Clustering fossils

chasing inhomogeneities to exploit 2PCF

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Theoretical and Observational Progress on Large-scale structure of the Universe
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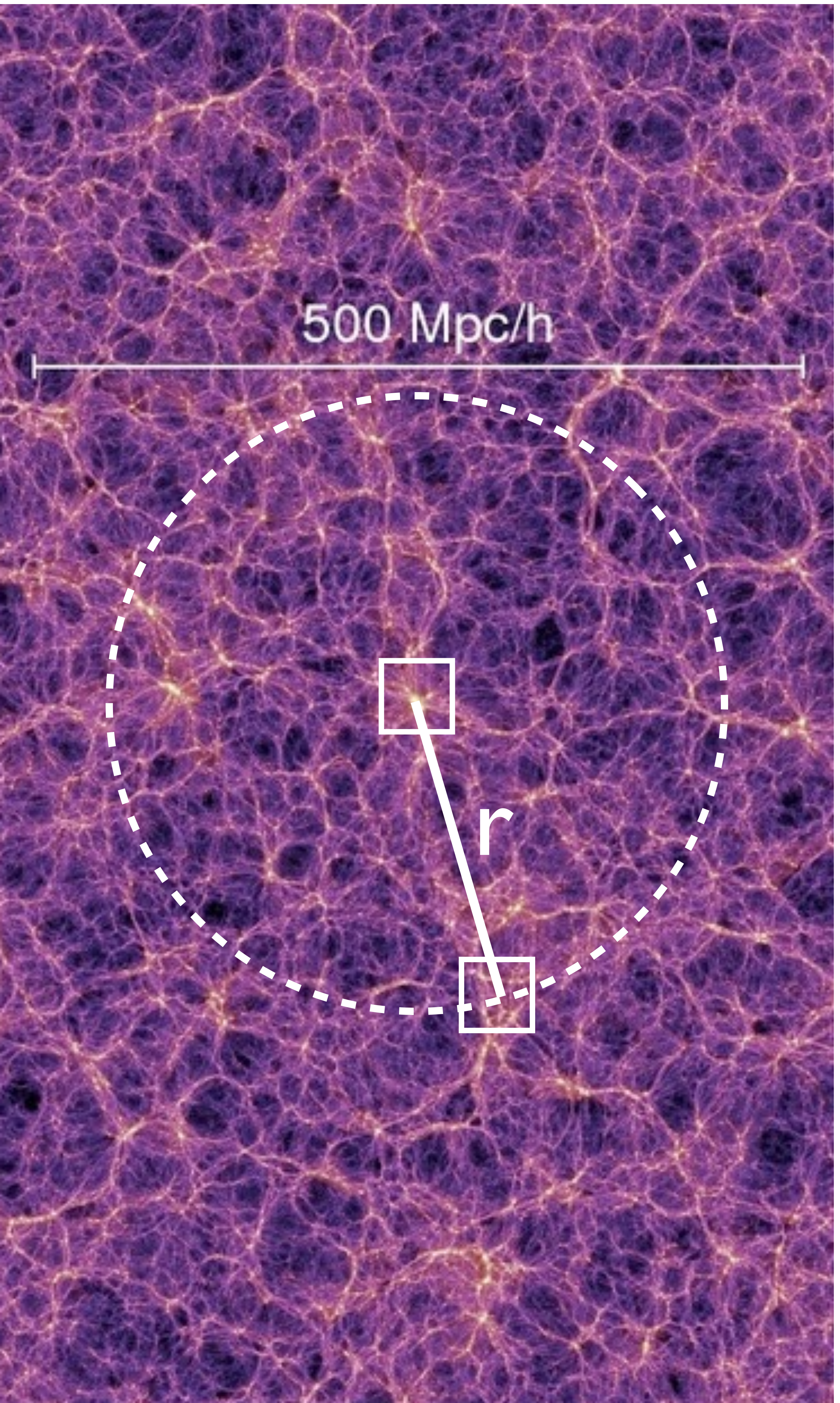
Dirac delta and homogeneity



- Two-point correlation function ξ(r) = excess number of pairs beyond random at separation r

$$\xi(\mathbf{r}) = \langle \delta(\mathbf{x})\delta(\mathbf{x}+\mathbf{r})\rangle$$

statistical homogeneity (translational invariance)

where δ(x) is the density contrast, excess number density beyond mean:

δ(x) = density(x)/(mean density) - 1

- Power spectrum is the Fourier transform of it:

$$\langle \delta(\mathbf{k})\delta(\mathbf{k}')\rangle = (2\pi)^3 P(\mathbf{k})\delta^D(\mathbf{k}+\mathbf{k}')$$

Parameterizing inhomogeneity

- Deviation from statistical homogeneity in the two-point functions will be evident in the off-diagonal correlation:

$$\langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\rangle|_{\mathbf{k}_1+\mathbf{k}_2\neq 0} \neq 0$$

- Q: How does the inhomogeneity appear?
  - A way to ***organize*** the off-diagonal correlations: **K = -(k$_1$+k$_2$)**

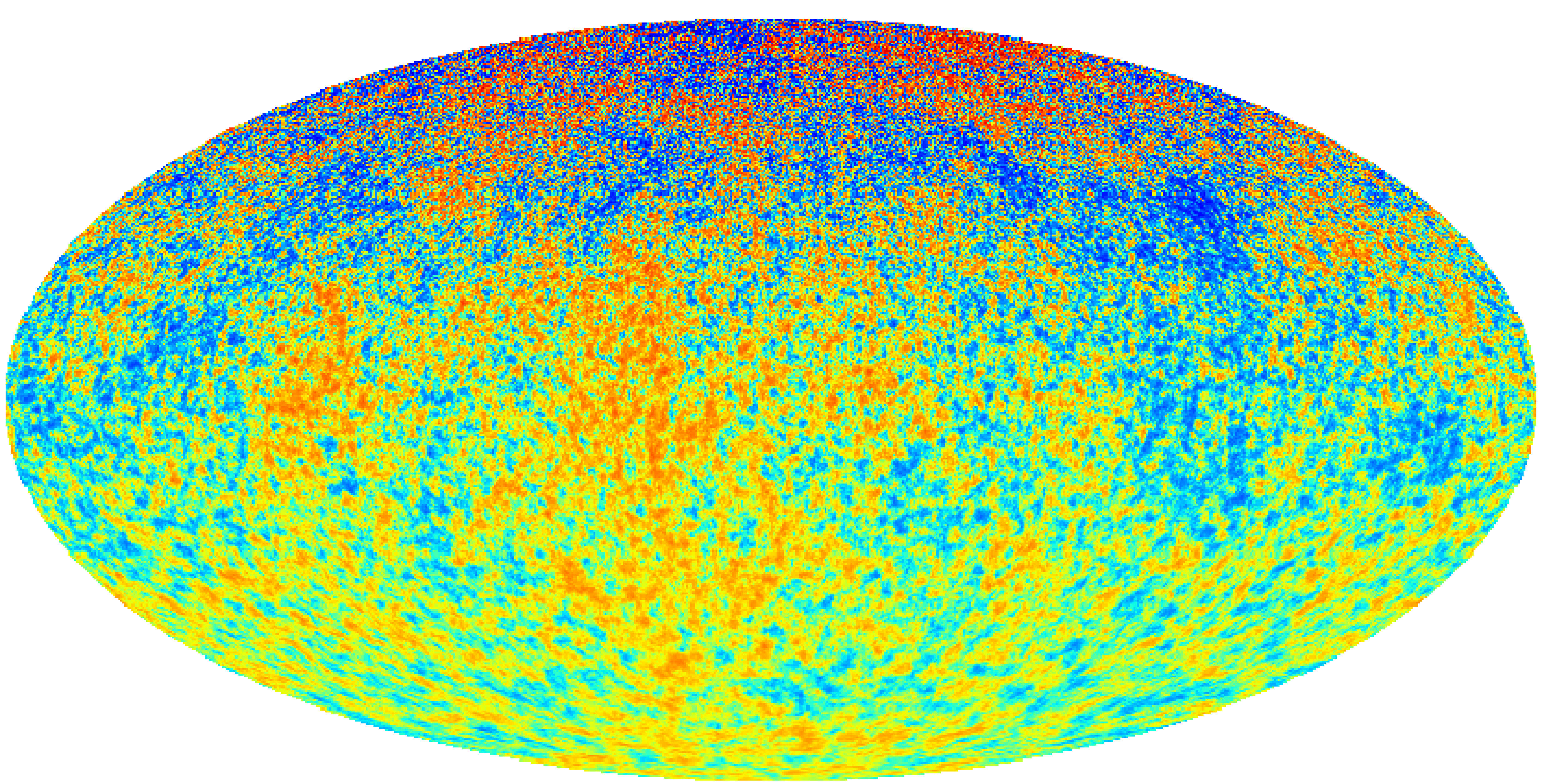
$$\langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\rangle = VP(\mathbf{k}_1)\delta^D_{\mathbf{k}_1+\mathbf{k}_2} + \sum_{\mathbf{K}} f(\mathbf{k}_1,\mathbf{k}_2,\mathbf{K})\delta^D_{\mathbf{k}_1+\mathbf{k}_2+\mathbf{K}}$$

  - The pattern of inhomogeneity is encoded in the function f!

cf. parameterizing anisotropy

- This is analogous to the BiPoSH (bipolar spherical harmonic) expansion to characterize the statistical anisotropy:

$$\langle a_{\ell m} a_{\ell' m'} \rangle = C_\ell \delta_{\ell\ell'} \delta_{mm'} + \sum_{JM} (-1)^{m'} \langle \ell, m; \ell', -m' | J, M \rangle A^{JM}_{\ell\ell'}$$



Example:
If we were to move with β~1 w.r.t. CMB rest frame, CMB would be statistically anisotropic (J=1, M=0) with

$$A^{10}_{\ell\ell'} > 0$$

What makes $\xi(r)$ inhomogeneous?

- Unknown systematics of the survey
  - If something varies over the survey volume and that something modulates the amplitude of clustering
- Our Universe might be intrinsically inhomogeneous
  - No compelling evidence so far, therefore, must be small!
- higher-order correlation functions
  - Non-linearities (e.g. position dependent power spectrum)
  - Primordial three-point function → clustering fossil

Jeong & Kamionkowski (2012)

Non-Gaussianity and ~~homogeneity~~ (local)

- IF we have a non-linear coupling between primordial density fluctuations and a spectator field h_p (JK coupling):

power spectrum of new field

$$\langle \delta_i(\mathbf{k}_1)\delta_i(\mathbf{k}_2)h_p(\mathbf{K})\rangle = VP_p(K)f_p(\mathbf{k}_1,\mathbf{k}_2)\varepsilon^p_{ij}k^i_1k^j_2\delta^D_{\mathbf{k}_1+\mathbf{k}_2+\mathbf{K}}$$

coupling amplitude

polarization basis (scalar, vector, tensor,…)

- THEN, density power spectrum we observe now has ***non-zero off-diagonal*** components: Fossil equation

$$\langle \delta_i(\mathbf{k}_1)\delta_i(\mathbf{k}_2)\rangle|_{h_p(\mathbf{K})} = VP_i(\mathbf{k}_1)\delta^D_{\mathbf{k}_1+\mathbf{k}_2} + h^*_p(\mathbf{K})f_p(\mathbf{k}_1,\mathbf{k}_2)\varepsilon^p_{ij}k^i_1k^j_2\delta^D_{\mathbf{k}_1+\mathbf{k}_2+\mathbf{K}}$$

Why called clustering fossils?

- Inflaton(s) : a scalar field(s) responsible for inflation
- But, inflaton might not be alone. Many inflationary models need/ introduce additional fields. But, direct detection of such fields turns out to be very hard:
  - Additional Scalar: may not contribute seed fluctuations
  - Vector: decays as 1/[scale factor]
  - Tensor: decays after coming inside of comoving horizon
- Clustering fossils may be the only way of detecting them!

SVT can be distinguished with $\varepsilon^p{}_{ij}$

- In a symmetric 3x3 tensor, we have 6 degrees of freedom, which are further decomposed by Scalar, Vector and Tensor polarization modes.
- They are orthogonal: $\epsilon^p_{ij}\epsilon^{p',ij} = 2\delta_{pp'}$
  - Scalar (p=0,z): $\epsilon^0_{ij} \propto \delta_{ij} \qquad \epsilon^z_{ij}(\boldsymbol{K}) \propto K_i K_j - K^2/3$
  - Vector (p=x,y): $\epsilon^{x,y}_{ij}(\boldsymbol{K}) \propto \frac{1}{2}(K_i e_j + K_j e_i)$ where $K_i e_i = 0$
  - Tensor [Gravitational Waves](p=x,+): transverse and traceless

$$K_i \epsilon^{+,\times}_{ij}(\boldsymbol{K}) = 0 \qquad \delta_{ij}\epsilon^{+,\times}_{ij}(\boldsymbol{K}) = 0$$

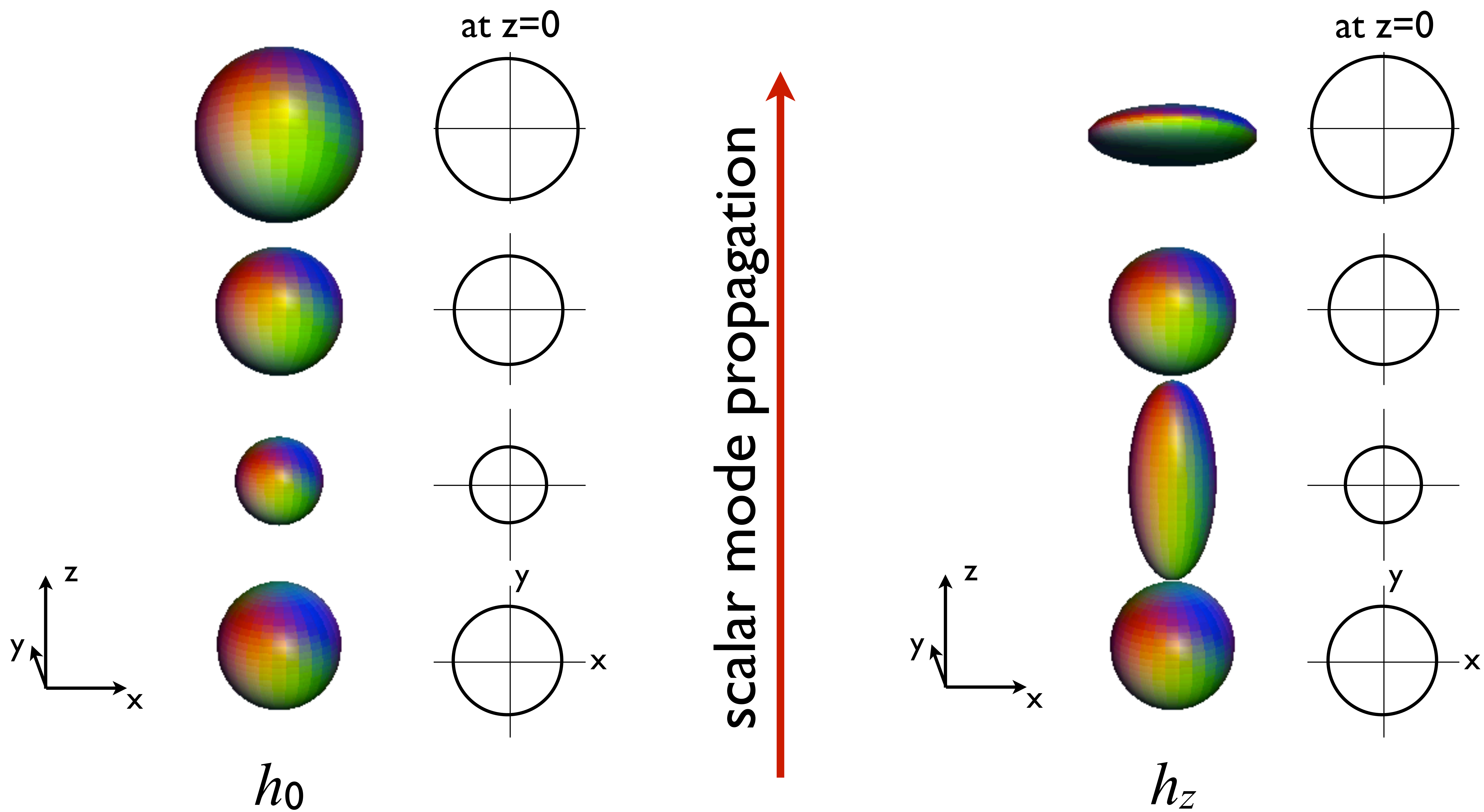
Effect of fossils on 2PCF

$$\langle \delta_i(\mathbf{k}_1)\delta_i(\mathbf{k}_2)\rangle|_{h_p(\mathbf{K})} = VP_i(\mathbf{k}_1)\delta^D_{\mathbf{k}_1+\mathbf{k}_2} + h^*_p(\mathbf{K})f_p(\mathbf{k}_1,\mathbf{k}_2)\varepsilon^p_{ij}k^i_1 k^j_2 \delta^D_{\mathbf{k}_1+\mathbf{k}_2+\mathbf{K}}$$

- Statistical homogeneity is broken in the presence of the spectator field $h_p(\mathbf{K})$.
- Depending on the polarization, the way that the spectator affects clustering is different. How?
  - I will show a rotation view of equi-correlation-function surface when $h_p(\mathbf{K})$ propagates upward.
  - Without $h_p(\mathbf{K})$, we expect that it should be spherical.

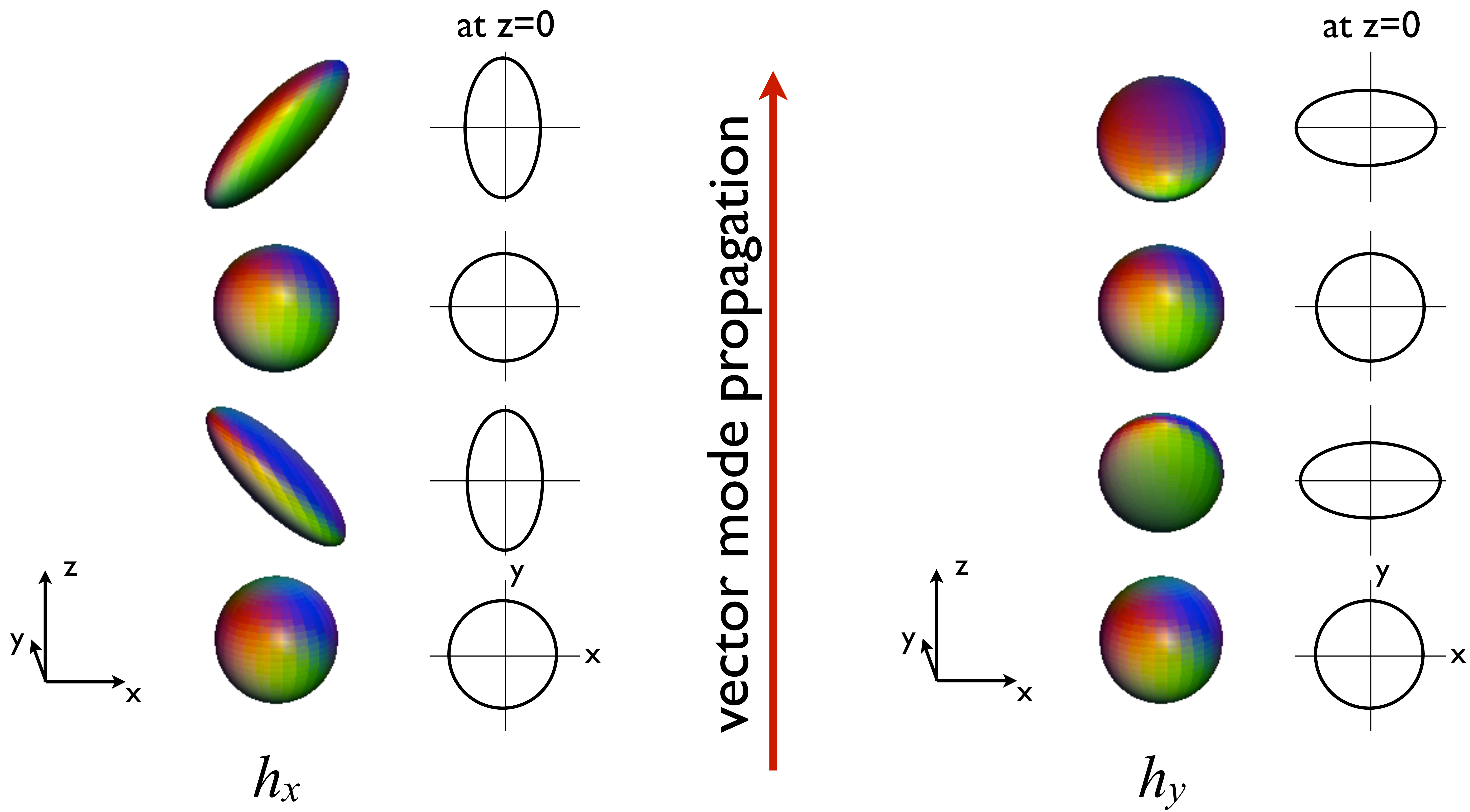
Jeong & Kamionkowski (2012)

ξ(**r**) with single scalar mode (p=0,z)



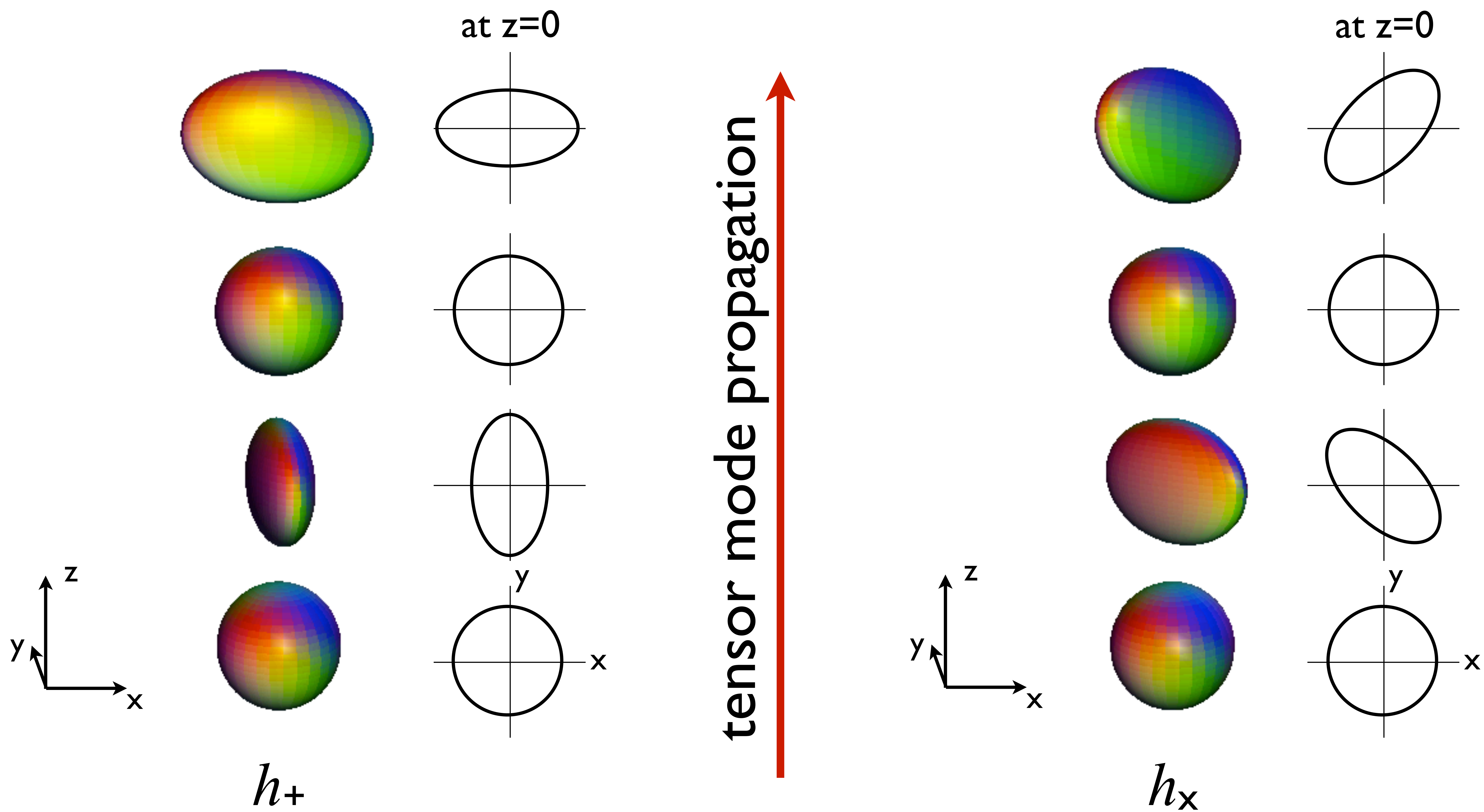
Jeong & Kamionkowski (2012)

ξ(r) with single vector mode (p=x,y)



Jeong & Kamionkowski (2012)

ξ(**r**) with single tensor mode (p=+,x)



Example: tensor clustering fossils

- For the *single-field slow-roll inflation* models ($k_t=k_1+k_2+k_3$),
  Maldacena (2003)

$$B_{\zeta\zeta h_p}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)=\frac{1}{2}\left[\frac{P_\zeta(k_1)}{k_2^3}+\frac{P_\zeta(k_2)}{k_1^3}\right]P_{h_p}(k_3)\varepsilon^p_{ij}k_1^ik_2^j\left[-k_t+\frac{k_1k_2+k_2k_3+k_3k_1}{k_t}+\frac{k_1k_2k_3}{k_t^2}\right]$$

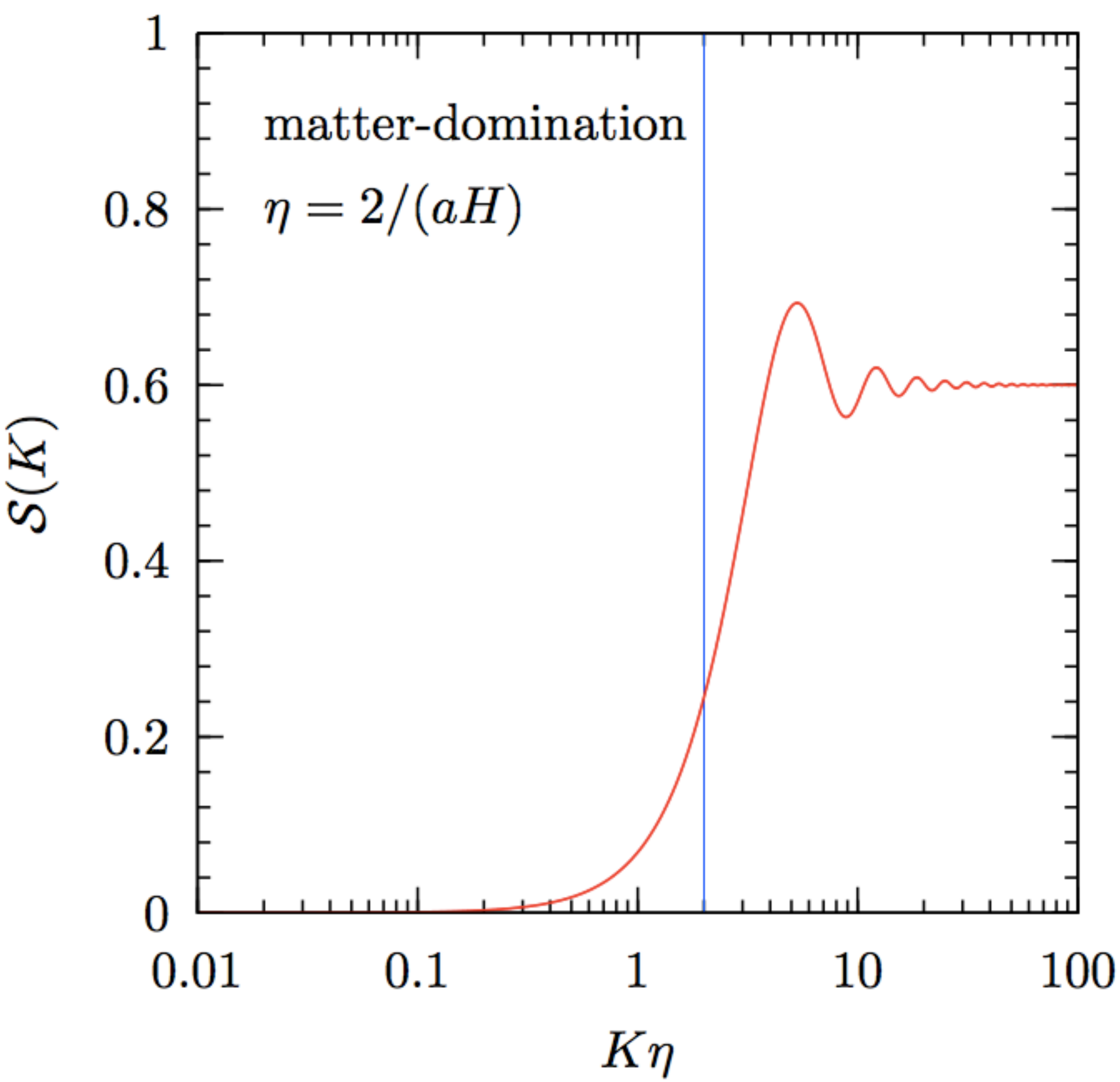
↓ squeeze limit ($k_1\approx k_2\gg k_3$)

$$\left(\frac{4-n_s}{2}\right)P_\zeta(k_1)P_{h_p}(k_3)\frac{\varepsilon^p_{ij}k_1^ik_1^j}{k_1^2}\equiv-\frac{1}{2}\frac{d\ln P_\zeta(k)}{d\ln k}P_\zeta(k_1)P_{h_p}(k_3)\varepsilon^p_{ij}\hat{k}_1^i\hat{k}_1^j$$

- In the squeeze limit, long-wavelength tensor field rescales small scale wave-vector: $k^2 \rightarrow k^2 - h_{ij}k_ik_j$ (or length $x^2 \rightarrow x^2 + h_{ij}x_ix_j$)!
- *Note*: the local observer (use physical ruler, not the coordinate ruler) will not see the effect!

Dai, Jeong & Kamionkowski (2013)

Interaction @ horizon crossing



- After inflation, tensor (long) modes re-enters horizon, and interact with density (small) modes:

$$\delta_{\rm int.}(\mathbf{k}) = -2S(K)h_p(K)\varepsilon^p_{ij}(\hat{\mathbf{K}})\hat{\mathbf{k}}_i\hat{\mathbf{k}}_j T(k)\zeta_p(\mathbf{k})$$

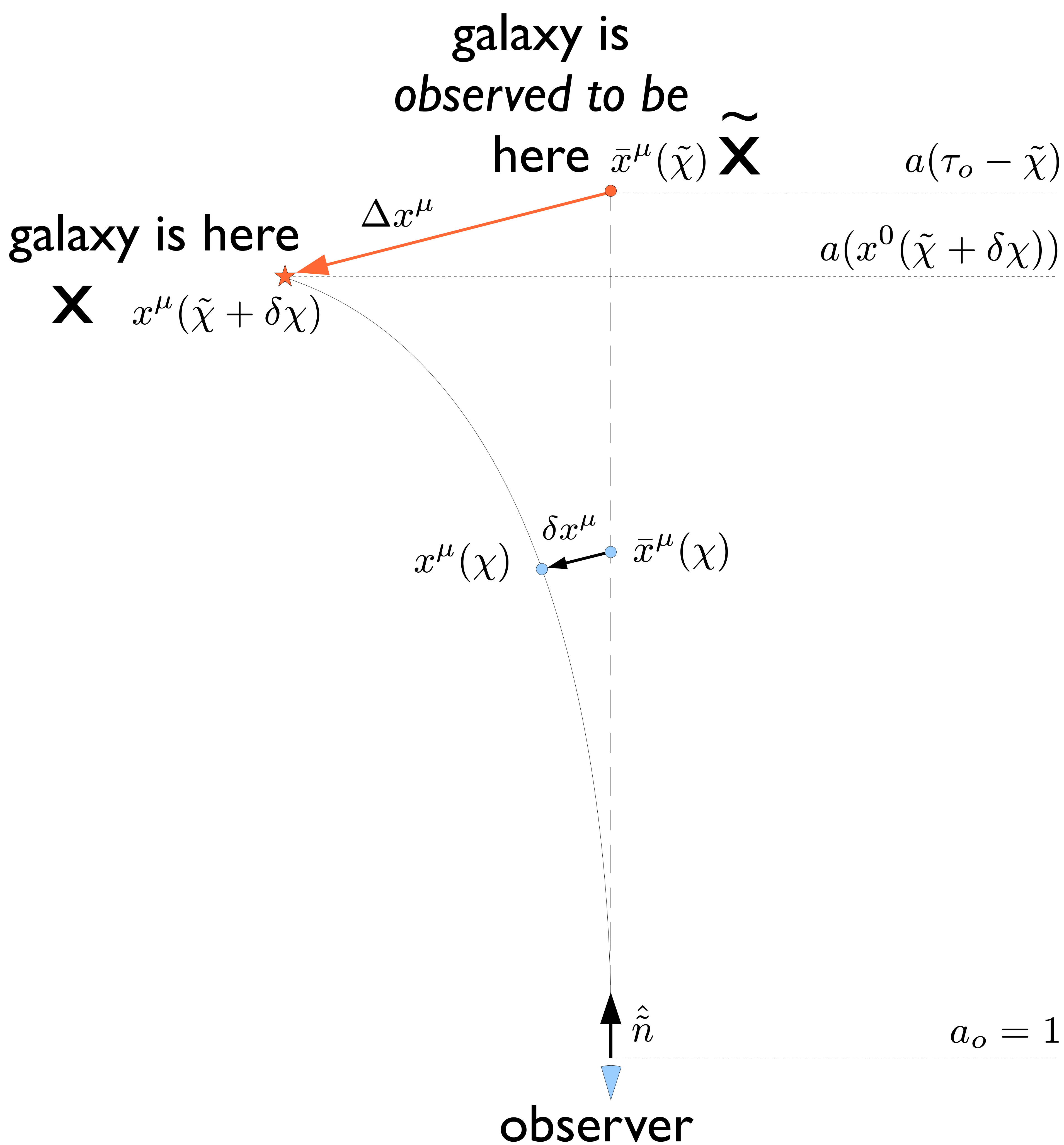
- Note that the influence dies out as tensor mode itself decays after horizon re-entry.

$$S(K) \simeq \frac{3}{5}\left[1 - \exp\left(-\frac{5}{42}K^2\eta^2\right)\right]$$

Jeong & Schmidt (2012)

$$ds^2 = a^2(\eta)\left[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$$

Light deflection due to GW



- Deflection of photon changes the observed location of galaxies.
- Geodesic equation gives Δx^μ
- On large scales (K→0), the displacement field is

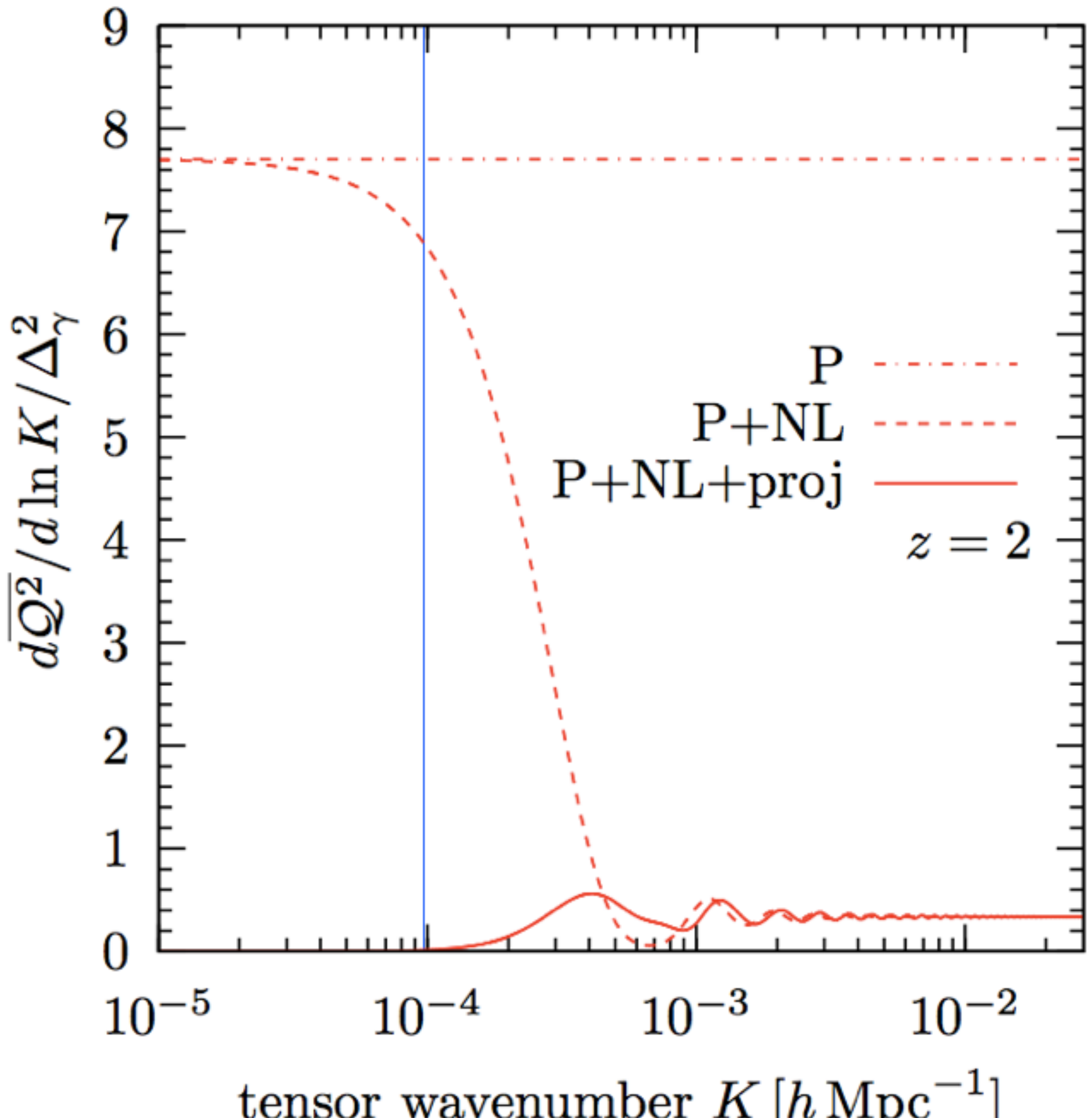
$$\Delta t \to 0$$
$$\Delta x^i \to -\frac{1}{2} h_0^{ij} x_j$$

- This effect cancels the super-horizon contributions!

Dai, Jeong & Kamionkowski (2013)
Schmidt et al. (2013)

Observable fossil amplitude

$$\langle \delta_g(\mathbf{k}_1)\delta_g(\mathbf{k}_2)\rangle \simeq P_g(k_1)\delta^D_{\mathbf{k}_1+\mathbf{k}_2} + \left[\frac{1}{2}(1-T_\gamma)\frac{\mathrm{d}\ln P_\delta(k_1)}{\mathrm{d}\ln k_1} + 2S(K)\right] P_g(k_1)h_p(K)\varepsilon^p_{ij}k_1^i k_1^j \delta^D_{\mathbf{k}_1+\mathbf{k}_2+\mathbf{K}}$$



- Quadrupole power spectrum contribution (when $K \ll k_F$) from single-field slow-roll inflation
- large-scale (super-horizon) fossils cancel *completely* with projection
- small-scale fossil cancels *partially* with tensor-scalar interaction around horizon crossing

Fossils from other inflation models

- The large-scale cancelation happens only for the SFSR models
  - With scalar-scalar-tensor correlation different from SFSR
    - Power quadrupole can constrain k_{min} (beginning of inflation)
    - clustering fossil signal can be big!
  - e.g.
    Solid inflation

Dimastrogiovanni, Fasiello, Jeong & Kamionkowski (2014)

$$\mathcal{B}_{c/c} = \frac{3}{2}\frac{\mathcal{R}}{\epsilon}P_\zeta(k)P_h(K)$$

Quasi-single field inflation

Dimastrogiovanni, Fasiello & Kamionkowski (2015)

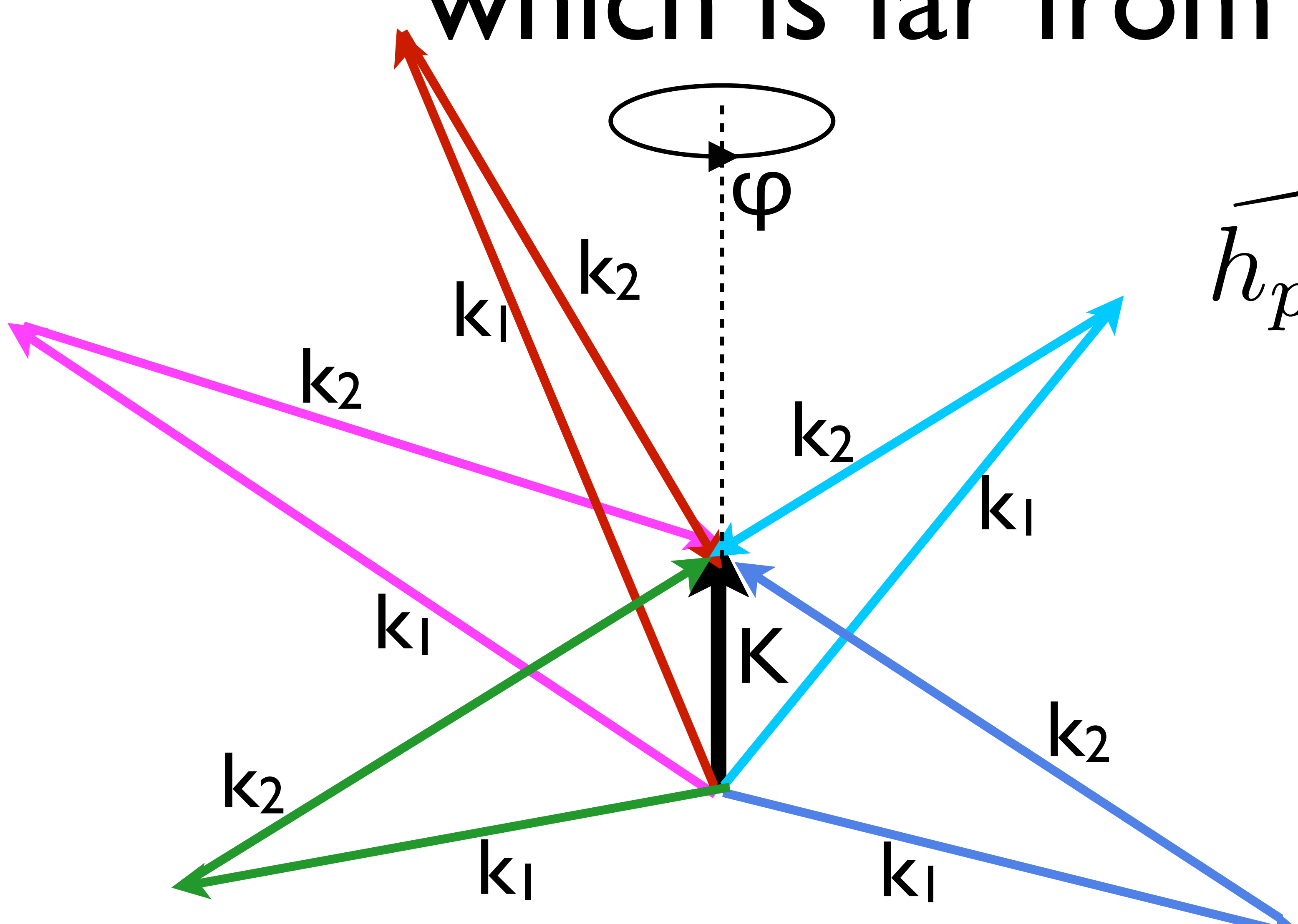
$$\mathcal{B}_{c/c} = -\frac{\pi^2}{2}w(\nu)\frac{\dot{\theta}_0^2}{H^2}P_\zeta(k)P_h(K)$$

LSS fossil estimator: naive

- Let's start from Fossil equation

$$\langle\delta(\boldsymbol{k}_1)\delta(\boldsymbol{k}_2)\rangle|_{h_p(\boldsymbol{K})} = h_p(\boldsymbol{k}_1+\boldsymbol{k}_2)f_p(\boldsymbol{k}_1,\boldsymbol{k}_2)\epsilon^p_{ij}k_1^ik_2^j\delta^D_{\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{K}}$$

- Rearranging it a bit, we get a naive estimator for the new field, which is far from optimal:



$$\widehat{h_p(\boldsymbol{K})} = \sum_{\boldsymbol{k}_1+\boldsymbol{k}_2=\boldsymbol{K}} \frac{\delta(\boldsymbol{k}_1)\delta(\boldsymbol{k}_2)}{f_p(\boldsymbol{k}_1,\boldsymbol{k}_2)\epsilon^p_{ij}k_1^ik_2^j}$$

Azimuthal(φ)-dependence, [cos(sφ)] s=spin, distinguishes scalar from vector from tensor geometrically!

Jeong & Kamionkowski (2012)

Optimal estimator (single mode)

- Inverse-variance weighting gives an optimal estimator for a single mode

$$\widehat{h_p(\mathbf{K})} = P^n_p(\mathbf{K}) \sum_{\mathbf{k}} \frac{f_p^*(\mathbf{k}, \mathbf{K}-\mathbf{k}) \epsilon^p_{ij} k^i (K-k)^j}{2V P^{\rm tot}(k) P^{\rm tot}(|\mathbf{K}-\mathbf{k}|)} \delta(\mathbf{k}) \delta(\mathbf{K}-\mathbf{k})$$

- With a noise power spectrum (P_{tot} = P_{galaxy} + P_{noise})

$$P^n_p(K) = \left[\sum_{\mathbf{k}} \frac{\left| f_p(\mathbf{k}, \mathbf{K}-\mathbf{k}) \epsilon^p_{ij} k^i (K-k)^j \right|^2}{2V P^{\rm tot}(k) P^{\rm tot}(|\mathbf{K}-\mathbf{k}|)} \right]^{-1}$$

Jeong & Kamionkowski (2012)

Optimal estimator for the power amplitude A_h

- For a stochastic background of new fields with power spectrum $P_p(K)=A_h P_h^f(K)$, we optimally summed over different K-modes to estimate the amplitude by (w/ NULL hypothesis):

$$\widehat{A_h} = \sigma_h^2 \sum_{\boldsymbol{K},p} \frac{\left[P_h^f(K)\right]^2}{2\left[P_p^n(K)\right]^2} \left(\frac{\left|\widehat{h_p(\boldsymbol{K})}\right|^2}{V} - P_p^n(K) \right)$$

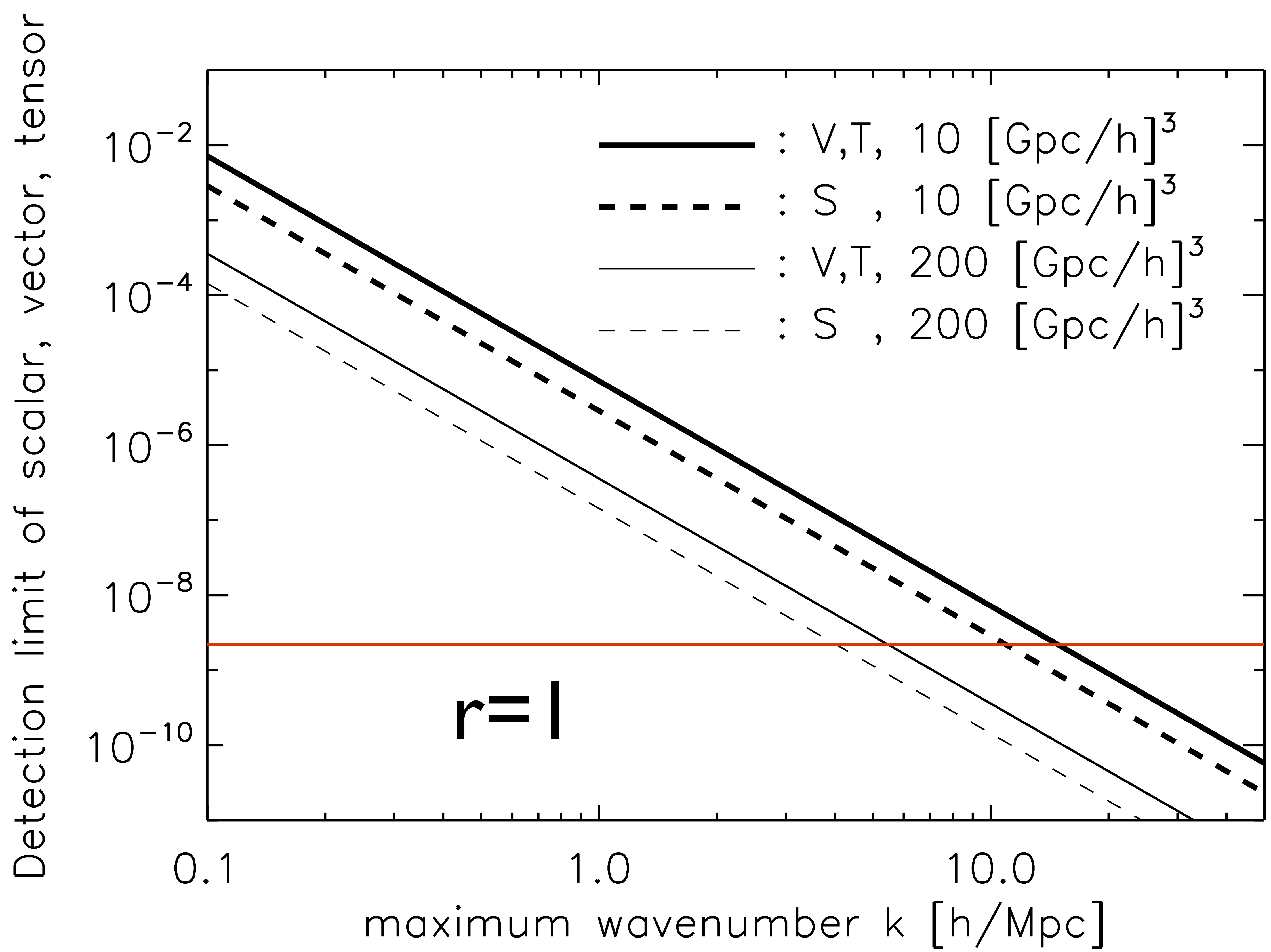
- Here, the minimum uncertainty of measuring amplitude is

$$\sigma_h^{-2} = \sum_{\boldsymbol{K},p} \left[P_h^f(K)\right]^2 / 2\left[P_p^n(K)\right]^2$$

Jeong & Kamionkowski (2012)

Order-of-magnitude calculation

- For the SFSR inflation models (Maldacena, 2003)



- projected 3-sigma (99% C.L.) detection limit with galaxy survey parameters
- To detect the gravitational wave, we need a large dynamical range
- Current and future survey should set a limit on primordial V and T (and higher-spin fields)!

Conclusion

- Off-diagonal correlators are the place to look at the signature for the spatial inhomogeneity.
- "Clustering fossil" is a way to look at primordial spectator fields that existed during the early time
    - requires large dynamical range to beat the small signal (e.g. 21cm). We can distinguish scalar/vector/tensor fossils.
    - Also, interesting potential to probe higher spin field.
- *We already have data, shall we dig for clustering fossils?*
- Systematics: survey systematics, non-linearities, non-Gaussianities