Leonardo Senatore (Stanford)

Aspects of the Effective Field Theory of Large Scale Structures

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Making of Large Scale Structures Science a High Precision Science



The EFTofLSS: A well defined perturbation theory

• Non-linearities at short scale



With this



With this



With this



The Theory of the Universe

- Useful or not, this is the correct description of the long distance universe
 - for oceans waves, we describe water as a fluid
 - not as a set of molecules hitting each other



• similarly the long distance universe *is* the system described by the EFTofLSS

Normal Approach: numerics

• Just simulate the full universe (such as water molecules to simulate ocean waves)



Why numerics are not enough

- they do not give the simple description of the system
- In principle, we can simulate the clustering of dark matter with N-body sims

• But

- simulations with dark matter are very slow
 - systematic error of order 1%

A. Schneider, R. Teyssier, ... V. Springel et al. 1503

• we cannot simulate baryons: we can only `model' them

–As a proof, SDSS stops analyzing data at $k \simeq 0.1 \, h {\rm Mpc}^{-1}$



Numerics have been great

• Do not misunderstand me:

-numerical simulations have provided some of the most beautiful history-making discoveries:

- dark matter is cold
- structures form from small to big
- ...

many due to Simon White here!

• But I believe, after these giants, we live in hard times

-and to make further progress, high precision is required

• N-body sims do not seem, to me, the only appropriate tool.

Idea of the Effective Field Theory

Consider a dielectric material

- Very complicated on atomic scales d_{atomic}
- On long distances $d \gg d_{\text{atomic}}$

-we can describe atoms with their gross characteristics

• polarizability $\vec{d}_{\text{dipole}} \sim \alpha \vec{E}_{\text{electric}}$: average response to electric field

-we are led to a uniform, smooth material, with just some macroscopic properties

- we simply solve dielectric Maxwell equations, we do not solve for each atom.
- The universe looks like a dielectric



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GR EM

Dielectric Fluid



Construction of the Effective Field Theory

The Effective ~Fluid

–In history of universe Dark Matter moves about $1/k_{\rm NL} \sim 10 \,{\rm Mpc}$

- it is an effective fluid-like system with mean free path ~ $1/k_{\rm NL} \sim 10 \,{
 m Mpc}$
- it interacts with gravity so matter and momentum are conserved
- Skipping subtleties, the resulting equations are equivalent to fluid-like equations

$$\nabla^2 \Phi_l = H^2 \frac{\delta \rho_l}{\rho}$$
$$\partial_t \rho_l + H \rho_l + \partial_i \left(\rho_l v_l^i \right) = 0$$
$$\dot{v}_l^i + H v_l^i + v_l^j \partial_j v_l^i = \frac{1}{\rho} \partial_j \tau_{ij}$$

with Baumann, Nicolis and Zaldarriaga JCAP 2012 with Carrasco and Hertzberg JHEP 2012 with Porto and Zaldarriaga JCAP1405

-short distance physics appears as a non trivial stress tensor for the long-distance fluid

$$\tau_{ij} \sim \delta_{ij} \rho_{\text{short}} \left(v_{\text{short}}^2 + \Phi_{\text{short}} \right)$$

Dealing with the Effective Stress Tensor

• Take expectation value over short modes (integrate them out)

$$\langle \tau_{ij} \rangle_{\text{long-fixed}} \sim \delta_{ij} \left[p_0 + c_s \,\delta \rho_l + \mathcal{O}\left(\frac{\partial}{k_{\text{NL}}}, \partial_i v_l^i, \delta \rho_l^2, \ldots\right) + \Delta \tau \right]$$

• We obtain equations containing only long-modes

$$\nabla^{2} \Phi_{l} = H^{2} \frac{\delta \rho_{l}}{\rho}$$

$$\partial_{t} \rho_{l} + H \rho_{l} + \partial_{i} \left(\rho_{l} v_{l}^{i} \right) = 0$$

$$\dot{v}_{l}^{i} + H v_{l}^{i} + v_{l}^{j} \partial_{j} v_{l}^{i} = \frac{1}{\rho} \partial_{j} \tau_{ij}$$

$$\langle \tau_{ij} \rangle_{\text{long-fixed}} \sim \delta_{ij} \left[p_{0} + c_{s} \delta \rho_{l} + \mathcal{O} \left(\frac{\partial}{k_{\text{NL}}}, \partial_{i} v_{l}^{i}, \delta \rho_{l}^{2}, \ldots \right) + \Delta \tau \right]$$

 k_{NL}

Λ

• How many terms to keep?

-each term contributes as an extra factor of

$$\frac{\delta\rho_l}{\rho} \sim \frac{k}{k_{\rm NL}}$$

• we keep as many as required precision

•
$$\Rightarrow$$
 manifest expansion in $\frac{k}{k_{\rm NL}} \ll 1$



• The EFTofLSS is a *theory* and not a *model*

• no guess-work, no intuition

-It Taylor expands in well defined, small parameters

- Order by order improvement
- We can estimate the theory error
- We can compute the next order and the result will improve

-Several observables are connected

-all of this does not happen for a model

How do we know the EFTofLSS is right?

• The EFTofLSS is *the* theory of the long distance universe

-By using only the symmetries of the problem, the Effective Field Theory correctly describes the LSS

-in this sense, it is manifestly correct

-this is not presumption

-History of physics have thought us that this is possible

- GR is the EFT of a spin-2 particle
- E&M for dielectrics is an EFT

— . . .

– The Chiral Lagrangian is an EFT

• All these theories have free parameters, but these parameters are guaranteed to make the theory approach the truth order by order in a perturbative expansion

• In the EFT we can solve iteratively $\delta_{\ell}, v_{\ell}, \Phi_{\ell} \ll 1$

$$\nabla^{2} \Phi_{l} = H^{2} \frac{\delta \rho_{l}}{\rho}$$

$$\partial_{t} \rho_{l} + H \rho_{l} + \partial_{i} \left(\rho_{l} v_{l}^{i} \right) = 0$$

$$\dot{v}_{l}^{i} + H v_{l}^{i} + v_{l}^{j} \partial_{j} v_{l}^{i} = \frac{1}{\rho} \partial_{j} \tau_{ij}$$

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-need to renormalize

- as loops with short-distance mode not under control:
- crucial difference wrt former techniques



Connecting with the Eulerian Treatment

• When we solve iteratively these equations in $\delta_{\ell}, v_{\ell}, \Phi_{\ell} \ll 1$,

-this corresponds to expanding in two parameters:

 $\epsilon_{\text{long displacement}}(k) \sim k^2 \int^k d^3q \; \frac{P(q)}{q^2}$

 $\epsilon_{\rm tidal}(k) \sim \int^{\kappa} d^3q \ P(q)$

Effect of Long Overdensities

Effect of Long Displacements

- –Displacement from long modes, longer than the BAO, cancel in P(k) by GR
- ⇒ they are important only for the BAO with Zaldarriaga JCAP1502 see originally Scoccimarro and Frieman 9609047



Perturbation Theory in our Universe

• Our universe has more than one scale: parameters scale differently.



 $\epsilon_{\text{long displacement is of order one}}$ for low k 's, but being IR dominated, its contribution can be treated non-perturbatively

Since displacements displace (they do not deform) effect is kinematical and not dynamical (so conceivable to resum)

• After IR-resummation, and after renormalization, each loop goes as power of $(\epsilon_{tidal})^L$

with Zaldarriaga JCAP1502

Was the IR-resummation already done by Zeldovich?

You are reinventing the wheel

- assessment from a famous theorist

• Zeldovich had already `guessed' a way to get a solution by non-expanding in

 $\epsilon_{\rm long \ displacement}$

- *But*:
 - –Zeldovich approximation is an approximation in ϵ_{tidal}
 - It is just a super-clever, awesome, humbling.... approximation
 - but it does not tell us how to include the next correction in ϵ_{tidal} , it ignores it
 - two decades of failing in making Lagrangian PT work demonstrate this
- Our IR-resummation goes beyond Zeldovich, because it allows us to go to arbitrary order in ϵ_{tidal}
 - this is what I would call to understand the problem

with Zaldarriaga JCAP1502

Results for Dark Matter

EFT of Large Scale Structures

- Loop contributions from non-linear modes give non-sense results: we need to correct for them: renormalization (make the calculation UV-insensitive)
- At 1-loop $\partial^2 \tau_{ij} \sim c_s \ k^2 \delta(k)$
- At 2-loops, consider $\partial^2 \tau_{ij} \sim c_1 \ k^2 [\delta^2](k) + c_4 \ k^4 \delta(k)$



Estimate size of counterterms

by requiring cutoff independent result $P_{2 \text{ loop}}^{(\text{UV-safe},\Lambda=\infty)}$ + counterterms)/ $P_{2 \text{ loop}}^{(\text{UV-safe}, \Lambda=2)}$ 1.6 --- $c_1^{(\text{UV})} = c_4^{(\text{UV})} = c_{\text{stoch}}^{(\text{UV})} = 0$ 1.4 ---- best-fit values for $c_1^{(UV)}, c_4^{(UV)}, c_{stoch}^{(UV)}$ 1.2 1.0 0.8 0.6 0.4 0.2 0.0 0.4 0.6 0.8 1.0with Foreman and Perrier 1507 $k [h \text{ Mpc}^{-1}]$

• \Rightarrow At two-loops, with precise data, 3 counterterms are needed, and we estimate size

• The fact that this works is another proof that the EFTofLSS is correct

EFT of Large Scale Structures

with Foreman and Perrier 1507

• At 2-loops, we need speed of sound & quadratic & higher-derivative counterterm:

$$\partial^2 \tau_{ij} \sim c_s \ k^2 \delta(k) + c_1 \ k^2 [\delta^2](k) + c_4 \ k^4 \delta(k)$$

- How to choose for them?
 - Fit them to data
 - How to get sure we do not overfit?
 - As data increase, the improved measurement of parameters should be compatible with measurement with less data

EFT of Large Scale Structures at Two Loops

 $\partial^2 \tau_{ij} \sim c_s \ k^2 \delta(k) + c_1 \ k^2 [\delta^2](k) + c_4 \ k^4 \delta(k)$

- As data increase, the improved measurement of parameters should be compatible with measurement with less data
- . If we fit until $k \sim 0.32 \, h \, {\rm Mpc}^{-1}$ we are not overfitting



with Foreman and Perrier 1507



- k-reach pushed to $k \sim 0.34 \, h \, {\rm Mpc}^{-1}$, cosmic variance $\sim 10^{-3}$
- Order by order improvement $\left(\frac{k}{k_{\rm NL}}\right)^L$
- Huge gain wrt former theories

with Carrasco, Foreman and Green JCAP1407 with Zaldarriaga JCAP1502 with Foreman and Perrier 1507



- Are we overfitting?
 - Fitting procedure constructed in order not to overfit
 - Size of counterterms compatible with expectations from UV-insensitivity
 - Theory error estimated by imposing 1σ compatibility of measurement of parameters as we increase k_{fit}
 - If we set $P_{2-\text{loop}} = 0$, then fit to data is very bad





• All former theories, RPT, LPT,.... differ from SPT just by the IR-resummation

• \implies by GR, IR-modes cancel in P(k), so cannot change the UV-reach of the theory

- they just change the BAO, which are 2% oscillations in k-space
- So, if you see plots where RPT is improving the UV-reach wrt SPT, it is not *just* IR-resummation, but something else which, to me, is physically not derived nor justified
 - at this point, you can call RPT as you wish (fitting function? ansatz?...you choose)
 - it does not seem to me a well defined theory.



• In former two-loop EFT calculation, the k-reach had been estimated to potentially reach $k \sim 0.4 - 0.6 h \,\mathrm{Mpc^{-1}}$ with only the c_s parameter.

• Using Coyote-emulator data, 2% sys. error bars



- More precise data show that the C_s parameter is 30% different than from Coyote
 - reduces the k-reach a bit more than expected (not by much though)
- Baldauf and Zaldarriaga 1507, 1507, with Foreman and Hideki 1507 • It is compulsory that with more precise data (0.1%), the k-reach is decreased (look linear theory failing at $k \sim 0.03 h \,\mathrm{Mpc}^{-1}$!) and more counterterms are needed:
 - k-reach makes sense as concept only after specifying the precision of the data
- The story has not been changing apart for better measurement of the parameters

Precision at low k's



- k-reach is not everything. Precision at low k's is also important and great
 - no matter the k-reach, at low k's very fast convergence.
- Look where linear theory fails!, $k \sim 0.03 h \,\mathrm{Mpc^{-1}}$, and these are Euclid-like error bars!
- we can see that order by order, at low k's, the EFT converges!
 - former techniques and N-body sims do not converge to this accuracy

In the EFTofLSS we need parameters. Let us measure them from small N-body Simulations!

with Carrasco and Hertzberg JHEP 2012

Measuring parameters from N-body sims.

• The EFT parameters can be measured from small N-body simulations, using UV theory

-similar to what happens in QCD: lattice sims

• We measure c_s using the dark matter particles:



• Agreement with fitting from Power Spectrum directly

with Carrasco and Hertzberg JHEP 2012

 $\frac{d c_s}{d\Lambda} = \frac{d}{d\Lambda} \int^{\Lambda} d^3k \ P_{13}(k)$

Other Observables

Other Observables

-Since this is a theory and not a model

-prediction for other observables from same parameters

-3point function

-very non-trivial function of three variables!

with Angulo, Foreman and Schmittful **1406** see also Baldauf et al. **1406**

-Momentum

-They all work as they should

with Carrasco, Foreman and Green JCAP1407

-Vorticity Spectrum

with Carrasco, Foreman and Green JCAP1407

-agrees with most accurate measurements in simulations

Pueblas and Scoccimarro **0809** Hahn, Angulo, Abel **1404**



Analytic Prediction of Baryon Effects

with Lewandowski and Perko JCAP1502

Baryonic effects

• When stars explode, baryons behave differently than dark matter



• They cannot be reliably simulated due to large range of scales

- Main idea for EFT for dark matter:
 - since in history of universe Dark Matter moves about $1/k_{\rm NL} \sim 10 \,{
 m Mpc}$
 - \implies it is an effective fluid-like system with mean free path $\sim 1/k_{\rm NL}$
- Baryons heat due to star formation, but they do not move much:
 - indeed, from observations in clusters, we know that they move

 $1/k_{\rm NL(B)} \sim 1/k_{\rm NL} \sim 10 \,{\rm Mpc}$

• \Rightarrow it is an effective fluid with similar mean free path

-Universe with CDM+Baryons \implies EFTofLSS with 2 species

• The effective force on baryons: expand force in long-wavelength fields:

$$\partial^2 \tau_b + \partial \gamma_b \sim c_s^2 \,\partial^2 \delta_l + c_\star \,\partial^2 \delta_l + \dots$$

gravity-induced pressure

star formation-induced pressure



-and it seems to work as expected

Halos Power and Bispectrum

Senatore (alone) **1406** with Angulo, Fasiello and Vlah **1503**

Halos in the EFTofLSS

- Similar considerations apply to biased tracers:
 - Halo formation depends on fields evaluated on past history on past path

$$\delta_{M}(\vec{x},t) \simeq \int^{t} dt' \ H(t') \left[\bar{c}_{\partial^{2}\phi}(t,t') \ \frac{\partial^{2}\phi(\vec{x}_{\mathrm{fl}},t')}{H(t')^{2}} \right]$$
Senatore **1406**
Mirbabahi, Schmidt, Zaldarriaga **1412**
$$+ \bar{c}_{\partial_{i}v^{i}}(t,t') \ \frac{\partial_{i}v^{i}(\vec{x}_{\mathrm{fl}},t')}{H(t')} + \bar{c}_{\partial_{i}\partial_{j}\phi\partial^{i}\partial^{j}\phi}(t,t') \ \frac{\partial_{i}\partial_{j}\phi(\vec{x}_{\mathrm{fl}},t')}{H(t')^{2}} \frac{\partial^{i}\partial^{j}\phi(\vec{x}_{\mathrm{fl}},t')}{H(t')^{2}} + \dots \right]$$

• this generalizes and completes McDonald and Roy 0902

• this correctly parametrizes assembly bias

- Since evolution is k-independent, we can formally evaluate the integrals, to obtain only 7 parameters for
 - at 1-loop power spectrum
 - tree level bispectrum
 - tree level trispectrum

Halos in the EFTofLSS with

- We compare $P_{hh}^{1-\text{loop}}$, $P_{hm}^{1-\text{loop}}$, B_{hhh}^{tree} , B_{hhm}^{tree} , B_{hmm}^{tree} using 7 bias parameters
 - Fit works up to $k \simeq 0.3 \, h \text{Mpc}^{-1}$ for 1-loop and $k \simeq 0.15 \, h \text{Mpc}^{-1}$ at tree-level (for low bins, with large theory uncertainties): as it should



- the 3pt function measures very well the bias coefficients (there is a lot of data)
 - 4pt function is predicted
- Similar formulas just worked out for redshift space distortions

with Zaldarriaga 1409



- A manifestly well-defined perturbation theory $\left(\frac{k}{k_{\rm NL}}\right)^L$
- we match until $k \sim 0.34 \, h \, \text{Mpc}^{-1}$, as where we should stop fitting -there are $\sim 10^2$ more quasi linear modes than previously believed! -huge impact on possibilities, for ex: $f_{\text{NL}}^{\text{equil., orthog.}} \lesssim 1$, neutrinos, dark energy.
- This is an huge opportunity and a challenge for us.

Conclusions

- The EFTofLSS: a novel and powerful way to analytically describe Large Scale Structures
 - -It describes something true, the real universe: many application for astrophysics
 - -It uses novel techniques that come from particle physics
 - Loops, divergencies, counterterms and renormalization, IR divergencies
 - Measurements in Simulations (lattice) and lattice-running
- Many calculations and verifications to do
- Huge opportunity for complementarity with simulations
 - -Maybe do simulations focussed to convey the EFT parameters?!
- If success continues, revolution in our expectations for next generation experiments





Extra

A subtlety: non-locality in Time

This EFT is non-local in time

• For local EFT, we need hierarchy of scales.

-In space we are ok





-In time we are not ok: all modes evolve with time-scale of order Hubble



with Carrasco, Foreman and Green 1310

Carroll, Leichenauer, Pollak 1310

• \Rightarrow The EFT is local in space, non-local in time

 $\langle \tau_{ij} \rangle_{\delta_l} \sim \int dt' \ K(t,t') \ \partial^2 \phi(x_{\rm fl},t')$

-Technically it does not affect much because the linear propagator is local in space

A Non-Renormalization Theorem

A non-renormalization theorem

• Can the short distance non-linearities change completely the overall expansion rate of the universe, possibly leading to acceleration without Λ ?



- In terms of the short distance perturbation, the effective stress tensor reads $\tau_{00} \sim (\text{mass} + \text{kinetic energy} + \text{gravity potential energy})$ $\tau_{ii} \sim (2 \text{ kinetic energy} + \text{gravity potential energy})$
- when objects virialize, induced pressure vanish $\langle \rho_S \left(2v_S^2 + \Phi_S \right) \rangle_{\text{virialized}} \to 0$

-ultraviolet modes do not contribute (like in SUSY)

• The backreaction is dominated by modes at the virialization scale

$$\tau_{l,ij} \sim \partial_t^2 \left(x^2 \tau_{l,00} \right) \sim \frac{H^2}{k_{\rm NL}^2} \tau_{l,00} \sim 10^{-5} \tau_{l,00} \qquad \Rightarrow \quad w_{\rm induced} \sim 10^{-5}$$

with Baumann, Nicolis and Zaldarriaga JCAP 2012

Perturbation theory

• Since equations are non-linear, we obtain convolution integrals (loops)

$$\delta^{(n)} \sim \int \text{GreenFunction} \times \text{Source}^{(n)} \left[\delta^{(1)}, \delta^{(2)}, \dots, \delta^{(n-1)} \right]$$

$$\Rightarrow \quad \delta^{(2)}(k_l) \sim \int d^3k_s \ \delta^{(1)}(k_s) \ \delta^{(1)}(k_l - k_s) \ , \quad \Rightarrow \quad \langle \delta_l^2 \rangle \sim \int d^3k_s \ \langle \delta_s^{(1)2} \rangle^2$$



• Regularization and renormalization of loops (no-scale universe) $P_{11}(k) = \frac{1}{k_{\rm NL}^3} \left(\frac{k}{k_{\rm NL}}\right)^n$ -evaluate with cutoff:

$$P_{1-\text{loop}} = c_1^{\Lambda} \left(\frac{\Lambda}{k_{\text{NL}}}\right) \left(\frac{k}{k_{\text{NL}}}\right)^2 P_{11} + c_1^{\text{finite}} \left(\frac{k}{k_{\text{NL}}}\right)^3 P_{11} + \text{subleading in } \frac{k}{k_{\text{NL}}}$$

- divergence (we extrapolated the equations where they were not valid anymore)

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- we need to add effect of stress tensor $\tau_{ij} \supset c_s^2 \, \delta \rho$

$$P_{11, c_s} = c_s \left(\frac{k}{k_{\rm NL}}\right)^2 P_{11}$$
, choose $c_s = -c_1^{\Lambda} \left(\frac{\Lambda}{k_{\rm NL}}\right) + c_{s, \text{finite}}$

$$\implies P_{1-\text{loop}} + P_{11, c_s} = c_{s, \text{finite}} \left(\frac{k}{k_{\text{NL}}}\right)^2 P_{11} + c_1^{\text{finite}} \left(\frac{k}{k_{\text{NL}}}\right)^3 P_{11} + \text{subleading in } \frac{k}{k_{\text{NL}}}$$

-we just re-derived renormalization

-after renormalization, result is finite and small

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The EFTofLSS at high-z

with Foreman **1503** with Foreman and Perrier **1507**

All redshifts



Results 2-loop IR-resummed

- UV reach improves at high-z 1.02 1.01 • Theory error gets smaller 1.00 0.99 0.98 1.02 1.01 0.99 z = 10.98 1.02 1.011.00 0.99 z = 20.98 0.5 1.5 0.0 k [h Mpc⁻¹]
- The gain wrt former techniques is huge
- Time dependence of C_s , C_1 , C_4 is measured (only 12 parameters for all z's)
 - size compatible with UV expectations
- \Rightarrow we can do CMB lensing analytically up to high ell.
 - and similarly galaxy lensing
 - C_s detected detected with high sensitivity by upcoming CMB experiments

IR-effects

The Effect of Long-modes on Shorter ones

• In Eulerian treatment



The Effect of Long-modes

- Add a long `trivial' force (trivial by GR)
- This tells you that one can resum the IR modes: this is the Lagrangian treatment



• The two species conserve mass, but exchange momentum (through gravity):

$$\begin{split} \nabla^2 \phi &= \frac{3}{2} H_0^2 \frac{a_0^3}{a} (\Omega_c \delta_c + \Omega_b \delta_b) \\ \dot{\delta}_c &= -\frac{1}{a} \partial_i ((1 + \delta_c) v_c^i) \\ \dot{\delta}_b &= -\frac{1}{a} \partial_i ((1 + \delta_b) v_b^i) \\ \partial_i \dot{v}_c^i + H \partial_i v_c^i + \frac{1}{a} \partial_i (v_c^j \partial_j v_c^i) + \frac{1}{a} \partial^2 \phi = -\frac{1}{a} \partial_i (\partial \tau_\rho)_c^i + \frac{1}{a} \partial_i (\gamma_c)_c^i \\ \partial_i \dot{v}_b^i + H \partial_i v_b^i + \frac{1}{a} \partial_i (v_b^j \partial_j v_b^i) + \frac{1}{a} \partial^2 \phi = -\frac{1}{a} \partial_i (\partial \tau_\rho)_b^i + \frac{1}{a} \partial_i (\gamma_b)_b^i \\ \end{split}$$

$$(\partial \tau_{\rho})^{i}_{\sigma} = \frac{1}{\rho_{\sigma}} \partial_{j} \tau^{ij}_{\sigma} , \qquad (\gamma)^{i}_{c} = \frac{1}{\rho_{c}} V^{i} , \qquad (\gamma)^{i}_{b} = -\frac{1}{\rho_{b}} V^{i}$$

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• The two species conserve mass, but exchange momentum (through gravity):

$$\begin{split} \nabla^2 \phi &= \frac{3}{2} H_0^2 \frac{a_0^3}{a} (\Omega_c \delta_c + \Omega_b \delta_b) \\ \dot{\delta}_c &= -\frac{1}{a} \partial_i ((1 + \delta_c) v_c^i) \\ \dot{\delta}_b &= -\frac{1}{a} \partial_i ((1 + \delta_b) v_b^i) \end{split}$$
Each-species' mass conservation
$$\dot{\delta}_b &= -\frac{1}{a} \partial_i ((1 + \delta_b) v_b^i) \\ \partial_i \dot{v}_c^i + H \partial_i v_c^i + \frac{1}{a} \partial_i (v_c^j \partial_j v_c^i) + \frac{1}{a} \partial^2 \phi = -\frac{1}{a} \partial_i (\partial \tau_\rho)_c^i + \frac{1}{a} \partial_i (\gamma)_c^i , \\ \partial_i \dot{v}_b^i + H \partial_i v_b^i + \frac{1}{a} \partial_i (v_b^j \partial_j v_b^i) + \frac{1}{a} \partial^2 \phi = -\frac{1}{a} \partial_i (\partial \tau_\rho)_b^i + \frac{1}{a} \partial_i (\gamma)_b^i , \end{split}$$

$$(\partial \tau_{\rho})^{i}_{\sigma} = \frac{1}{\rho_{\sigma}} \partial_{j} \tau^{ij}_{\sigma} , \qquad (\gamma)^{i}_{c} = \frac{1}{\rho_{c}} V^{i} , \qquad (\gamma)^{i}_{b} = -\frac{1}{\rho_{b}} V^{i}$$

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$$\begin{split} \nabla^2 \phi &= \frac{3}{2} H_0^2 \frac{a_0^3}{a} (\Omega_c \delta_c + \Omega_b \delta_b) & \text{Stress tensor like term:} \\ \dot{\delta}_c &= -\frac{1}{a} \partial_i ((1 + \delta_c) v_c^i) & \text{two derivatives from momentum conservation} \\ \dot{\delta}_b &= -\frac{1}{a} \partial_i ((1 + \delta_b) v_b^i) & & & & \\ \partial_i \dot{v}_c^i + H \partial_i v_c^i + \frac{1}{a} \partial_i (v_c^j \partial_j v_c^i) + \frac{1}{a} \partial^2 \phi = -\frac{1}{a} \partial_i (\partial \tau_\rho)_c^i + \frac{1}{a} \partial_i (\gamma_c)_c^i \\ \partial_i \dot{v}_b^i + H \partial_i v_b^i + \frac{1}{a} \partial_i (v_b^j \partial_j v_b^i) + \frac{1}{a} \partial^2 \phi = -\frac{1}{a} \partial_i (\partial \tau_\rho)_b^i + \frac{1}{a} \partial_i (\gamma_b)_b^i \\ & & & \\ \partial_i \dot{v}_b^i + H \partial_i v_b^i + \frac{1}{a} \partial_i (v_b^j \partial_j v_b^i) + \frac{1}{a} \partial^2 \phi = -\frac{1}{a} \partial_i (\partial \tau_\rho)_b^i + \frac{1}{a} \partial_i (\gamma_b)_b^i \\ & & \\ & & \\ & & \\ \partial_i \dot{v}_b^i + H \partial_i v_b^i + \frac{1}{a} \partial_i (v_b^j \partial_j v_b^i) + \frac{1}{a} \partial^2 \phi = -\frac{1}{a} \partial_i (\partial \tau_\rho)_b^i + \frac{1}{a} \partial_i (\gamma_b)_b^i \\ & & \\ & \\ & & \\$$

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• The two species conserve mass, but exchange momentum (through gravity):

$$\begin{split} \nabla^2 \phi &= \frac{3}{2} H_0^2 \frac{a_0^3}{a} (\Omega_c \delta_c + \Omega_b \delta_b) \text{ No Stress-tensor-like term:} \\ &\text{only one derivative term,} \\ \dot{\delta}_c &= -\frac{1}{a} \partial_i ((1 + \delta_c) v_c^i) &\text{it cancel in the sum (overall momentum cons.)} \\ \dot{\delta}_b &= -\frac{1}{a} \partial_i ((1 + \delta_b) v_b^i) \\ \partial_i \dot{v}_c^i + H \partial_i v_c^i + \frac{1}{a} \partial_i (v_c^j \partial_j v_c^i) + \frac{1}{a} \partial^2 \phi = -\frac{1}{a} \partial_i (\partial \tau_\rho)_c^i + \frac{1}{a} \partial_i (\gamma)_c^i , \\ \partial_i \dot{v}_b^i + H \partial_i v_b^i + \frac{1}{a} \partial_i (v_b^j \partial_j v_b^i) + \frac{1}{a} \partial^2 \phi = -\frac{1}{a} \partial_i (\partial \tau_\rho)_b^i + \frac{1}{a} \partial_i (\gamma)_b^i , \end{split}$$

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