A complete parametrization of galaxy bias

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Mirbabayi, FS, Zaldarriaga (MSZ), arXiv:1412.5169

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Clarification: what do I mean by bias ?

- I mean a *perturbative bias expansion:*

$$\delta_g(\mathbf{x},\tau) = \sum_O b_O(\tau) O(\mathbf{x},\tau)$$

- Goal is to identify which operators O and corresponding bias parameters b_O we need to keep
    - at each order in perturbation theory (PT)
    - be agnostic: should apply to *any tracer*
- Why ? PT is the only approach that allows us a *rigorous error control* on our theory prediction - for any tracer

Bias: open questions

- Historically, “local bias” ansatz:

$$O \in \{\delta,\, \delta^2, \ldots\}$$

Fry & Gaztanaga

- Recently, has become clear that we need to include biasing with respect to *tidal field*

$$O \in \{(K_{ij})^2,\, (K_{ij})^3,\, \delta(K_{ij})^2, \ldots\}$$

$$K_{ij} = \left(\partial_i \partial_j - \frac{1}{3}\delta_{ij}\nabla^2\right)\Phi$$

  - Often referred to as “nonlocal bias”

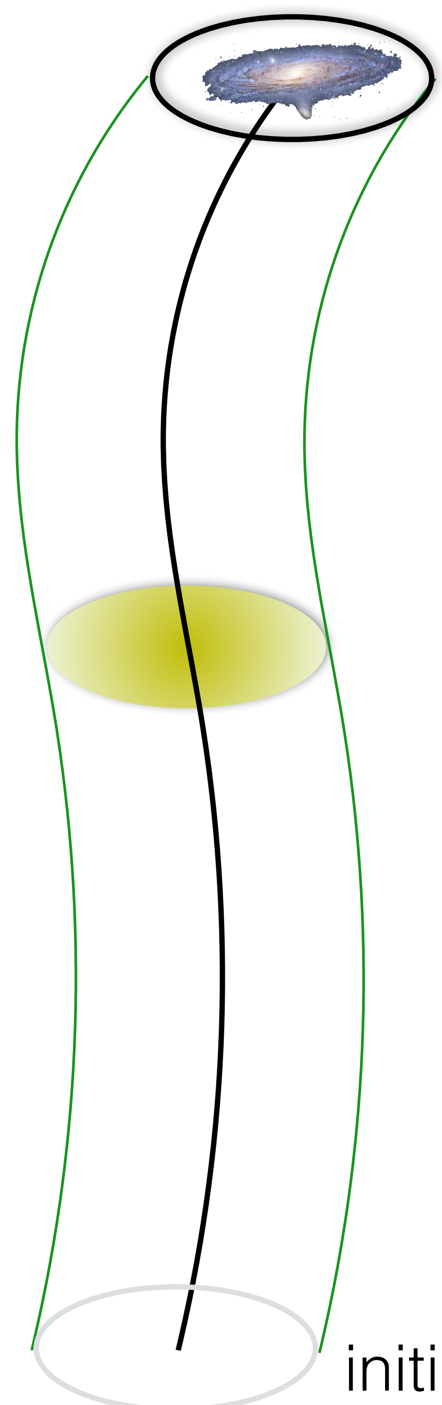
McDonald & Roy
Chan et al, Baldauf et al

Bias: open questions

- We can thus use set of operators $O \in \{\delta, \delta^2, (K_{ij})^2, \dots\}$
- *But this leaves several questions:*
    - How do we know it is complete (describes any tracer ?)
    - Lagrangian vs Eulerian biasing (evaluate *O*'s at initial or final time ?)
    - What about scale-dependent bias ?
    - And velocity bias ?

A general framework for bias

- Local galaxy density n_g can only depend on local observables, determined by *equivalence principle*:
    - density δ and tidal field K_{ij} *
- However, in general n_g will depend on these observables *along the entire past trajectory* (geodesic)
    - Equivalently, galaxy density depends on time derivatives of local observables



* also, spatial derivatives -> later; assume Gaussian adiab. IC here

General bias expansion

- That is, set of operators should include δ, K_{ij} as well as $D^n/Dt^n\{\delta^2, K_{ij}, \ldots\}$, where *D/Dt is convective (or Lagrangian) time derivative*
- Want to work out which terms to keep at given order in PT: i.e., need a *complete non-redundant set* (without double-counting) of operators
- MSZ give Eulerian and Lagrangian examples
  - Key trick: use that in PT,

$$\delta = D(t)\delta^{[1]}(\mathbf{x}) + D^2(t)\delta^{[2]}(\mathbf{x}) + \cdots$$

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General bias expansion

- Example: up to third order,

$$\delta_g(\mathbf{x}_{\mathrm{fl}}(\tau),\tau) = \sum_{n=1}^{3} \frac{b_n^E}{n!}\delta^n + \sum_{n=2}^{3} \frac{b_{K^n}^E}{n!}\mathrm{Tr}\left[K_{ij}^n\right] + \frac{1}{6} b_{\delta K^2}^E \delta\, \mathrm{Tr}\left[K_{ij}^2\right]$$
$$+ \frac{1}{6} b_{\mathrm{nloc}}^E K^{ij} \frac{\partial_i \partial_j}{\nabla^2}\left(\delta^2 - \frac{3}{2}\mathrm{Tr}[K_{ij}^2]\right)$$

Nonlocal* term, induced by time evolution

(this term was introduced in different form before; truly new operators from evolution appear at 4th order)

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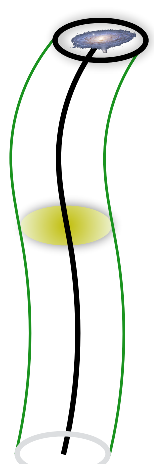
Virtues of a complete bias parametrization

- *Unambiguous set of bias parameters* at each order in PT
    - E.g., want to calculate *N*-pt function to *m* loops ? This tells you exactly how many and which biases you need
- Works *equivalently in Eulerian and Lagrangian space* (or anything in between)
    - Does not make any assumptions about where in time halo/galaxy formation happens



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Higher derivative biases

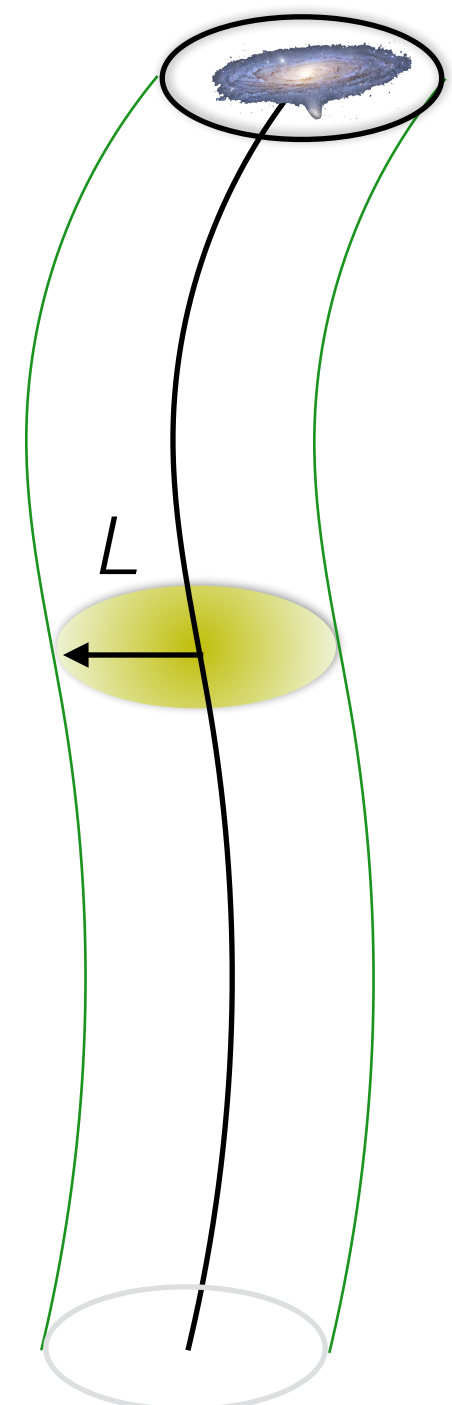
- Treatment so far is valid if density and tidal field perturbations are *effectively spatially constant* as far as the local galaxy is concerned: *lowest order in spatial derivatives*

- However, galaxies will care about detailed matter distribution within in some finite region ~*L* around them

- Dependence on matter distribution (a functional) can be expanded in terms of spatial derivatives:

$$\delta_g \supset L^2\partial^2\delta,\ L^4\partial^4\delta,\ L^2\partial_i\delta\partial^i\delta\cdots$$

↘ Scale-dep. bias ~k^2L^2



Virtues of higher derivative biases

- Physically, they are there (e.g. required for consistency by renormalization)

- By marginalizing over L, and coefficients, we effectively *smoothly cut off information on small scales* which depend on details of galaxy formation, feedback, etc.

  - Fitting $P(k)$ and $\xi(r)$ then amounts to the same information - not true for sharp k_{max} and r_{min} !

- Given matter $P(k)$ and L, *theory tells us how many higher derivative biases we need* at desired order in PT

Remark: what is “nonlocal” bias ?

- All operators in bias expansion have to be local observables - in this sense, *bias is always local!*

- Beyond this, it is a matter of definition: nonlocal bias is…

    - anything that is not a power of δ, e.g. $(K_{ij})^2$ *(traditional bias literature)*

    - anything that is nonlocal in $\partial_i\partial_j\Phi$, e.g. $K^{ij}\frac{\partial_i\partial_j}{\nabla^2}\left(\delta^2 - \frac{3}{2}\mathrm{Tr}[K_{ij}^2]\right)$ *(some current literature)*

        - These latter terms are local observables because they are convective time derivatives of local observables

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For completeness…

- In general, also have to allow for *stochasticity*. Each operator should come, in addition, multiplied by a stochastic field ε.

For example, up to third order,

$$\delta_g^{\rm stoch} = \epsilon_0^* + \epsilon_\delta^E \delta + \frac{1}{2}\epsilon_{\delta^2}^E \delta^2 + \frac{1}{2}\epsilon_{K^2}^E {\rm Tr}\left[K_{ij}^2\right]$$

where ε_i are completely characterized by 1-pt PDF

- *Velocity bias:* at lowest order in derivatives, the equivalence principle requires that there is *no velocity bias*. Leading correction is of the form $\mathbf{v}_g = \mathbf{v} + L^2 \partial^2 \mathbf{v}$

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Summary (I)

- There exists a *unique bias expansion* which describes the relation between a general tracer and matter perturbations - based only on homogeneity/isotropy and the equivalence principle.

- These should allow for rigorous cosmology constraints from galaxy (and Lyα) statistics on quasilinear scales, *without making any assumptions* about galaxy formation, HOD, etc.

- There are *two cut-offs of the perturbative description*: the nonlinear scale where $\delta \sim 1$, and *L,* the scale over which galaxy formation happens. Which one is bigger is still unknown ! (and presumably depends on galaxy sample)

Summary (II)

- Of course, this results in a large number of bias parameters: here, *simulations and semi-analytics* can be extremely useful by constraining relations between bias parameters*

- Further topics not covered here: (please ask!)

  - rigorous embedding in GR context

  - connection to initial conditions (f_{NL} in single field…)

  - application to intrinsic alignments

*For precision measurements of b_{δ^n}, see Titouan Lazeyras' poster!