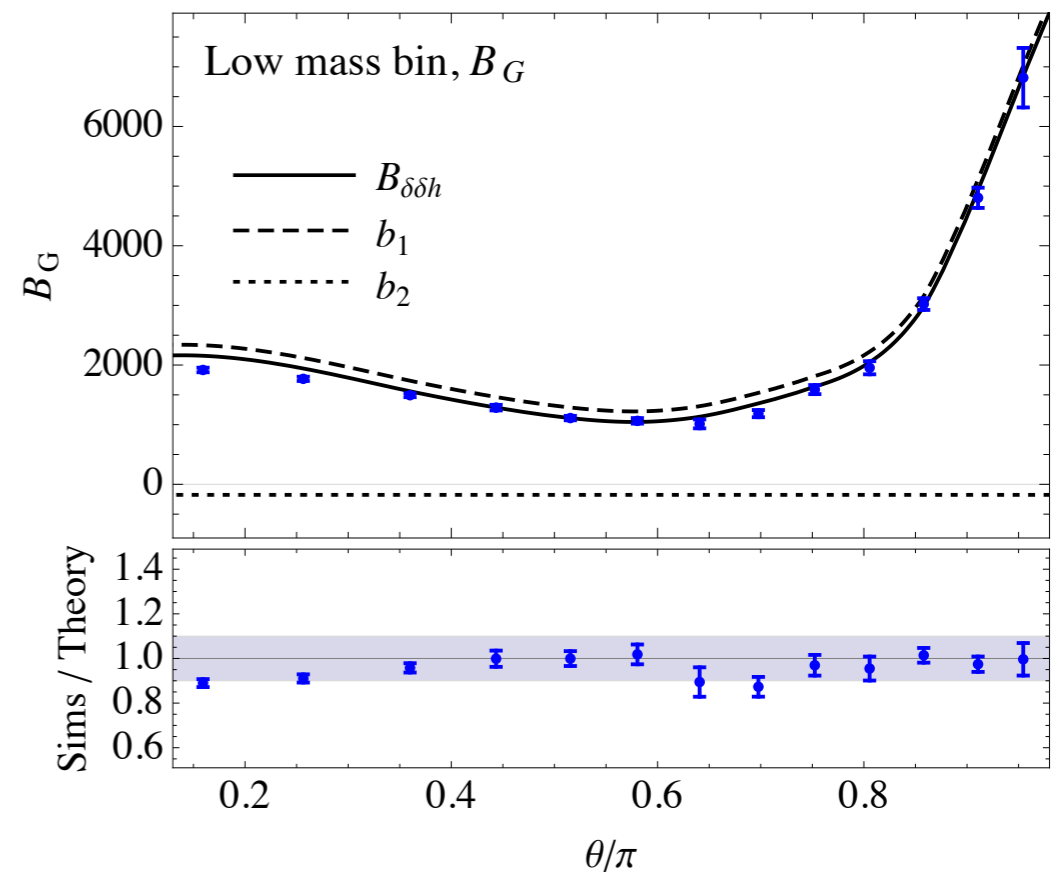
$\rightarrow b_1$

We fit for  $b_{1,G}$ ,  $b_{2,G}$ ,  $\Delta b_{1,si}$  and  $\Delta b_{2,si}$  **all** triangular configurations up to  $k = 0.07 \text{ h/Mpc}$



$\rightarrow \Delta b_{1,si}$

$b_2 \leftarrow$



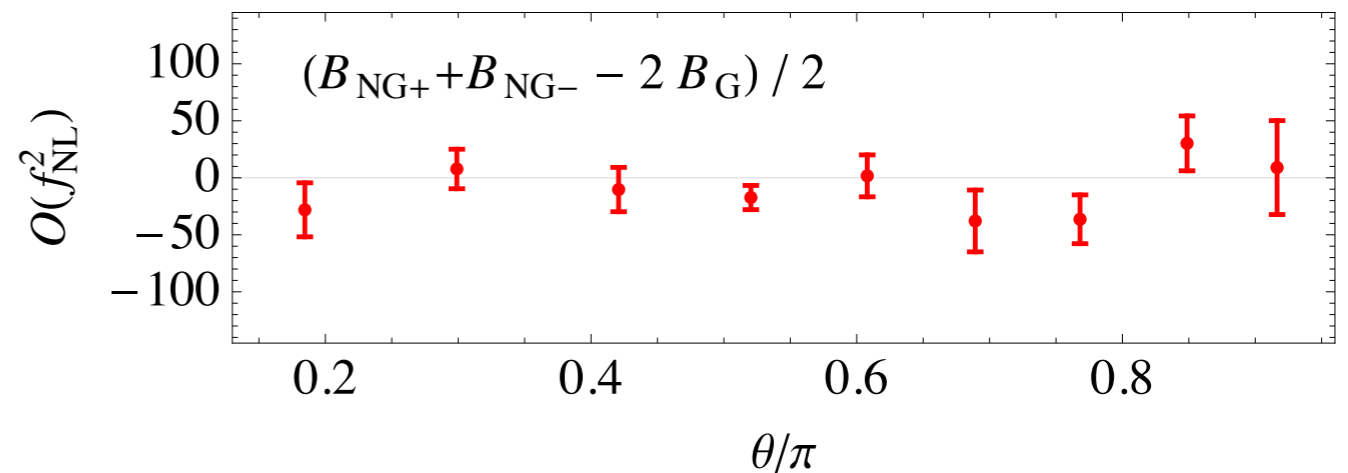
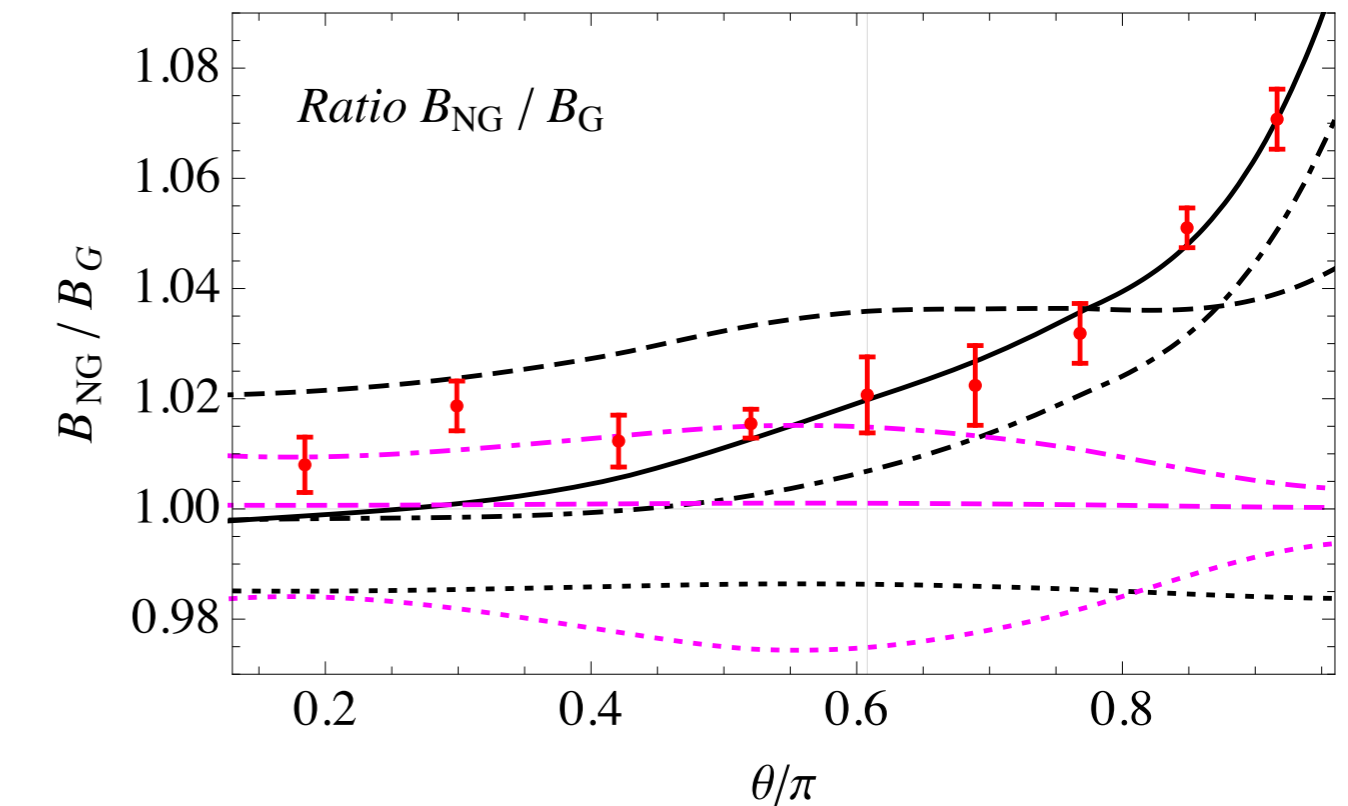
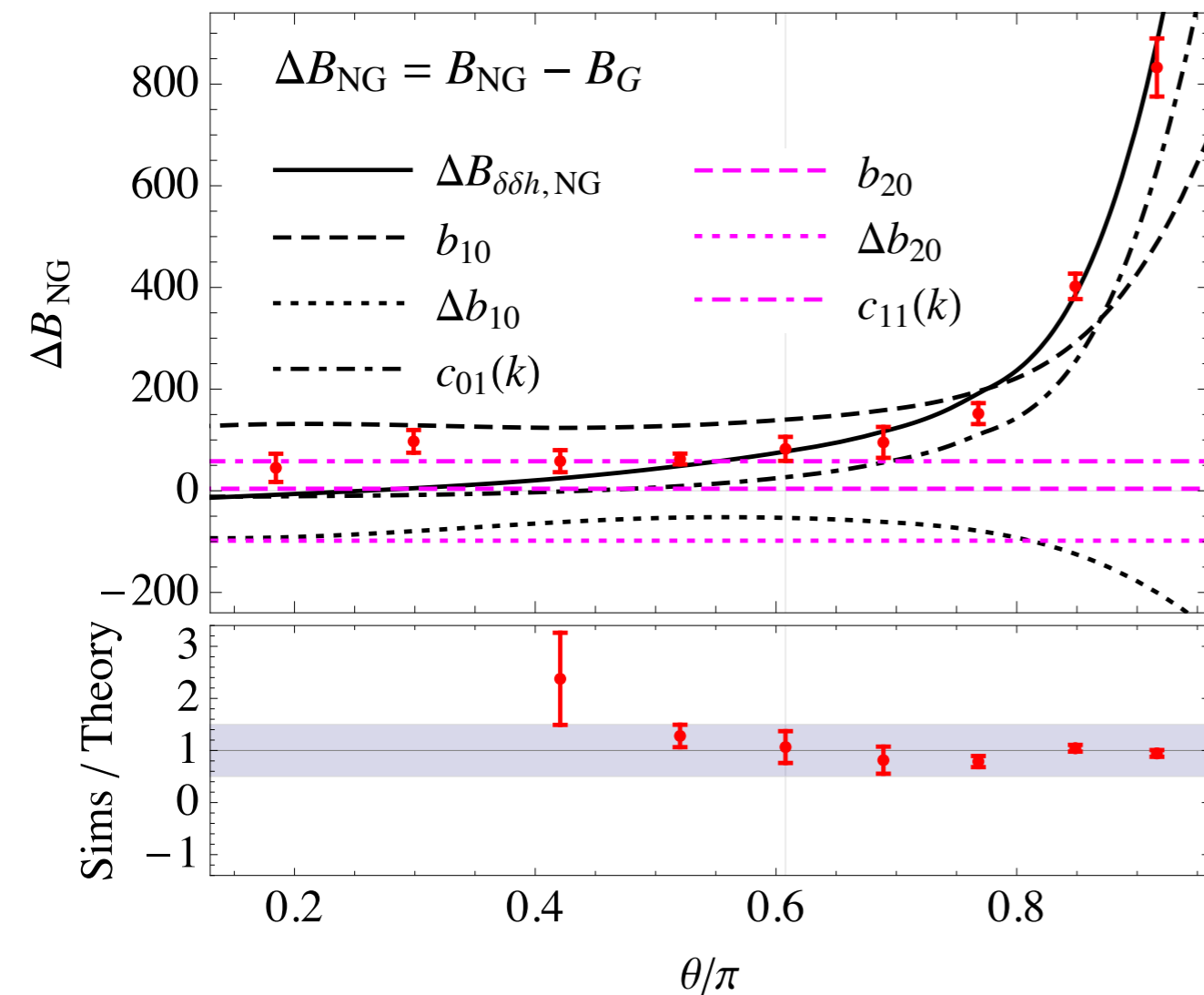
# The Halo Bispectrum: *theory vs. simulations*

## Matter-matter-halo bispectrum:

$$B_{mmh}(k_1, k_2; k_3) = b_1(f_{NL}, k) B(k_1, k_2, k_3) + b_2(f_{NL}, k_1, k_2) P(k_1) P(k_2)$$

$B(k_1, k_2, \theta)$  as a function of  $\theta$  with  $k_1 = 0.05 \text{ h/Mpc}$ ,  $k_2 = 0.07 \text{ h/Mpc}$

$M > 1.6 \times 10^{13} h^{-1} M_\odot$



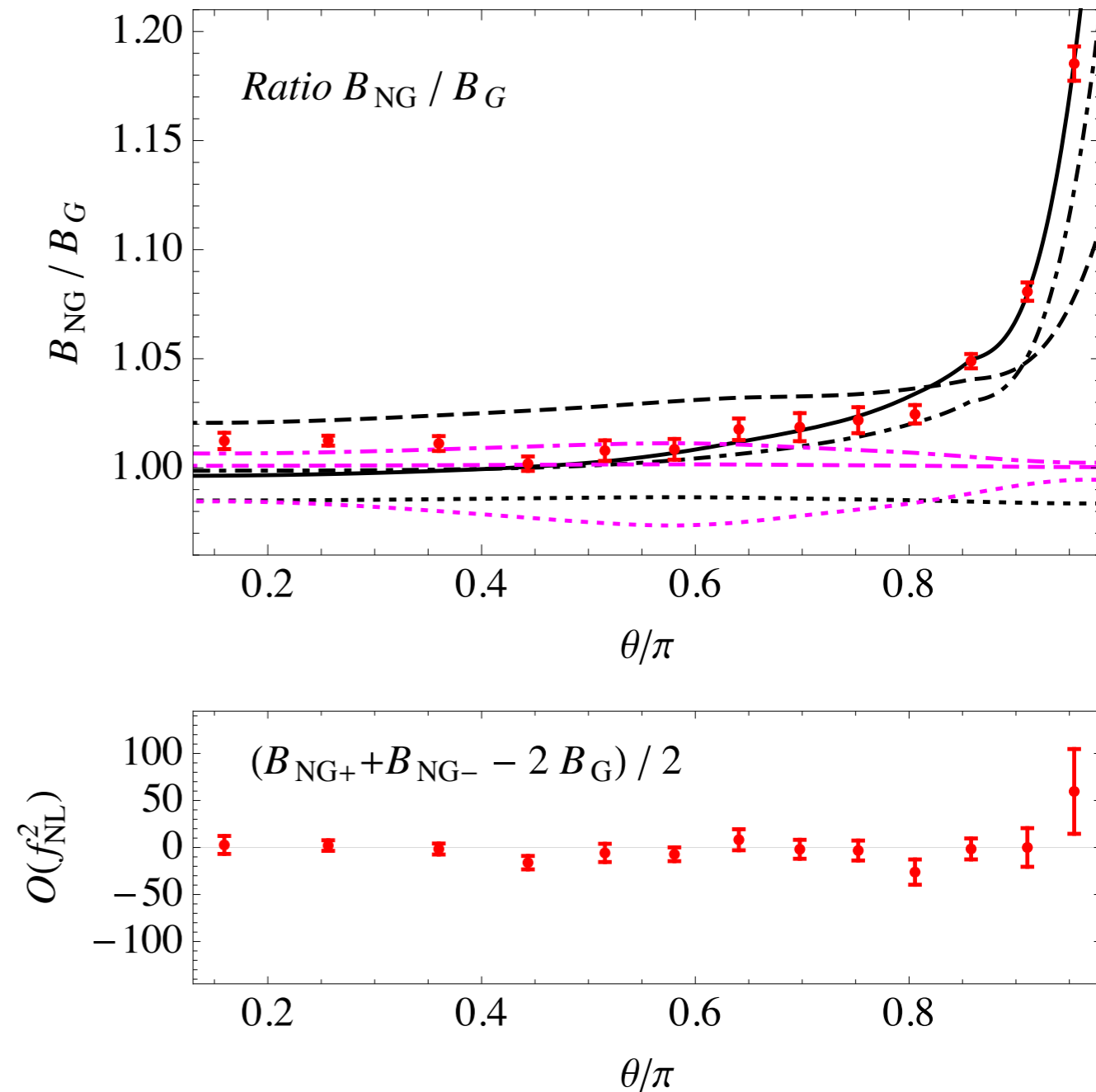
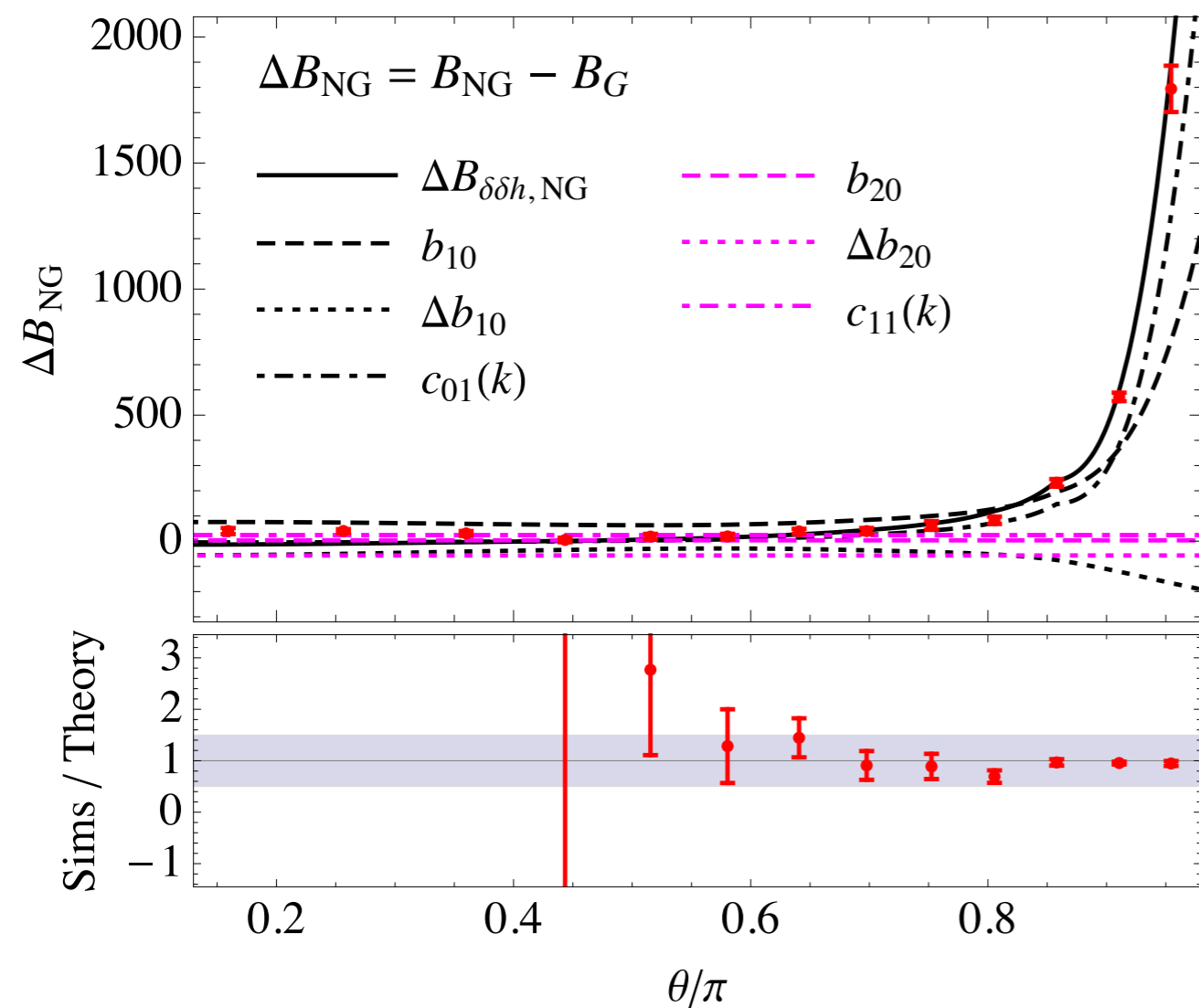
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$B(k_1, k_2, \theta)$  as a function of  $\theta$  with  $k_1 = 0.07 h/\text{Mpc}$ ,  $k_2 = 0.08 h/\text{Mpc}$

$M > 1.6 \times 10^{13} h^{-1} M_\odot$



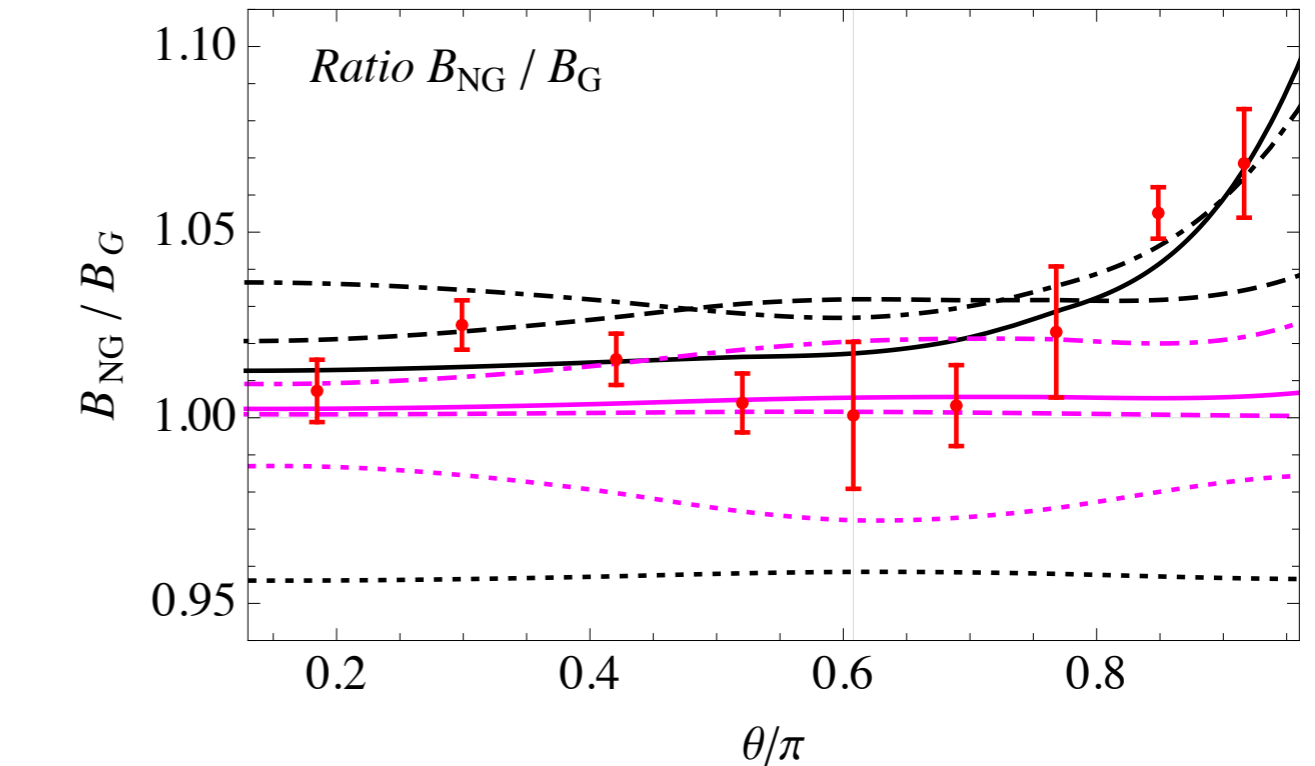
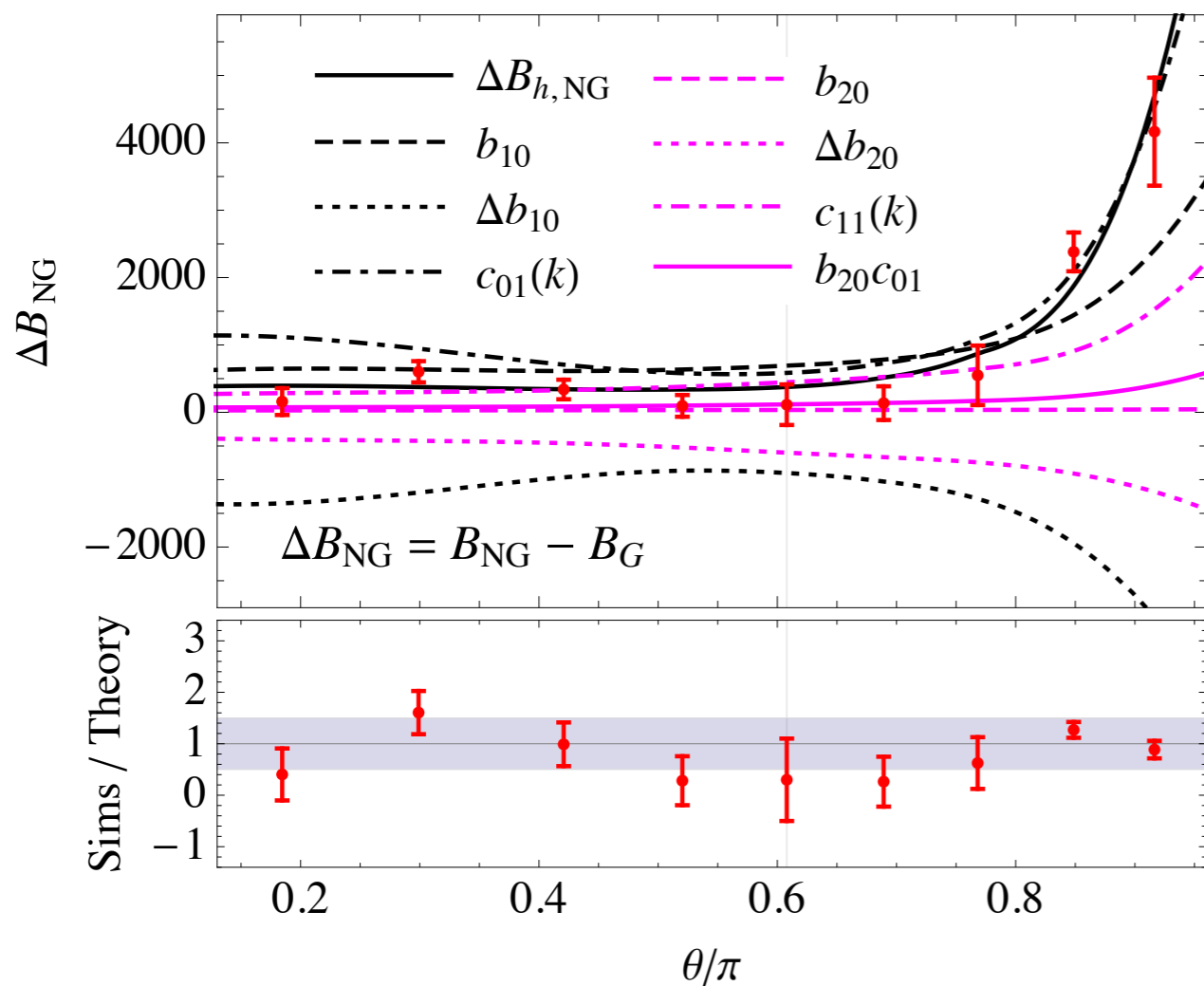
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$B(k_1, k_2, \theta)$  as a function of  $\theta$  with  $k_1 = 0.05 h/\text{Mpc}$ ,  $k_2 = 0.07 h/\text{Mpc}$

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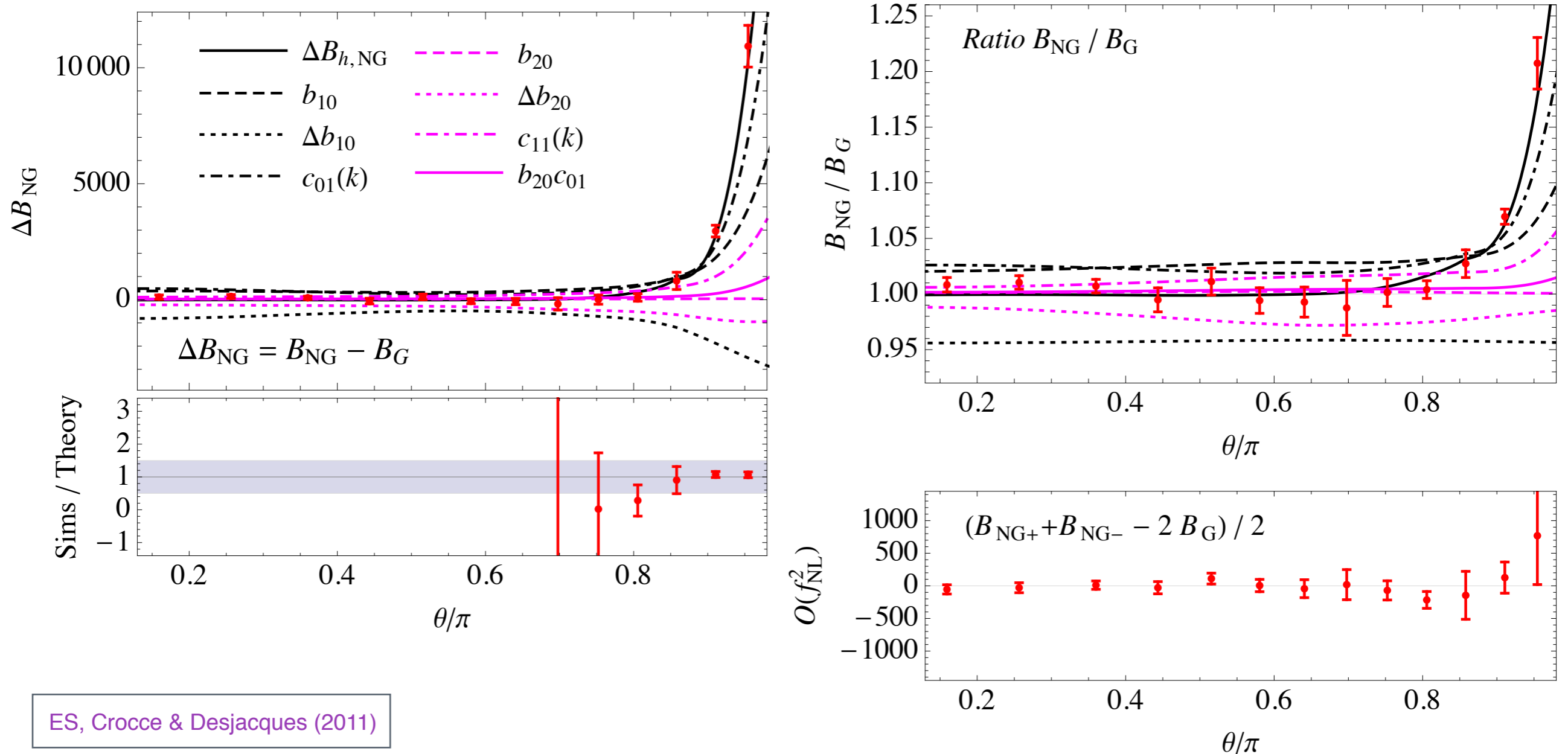
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$B(k_1, k_2, \theta)$  as a function of  $\theta$  with  $k_1 = 0.07 h/\text{Mpc}$ ,  $k_2 = 0.08 h/\text{Mpc}$

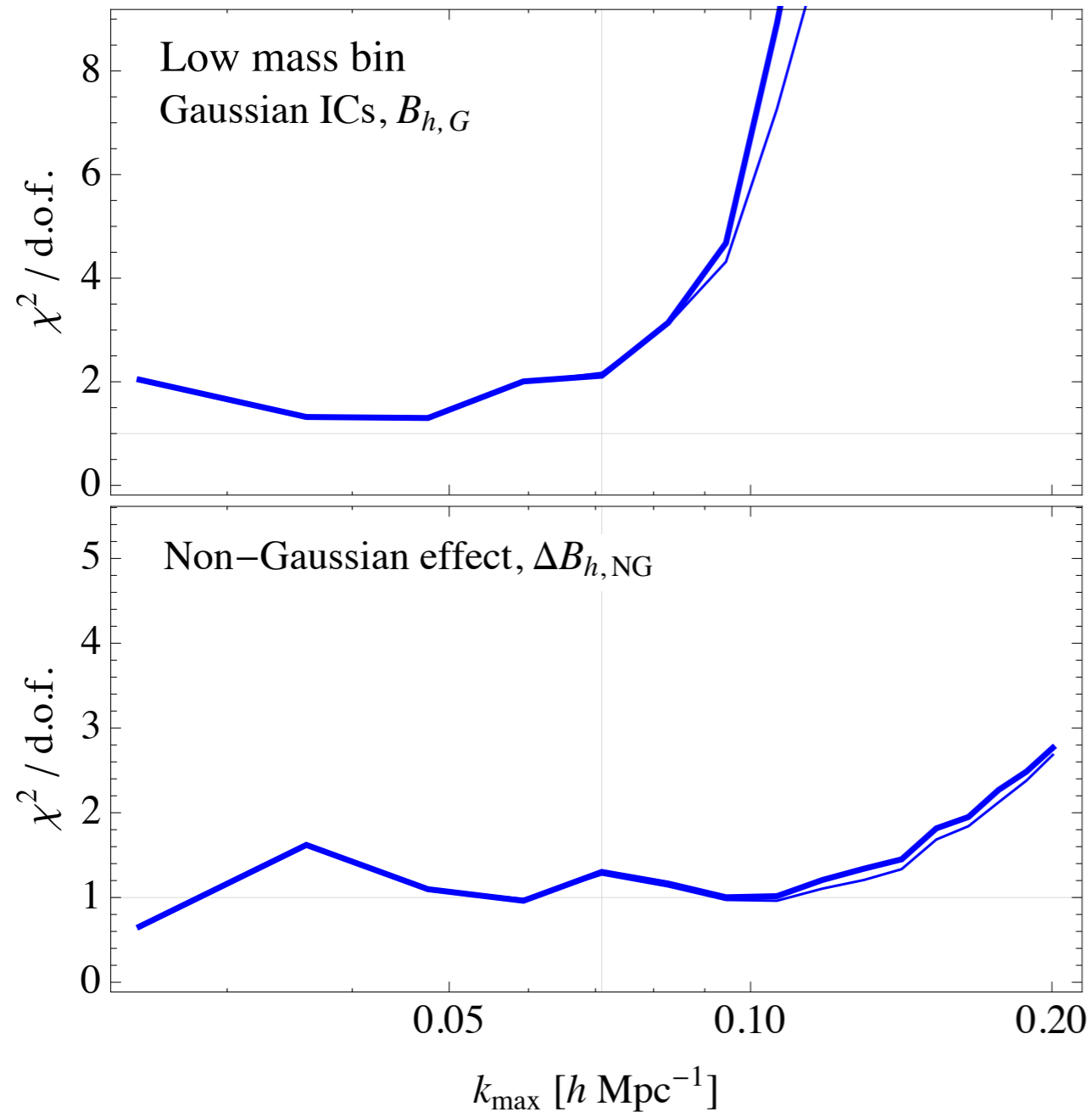
$M > 1.6 \times 10^{13} h^{-1} M_\odot$



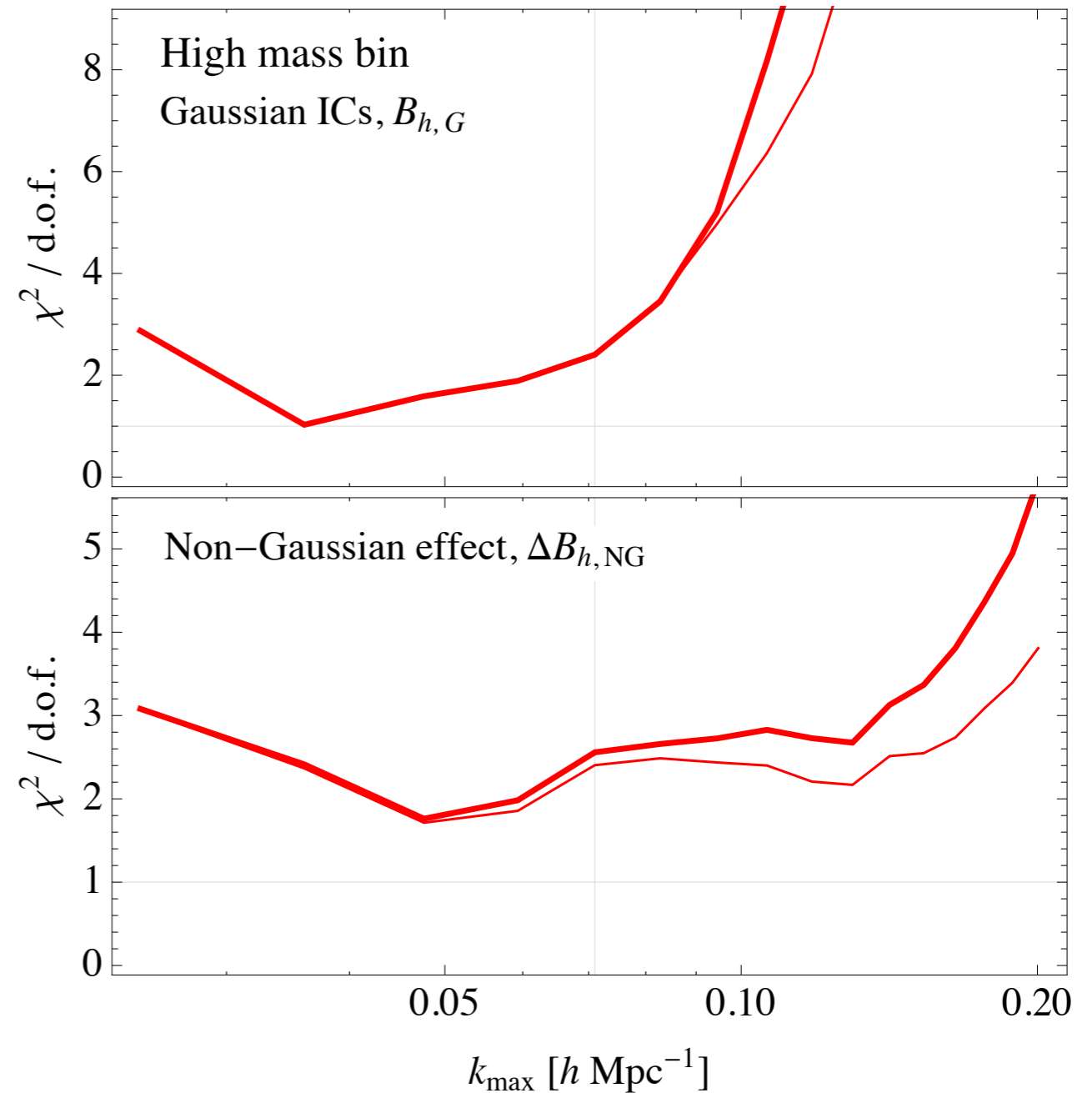
# The Halo Bispectrum: *theory vs. simulations*

$\chi^2$ , for all triangles, as a function of  $k_{\max}$

$8.8 \times 10^{12} h^{-1} M_{\odot} < M < 1.6 \times 10^{13} h^{-1} M_{\odot}$



$M > 1.6 \times 10^{13} h^{-1} M_{\odot}$

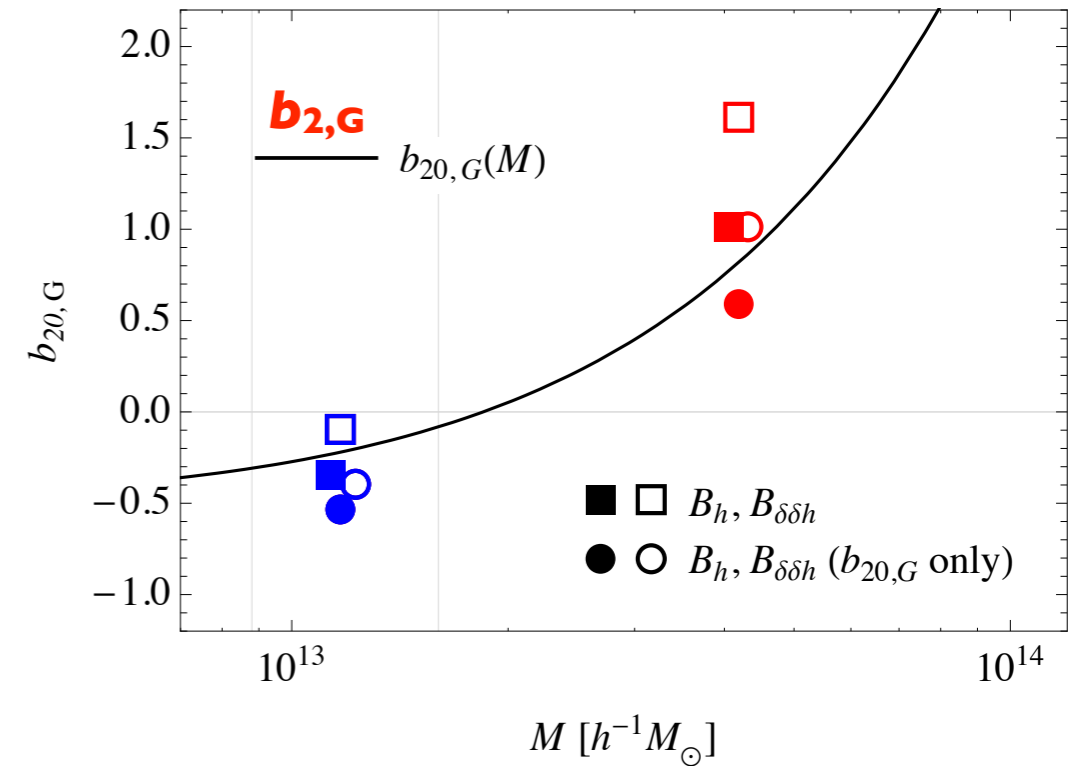
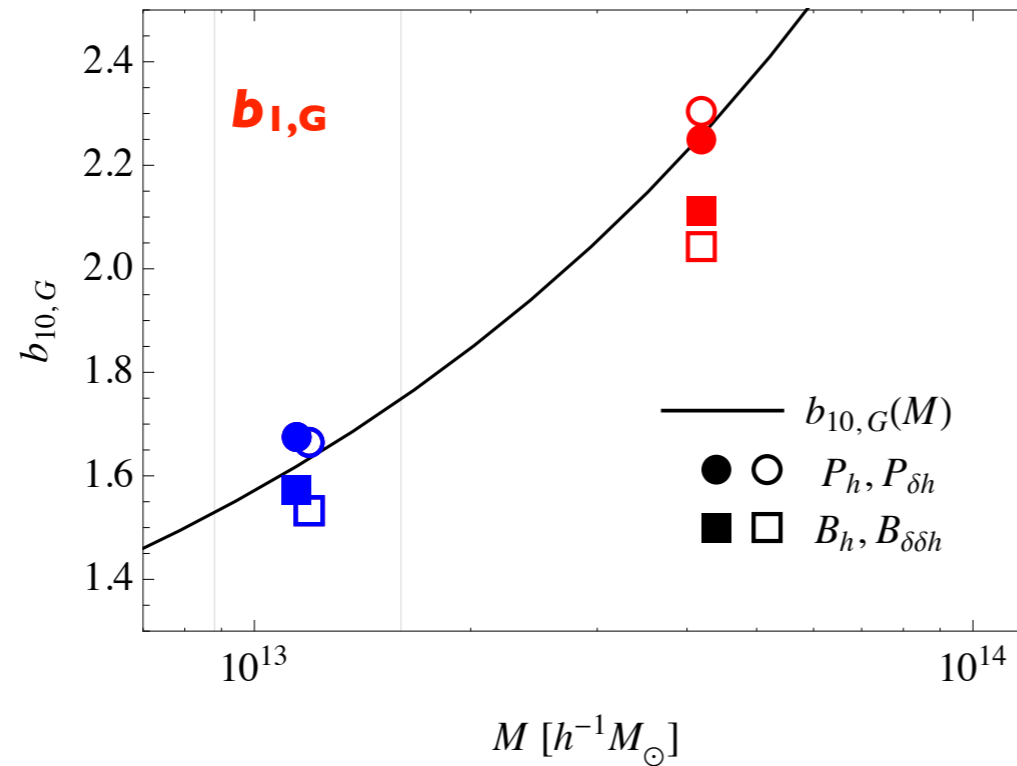




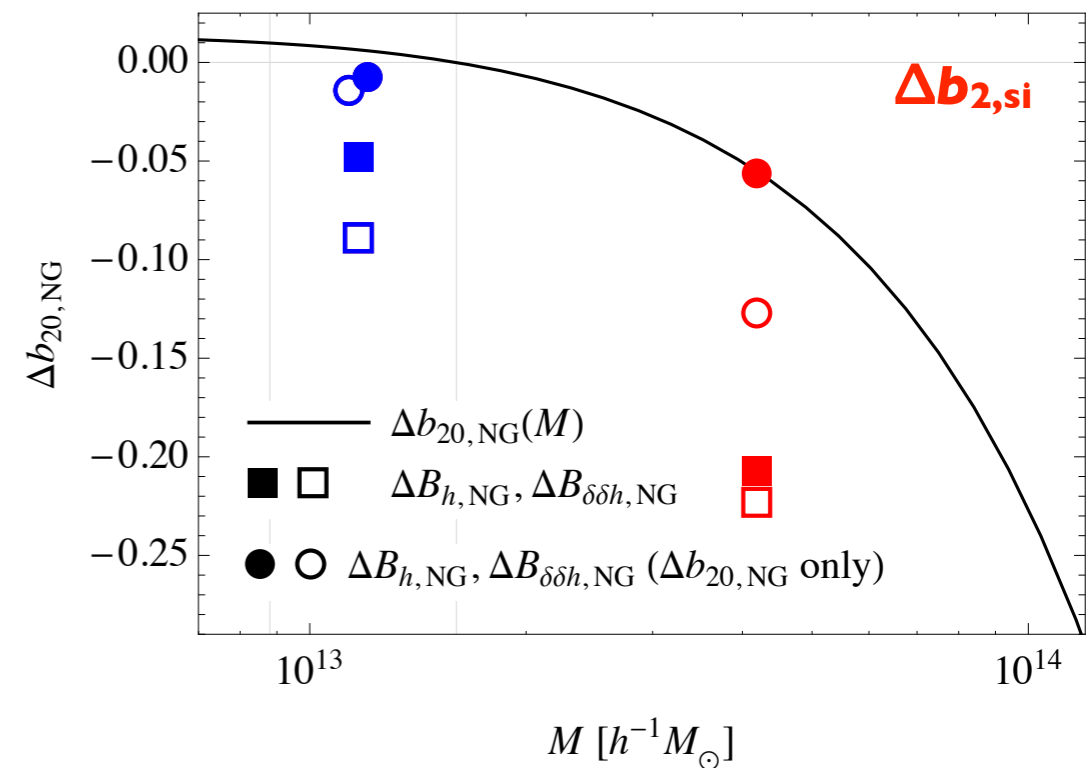
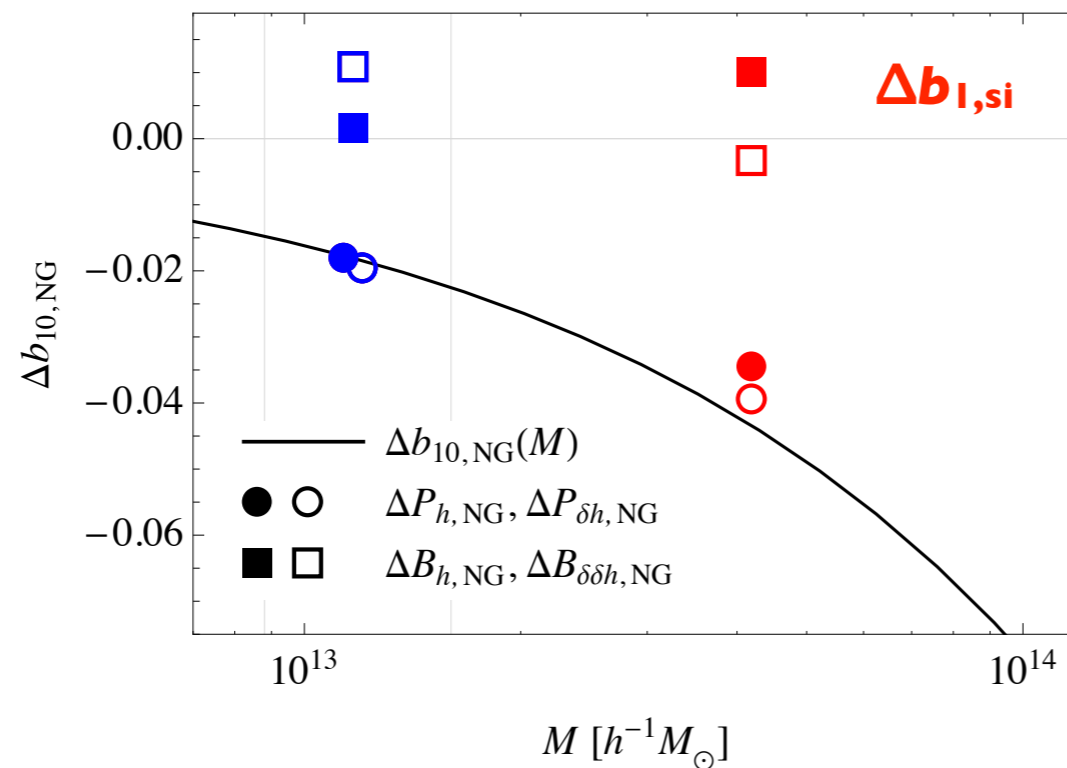
# The Halo Bispectrum: *theory vs. simulations*

Best-fit bias parameters and their peak-background split predictions

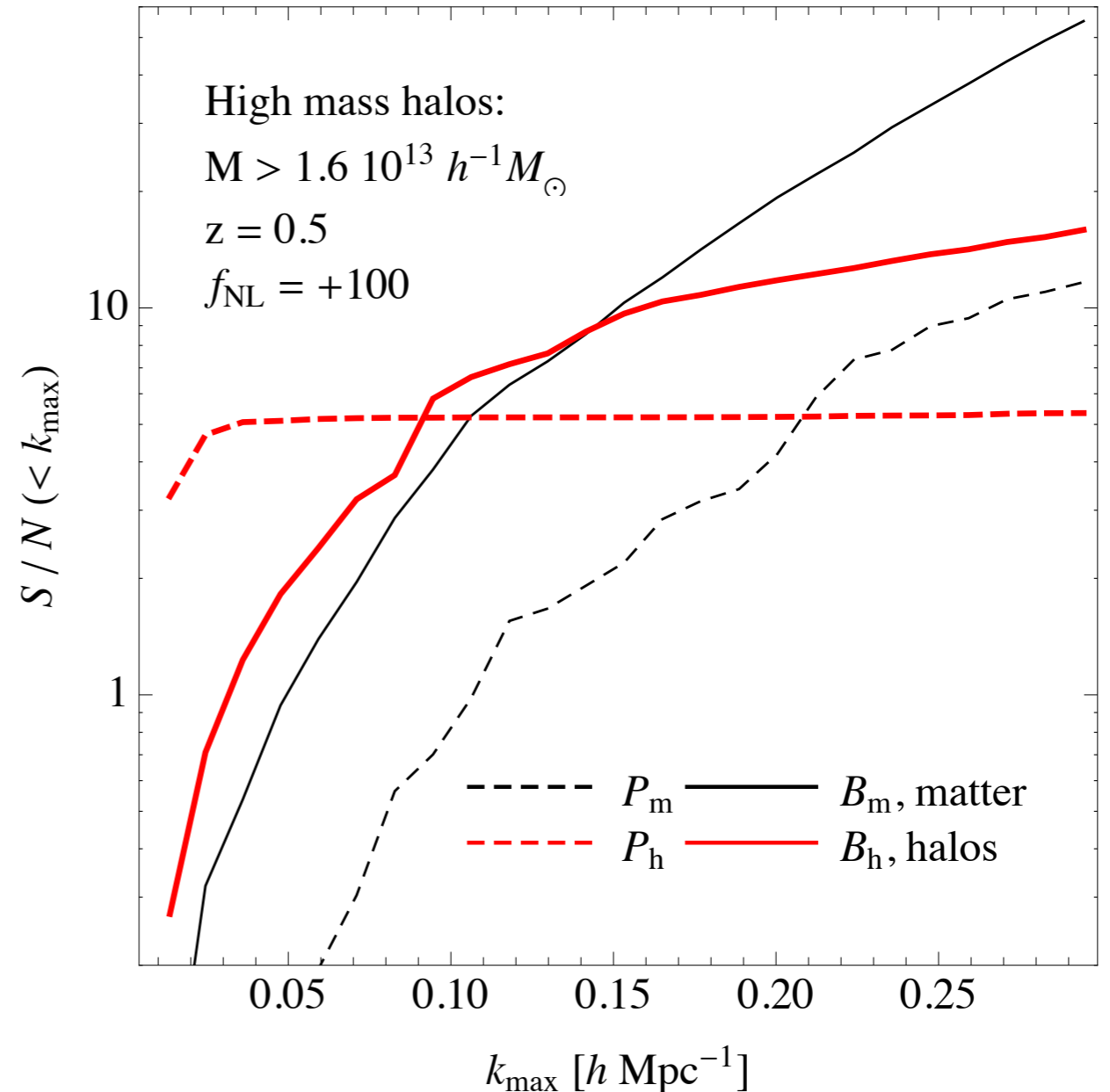
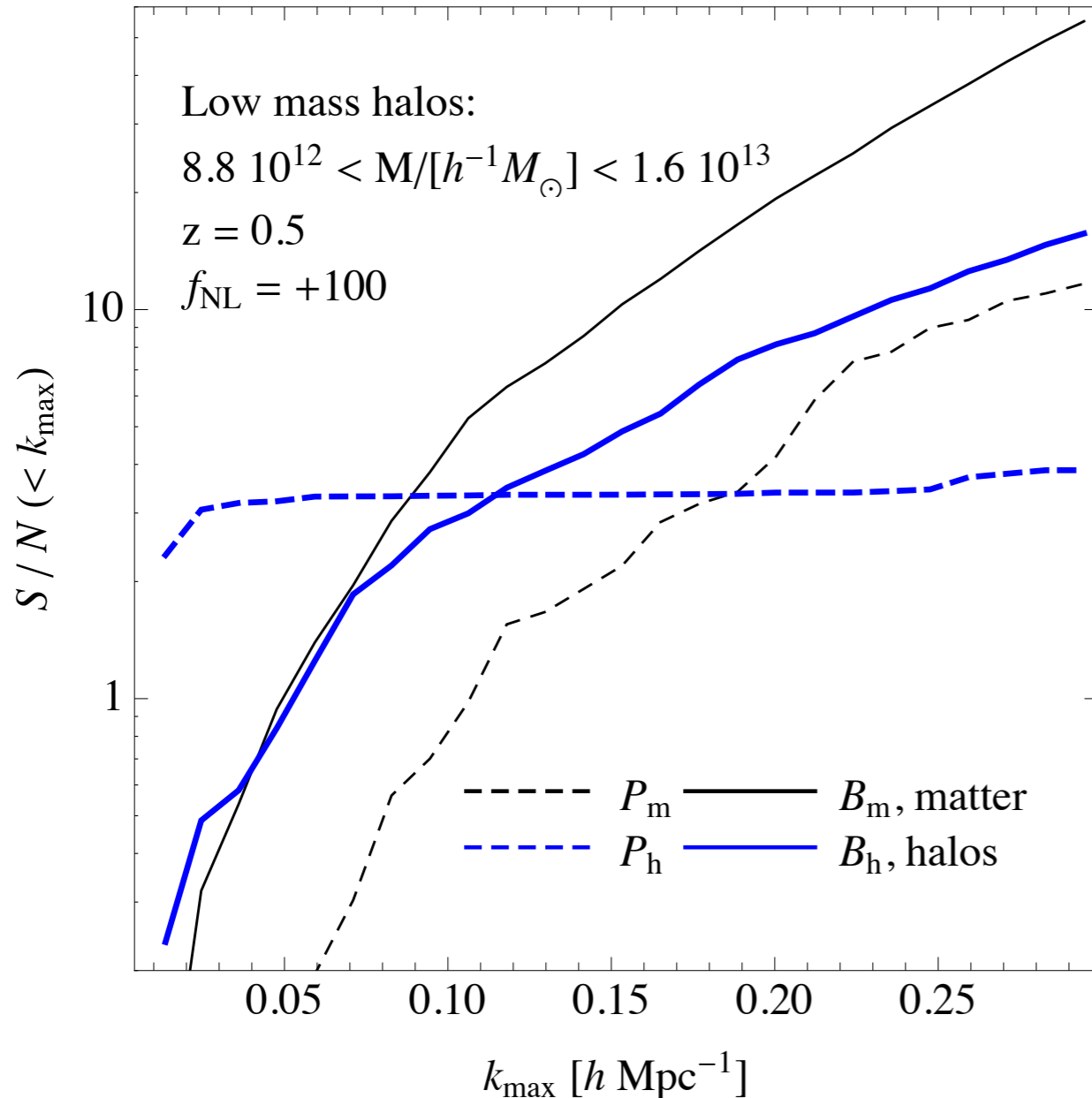
Gaussian halo bias



Non-Gaussian, scale-independent, halo bias corrections



# Halo Power Spectrum vs. Halo Bispectrum



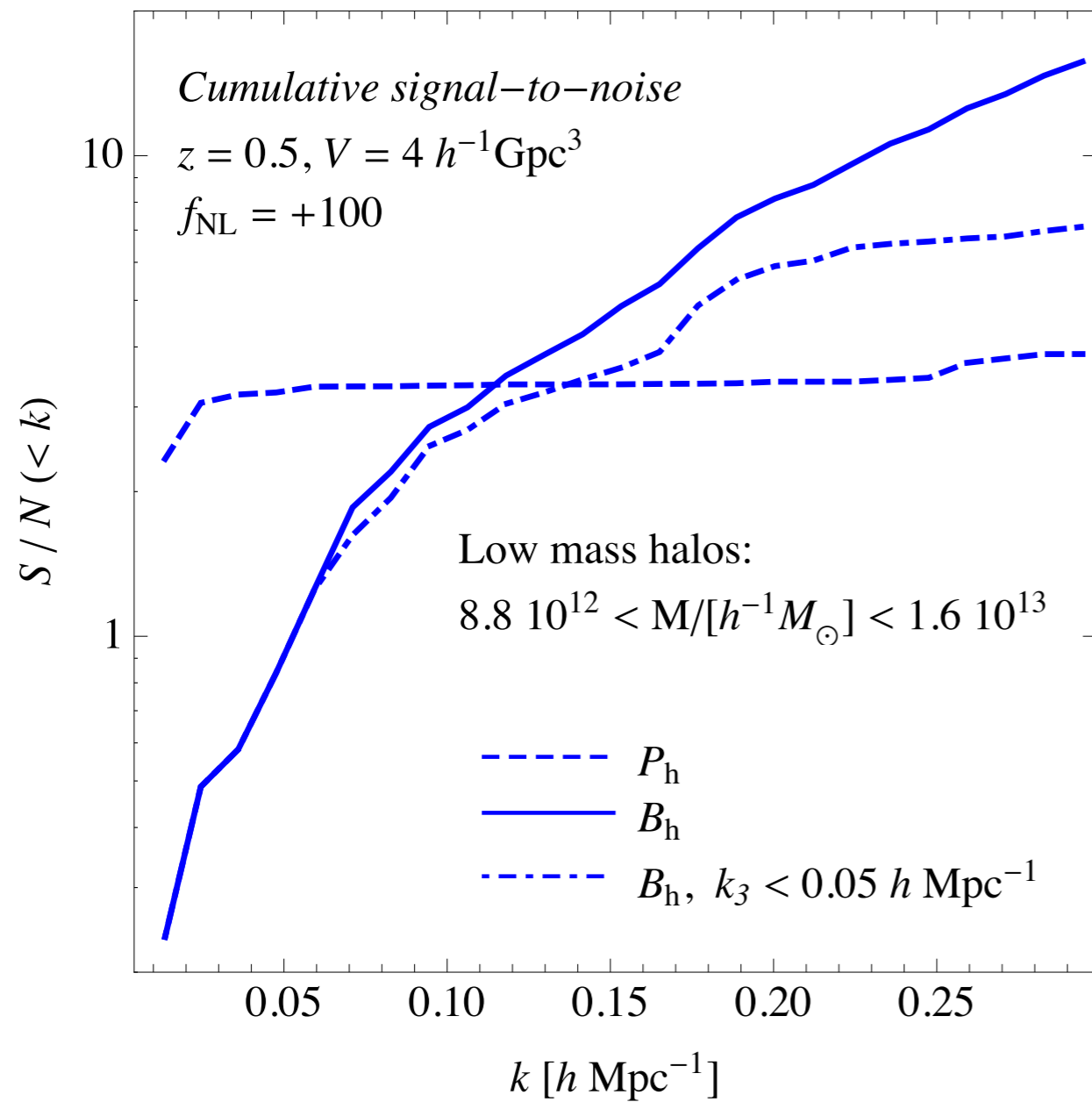
Cumulative signal-to-noise for the effect of NG initial conditions on matter and galaxy correlators (P & B)

Sum of all configurations up to  $k_{\text{max}}$

$$\left(\frac{S}{N}\right)_P^2 = \sum_k^{k_{\text{max}}} \frac{(P_{\text{NG}} - P_G)^2}{\Delta P^2} \quad \left(\frac{S}{N}\right)_B^2 = \sum_{k_1, k_2, k_3}^{k_{\text{max}}} \frac{(B_{\text{NG}} - B_G)^2}{\Delta B^2}$$

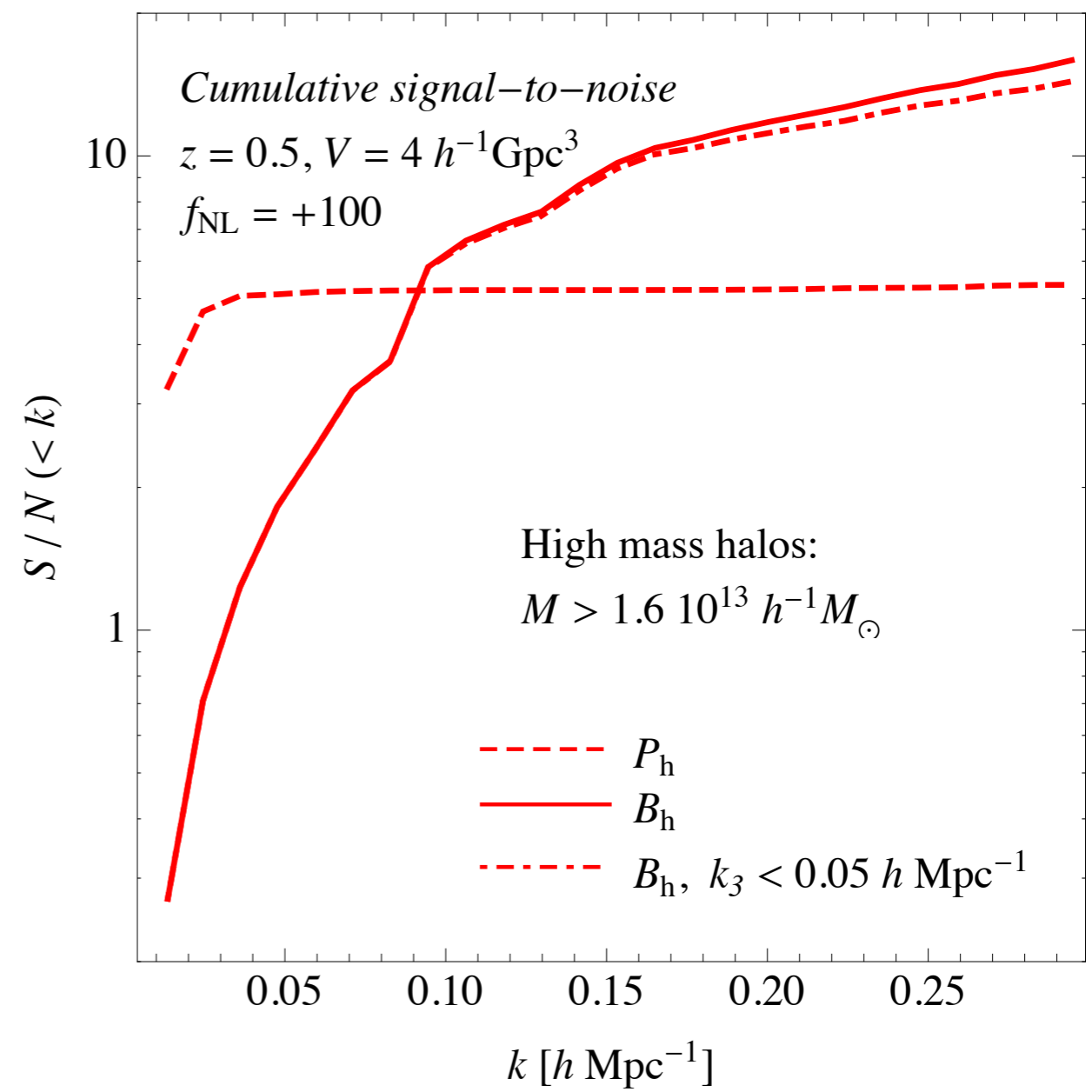
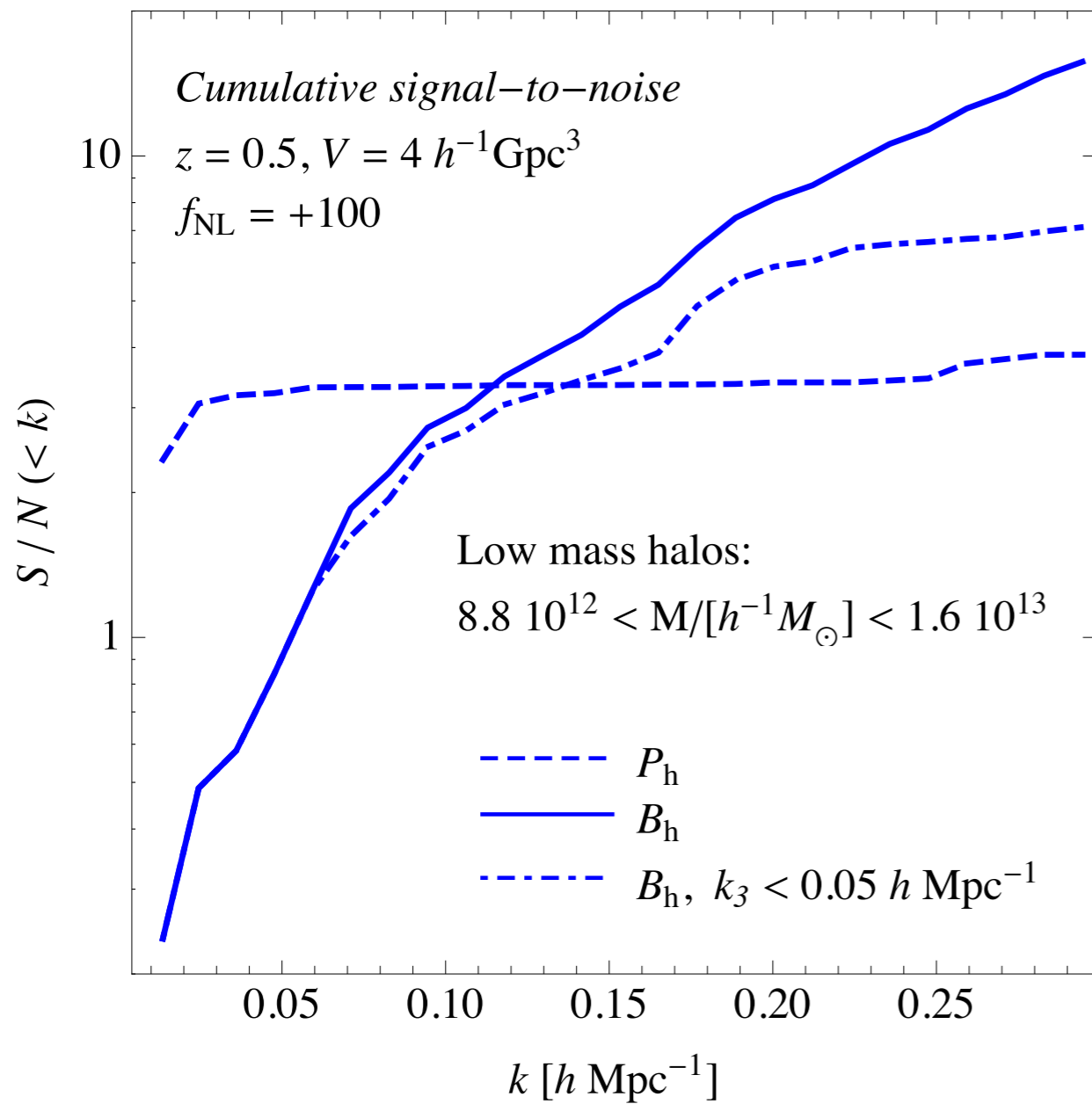
The cumulative NG effect is comparable at mildly nonlinear scales

# Halo Power Spectrum vs. Halo Bispectrum



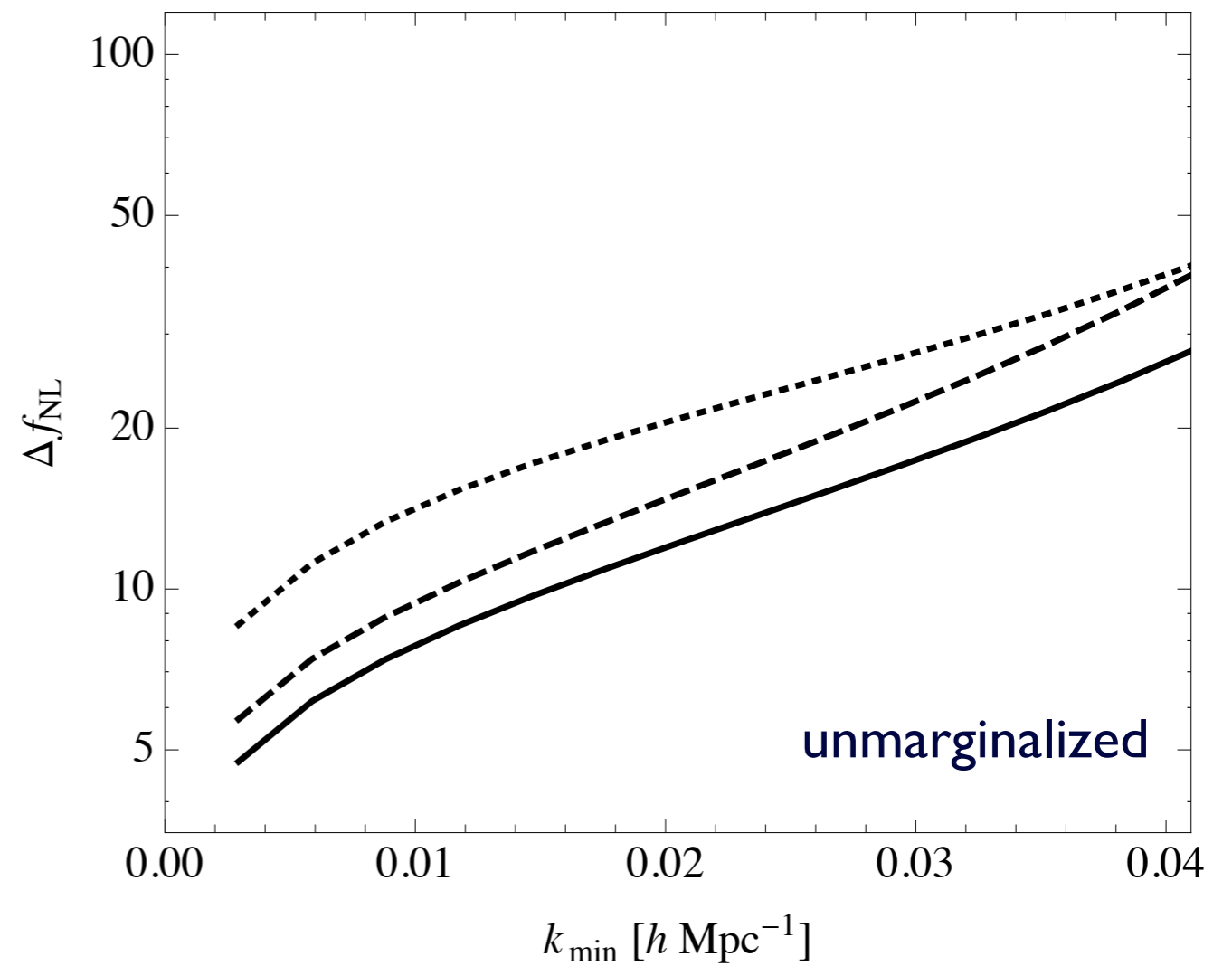
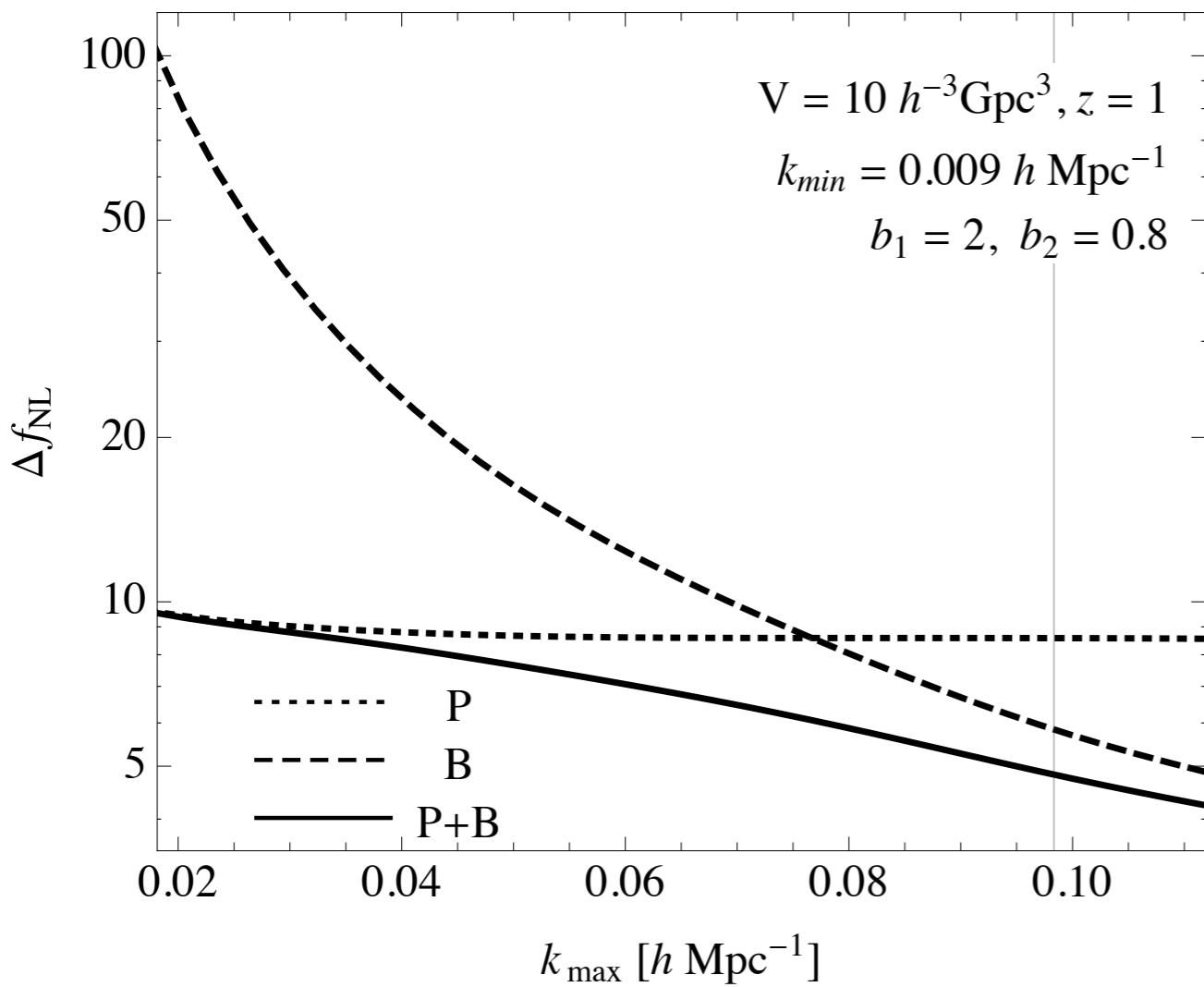
What is the signal in squeezed configurations?

# Halo Power Spectrum vs. Halo Bispectrum

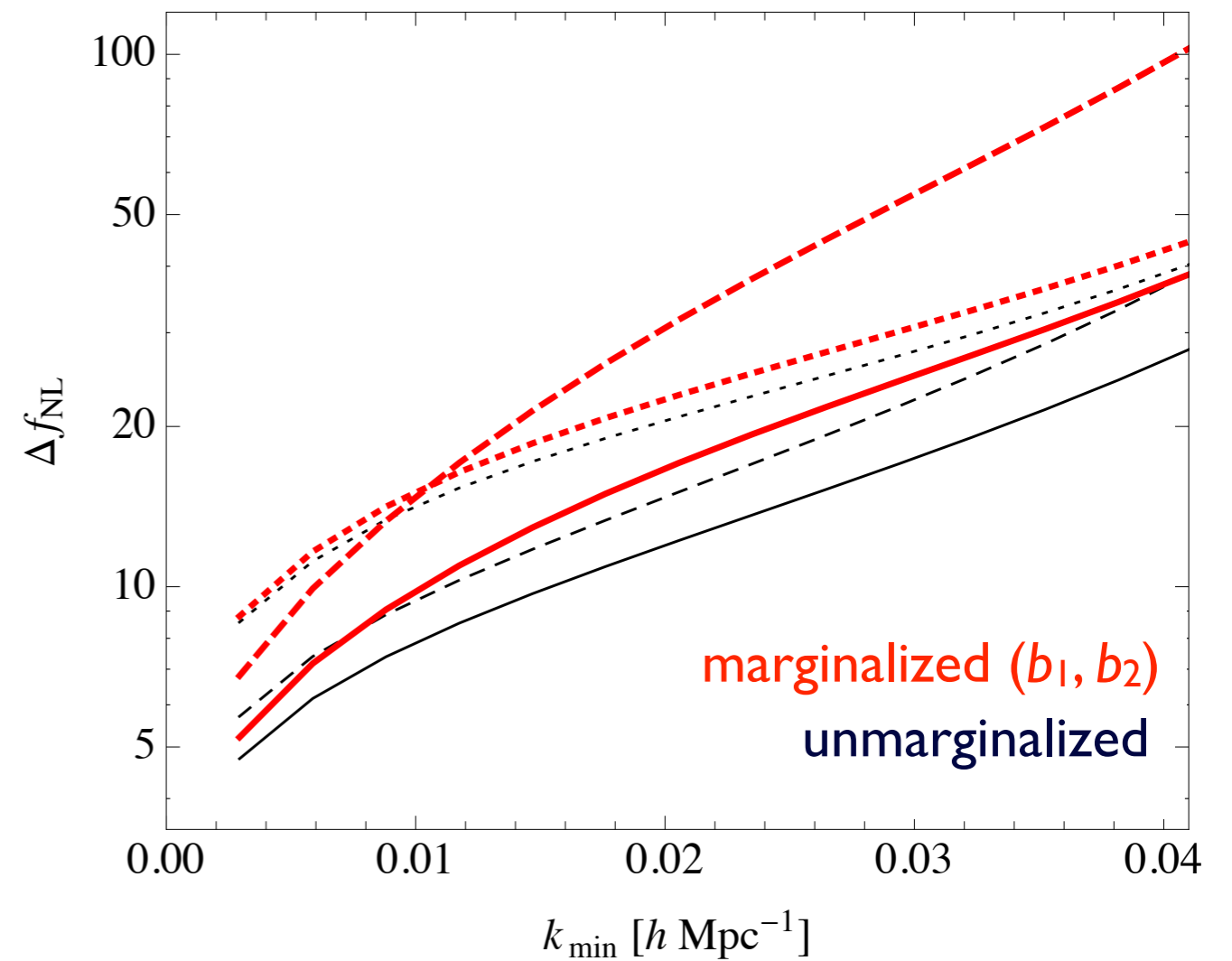
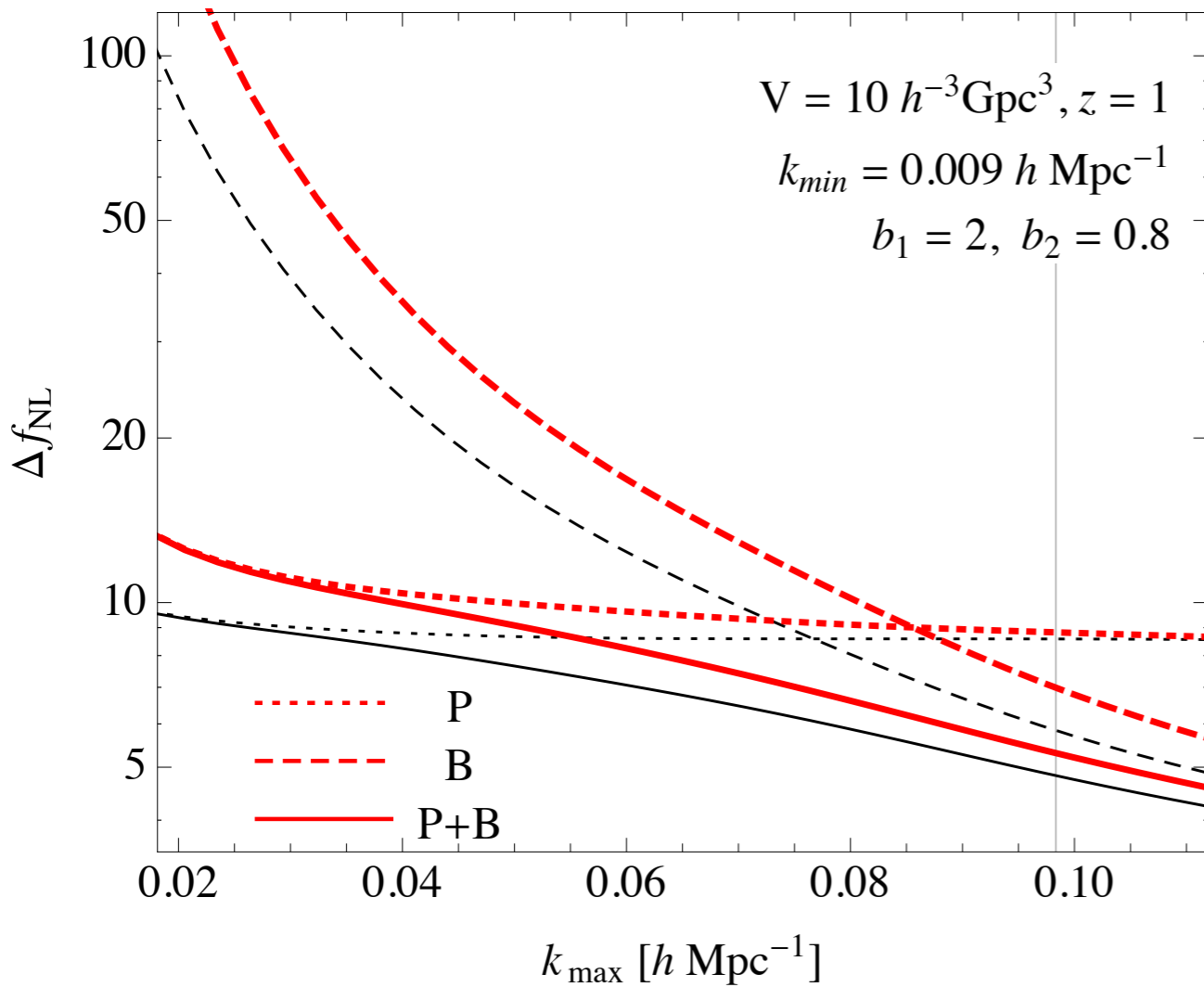


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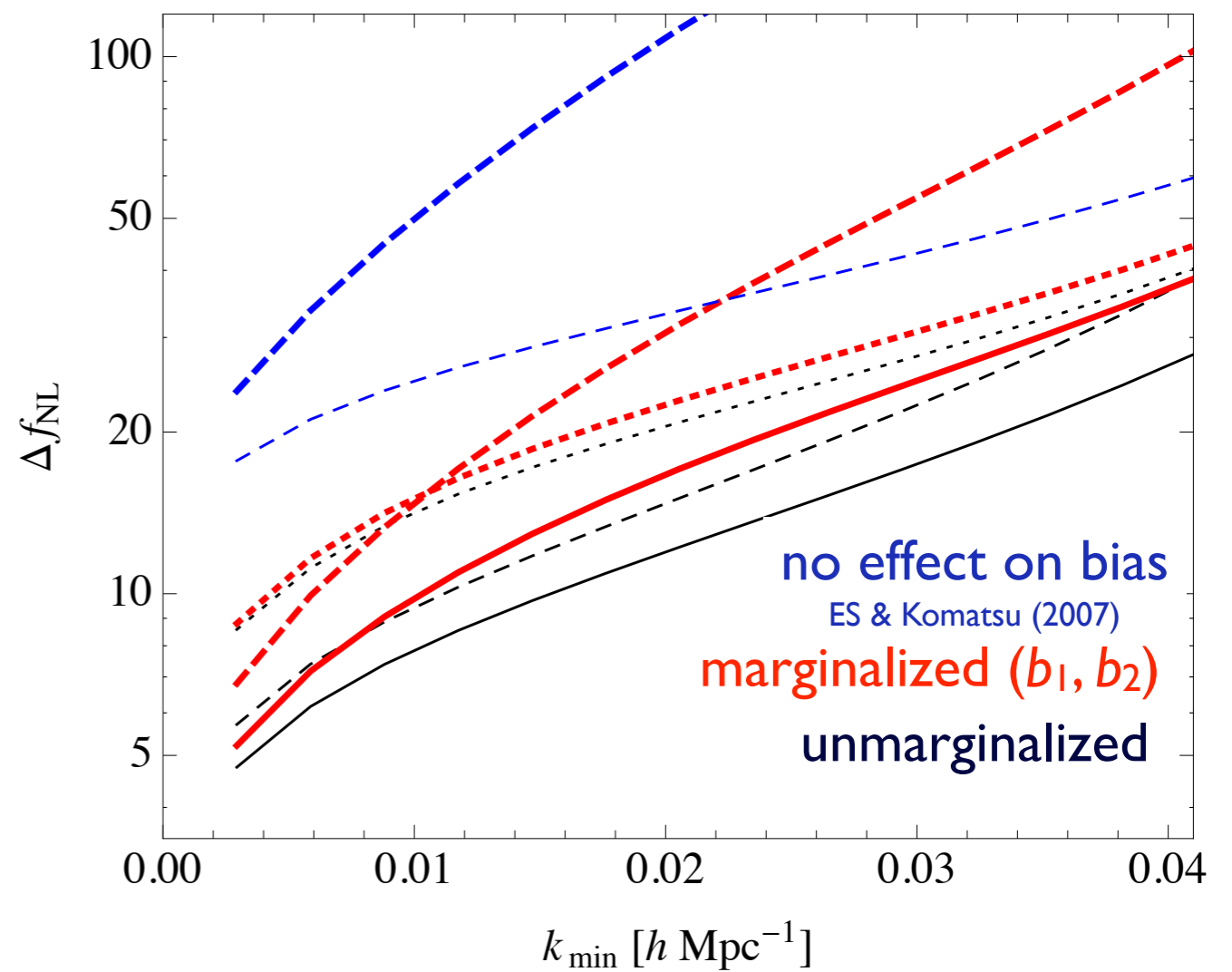
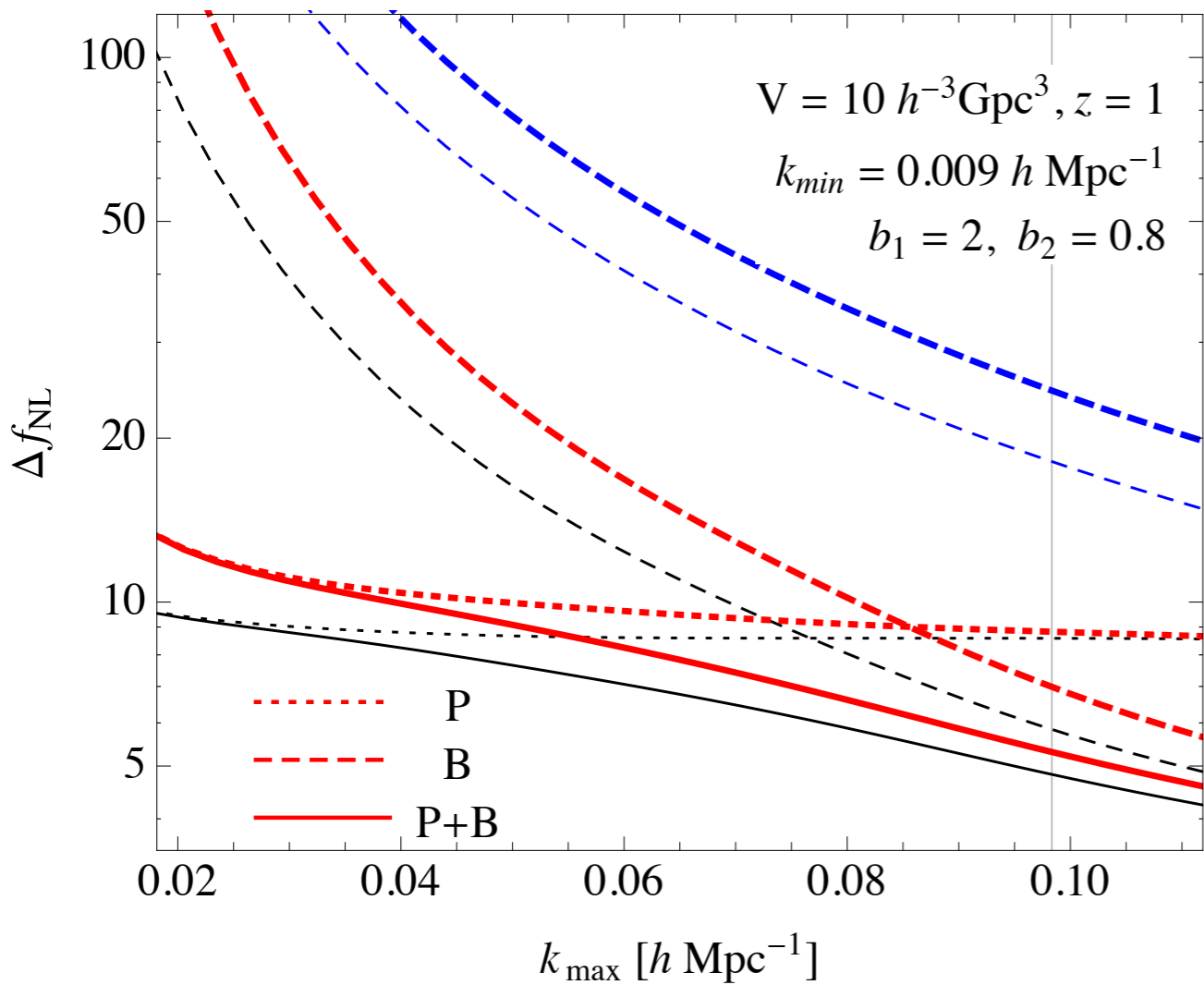
# The uncertainty on $f_{\text{NL}}$ (local) from Power Spectrum & Bispectrum (& both)



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# The uncertainty on $f_{\text{NL}}$ (local) from Power Spectrum & Bispectrum (& both)



# The matter bispectrum at small scales



# Matter Power Spectrum

In Perturbation Theory ...

$$P = P_0 + P_G^{loop}[P_0] + P_{NG}^{loop}[P_0, B_0]$$

**matter power spectrum**

Linear power spectrum

Gravity-induced contributions  
(depending on  $P_0$  alone)

**Additional** gravity-induced contributions  
present *only* for NG initial conditions ( $B_0$ )

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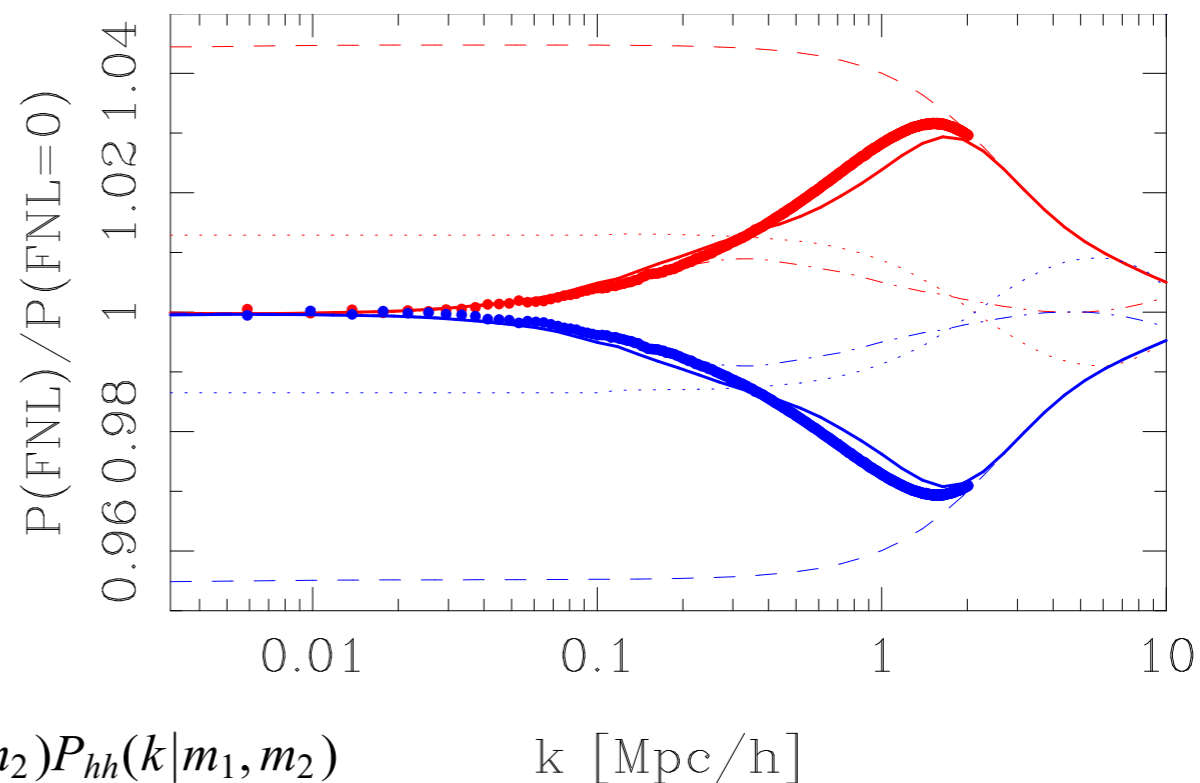
Gravity-induced contributions (depending on  $P_0$  alone)

**Additional** gravity-induced contributions present *only* for NG initial conditions ( $B_0$ )

**Few percent effect at small scales for allowed values of  $f_{NL}$**

Ratio of the non-Gaussian to the Gaussian power spectrum for  $f_{NL} = \pm 100$  (local) at  $z = 1$

Smith, Desjacques & Marian (2010)



In the Halo Model:

$$P(k) = P^{1h}(k) + P^{2h}(k), \quad \text{where}$$

$$P^{1h}(k) = \int dm n(m) \left(\frac{m}{\bar{\rho}}\right)^2 |u(k|m)|^2$$

$$P^{2h}(k) = \int dm_1 n(m_1) \left(\frac{m_1}{\bar{\rho}}\right) u(k|m_1) \int dm_2 n(m_2) \left(\frac{m_2}{\bar{\rho}}\right) u(k|m_2) P_{hh}(k|m_1, m_2)$$

$k$  [Mpc/h]

# The matter **bispectrum** and PNG: *small scales*

In Perturbation Theory ...

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matter power spectrum

Linear power spectrum

Gravity-induced contributions (depending on  $P_0$  alone)

**Additional** gravity-induced contributions present *only* for NG initial conditions ( $B_0$ )

$$B = B_0 + B_G^{tree}[P_0] + B_G^{loop}[P_0] + B_{NG}^{loop}[P_0, B_0]$$

& bispectrum

**Primordial component**

**Nonlinear corrections are *also* affected by the initial conditions!**

# The matter bispectrum and PNG: *small scales*

$$B = B_0 + B_G^{tree}[P_0] + B_G^{loop}[P_0] + B_{NG}^{loop}[P_0, B_0]$$

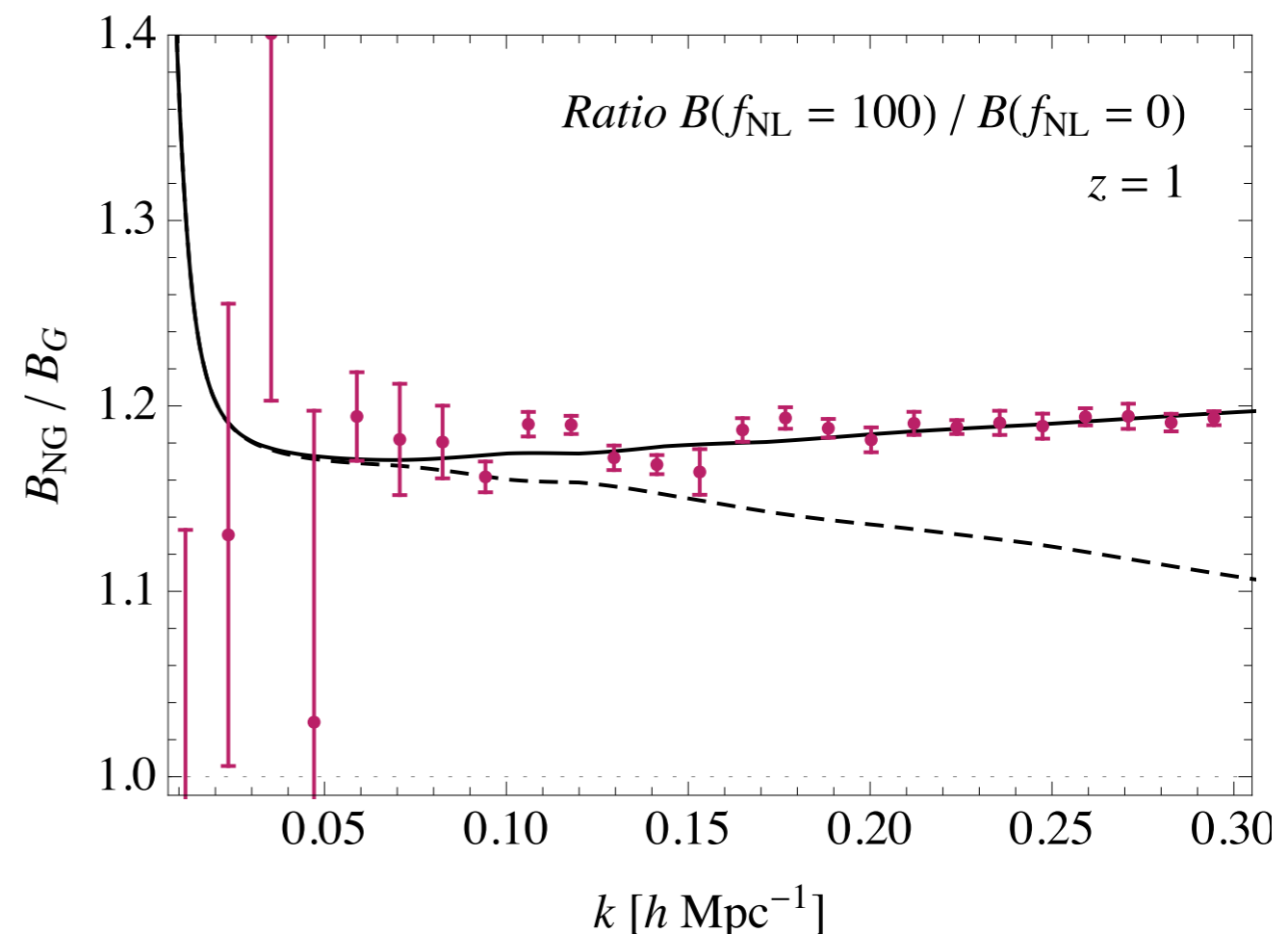
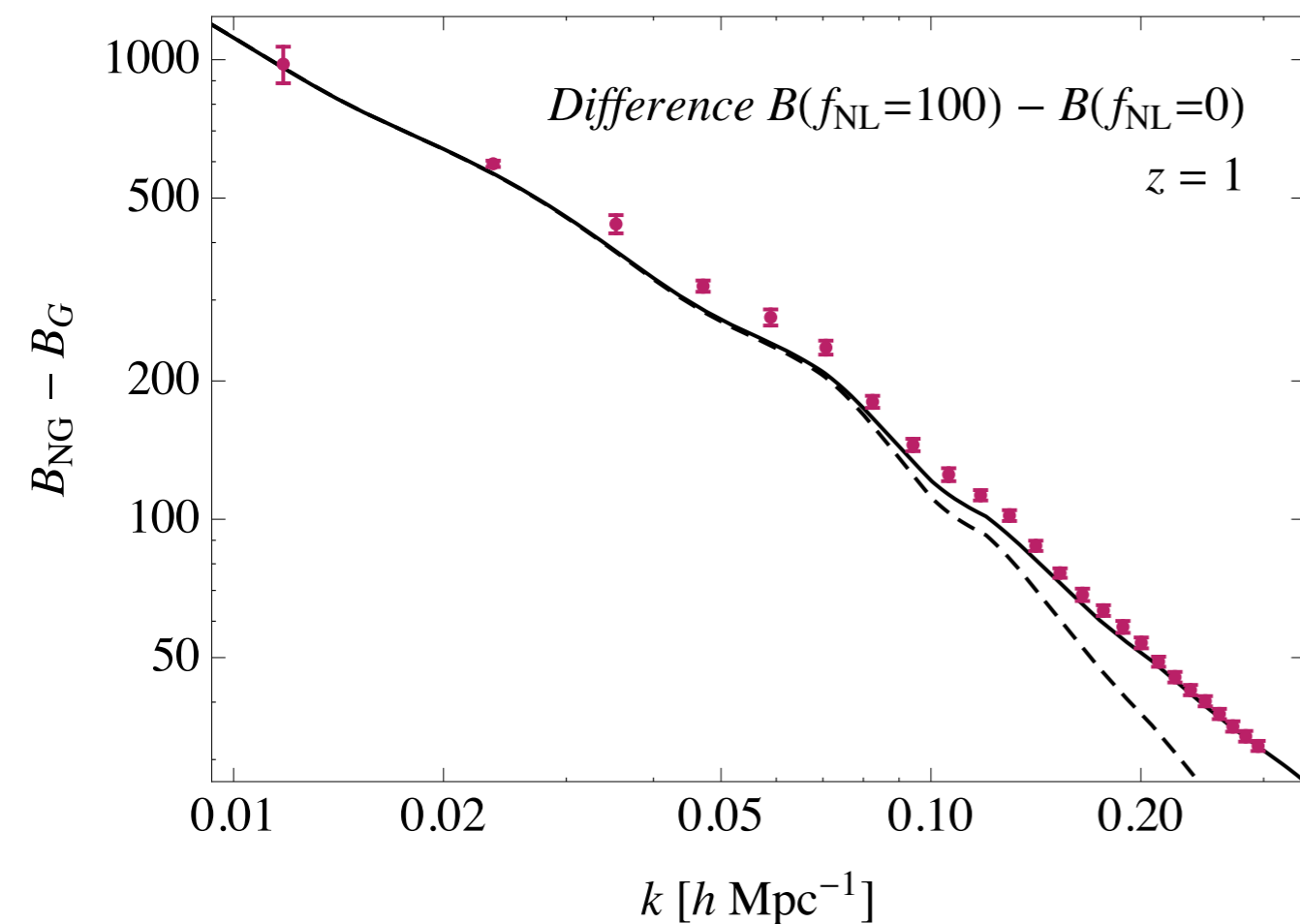
↓  
Primordial  
component

↓ Gravity-induced  
contributions

↓ **Additional** gravity-induced contributions  
present for NG initial conditions ( $B_0$ )

Squeezed configurations  $B(\Delta k, k, k)$   
as a function of  $k$  with  $\Delta k = 0.01 h/\text{Mpc}$

ES (2009)  
ES, Crocce & Desjacques (2010)



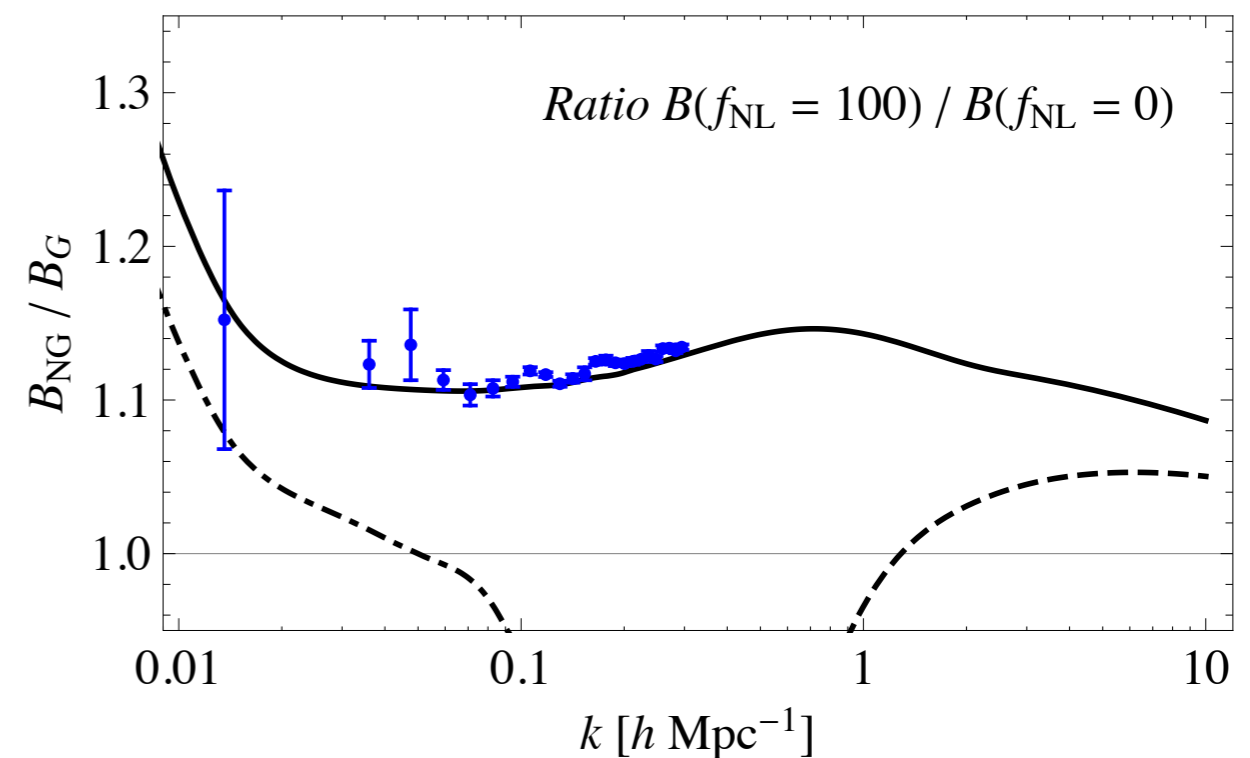
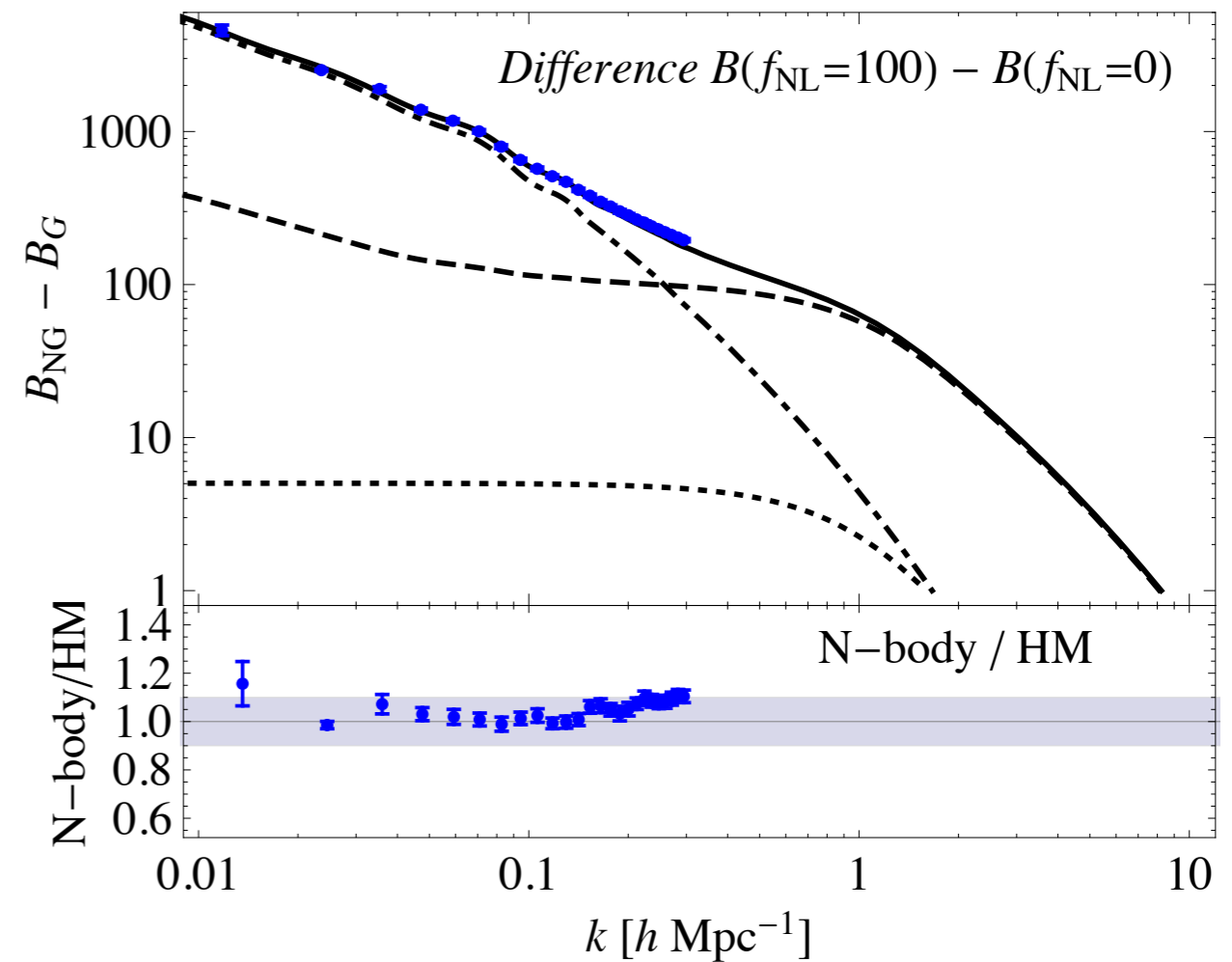
# The matter bispectrum and PNG: *even smaller scales*

## Beyond PT: *The Halo Model*

There is a **significant effect** of NG initial conditions of about 5-15% on all triangles, at **small scales** and at **late times** for  $f_{\text{NL}} = 100$

Squeezed configurations  $B(\Delta k, k, k)$   
as a function of  $k$  with  $\Delta k = 0.01 h/\text{Mpc}$

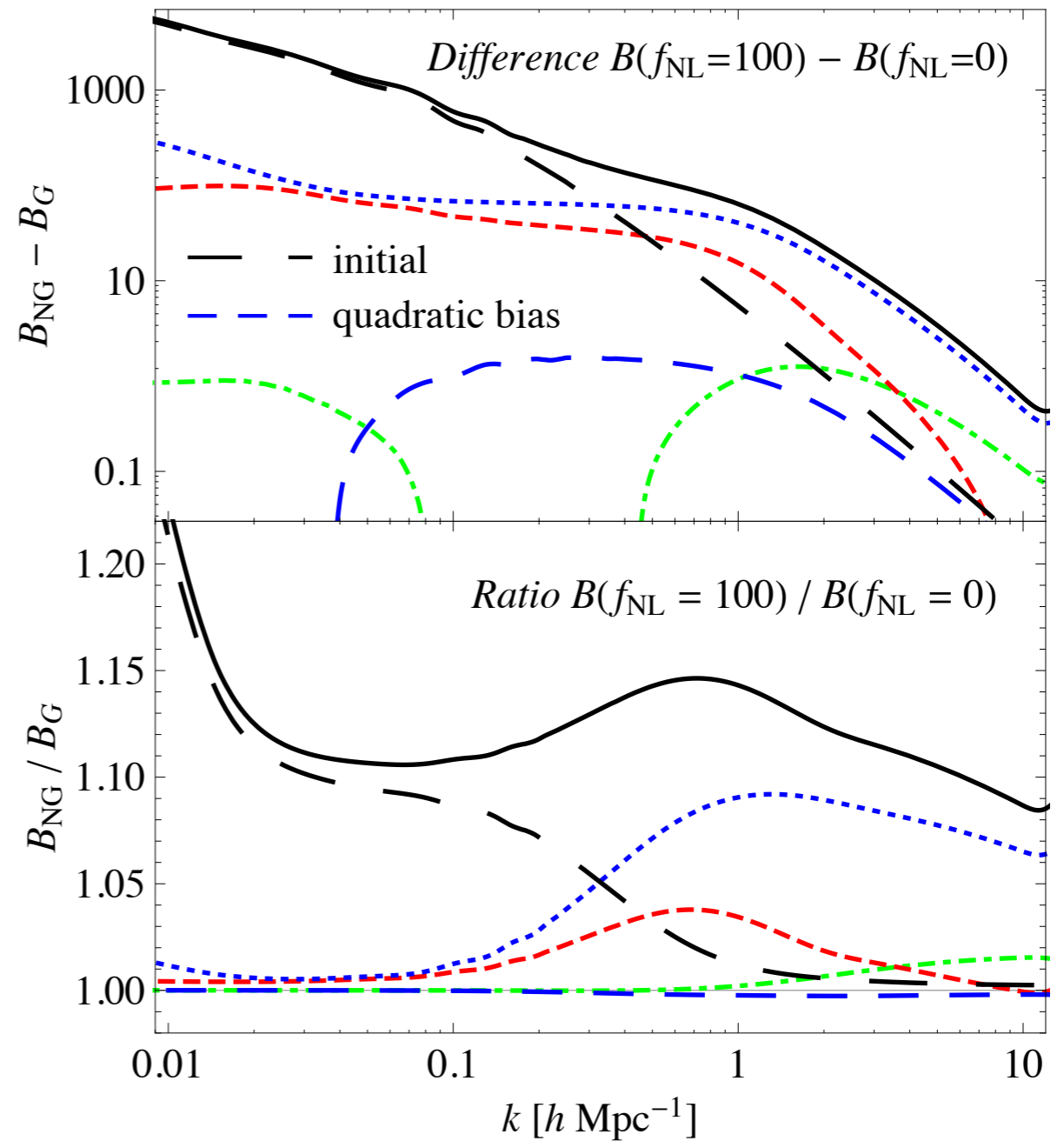
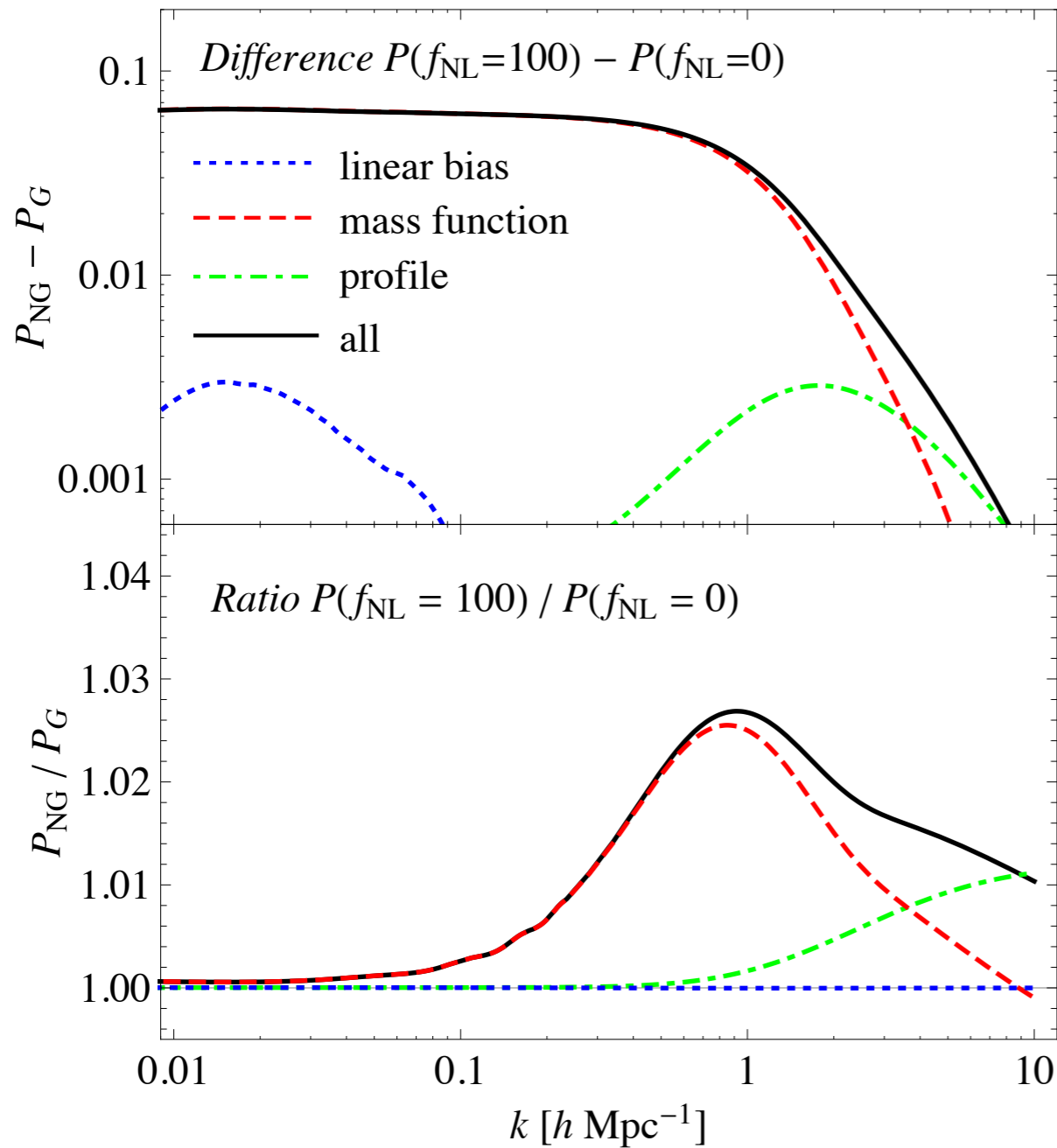
Figuera, ES, Riotto & Vernizzi (2012)



# The matter bispectrum and PNG: *even smaller scales*

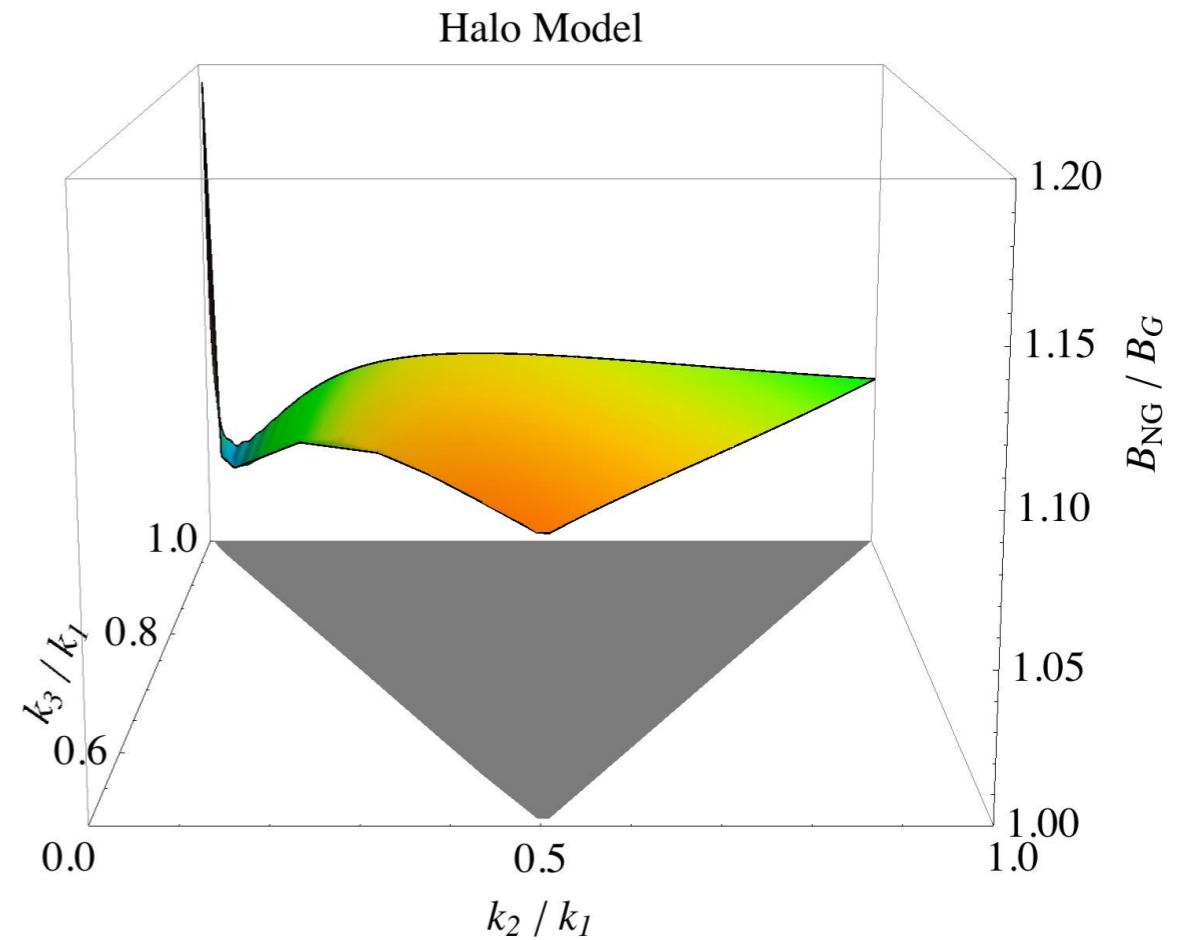
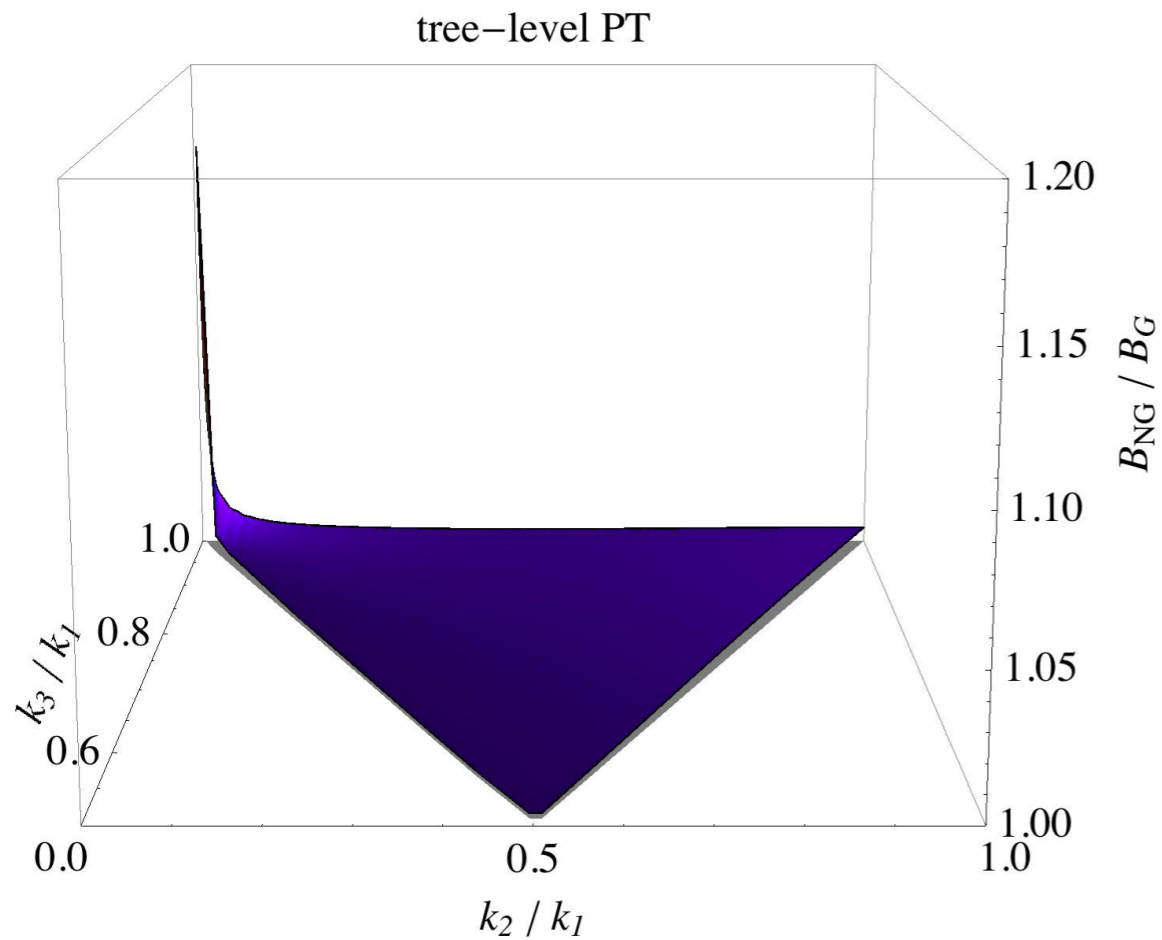
## Beyond PT: *The Halo Model*

Squeezed configurations  $B(\Delta k, k, k)$   
as a function of  $k$  with  $\Delta k = 0.01 h/\text{Mpc}$



# The matter bispectrum and PNG: *even smaller scales*

## Beyond PT: *The Halo Model*



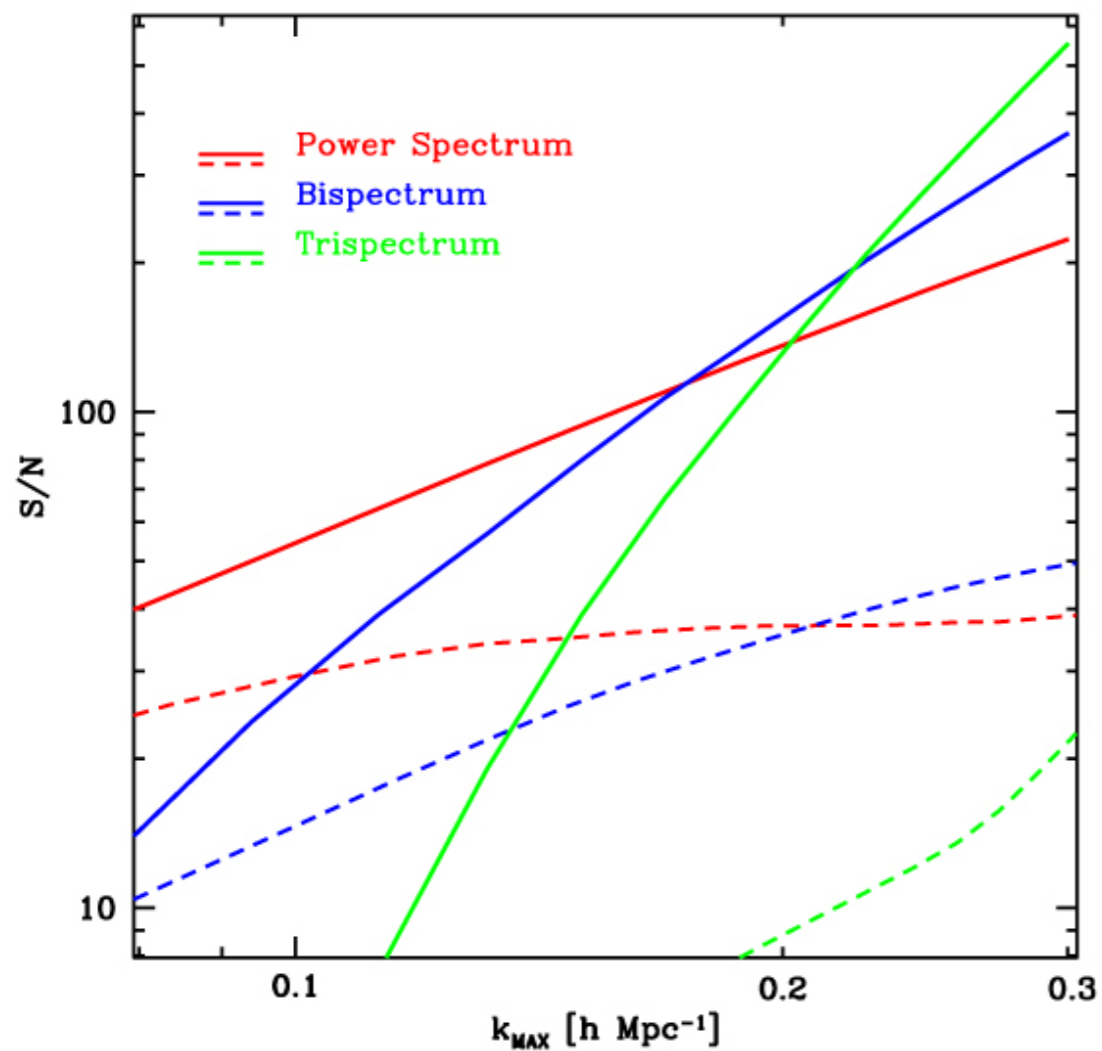
$k_1 = 3 h/\text{Mpc}$

$$f_{\text{NL}} = 0$$



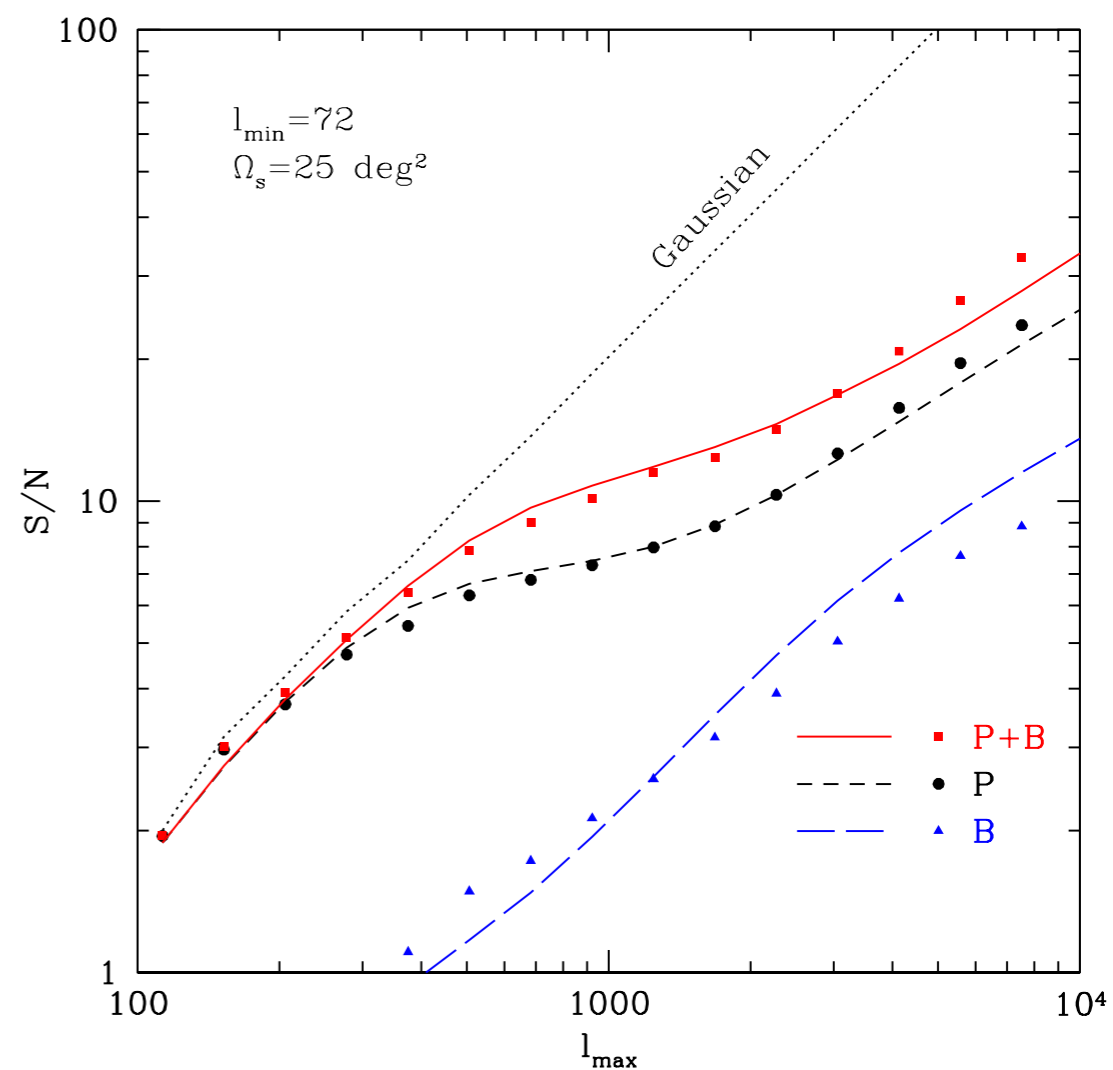
$$f_{\text{NL}} = 0$$

galaxies



ES & Scoccimarro (2005)

weak lensing



Kayo, Takada & Jain (2012)

# Conclusions

- We do have a good understanding of the **multiple effects of PNG on the galaxy bispectrum** at large scales (with room for improvement!)
- The **impact of NG on nonlinear evolution of structure** is significant, particularly in terms of the **matter bispectrum**: can this be detected in weak lensing surveys?
- A **complete analysis of the large-scale structure** (e.g. galaxy power spectrum *and* bispectrum) **can do better than power spectrum alone**: smaller uncertainties on NG parameters **for** virtually **any model of non-Gaussianity**

# Galaxy bias and the galaxy power spectrum

Dalal *et al.* (2008):

**The bias of galaxies receives a significant *scale-dependent* correction for NG initial conditions of the *local* type**

$$P_g(k) = [b_1 + \Delta b_1(f_{NL}, k)]^2 P(k)$$

↓  
“Gaussian”  
bias

↓  
Scale-dependent correction  
due to local non-Gaussianity

$$\Delta b_{1,NG}(f_{NL}, k) = \frac{2f_{NL}(b_1 - 1)\delta_c}{M(k)}$$

$$M(k) = \frac{2}{3} \frac{D(z)T(k)}{\Omega_m H_0^2} k^2$$

# Galaxy bias and the galaxy power spectrum

**The bias of galaxies receives a scale-dependent correction for NG initial conditions of any type**

Matarrese & Verde (2008)  
Desjacques, Schmidt & Jeong (2011)  
Scoccimarro *et al.* (2011)

$$P_g(k) = [b_1 + \Delta b_1(f_{NL}, k)]^2 P(k)$$

↓  
“Gaussian”  
bias

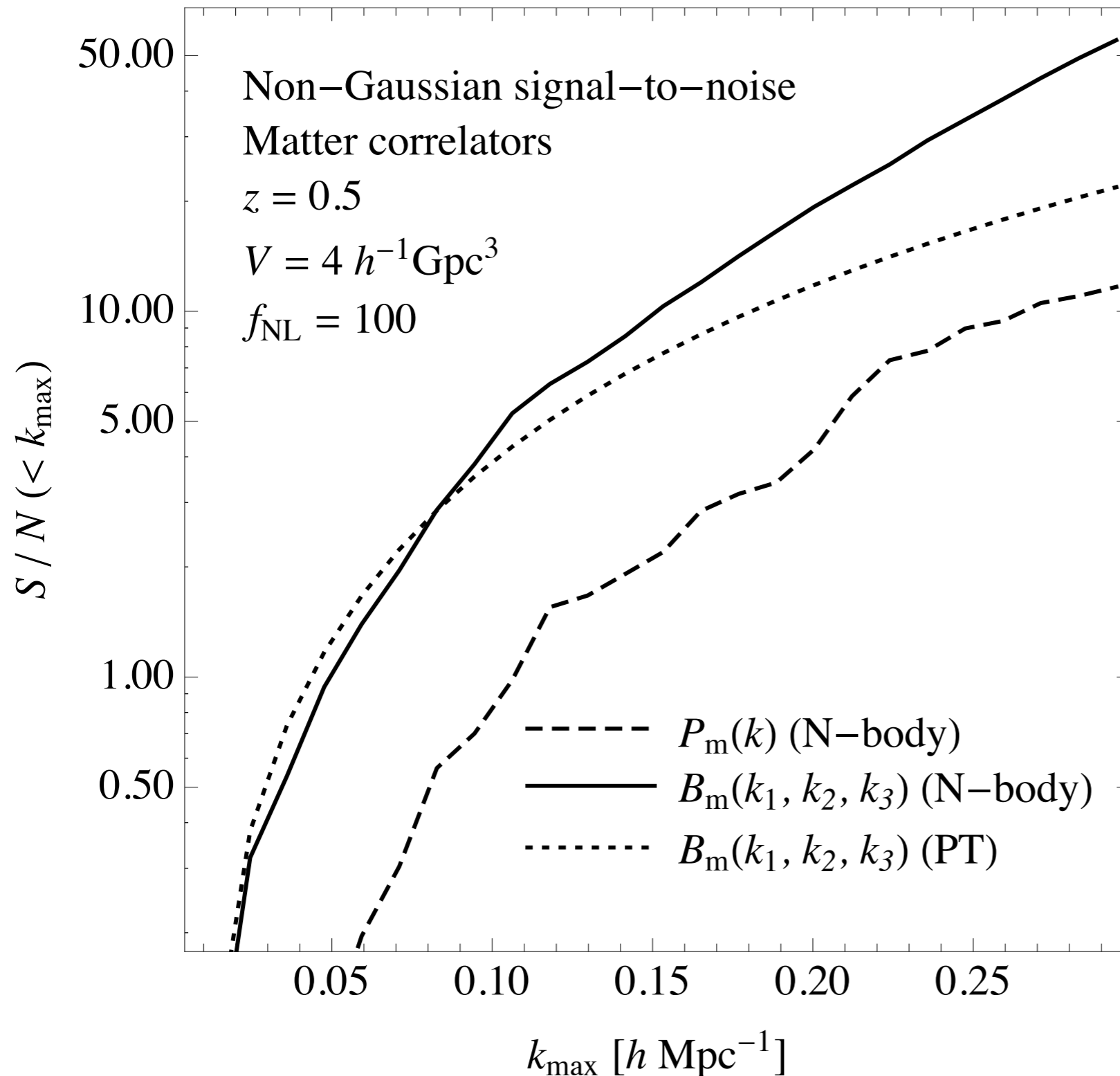
↓  
Scale-dependent correction

$$\Delta b_{1,NG}(f_{NL}, k) = \frac{(b_1 - 1)\delta_c}{2M(k)} I(k, m) + \frac{1}{M(k, z)} \frac{\partial I(k, m)}{\partial \ln \sigma_m^2}$$

$$M(k) = \frac{2}{3} \frac{D(z)T(k)}{\Omega_m H_0^2} k^2$$

$$I(k, m) \sim \int d^3 q [\dots] B_\Phi(k, q, |\vec{k} - \vec{q}|) \longrightarrow \text{Initial bispectrum}$$

# Matter correlators with non-Gaussian initial conditions



*Cumulative signal-to-noise for the effect of NG initial conditions*

*Sum of all configurations up to  $k_{\text{max}}$*

$$\left(\frac{S}{N}\right)_P^2 = \sum_k^{k_{\text{max}}} \frac{(P_{\text{NG}} - P_G)^2}{\Delta P^2}$$

$$\left(\frac{S}{N}\right)_B^2 = \sum_{k_1, k_2, k_3}^{k_{\text{max}}} \frac{(B_{\text{NG}} - B_G)^2}{\Delta B^2}$$

- Both the direct contribution of  $B_0$  and its effect on the nonlinear corrections are important
- The effect of PNG on the matter bispectrum is more significant than on the power spectrum

# The matter bispectrum and PNG: *small scales*

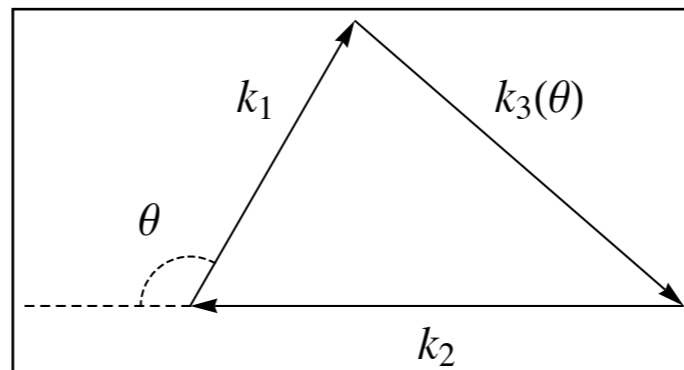
$$B = B_0 + B_G^{tree}[P_0] + B_G^{loop}[P_0] + B_{NG}^{loop}[P_0, B_0]$$

Primordial component

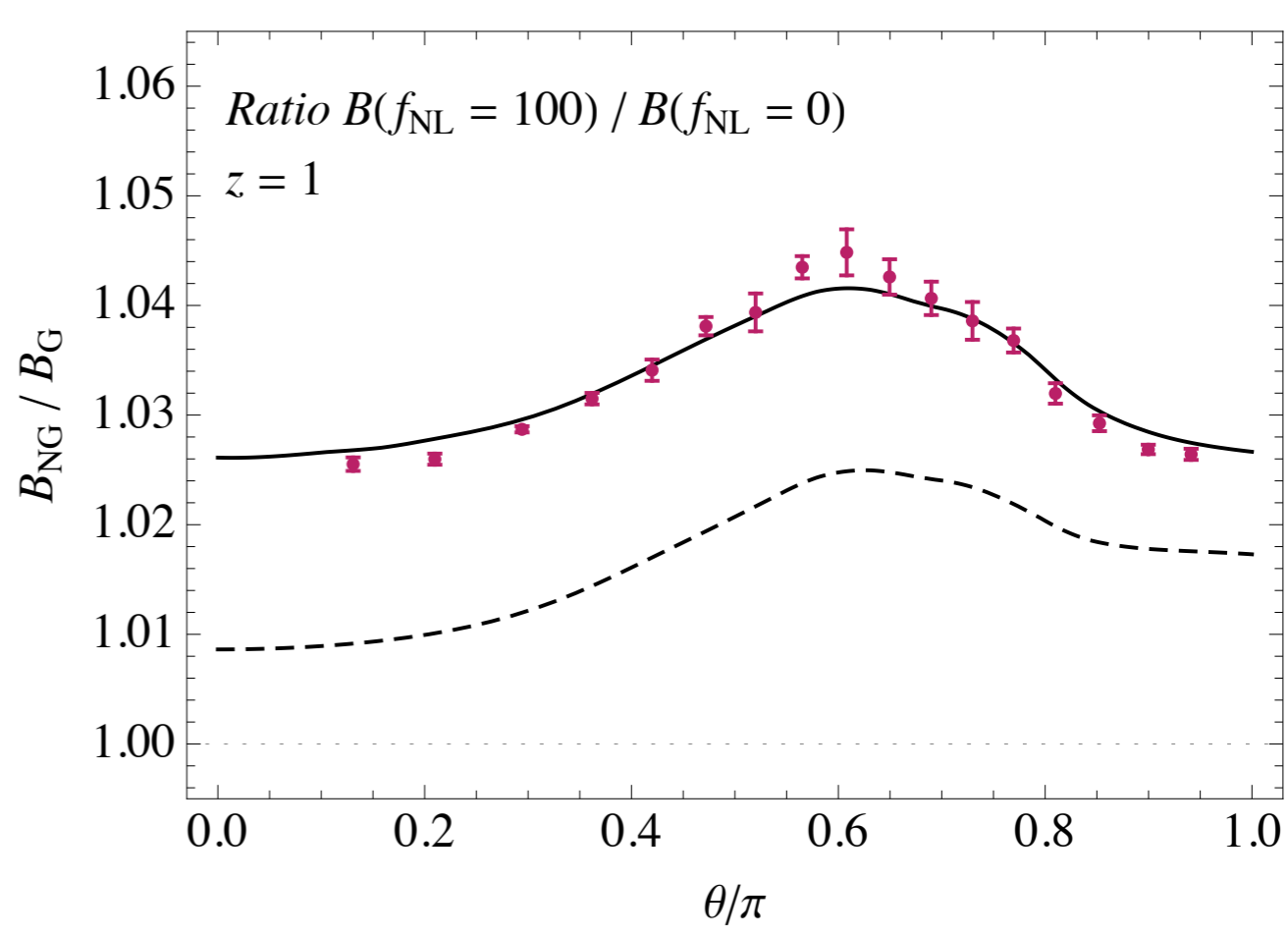
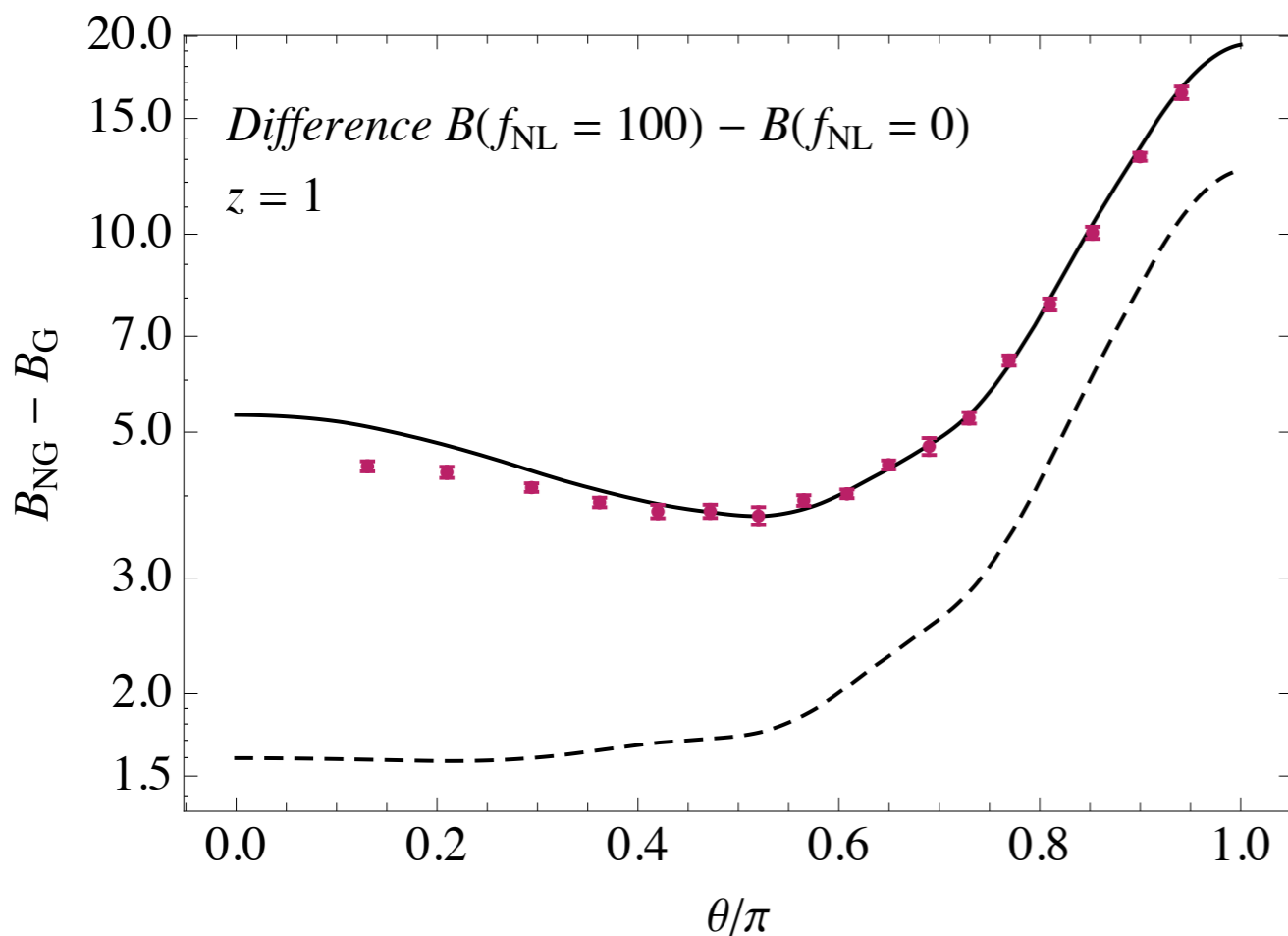
Gravity-induced contributions

**Additional** gravity-induced contributions present for NG initial conditions ( $B_0$ )

Generic configurations  $B(k_1, k_2, \theta)$  as a function of  $\theta$  with  $k_1 = 0.1 h/\text{Mpc}$ ,  $k_2 = 1.5 k_1$



ES (2009)  
ES, Crocce & Desjacques (2010)



# The matter **bispectrum** and PNG: *even smaller scales*

## Beyond PT: *The Halo Model*

$$B(k_1, k_2, k_3) = B_{3h}(k_1, k_2, k_3) + B_{2h}(k_1, k_2, k_3) + B_{1h}(k_1, k_2, k_3),$$

$$B_{3h}(k_1, k_2, k_3, z) = \frac{1}{\bar{\rho}^3} \left[ \prod_{i=1}^3 \int d m_i n(m_i, z) \hat{\rho}(m_i, z, k_i) \right] B_h(k_1, m_1; k_2, m_2; k_3, m_3; z),$$

$$B_{2h}(k_1, k_2, k_3, z) = \frac{1}{\bar{\rho}^3} \int d m n(m, z) \hat{\rho}(m, z, k_1) \int d m' n(m', z) \hat{\rho}(m', z, k_2) \hat{\rho}(m', z, k_3) \\ \times P_h(k_1, m, m', z) + \text{cyc.},$$

$$B_{1h}(k_1, k_2, k_3, z) = \frac{1}{\bar{\rho}^3} \int d m n(m, z) \hat{\rho}(k_1, m, z) \hat{\rho}(k_2, m, z) \hat{\rho}(k_3, m, z).$$

$$B_h(k_1, m_1; k_2, m_2; k_3, m_3; z) = b_1(m_1) b_1(m_2) b_1(m_3) B(k_1, k_2, k_3) \\ + [b_1(m_1) b_1(m_2) b_2(m_3) P(k_1) P(k_2) + \text{cyc.}]$$

# Galaxy bias and the galaxy power spectrum

Dalal *et al.* (2008):

**The bias of galaxies receives a significant *scale-dependent* correction for NG initial conditions of the *local* type**

$$P_g(k) = [b_1 + \Delta b_1(f_{NL}, k)]^2 P(k)$$

↓  
“Gaussian”  
bias

↓  
Scale-dependent correction  
due to local non-Gaussianity

QSOs+LRGs: **-31 < f<sub>NL</sub> < 70** (95% CL)

[Slosar *et al.* (2008)]

AGNs+QSOs+LRGs: **8 < f<sub>NL</sub> < 88** (95% CL)

[Xia *et al.* (2011)]

high-redshift sources: quasars & AGNs

**CMB limits (95% CL): -10 < f<sub>NL</sub> < 74**

[Komatsu *et al.* (2009)]

Limits from LSS are already competitive with the CMB!

(at least for the local model ...)

From EUCLID we expect:

$$\Delta f_{NL} \sim 5$$

from the 3D power spectrum alone or better with multitracers

[e.g. Giannantonio *et al.* (2011), Seljak (2009)  
Hamaus *et al.* (2011)]