Max Planck Institute
for Astrophysics

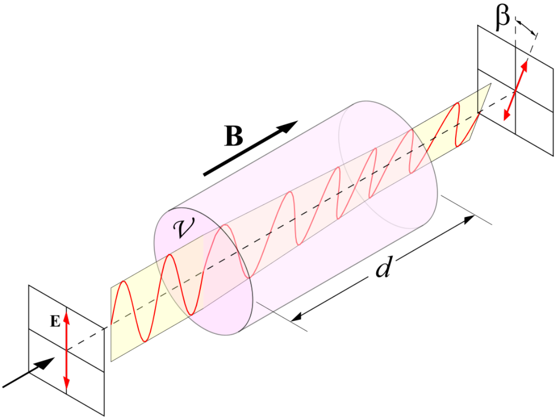
The Galactic Faraday sky
–
What it is, how it's done, and why it's useful

Niels Oppermann

with
G. Robbers, T.A. Enßlin, H. Junklewitz, M.R. Bell, A. Bonafede, R. Braun, J.-A.C. Brown, T.E. Clarke, I.J. Feain, B.M. Gaensler, A. Hammond, L. Harvey-Smith, G. Heald, M. Johnston-Hollitt, U. Klein, P.P. Kronberg, S.A. Mao, N.M. McClure-Griffiths, S.P. O'Sullivan, L. Pratley, T. Robishaw, S. Roy, D.H.F.M. Schnitzeler, C. Sotomayor-Beltran, J. Stevens, J.M. Stil, C. Sunstrum, A. Tanna, A.R. Taylor, and C.L. Van Eck

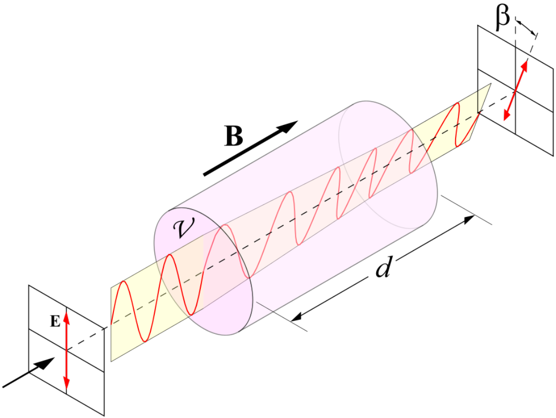
Polarized Foreground Workshop, Garching, 2012-11-26

What it is



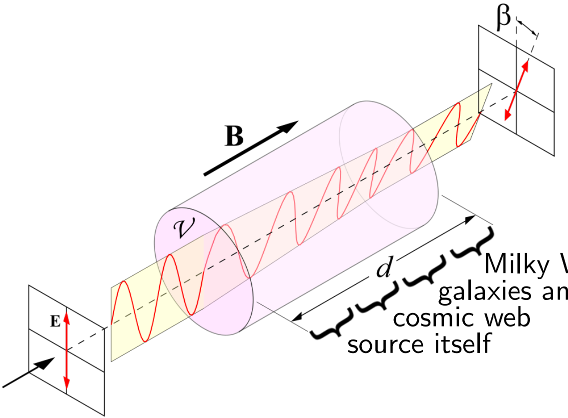
$$\mathrm{d}\beta \propto \lambda^2 n_\mathrm{e}(\vec{x}) B_r(\vec{x}) \mathrm{d}r$$

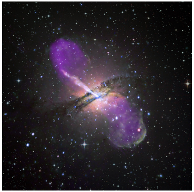
$$\Rightarrow \quad \beta \propto \lambda^2 \int_{r_\mathrm{source}}^{0} n_\mathrm{e}(\vec{x}) B_r(\vec{x}) \mathrm{d}r$$



$$\text{Faraday depth: } \phi \propto \int_{r_{\rm source}}^{0} n_{\rm e}(\vec{x}) B_r(\vec{x}) \mathrm{d}r$$

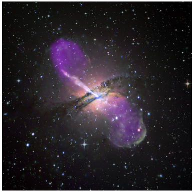
$$\beta = \phi \lambda^2$$

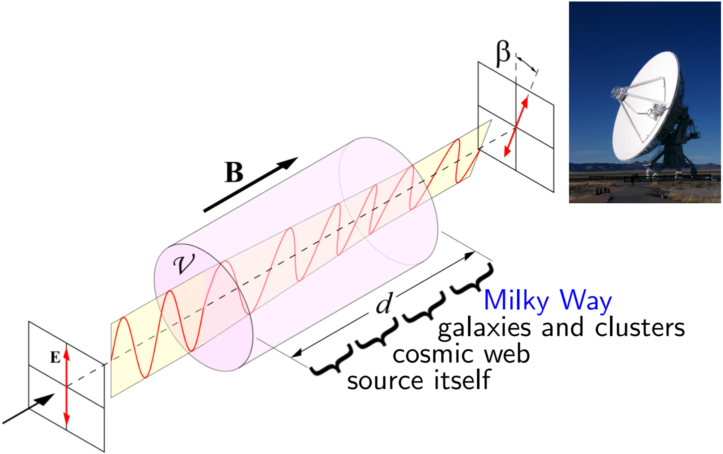




Faraday depth: $\phi \propto \int_{r_{\text{source}}}^{0} n_{\mathrm{e}}(\vec{x}) B_r(\vec{x}) \mathrm{d}r$

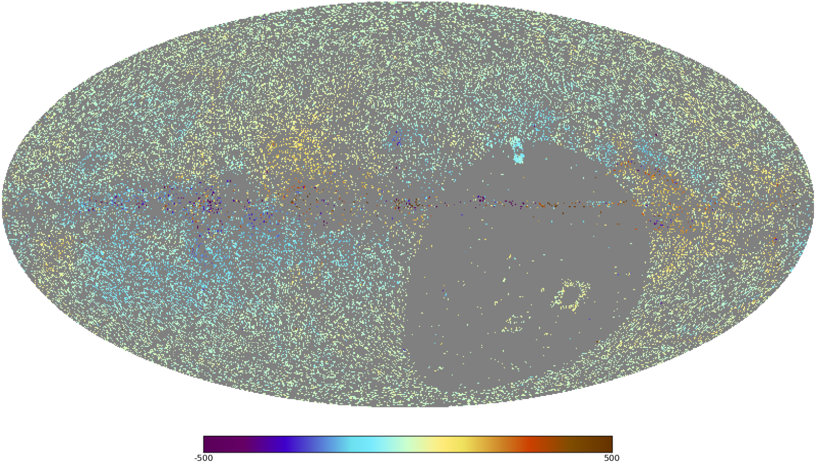
$$\beta = \phi \lambda^2$$



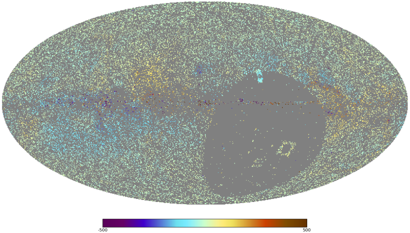


Galactic Faraday depth:

$$\phi \propto \int_{r_{\mathrm{MilkyWay}}}^{0} n_{\mathrm{e}}(\vec{x}) B_r(\vec{x}) \mathrm{d}r$$



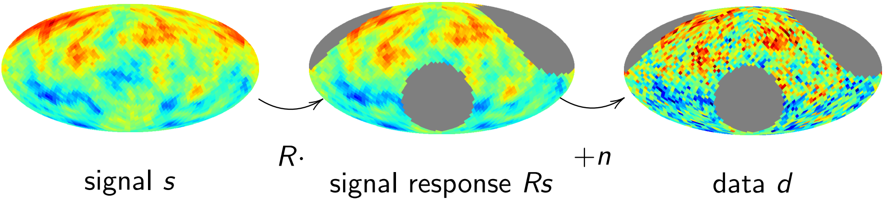
41 330 data points



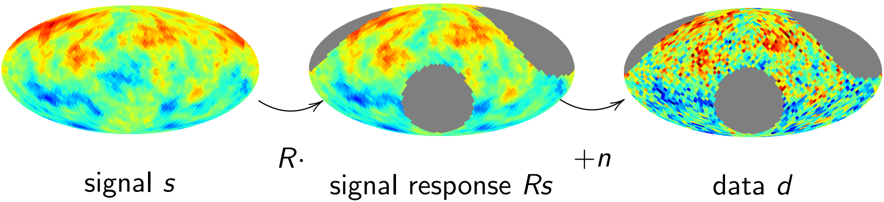
Challenges

- Regions without data
- Uncertain error bars:
  - complicated observations
  - $n\pi$-ambiguity
  - extragalactic contributions unknown

How it's done



$$\boxed{d = Rs + n}$$

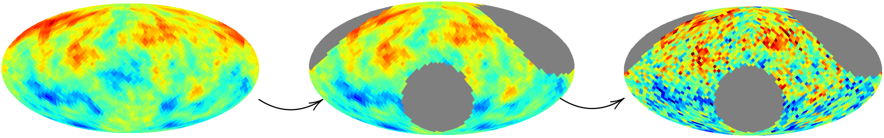


$$\boxed{d = Rs + n}$$

$$\mathcal{P}(s) = \mathcal{G}(s, S)$$

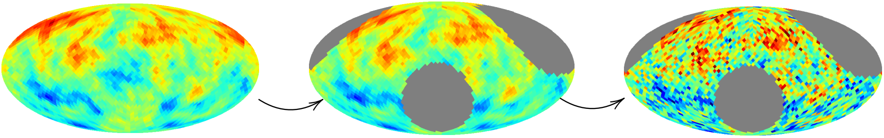
$$\mathcal{P}(n) = \mathcal{G}(n, N)$$

$$\mathcal{G}(s, S) = \frac{1}{|2\pi S|^{1/2}} \exp\left[\frac{1}{2} s^\dagger S^{-1} s\right]$$



$$d = Rs + n$$

$$m = \int \mathcal{D}s \; s \; \mathcal{P}(s|d)$$



$R\cdot$ $+n$

signal s | signal response Rs | data d

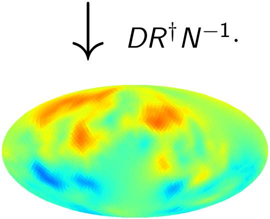
Wiener Filter

$$d = Rs + n$$

$$m = \int \mathcal{D}s \; s \; \mathcal{P}(s|d)$$

$$m = Dj, \text{ where } \quad \begin{array}{l} j = R^\dagger N^{-1} d \\ D = \left(S^{-1} + R^\dagger N^{-1} R\right)^{-1} \end{array}$$

$DR^\dagger N^{-1}\cdot$



$$S(\hat{n}, \hat{n}') = \int \mathcal{D}s \; s(\hat{n}) s(\hat{n}') \mathcal{P}(s)$$

$$\Rightarrow S_{(\ell m),(\ell' m')} = \int \mathcal{D}s \; s_{\ell m} s^*_{\ell' m'} \mathcal{P}(s)$$

$$
\begin{aligned}
S(\hat{n}, \hat{n}') &= \int \mathcal{D}s \; s(\hat{n}) s(\hat{n}') \mathcal{P}(s) \\
&= S(\hat{n} \cdot \hat{n}') \\
\Rightarrow S_{(\ell m),(\ell' m')} &= \int \mathcal{D}s \; s_{\ell m} s^*_{\ell' m'} \mathcal{P}(s) \\
&= \delta_{\ell\ell'} \delta_{mm'} C_\ell
\end{aligned}
$$

$\hookrightarrow$ angular power spectrum

$$
\begin{aligned}
S(\hat{n},\hat{n}') &= \int \mathcal{D}s\ s(\hat{n})s(\hat{n}')\mathcal{P}(s) \\
&= S(\hat{n}\cdot\hat{n}') \\
\Rightarrow S_{(\ell m),(\ell' m')} &= \int \mathcal{D}s\ s_{\ell m}s^{*}_{\ell' m'}\mathcal{P}(s) \\
&= \delta_{\ell\ell'}\delta_{mm'}\,C_\ell
\end{aligned}
$$

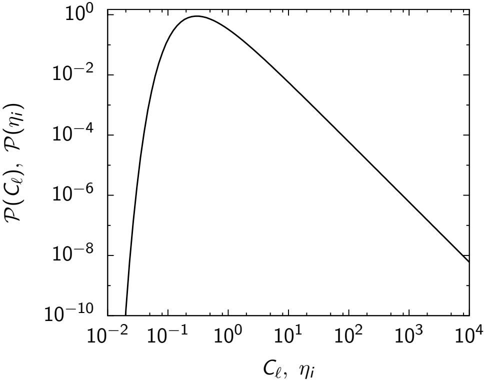
$\hookrightarrow$ angular power spectrum

$$N_{ij} = \delta_{ij}\sigma_i^2$$

(uncorrelated noise)

$$S_{(\ell m),(\ell' m')} = \delta_{\ell\ell'}\delta_{mm'} C_\ell \qquad N_{ij} = \delta_{ij}\eta_i\sigma_i^2$$

$$S_{(\ell m),(\ell' m')} = \delta_{\ell\ell'}\delta_{mm'} C_\ell \qquad N_{ij} = \delta_{ij}\eta_i\sigma_i^2$$



$\Rightarrow$ marginalize over all possible parameters

Problem: $\mathcal{P}(s|d)$ is non-Gaussian.
Solution: Find Gaussian $\mathcal{G}(s - m, D)$, that best approximates $\mathcal{P}(s|d)$.

Problem: $\mathcal{P}(s|d)$ is non-Gaussian.
Solution: Find Gaussian $\mathcal{G}(s-m, D)$, that best approximates $\mathcal{P}(s|d)$.

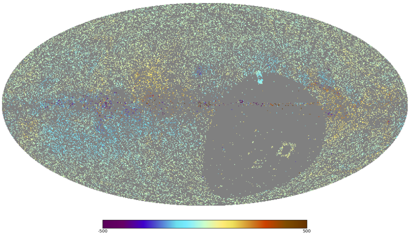
Extended Critical Filter

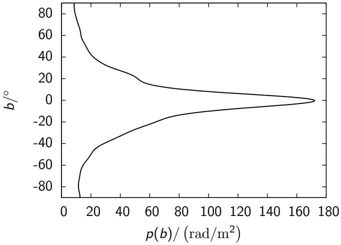
$$m = Dj, \quad D = \left[\sum_\ell C_\ell^{-1} S_\ell^{-1} + \sum_i \eta_i^{-1} R^\dagger N_i^{-1} R \right]^{-1},$$

$$j = \sum_i \eta_i^{-1} R^\dagger N_i^{-1} d$$

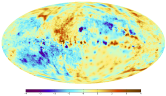
$$C_\ell = \frac{1}{\alpha_\ell + \ell - 1/2} \left[q_\ell + \frac{1}{2} \mathrm{tr} \left(\left(m m^\dagger + D \right) S_\ell^{-1} \right) \right]$$

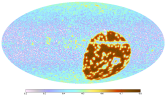
$$\eta_i = \frac{1}{\alpha_i} \left[q_i + \frac{1}{2} \mathrm{tr} \left(\left((d - Rm)(d - Rm)^\dagger + RDR^\dagger \right) N_i^{-1} \right) \right]$$

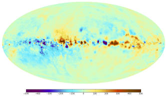


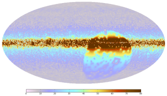


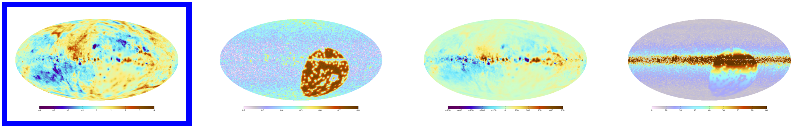
- Approximate $s(b, l) := \frac{\phi(b,l)}{p(b)}$ as a statistically isotropic Gaussian field
- R: multiplication with $p(b)$ and projection on directions of sources
- $N_{ij} = \delta_{ij}\eta_i\sigma_i^2$


−4 −3 −2 −1 0 1 2 3 4

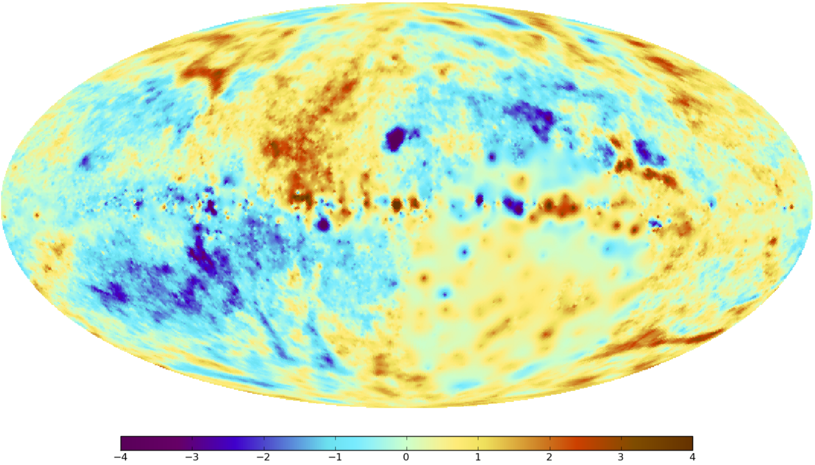

0.2 0.3 0.4 0.5 0.6 0.7 0.8


−500 −400 −300 −200 −100 0 100 200 300 400 500

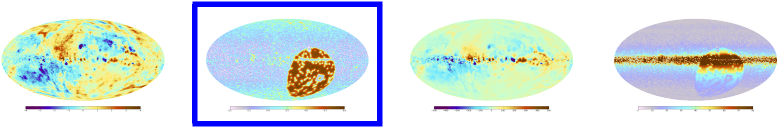

0 10 20 30 40 50 60 70 80



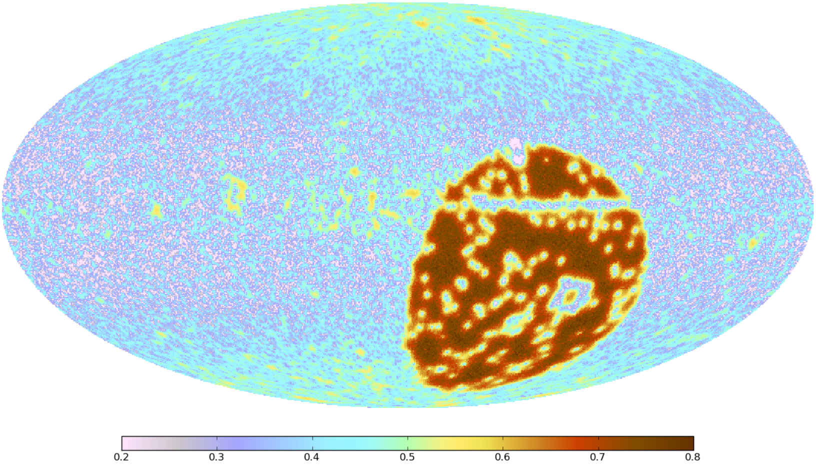
posterior mean of the signal



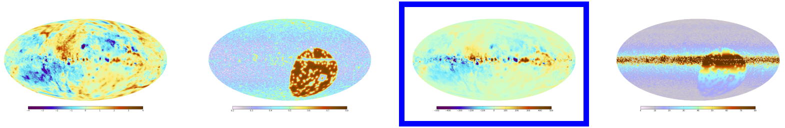
m



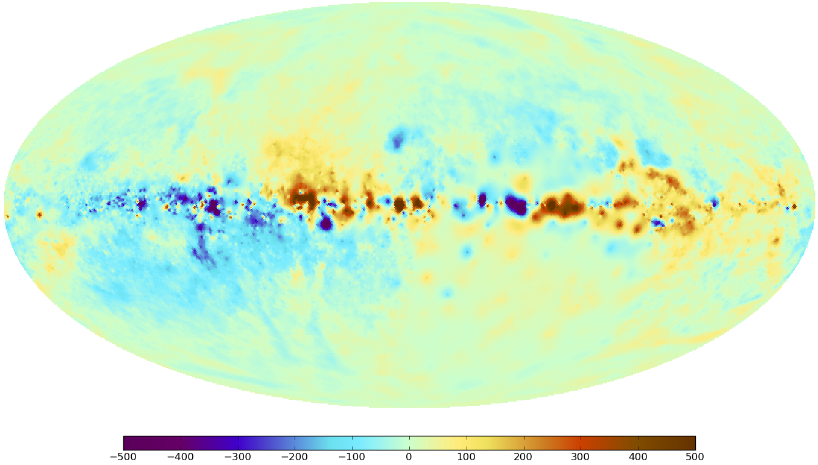
uncertainty of the signal map



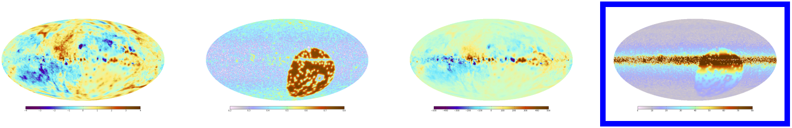
$$\sqrt{\mathrm{diag}\,(D)}$$



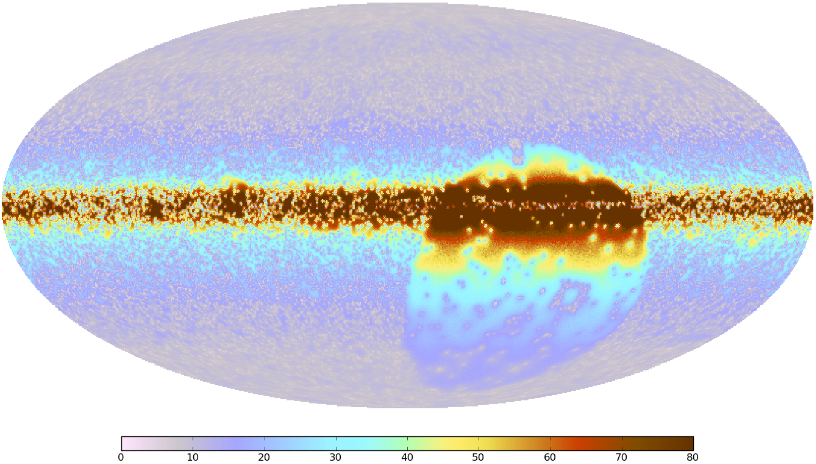
posterior mean of the Faraday depth



$pm\,/(\mathrm{rad}/\mathrm{m}^2)$

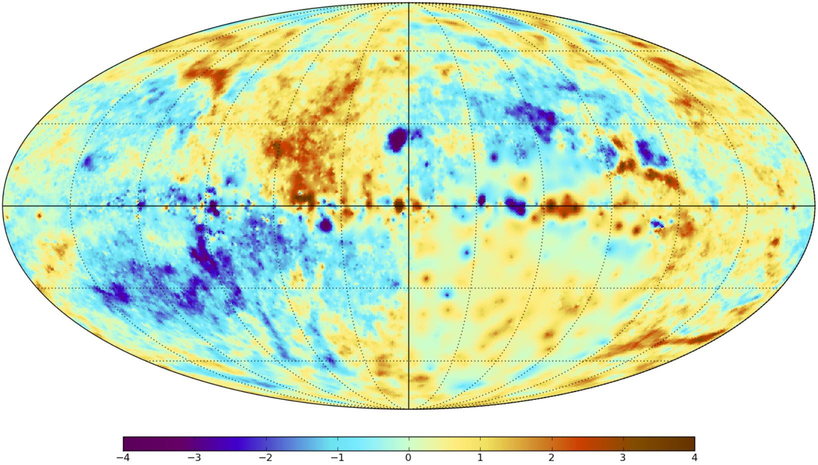


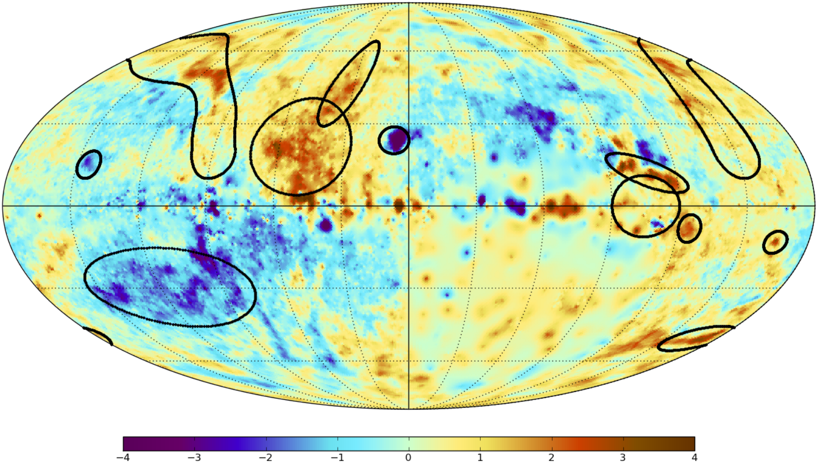
uncertainty of the Faraday depth

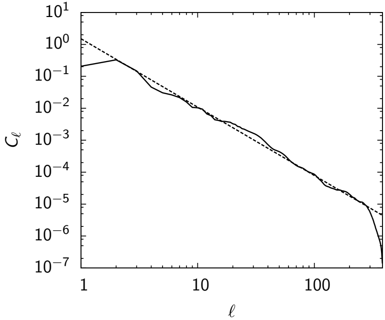


$p\sqrt{\mathrm{diag}\,(D)}\,/(\mathrm{rad}/\mathrm{m}^2)$

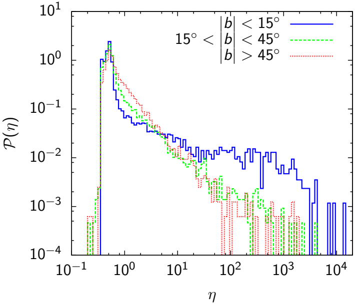
Why it's useful


−4
−3
−2
−1
0
1
2
3
4

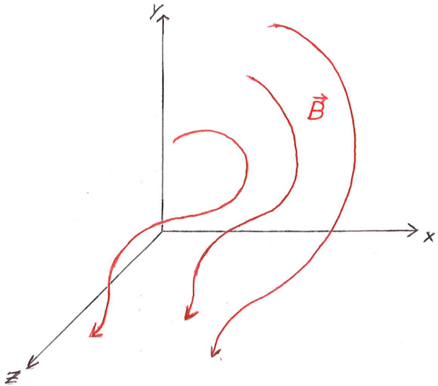

−4
−3
−2
−1
0
1
2
3
4



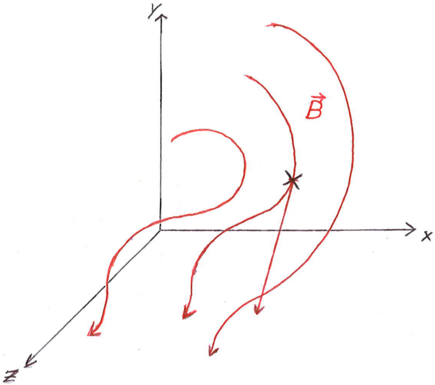
$$C_\ell \propto \ell^{-2.17}$$

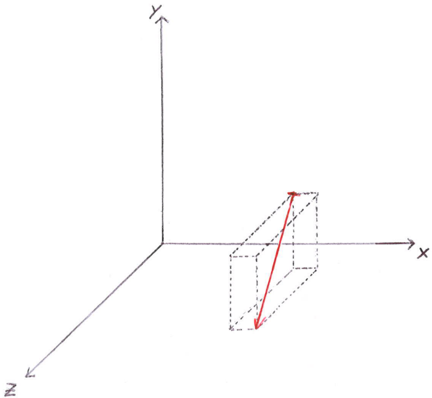


$$N_{ij} = \langle n_i n_j \rangle = \delta_{ij} \eta_i \sigma_i^2$$

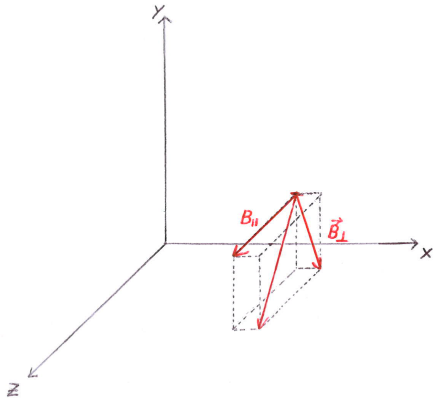


y
x
z
$\vec{B}$

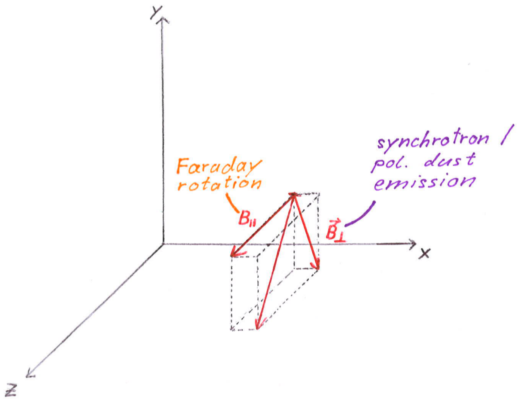

y
x
z
$\vec{B}$

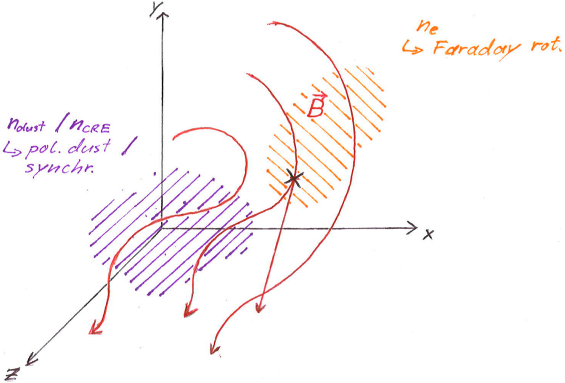


Y
X
Z



Y
X
Z
$B_{\parallel}$
$\vec{B}_{\perp}$


Y
X
Z
Faraday rotation
B∥
B⊥
synchrotron / pol. dust emission



y
x
z
n_{dust} / n_{CRE}
↳ pol. dust /
synchr.
n_e
↳ Faraday rot.
$\vec{B}$

Summary

- New map of the Galactic contribution to Faraday depth
- Extragalactic contributions filtered out via spatial correlation structure
- Potential for studies of
  - Interstellar medium
  - Galactic magnetic field
  - Extragalactic sources

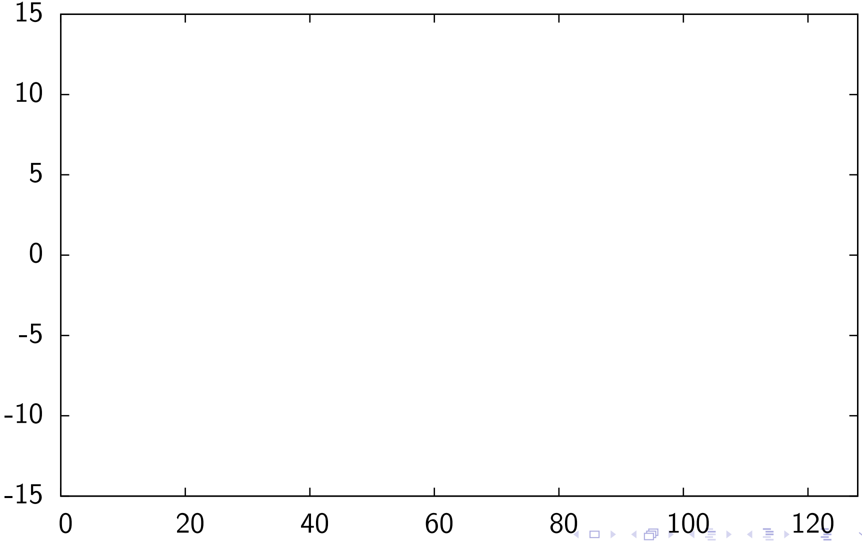
All results available at
http://www.mpa-garching.mpg.de/ift/faraday/

Backup

1D test case

Assumptions:

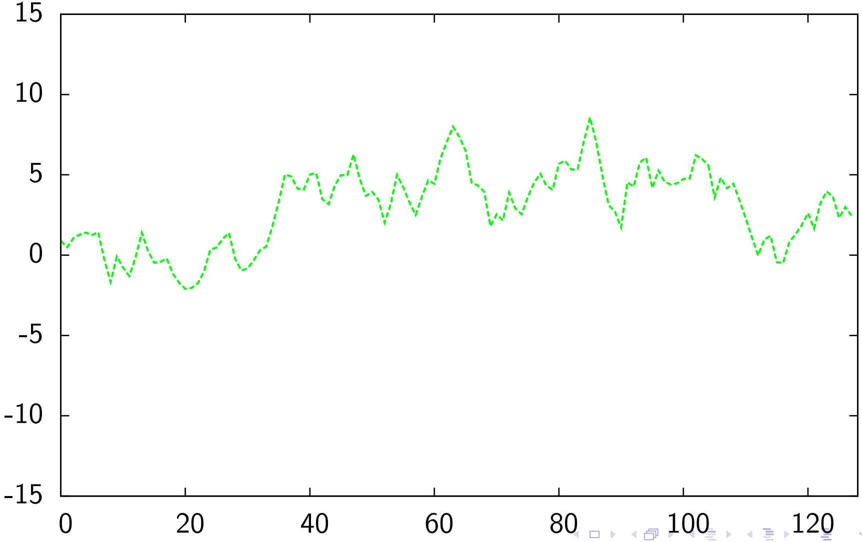
-
-



1D test case

Assumptions:

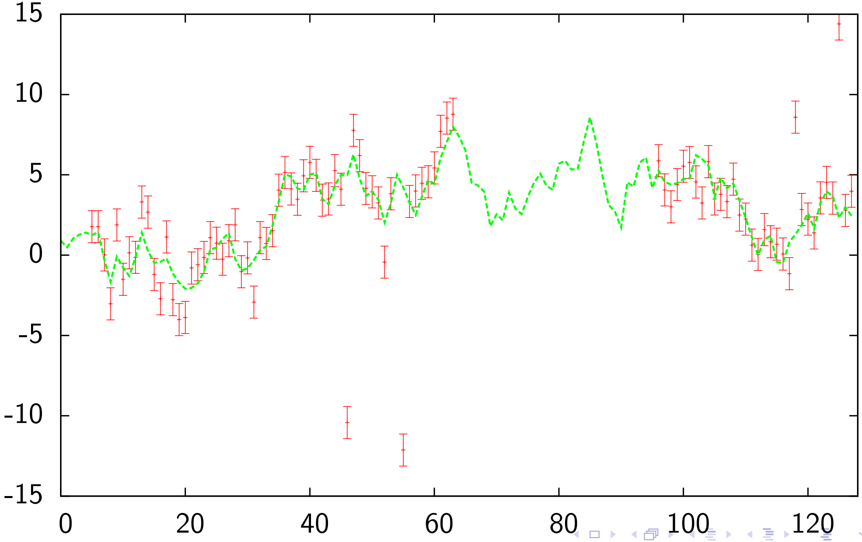
- signal field statistically homogeneous Gaussian random field
-



1D test case

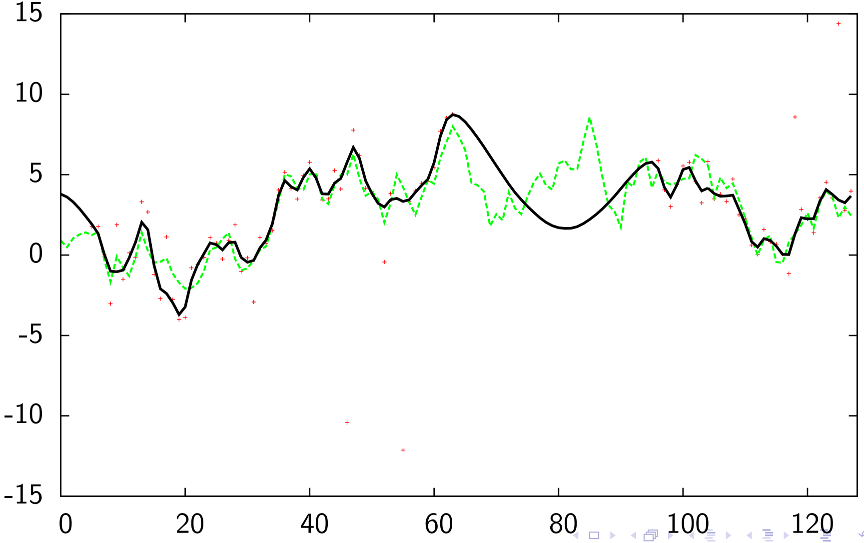
Assumptions:

- signal field statistically homogeneous Gaussian random field
- noise uncorrelated, Gaussian



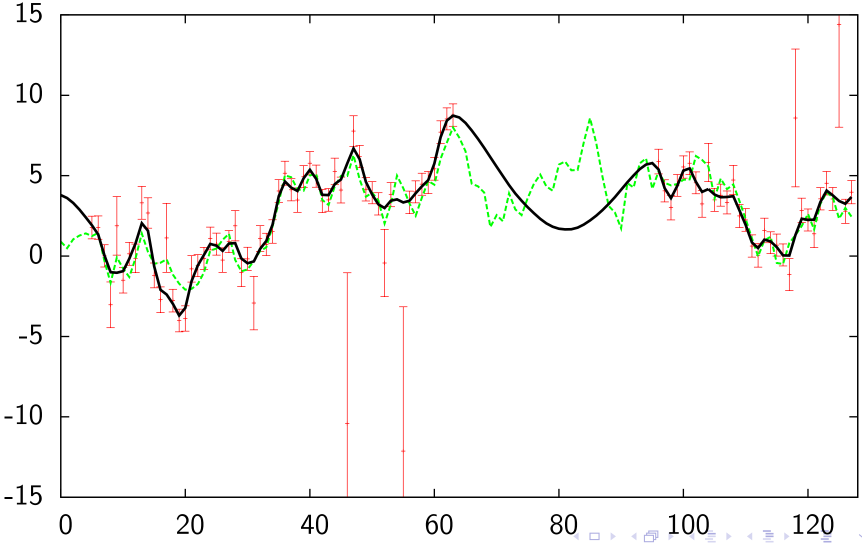
1D test case

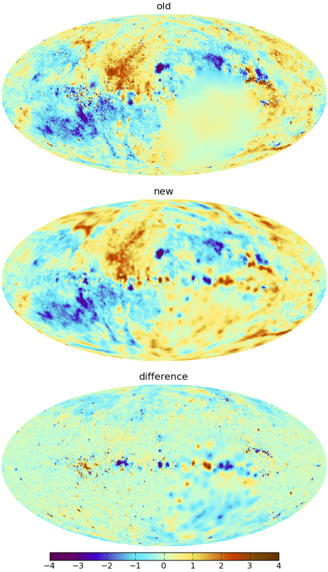
- Reconstruct (iteratively):
  signal, power spectrum, noise variance

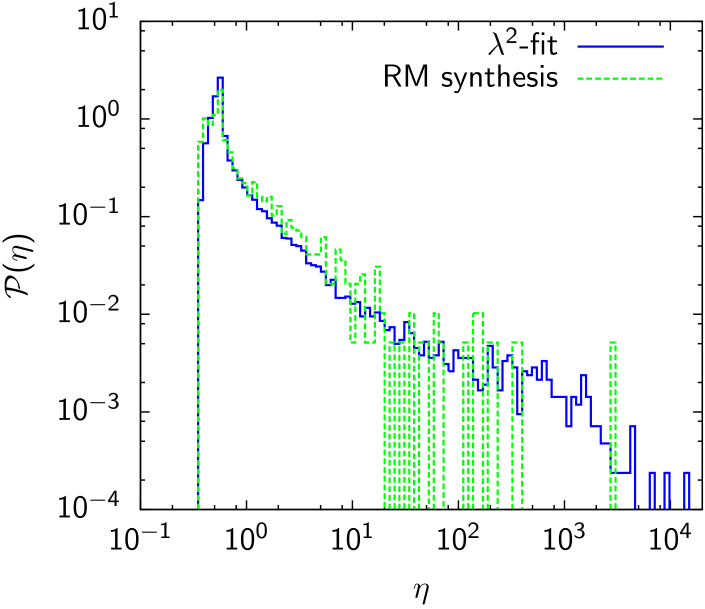


1D test case

- Reconstruct (iteratively):
  signal, power spectrum, noise variance




old
new
difference
−4
−3
−2
−1
0
1
2
3
4


λ^2-fit
RM synthesis
$\mathcal{P}(\eta)$
η
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}