CMB non-Gaussianities from adiabatic & isocurvature perturbations

David Langlois
(APC, Paris)

Astroparticules
et Cosmologie

Outline

1. Primordial isocurvature modes
2. Generalized CMB angular bispectra and Planck prospects
3. Isocurvature non-Gaussianities from a curvaton scenario

Mainly based on DL & B. van Tent, JCAP 1207 (2012) 040 [arXiv:1204.5042]

Adiabatic & isocurvature perturbations

- **Several matter components** in the early Universe:
  photons, neutrinos, CDM, baryons $(X = \gamma, \nu, c, b)$

- Initial conditions for the perturbations determined by **the gauge-invariant quantities**

$$\zeta_X \equiv -\psi - \frac{H}{\dot{\rho}_X}\delta\rho_X$$

- Usual assumption: **adiabatic** initial conditions

  characterized by $$\frac{\delta n_c}{n_c} = \frac{\delta n_b}{n_b} = \frac{\delta n_\nu}{n_\nu} = \frac{\delta n_\gamma}{n_\gamma}$$

  or, equivalently, $$\zeta_c = \zeta_b = \zeta_\nu = \zeta_\gamma = \zeta \qquad \left[\zeta \equiv -\psi - \frac{H}{\dot{\rho}}\delta\rho\right]$$

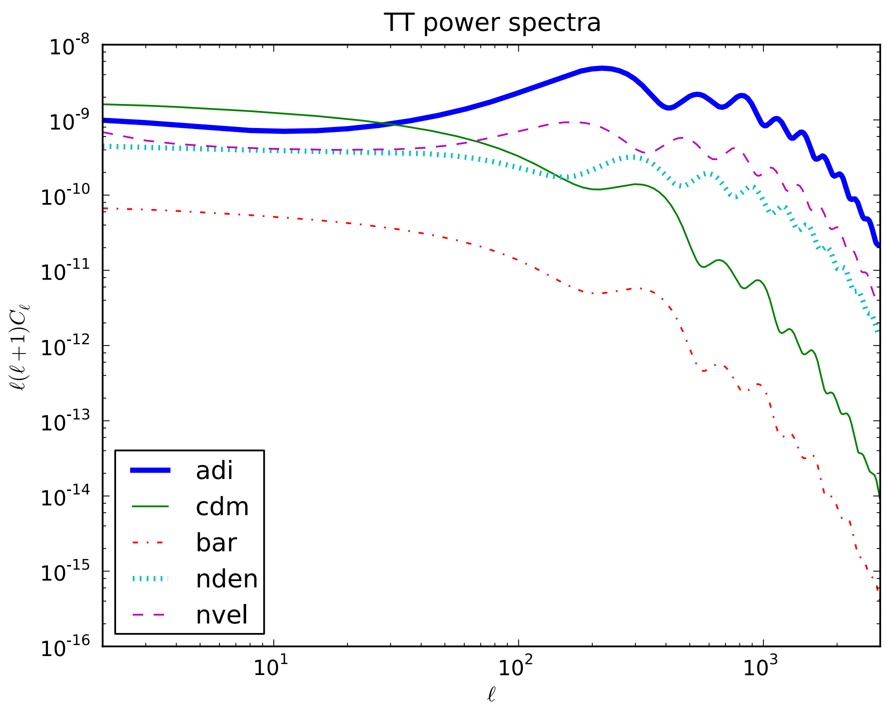
Adiabatic & isocurvature perturbations

- But multi-field inflation can lead to **isocurvature** perturbations

$$S_X \equiv 3(\zeta_X - \zeta_\gamma) \neq 0$$

- **Adiabatic** and **isocurvature** perturbations are characterized by **different transfer functions**.

$$C_\ell^{(I)} = \frac{2}{\pi} \int k^2 dk \left[g_\ell^I(k)\right]^2 P(k)$$



Isocurvature perturbations

- Adiabatic and isocurvature perts can be **correlated**.

- Present constraints on the CDM isocurvature fraction

[WMAP7+BAO+SN]

$$\frac{\mathcal{P}_S}{\mathcal{P}_\zeta} = \alpha \equiv \frac{a}{1-a} \qquad a_0 < 0.064 \quad (95\%\mathrm{CL})$$

$$a_1 < 0.0037 \quad (95\%\mathrm{CL})$$

depending on the correlation $\mathcal{C} \equiv \frac{\mathcal{P}_{S,\zeta}}{\sqrt{\mathcal{P}_S \mathcal{P}_\zeta}}$

- **Non-Gaussianities from isocurvature modes ?**
  - If isocurvature modes exist, can they contribute to NG ?
  - What would be their observational signature in the CMB ?

CMB anisotropies

- Combination of adiabatic and isocurvature perturbations

$$X^I = \{\zeta, S\}$$

- Temperature anisotropies

$$\frac{\Delta T}{T} = \sum_{lm} a_{lm} Y_{lm}, \qquad a_{lm} = 4\pi(-i)^l \int \frac{d^3\mathbf{k}}{(2\pi)^3} \left(\sum_I X^I(\mathbf{k}) g_l^I(k) \right) Y_{lm}^*(\hat{\mathbf{k}})$$

- Angular power spectrum

$$C_l = \langle a_{lm}\, a_{lm}^* \rangle = \sum_{I,J} \frac{2}{\pi} \int_0^\infty dk\, k^2 g_l^I(k) g_l^J(k) P_{IJ}(k)$$

with $\langle X^I(\mathbf{k}_1) X^J(\mathbf{k}_2) \rangle \equiv (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2) P_{IJ}(k_1)$

Angular bispectrum

Komatsu & Spergel 01; Bartolo, Matarrese & Riotto 02 ….

- Three-point function

$$\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle = b_{l_1 l_2 l_3} \int d^2\mathbf{n}\, Y_{l_1 m_1}(\mathbf{n})\, Y_{l_2 m_2}(\mathbf{n})\, Y_{l_3 m_3}(\mathbf{n})$$

with he **reduced angular bispectrum**

$$b_{l_1 l_2 l_3} = \sum_{I,J,K} \left(\frac{2}{\pi}\right)^3 \int k_1^2\, dk_1 \int k_2^2\, dk_2 \int k_3^2\, dk_3\; g^I_{l_1}(k_1)\, g^J_{l_2}(k_2)\, g^K_{l_3}(k_3)$$
$$B_{IJK}(k_1, k_2, k_3) \int_0^\infty r^2 dr\, j_{l_1}(k_1 r)\, j_{l_2}(k_2 r)\, j_{l_3}(k_3 r)$$

which depends on the **generalized primodial bispectra**

$$\langle X^I(\mathbf{k}_1) X^J(\mathbf{k}_2) X^K(\mathbf{k}_3) \rangle \equiv (2\pi)^3 \delta(\Sigma_i \mathbf{k}_i) B^{IJK}(k_1, k_2, k_3)$$

Link with multi-field inflation

- The « primordial » perturbations can be related to the scalar field fluctuations (assumed here to be quasi-Gaussian)

$$X^I = N^I_a \delta\phi^a + \frac{1}{2} N^I_{ab} \delta\phi^a \delta\phi^b + \dots$$

$$\langle \delta\phi^a(\mathbf{k}) \delta\phi^b(\mathbf{k}') \rangle = (2\pi)^3 \, \delta^{ab} P_{\delta\phi}(k) \, \delta(\mathbf{k}+\mathbf{k}') \qquad P_{\delta\phi}(k) = \frac{2\pi^2}{k^3} \left(\frac{H_*}{2\pi} \right)^2 ,$$

- The generalized bispectra can thus be written as

$$B^{IJK}(k_1,k_2,k_3) = \tilde{f}^{I,JK}_{\rm NL} P_\zeta(k_2) P_\zeta(k_3) + \tilde{f}^{J,KI}_{\rm NL} P_\zeta(k_1) P_\zeta(k_3) + \tilde{f}^{K,IJ}_{\rm NL} P_\zeta(k_1) P_\zeta(k_2)$$

$$\tilde{f}^{I,JK}_{\rm NL} = \delta^{ac} \, \delta^{bd} \, N^I_{ab} \, N^J_c \, N^K_d \, \left(\frac{P_{\delta\phi}}{P_\zeta} \right)^2$$

Angular bispectrum

- Substituting

$$B^{IJK}(k_1, k_2, k_3) = \tilde{f}^{I,JK} P_\zeta(k_2) P_\zeta(k_3) + \tilde{f}^{J,KI} P_\zeta(k_1) P_\zeta(k_3) + \tilde{f}^{K,IJ} P_\zeta(k_1) P_\zeta(k_2)$$

one finds that the reduced bispectrum is of the form

$$b_{l_1 l_2 l_3} = \sum_{I,J,K} \tilde{f}_{\rm NL}^{I,JK} \; b_{l_1 l_2 l_3}^{I,JK}$$

Langlois & van Tent 11

- **Six distinct angular bispectra** [AD + 1 ISO]

$$(I, JK) = \{(\zeta, \zeta\zeta), (\zeta, \zeta S), (\zeta, SS), (S, \zeta\zeta), (S, \zeta S), (S, SS)\}$$

$$b_{l_1 l_2 l_3}^{I,JK} = 3 \int_0^\infty r^2 dr \alpha_{(l_1}^I(r) \beta_{l_2}^J(r) \beta_{l_3)}^K(r)$$

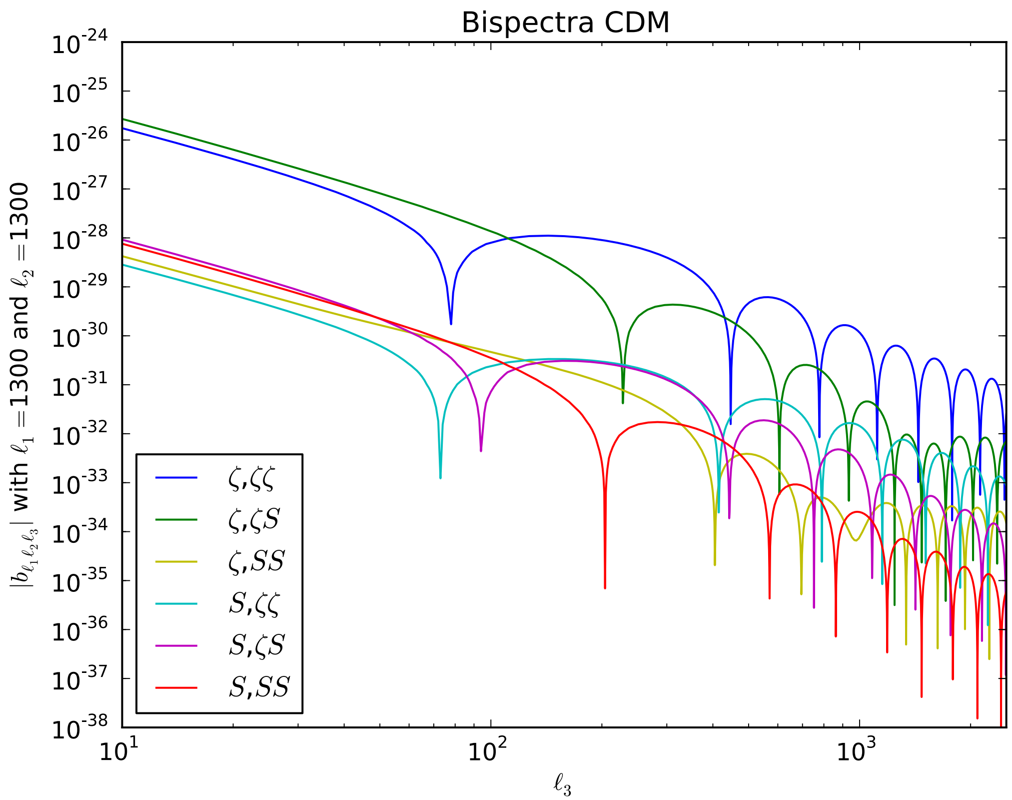
$$\alpha_l^I(r) \equiv \frac{2}{\pi} \int k^2 dk j_l(kr) g_l^I(k)$$

$$\beta_l^I(r) \equiv \frac{2}{\pi} \int k^2 dk j_l(kr) g_l^I(k) P_\zeta(k)$$

Angular bispectrum

$$b^{I,JK}_{l_1 l_2 l_3} = 3\int_0^\infty r^2 dr\, \alpha^I_{(l_1}(r)\beta^J_{l_2}(r)\beta^K_{l_3)}(r)$$

$$\ell_1 = \ell_2 = 1300$$



Angular bispectrum

- Angle-averaged bispectrum

$$
\begin{aligned}
B_{\ell_1\ell_2\ell_3} &\equiv \sum_{m_1,m_2,m_3} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} B^{m_1m_2m_3}_{\ell_1\ell_2\ell_3} \\
&= \sqrt{\frac{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)}{4\pi}} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix} b_{\ell_1\ell_2\ell_3} \,,
\end{aligned}
$$

- Total bispectrum

$$
B_{l_1l_2l_3} = \sum_i \tilde{f}^{(i)}_{\rm NL}\, B^{(i)}_{l_1l_2l_3}
$$

with $i = \{(\zeta,\zeta\zeta),(\zeta,\zeta S),(\zeta,SS),(S,\zeta\zeta),(S,\zeta S),(S,SS)\}$

CMB constraints

- Minimization of

$$\chi^2 = \langle B^{obs} - \sum_i \tilde{f}_{\rm NL}^{(i)} B^{(i)}, B^{obs} - \sum_i \tilde{f}_{\rm NL}^{(i)} B^{(i)} \rangle$$

$$\langle B, B' \rangle \equiv \sum_{l_i} \frac{B_{l_1 l_2 l_3}\, B'_{l_1 l_2 l_3}}{\sigma^2_{l_1 l_2 l_3}}$$

(simplified version)

$$\sigma^2_{l_1 l_2 l_3} \equiv \langle B^2_{l_1 l_2 l_3} \rangle - \langle B_{l_1 l_2 l_3} \rangle^2 \approx \Delta_{l_1 l_2 l_3} C_{l_1} C_{l_2} C_{l_3}$$

- Parameters = solutions of $\sum_j \langle B^{(i)}, B^{(j)} \rangle \, \tilde{f}_{\rm NL}^{(j)} = \langle B^{(i)}, B^{obs} \rangle$

- Fisher matrix: $F_{ij} \equiv \langle B^{(i)}, B^{(j)} \rangle$

Fisher matrix (cdm iso)

- 6 parameters: $i = \{(\zeta,\zeta\zeta),(\zeta,\zeta S),(\zeta,SS),(S,\zeta\zeta),(S,\zeta S),(S,SS)\}$

- Fisher matrix

$(\zeta,\zeta\zeta)$	$(\zeta,\zeta S)$	(ζ,SS)	$(S,\zeta\zeta)$	$(S,\zeta S)$	(S,SS)
$3.9\,(2.5)\times 10^{-2}$	$4.5\,(3.6)\times 10^{-2}$	$2.3\,(2.1)\times 10^{-4}$	$2.4\,(1.6)\times 10^{-4}$	$6.9\,(4.3)\times 10^{-4}$	$5.3\,(3.1)\times 10^{-4}$
-	$7.1\,(6.0)\times 10^{-2}$	$5.3\,(3.8)\times 10^{-4}$	$3.8\,(2.1)\times 10^{-4}$	$11\,(7.4)\times 10^{-4}$	$8.8\,(5.5)\times 10^{-4}$
-	-	$28\,(6.4)\times 10^{-5}$	$16\,(3.7)\times 10^{-5}$	$33\,(9.5)\times 10^{-5}$	$11\,(5.0)\times 10^{-5}$
-	-	-	$15\,(3.0)\times 10^{-5}$	$22\,(5.8)\times 10^{-5}$	$7.5\,(3.2)\times 10^{-5}$
-	-	-	-	$5.1\,(1.6)\times 10^{-4}$	$2.4\,(1.0)\times 10^{-4}$
-	-	-	-	-	$21\,(8.3)\times 10^{-5}$

- Statistical uncertainty on the parameters

$$\Delta \tilde{f}^i = \sqrt{(F^{-1})_{ii}} = \{9.6, 7.1, 160, 150, 180, 140\}\,.$$

- Without polarization: $\Delta \tilde{f}^i = \{17, 11, 980, 390, 1060, 700\}$

- **Purely iso NG** seen as $\tilde{f}^{(1)} = (F_{16}/F_{11})\tilde{f}^{(6)} \simeq 10^{-2}\,\tilde{f}^{(6)}$
  in a purely adiab template.

CMB constraints

- Parameter uncertainties for the other isocurvature modes

  - Baryon isocurvature mode:

$$\Delta \tilde{f}^i = \{9.6, 35, 4000, 720, 4300, 16600\}$$

  - Neutrino isocurvature density mode

$$\Delta \tilde{f}^i = \{28, 36, 190, 150, 240, 320\}$$

  - Neutrino isocurvature velocity mode

$$\Delta \tilde{f}^i = \{25, 22, 85, 81, 77, 71\}$$

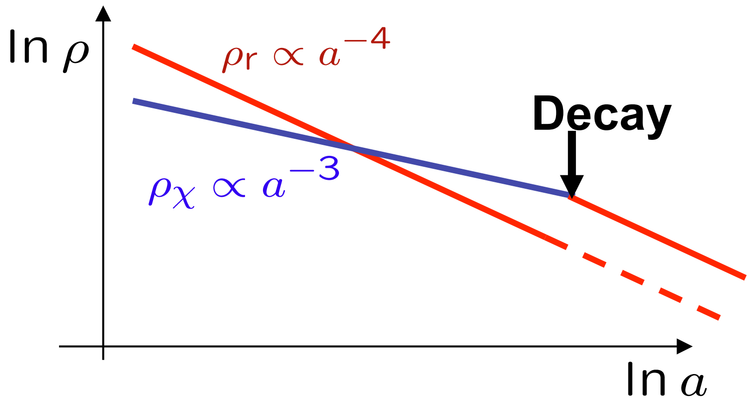
Langlois & van Tent 12

The curvaton scenario

Mollerach (1990); Linde & Mukhanov (1997) ;
Enqvist & Sloth; Lyth & Wands; Moroi & Takahashi (2001)

- Light scalar field during inflation (when H > m),

  which later oscillates (when H < m), and finally decays.



- **Mixed curvaton-inflaton** scenario: both inflaton and curvaton fluctuations contribute to the observable perturbations.

DL & Vernizzi 04

- **Residual isocurvature** perturbations

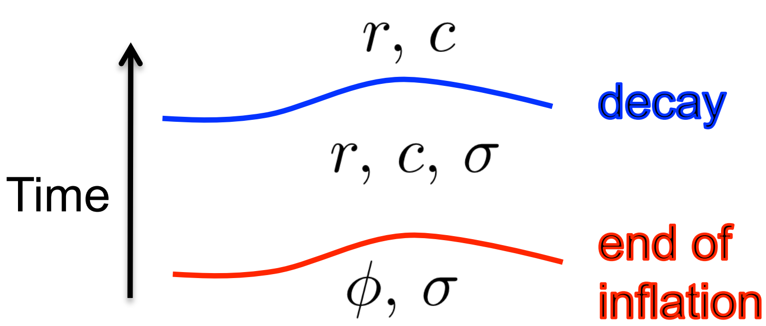
Lyth, Ungarelli & Wands 02

Mixed curvaton-inflaton scenario

Simple example: radiation + cdm + single curvaton

- **Curvaton fluctuations** (potential $V(\sigma) = \frac{1}{2} m^2 \sigma^2$)
  - inflation: $\delta\sigma_* \simeq \frac{H_*}{2\pi}$
  - oscillating phase: $\rho_\sigma = m^2 \sigma^2 \implies \frac{\delta\rho_\sigma}{\rho_\sigma} = \hat{S} + \frac{1}{4}\hat{S}^2, \quad \hat{S} \equiv 2\frac{\delta\sigma_*}{\sigma_*}$

- **Decay**



$$\rho_{r+} = \rho_{r-} + \gamma_r \, \rho_\sigma$$

$$\rho_{c+} = \rho_{c-} + (1 - \gamma_r) \, \rho_\sigma$$

- Parameters: , ,

“Primordial” perturbations

$$X^I = N^I_a \delta\phi^a + \frac{1}{2} N^I_{ab} \delta\phi^a \delta\phi^b + \ldots$$

After the decay (assuming $\zeta_{c-} = \zeta_{r-} = \zeta_{\rm inf}$ and $\Omega_c \ll 1$), the perturbations up to 2$^{\rm nd}$ order are

[DL & Lepidi 11]

$$\zeta_{\rm r} = \zeta_{\rm inf} + z_1 \hat{S} + \frac{1}{2} z_2 \hat{S}^2 \qquad z_1 = \frac{r}{3}, \quad z_2 = \frac{r}{6} \mathcal{F}(r, \xi)$$

$$S_c = s_1 \hat{S} + \frac{1}{2} s_2 \hat{S}^2 \qquad s_1 = f_c - r, \quad s_2 = \frac{1}{2} f_c (1 - 2 f_c) - \frac{r}{2} \mathcal{G}(r, \xi)$$

where the transfer coefficients depend on the parameters:

$$f_c \equiv \frac{(1-\gamma_r)\Omega_\sigma}{\Omega_c + (1-\gamma_r)\Omega_\sigma} \qquad \xi \equiv \frac{\gamma_r}{1 - (1-\gamma_r)\Omega_\sigma}, \qquad r \equiv \frac{3\Omega_\sigma}{4 - \Omega_\sigma} \xi$$

Power spectra

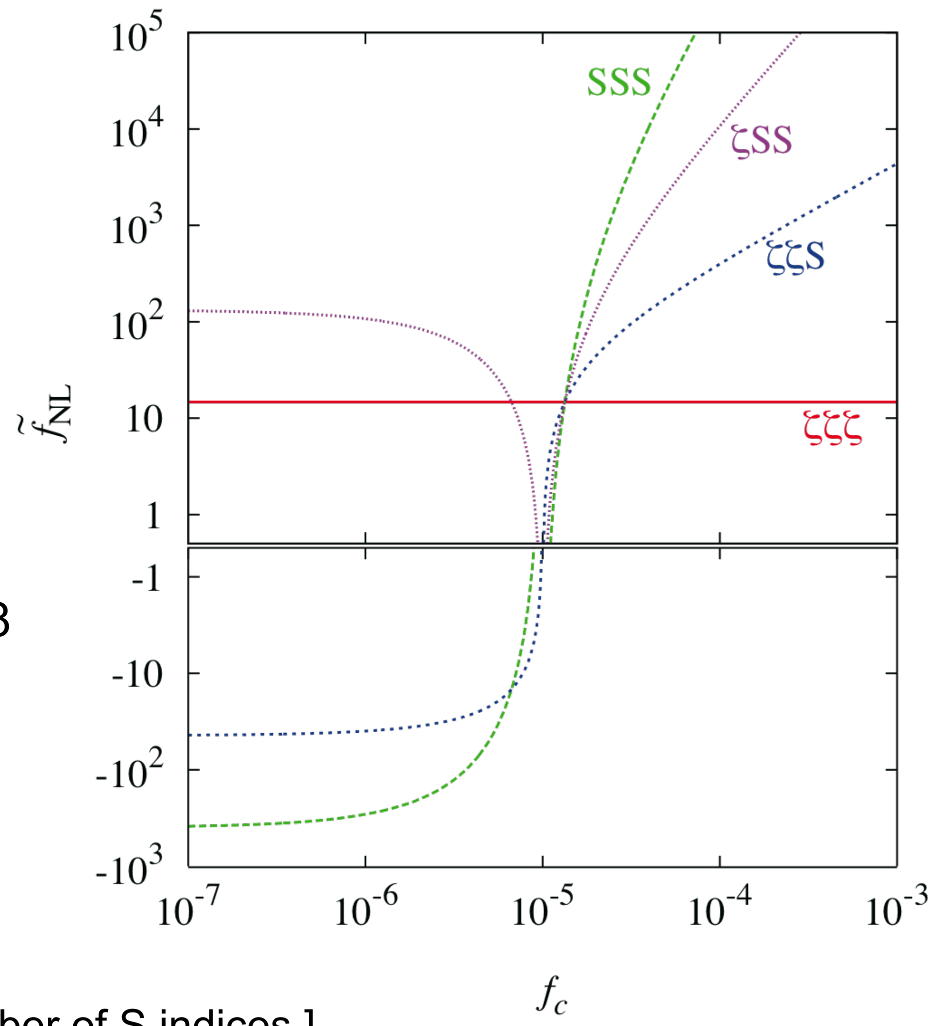
- Curvature: $\mathcal{P}_{\zeta_{\rm r}} = \mathcal{P}_{\zeta_{\rm inf}} + \frac{r^2}{9}\mathcal{P}_{\hat{S}}\,,$ $\qquad \Xi \equiv \frac{(r^2/9)\mathcal{P}_{\hat{S}}}{\mathcal{P}_{\zeta_{\rm inf}} + (r^2/9)\mathcal{P}_{\hat{S}}}$
- Isocurvature: $\mathcal{P}_{S_c} = (f_c - r)^2 \mathcal{P}_{\hat{S}}$
- Correlation: $\mathcal{C} = \frac{\mathcal{P}_{S_c,\zeta_r}}{\sqrt{\mathcal{P}_{S_c}\mathcal{P}_{\zeta_r}}} = \varepsilon_f \sqrt{\Xi},$ $\qquad \varepsilon_f \equiv \mathrm{sgn}(f_c - r)$

The observational constraint on $\alpha = \frac{\mathcal{P}_{S_c}}{\mathcal{P}_{\zeta_r}} = 9\left(1 - \frac{f_c}{r}\right)^2 \Xi$
is satisfied if

$$\Xi \ll 1 \quad \text{or} \quad |f_c - r| \ll r$$

Bispectra

- If $r \ll 1$, $\tilde{f}_{\rm NL}^{\zeta,\zeta\zeta} \simeq \dfrac{3\,\Xi^2}{2r}$
- This is dominant if $|f_c - r| \ll r$
- But for $\Xi \ll 1$, one finds:
  1. If $f_c \ll r \ll 1$
     $$\tilde{f}_{\rm NL}^{I,JK} \simeq (-3)^{\mathcal{I}_S}\, \tilde{f}_{\rm NL}^{\zeta,\zeta\zeta}$$
     DL, Vernizzi & Wands 08
  2. If $r \ll f_c \ll 1$
     $$\tilde{f}_{\rm NL}^{I,JK} \simeq \left(\frac{3f_c}{r}\right)^{\mathcal{I}_S}\, \tilde{f}_{\rm NL}^{\zeta,\zeta\zeta}$$



[$\mathcal{I}_S$ = number of S indices]

Generalized trispectra

- Generalized coefficients:

  Eight $\tilde{g}_{\rm NL}^{I,JKL}$

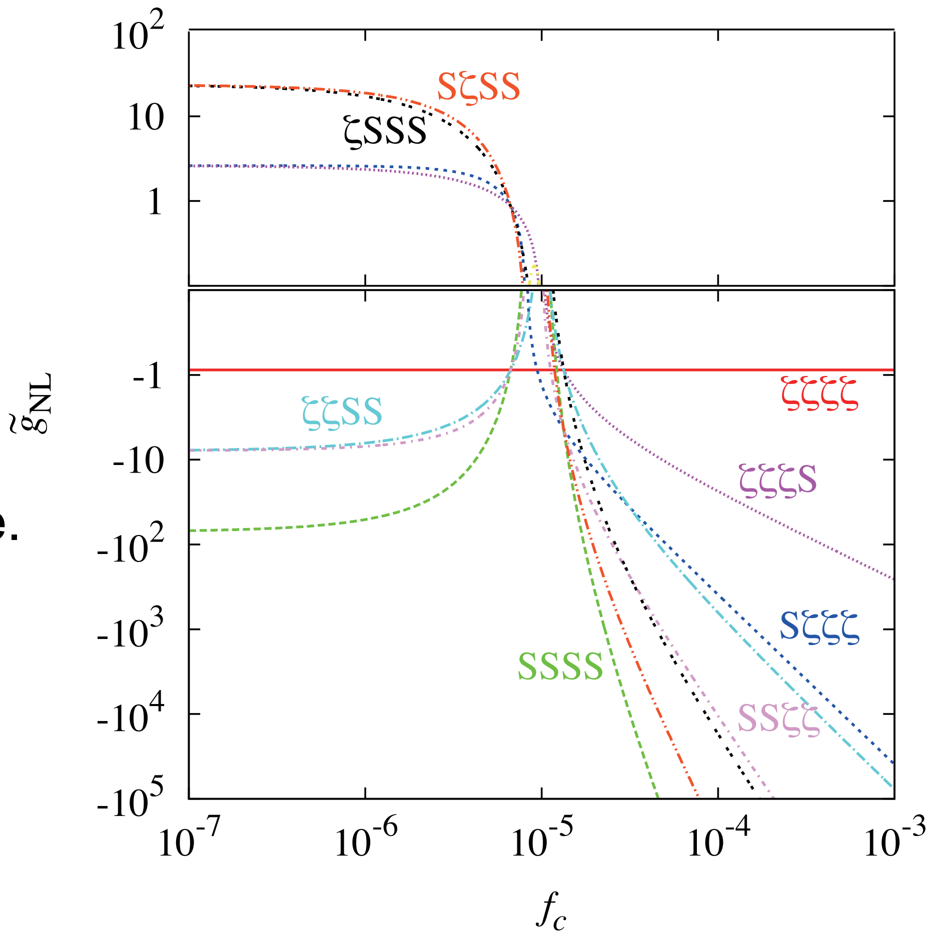
  Nine $\tau_{\rm NL}^{IJ,KL}$

- If $|f_c - r| \ll r$, the purely curvature contributions dominate.

- If $\Xi \ll 1$, the isocurvature contributions dominate.

- Remark: generalized relations between the $\tau_{\rm NL}$ and $\tilde{f}_{\rm NL}$

DL & Takahashi 2011



Conclusions

- With adiabatic and isocurvature initial perturbations, the local bispectrum is the sum of six distinct shapes:
  - purely adiabatic shape
  - purely isocurvature shape
  - four shapes from adiabatic-isocurvature correlations

- For the trispectrum, one finds 9 $\tau_{\rm NL}$ - like coefficients and 8 $g_{\rm NL}$ - like coefficients.

- One can find models where isocurvature non-Gaussianities dominate the purely adiabatic one.

- Constraints from Planck data ?