Primordial Gravitational Waves: Theoretical Expectations

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How predictive is Cosmic Inflation?

VERY predictive, unless....

THEORY OF COSMOLOGICAL

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V.F. M

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NBERGER[a]

$$S = \int [\tfrac{1}{2}\chi_{;\mu}\chi^{;\mu} - V(\chi) + \tfrac{1}{2}\varphi_{;\mu}\varphi^{;\mu} - \tfrac{1}{2}m_0^2\varphi^2 - V_1(\chi, \varphi, \ldots)]\sqrt{-g}\, \mathrm{d}^4x ,$$

Note that the effect discussed in this section can arise for both adiabatic and entropy perturbations. it is possible to obtain a suppression of the long-wavelength part of cosmological perturbation nontrivial spectra with mountains and valleys can also be obtained [redacted] It is also possible to generate non-Gaussian fluctuations [redacted]

However, this procedure is extremely unappealing since it implies a complete loss of predictability. [redacted]

Inflation is THE theory only when it is understood as the stage of unbroken accelerated expansion due to the same ingridient which is responsible for quantum fluctuations.

Otherwise it is rubbish without any predictions!!!

In this case it is unbeatable as predictive theory because it allows us to calculated the effect of amplification of quantum fluctuations in completely controlable weak coupling regimes

while most alternatives cannot even compete with "rubbish inflation" in a sense of controlable reproduction of outcome for quantum fluctuations

COSMOLOGY - Theology = $\exp(Ht)$

during at least 70 H^{-1}, but less than 10^6 H^{-1} $\rightarrow$

no any problems with predictions, which could falsify the theory in Popper's sense

Inflation is not a unique theory, but rather a class of models based on similar principles.

WRONG!

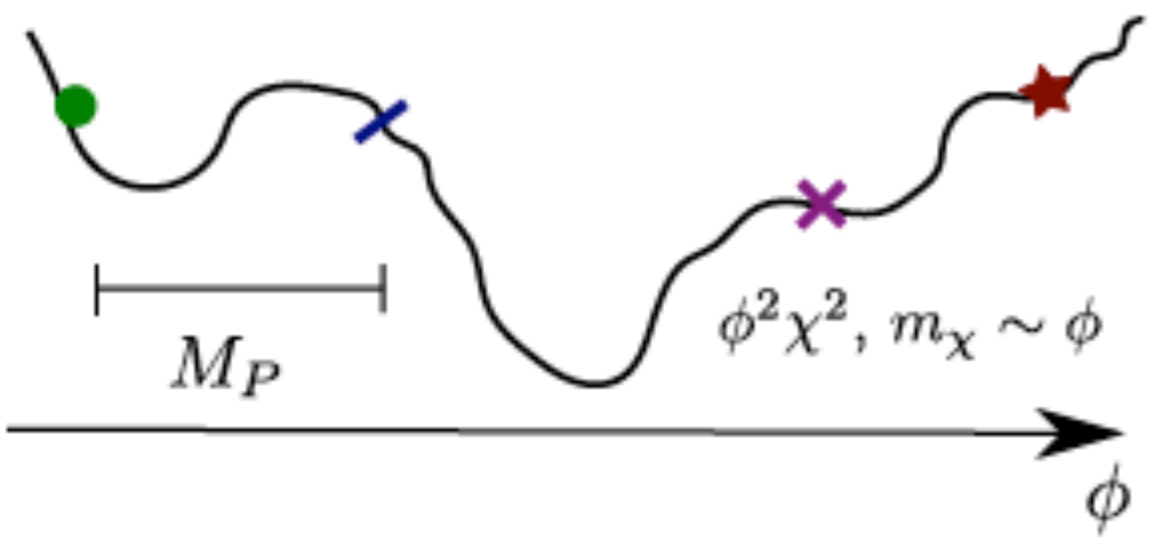
The only purpose of inflationary models relevant for observation is a maping

$$V(\varphi) \quad \text{to} \quad p \approx -\varepsilon$$

and this maping happened to be not crucial for robust predictions but important only for excluding definite potentials $V(\varphi)$, which anyway we will never be able to verify in any other independent experiments

$$V(\tau,\theta) = \frac{12W_0^2\xi}{(4\mathcal{V}_m - \xi)(2\mathcal{V}_m + \xi)^2} + \frac{D_1 + 12e^{-2a_2\tau}\xi A_2^2}{(4\mathcal{V}_m - \xi)(2\mathcal{V}_m + \xi)^2} + \frac{D_2 + \frac{16(a_2A_2)^2}{3\alpha\lambda_2}\sqrt{\tau}e^{-2a_2\tau}}{(2\mathcal{V}_m + \xi)} \tag{25}$$

$$+\frac{D_3 + 32e^{-2a_2\tau}a_2A_2^2\tau(1 + a_2\tau)}{(4\mathcal{V}_m - \xi)(2\mathcal{V}_m + \xi)} + \frac{D_4 + 8W_0A_2e^{-a_2\tau}\cos(a_2\theta)}{(4\mathcal{V}_m - \xi)(2\mathcal{V}_m + \xi)}\left(\frac{3\xi}{(2\mathcal{V}_m + \xi)} + 4a_2\tau\right) + \frac{\beta}{\mathcal{V}_m^2}.$$



What is relevant for predictions?

$-\varepsilon$ energy density

$-p$ pressure

$$1+w \equiv \frac{\varepsilon+p}{\varepsilon} \ll 1$$

during last 70 e-folds $(a = a_f \cdot e^{-N})$

$a)$ $1+w \ll 1$ for $N \gg 1$

$b)$ $1+w \approx O(1)$ for $N \simeq O(1)$

$c)$ $1+w$ is a smooth function of N

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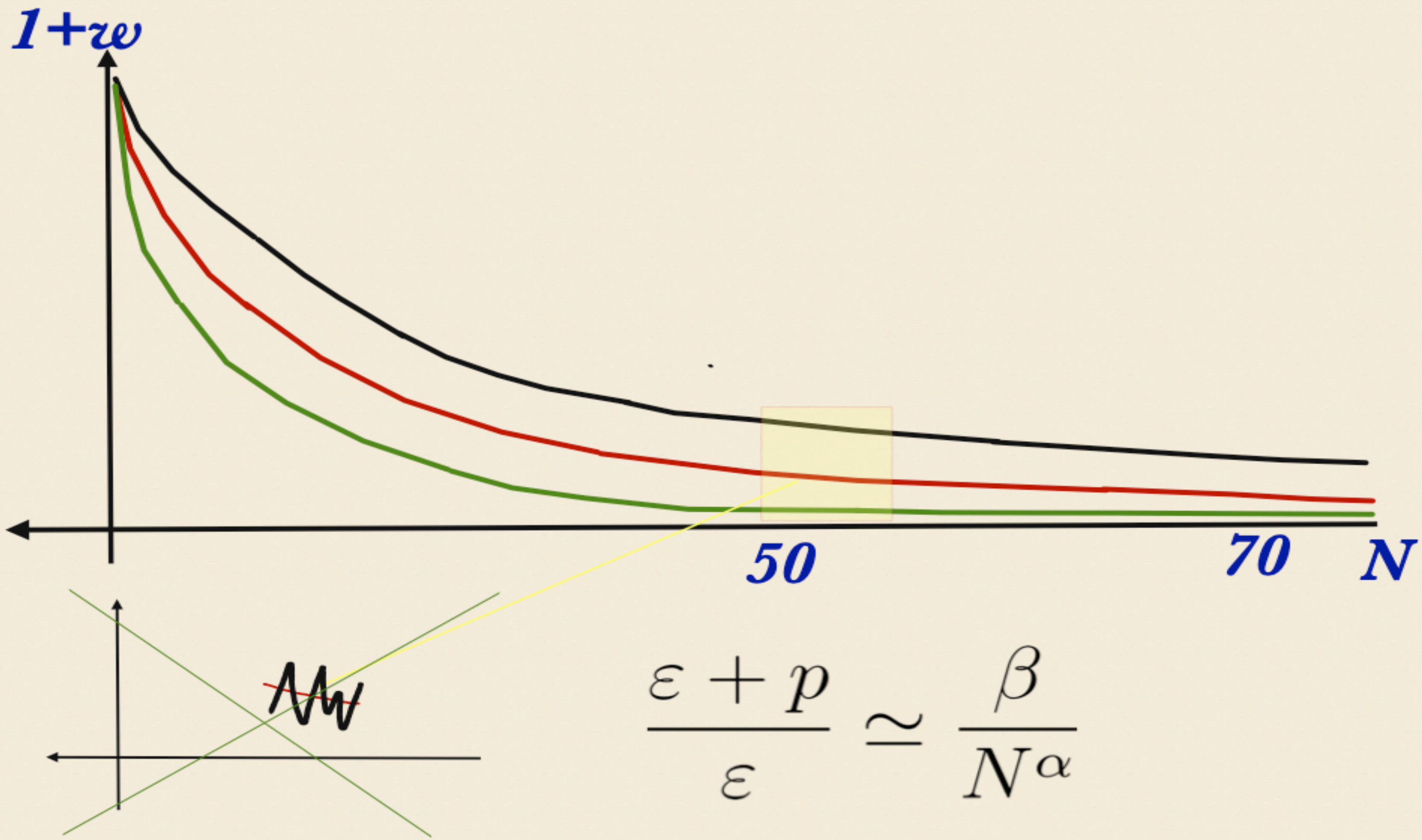


$$\frac{\varepsilon + p}{\varepsilon} \simeq \frac{\beta}{N^{\alpha}}$$

PREDICTIONS

("smoking guns"-nonconfirming any of them would falsify THE theory)

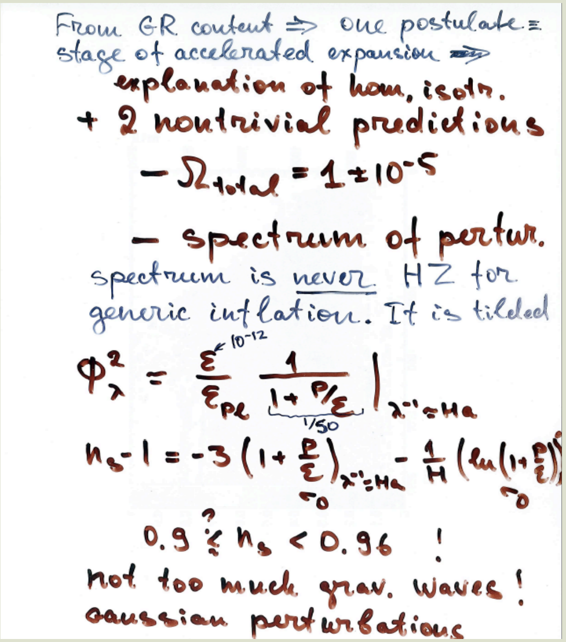
- flat universe
- adiabatic perturbations
- small non-gaussianity ($f_{NL} \sim O(1)$)
- red-tilted spectrum

$$\Phi^2 \propto \lambda^{1-n_S}$$

$$1-n_S = 3(1+w) - \frac{d\ln(1+w)}{dN} = \frac{3\beta}{N^\alpha} + \frac{\alpha}{N}$$

of the faint ripples that we detect in the cosmic microwave background (CMB). First, the ripples should be nearly scale-invariant, meaning that they have nearly the same intensity at

The theory always predicts red-tilted spectrum



Cambridge, 2000

[1]. Contrary to an erroneous belief inflation does not predict the scale-invariant, Harrison-Zel'dovich spectrum. The spectral index should be in the range of $0.92 < n_s < 0.97$. The physical

V. Mukhanov, CMB, Quantum Fluctuations
and the Predictive Power of Inflation,
arXiv:astro-ph/0303077 (2003)

Red-tilted log spectrum (MC, H, 1981-1982) $\rightarrow$

$$n_S = 1 - \frac{A}{\ln(B\lambda_{gal} / \lambda_{CMB})},$$

where $A > 1{,}5$ and $B \simeq 1 - 100$ depending on $50 < N < 55 \rightarrow$

$$n_S < 0.97$$

irrespective of any particular model!

L.P. 9/6/2003:

We are writing a proposal to get money to do our small angular scale CMB experiment. If I say that simple models of inflation require n_s=0.95+/-0.03 (95\% cl) is it correct?

I'm especially interested in the error. Specifically, if n_s=0.99 would you throw in the towel on inflation?

V.M. 9/8/2003

The "robust" estimate for spectral index for inflation is 0.92<n_s<0.97.

The upper bound is more robust than lower. The physical reason for the deviation of spectrum from the flat one is the nessesity to finish inflation....

If you find n_s=0.99 +/- 0.01 (3 sigma) I would throw in the towel on inflation.

The unavodable uncertainty in B is bad news for "model bilders"!

It leads to theoretical uncertainty in prediction of n_s of order 0.005 for any model of inflaton and hence further increasing of expermental accuracy in n_s will not no help us much in model selecton

Did the current CMB measurements proved that gravitational field is quantized?

Yes!

Scalar perturbations For scalar perturbations the metric takes the form

$$ds^2 = a^2 \left[(1 + 2\hat{\phi})\, d\eta^2 + 2\hat{B}_{,i} dx^i d\eta - \left((1 - 2\hat{\psi})\, \delta_{ij} - 2\hat{E}_{,ij}\right) dx^i dx^j\right].$$

$$\hat{\delta\varepsilon}$$

$$5-2=3$$

↑

coordinate modes

Further predictions ("non-smoking guns"):

- Primordal gravitational waves (*getting smoking*)
- Nongaussianities due to nonlinearity of

Einstein equation (3,4,...points correlaton functions)

There must be primordial gravitatonal waves

$$r \equiv \frac{T}{S} = 24 \cdot (1 + p / \varepsilon) = \frac{\beta}{N^{\alpha}}$$

No a priori low bound on their ampltude!

But when n_s is measured we have lower bound on r