Aspects of EFT, and its effects to the BAO
&
Biasing in the EFT framework

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Contents

- Aspects of PT, EFT, and effects on the BAO
- Halo Power Spectrum and Bispectrum in EFT

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- Aspects of PT, EFT, and effects on the BAO

with:
Man Yat Chu (Berkeley),
Yu Feng (Berkeley),
Uros Seljak (Berkeley)

Gravitational clustering of dark matter

Evolution of collisionless particles - Vlasov equation:

$$\frac{df}{d\tau} = \frac{\partial f}{\partial \tau} + \frac{1}{m}\mathbf{p} \cdot \nabla f - am\nabla\phi \cdot \nabla_p f = 0,$$

and $\nabla^2\phi = 3/2\mathcal{H}\Omega_m\delta$.

Integral moments of the distribution function:

mass density field & mean streaming velocity field

$$\rho(\mathbf{x}) = ma^{-3}\int d^3p\, f(\boldsymbol{x}, \boldsymbol{p}), \qquad v_i(\mathbf{x}) = \frac{\int d^3p\, \frac{p_i}{am} f(\boldsymbol{x}, \boldsymbol{p})}{\int d^3p\, f(\boldsymbol{x}, \boldsymbol{p})},$$

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and $\nabla^2\phi = 3/2\mathcal{H}\Omega_m\delta$.

Eulerian framework - fluid approximation:

$$\frac{\partial\delta}{\partial\tau} + \nabla\cdot[(1+\delta)\mathbf{v}] = 0$$
$$\frac{\partial v_i}{\partial\tau} + \mathcal{H}v_i + \mathbf{v}\cdot\nabla v_i = -\nabla_i\phi - \frac{1}{\rho}\nabla_i(\rho\sigma_{ij}),$$

where σ_{ij} is the velocity dispersion.
σ_{ij} is reintroduced in EFT framework after integrating out the short modes.

[Baumann et al 2010, Carrasco et al. 2012]

PS and transfer function approach

Transfer function

$$\tilde{T}_{\rm 1LPT}(k) = \frac{\langle \delta_{\rm 1LPT}\delta_{\rm dm}\rangle}{\langle \delta_{\rm 1LPT}\delta_{\rm 1LPT}\rangle},$$

with 1LPT and dark matter density perturbations in Fourier space. Cross-correlation coefficient

$$r^2_{\rm 1LPT} = \frac{\langle \delta_{\rm 1LPT}\delta_{\rm dm}\rangle^2}{\langle \delta_{\rm 1LPT}\delta_{\rm 1LPT}\rangle\langle \delta_{\rm dm}\delta_{\rm dm}\rangle}.$$

The corresponding stochastic power $P_J(k)$ is defined as

$$P_J(k) = \left[1 - r^2_{\rm 1LPT}(k)\right] P_{\rm dm}(k).$$

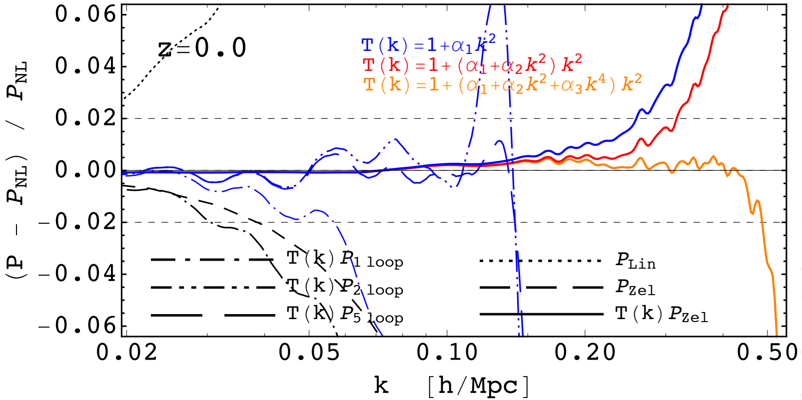
Non-perturbative effects on the power spectrum then it is simpler to define,

$$\frac{P_{\rm dm}(k)}{P_{\rm 1LPT}(k)} \equiv T^2_{\rm 1LPT}(k) = 1 + \alpha_{\rm 1LPT}(k) k^2 \equiv \left(1 + \sum_{i=1}^{\infty} \alpha_{i,{\rm 1LPT}} k^{2i}\right).$$

Note that $T(k)$ includes stochasticity and there is no guarantee that it can be expanded in terms of even powers of k, although we expect that at low k the leading term is $\alpha_{1,1LPT}k^2$.

Clustering in 1D

1D case studied recently in: [Matthew McQuinn & Martin White 2015]



Extensions to 3D: models

Define one-loop SPT-EFT parameter $\alpha_{\mathrm{SPT,1-loop}}(k)$

$$P_{\mathrm{dm}}(k) = D_+^2 P_L(k) \left[1 + \alpha_{\mathrm{SPT,1-loop}}(k) k^2\right] + D_+^4 P_{\mathrm{SPT,1-loop}},$$

IR-SPT resummation model: [Carrasco et al. 2012]

$$P_{\mathrm{dm}}(k) = P_{\mathrm{nw,L}}(k) + P_{\mathrm{nw,SPT,1-loop}}(k) + \alpha_{\mathrm{SPT,1-loop,IR}}(k) k^2 P_{\mathrm{nw,L}}(k) \\ + e^{-k^2\Sigma^2}\left(\Delta P_{\mathrm{w,SPT,1-loop}}(k) + \left(1 + (\alpha_{\mathrm{SPT,1-loop,IR}} + \Sigma^2)k^2\right)\Delta P_{\mathrm{w,L}}(k)\right).$$

Hybrid model: [Baldauf et al. 2015]

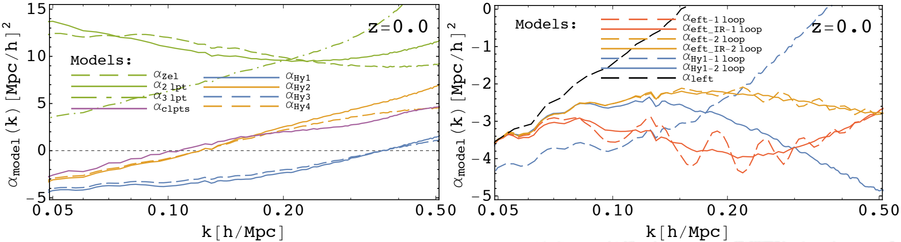
$$P_{\mathrm{dm}}(k) = P_{\mathrm{1LPT}}(k)\left(1 + \alpha_{\mathrm{1LPT,1-loop}}(k) k^2\right) \\ + \left(P_{\mathrm{SPT,1-loop}}(k) - P_{\mathrm{1LPT,1-loop}}(k)\right)_{IR}$$

Extensions to 3D

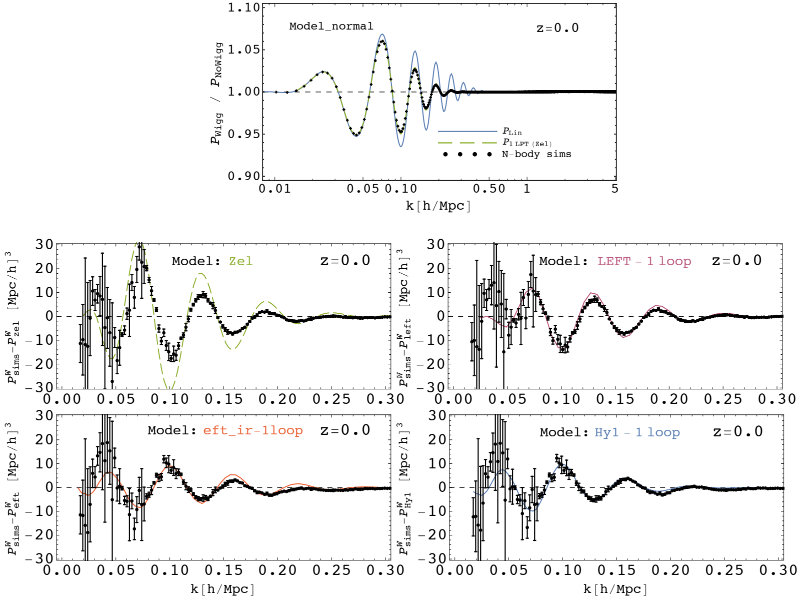
Positive corrections relative to 1LPT or 2LPT:

$$P_{\rm dm} = P_{\rm iLPT}\left(1+\alpha_{\rm iLPT}(k)k^2\right),$$

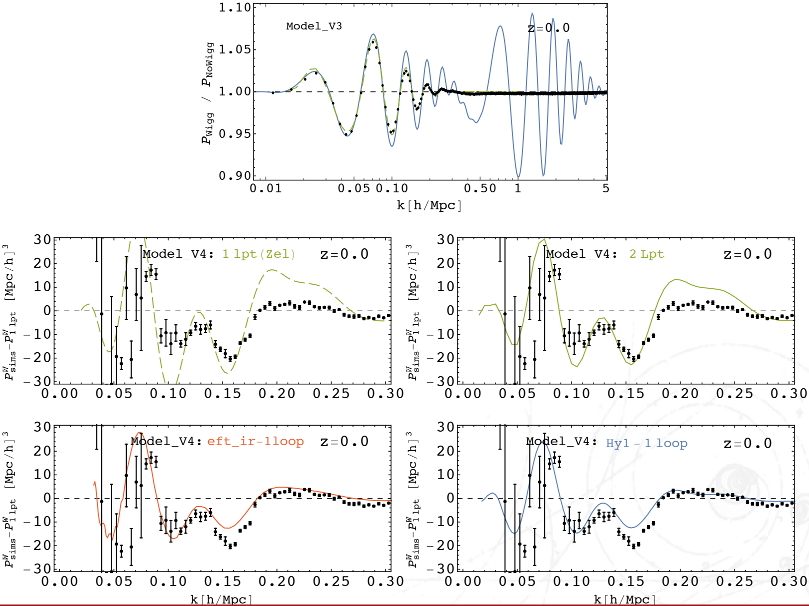
Stochasticity for 2LPT suppressed. [Tassev et. al. 2011,Baldauf et. al. 2015]



Wiggle residuals in our schemes: BAO

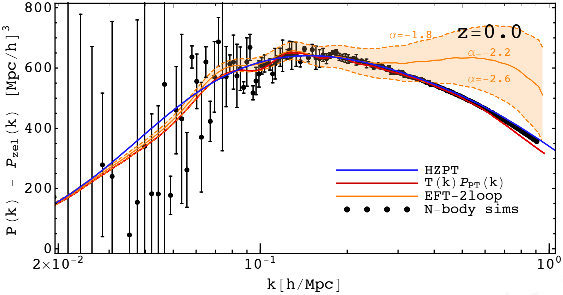


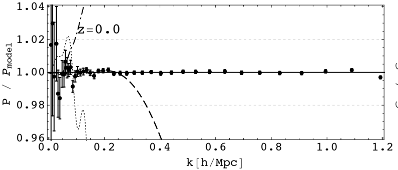
Wiggle residuals in our schemes: BAO+

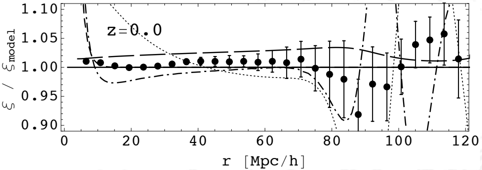


Interpreting the dark matter power spectrum

Halo-Zeldovich model : $P(k) = P_{Zel}(k) + P_{BB}(k)$





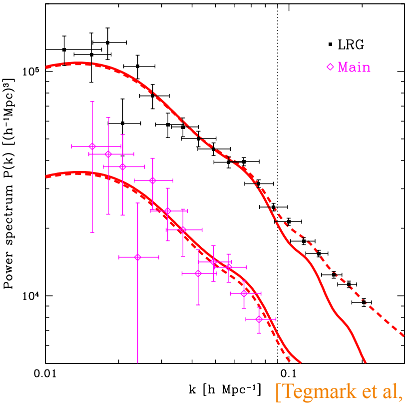


Contents

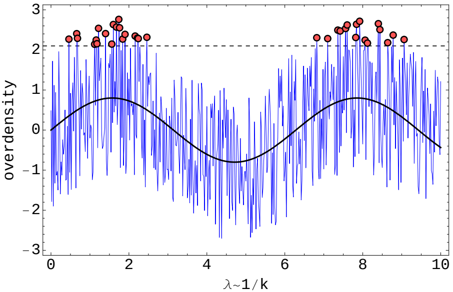
- Halo Power Spectrum and Bispectrum in EFT

with:
Raul Angulo (CEFCA),
Matteo Fasiello (Stanford),
Leonardo Senatore (Stanford)

Galaxies form at high density peaks of initial matter density:
rare peaks exhibit higher clustering!



[Tegmark et al, 2006]



- Tracer detriments the amplitude: $P_g(k) = b^2 P_m(k) + \ldots$
- Understanding bias is crucial for understanding the galaxy clustering

Earlier approaches to halo biasing

Local biasing model: halo field is a function of just DM density field

$$\delta_{\rm h} = c_\delta \delta + c_{\delta^2}\left(\delta^2 - \left\langle \delta^2 \right\rangle\right) + c_{\delta^3}\delta^3 + \dots$$

[Fry & Gaztanaga 1993]

Quasi-local (in space) relation of the halo density field to the dark matter

[McDonald & Roy 2008, Assassi et. al. 2014]

$$\begin{aligned}\delta_{\rm h}(\mathbf{x}) &= c_\delta\delta(\mathbf{x}) + c_{\delta^2}\delta^2(\mathbf{x}) + c_{\delta^3}\delta^3(\mathbf{x}) \\ &+ c_{s^2}s^2(\mathbf{x}) + c_{\delta s^2}\delta(\mathbf{x})s^2(\mathbf{x}) + c_\psi\psi(\mathbf{x}) + c_{st}s(\mathbf{x})t(\mathbf{x}) + c_{s^3}s^3(\mathbf{x}) \\ &+ c_\epsilon\epsilon + \dots,\end{aligned}$$

with effective ('Wilson') coefficients c_l and variables:

$$s_{ij}(\mathbf{x}) = \partial_i\partial_j\phi(\mathbf{x}) - \frac{1}{3}\delta^{\rm K}_{ij}\delta(\mathbf{x}), \qquad t_{ij}(\mathbf{x}) = \partial_i v_j - \frac{1}{3}\delta^{\rm K}_{ij}\theta(\mathbf{x}) - s_{ij}(\mathbf{x}),$$

$$\psi(\mathbf{x}) = [\theta(\mathbf{x}) - \delta(\mathbf{x})] - \frac{2}{7}s(\mathbf{x})^2 + \frac{4}{21}\delta(\mathbf{x})^2,$$

where ϕ is the gravitational potential, and white noise (stochasticity) ϵ.

Effective field theory of biasing

Non-local (time) and quasi-local (spece) relation of the halo density field to the dark matter

[Senatore 2014]

$$\delta_h(\boldsymbol{x},t) \simeq \int^t dt'\; H(t')\; \Big[\bar{c}_\delta(t,t')\;\; :\delta(\boldsymbol{x}_{\rm fl},t'):$$
$$+\,\bar{c}_{\delta^2}(t,t')\;\; :\delta(\boldsymbol{x}_{\rm fl},t')^2: + \bar{c}_{s^2}(t,t')\;\; :s^2(\boldsymbol{x}_{\rm fl},t'):$$
$$+\,\bar{c}_{\delta^3}(t,t')\;\; :\delta(\boldsymbol{x}_{\rm fl},t')^3: + \bar{c}_{\delta s^2}(t,t')\;\; :\delta(\boldsymbol{x}_{\rm fl},t')s^2(\boldsymbol{x}_{\rm fl},t'): + \dots$$
$$+\,\bar{c}_\epsilon(t,t')\;\epsilon(\boldsymbol{x}_{\rm fl},t') + \bar{c}_{\epsilon\delta}(t,t')\;\; :\epsilon(\boldsymbol{x}_{\rm fl},t')\delta(\boldsymbol{x}_{\rm fl},t'): + \dots$$
$$+\,\bar{c}_{\partial^2\delta}(t,t')\;\frac{\partial^2_{x_{\rm fl}}}{k_M^2}\delta(\boldsymbol{x}_{\rm fl},t') + \dots\Big]$$

Novice consideration of non-local in time formation, which depends on fields evaluated on past history on past path:

$$\boldsymbol{x}_{\rm fl}(\boldsymbol{x},\tau,\tau') = \boldsymbol{x} - \int_{\tau'}^{\tau} d\tau''\;\boldsymbol{v}(\tau'',\boldsymbol{x}_{\rm fl}(\boldsymbol{x},\tau,\tau''))$$

Effective field theory of biasing

New physical scale $k_M \sim 2\pi \left(\frac{4Pi}{3}\frac{\rho_0}{M}\right)^{1/3}$, which can be different then k_{NL}. We look at the correlations at $k \ll k_M$.

Each order in perturbation theory we get new bias coefficients:

$$\delta_{\rm h}(k,t) = c_{\delta,1}\left[\delta^{(1)}(k,t) + \text{flow terms}\right] + c_{\delta,2}\left[\delta^{(2)}(k,t) + \text{flow terms}\right] + \dots$$

Emergence of degeneracy: choice of most convenient basis

Turns out that at one loop 2-pt and tree level 3-pt function LIT and non-LIT are degenerate- this is no longer the case at higher loops or when 4-pt function is considered.

Effective field theory of biasing

Independent operators in the`Basis of Descendants':

(1)st order: $\left\{\mathbb{C}^{(1)}_{\delta,1}\right\}$

(2)nd order: $\left\{\mathbb{C}^{(2)}_{\delta,1},\ \mathbb{C}^{(2)}_{\delta,2},\ \mathbb{C}^{(2)}_{\delta^2,1}\right\}$

(3)rd order: $\left\{\mathbb{C}^{(3)}_{\delta,1},\ \mathbb{C}^{(3)}_{\delta,2},\ \mathbb{C}^{(3)}_{\delta,3},\ \mathbb{C}^{(3)}_{\delta^2,1},\ \mathbb{C}^{(3)}_{\delta^2,2},\ \mathbb{C}^{(3)}_{\delta^3,1}, \mathbb{C}^{(3)}_{\delta,3_{c_s}}\ \mathbb{C}^{(3)}_{s^2,2}\right\}$

Stochastic: $\left\{\mathbb{C}_{\epsilon},\ \mathbb{C}^{(1)}_{\delta\epsilon,1}\right\}$

We compare $P^{1-\text{loop}}_{\text{hh}}$, $P^{1-\text{loop}}_{\text{hm}}$, $B^{\text{tree}}_{\text{hhh}}$, $B^{\text{tree}}_{\text{hhm}}$, $B^{\text{tree}}_{\text{hmm}}$ statistics
Renormalization! (takes care of short distance physics has at long distances of interest)
In practice, $\tilde{c}_{\delta,1}$ is a bare parameter, the sum of a finite part and a counterterm:

$$\tilde{c}_{\delta,1} = \tilde{c}_{\delta,1,\,\text{finite}} + \tilde{c}_{\delta,1,\,\text{counter}},$$

After renormalization we end up with using 7 finite bias parameters b_i (coefficients in EFT).

Observables: P_{hm}, P_{hh}, B_{hmm}, B_{hhm}, B_{hhh}

Example: Halo-Matter Power Spectrum (one loop)

$$\begin{aligned}
P_{hm}(k) =& b_{\delta,1}(t) \Bigg(P_{11}(k) + 2\int \frac{d^3q}{(2\pi)^3} F_s^{(2)}(\boldsymbol{k}-\boldsymbol{q},\boldsymbol{q})\, \widehat{c}^{(2)}_{\delta,1,s}(\boldsymbol{k}-\boldsymbol{q},\boldsymbol{q})\, P_{11}(q) P_{11}(|\boldsymbol{k}-\boldsymbol{q}|) \\
&\quad + 3P_{11}(k) \int \frac{d^3q}{(2\pi)^3} \left(F_s^{(3)}(\boldsymbol{k},-\boldsymbol{q},\boldsymbol{q}) + \widehat{c}^{(3)}_{\delta,1,s}(\boldsymbol{k},-\boldsymbol{q},\boldsymbol{q}) \right) P_{11}(q) \Bigg) \\
&+ b_{\delta,2}(t)\, 2\int \frac{d^3q}{(2\pi)^3} F_s^{(2)}(\boldsymbol{k}-\boldsymbol{q},\boldsymbol{q}) \left(F_s^{(2)}(\boldsymbol{k}-\boldsymbol{q},\boldsymbol{q}) - \widehat{c}^{(2)}_{\delta,1,s}(\boldsymbol{k}-\boldsymbol{q},\boldsymbol{q}) \right) \\
&\qquad \times P_{11}(q) P_{11}(|\boldsymbol{k}-\boldsymbol{q}|) \\
&+ b_{\delta,3}(t) 3P_{11}(k) \int \frac{d^3q}{(2\pi)^3} \left(\widehat{c}^{(3)}_{\delta,3,s}(\boldsymbol{k},-\boldsymbol{q},\boldsymbol{q}) \right) P_{11}(q) \\
&+ b_{\delta^2}(t) 2\int \frac{d^3q}{(2\pi)^3} F_s^{(2)}(\boldsymbol{k}-\boldsymbol{q},\boldsymbol{q}) P_{11}(q) P_{11}(|\boldsymbol{k}-\boldsymbol{q}|) \\
&+ \left(b_{c_s}(t) - 2(2\pi) c^2_{s(1)}(t) b_{\delta,1}(t) \right) \frac{k^2}{k^2_{\mathrm{NL}}} P_{11}(k)
\end{aligned}$$

Error estimates and bias fits

Error bars of the theory are given by the higher loop estimates:

e.g. $\Delta P_{hm} \sim (2\pi)\, b_1 \left(\frac{k}{k_{\rm NL}}\right)^3 P_{11}(k)$.

This determines the theory reach $k_{\rm max}$.

$k_{\rm max}$ [h/Mpc]	bin0	bin1
mm	0.22 – 0.31	0.22 – 0.31
hm	0.24 – 0.35	0.22 – 0.35
hh	0.19 – 0.32	0.17 – 0.30
mmm	0.14 – 0.22	0.14 – 0.22
hmm	0.13 – 0.22	0.13 – 0.22
hhm	0.13 – 0.22	0.13 – 0.22
hhh	0.13 – 0.21	0.13 – 0.21

Fits to N-body simulations:

bin_1: $k_{\rm min}$=0.04h/Mpc, $k_{\rm max}$=0.11h/Mpc						
hm	hh	hmm	hhm	hhh	χ^2	p
+	+	−	−	−	0.0372	1.000
+	+	+	−	−	0.662	0.9937
+	+	−	+	−	0.615	0.9982
+	+	−	−	+	0.730	0.9724
+	+	+	+	−	0.849	0.8911
+	+	+	−	+	0.846	0.8963
+	+	−	+	+	1.17	0.09115
+	+	+	+	+	1.13	0.1105

Most of the constraint comes form the 3-pt function.

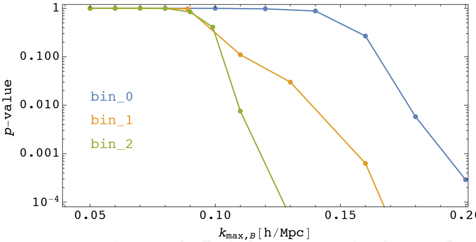
Fits to 3-pt and 4-pt function would enable full predictivity for 2-pt function.

EFT of biased tracers: bias fits

Error bars of the theory are given by the higher loop estimates:
e.g. $\Delta P_{hm} \sim (2\pi)\, b_1 \left(\frac{k}{k_{\rm NL}}\right)^3 P_{11}(k)$.
This determines the theory reach $k_{\rm max}$.

	bin0	bin1
$b_{\delta,1}$	1.00 ± 0.01	1.32 ± 0.01
$b_{\delta,2}$	0.23 ± 0.01	0.52 ± 0.01
$b_{\delta,3}$	0.48 ± 0.12	0.66 ± 0.13
b_{δ^2}	0.28 ± 0.01	0.30 ± 0.01
b_{c_s}	0.72 ± 0.16	0.27 ± 0.17
$b_{\delta\epsilon}$	0.31 ± 0.08	0.76 ± 0.17
Const_ϵ	5697 ± 108	10821 ± 169

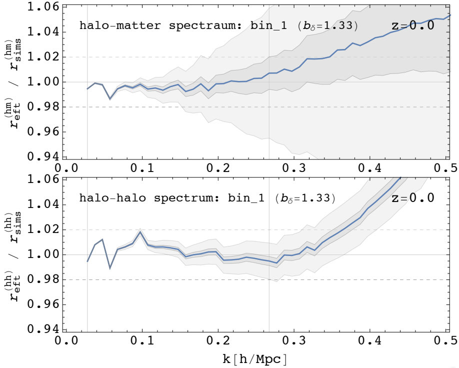
Characteristic sharp drop in the p-value after the maximal Bispectrum scale $k_{\max,B}$



Within these scales EFT results fit the data well, and then fail after crossing this scales.

Halo PS & BIS results (bin 1)

Comparison to N-body simulations: Power Spectrum fitted up to $k < 0.26\text{Mpc}/h$ and Bispectrum up to $k < 0.11\text{Mpc}/h$



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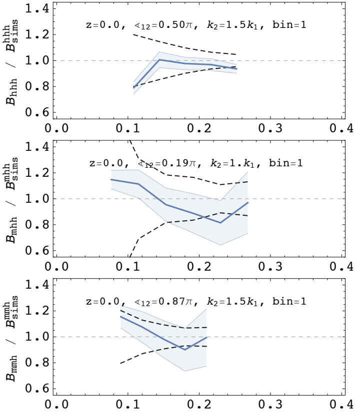

Summary

Part I:

- Transfer function offers a intuitive outlook to dark matter nonlinear clustering.
- All constructed models can predict various wiggle shapes.
- Matching of EFT schemes at low k and Halo model motivated models offers good parameterizations for wide range of scales.

Part II:

- EFT gives a consistent expansion in $(k/k_{\rm NL})^2$, and for halos also in $(k/k_{\rm M})^2$, nonlocal effect in time and space included
- EFT approach is well suited for galaxy clustering (one-loop power spectra $k \sim 0.3h/$Mpc, tree level bispectra $k \sim 0.1 - 0.15h/$Mpc)
- Consistent description of five different observables ($P_{\rm hm}$, $P_{\rm hh}$, $B_{\rm hmm}$, $B_{\rm hhm}$, $B_{\rm hhh}$) with seven bias parameters.