title:

Takahiro Morishima
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TOHOKU UNIVERSITY

title:

Development of component separation scheme based on hierarchical Bayes method

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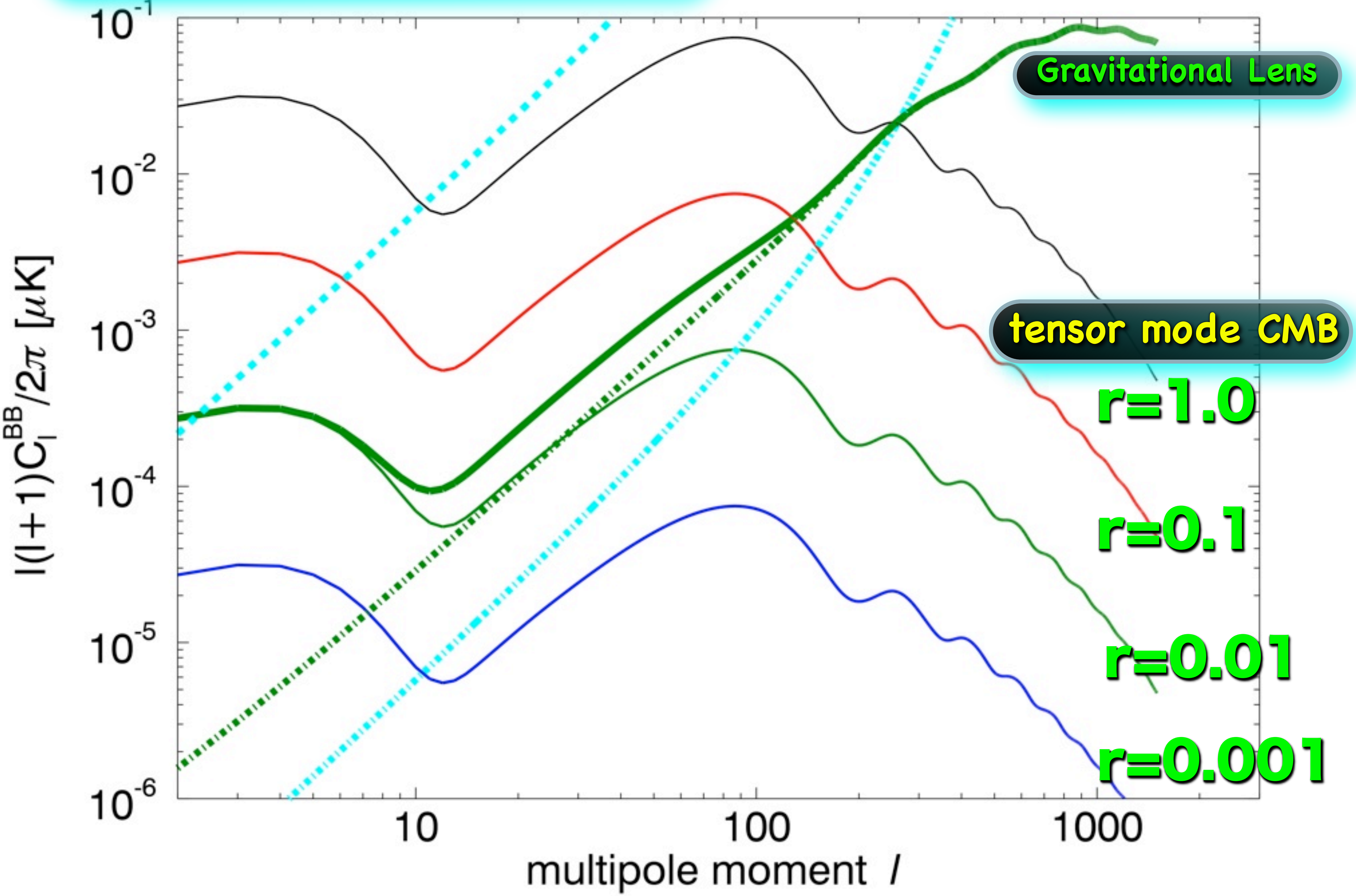
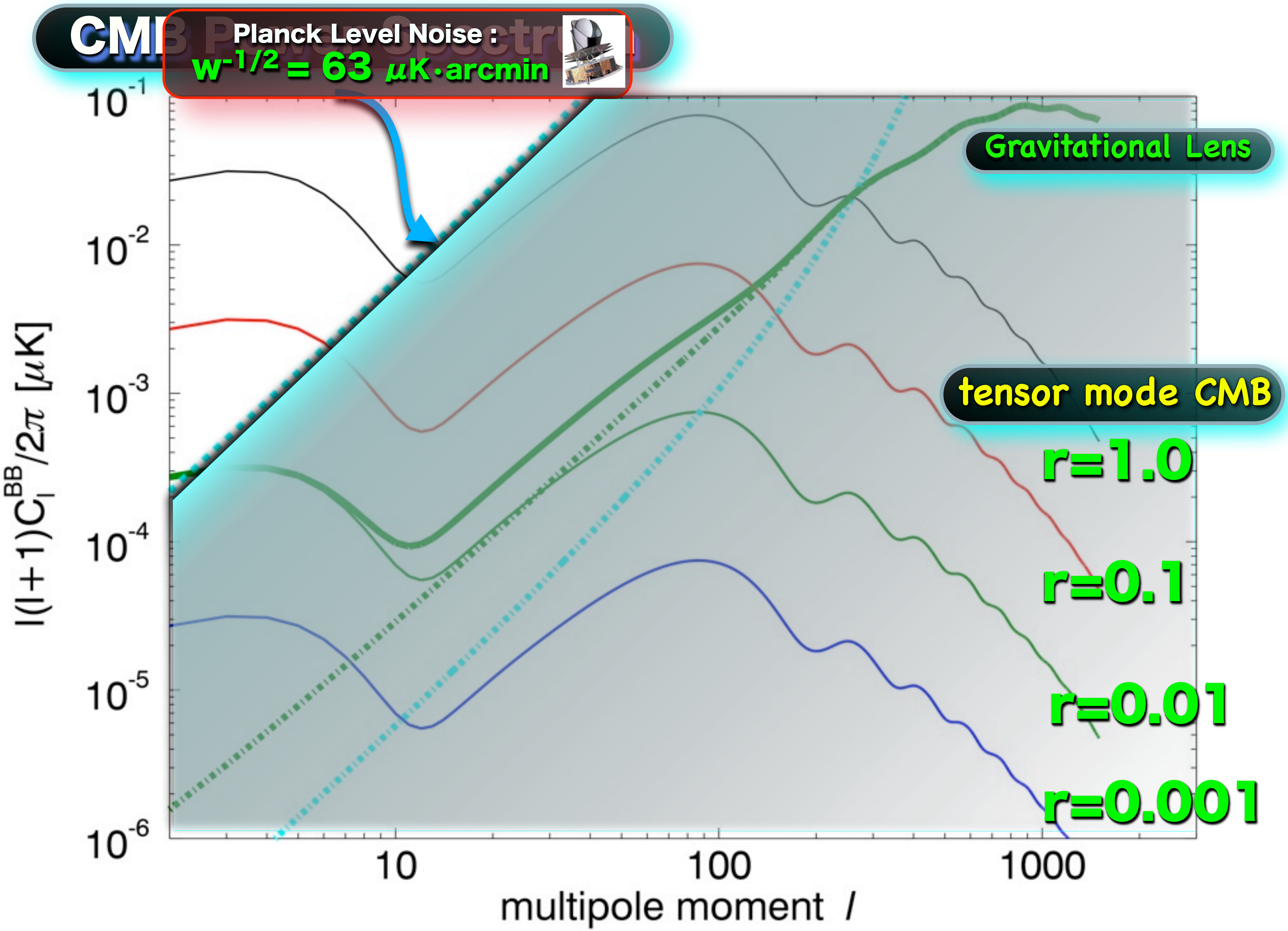
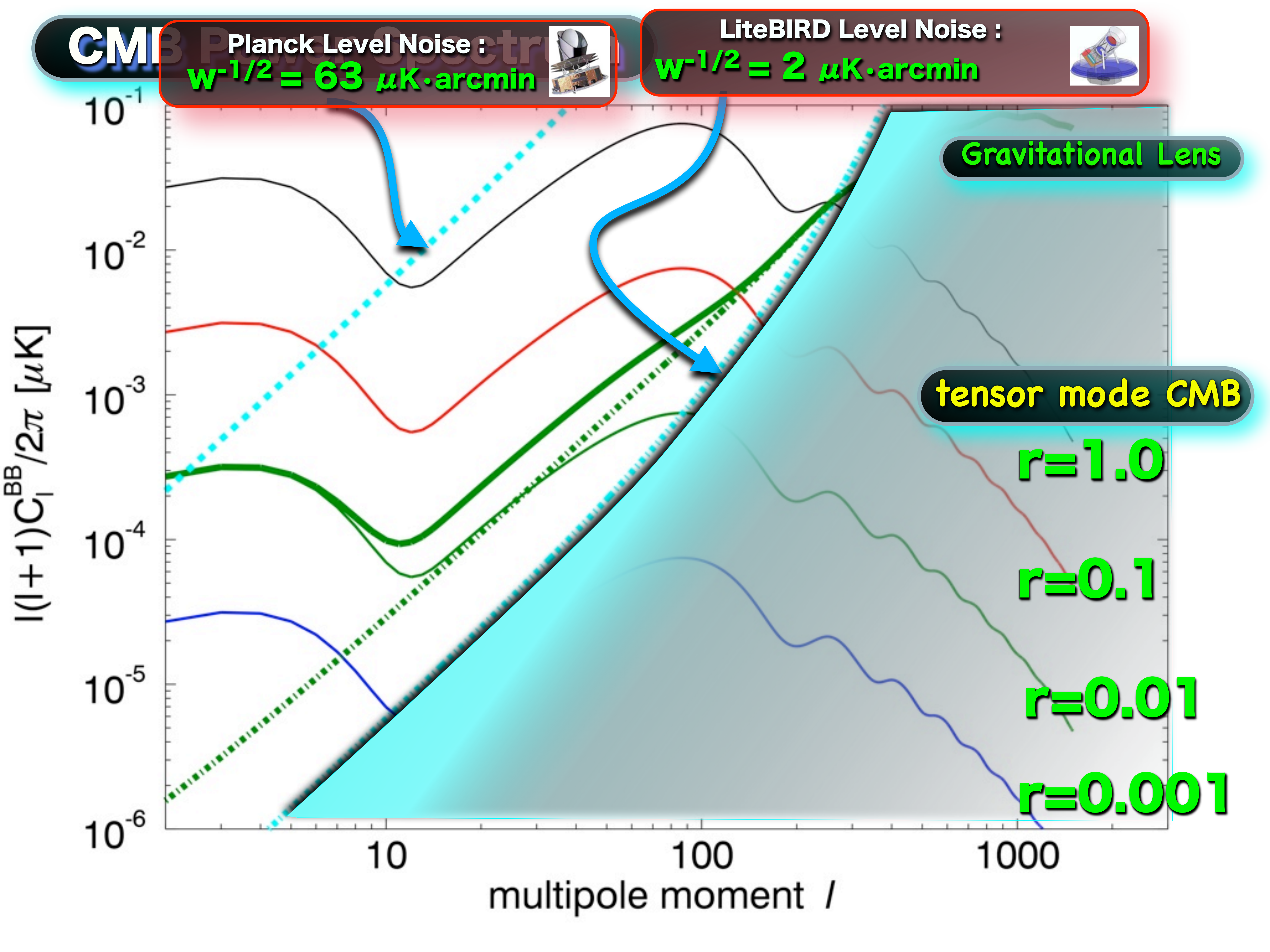
CMB Power Spectrum

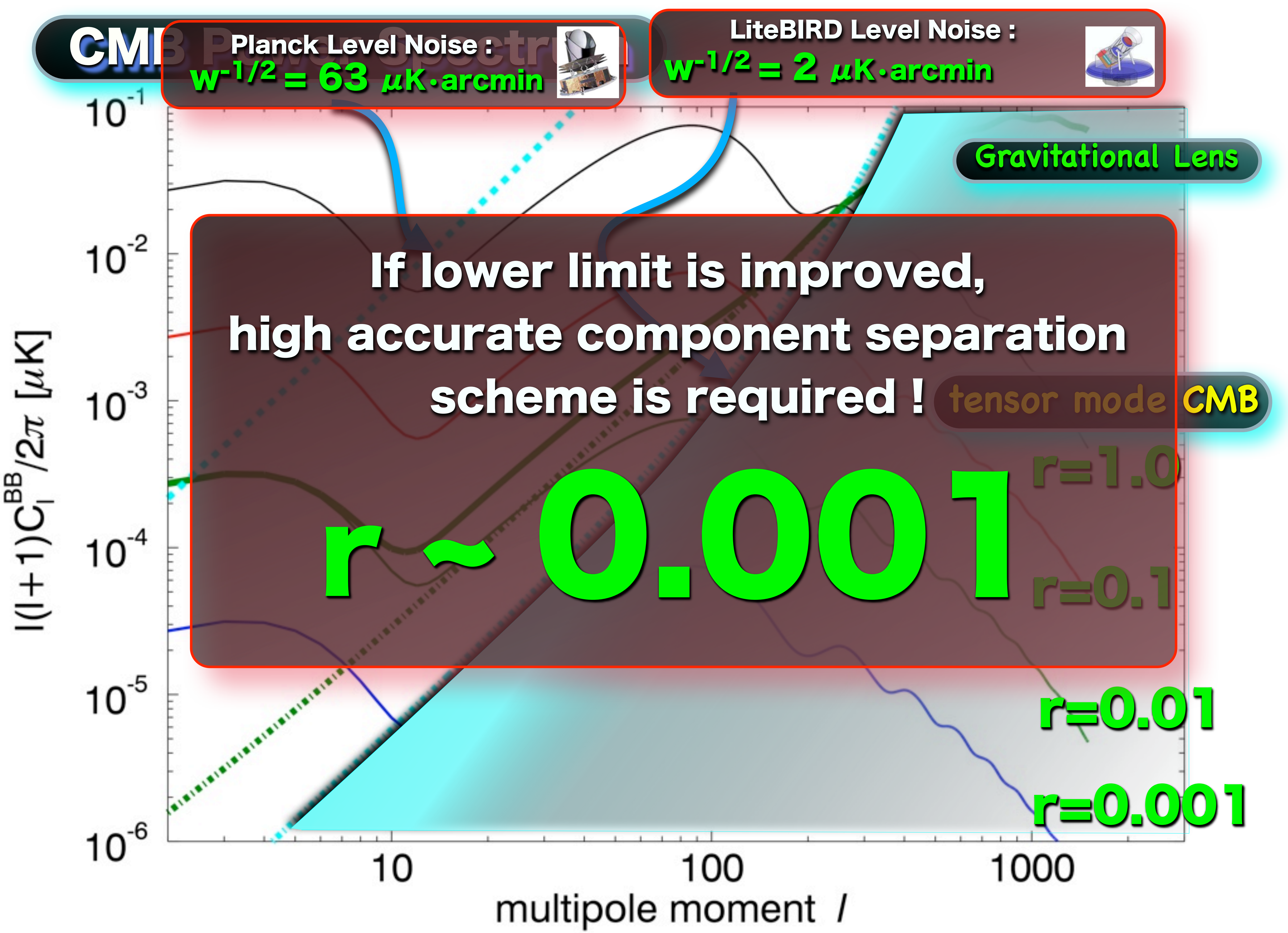




CMB
Planck Level Noise :
$w^{-1/2} = 63\ \mu K \cdot arcmin$
Gravitational Lens
tensor mode CMB
r=1.0
r=0.1
r=0.01
r=0.001
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
$l(l+1)C_l^{BB}/2\pi\ [\mu K]$
10
100
1000
multipole moment l



CMB Power Spectrum
Planck Level Noise :
w^-1/2 = 63 μK·arcmin
LiteBIRD Level Noise :
w^-1/2 = 2 μK·arcmin
Gravitational Lens
tensor mode CMB
r=1.0
r=0.1
r=0.01
r=0.001
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10
100
1000
multipole moment l
l(l+1)C_l^BB/2π [μK]


CMB Power Spectrum
Planck Level Noise :
$w^{-1/2}$ = 63 μK·arcmin
LiteBIRD Level Noise :
$w^{-1/2}$ = 2 μK·arcmin
Gravitational Lens
If lower limit is improved,
high accurate component separation
scheme is required !
tensor mode CMB
r ~ 0.001
r=1.0
r=0.1
r=0.01
r=0.001
$l(l+1)C_l^{BB}/2\pi$ [μK]
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10
100
1000
multipole moment l

philosophy:

component separation scheme which is able to evaluate the systematics introduced by incorporating the physics of the foregrounds quantitatively.

"no mask"

"template free"

We currently choose hierarchical bayesian method

toward LiteBIRD Sky Model (LSM)

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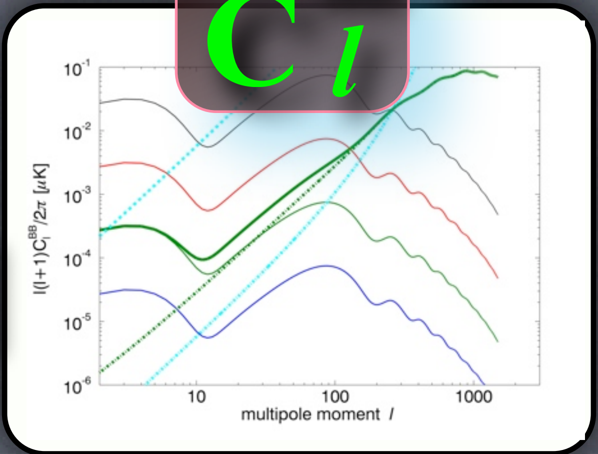
simulation data:

LiteBIRD Sky Model (LSM)

CMB component set

scalar+tensor+glens component including
which is produced by CAMB
(r=1.0, 0.1, 0.01, 0.001)

C_l^{BB}



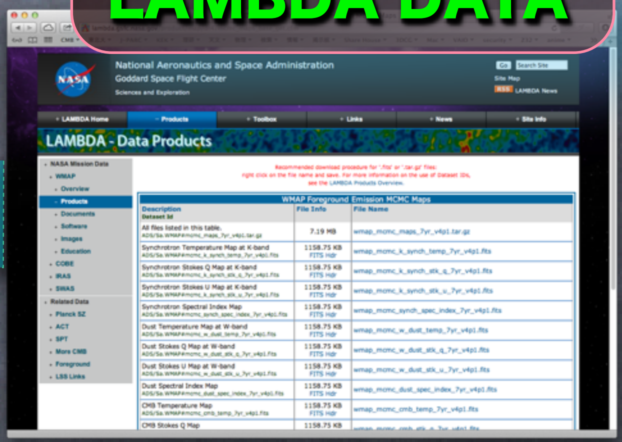
+

Synchrotoron Q-map
Synchrotoron U-map
030 GHz
044 GHz
070 GHz
100 GHz

Synchrotoron component set

WMAP7yr MCMC foreground maps
(only polarized foreground maps
enable to access)

LAMBDA DATA



+

Dust component

no dust (in this simulation)

AKARI

= total

Nside=64
(effective $l<60$)

smoothing=1°

simulation data

LiteBIRD Sky Model (LSM)

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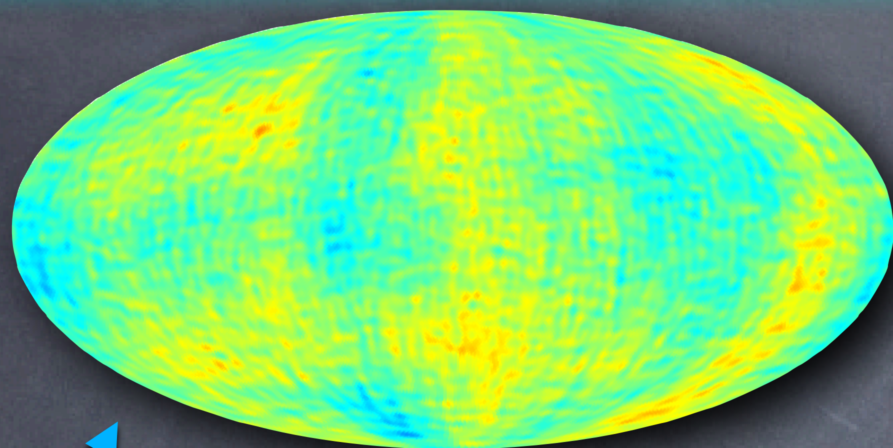
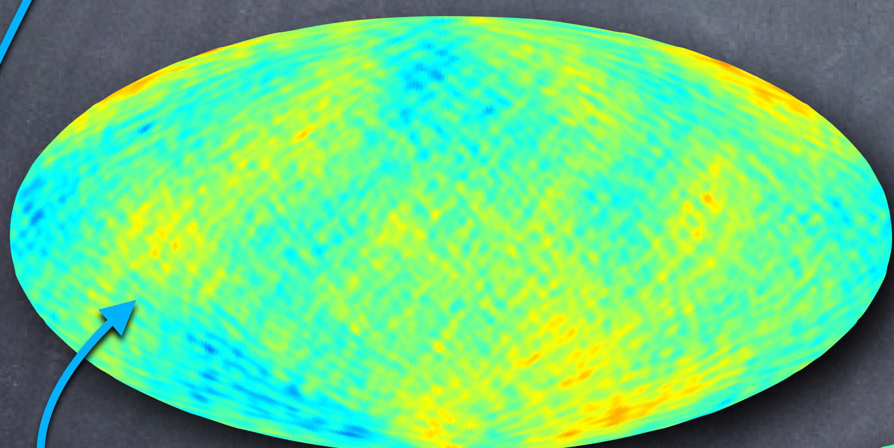
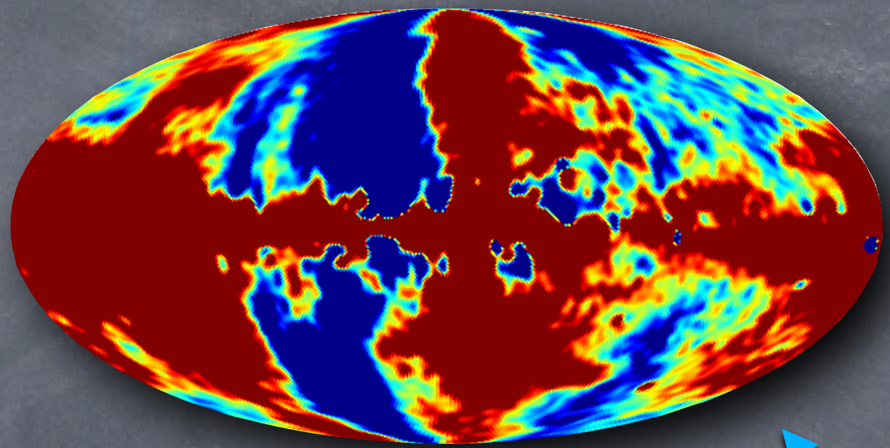
true maps:



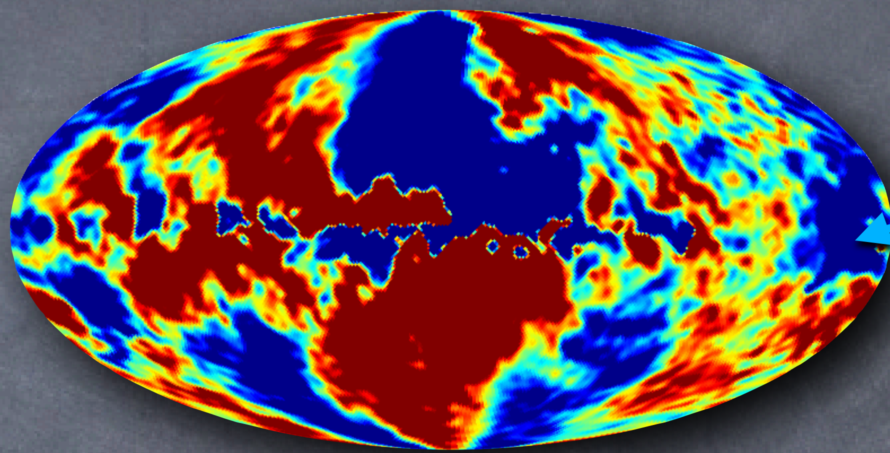
CMB Q-map



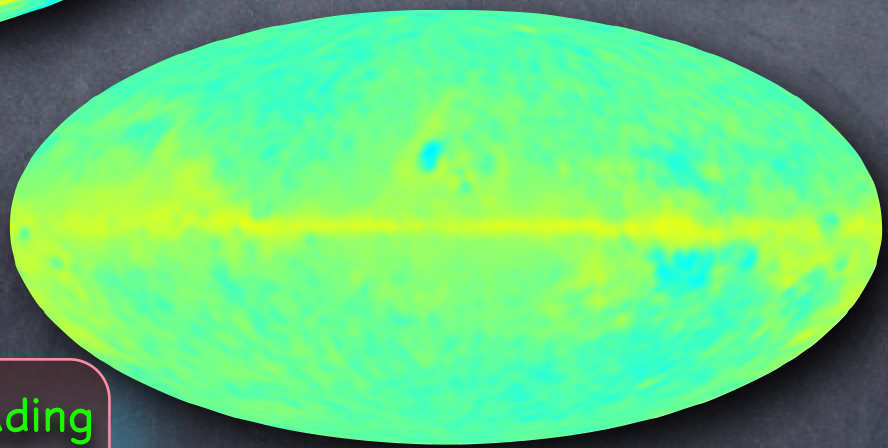
CMB U-map



Synchrotoron Q-map (ν_0=23GHz)



Synchrotoron U-map (ν_0=23GHz)



Synchrotoron Spectral INDEX-map

scalar+tensor+glens component including which is produced by CAMB (r=1.0, 0.1, 0.01, 0.001)

WMAP7yr MCMC foreground maps (only polarized foreground maps enable to access)

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simulation data:

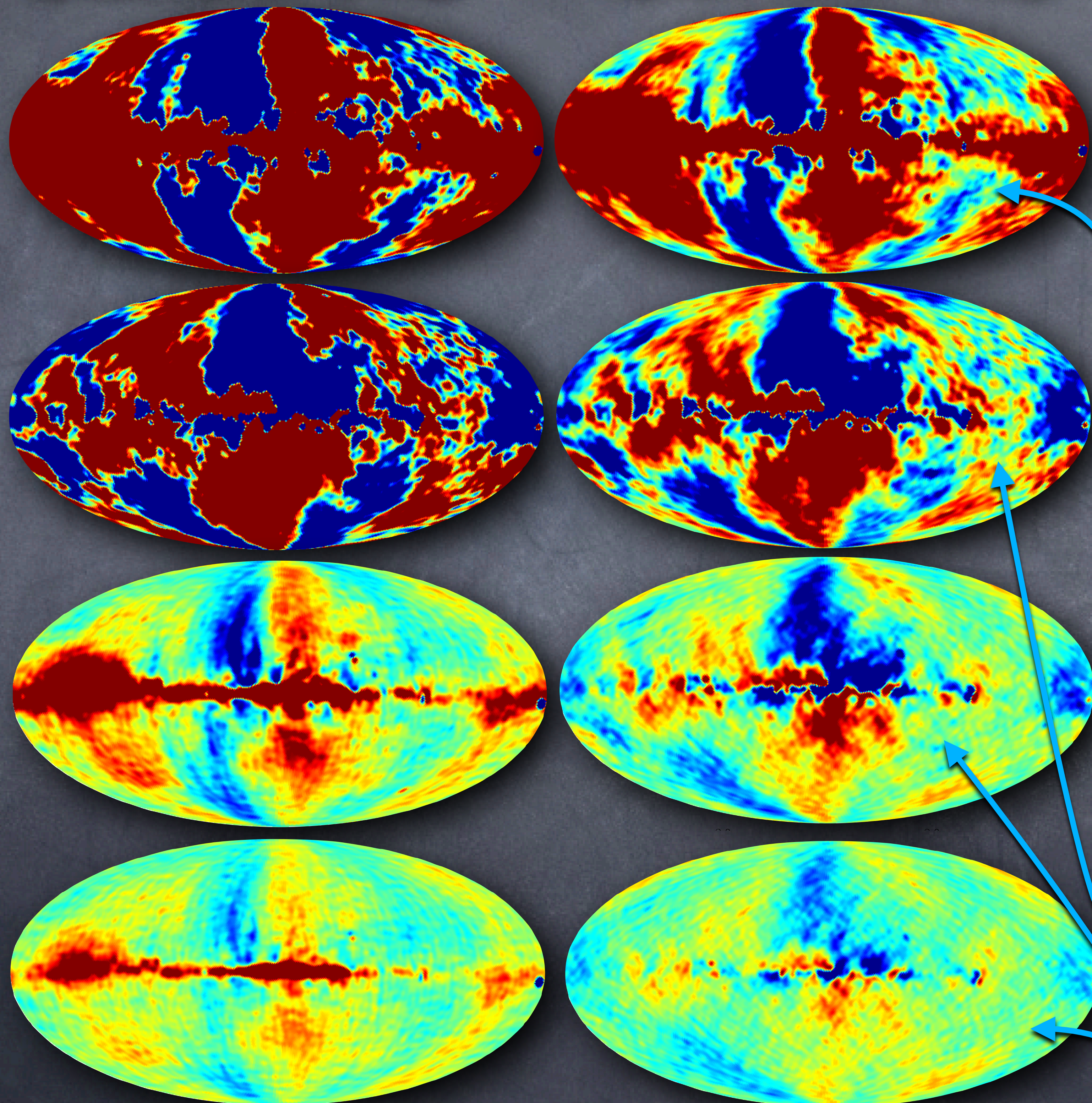
Synchrotoron Q-map | Synchrotoron U-map

030 GHz

044 GHz

070 GHz

100 GHz



CMB component

scalar+tensor+glens component including which is produced by CAMB (r=1.0, 0.1, 0.01, 0.001)

+

Synchrotoron component

WMAP7yr MCMC foreground maps (only polarized foreground maps enable to access)

+

Dust component

(no dust)

=

total

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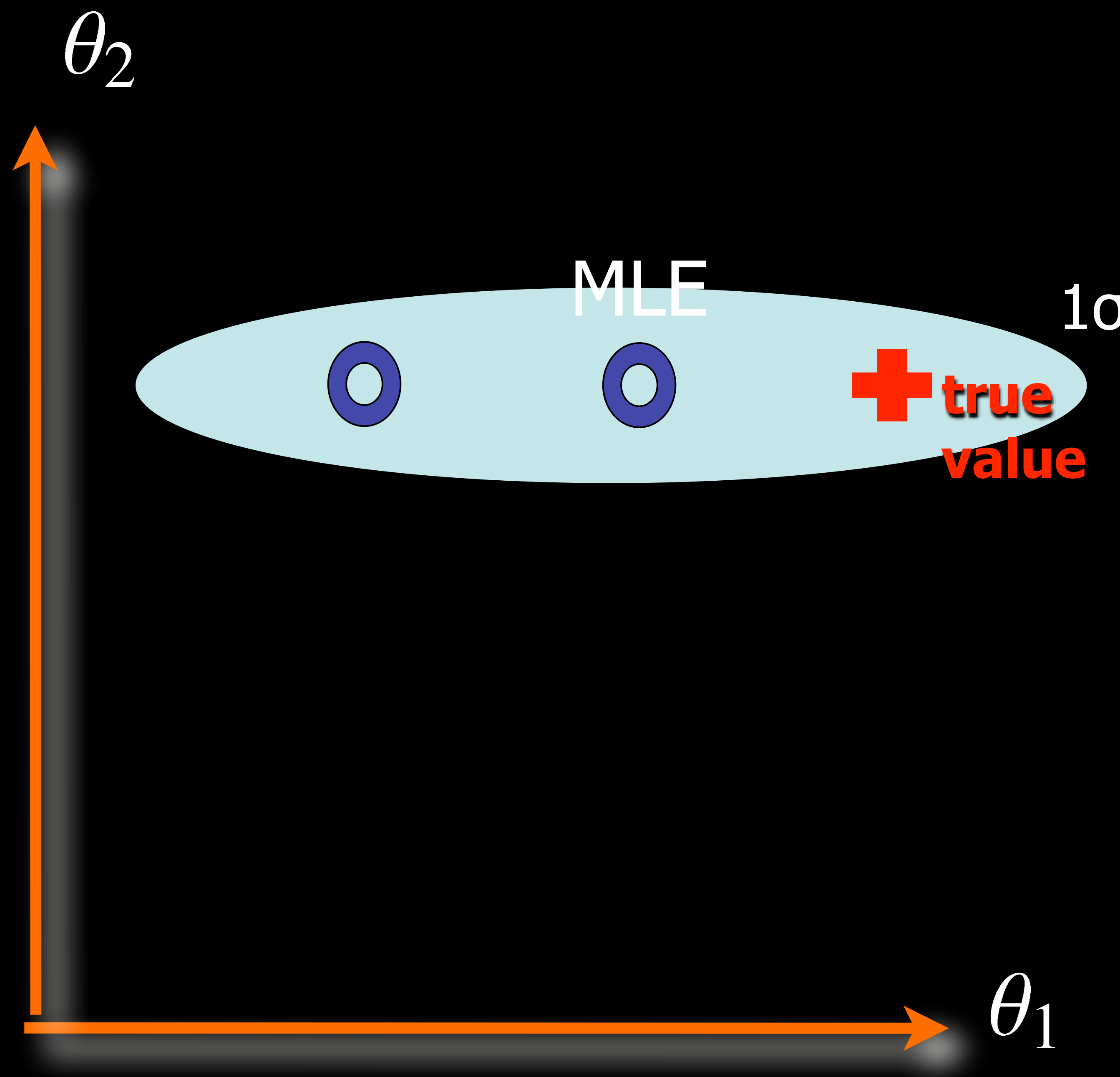
Hierarchical Bayes method

Takahiro Morishima (Astronomical Institute, Graduate School of Science, Tohoku University)

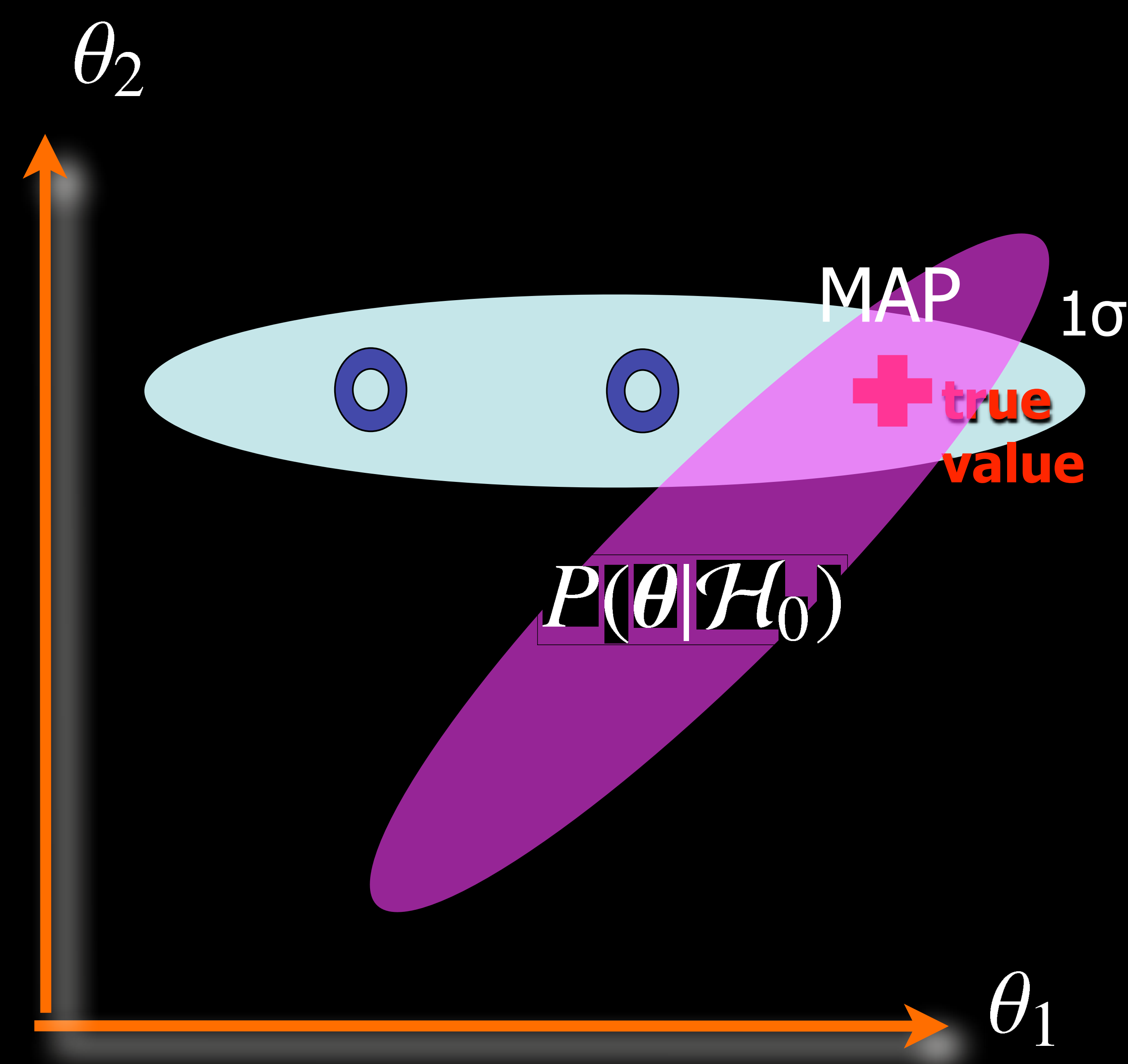
Hierarchical Bayes method

$$\log P(\mathbf{d}|\boldsymbol{\theta})$$



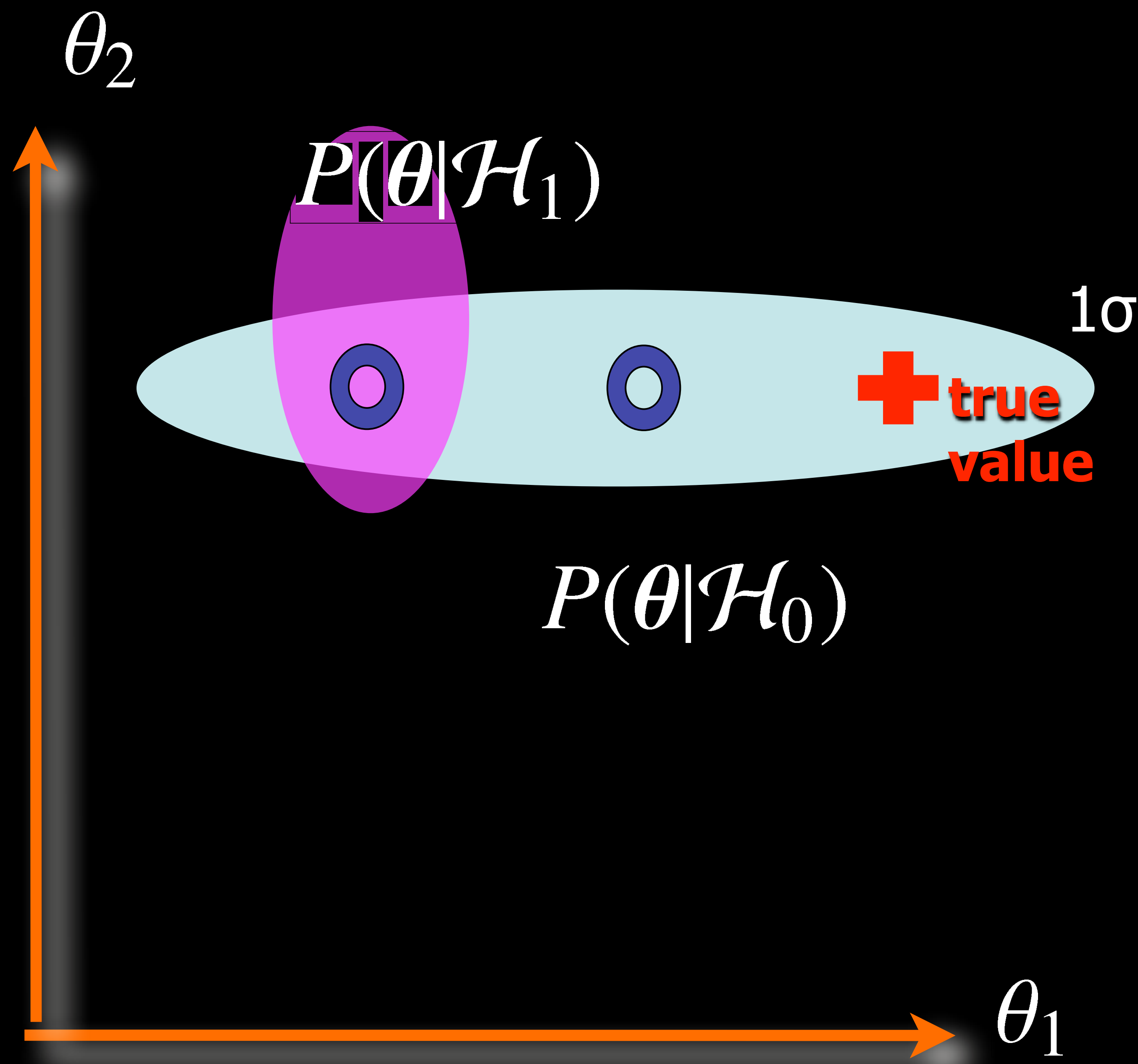
Hierarchical Bayes method

$$\log P(\mathbf{d}|\boldsymbol{\theta}) + \log P(\boldsymbol{\theta}|\mathcal{H}_0)$$



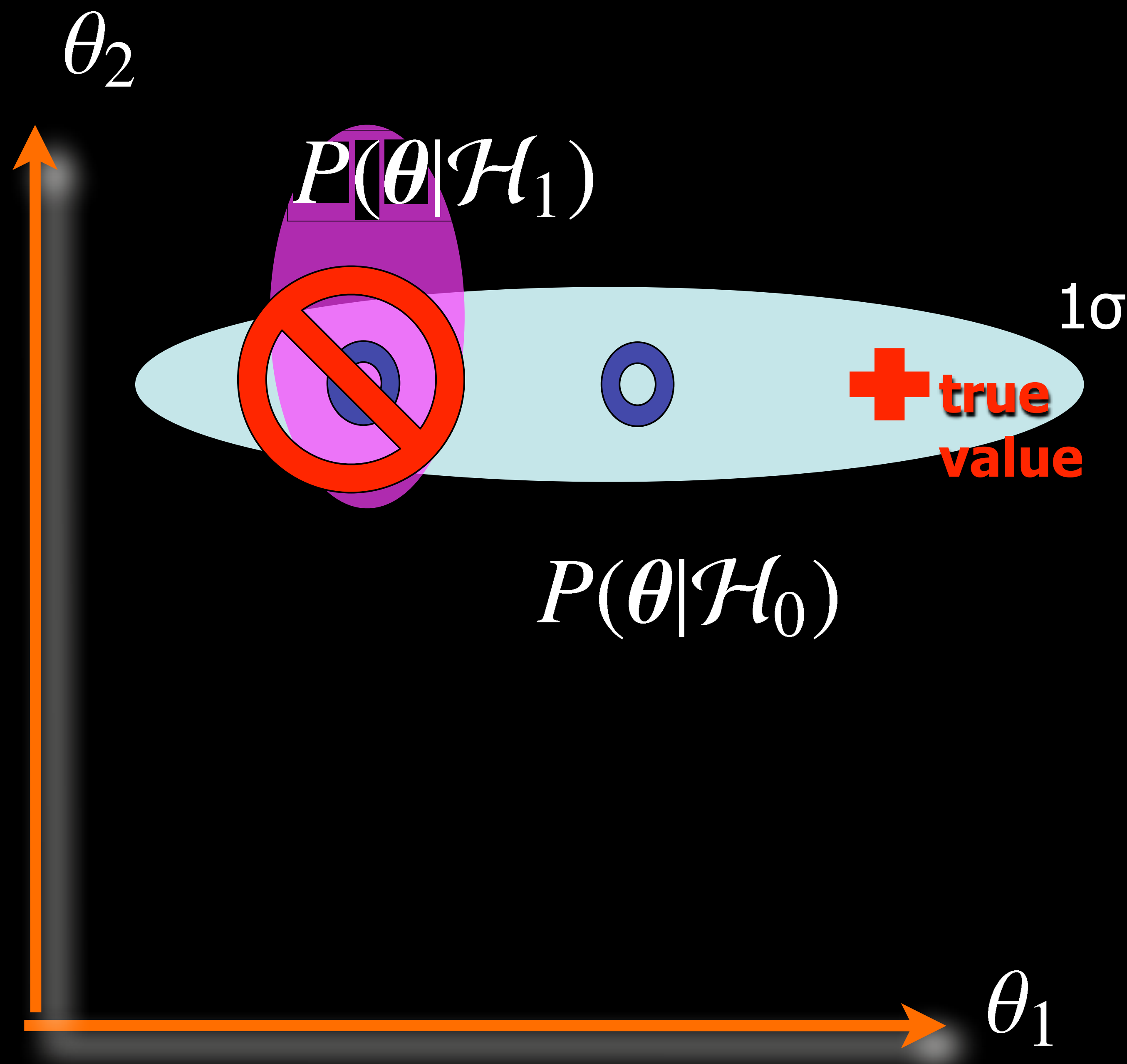
Hierarchical Bayes method

$$\log P(\mathbf{d}|\boldsymbol{\theta}) + \log P(\boldsymbol{\theta}|\mathcal{H}_1)$$



Hierarchical Bayes method

$$\log P(\mathbf{d}|\boldsymbol{\theta}) + \log P(\boldsymbol{\theta}|\mathcal{H}_1)$$



Hierarchical Bayes

Goodness of Selected models are statistically evaluated by using marginal log likelyhood

$$P(\mathcal{H}_i|\mathbf{d}) \propto P(\mathbf{d}|\mathcal{H}_i) = \int d\boldsymbol{\theta}\, P(\mathbf{d}|\boldsymbol{\theta}, \mathcal{H}_i) P(\boldsymbol{\theta}|\mathcal{H}_i)$$

$$E(\lambda) = \int d\boldsymbol{\theta}\, P(\mathbf{d}|\boldsymbol{\theta}, \lambda) P(\boldsymbol{\theta}|\lambda)$$

(marginal log liklihood) = - log E(λ)

= Evidence

Foreground priors

1. Spectral Index prior
2. Jeffreys' Ignorance prior

CMB prior

Gaussianity

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Spectral index prior:

$$P^{w_G}(\beta|\Delta\beta)\;; \qquad P(\beta|\Delta\beta) \sim \exp\left[-\frac{(\beta - \beta_{\rm prior})^2}{2\Delta\beta^2}\right]$$

β ~ -3

variance
(Hyper-parametrize)

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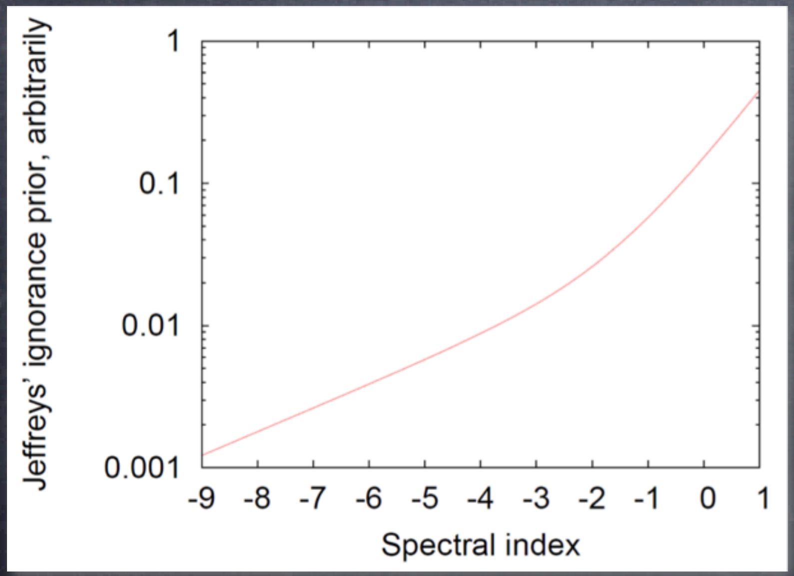
Jeffreys' Ignorance prior:

$$P_{\mathrm{J}}^{w_{\mathrm{J}}}(\theta)\,; \quad P_{\mathrm{J}}(\theta) \sim \sqrt{F_{\theta\theta}} = \sqrt{-\left\langle \frac{\partial^2 \ln \mathcal{L}}{\partial \theta^2} \right\rangle}$$



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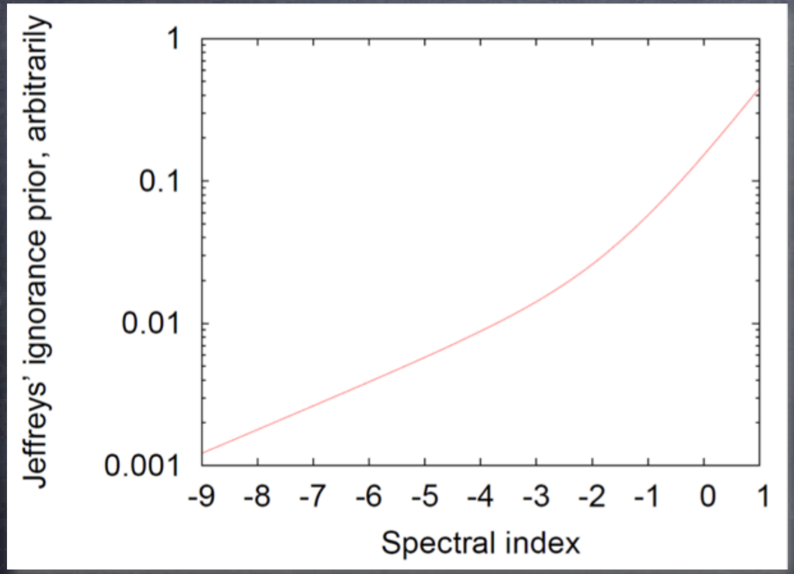
Jeffreys' Ignorance prior:

"Jeffrey's prior is the prior in the case of no prior."

$$P_{\mathrm{J}}^{w_{\mathrm{J}}}(\theta)\ ; \qquad P_{\mathrm{J}}(\theta) \sim \sqrt{F_{\theta\theta}} = \sqrt{-\left\langle \frac{\partial^2 \ln \mathcal{L}}{\partial \theta^2} \right\rangle}$$



if linear parameter, above derivative is const.

if non-linear parameter, Jeffrey's prior is effective.

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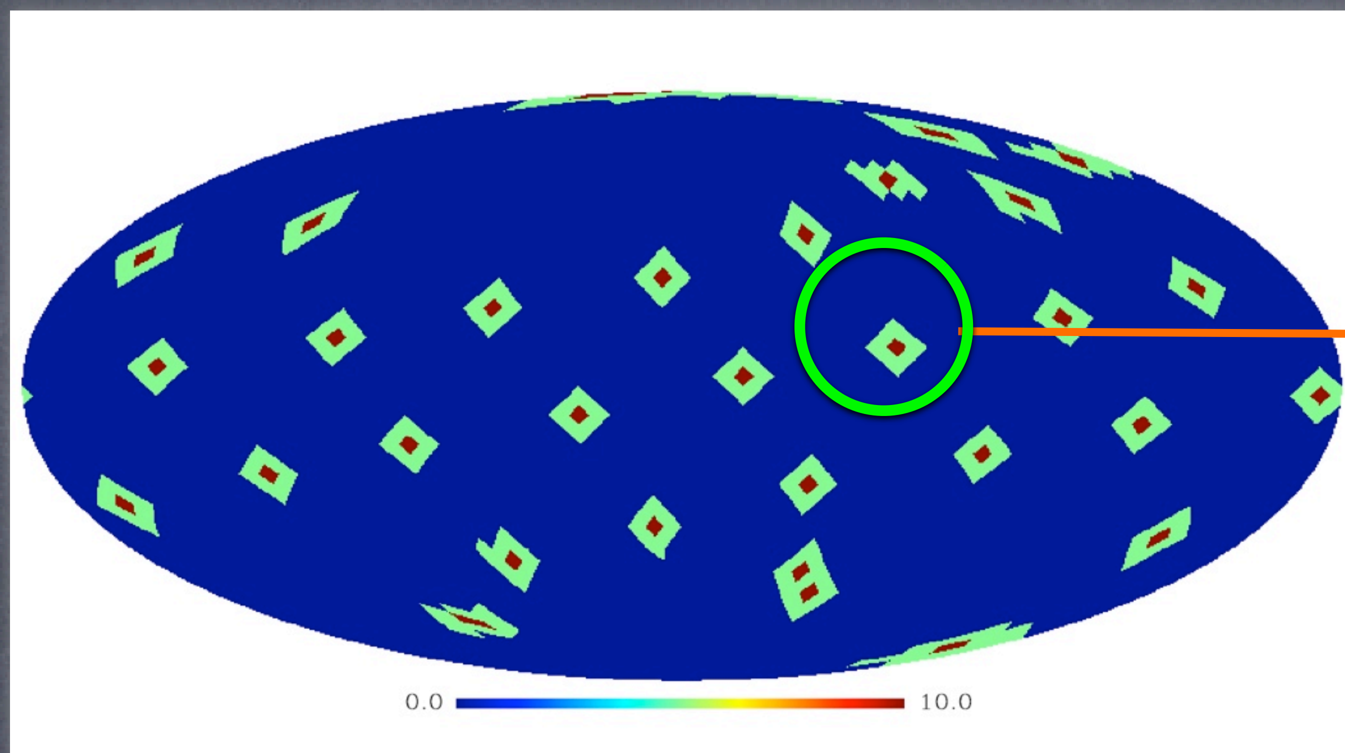
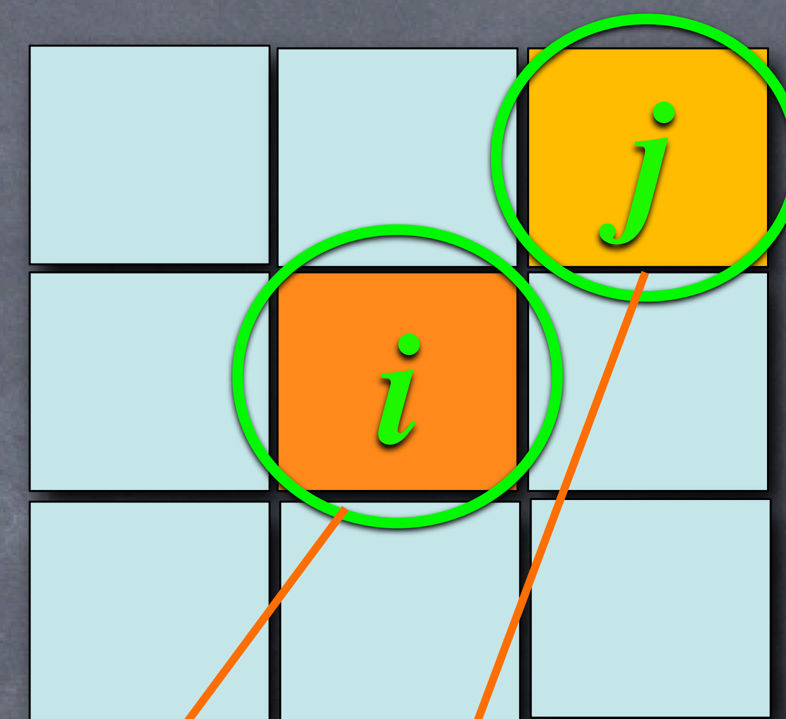
To take into account our knowledge of spatial continuty of intensity distribution of foreground:

Markov Random Field prior

MRF prior:





$$\overset{n+1}{P}(\boldsymbol{\theta}_i|\alpha) \propto \exp\left[-\frac{1}{2}\alpha \sum_{\nu, j\in C} \frac{\left(\overset{n+1}{f_\nu}(\boldsymbol{\theta}_i) - \overset{n+1}{f_\nu}(\boldsymbol{\theta}_j)\right)^2}{[\, a\, \overset{n}{f_\nu}(\theta_i)\,]^2}\right]$$

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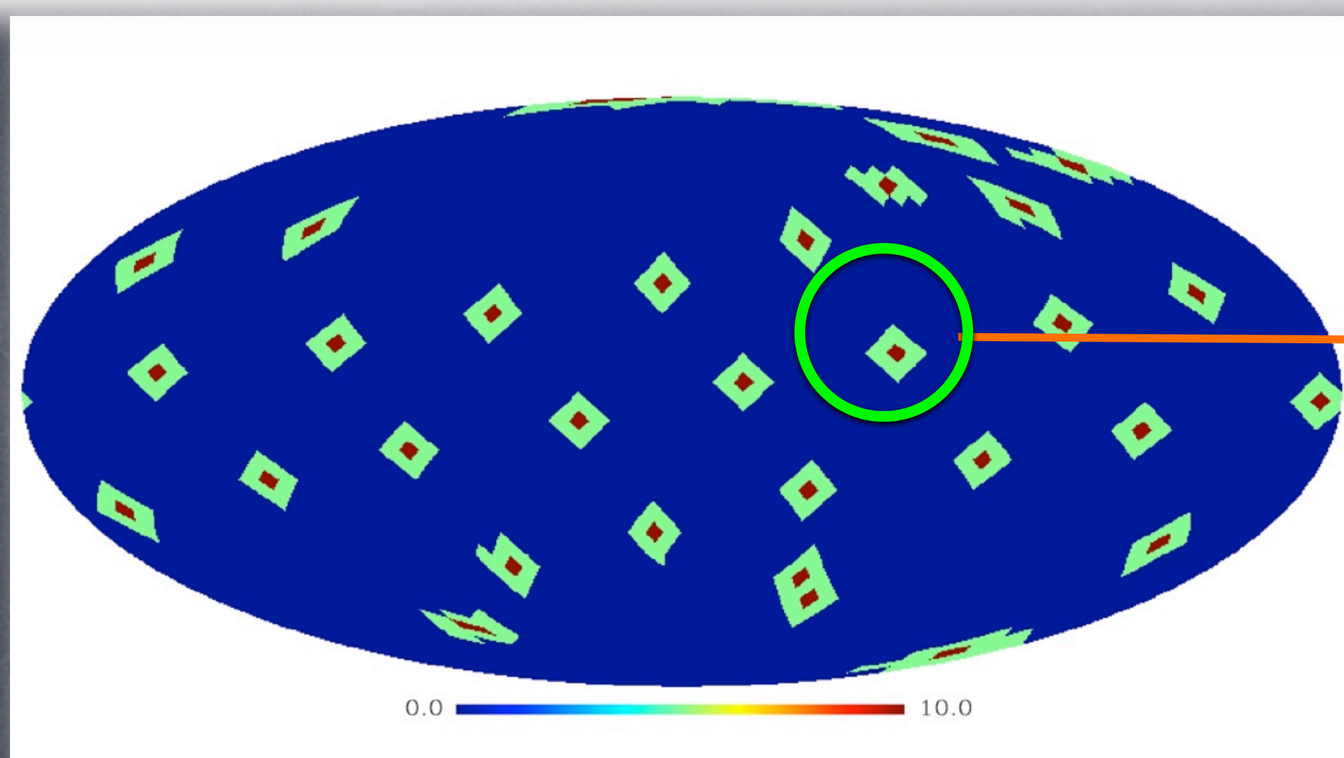
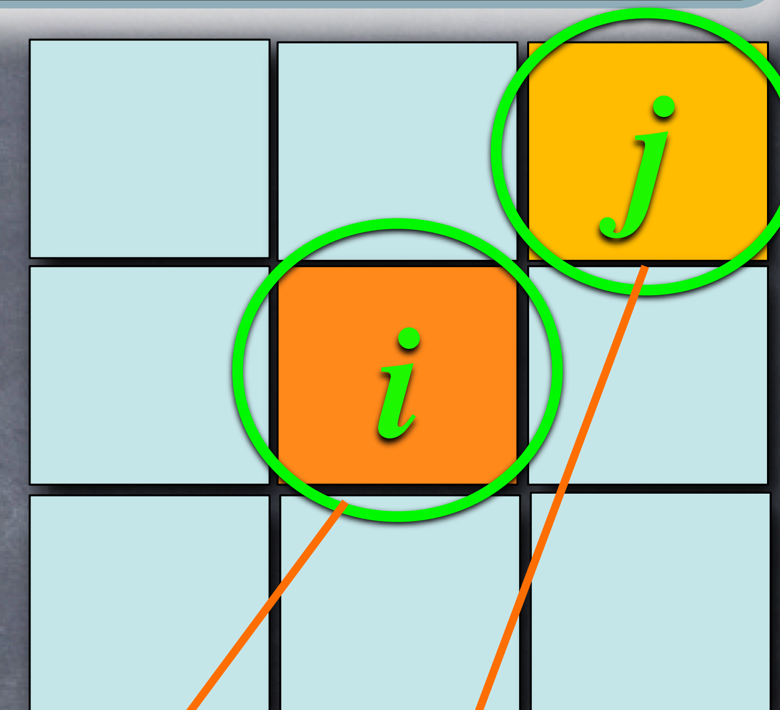
MRF prior:

In foreground components

"the neighbouring each pixel tend to take the same value."

(= synchrotoron distribute must be continuous !)





deviation

$$P^{n+1}(\boldsymbol{\theta}_i|\alpha) \propto \exp\left[-\frac{1}{2}\alpha \sum_{\nu, j\in C} \frac{\left(f_\nu^{n+1}(\boldsymbol{\theta}_i) - f_\nu^{n+1}(\boldsymbol{\theta}_j)\right)^2}{[\, a\, f_\nu^{n}(\theta_i)\,]^2}\right]$$

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How to gather all priors:

$$P(\boldsymbol{\theta}|\mathbf{d},\mathbf{w},\lambda) \propto P(\mathbf{d}|\boldsymbol{\theta}) \times P^{w_1}(\boldsymbol{\theta}|\lambda_1)P^{w_2}(\boldsymbol{\theta}|\lambda_2)\cdots P^{w_n}(\boldsymbol{\theta}|\lambda_n)$$

Exponents are also treated as hyper parameters which control weight of each prior

$$P(\mathbf{d}|\mathbf{w},\lambda)$$

How to gather all priors:

Hyper parameters

$$P(\boldsymbol{\theta}|\mathbf{d}, \mathbf{w}, \lambda) \propto P(\mathbf{d}|\boldsymbol{\theta}) \times P^{w_1}(\boldsymbol{\theta}|\lambda_1)P^{w_2}(\boldsymbol{\theta}|\lambda_2)\cdots P^{w_n}(\boldsymbol{\theta}|\lambda_n)$$

Exponents are also treated as hyper parameters which control weight of each prior

$$P(\mathbf{d}|\mathbf{w}, \lambda)$$

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How to gather all priors:

Hyper parameters

Exponent

$$P(\theta|\mathbf{d}, \mathbf{w}, \lambda) \propto P(\mathbf{d}|\theta) \times P^{w_1}(\theta|\lambda_1) P^{w_2}(\theta|\lambda_2) \cdots P^{w_n}(\theta|\lambda_n)$$

Exponents are also treated as hyper parameters which control weight of each prior

$$P(\mathbf{d}|\mathbf{w}, \lambda)$$

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Results applying for temperature fluctuation

Noise per pixel = 10 μK
Nside=64

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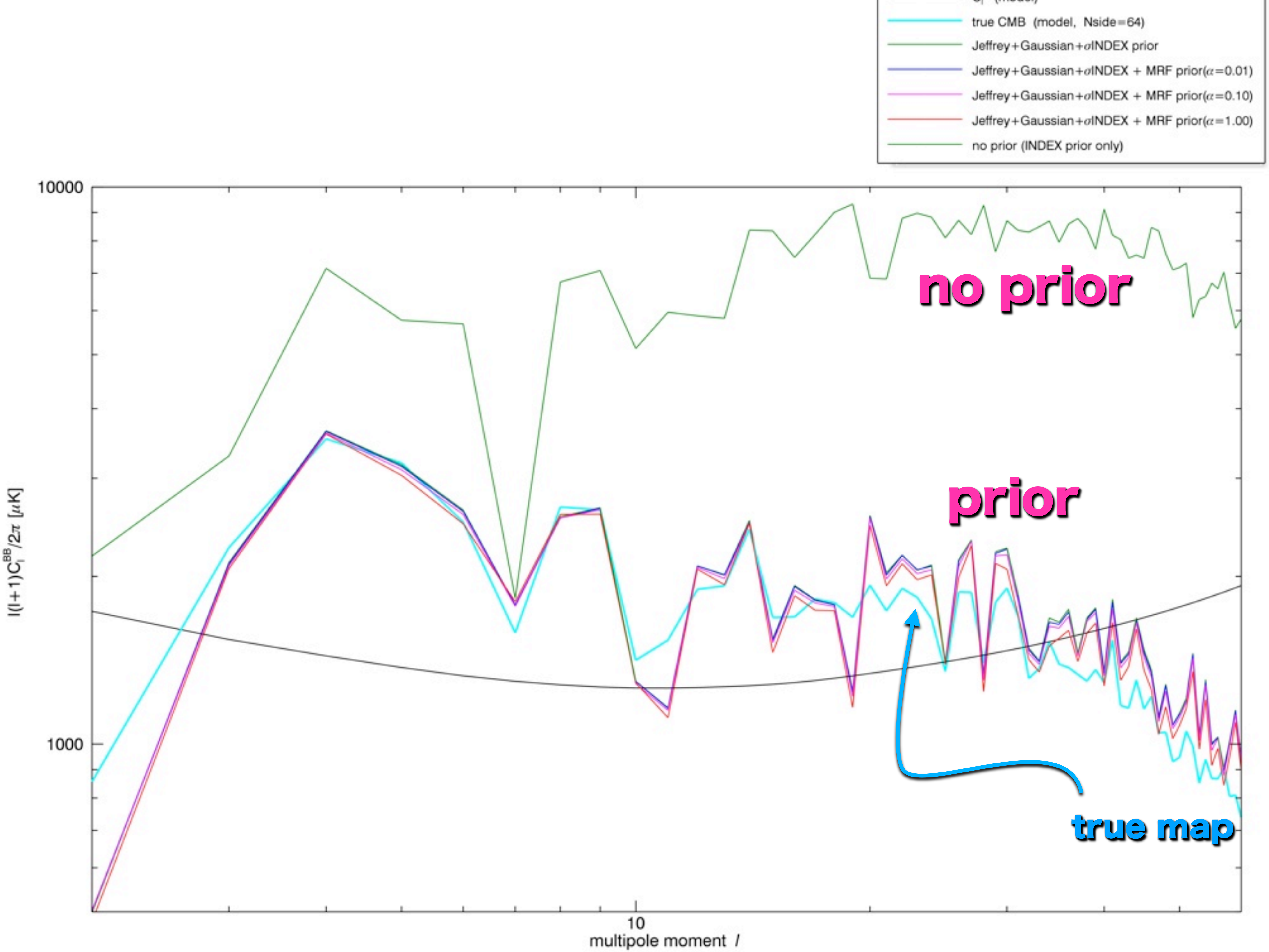
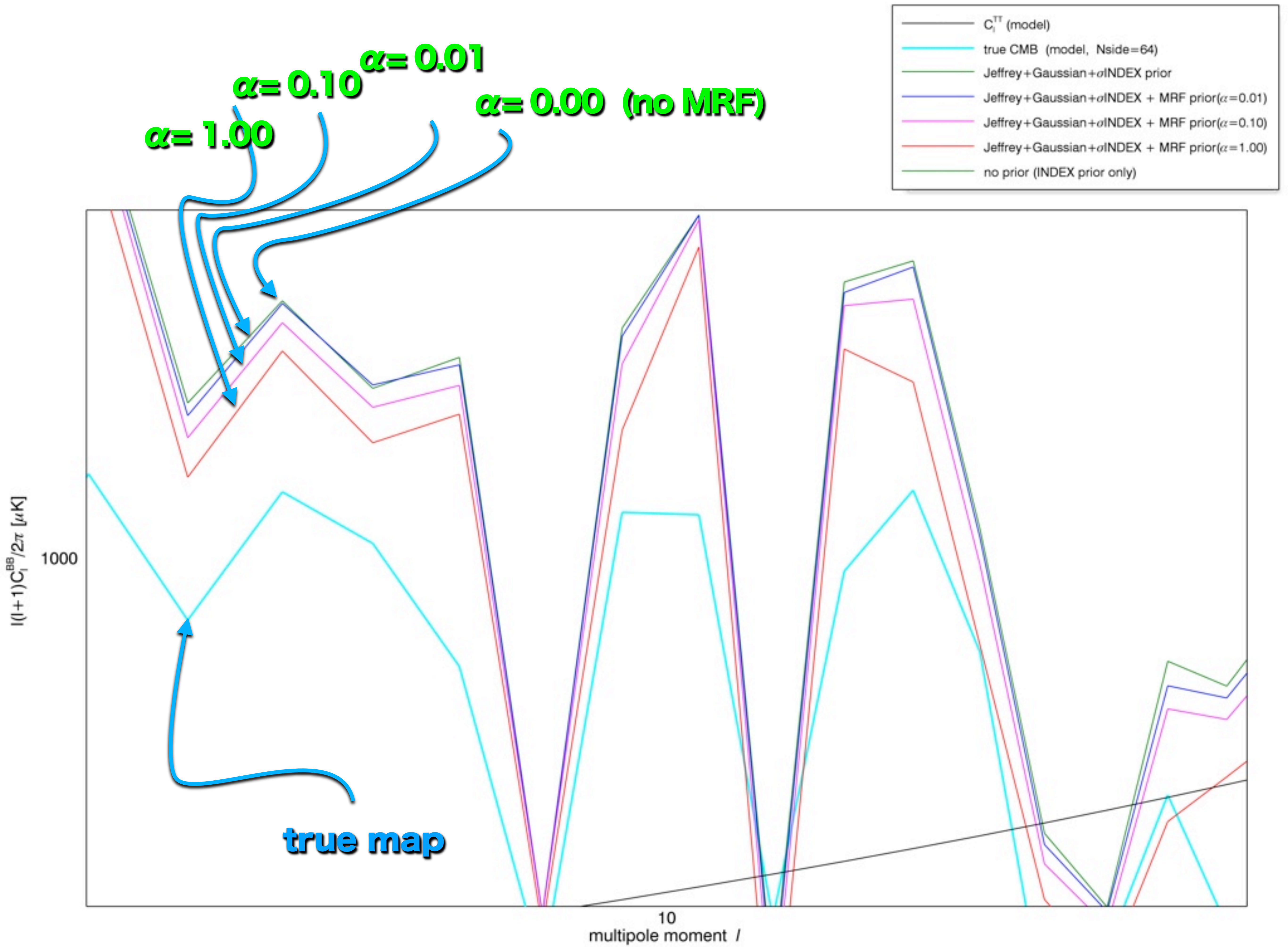


true CMB (model, Nside=64)
Jeffrey+Gaussian+σINDEX prior
Jeffrey+Gaussian+σINDEX + MRF prior(α=0.01)
Jeffrey+Gaussian+σINDEX + MRF prior(α=0.10)
Jeffrey+Gaussian+σINDEX + MRF prior(α=1.00)
no prior (INDEX prior only)
10000
1000
l(l+1)C_l^BB/2π [μK]
10
multipole moment l
no prior
prior
true map



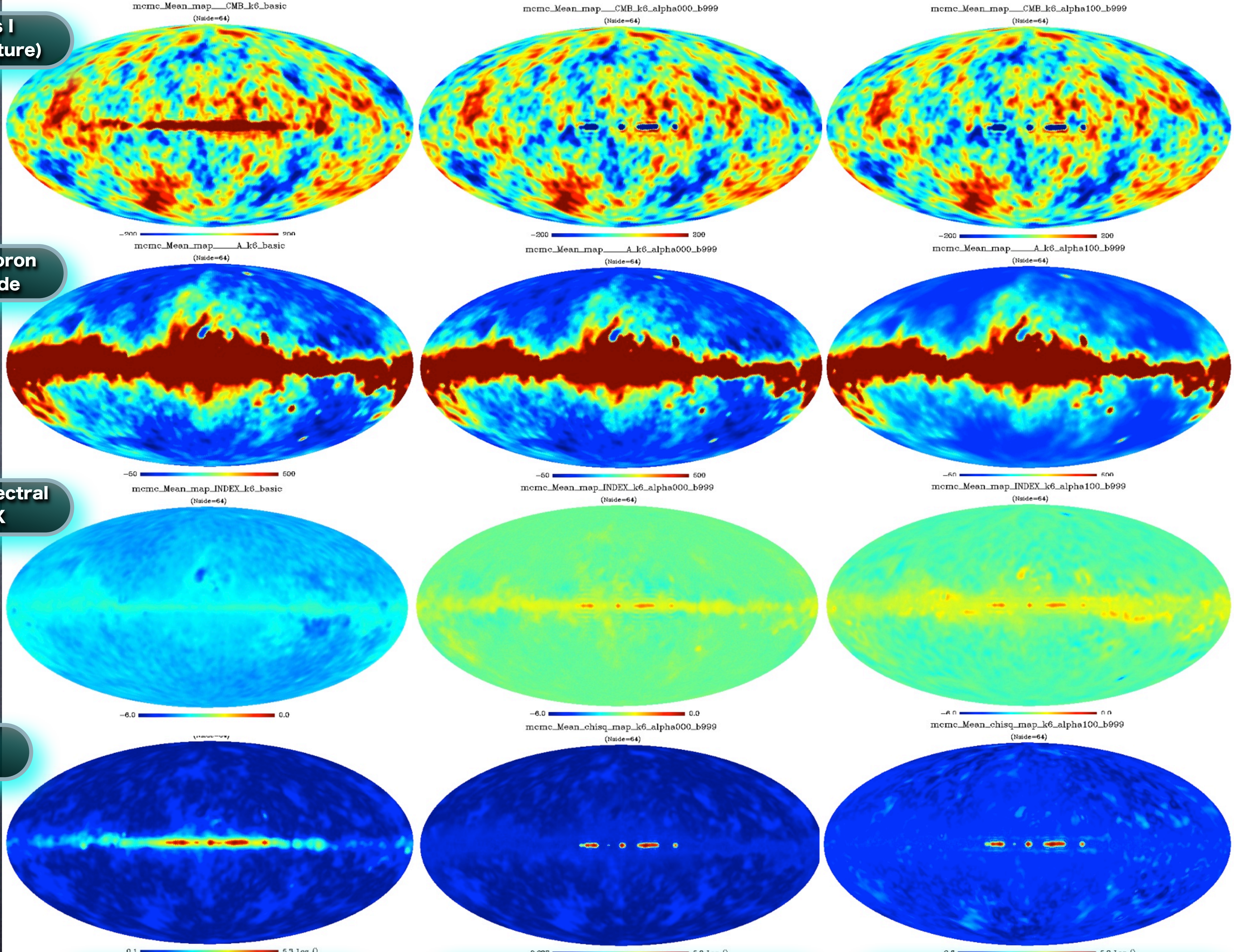
C_l^{TT} (model)
true CMB (model, Nside=64)
Jeffrey+Gaussian+σINDEX prior
Jeffrey+Gaussian+σINDEX + MRF prior(α=0.01)
Jeffrey+Gaussian+σINDEX + MRF prior(α=0.10)
Jeffrey+Gaussian+σINDEX + MRF prior(α=1.00)
no prior (INDEX prior only)
α= 0.01
α= 0.10
α= 0.00 (no MRF)
α= 1.00
true map
1000
$l(l+1)C_l^{BB}/2\pi$ [μK]
10
multipole moment l

Stokes I (temperature)

Synchrotoron Amplitude

Synch. spectral INDEX

χ^2



no prior (INDEX prioronly)

with prior (INDEX, Gaussian, Jeffrey prior)

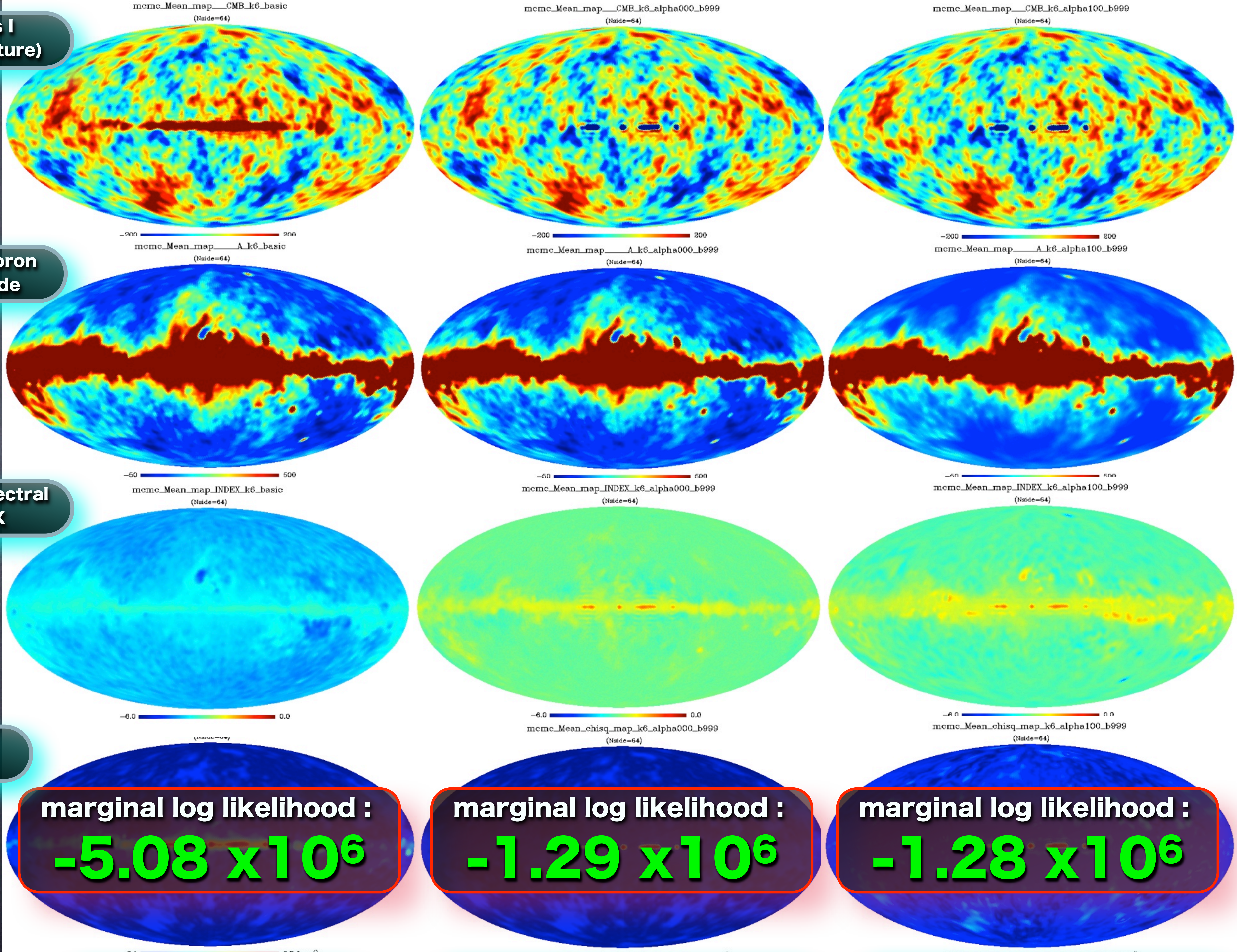
with prior (INDEX, Gaussian, Jeffrey + MRF prior)

Stokes I (temperature)

Synchrotoron Amplitude

Synch. spectral INDEX

χ^2

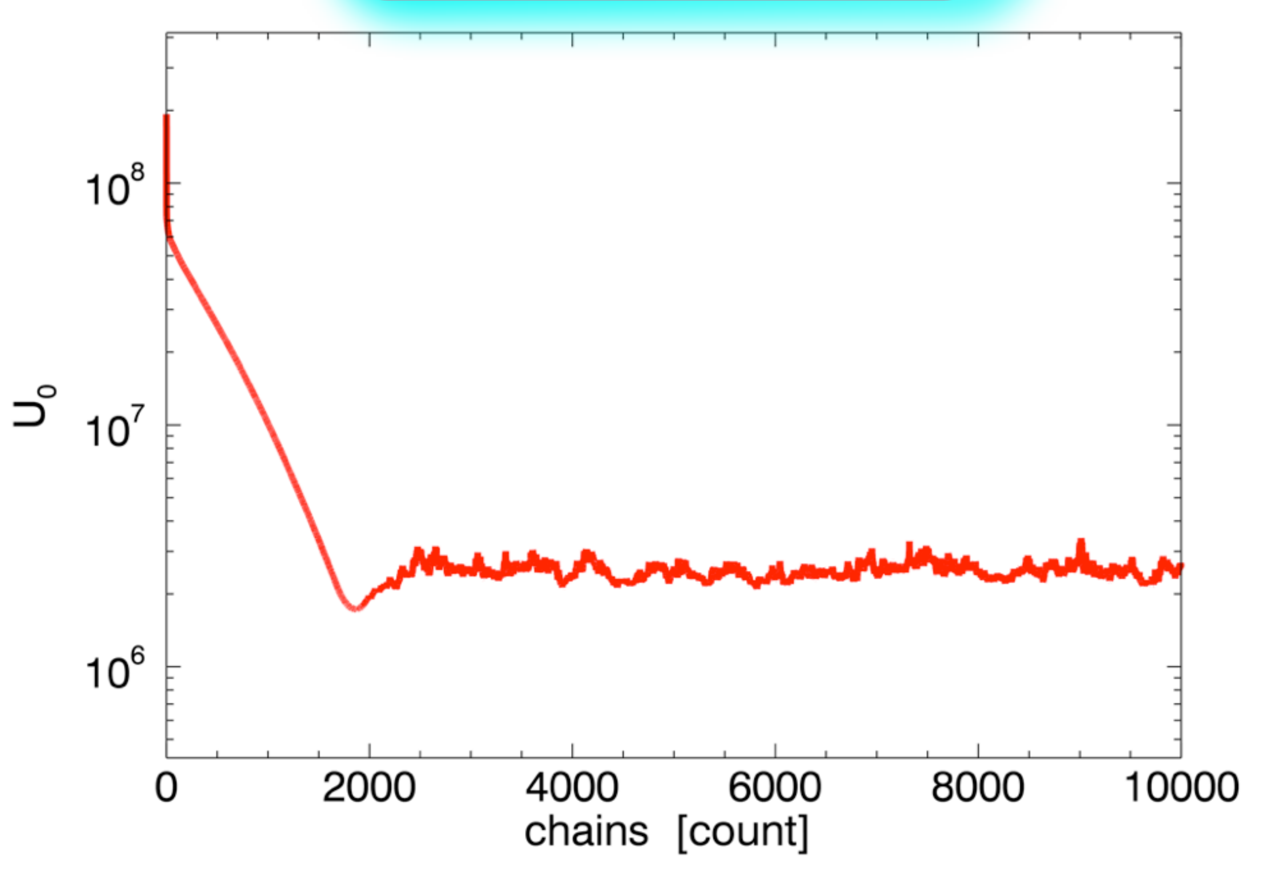


marginal log likelihood : -5.08×10^6	marginal log likelihood : -1.29×10^6	marginal log likelihood : -1.28×10^6
no prior (INDEX prioronly)	with prior (INDEX, Gaussian, Jeffrey prior)	with prior (INDEX, Gaussian, Jeffrey + MRF prior)

$$U(\mathbf{d}_\nu, \mathbf{s}, \mathbf{f}_\nu) = \sum_\nu (\mathbf{d}_\nu - \mathbf{A}\mathbf{s} - \mathbf{f}_n u)^t \mathbf{N}_\nu^{-1} (\mathbf{d}_\nu - \mathbf{A}\mathbf{s} - \mathbf{f}_\nu) + \mathbf{s}^t \mathbf{S}^{-1} \mathbf{s}$$

$$\mathbf{d}_\nu = \mathbf{A}_\nu \mathbf{s} + \mathbf{f}_\nu + \mathbf{n}_\nu \qquad P(\mathbf{s}|C_l, \mathbf{d}) \propto e^{-\frac{1}{2}(\mathbf{s}-\hat{\mathbf{s}})(\mathbf{S}^{-1}+\mathbf{N}^{-1})(\mathbf{s}-\hat{\mathbf{s}})}$$

potential U_0



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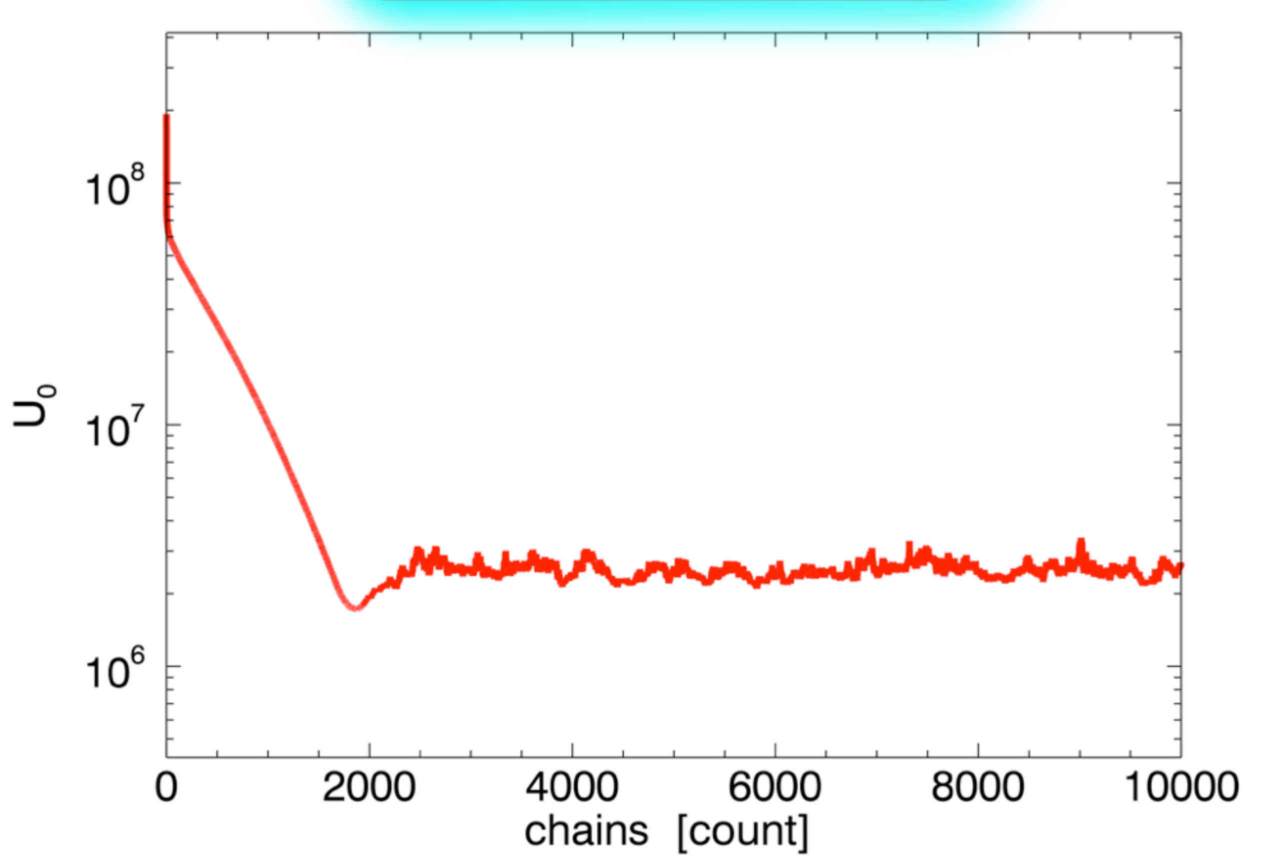
$$U(\mathbf{d}_\nu, \mathbf{s}, \mathbf{f}_\nu) = \sum_\nu (\mathbf{d}_\nu - \mathbf{A}\mathbf{s} - \mathbf{f}_n u)^t \mathbf{N}_\nu^{-1} (\mathbf{d}_\nu - \mathbf{A}\mathbf{s} - \mathbf{f}_\nu) + \mathbf{s}^t \mathbf{S}^{-1} \mathbf{s}$$

$$\underbrace{\mathbf{d}_\nu}_{\text{data}} = \mathbf{A}_\nu \underbrace{\mathbf{s}}_{\text{CMB}} + \underbrace{\mathbf{f}_\nu}_{\text{fg}} + \underbrace{\mathbf{n}_\nu}_{\text{noise}} \qquad P(\mathbf{s}|C_l, \mathbf{d}) \propto e^{-\frac{1}{2}(\mathbf{s}-\hat{\mathbf{s}})(\mathbf{S}^{-1}+\mathbf{N}^{-1})(\mathbf{s}-\hat{\mathbf{s}})}$$

potential U_0



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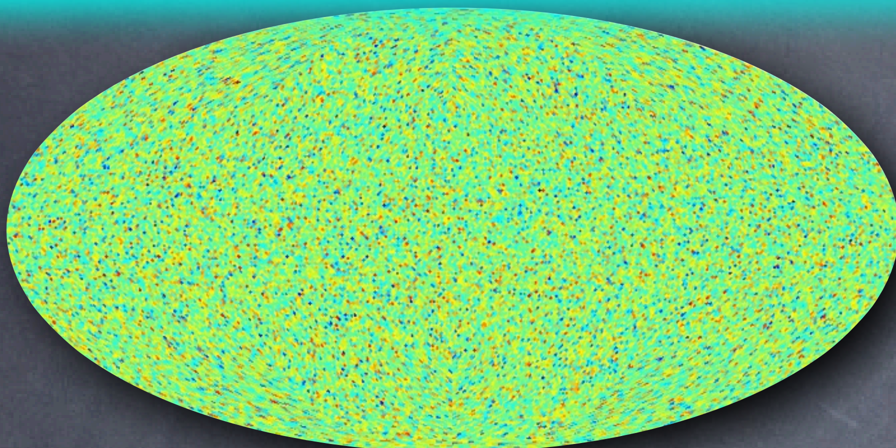
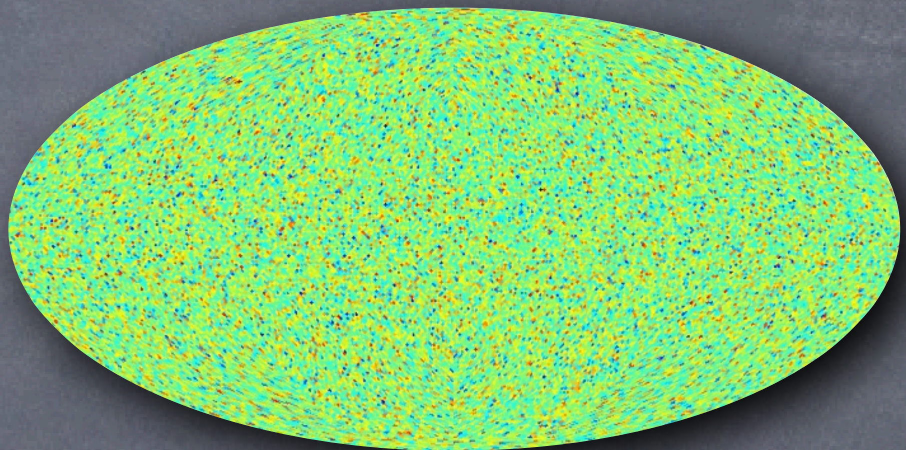
Results applying for polarization data

$w^{-1/2}$ = 2 μK・arcmin
Nside=64

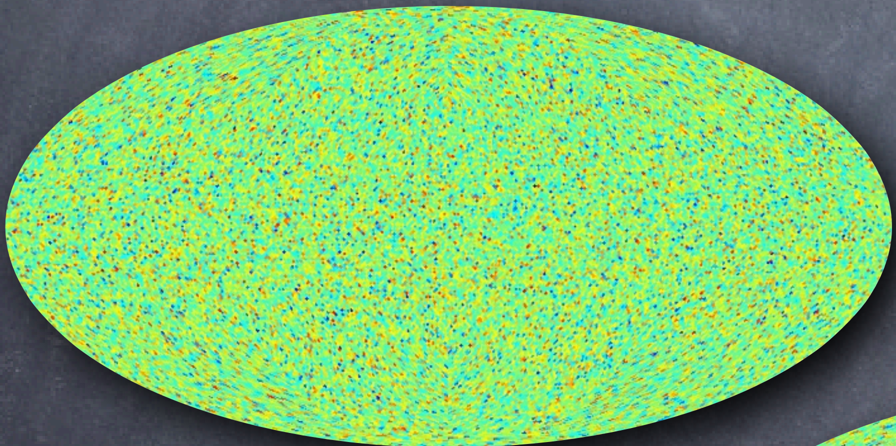
initial maps: (MCMC start)



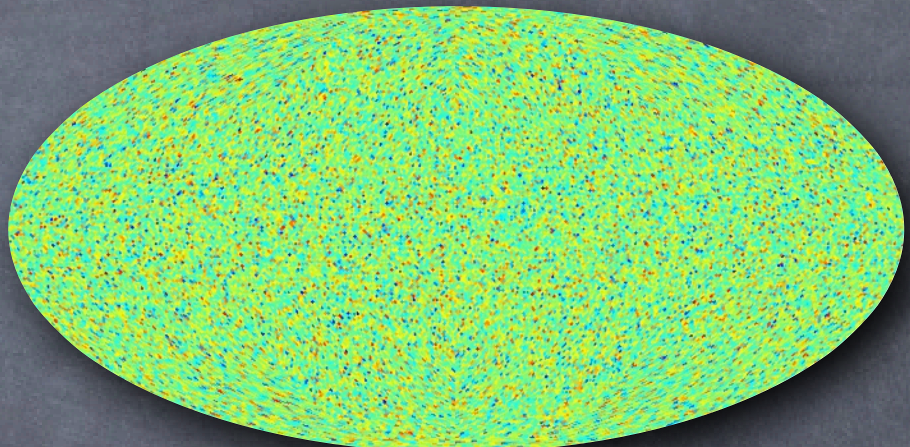
CMB Q-map



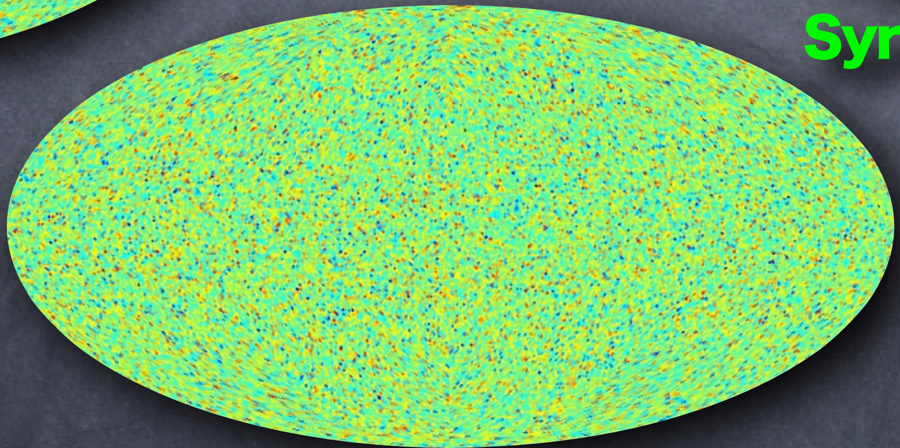
Synchrotoron Q-map



CMB U-map

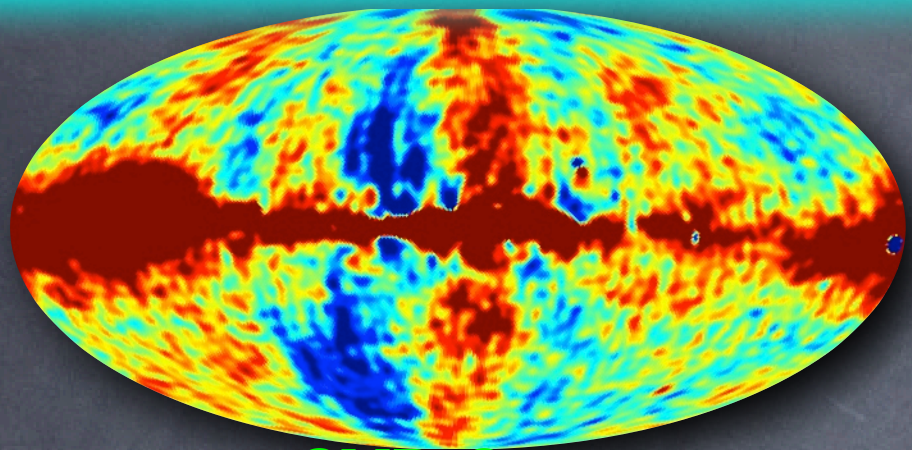


Synchrotoron U-map

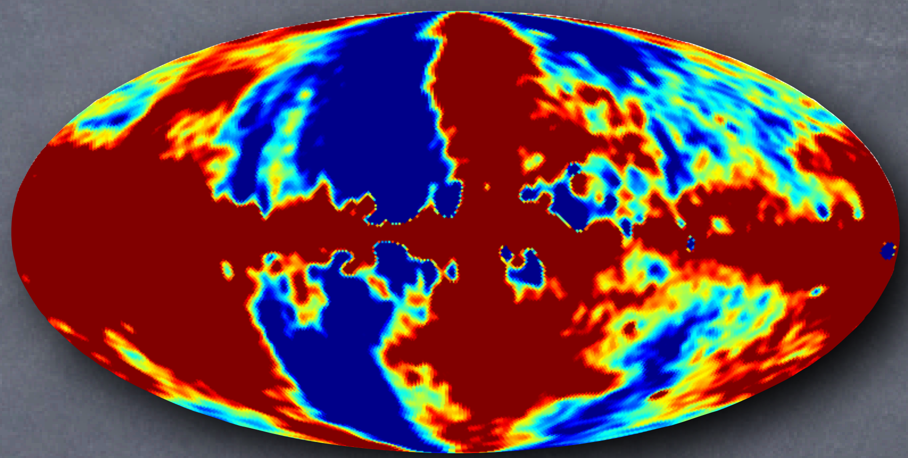


Synchrotoron Spectral INDEX-map

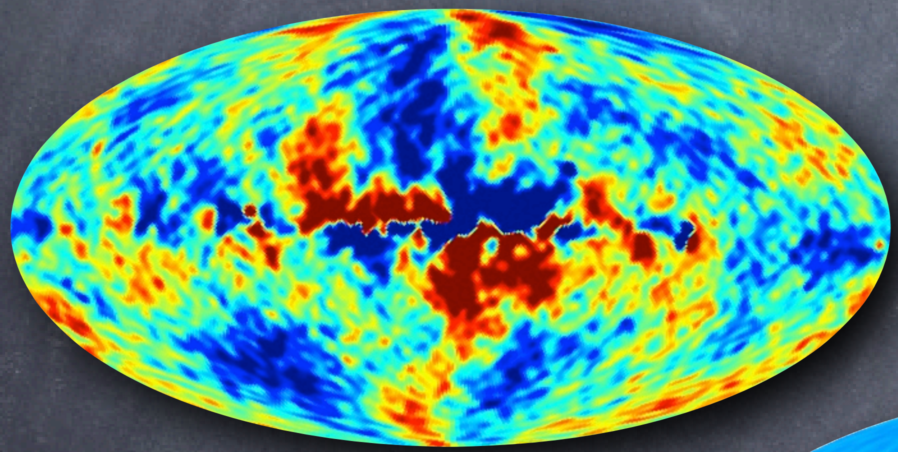
result maps: (no prior)



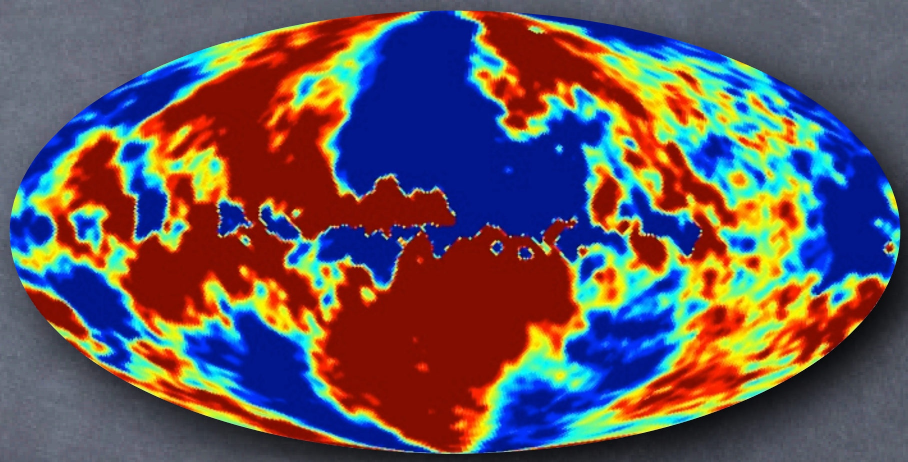
CMB Q-map



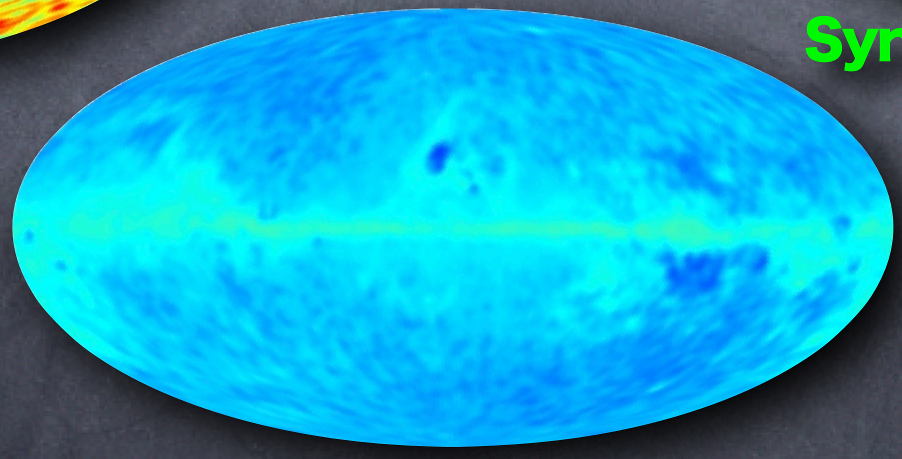
Synchrotoron Q-map



CMB U-map



Synchrotoron U-map



Synchrotoron Spectral INDEX-map

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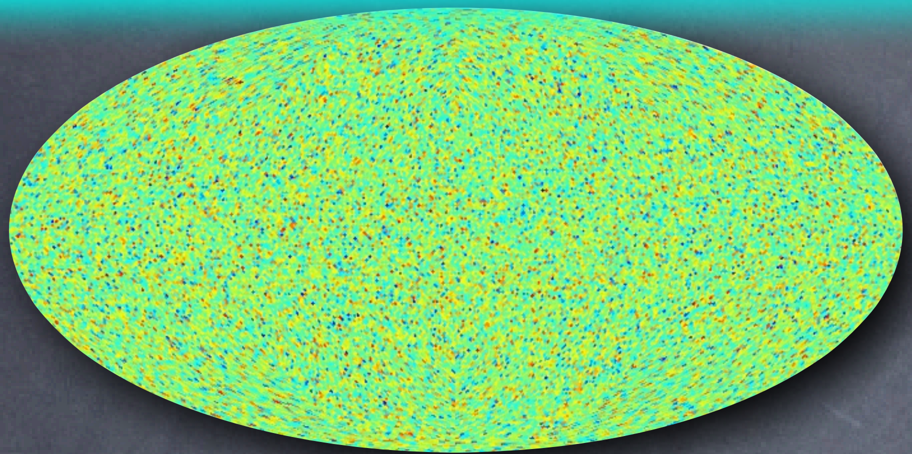
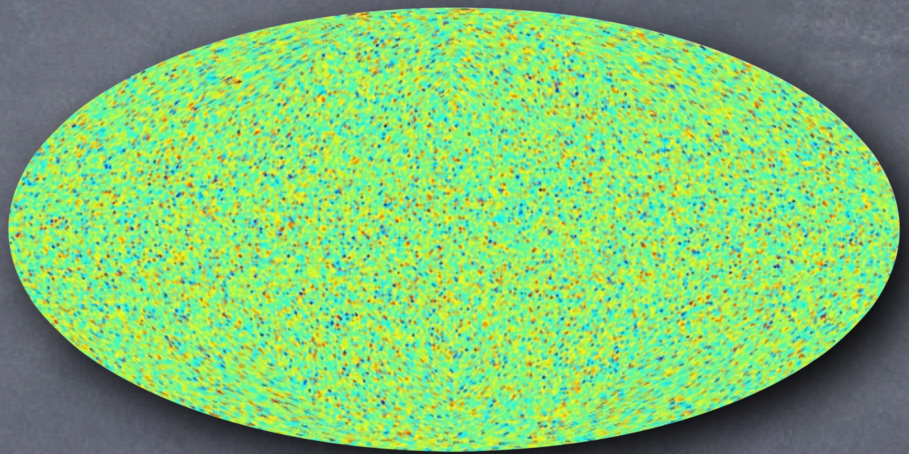
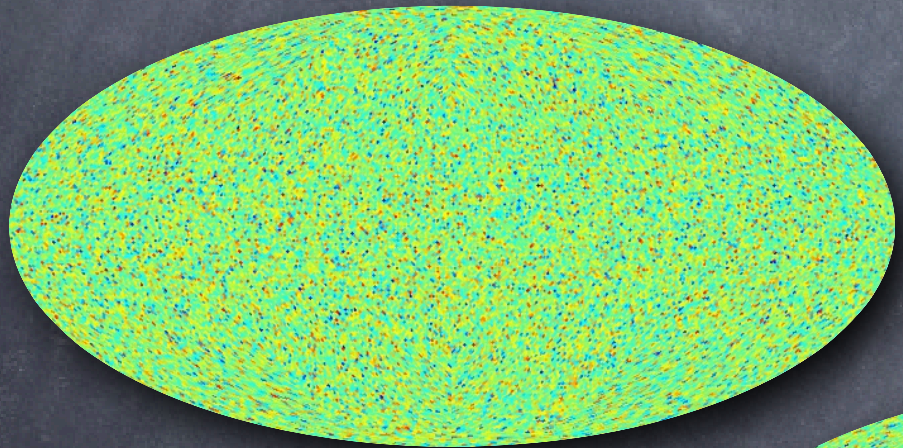
initial maps:
(MCMC start)



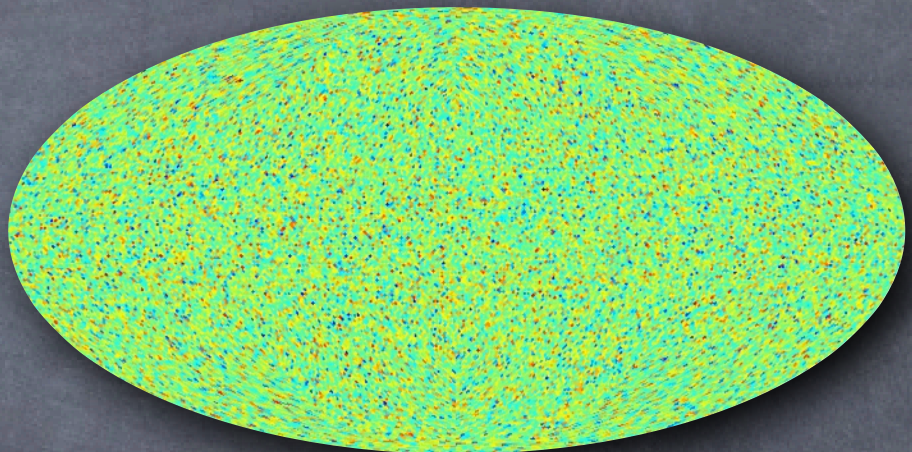
CMB Q-map



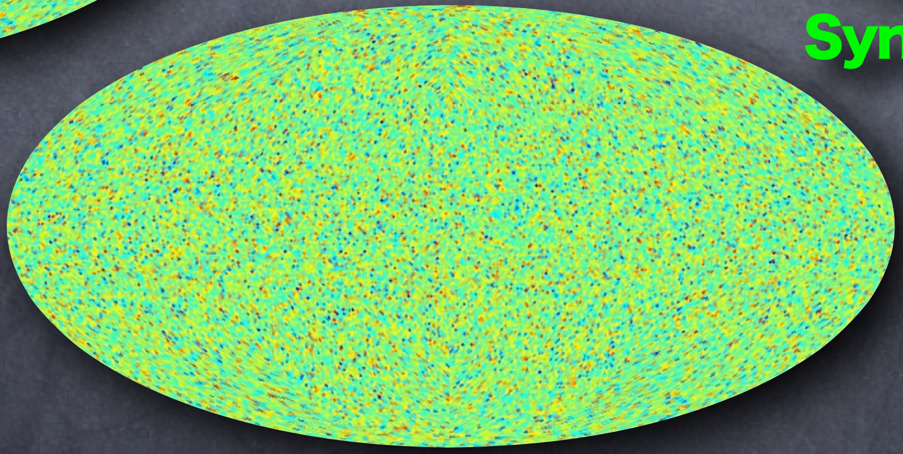
Synchrotoron Q-map



CMB U-map



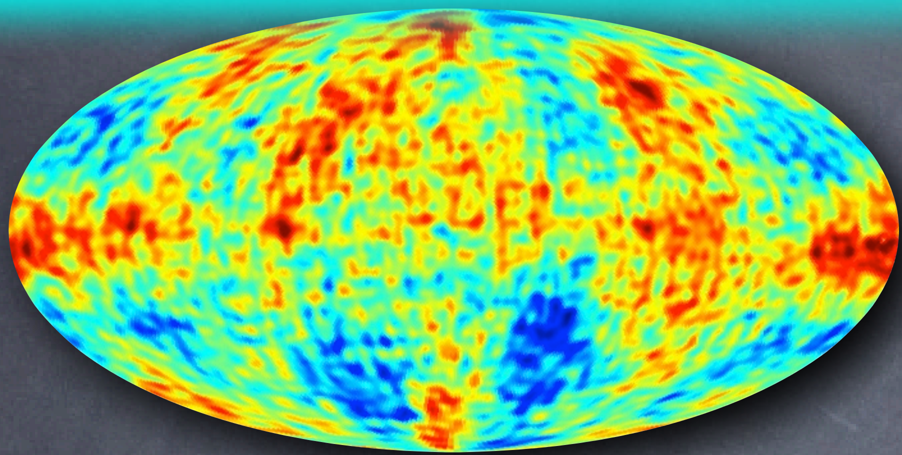
Synchrotoron U-map



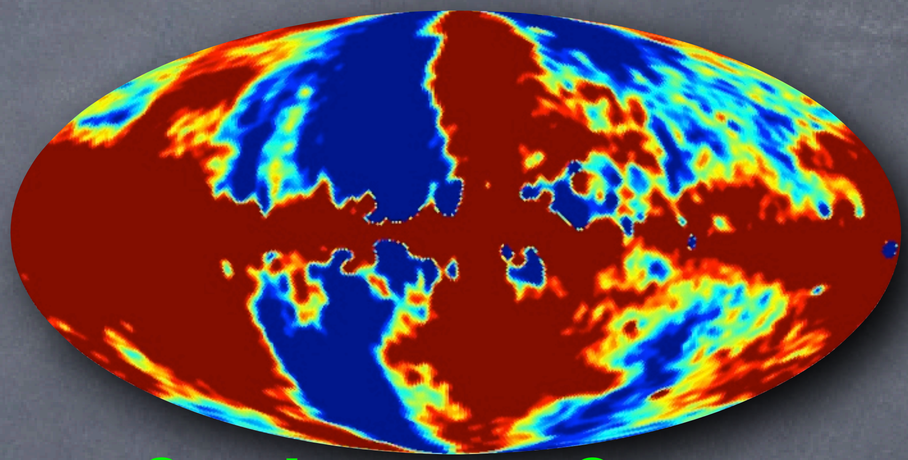
Synchrotoron Spectral
INDEX-map

Takahiro Morishima (Astronomical Institute, Graduate School of Science, Tohoku University)

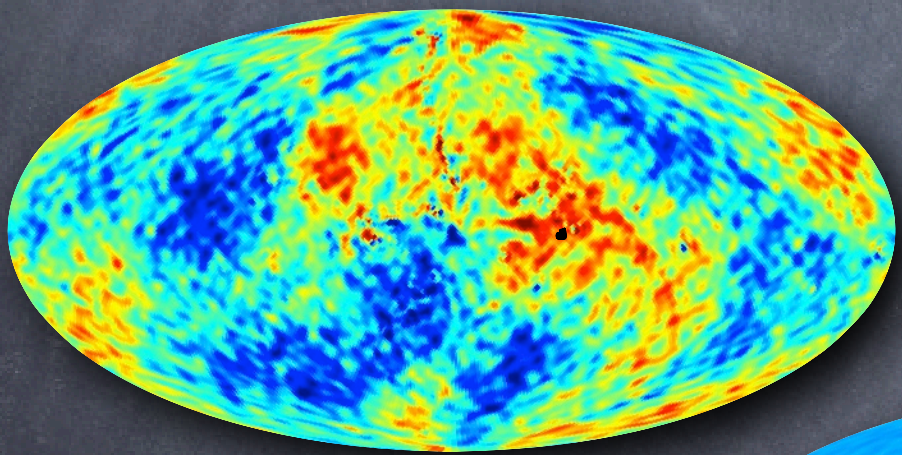
result maps: (with prior)



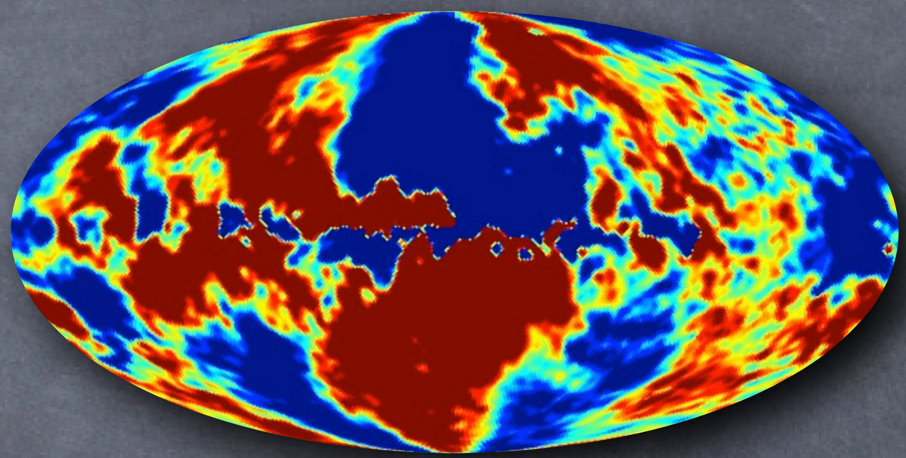
CMB Q-map



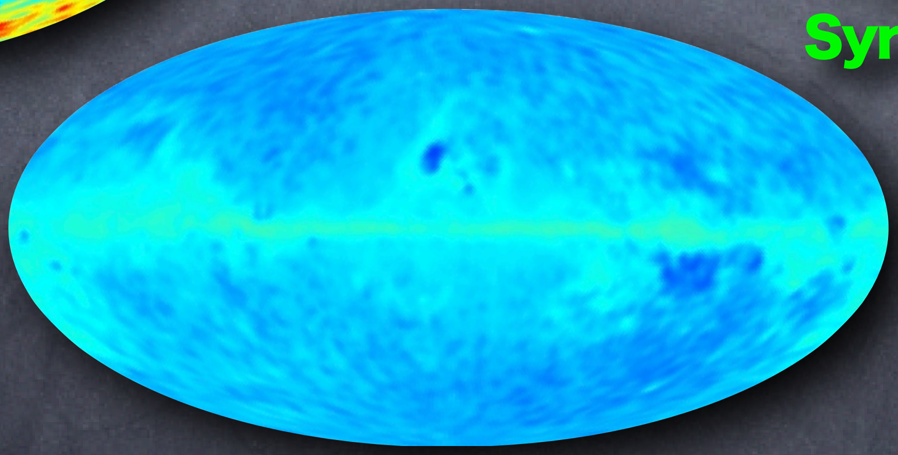
Synchrotoron Q-map



CMB U-map



Synchrotoron U-map

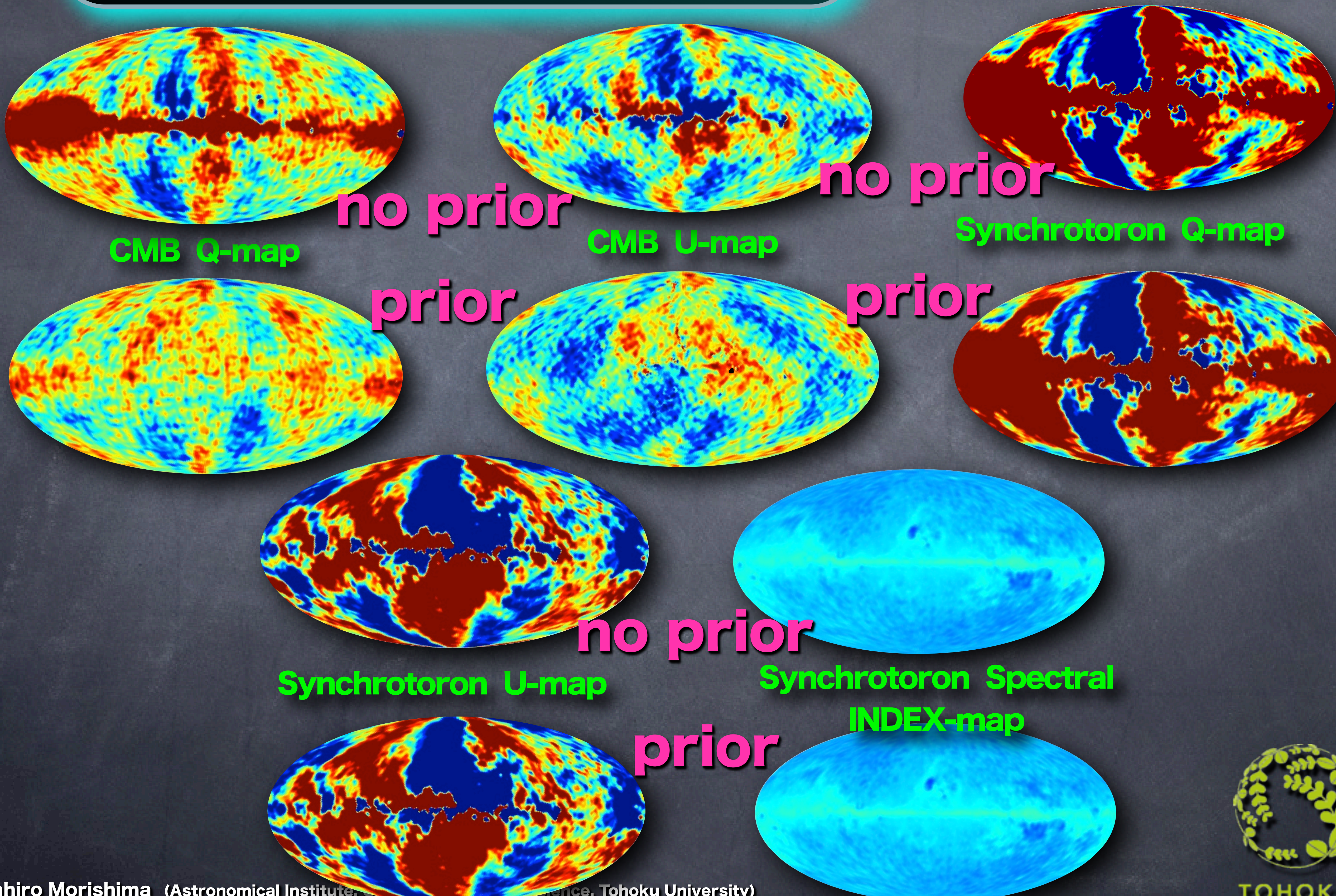


Synchrotoron Spectral INDEX-map

Takahiro Morishima (Astronomical Institute, Graduate School of Science, Tohoku University)

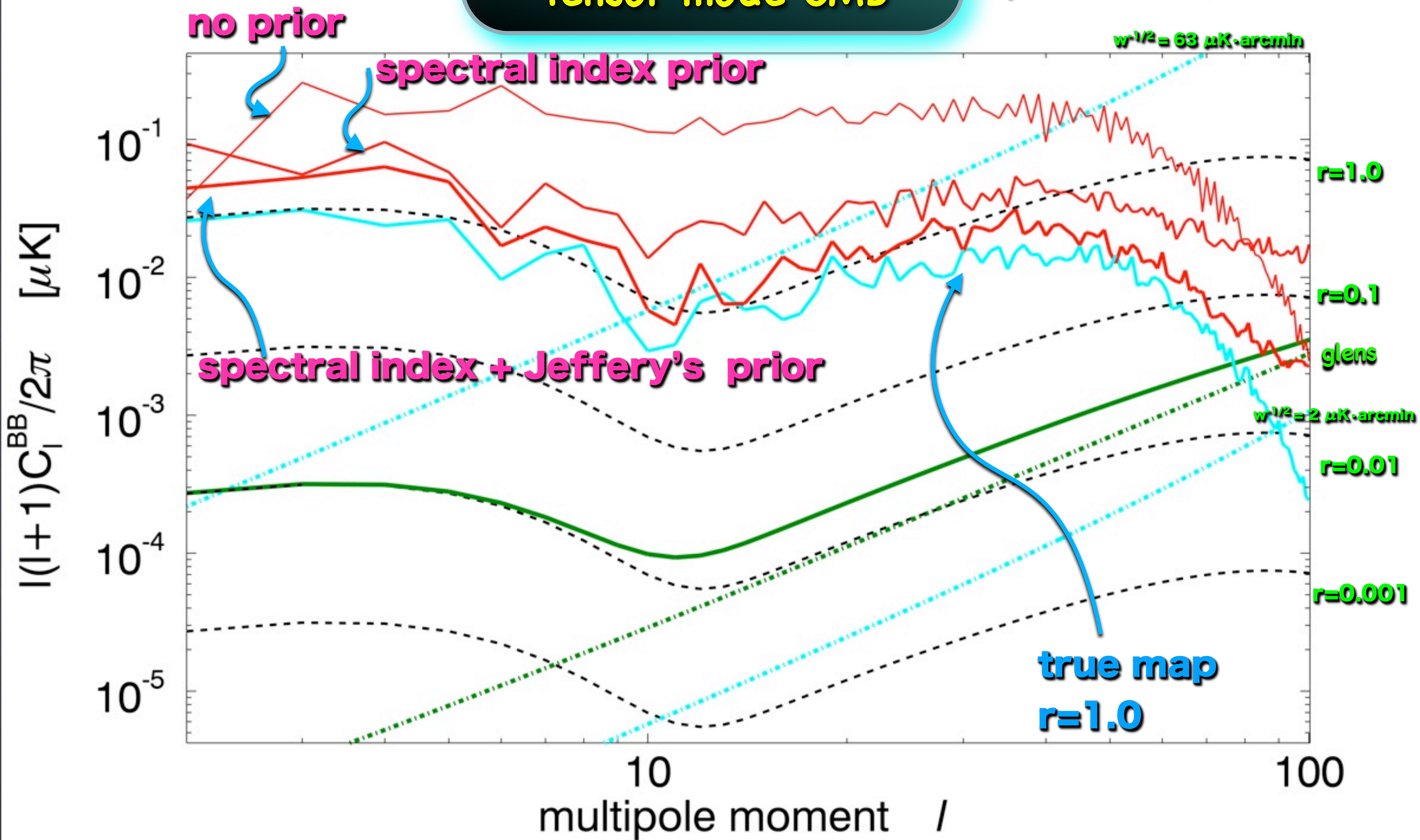
TOHOKU UNIVERSITY

result maps compare: (no prior, with prior)



Power Spectrum tensor mode CMB

preliminary results



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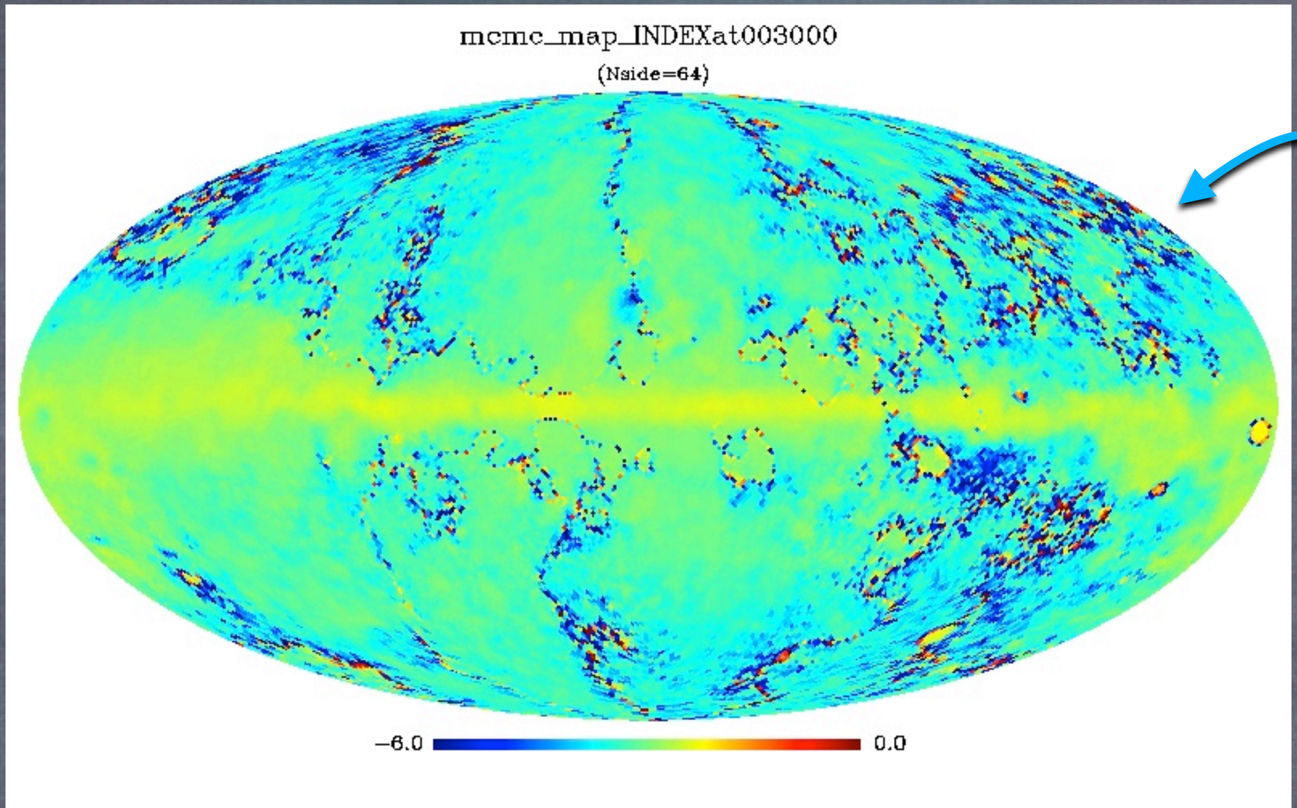
Power Spectrum tensor mode CMB

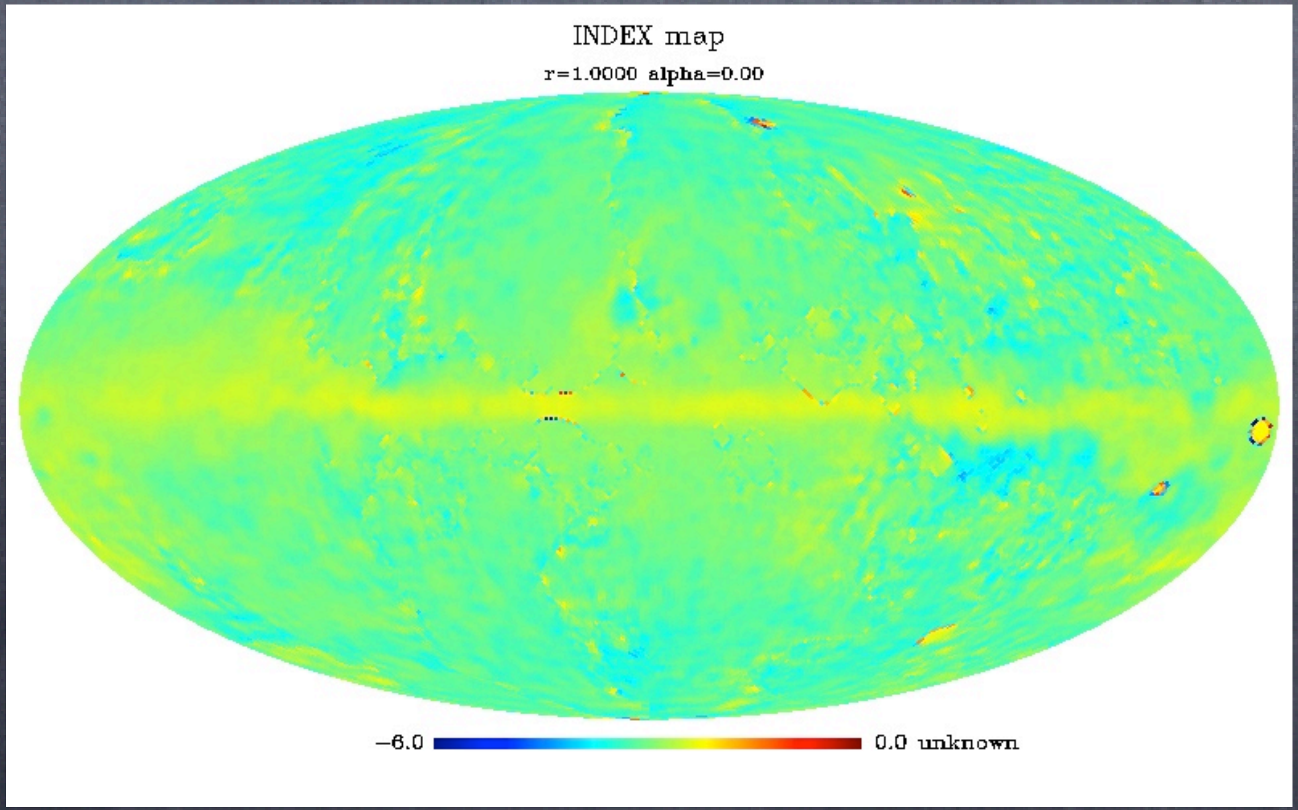
preliminary results

MRF prior



uncertain noise

no MRF prior



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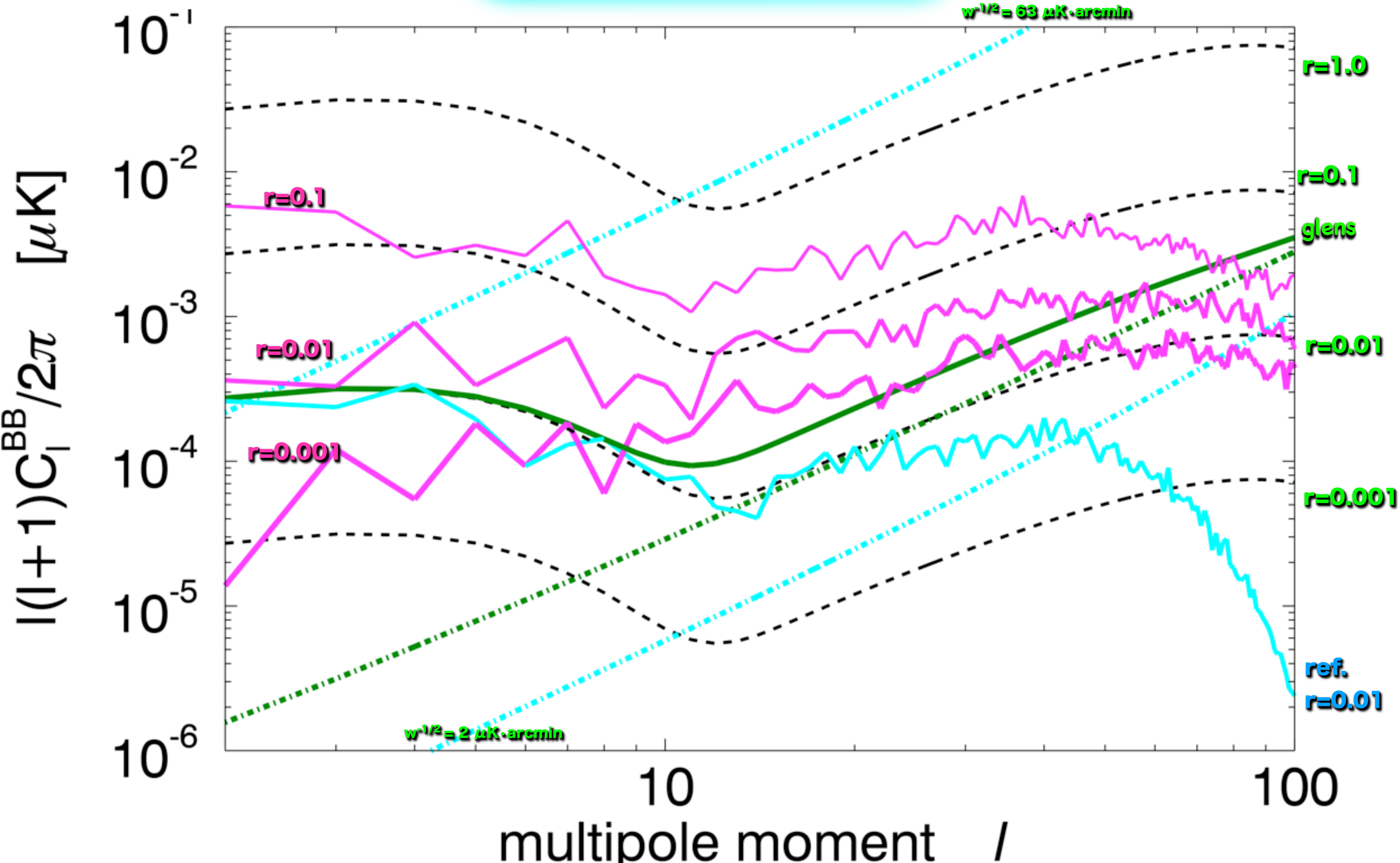
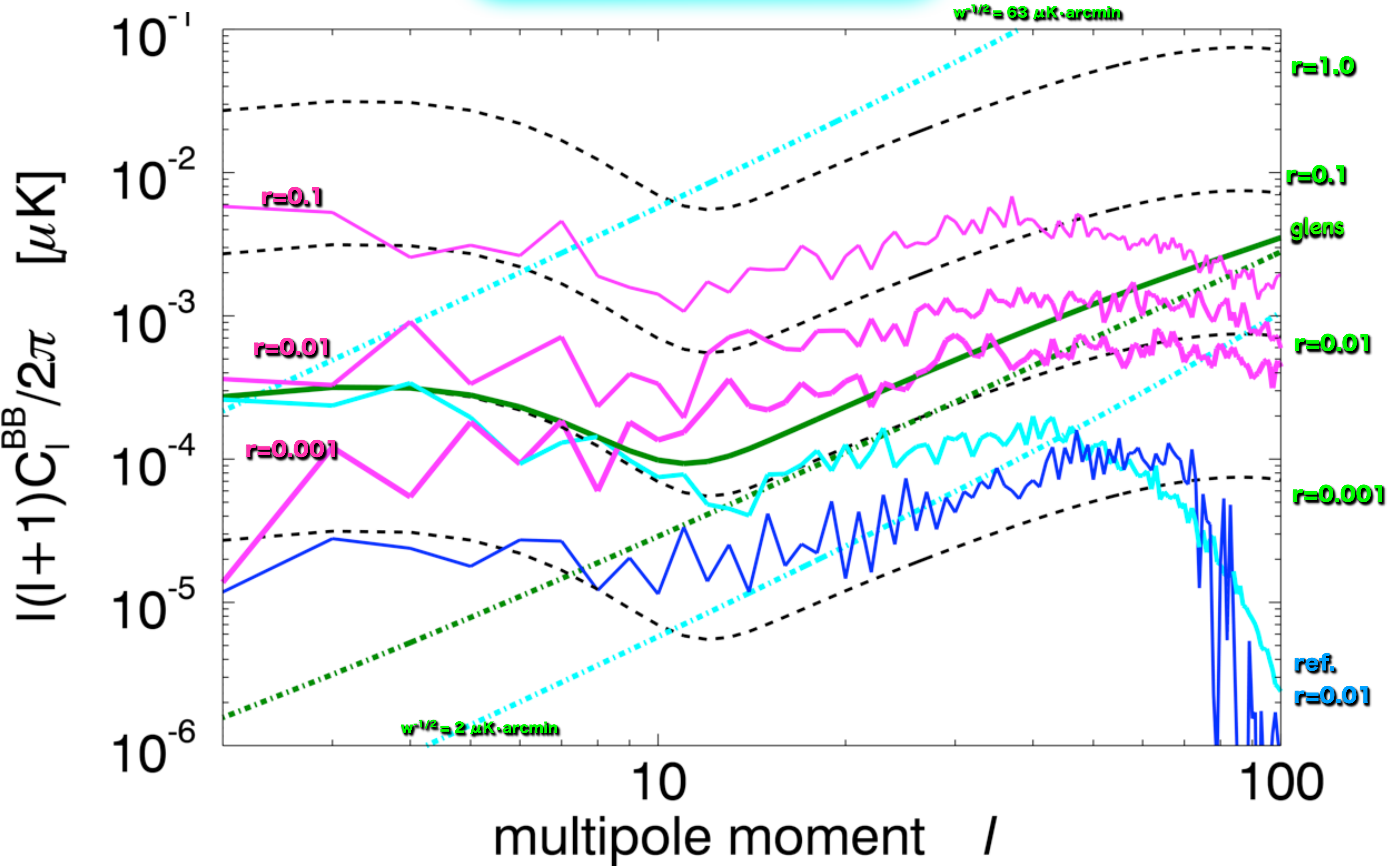
Power Spectrum tensor mode CMB

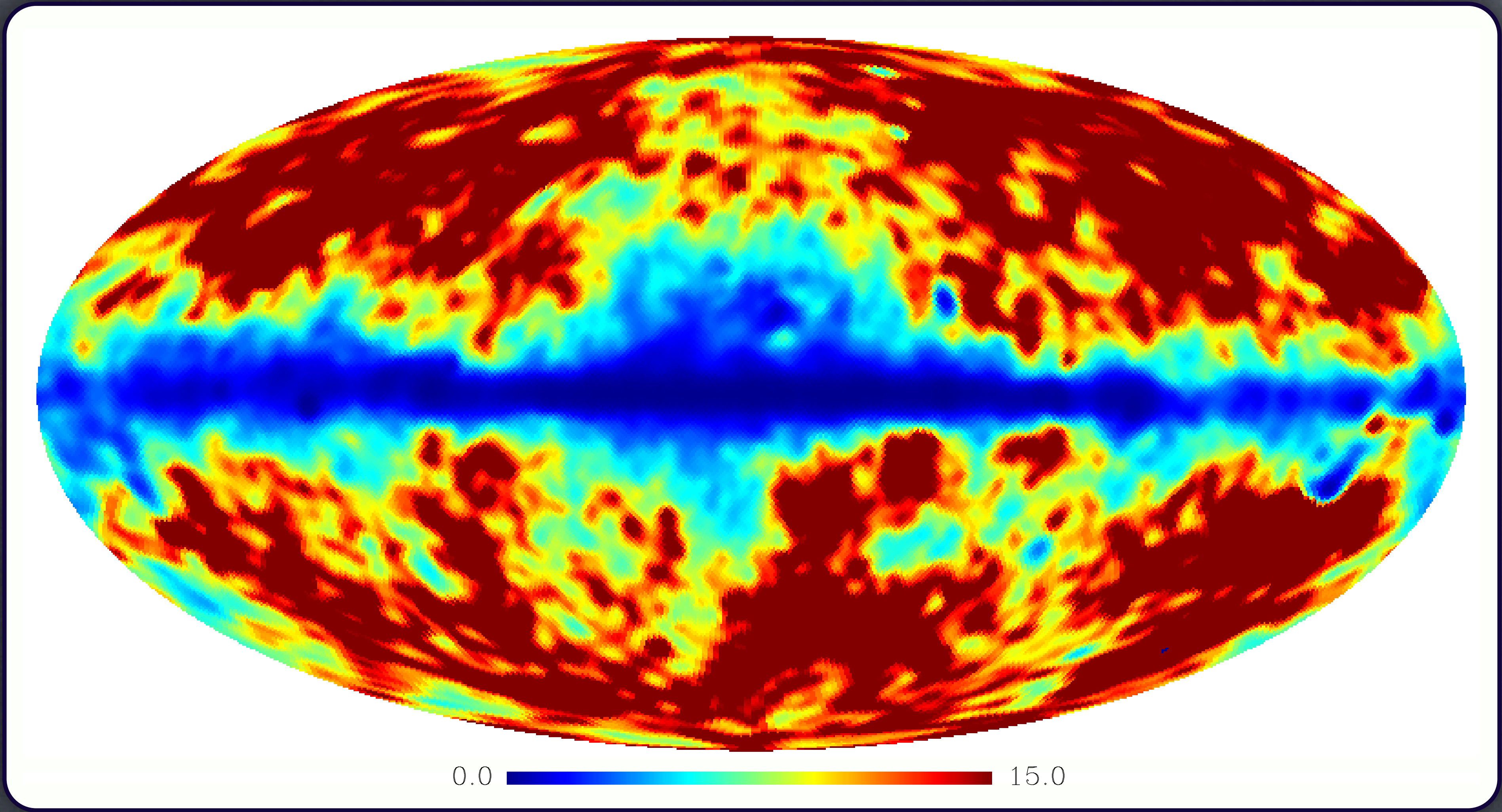
preliminary results



Power Spectrum tensor mode CMB

preliminary results





prior sample

SUMMARY

Component separation scheme based on hierachical Bayesian has been developped as for one of concrete example of the scheme which is able to take into account the physical knowledge of fg.

MRF prior is proposed to take into account the spatially correlated nature of fg.
For temperature fluctuation, MRF works well.
For polatization, spectral index and Jeffery's prior work effectively but MRF makes situation worse.

Further optimization of synchrotron priors are required.

How the updated knowledge of the Galactic Magnetic Field and the dust are taken into account in the component separation scheme statistically is next challenging topics.

END