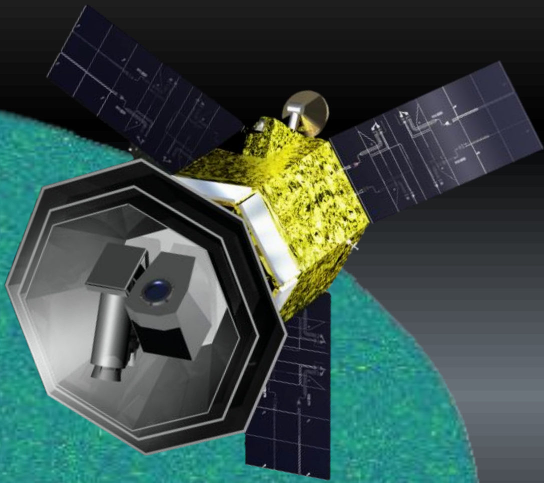
Simultaneous determination of the cosmic birefringence and miscalibrated polarisation angles from CMB experiments

DOI: 10.1093/ptep/ptz079



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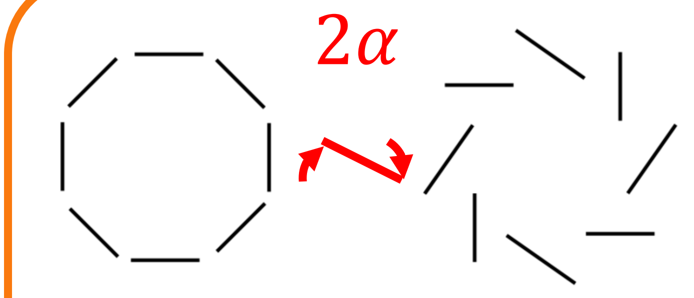
Core-to-Core Program

Introduction:



- Miscalibration of detector rotation angle (α) creates spurious B−mode from E−mode

$$C_\ell^{BB,o} = C_\ell^{EE}\sin(2\alpha) + C_\ell^{BB}cos(2\alpha) \quad \cdots (1)$$

observed before detectors

We need to determine α to calibrate rotation angle

- In past experiments, this α was calculated assuming that EB correlation of CMB is zero:

$$C_\ell^{EB,o} = \frac{1}{2}\left(C_\ell^{EE,CMB} - C_\ell^{BB,CMB}\right)\sin(4\alpha) \quad \cdots (2)$$

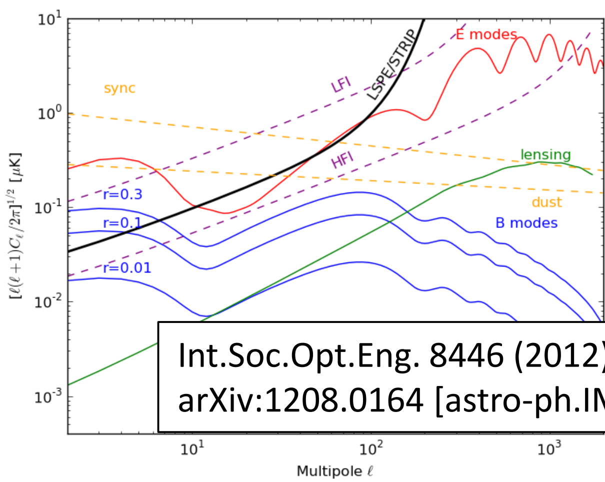
From theory

"Self-calibration" by Brian G. Keating et al.（2013）

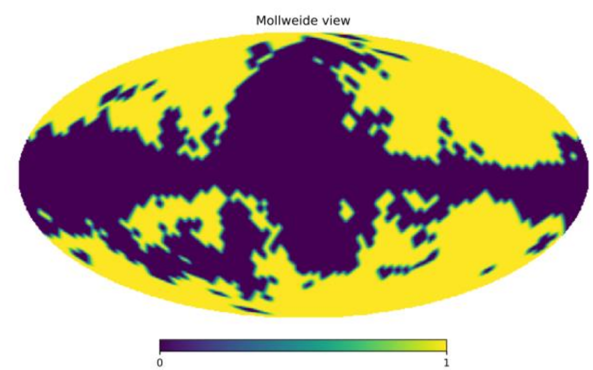
Some issues in “self-calibration”

CMB should be dominant

- Because foreground signals also have α,
  - We need to know the foreground model
  - We need to mask the Galactic plane



Int.Soc.Opt.Eng. 8446 (2012) 84467C
arXiv:1208.0164 [astro-ph.IM]



Cosmological *EB* correlation should be zero

- We lose sensitivity to cosmic birefringence

To solve these issues

We relate observed *E*- and *B*- modes to the intrinsic ones as

$$E^{o}_{\ell,m} = E_{\ell,m}\cos(2\alpha) - B_{\ell,m}\sin(2\alpha)$$
$$B^{o}_{\ell,m} = E_{\ell,m}\sin(2\alpha) + B_{\ell,m}\cos(2\alpha) \quad \cdots (3)$$

From these equations, we find

$$C_{\ell}^{EB,o} = \frac{1}{2}\left(C_{\ell}^{EE,o} - C_{\ell}^{BB,o}\right)\tan(4\alpha) + \frac{C_{\ell}^{EB}}{\cos(4\alpha)} \quad \cdots (4)$$

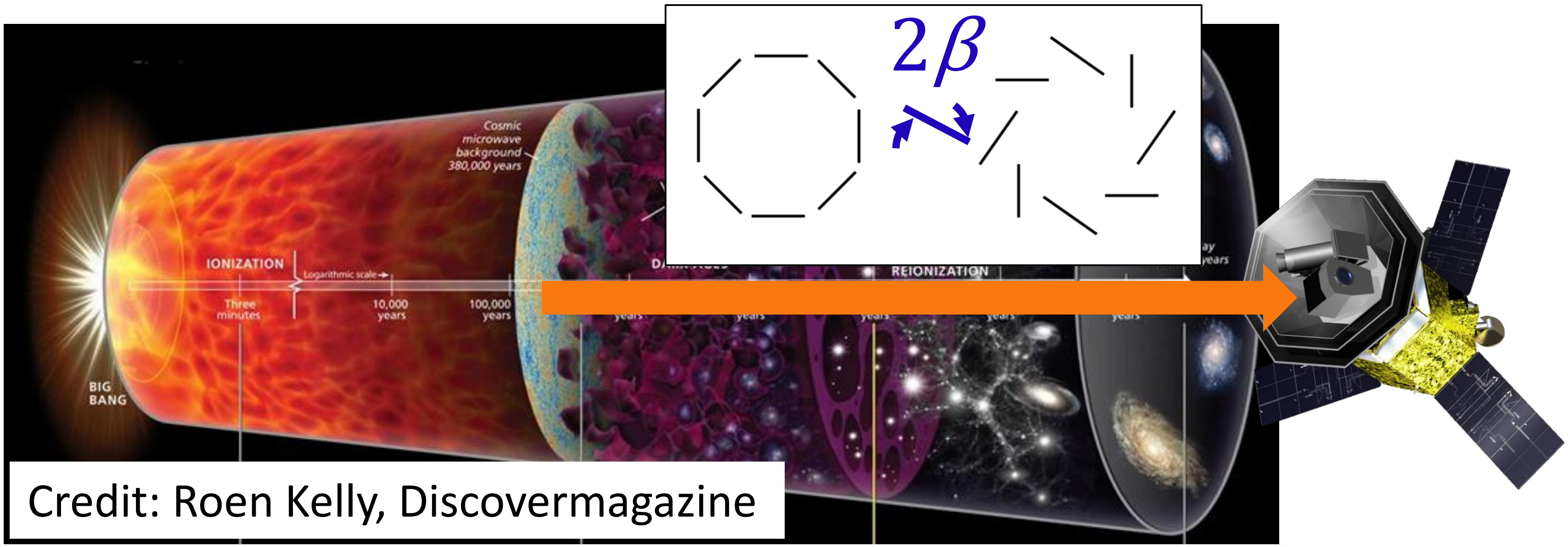
G. B. Zhao et al.（2015）

Our work

- ➢ We can estimate α with only observed data
- ➢ If we assume theory CMB power spectra, we can estimate an additional angle!

Cosmic birefringence

During the long travel from the recombination era to Earth, CMB can be rotated by some physics (e.g. axionic fields)



Credit: Roen Kelly, Discovermagazine

In this case,

- Foreground term: rotated only by α
- CMB term: rotated by $\alpha + \beta$

Equations including birefringence rotation:

The coefficients become

$$E^{\rm o}_{\ell,m} = E^{\rm fg}_{\ell,m}\cos(2\alpha) - B^{\rm fg}_{\ell,m}\sin(2\alpha) + E^{\rm CMB}_{\ell,m}\cos(2\alpha+2\beta) - B^{\rm CMB}_{\ell,m}\sin(2\alpha+2\beta) + E^{\rm N}_{\ell,m}$$

$$B^{\rm o}_{\ell,m} = E^{\rm fg}_{\ell,m}\sin(2\alpha) + B^{\rm fg}_{\ell,m}\cos(2\alpha) + E^{\rm CMB}_{\ell,m}\sin(2\alpha+2\beta) + B^{\rm CMB}_{\ell,m}\cos(2\alpha+2\beta) + B^{\rm N}_{\ell,m}$$

$\cdots (5)$

From them, we derived

$$\begin{aligned}
&\left(C_\ell^{EB,\rm o} - \frac{\tan(4\alpha)}{2}\left(C_\ell^{EE,\rm o} - C_\ell^{BB,\rm o}\right)\right)\cos(4\alpha) = \\
&\quad (C_\ell^{EE,\rm CMB} - C_\ell^{BB,\rm CMB})\sin(4\beta)/2 \\
&\quad - (C_\ell^{EE,\rm N} - C_\ell^{BB,\rm N})\sin(4\alpha)/2 \\
&\quad + C_\ell^{EB,\rm fg} + C_\ell^{EB,\rm N}\cos(4\alpha) + C_\ell^{EB,\rm CMB}\cos(4\beta) \\
&\quad + (C_\ell^{E^{\rm fg}B^{\rm CMB}} + C_\ell^{E^{\rm CMB}B^{\rm fg}})\cos(2\beta) + (C_\ell^{E^{\rm fg}E^{\rm CMB}} - C_\ell^{B^{\rm fg}B^{\rm CMB}})\sin(2\beta) \\
&\quad + (C_\ell^{E^{\rm fg}B^{\rm N}} + C_\ell^{B^{\rm fg}E^{\rm N}})\cos(2\alpha) - (C_\ell^{E^{\rm fg}E^{\rm N}} - C_\ell^{B^{\rm fg}B^{\rm N}})\sin(2\alpha) \\
&\quad + (C_\ell^{E^{\rm CMB}B^{\rm N}} + C_\ell^{B^{\rm CMB}E^{\rm N}})\cos(2\alpha - 2\beta) \\
&\quad - (C_\ell^{E^{\rm CMB}E^{\rm N}} - C_\ell^{B^{\rm CMB}B^{\rm N}})\sin(2\alpha - 2\beta)\,.
\end{aligned}$$

$\cdots (6)$

Equations including birefringence rotation:

If we take an ensemble average

$$\langle C_\ell^{EB,\mathrm{o}}\rangle = \frac{\tan(4\alpha)}{2}\left(\langle C_\ell^{EE,\mathrm{o}}\rangle - \langle C_\ell^{BB,\mathrm{o}}\rangle\right) + \frac{\sin(4\beta)}{2\cos(4\alpha)}\left(\langle C_\ell^{EE,\mathrm{CMB}}\rangle - \langle C_\ell^{BB,\mathrm{CMB}}\rangle\right) \quad \cdots (7)$$

$$+ \frac{1}{\cos(4\alpha)}\langle C_\ell^{EB,\mathrm{fg}}\rangle + \frac{\cos(4\beta)}{\cos(4\alpha)}\langle C_\ell^{EB,\mathrm{CMB}}\rangle\,.$$

Assume these to be zero

Therefore, we can determine both miscalibration and birefringence-rotation angles simultaneously!

Construct a likelihood for determination of α and β

If we take an ensemble average

$$\langle C_\ell^{EB,\mathrm{o}}\rangle = \frac{\tan(4\alpha)}{2}\left(\langle C_\ell^{EE,\mathrm{o}}\rangle - \langle C_\ell^{BB,\mathrm{o}}\rangle\right) + \frac{\sin(4\beta)}{2\cos(4\alpha)}\left(\langle C_\ell^{EE,\mathrm{CMB}}\rangle - \langle C_\ell^{BB,\mathrm{CMB}}\rangle\right) \quad \cdots (7)$$

$$+ \frac{1}{\cos(4\alpha)}\langle C_\ell^{EB,\mathrm{fg}}\rangle + \frac{\cos(4\beta)}{\cos(4\alpha)}\langle C_\ell^{EB,\mathrm{CMB}}\rangle .$$

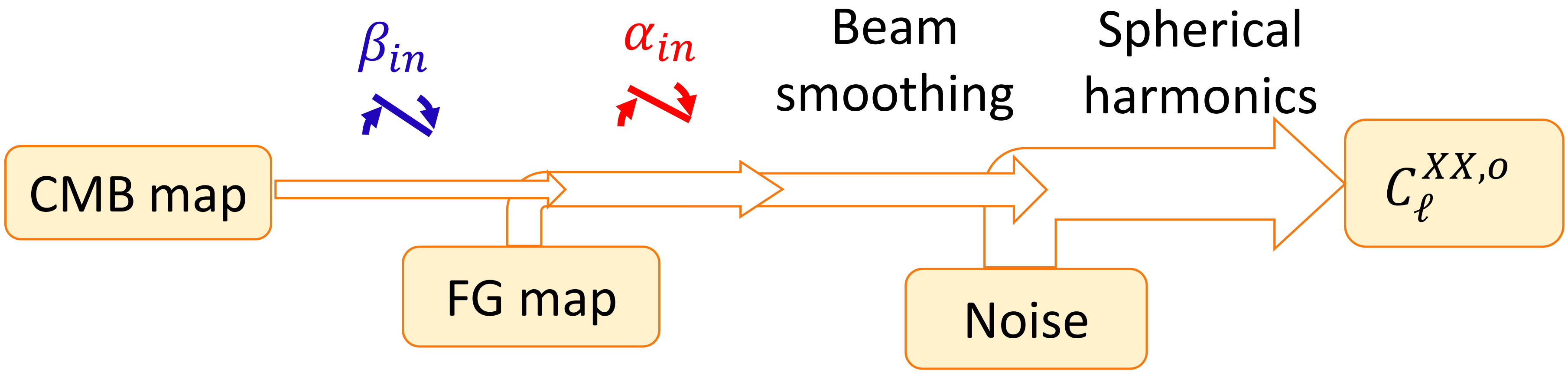
Assume these to be zero

$$-2\ln\mathcal{L} = \sum_{\ell=2}^{\ell_{\max}} \frac{\left[C_\ell^{EB,\mathrm{o}} - \frac{\tan(4\alpha)}{2}\left(C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}}\right) - \frac{\sin(4\beta)}{2\cos(4\alpha)}\left(C_\ell^{EE,\mathrm{CMB}} - C_\ell^{BB,\mathrm{CMB}}\right)\right]^2}{\mathrm{Var}\left(C_\ell^{EB,\mathrm{o}} - \frac{\tan(4\alpha)}{2}\left(C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}}\right)\right)} \quad \cdots (8)$$

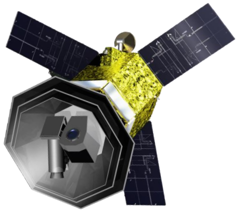
Minimise $-2\ln\mathcal{L}$ to determine α and β

Sky simulation setup for the validation

- Components
  - Thermal dust: modified black body
  - Synchrotron: simple power law
  - CMB: tensor-to-scalar ratio r = 0
  - Noise: white noise with LiteBIRD polarisation sensitivity
- N_{side} is 512 and l_{max} is 1024
- Compute EB power spectra from full-sky maps



Sky simulation setup for the validation: LiteBIRD



We extract representative 4 frequencies to show how the method works

	Frequency	polarisation sensitivity (uK')	Beam size in FWHM (arcmin)
Synchrotron	50	24.0	48
CMB	119	7.6	25
Dust + CMB	195	5.8	20
Dust	235	7.7	19

LiteBIRD parameters extracted from M. Hazumi et al., J. Low Temp. Phys. 194, 443 (2019).

Variance

- With full-sky power spectra (not cut-sky pseudo power spectra), we can calculate variance exactly as

$$\begin{aligned}
&\mathrm{Var}\left[C_\ell^{EB,\mathrm{o}} - (C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}})\tan(4\alpha)/2\right] \\
&= \left\langle\left[C_\ell^{EB,\mathrm{o}} - (C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}})\tan(4\alpha)/2\right]^2\right\rangle - \langle C_\ell^{EB,\mathrm{o}} - (C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}})\tan(4\alpha)/2\rangle^2 \\
&= \frac{1}{2\ell+1}\langle C_\ell^{EE}\rangle\langle C_\ell^{BB}\rangle + \frac{\tan^2(4\alpha)}{4}\frac{2}{2\ell+1}\left(\langle C_\ell^{EE}\rangle^2 + \langle C_\ell^{BB}\rangle^2\right) \\
&\quad - \tan(4\alpha)\frac{2}{2\ell+1}\langle C_\ell^{EB}\rangle\left(\langle C_\ell^{EE}\rangle - \langle C_\ell^{BB}\rangle\right) + \boxed{\frac{1}{2\ell+1}\left(1-\tan^2(4\alpha)\right)\langle C_\ell^{EB}\rangle^2.}
\end{aligned} \quad \cdots (9)$$

$$= 0$$

- We approximate $\langle C_\ell^{XY}\rangle \approx C_\ell^{XY,o}$
- We ignore $\langle C_\ell^{EB}\rangle^2$ term because it's small and yields bias
  - Even if $\langle C_\ell^{EB}\rangle \approx 0$, $C_\ell^{EB,o}$ has a small non-zero value with fluctuation, and ${C_\ell^{EB,o}}^2$ yields bias

Before the simultaneous determination: α only case

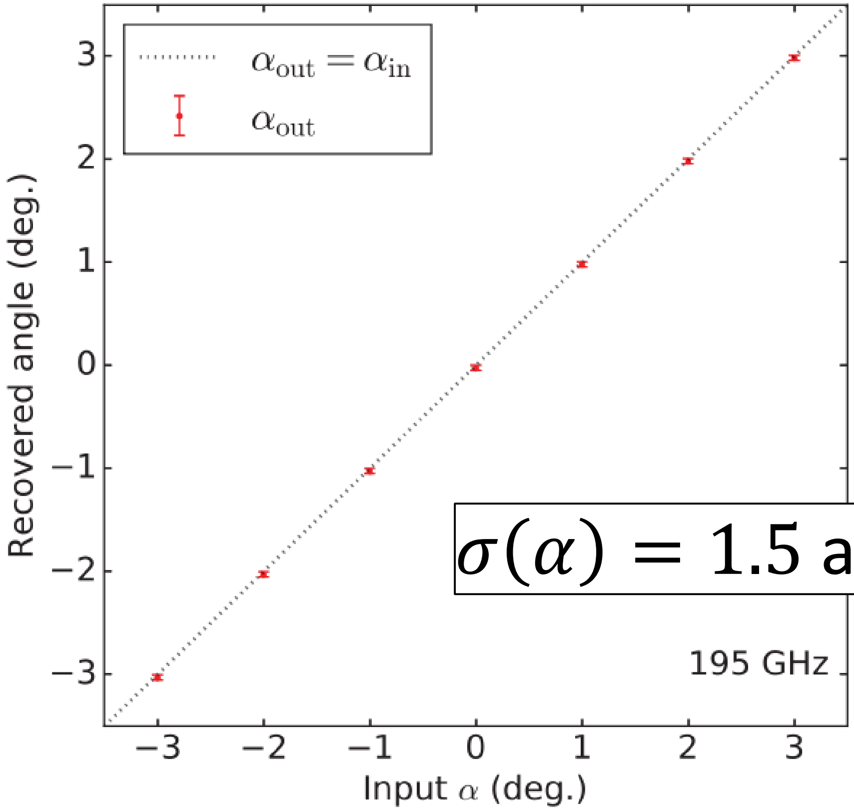
If we assume $\beta_{in}=0$, we can set β=0 in the Likelihood as,

$$-2\ln\mathcal{L} = \frac{\left[C_\ell^{EB,o} - \frac{1}{2}\left(C_\ell^{EE,o} - C_\ell^{BB,o}\right)\tan(4\alpha)\right]^2}{\mathrm{Var}\left(C_\ell^{EB,o} - \frac{1}{2}\left(C_\ell^{EE,o} - C_\ell^{BB,o}\right)\tan(4\alpha)\right)}. \qquad \cdots (10)$$

With this likelihood, we can determine α.

α only estimation at 195 GHz

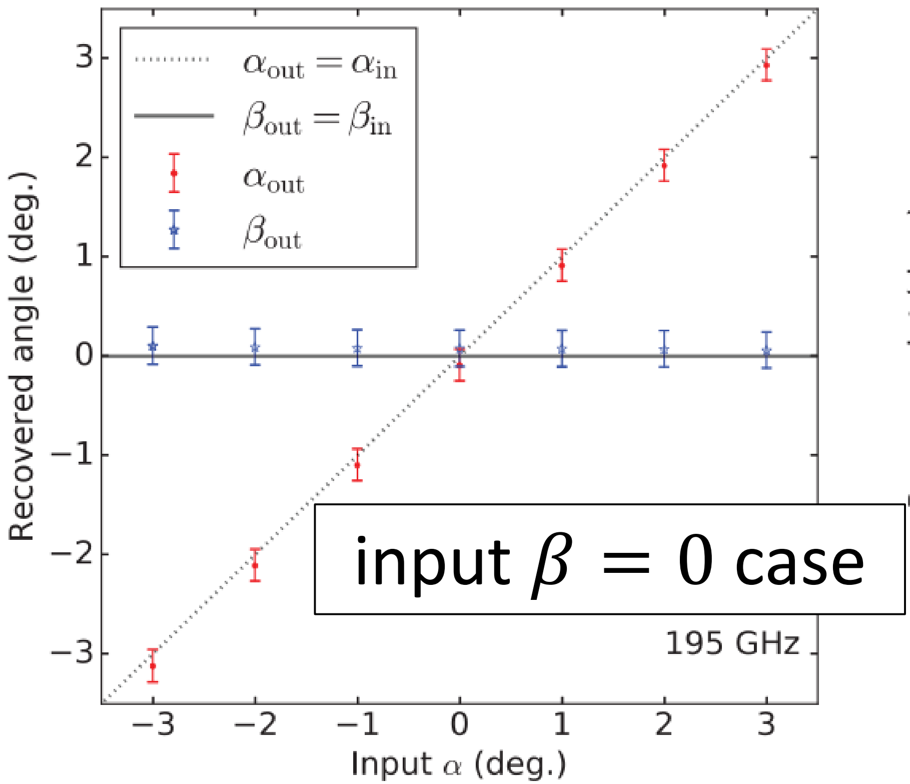
We set β_{in}=0 and try whether we can determine α_{in}

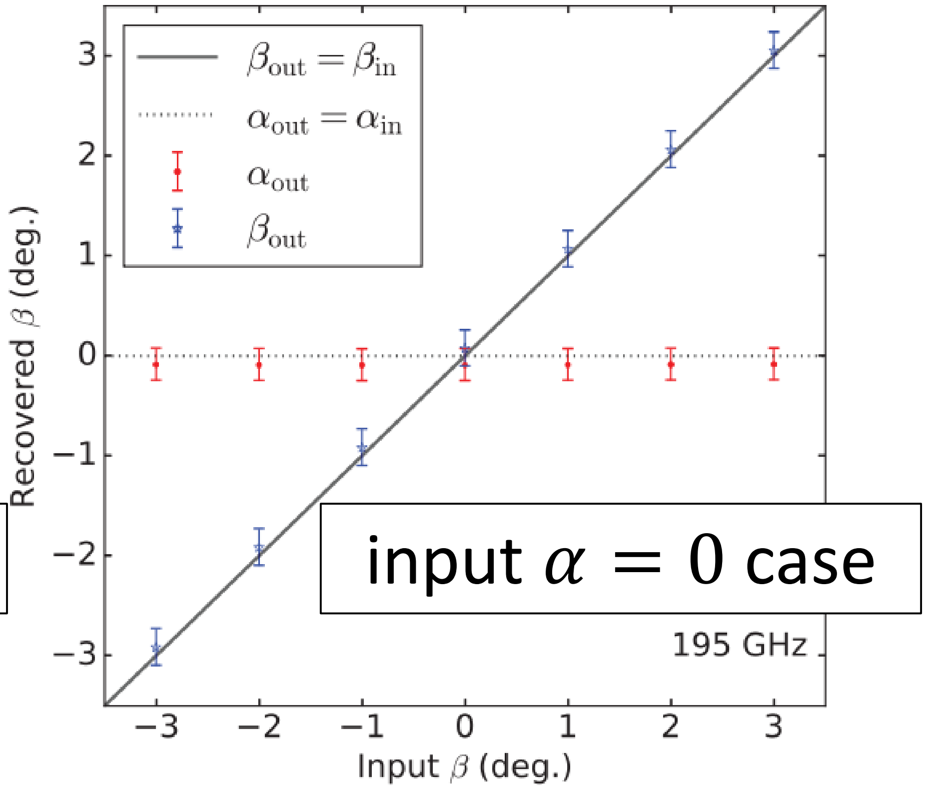


$\sigma(\alpha) = 1.5$ arcmin

We can recover the correct α without theoretical power spectra

Simultaneous determination at 195 GHz



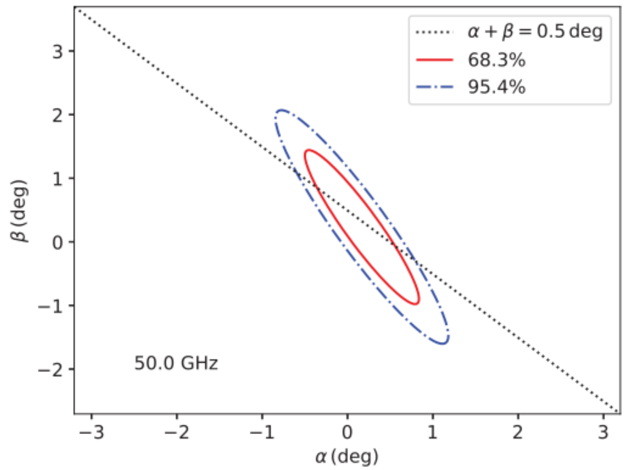


$\sigma(\alpha) = 9.6$ arcmin and $\sigma(\beta) = 11$ arcmin

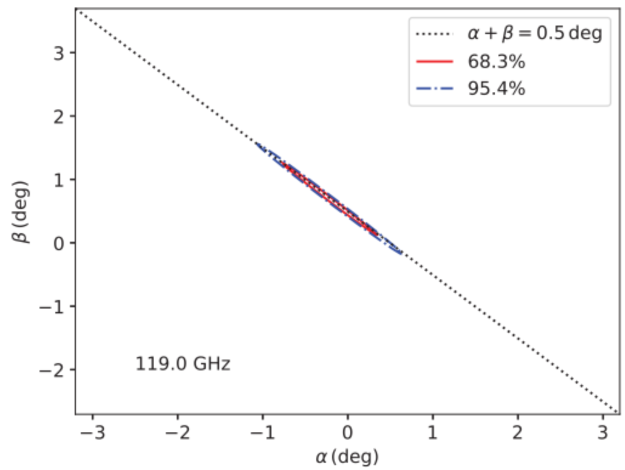
We can recover the correct α and β simultaneously

Correlation between α and β

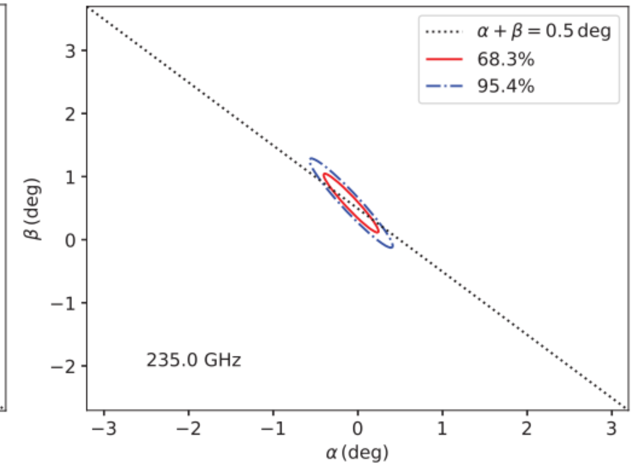
$$E^{\rm o}_{\ell,m} = E^{\rm fg}_{\ell,m}\cos(2\alpha) - B^{\rm fg}_{\ell,m}\sin(2\alpha) + E^{\rm CMB}_{\ell,m}\cos(2\alpha+2\beta) - B^{\rm CMB}_{\ell,m}\sin(2\alpha+2\beta)$$



Synchrotron channel
(50 GHz)



CMB channel
(119 GHz)



Dust channel
(235 GHz)

- CMB has a power to determine α+β
- FG has a power to determine α

Future prospect: Possibility to determine foreground *EB*

- In general, *EB* is related to *EE* and *BB* as

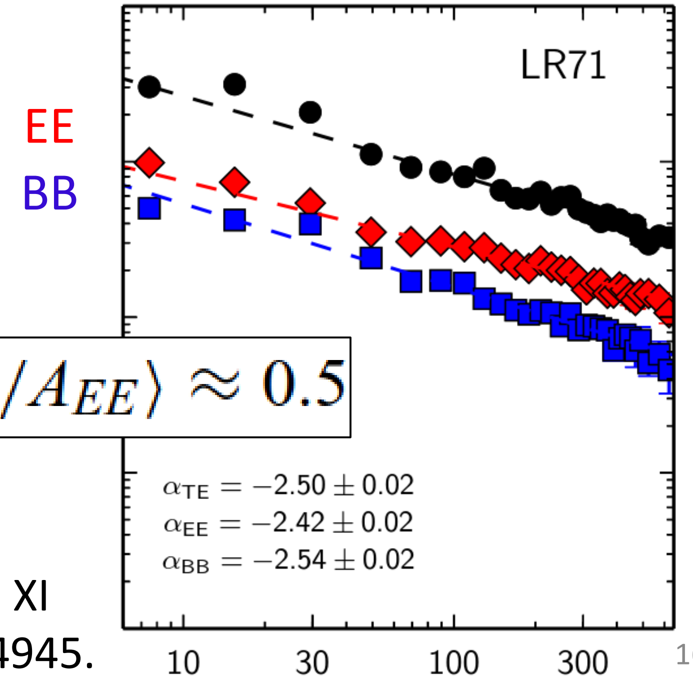
$$C_\ell^{EB} = f_c \sqrt{\langle C_\ell^{EE} \rangle \langle C_\ell^{BB} \rangle} \quad \cdots (11)$$

where f_c is a correlation coefficient.

- We assume *BB* is proportional to *EE* as

$$\langle C_\ell^{BB,fg} \rangle = \xi \langle C_\ell^{EE,fg} \rangle \quad \cdots (12)$$

  - This seems valid for thermal dust observed by Planck



$\langle A_{BB}/A_{EE} \rangle \approx 0.5$

Planck Collaboration XI (2018), arXiv:1801.04945.

Relate correlation parameters to rotation angle

- If *EB* correlation is small enough to meet

$$0 \le \frac{2f_c\sqrt{\xi}}{1-\xi} \le 1, \qquad \cdots(13)$$

we can put f_c and ξ into the rotation angle γ as,

$$\frac{\sin(4\gamma)}{2} = \frac{f_c\sqrt{\xi}}{1-\xi}. \qquad \cdots(14)$$

- Therefore, we can determine foreground EB correlation, if we give up measuring β

Summary

There was a consensus in the CMB community that the measurement of the cosmic birefringence and the polarization angle calibration cannot be done simultaneously.

We have shown that this is not the case.

We can determine the birefringence angle of order 10 arcmin (current 1σ bound is 30 arcmin)

This is a great news!

Backups

Table 1: Polarisation sensitivity and beam size of the LiteBIRD telescopes [15]

Frequency (GHz)	Polarization Sensitivity ($\mu K'$)	Beam Size in FWHM (arcmin)
40	37.5	69
50	24.0	56
60	19.9	48
68	16.2	43
78	13.5	39
89	11.7	35
100	9.2	29
119	7.6	25
140	5.9	23
166	6.5	21
195	5.8	20
235	7.7	19
280	13.2	24
337	19.5	20
402	37.5	17

Possibility to determine foreground *EB*

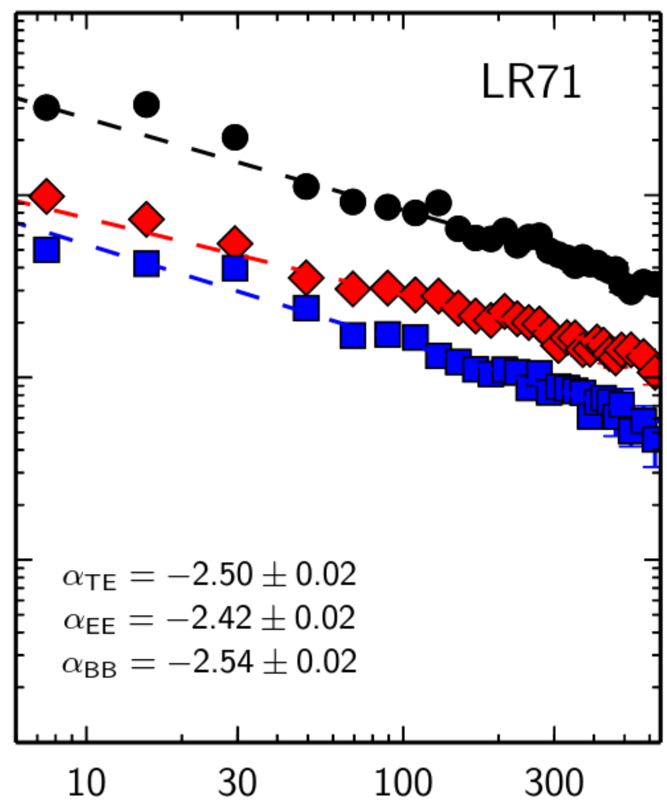
M. H. Abitbol, J. C. Hill, and B. R. Johnson, Mon. Not. Roy. Astron. Soc., 457(2), 1796–1803 (2016),
200 arXiv:1512.06834.

$$C_\ell^{dust,XY} = A^{XY}\left(\frac{\ell}{80}\right)^{-2.42} \quad (1)$$

$$C_{\ell,mult}^{dust,XY} = mC_\ell^{dust,XY} \quad (2)$$

$$C_{\ell,corr}^{dust,ZB} = f_c\sqrt{C_{\ell,mult}^{dust,ZZ}C_{\ell,mult}^{dust,BB}}, \quad (3)$$

where A^{XY} is the best-fitting amplitude, m is a multiplicative factor, f_c is a correlation fraction, $X, Y \in \{T, E, B\}$ and $Z \in \{T, E\}$.



Planck Collaboration XI (2018), arXiv:1801.04945.

Foreground

$$\langle C_\ell^{EB,\mathrm{fg}}\rangle = \frac{f_c\sqrt{\xi}}{1-\xi}\left(\langle C_\ell^{EE,\mathrm{fg}}\rangle - \langle C_\ell^{BB,\mathrm{fg}}\rangle\right).$$

→

$$\frac{\sin(4\gamma)}{2}\left(\left\langle C_\ell^{EE,fg}\right\rangle - \left\langle C_\ell^{BB,fg}\right\rangle\right)$$

$$E_{\ell,m}^{\mathrm{o,fg}} = E_{\ell,m}^{\mathrm{fg}}\cos(2\gamma) - B_{\ell,m}^{\mathrm{fg}}\sin(2\gamma),$$
$$B_{\ell,m}^{\mathrm{o,fg}} = E_{\ell,m}^{\mathrm{fg}}\sin(2\gamma) + B_{\ell,m}^{\mathrm{fg}}\cos(2\gamma),$$

Replace α-> α+γ
β->-γ
in birefringence estimation

$$\begin{aligned}
&\left(C_\ell^{EB,\mathrm{o}} - \frac{\tan(4\alpha)}{2}\left(C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}}\right)\right)\cos(4\alpha+4\gamma) = \\
&\quad (C_\ell^{EE,\mathrm{CMB}} - C_\ell^{BB,\mathrm{CMB}})\sin(-4\gamma)/2 \\
&\quad + C_\ell^{EB,\mathrm{fg}} + C_\ell^{EB,\mathrm{N}}\cos(4\alpha+4\gamma) + C_\ell^{EB,\mathrm{CMB}}\cos(-4\gamma) \\
&\quad - (C_\ell^{EE,\mathrm{N}} - C_\ell^{BB,\mathrm{N}})\sin(4\alpha+4\gamma)/2 \\
&\quad + (C_\ell^{E^{\mathrm{fg}}B^{\mathrm{CMB}}} + C_\ell^{E^{\mathrm{CMB}}B^{\mathrm{fg}}})\cos(-2\gamma) + (C_\ell^{E^{\mathrm{fg}}E^{\mathrm{CMB}}} - C_\ell^{B^{\mathrm{fg}}B^{\mathrm{CMB}}})\sin(-2\gamma) \\
&\quad + (C_\ell^{E^{\mathrm{fg}}B^{\mathrm{N}}} + C_\ell^{B^{\mathrm{fg}}E^{\mathrm{N}}})\cos(2\alpha+2\gamma) - (C_\ell^{E^{\mathrm{fg}}E^{\mathrm{N}}} - C_\ell^{B^{\mathrm{fg}}B^{\mathrm{N}}})\sin(2\alpha+2\gamma) \\
&\quad + (C_\ell^{E^{\mathrm{CMB}}B^{\mathrm{N}}} + C_\ell^{B^{\mathrm{CMB}}E^{\mathrm{N}}})\cos(2\alpha+4\gamma) \\
&\quad - (C_\ell^{E^{\mathrm{CMB}}E^{\mathrm{N}}} - C_\ell^{B^{\mathrm{CMB}}B^{\mathrm{N}}})\sin(2\alpha+4\gamma)
\end{aligned}$$

Equations including birefringence rotation – carefully-

We should not forget noise and other correlations!

$$E^{\rm o}_{\ell,m} = E^{\rm fg}_{\ell,m}\cos(2\alpha) - B^{\rm fg}_{\ell,m}\sin(2\alpha) + E^{\rm CMB}_{\ell,m}\cos(2\alpha+2\beta) - B^{\rm CMB}_{\ell,m}\sin(2\alpha+2\beta) + E^{\rm N}_{\ell,m},$$

$$B^{\rm o}_{\ell,m} = E^{\rm fg}_{\ell,m}\sin(2\alpha) + B^{\rm fg}_{\ell,m}\cos(2\alpha) + E^{\rm CMB}_{\ell,m}\sin(2\alpha+2\beta) + B^{\rm CMB}_{\ell,m}\cos(2\alpha+2\beta) + B^{\rm N}_{\ell,m}$$

Noise

$$\left(C_\ell^{EB,\rm o} - \frac{\tan(4\alpha)}{2}\left(C_\ell^{EE,\rm o} - C_\ell^{BB,\rm o}\right)\right)\cos(4\alpha) =$$

$$(C_\ell^{EE,\rm CMB} - C_\ell^{BB,\rm CMB})\sin(4\beta)/2$$

$$+ C_\ell^{EB,\rm fg} + C_\ell^{EB,\rm N}\cos(4\alpha) + C_\ell^{EB,\rm CMB}\cos(4\beta)$$

$$- (C_\ell^{EE,\rm N} - C_\ell^{BB,\rm N})\sin(4\alpha)/2$$

$$+ (C_\ell^{E^{\rm fg}B^{\rm CMB}} + C_\ell^{E^{\rm CMB}B^{\rm fg}})\cos(2\beta) + (C_\ell^{E^{\rm fg}E^{\rm CMB}} - C_\ell^{B^{\rm fg}B^{\rm CMB}})\sin(2\beta)$$

$$+ (C_\ell^{E^{\rm fg}B^{\rm N}} + C_\ell^{B^{\rm fg}E^{\rm N}})\cos(2\alpha) - (C_\ell^{E^{\rm fg}E^{\rm N}} - C_\ell^{B^{\rm fg}B^{\rm N}})\sin(2\alpha)$$

$$+ (C_\ell^{E^{\rm CMB}B^{\rm N}} + C_\ell^{B^{\rm CMB}E^{\rm N}})\cos(2\alpha - 2\beta)$$

$$- (C_\ell^{E^{\rm CMB}E^{\rm N}} - C_\ell^{B^{\rm CMB}B^{\rm N}})\sin(2\alpha - 2\beta)$$

Variance

$$\begin{aligned}
&\mathrm{Var}\left[C_\ell^{EB,\mathrm{o}} - (C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}})\tan(4\alpha)/2\right] \\
&= \langle\left[C_\ell^{EB,\mathrm{o}} - (C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}})\tan(4\alpha)/2\right]^2\rangle - \langle C_\ell^{EB,\mathrm{o}} - (C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}})\tan(4\alpha)/2\rangle^2 \\
&= \frac{1}{2\ell+1}\langle C_\ell^{EE}\rangle\langle C_\ell^{BB}\rangle + \frac{\tan^2(4\alpha)}{4}\frac{2}{2\ell+1}\left(\langle C_\ell^{EE}\rangle^2 + \langle C_\ell^{BB}\rangle^2\right) \\
&\quad - \tan(4\alpha)\frac{2}{2\ell+1}\langle C_\ell^{EB}\rangle\left(\langle C_\ell^{EE}\rangle - \langle C_\ell^{BB}\rangle\right) + \frac{1}{2\ell+1}\left(1-\tan^2(4\alpha)\right)\langle C_\ell^{EB}\rangle^2. \qquad \text{(A1)}
\end{aligned}$$

$$\begin{aligned}
&\mathrm{Var}\left[C_\ell^{EB,\mathrm{o}} - (C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}})\tan(4\alpha)/2\right] \\
&\approx \frac{1}{2\ell+1}C_\ell^{EE,\mathrm{o}}C_\ell^{BB,\mathrm{o}} + \frac{\tan^2(4\alpha)}{4}\frac{2}{2\ell+1}\left[(C_\ell^{EE,\mathrm{o}})^2 + (C_\ell^{BB,\mathrm{o}})^2\right] \\
&\quad - \tan(4\alpha)\frac{2}{2\ell+1}C_\ell^{EB,\mathrm{o}}\left(C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}}\right).
\end{aligned}$$