A framework for performance forecasting of the parametric component separation in the presence of systematic effects

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***C.Vergès et al, 2019*, in prep.**

Context

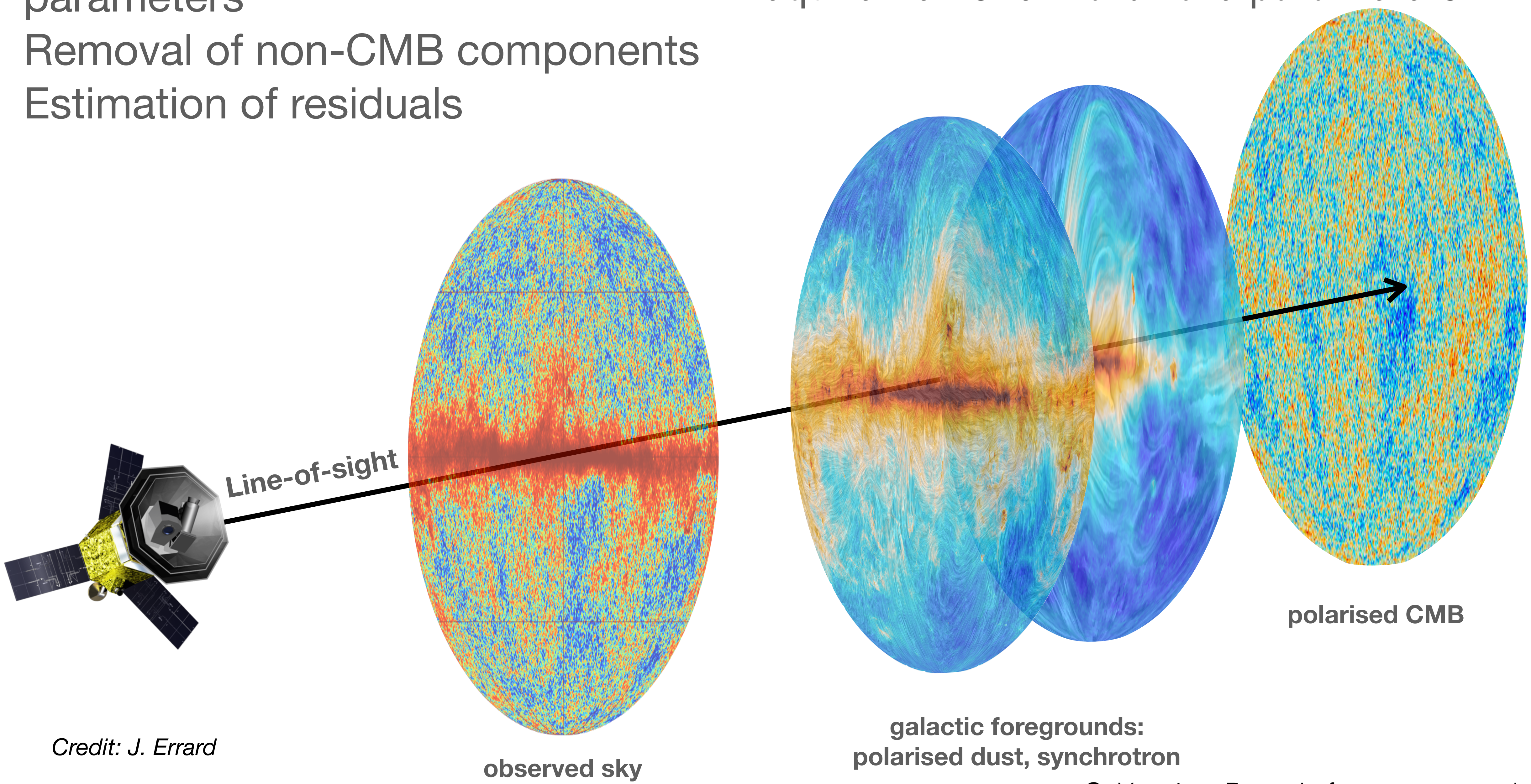
Component separation

Main steps

- Measurement in several frequency bands
- Estimation of components spectral parameters
- Removal of non-CMB components
- Estimation of residuals

⟶ How frequency-dependent instrumental effects affect component separation?

⟶ What are the calibration and precision requirements for hardware parameters?

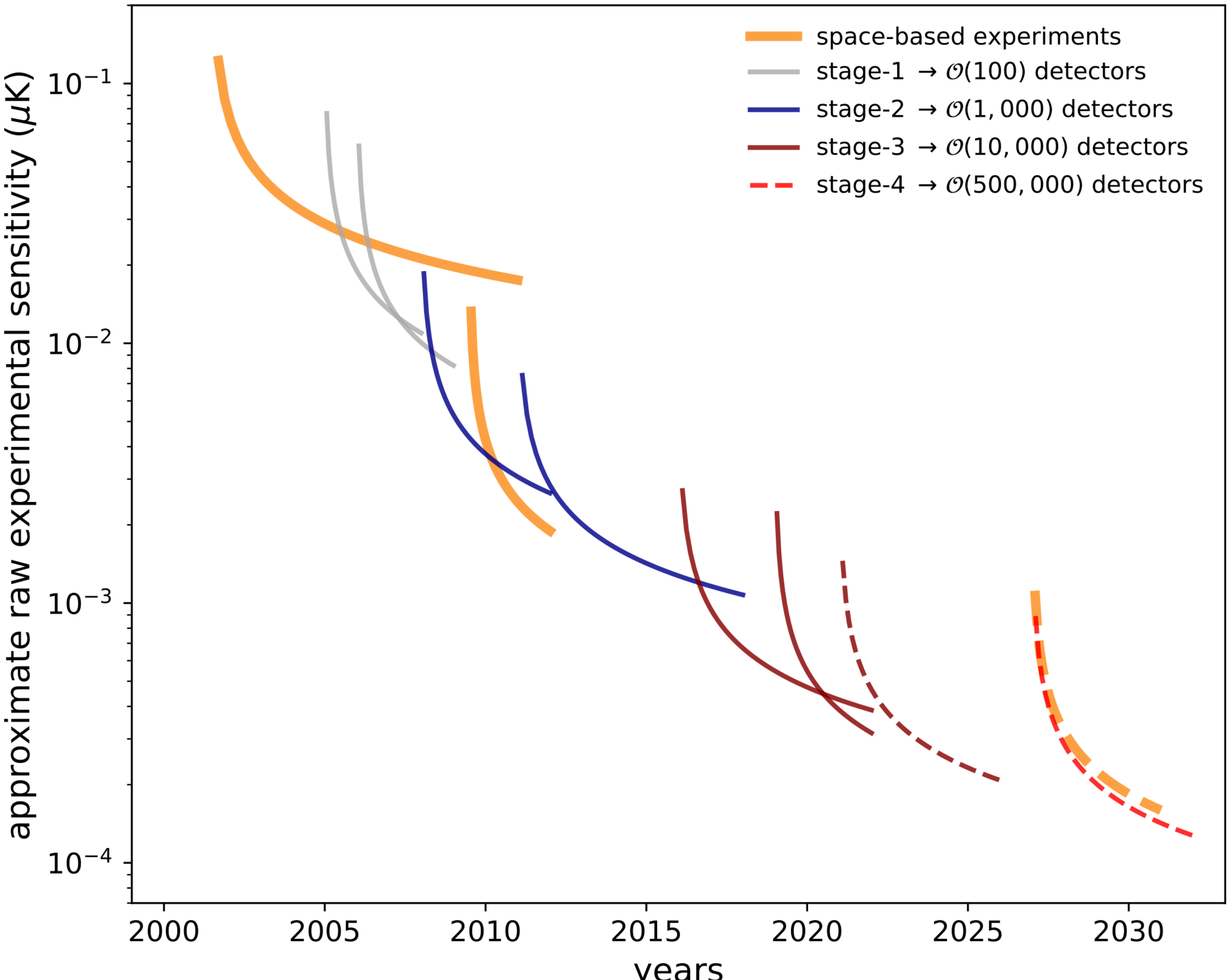


Credit: J. Errard

Context

New generation of CMB polarisation experiments

Increased sensitivity → increased complexity → need better mitigation of instrumental effects



Rotating HWP mitigates some systematics…

- long time scale effects (noise drifts)
- beam systematics
- bandpass mismatch

… but also introduces new ones

- frequency-dependent effects
- interplay with bandpasses

Updated by J. Errard

Framework

Instrument model

Monochromatic, single layer HWP

$$\mathbf{M}_{\text{layer}} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\delta & -\sin\delta \\ 0 & 0 & \sin\delta & \cos\delta \end{pmatrix}$$

$$\delta \equiv \frac{2\pi\theta_{\text{hwp}}\,|n_o - n_e|\nu}{c}$$

$$\mathbf{M}_{\text{monochromatic}} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Time domain data model

$$\mathbf{d}_t(\nu) = \mathrm{I}(\boldsymbol{\gamma}(t), \nu) + \cos(4\varphi_t)\,\mathrm{Q}(\boldsymbol{\gamma}(t), \nu) + \sin(4\varphi_t)\,\mathrm{U}(\boldsymbol{\gamma}(t), \nu)$$

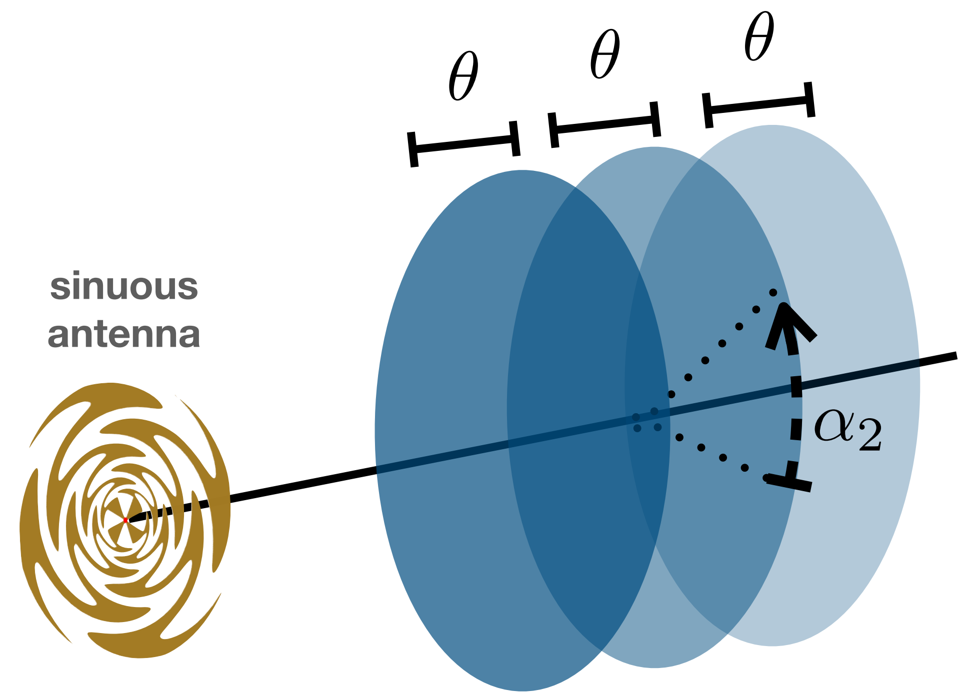
Map-making

cos(4ϕ) modulated term → Q
sin(4ϕ) modulated term → U

Broadband, multi layer HWP

$$\mathbf{M}_{\text{HWP}} = \mathbf{M}_{\text{layer}}(\theta)\mathbf{R}(-\alpha_2)\mathbf{M}_{\text{layer}}(\theta)\mathbf{R}(\alpha_2)\mathbf{M}_{\text{layer}}(\theta)$$

(e.g. *Bao et al, 2011, Komatsu et al, 2019*)



Sinuous antennas

Antenna with frequency dependent polarisation angle = **wobble angle** (*Suzuki, 2013*)

$$\mathbf{M} = \mathbf{M}_{\text{antenna}}\mathbf{R}(-2\varphi_t)\mathbf{M}_{\text{HWP}}\mathbf{R}(2\varphi_t)$$

$$\mathbf{d}_t(\nu) = \mathbf{M}_{00}(\nu)\,\mathrm{I}(\boldsymbol{\gamma}(t), \nu) + \mathbf{M}_{01}(\nu, \varphi_t)\,\mathrm{Q}(\boldsymbol{\gamma}(t), \nu) + \mathbf{M}_{02}(\nu, \varphi_t)\,\mathrm{U}(\boldsymbol{\gamma}(t), \nu)$$

Framework

Bandpass integration

Single layer HWP

$$\mathbf{M}_{\text{layer}} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\delta & -\sin\delta \\ 0 & 0 & \sin\delta & \cos\delta \end{pmatrix} \quad \delta \equiv \frac{2\pi\theta_{\text{hwp}}\,|n_o - n_e|\nu}{c}$$

Time domain data model

$$\mathbf{d}_t(\nu) = \int_{\nu_0}^{\nu_1} BP(\nu)\mathrm{I}(\boldsymbol{\gamma}(t),\nu) + \cos(4\varphi_t)\int_{\nu_0}^{\nu_1} BP(\nu)\, C_Q(\delta)\, \mathrm{Q}(\boldsymbol{\gamma}(t),\nu) + \sin(4\varphi_t)\int_{\nu_0}^{\nu_1} BP(\nu)\, C_U(\delta)\, \mathrm{U}(\boldsymbol{\gamma}(t),\nu)$$

+ sky only modulated terms

Multi-layer HWP + sinuous antennas

$$\mathbf{M} = \mathbf{M}_{\text{antenna}}\mathbf{R}(-2\varphi_t)\mathbf{M}_{\text{HWP}}\mathbf{R}(2\varphi_t)$$

Wobble parameters (amplitude and phase) → $\mathbf{M}_{\text{antenna}}$

HWP parameters (layer angle and thickness) → $\mathbf{M}_{\text{HWP}}$

$$\mathbf{d}_t(\nu) = \int_{\nu_0}^{\nu_1} BP(\nu)\mathrm{I}(\boldsymbol{\gamma}(t),\nu) + \cos(4\varphi_t)\int_{\nu_0}^{\nu_1} BP(\nu)\Big[C_Q(\delta)\,Q(\boldsymbol{\gamma}(t),\nu) + C_U(\delta)\,U(\boldsymbol{\gamma}(t),\nu)\Big] + \sin(4\varphi_t)\int_{\nu_0}^{\nu_1} BP(\nu)\Big[S_Q(\delta)\,Q(\boldsymbol{\gamma}(t),\nu) + S_U(\delta)\,U(\boldsymbol{\gamma}(t),\nu)\Big]$$

+ sky only modulated terms

→introduce leakage term/phase?

need to make assumption on the spectral and hardware parameters to define a phase, prior to component separation and map-making

→ potential bias on recovered frequency maps and parameters

→ can not account for spatial varying foregrounds parameters

Framework

Effective Stokes components

We rewrite the data model, based on modulation order…

$$
\begin{aligned}
\bar{\mathbf{d}}_t(\nu_c) \equiv\; & \mathbf{n}_t + \sum_{\substack{\text{comp=cmb,}\\ \text{dust,sync}}} \bar{\mathbf{M}}^{\text{comp}}_{00}(\nu_c,\nu_0)\,\mathrm{I}_{\text{comp}}(\gamma(t),\nu_0) \\
& + \sum_{\substack{\text{comp=cmb,}\\ \text{dust,sync}}} \left[\bar{\mathbf{C}}^{\text{comp}}_{01;\,0}(\nu_c,\nu_0)\,\mathrm{Q}_{\text{comp}}(\gamma(t),\nu_0) + \bar{\mathbf{C}}^{\text{comp}}_{02;\,0}(\nu_c,\nu_0)\,\mathrm{U}_{\text{comp}}(\gamma(t),\nu_0) \right] \times \cos 2\psi_t \\
& + \sum_{\substack{\text{comp=cmb,}\\ \text{dust,sync}}} \left[\bar{\mathbf{S}}^{\text{comp}}_{01;\,0}(\nu_c,\nu_0)\,\mathrm{Q}_{\text{comp}}(\gamma(t),\nu_0) + \bar{\mathbf{S}}^{\text{comp}}_{02;\,0}(\nu_c,\nu_0)\,\mathrm{U}_{\text{comp}}(\gamma(t),\nu_0) \right] \times \sin 2\psi_t \\
& + \sum_{\substack{\text{comp=cmb,}\\ \text{dust,sync}}} \left[\bar{\mathbf{C}}^{\text{comp}}_{01;\,4}(\nu_c,\nu_0)\,\mathrm{Q}_{\text{comp}}(\gamma(t),\nu_0) + \bar{\mathbf{C}}^{\text{comp}}_{02;\,4}(\nu_c,\nu_0)\,\mathrm{U}_{\text{comp}}(\gamma(t),\nu_0) \right] \times \cos(4\phi_t + 2\psi_t) \\
& + \sum_{\substack{\text{comp=cmb,}\\ \text{dust,sync}}} \left[\bar{\mathbf{S}}^{\text{comp}}_{01;\,4}(\nu_c,\nu_0)\,\mathrm{Q}_{\text{comp}}(\gamma(t),\nu_0) + \bar{\mathbf{S}}^{\text{comp}}_{02;\,4}(\nu_c,\nu_0)\,\mathrm{U}_{\text{comp}}(\gamma(t),\nu_0) \right] \times \sin(4\phi_t + 2\psi_t)
\end{aligned}
$$

Sky only modulated

HWP and sky modulated

The **HWP linearly mixes Stokes components**, and this mixing must be included in a **generalised component mixing matrix**

Framework

Effective Stokes components

… and define effective Stokes components

$$
\begin{aligned}
\bar{\mathbf{d}}_t(\nu_c) \equiv \mathbf{n}_t &+ \sum_{\substack{\mathrm{comp=cmb,}\\ \mathrm{dust,sync}}} \mathscr{I}(\gamma(t), \nu_c) \\
&+ \sum_{\substack{\mathrm{comp=cmb,}\\ \mathrm{dust,sync}}} \mathscr{C}_0(\gamma(t), \nu_c) \times \cos 2\psi_t \\
&+ \sum_{\substack{\mathrm{comp=cmb}\\ \mathrm{dust,sync}}} \mathscr{S}_0(\gamma(t), \nu_c) \times \sin 2\psi_t \\
&+ \sum_{\substack{\mathrm{comp=cmb,}\\ \mathrm{dust,sync}}} \mathscr{C}_4(\gamma(t), \nu_c) \times \cos(4\phi_t + 2\psi_t) \\
&+ \sum_{\substack{\mathrm{comp=cml}\\ \mathrm{dust,sync}}} \mathscr{S}_4(\gamma(t), \nu_c) \times \sin(4\phi_t + 2\psi_t)
\end{aligned}
$$

Sky only modulated

HWP and sky modulated

Linear combination of Q and U

Coefficients → depend on HWP, bandpasses, antennas and spectral parameters

Output of map-making are NOT Q and U maps

→ take this into account in the component separation process

Framework

Generalised data model

Standard approach

Multi frequency data set
Q and U maps
for each observed frequency

=

Standard
mixing
matrix

Single frequency maps
Q and U map
for each sky component

Parametrised by
spectral parameters

Generalisation

Multi frequency data set
$\mathcal{C}_0, \mathcal{S}_0, \mathcal{C}_4, \mathcal{S}_4$ integrated maps
for each observed frequency

=

Generalised
mixing
matrix

Single frequency maps
Q and U map
for each sky component

Parametrised by
spectral and hardware parameters

Framework

xTending xForecast

Errard et al, 2012, Stompor et al, 2016

Goal = Estimate residuals and *r* from foreground templates and a given instrumental configuration (frequency bands, noise levels)

Standard approach

- Only foregrounds are parametrised
- Parameters are foregrounds specific
- No Q and U mixing
- Average over noise realisations

Standard mixing matrix

$$A(\beta_d, T_d, \beta_s)$$

Generalisation

- Spectral parameters + **hardware parameters**
- Hardware parameters are **global**
- **CMB scaling** is parametrised
- Q and U are mixed into newly defined **effective Stokes components**
- **Priors** on hardware parameters
- **Parametric bandpass** integration
- Average over **noise + CMB realisations**

Generalised mixing matrix (single frequency)

$$\begin{bmatrix} \bar{\mathrm{C}}_{01;0} & \bar{\mathrm{C}}_{02;0} \\ \bar{\mathrm{S}}_{01;0} & \bar{\mathrm{S}}_{02;0} \\ \bar{\mathrm{C}}_{01;4} & \bar{\mathrm{C}}_{02;4} \\ \bar{\mathrm{S}}_{01;4} & \bar{\mathrm{S}}_{02;4} \end{bmatrix} A(\beta_d, T_d, \beta_s)$$

Framework

Pipeline

data model

$$\mathbf{d} = \mathscr{M}(\beta_d, T_d, \beta_s, \text{HWP, antenna, bandpasses})\,\mathbf{s}$$

$$= \{\mathscr{C}_0, \mathscr{S}_0, \mathscr{C}_4, \mathscr{S}_4\}$$

multi-frequency integrated component maps

we estimate sky components and hardware parameters on effective Stokes maps

Parameters estimation

$$\langle \mathcal{S}_{spec} \rangle = -\mathrm{tr} \sum_p \left\{ (\mathbf{N}_p^{-1} - \mathbf{P}_p)\left(\hat{\mathbf{d}}_p \hat{\mathbf{d}}_p^t + \mathbf{N}_p\right) \right\}$$

components and hardware parameters

Component separation

$$\hat{s} = (\mathscr{M} N^{-1} \mathscr{M})^{-1} \mathscr{M}^t N^{-1} \mathbf{d}$$

residuals → tensor-to-scalar ratio

$$C_\ell^{res} = C_\ell^{syst} + C_\ell^{stat}$$

mismatch between data and model (C_ℓ^{syst})

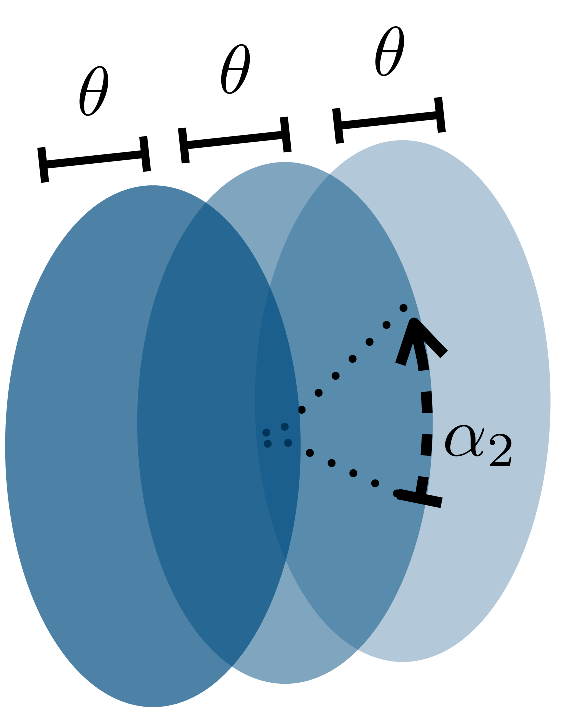
statistical uncertainties (C_ℓ^{stat})

Test case

As a test case, we model the hardware configuration based on the three Small Aperture Telescopes (SATs) of the Simons Observatory
(The Simons Observatory: Science Goals and Forecasts, 2018)

Assumed hardware configuration

- 6 frequency bands in 3 dichroic focal planes:
  {30 and 40 GHz}, {90 and 150 GHz}, {225 and 280 GHz}
- 3-layer achromatic HWP parameters
  - Angle of the central layer
  - Thickness of the layer
- Sinuous antennas

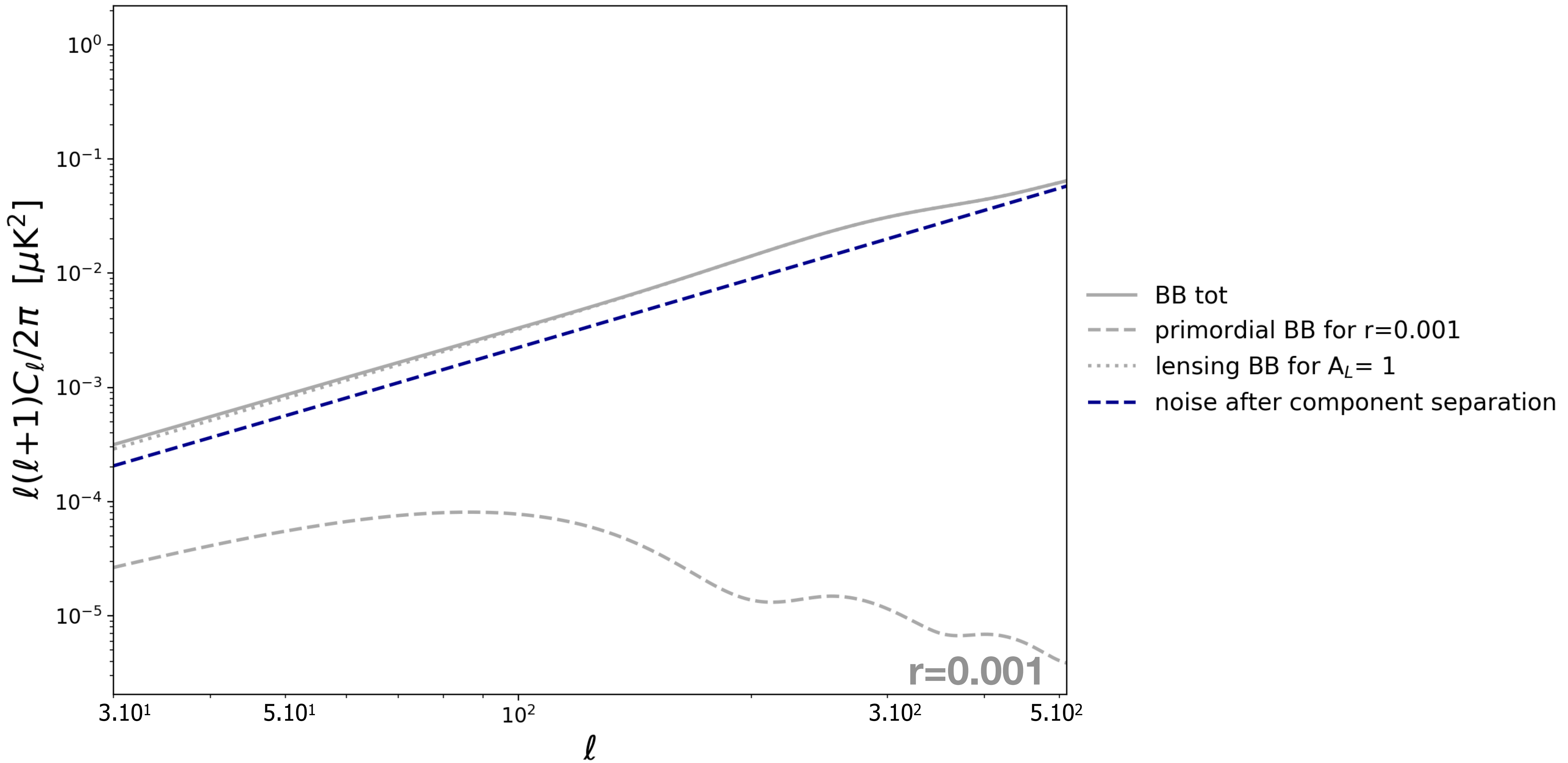


multi-layer HWP

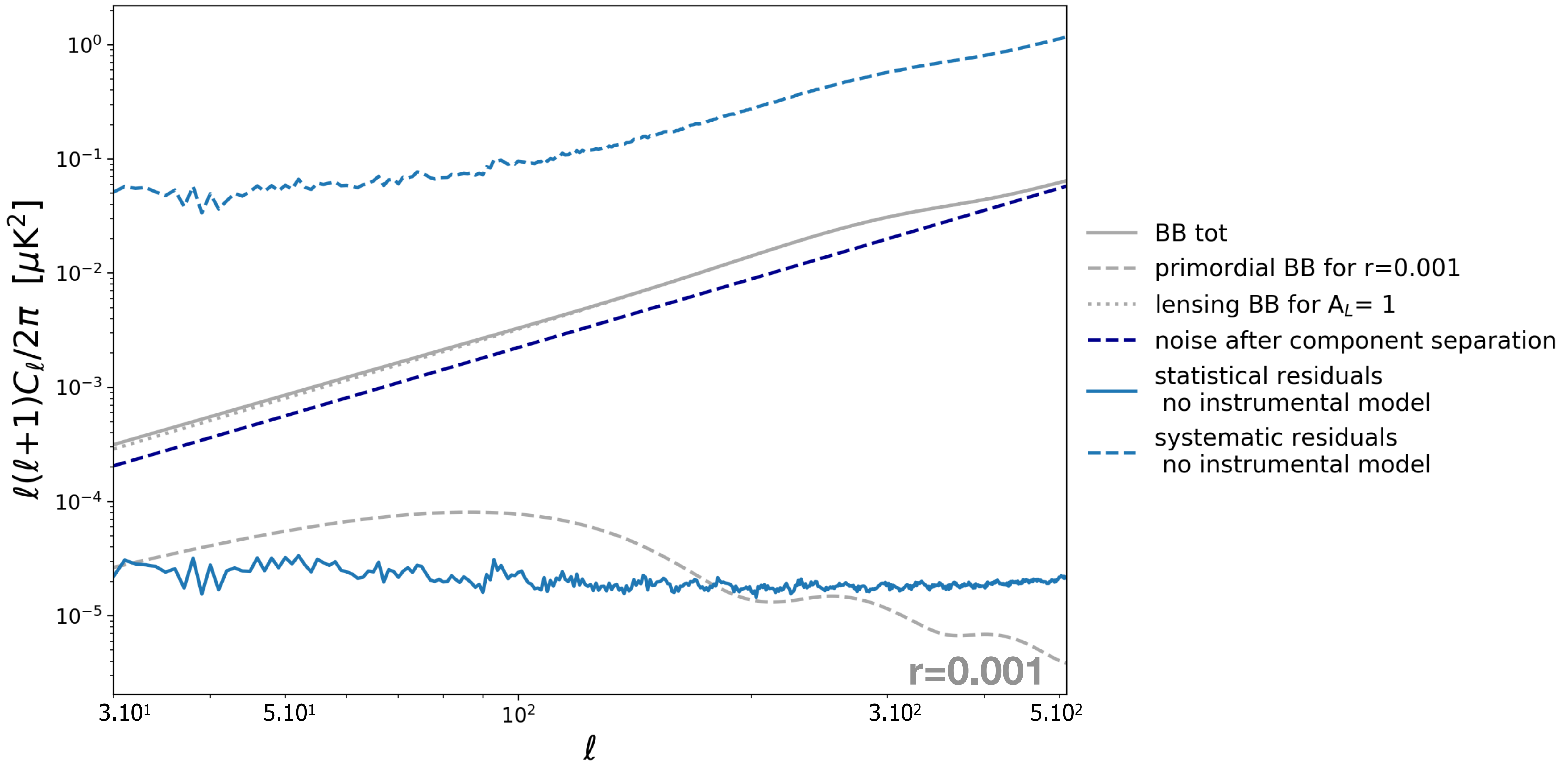
Other assumptions

- T_dust = 19.6K
- White noise only
- Perfectly known bandpasses
- Perfectly known wobble

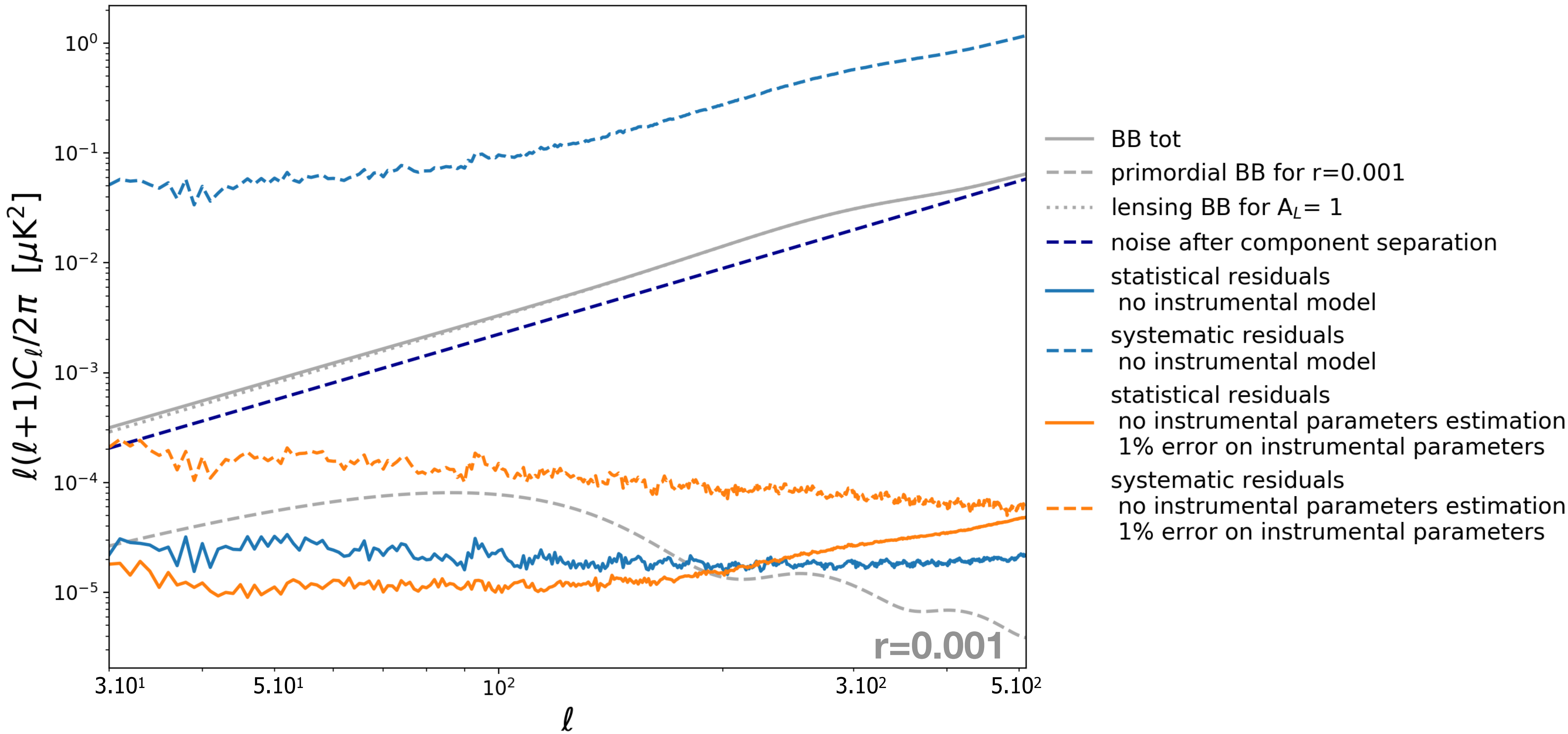
Example of results



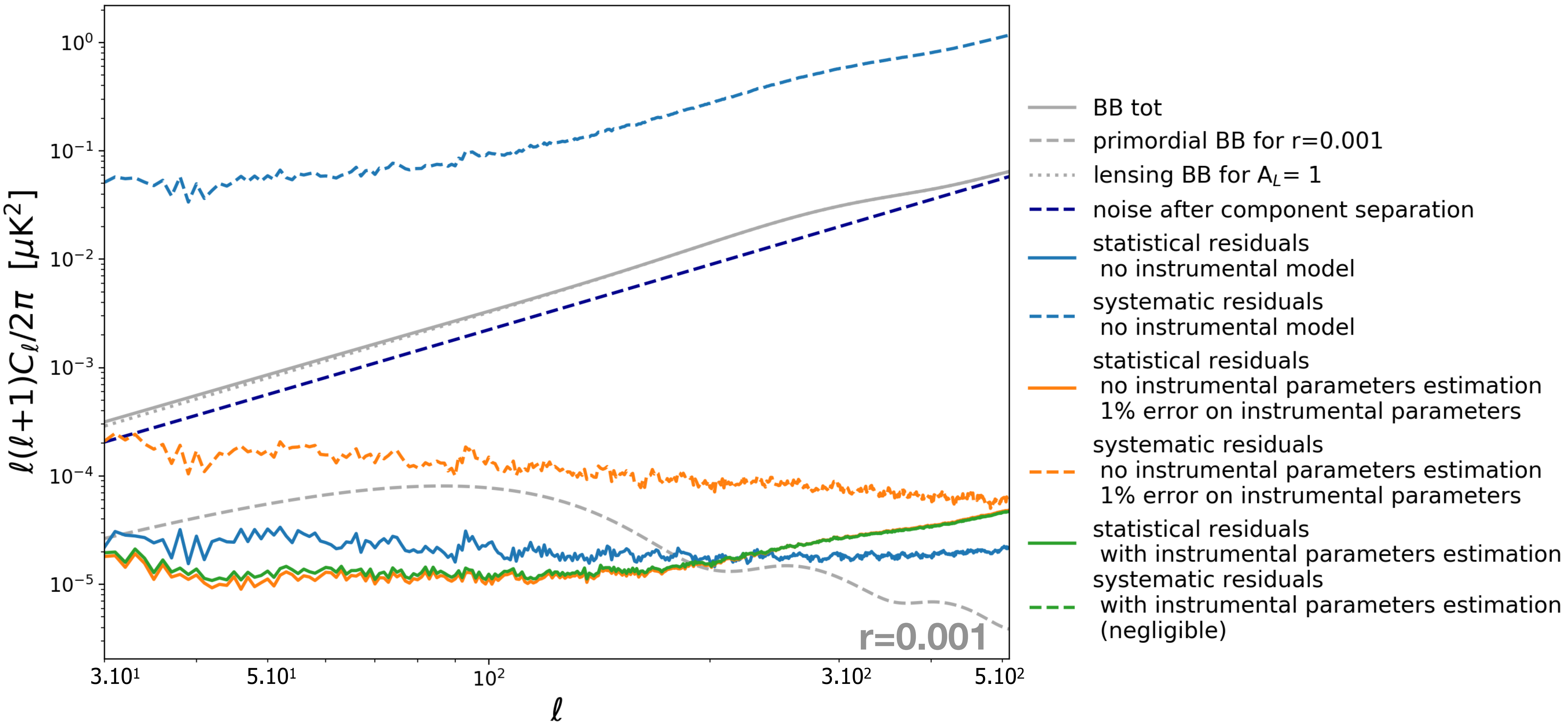
Example of results

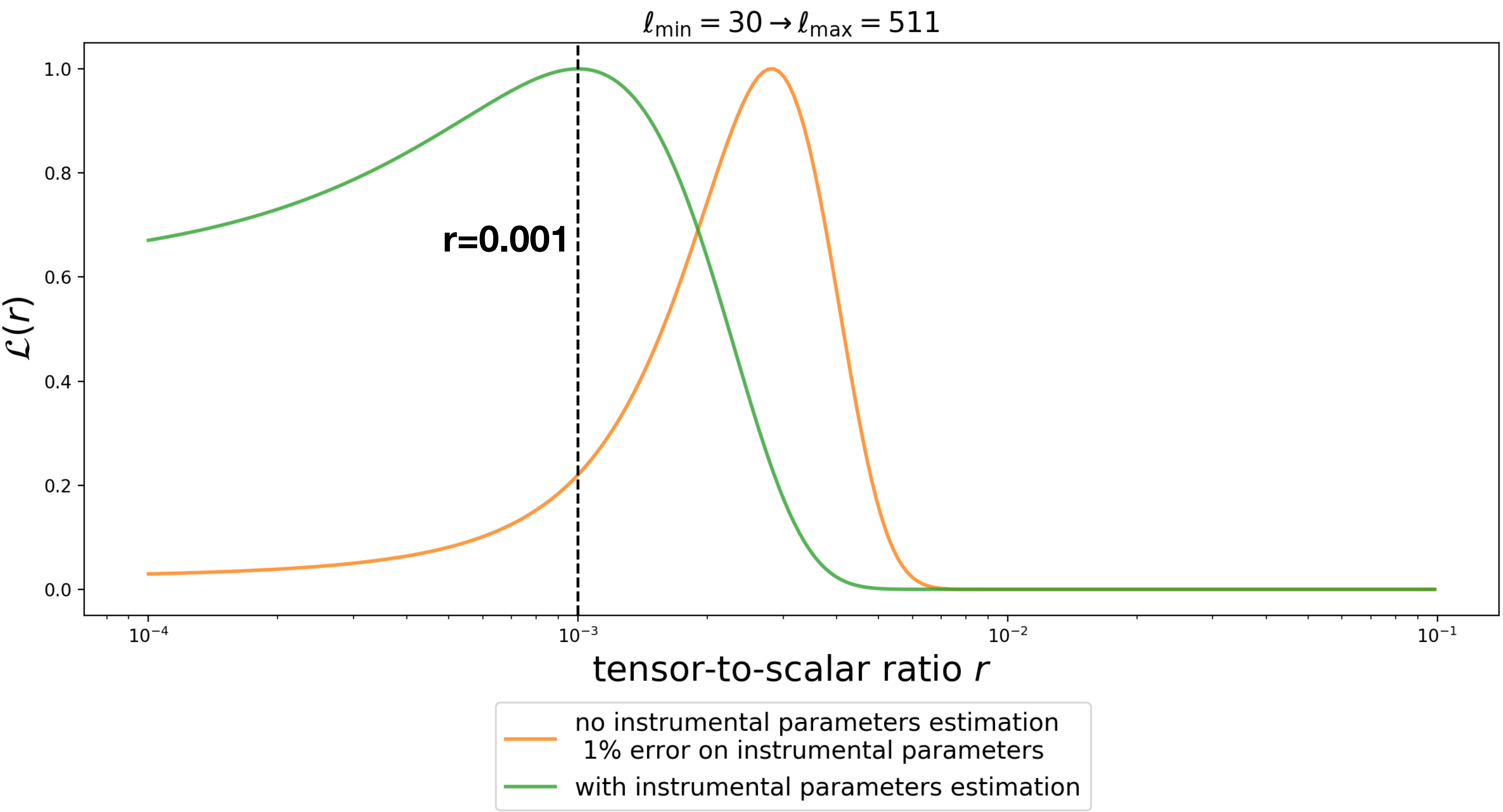


Example of results



Example of results





We marginalise over **HWP parameters**
We can correctly estimate these parameters with **no significant bias on *r* with the targeted precision**, using the generalised mixing matrix formalism.
At this stage, we **don't need to introduce any priors** on hardware parameters (self-calibration)

Conclusion & future steps

Need for accurate instrument model and accurate measurements of instrumental parameters

- Avoid bias in the component separation process
- As important as foreground model

I developed, implemented and demonstrated an end-to-end component separation framework incorporating instrumental effects

- Generalised data model including parametric bandpasses
- Component separation techniques accounting for systematic effects
- Flexible framework than can accommodate more complex models (for HWP or other elements)

To do list

- Go beyond the assumption that bandpasses and wobble are known, and estimate these parameters as well
- Introduce priors and determine the precision we need for calibration
- Study the performance of the method for realistic calibration strategies
- Implement the framework for use in publicly available component separation codes

… and more exciting features to come! Stay tuned for the paper ;)