The non-Gaussian signal in the galaxy bispectrum

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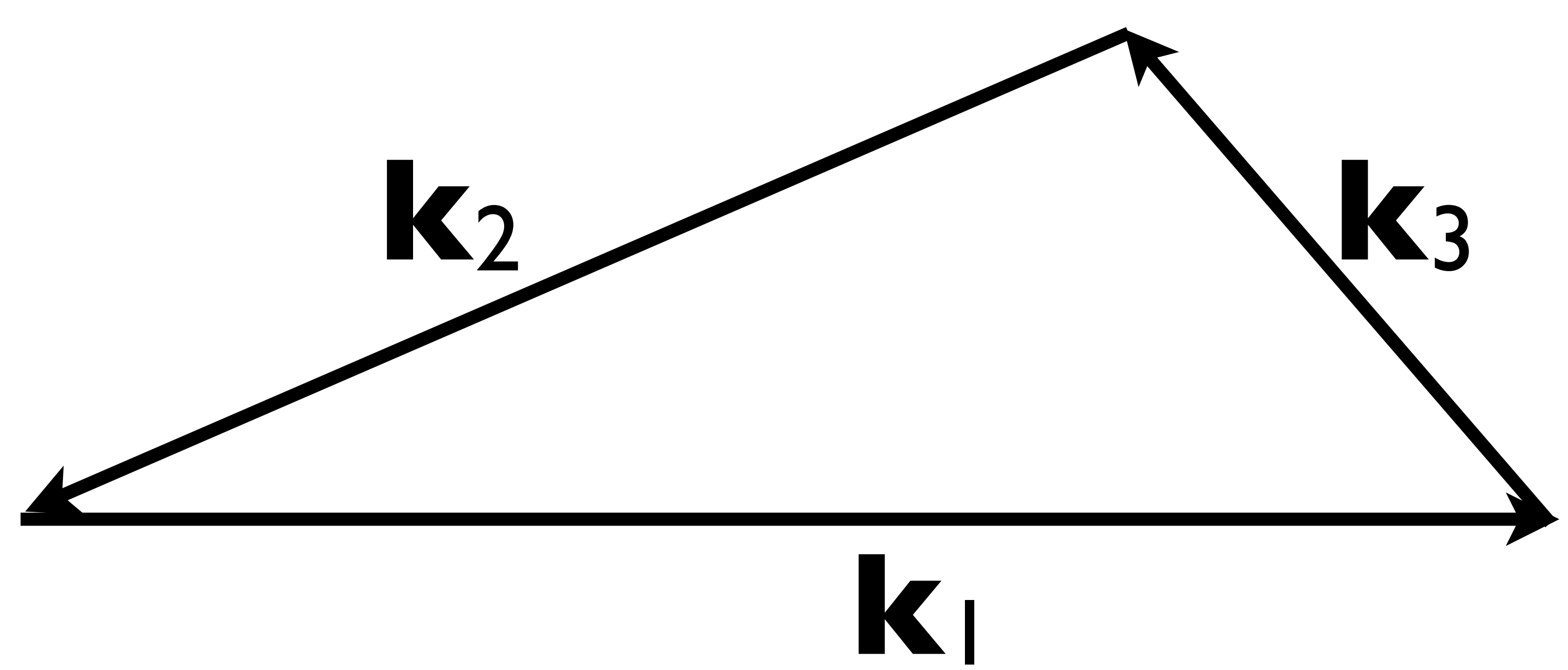
The galaxy bispectrum

- The galaxy bispectrum is the Fourier transform of the three point correlation function

$$\langle \delta_g(\boldsymbol{k}_1)\delta_g(\boldsymbol{k}_2)\delta_g(\boldsymbol{k}_3)\rangle = (2\pi)^3 \boldsymbol{B_g(k_1, k_2, k_3)}\delta^D(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)$$

- The galaxy bispectrum depends on triangular configurations.



Signal of the galaxy bispectrum

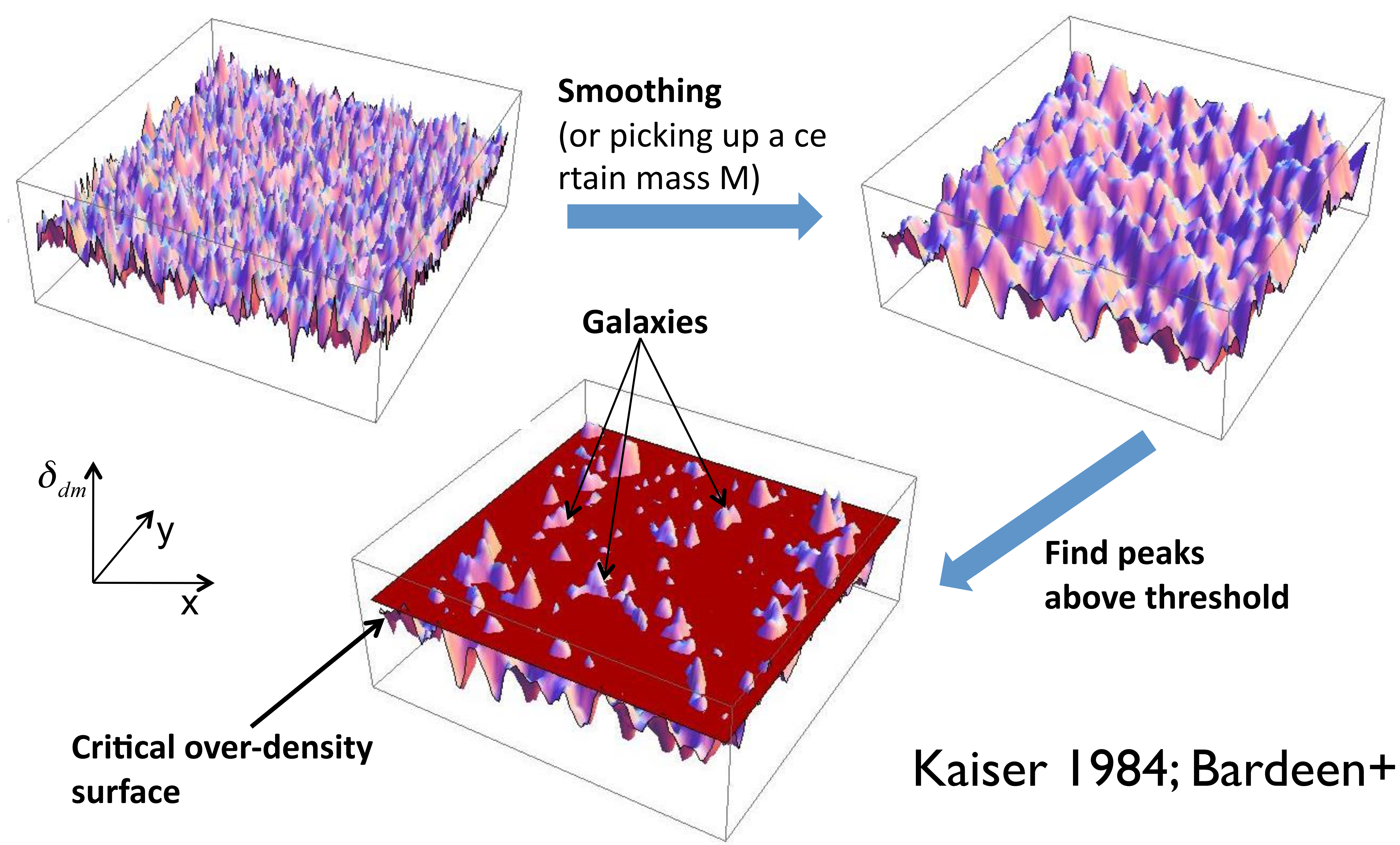
- In the Gaussian universe, the bispectrum for linear galaxy density field vanishes.
- Therefore, bispectrum signal consists of
  - Non-linear matter clustering
  - Non-linear bias
  - Non-linear redshift space distortion
  - Linearly evolved non-Gaussian matter bispectrum
  - Scale dependent non-Gaussian bias

The local bias assumption

$$\delta_g(\boldsymbol{x}) = \epsilon(\boldsymbol{x}) + f(\delta_m(\boldsymbol{x}))$$ Fry & Gaztanaga 1993

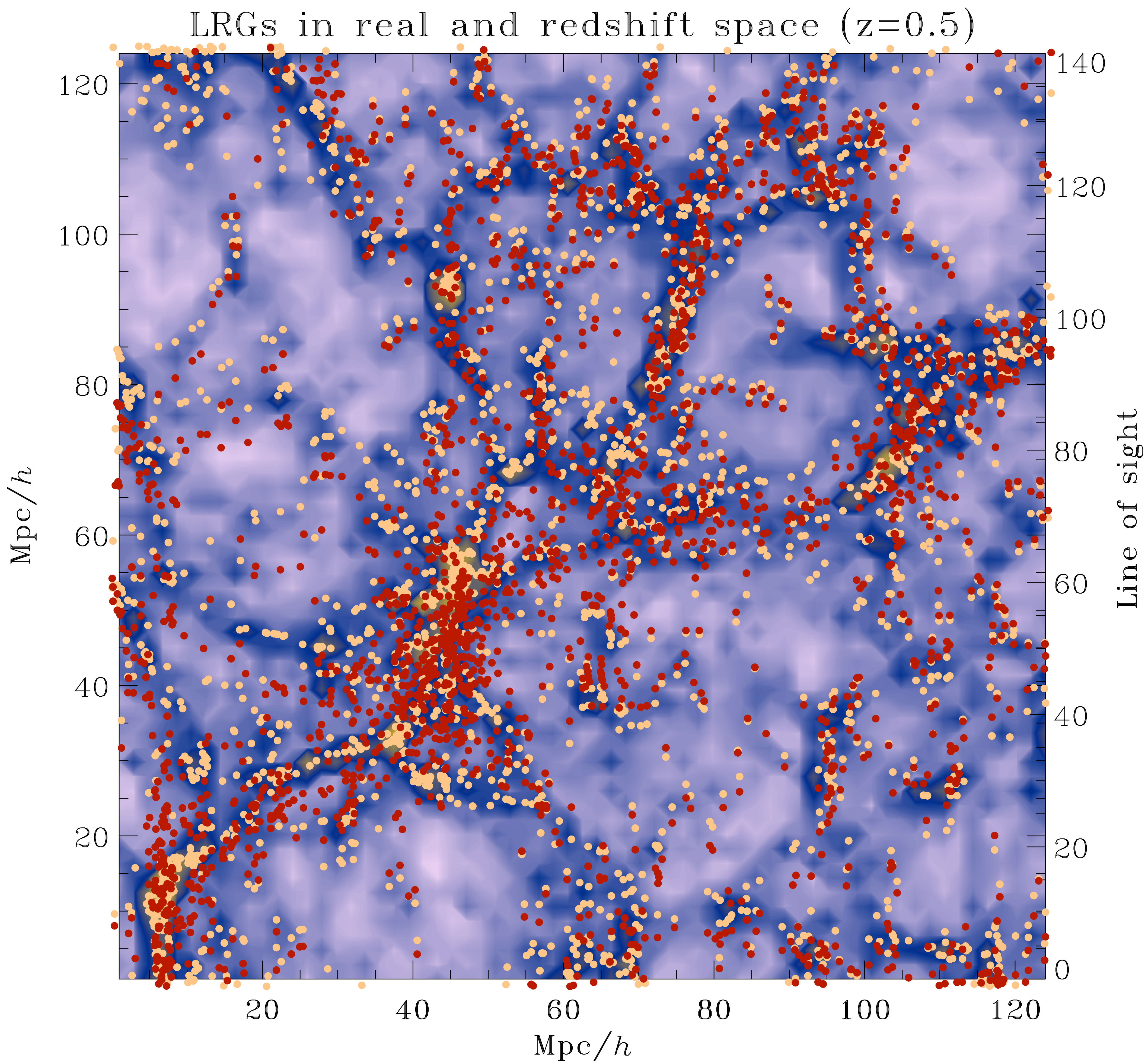
$$= \epsilon(\boldsymbol{x}) + c_1\delta_m(\boldsymbol{x}) + \frac{c_2}{2}\delta_m^2(\boldsymbol{x}) + \frac{c_3}{6}\delta_m^3(\boldsymbol{x}) + \cdots$$

e.g.
selecting
high peaks



Kaiser 1984; Bardeen+ 1986 (BBKS)

Redshift space distortion



- Blue : matter distribution
  orange: real space LRGs
  red: redshift space LRGs
- We infer the distance to galaxies by spectral shift.
- spectral shift = true redshift + peculiar velocity
- As peculiar velocity is correlated with density, it induce the systematic change in bispectrum.

MPA galaxy catalogues from Millennium Simulation, figure by Olivier Dore

Bispectrum from non-linearities

- For a generic non-linear density field (δ_L = linear)

$$\delta(\boldsymbol{k}) = K_1^{(s)}(\boldsymbol{k})\delta_L(\boldsymbol{k}) + \int \frac{d^3q_1}{(2\pi)^3} \int d^3q_2 \delta^D(\boldsymbol{k} - \boldsymbol{q}_1 - \boldsymbol{q}_2) K_2^{(s)}(\boldsymbol{q}_1, \boldsymbol{q}_2)\delta_L(\boldsymbol{q}_1)\delta_L(\boldsymbol{q}_2) + \cdots$$

  the leading order (tree-level) bispectrum is given by

$$B(k_1, k_2, k_3) = 2K_1^{(s)}(\boldsymbol{k}_1)K_1^{(s)}(\boldsymbol{k}_2)K_2^{(s)}(\boldsymbol{k}_1, \boldsymbol{k}_2)P_L(k_1)P_L(k_2) + (\text{cyclic})$$

- Next-to-leading order bispectrum requires fourth order kernel (same physics, just a bit messy):
  $B^{(1\text{-Loop})}$ = <114> + <123> + <222>

2nd order Kernel in PT

- Perturbation theory with local bias assumption reads the kernels : Heavens+(1998), Scoccimarro+(1999), Jeong (2010).

$$K_1^{(s)}(\boldsymbol{k}) = b_1 + f\mu^2$$

$$K_2^{(s)}(\boldsymbol{q}_1, \boldsymbol{q}_2) = \frac{b_2}{2} + b_1 F_2^{(s)}(\boldsymbol{q}_1, \boldsymbol{q}_2) + f\mu^2 G_2^{(s)}(\boldsymbol{q}_1, \boldsymbol{q}_2) + b_1 \frac{fk\mu}{2}\left[\frac{q_{1z}}{q_1^2} + \frac{q_{2z}}{q_2^2}\right] + \frac{(fk\mu)^2}{2}\frac{q_{1z}q_{2z}}{q_1^2 q_2^2}$$

z = line of sight direction, μ = k·z, and

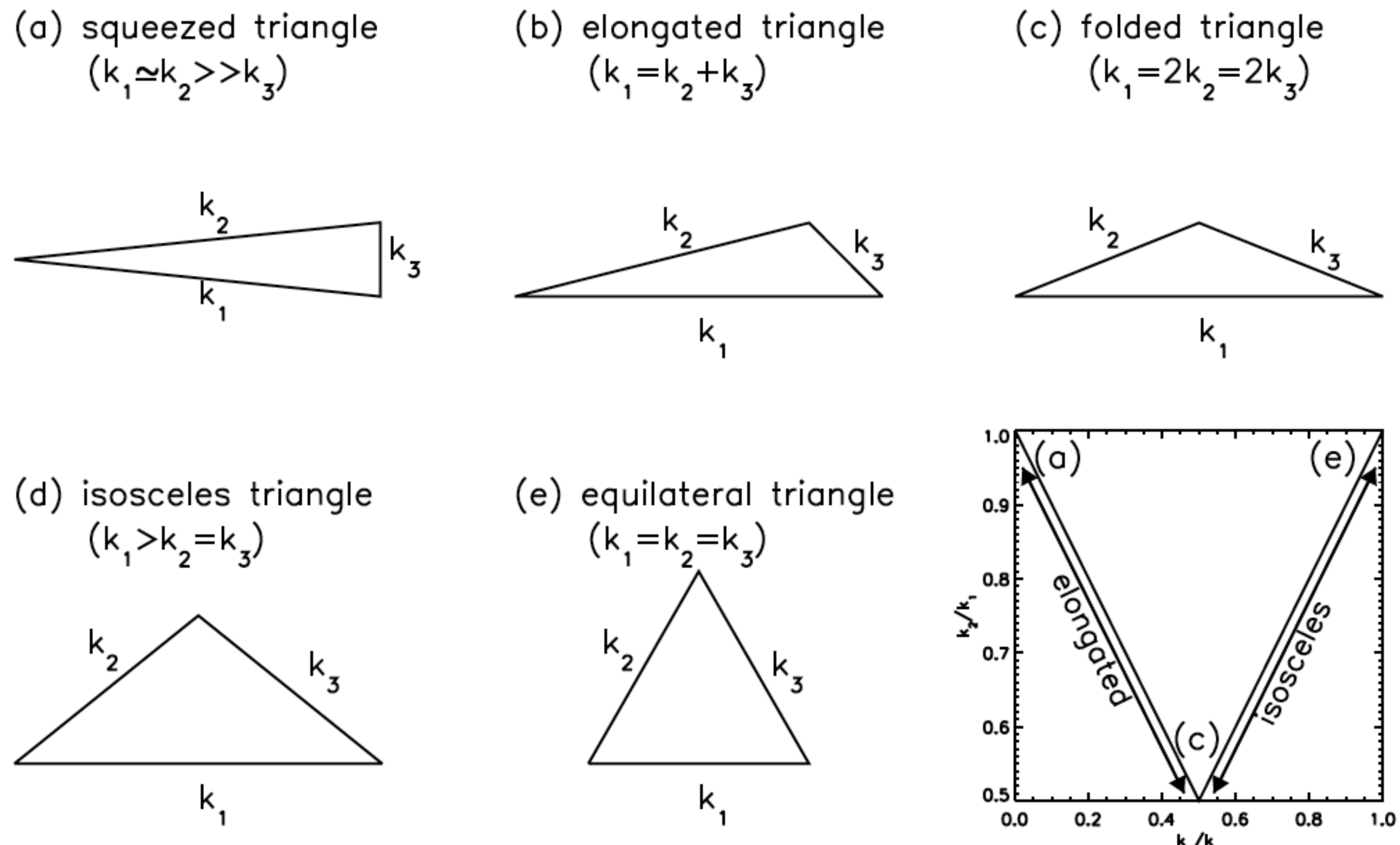
$$F_2^{(s)}(\boldsymbol{q}_1, \boldsymbol{q}_2) = \frac{5}{7} + \frac{2}{7}(\hat{q}_1 \cdot \hat{q}_2)^2 + \frac{\hat{q}_1 \cdot \hat{q}_2}{2}\left(\frac{q_1}{q_2} + \frac{q_2}{q_1}\right)$$

$$G_2^{(s)}(\boldsymbol{q}_1, \boldsymbol{q}_2) = \frac{3}{7} + \frac{4}{7}(\hat{q}_1 \cdot \hat{q}_2)^2 + \frac{\hat{q}_1 \cdot \hat{q}_2}{2}\left(\frac{q_1}{q_2} + \frac{q_2}{q_1}\right)$$

F_2 and G_2 are maximum when $q_1 // q_2$ and 0 when $q_1 = -q_2$

Jeong & Komatsu, 2009

Name of triangles ($k_1 \geq k_2 \geq k_3$)



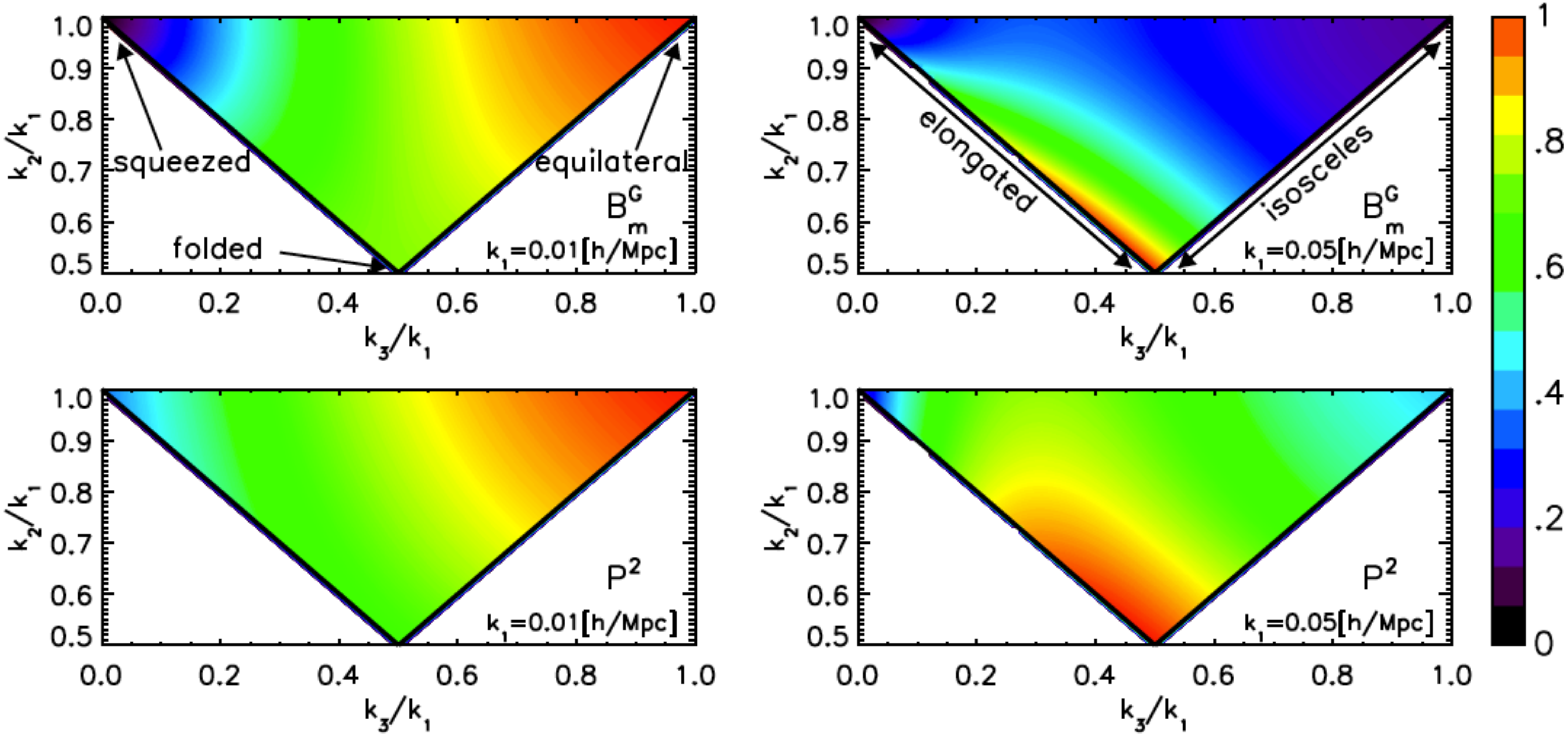
Jeong & Komatsu, 2009

Bispectrum of Gaussian Universe

- Non-linear terms peaks at Equilateral and Folded triangles.

non-linear gravity

non-linear bias



NG galaxy bispectrum

- MLB formula (Matarrese et al. 1986) reads (high-peak limit) non-Gaussian galaxy bispectrum in terms of matter bispectrum and trispectrum as (See also, Sefusatti 2009)

$$B_g(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = b_1^3\left[B_R(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) + \frac{b_2}{b_1}\left\{P_R(k_1)P_R(k_2) + (2\text{ cyclic})\right\}\right.$$
$$\left. + \frac{\delta_c}{2\sigma_R^2}\int\frac{d^3q}{(2\pi)^3}T_R(\boldsymbol{q},\boldsymbol{k}_1-\boldsymbol{q},\boldsymbol{k}_2,\boldsymbol{k}_3) + (2\text{ cylic})\right]$$

- Note that we assumes that galaxies form at the density threshold in the Eulerian (evolved) density field.

Jeong & Komatsu, 2009

Understanding bispectrum

linearly evolved primordial bispectrum ∝ f_{NL}

$$B_R(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = \mathcal{M}_R(k_1)\mathcal{M}_R(k_2)\mathcal{M}_R(k_3)B_\Phi(k_1, k_2, k_3) + 2F_2^{(s)}(\boldsymbol{k}_1, \boldsymbol{k}_2)P_R(k_1)P_R(k_2) + (2\ \text{cyclic})$$

matter bispectrum due to non-linear gravity

Gaussian terms

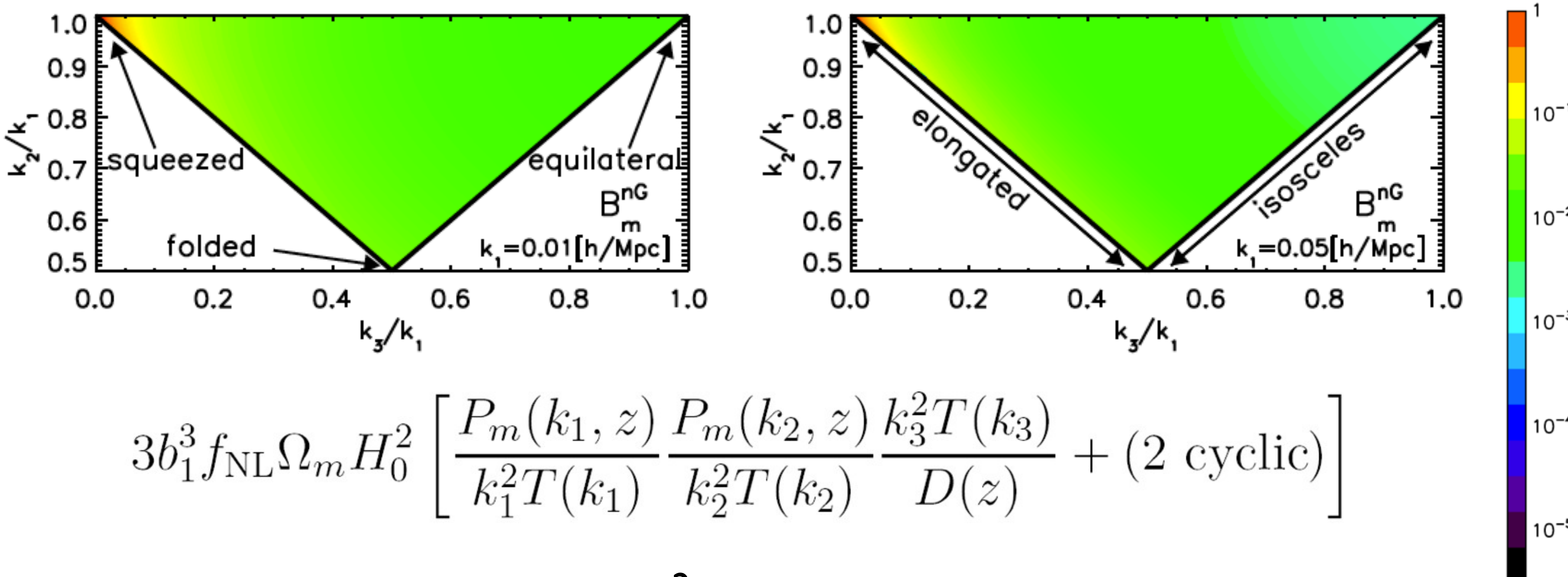
Matter B(k) non-linear bias term

$$B_g(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = b_1^3\left[B_R(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) + \frac{b_2}{b_1}\{P_R(k_1)P_R(k_2) + (2\ \text{cyclic})\} + \frac{\delta_c}{2\sigma_R^2}\int \frac{d^3q}{(2\pi)^3} T_R(\boldsymbol{q}, \boldsymbol{k}_1 - \boldsymbol{q}, \boldsymbol{k}_2, \boldsymbol{k}_3) + (2\ \text{cylic})\right]$$

"peak" correlation terms ∝ f_{NL}, g_{NL}, τ_{NL} ($\sim f_{NL}^2$)

Jeong & Komatsu, 2009

Matter bispectrum due to f_{NL}

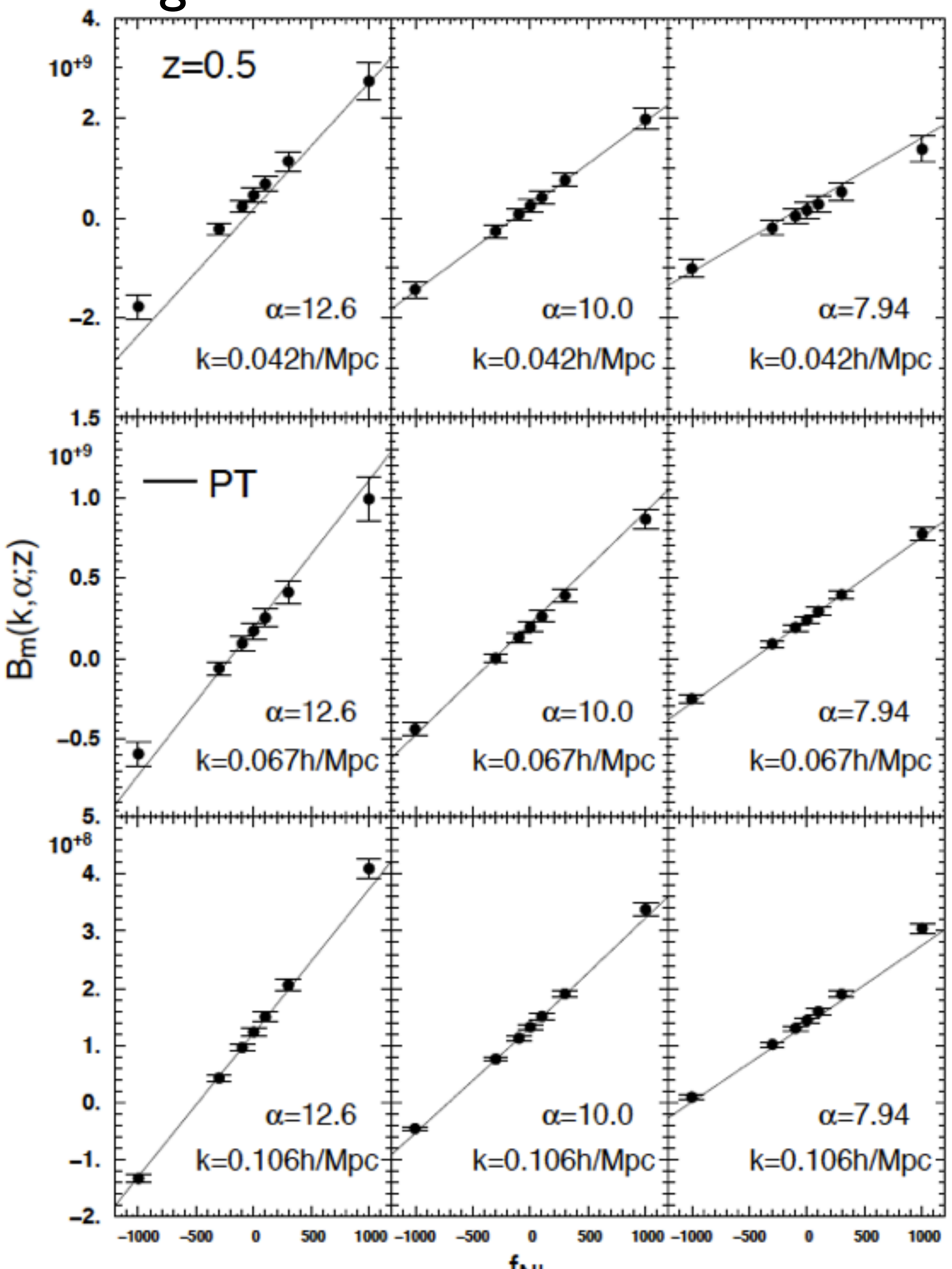


$$3b_1^3 f_{\rm NL}\Omega_m H_0^2\left[\frac{P_m(k_1,z)}{k_1^2T(k_1)}\frac{P_m(k_2,z)}{k_2^2T(k_2)}\frac{k_3^2T(k_3)}{D(z)}+(2\ \text{cyclic})\right]$$

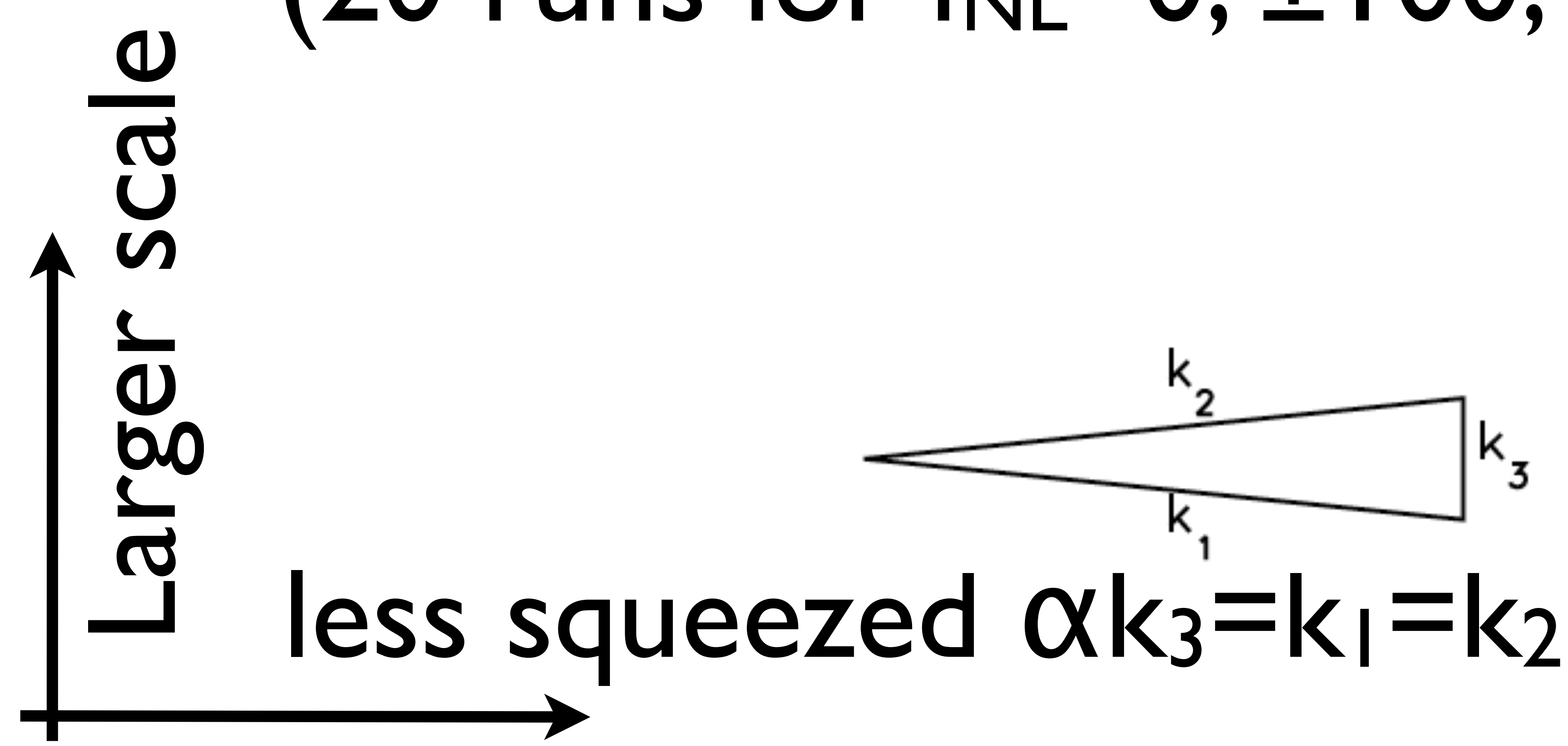
- Notice the factor of k^2 in the denominator.
- It sharply peaks at squeezed triangles!

N-body matter bispectrum

Figure from Nishimich et al. 2010



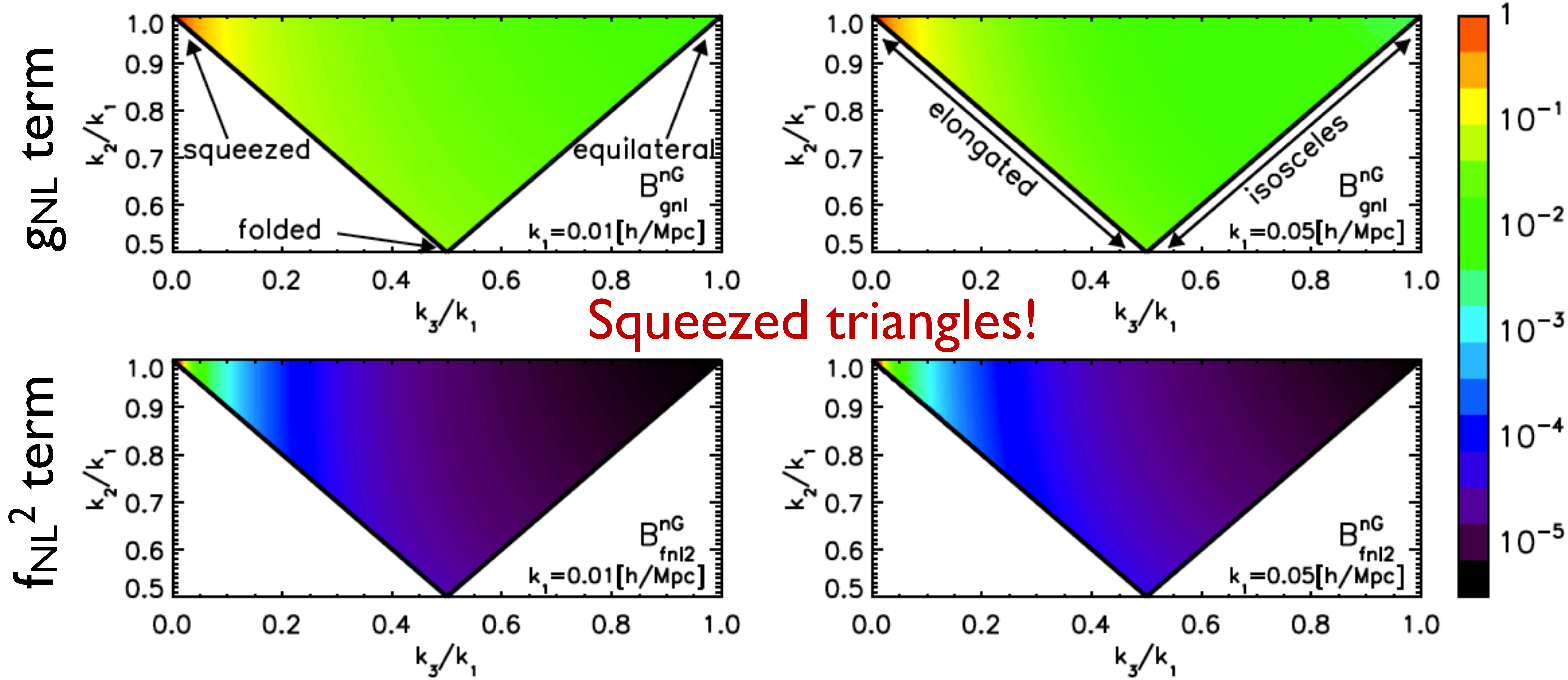
- Theory agrees with N-body!
- Solid = theoretical prediction
- data = 140 N-body simulations 512^3 particles in $(2\ [\mathrm{Gpc}/h])^3$, (20 runs for f_{NL}=0, ±100, ±300, ±1000)



Jeong & Komatsu, 2009

linearly evolved primordial T_Φ

$$T_R^{(1111)}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4) = \prod_{i=1}^{4} \mathcal{M}_{\mathcal{R}}(k_i) T_\Phi(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4) \ni \mathbf{g}_{\mathrm{NL}}, \tau_{\mathrm{NL}} (\text{or } \mathbf{f}_{\mathrm{NL}}^{\mathbf{2}})$$



Jeong & Komatsu, 2009

$T^{(1112)}(k_1,k_2,k_3,k_4)$

- This term is the matter trispetrum generated by non-linearly evolved primordial "quadspectrum"

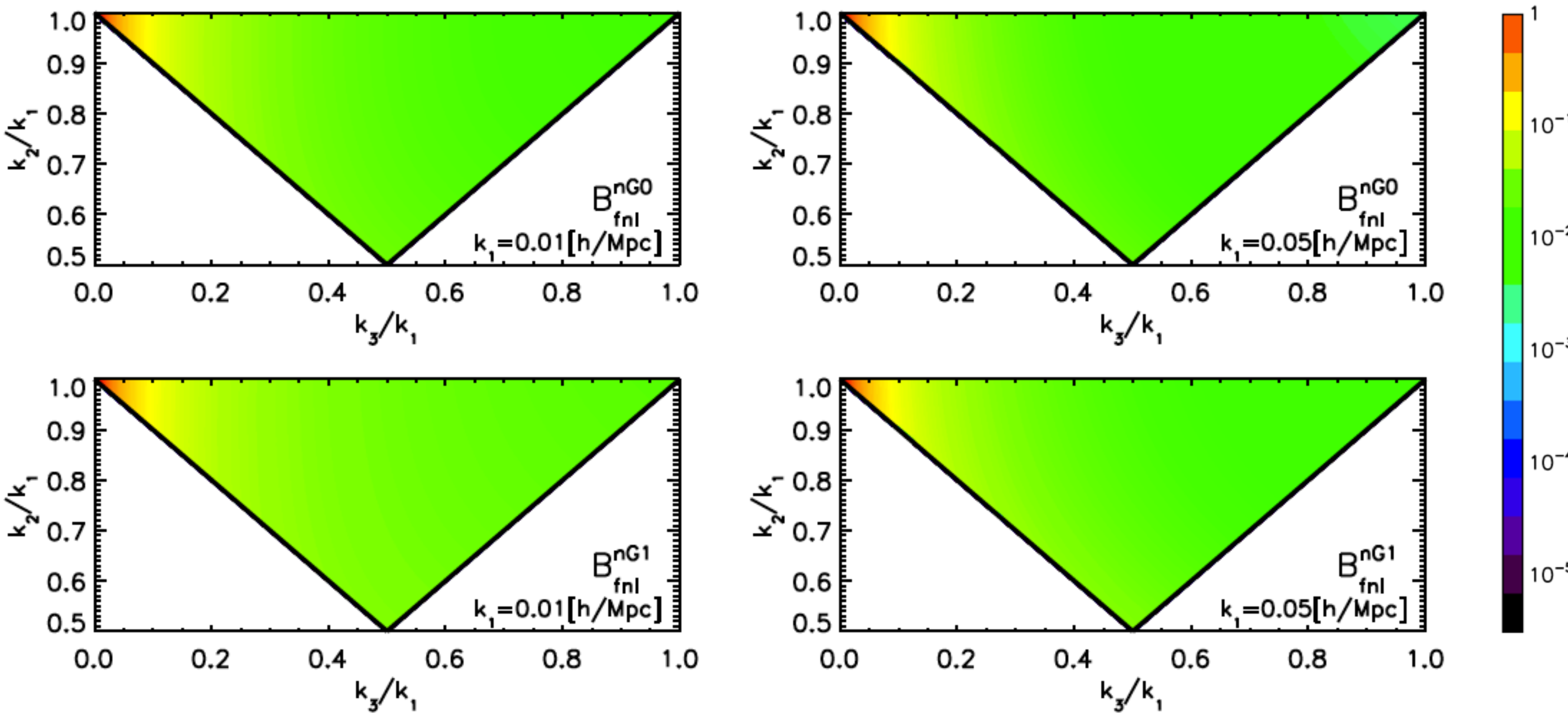
$$\langle\delta^{(1)}(\boldsymbol{k}_1)\delta^{(1)}(\boldsymbol{k}_2)\delta^{(1)}(\boldsymbol{k}_3)\delta^{(2)}(\boldsymbol{k}_4)\rangle$$

$$= \int \frac{d^3q}{(2\pi)^3} F_2^{(s)}(\boldsymbol{q}, \boldsymbol{k}_4 - \boldsymbol{q})\langle\delta^{(1)}(\boldsymbol{k}_1)\delta^{(1)}(\boldsymbol{k}_2)\delta^{(1)}(\boldsymbol{k}_3)\delta^{(1)}(\boldsymbol{k}_4 - \boldsymbol{q})\delta^{(1)}(\boldsymbol{q})\rangle$$

$$= (2\pi)^3 \Big[2\boxed{f_{\rm NL}} P_m(k_1)\mathcal{M}(k_3) \int d^3q \mathcal{M}(q)\mathcal{M}(|\boldsymbol{k}_4 - \boldsymbol{q}|)P_\phi(q)\left\{P_\phi(|\boldsymbol{k}_4 - \boldsymbol{q}|) + 2P_\phi(k_3)\right\}$$
$$\times F_2^{(s)}(\boldsymbol{q}, \boldsymbol{k}_4 - \boldsymbol{q})\delta^D(\boldsymbol{k}_{12}) + 4\boxed{f_{\rm NL}}\mathcal{M}(k_2)\mathcal{M}(k_3)\mathcal{M}(k_{14})P_m(k_1)F_2^{(s)}(-\boldsymbol{k}_1, \boldsymbol{k}_{14})$$
$$\times \left\{P_\phi(k_2)P_\phi(k_3) + P_\phi(k_2)P_\phi(k_{14}) + P_\phi(k_3)P_\phi(k_{14})\right\} + (\text{cyclic } 123)\Big] \delta^D(\boldsymbol{k}_{1234}).$$

Jeong & Komatsu, 2009

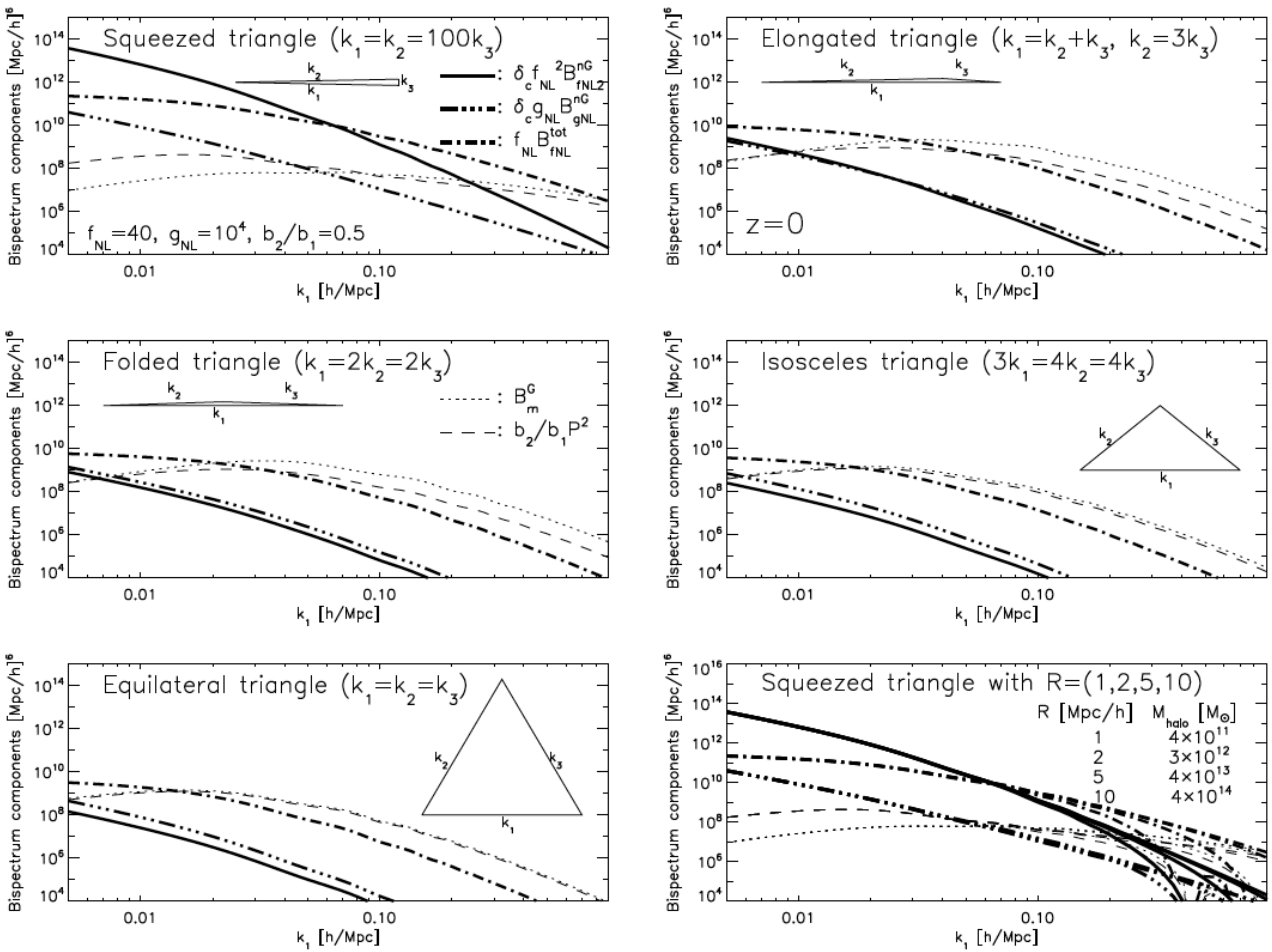
Shape of $T^{(1112)}(k_1,k_2,k_3,k_4)$



Again, Squeezed triangles!

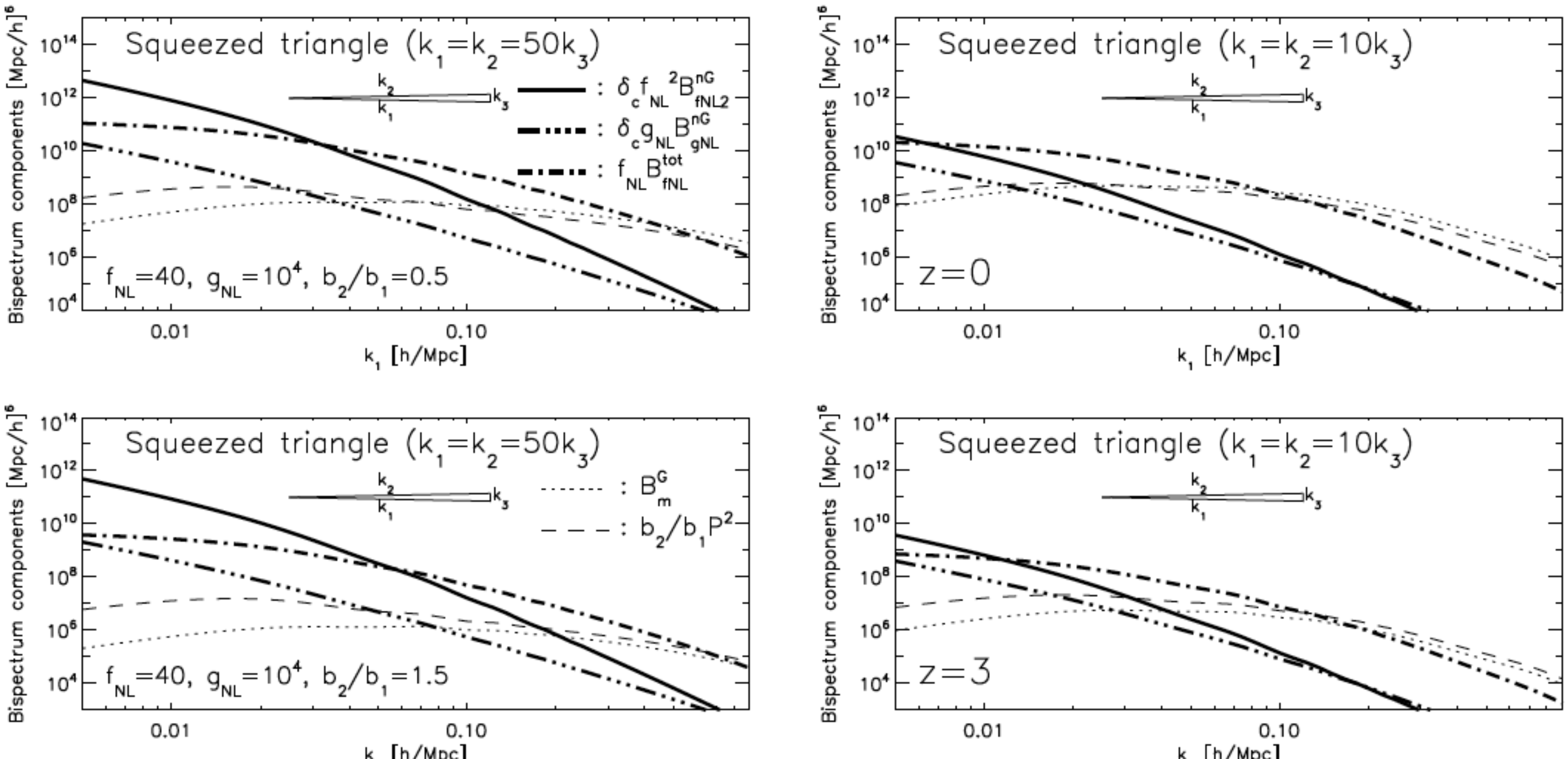
Jeong & Komatsu, 2009

nG terms are important !



Jeong & Komatsu, 2009

Result for mildly squeezed triangles

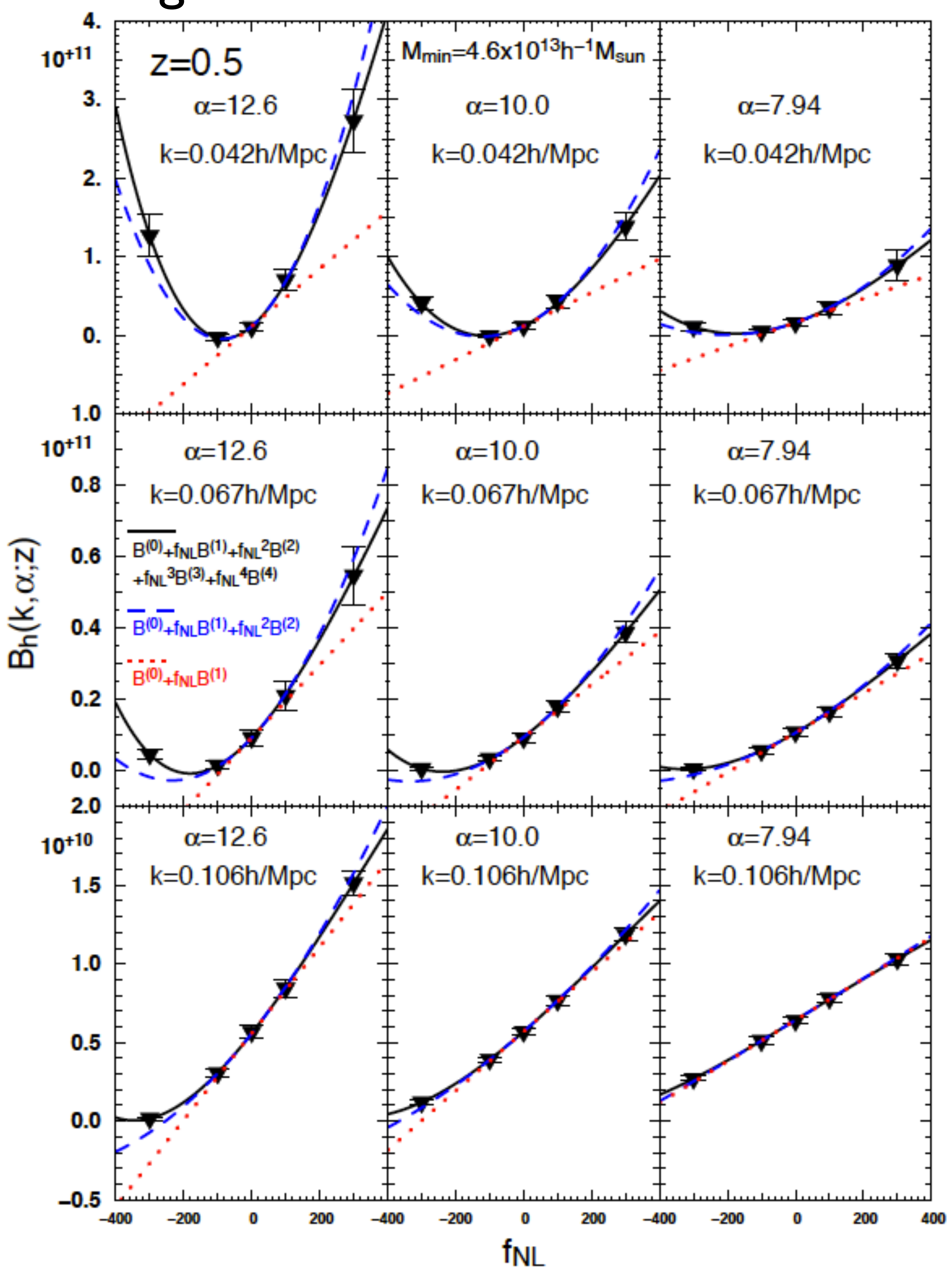


f_{NL}^2 and g_{NL} terms are more important at high-z

$\propto 1/D^2(z)$, [c.f. f_{NL} terms $\propto 1/D(z)$]

N-body result for squeezed B_g

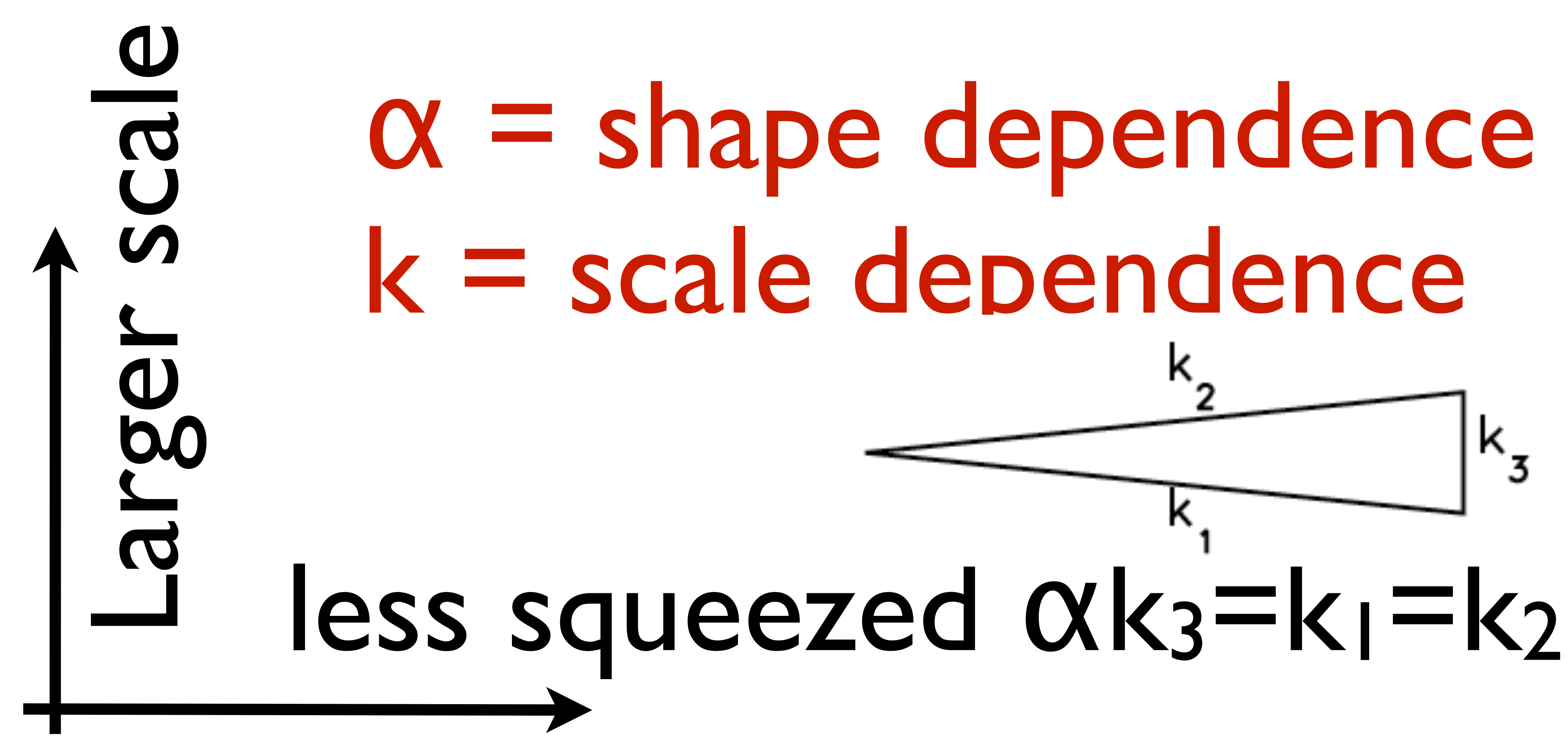
Figure from Nishimich et al. 2010



- In Nishimichi et al., 2010, they fit the resulting galaxy bispectrum by $B_g(k,\alpha)$ $=B^{(0)}(k,\alpha)+f_{NL}B^{(1)}(k,\alpha)+f_{NL}{}^2B^{(2)}(k,\alpha)$ and find a good agreement!

α = shape dependence

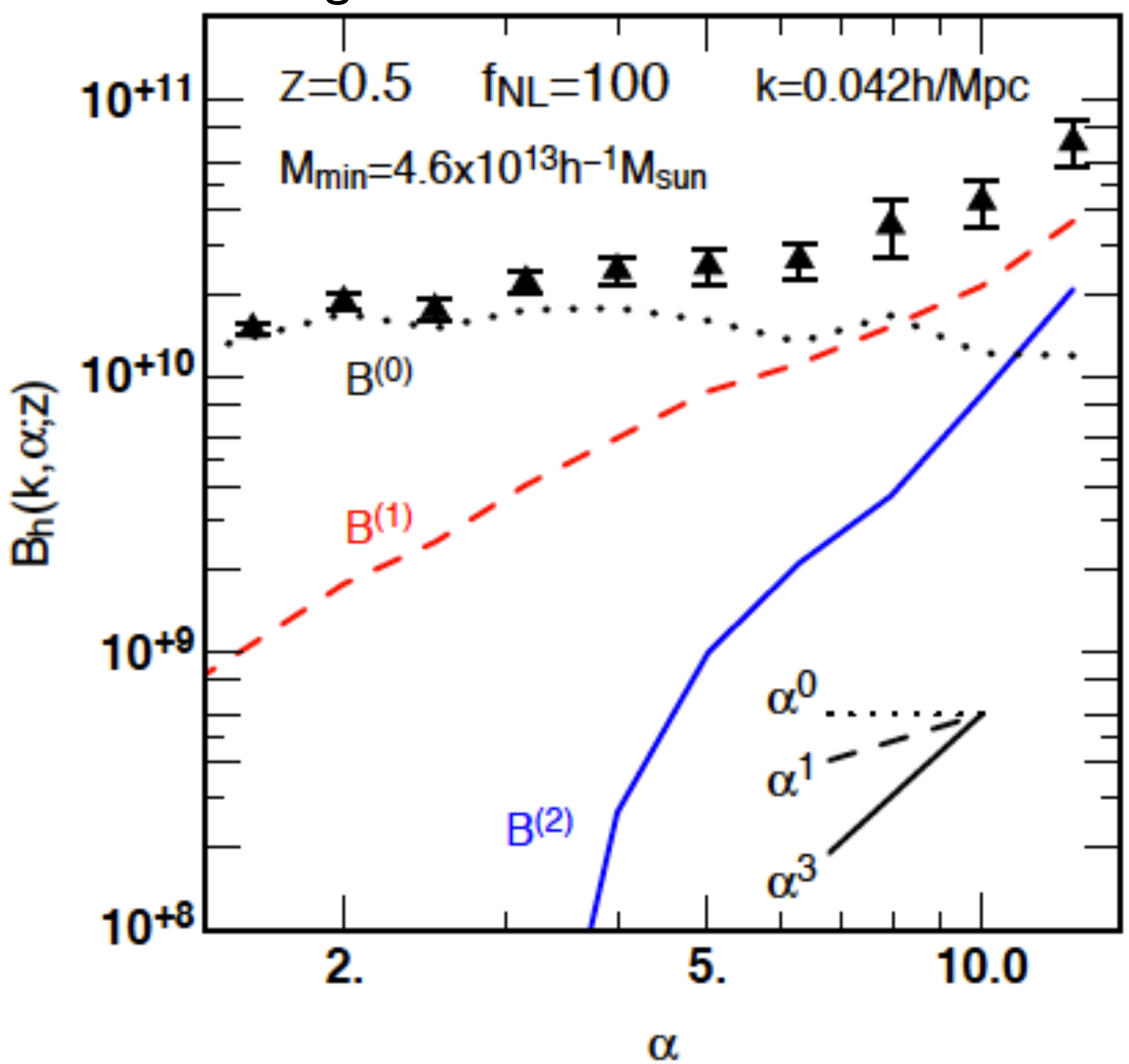
k = scale dependence



$\alpha k_3=k_1=k_2$

Shape dependence of B_g

Figure from Nishimich et al. 2010



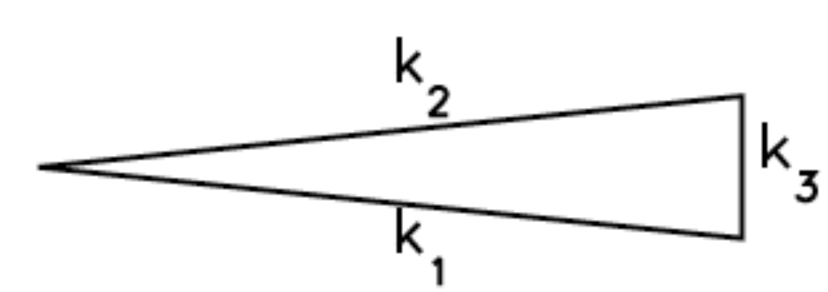
- Shape dependence agrees with the theory prediction in Jeong & Komatsu, 2009.

$B^{(0)}(\alpha) \propto \text{const.}$

$B^{(1)}(\alpha) \propto \alpha$

$B^{(2)}(k, \alpha) \propto \alpha^3$

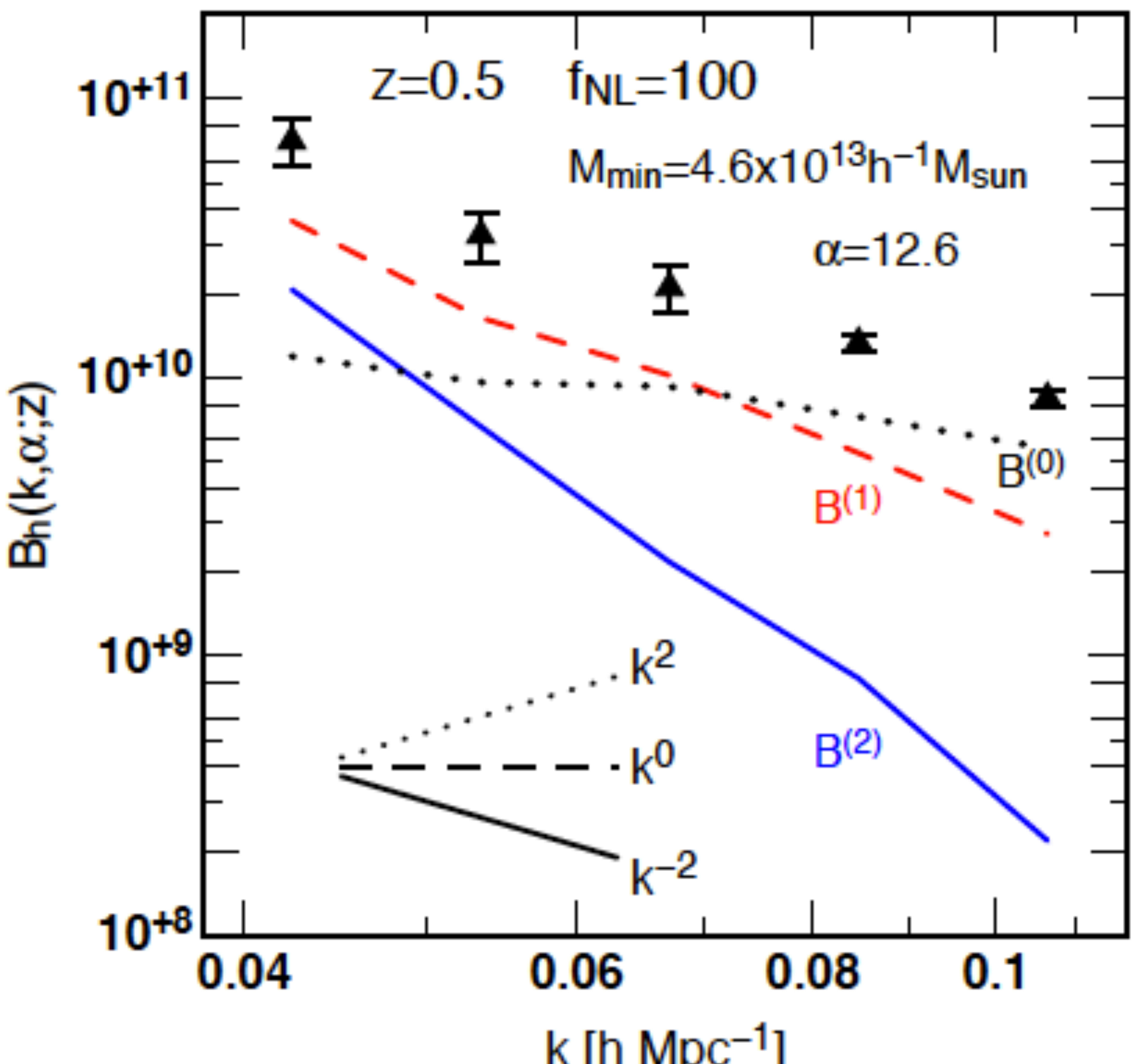
$B_g(k,\alpha) = B^{(0)}(k, \alpha) + f_{NL} B^{(1)}(k, \alpha) + f_{NL}^2 B^{(2)}(k, \alpha)$



$\alpha k_3 = k_1 = k_2 = k$

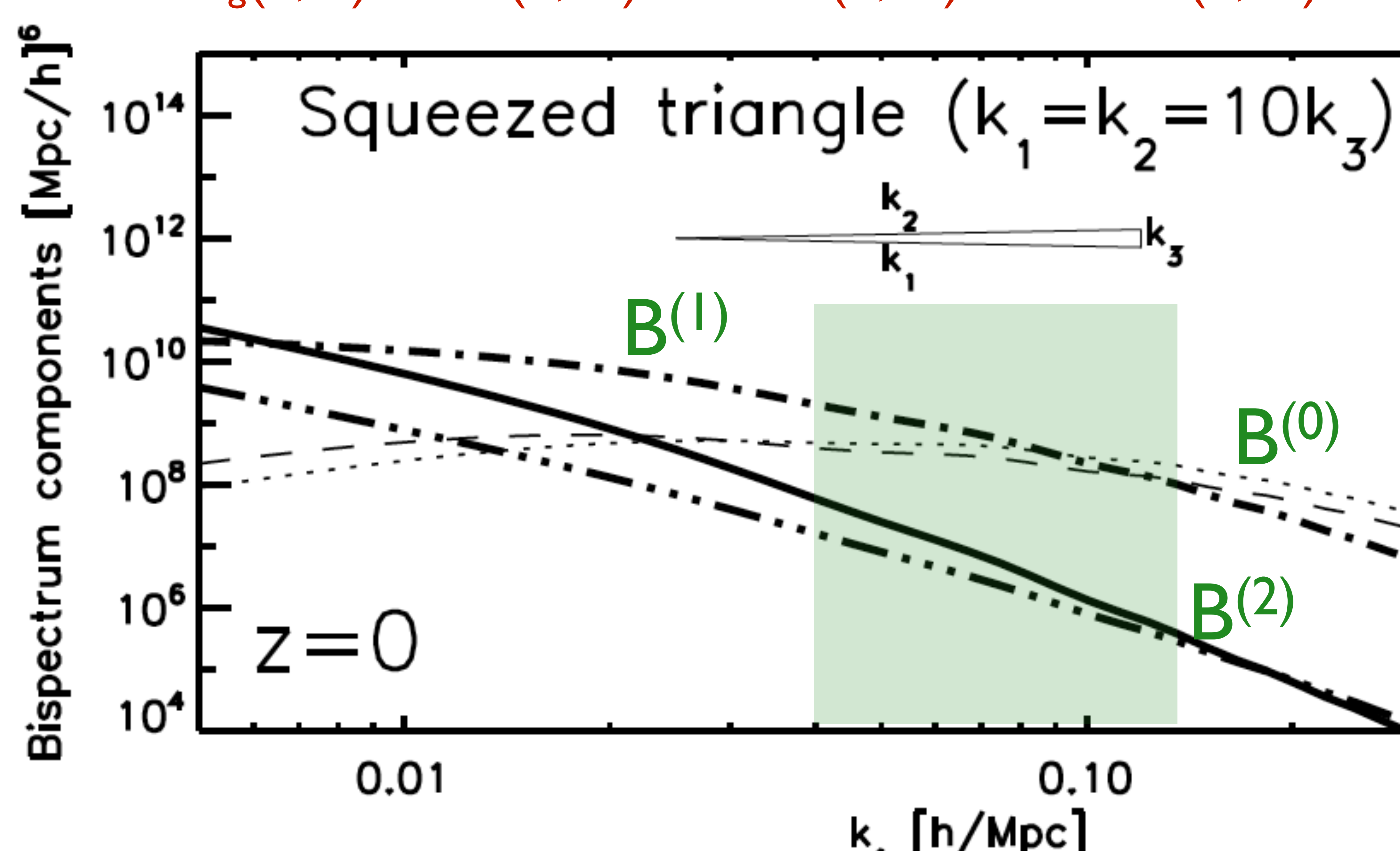
Scale dependence of B_g

Figure from Nishimich et al. 2010



- Scale dependence also agrees with Jeong & Komatsu, 2009.

$B_g(k,\alpha) = B^{(0)}(k, \alpha) + f_{NL} B^{(1)}(k, \alpha) + f_{NL}^2 B^{(2)}(k, \alpha)$



Prediction for galaxy surveys

- Uncertainty after marginalizing over bias parameters. (Planck=5)

	z	V [Gpc/h]^3	n_g 10^{-5}[h/Mpc]^3	k_{max} [h/Mpc]	Δf_{NL} P(k)	Δf_{NL} Bk	Δf_{NL} Bk	$\Delta \tau_{NL}$ Bk
SDSS LRG	0.315	1.48	136	0.1	41.80	5.62	12.12	28132
BOSS	0.35	5.66	26.6	0.1	21.25	3.34	5.41	6300
HETDEX	2.7	2.07	38.65	0.2	12.4	3.65	5.19	4240
CIP	2.25	6.54	500	0.2	7.86	**1.03**	1.35	**952**
BigBOSS LRG	0.5	13.1	30	0.1	11.59	2.27	3.13	2399
BigBOSS QSO	2.15	138.2	5	0.1	7.80	17.02	17.11	1500
WFIRST	1.5	107.3	93.7	0.1	2.73	**1.11**	1.18	**322**
EUCLID	1.0	102.9	156	0.1	3.70	**0.92**	1.00	**293**

Caution!!

- Signal : assumed thresholded regions
  Although shape and scale dependence is correct, amplitude of non-Gaussian correction is smaller for normal galaxies
- Noise : assumed Gaussian, diagonal error
  Covariance between different triangles are ignored
- Survey : ignored window function effect
  assumed cubic survey volume with constant mean number density
- For more realistic forecast, please wait for Emiliano's talk!

To measure NG from bispectrum

- Signal : Detailed comparison between theory and N-body simulations is needed (RSD, non-local NG)
- Noise : NG covariance among triangles, optimal estimator
- Mock : How to generate mock from given Pk and Bk? How do we resolve kernel ambiguity? Need N-body? #LPT?
- Random/Correlated errors in window function (e.g. extinction, transparency) can mimic NG.
- Wide-angle effect has to be included for measuring very long wavelength mode

Conclusion

- The real meat (humus, if you are vegetarian) is in the galaxy bispectrum.
- Therefore, we need to work hard to measure NG from the galaxy bispectrum!!