A 2020s Vision of CMB Lensing

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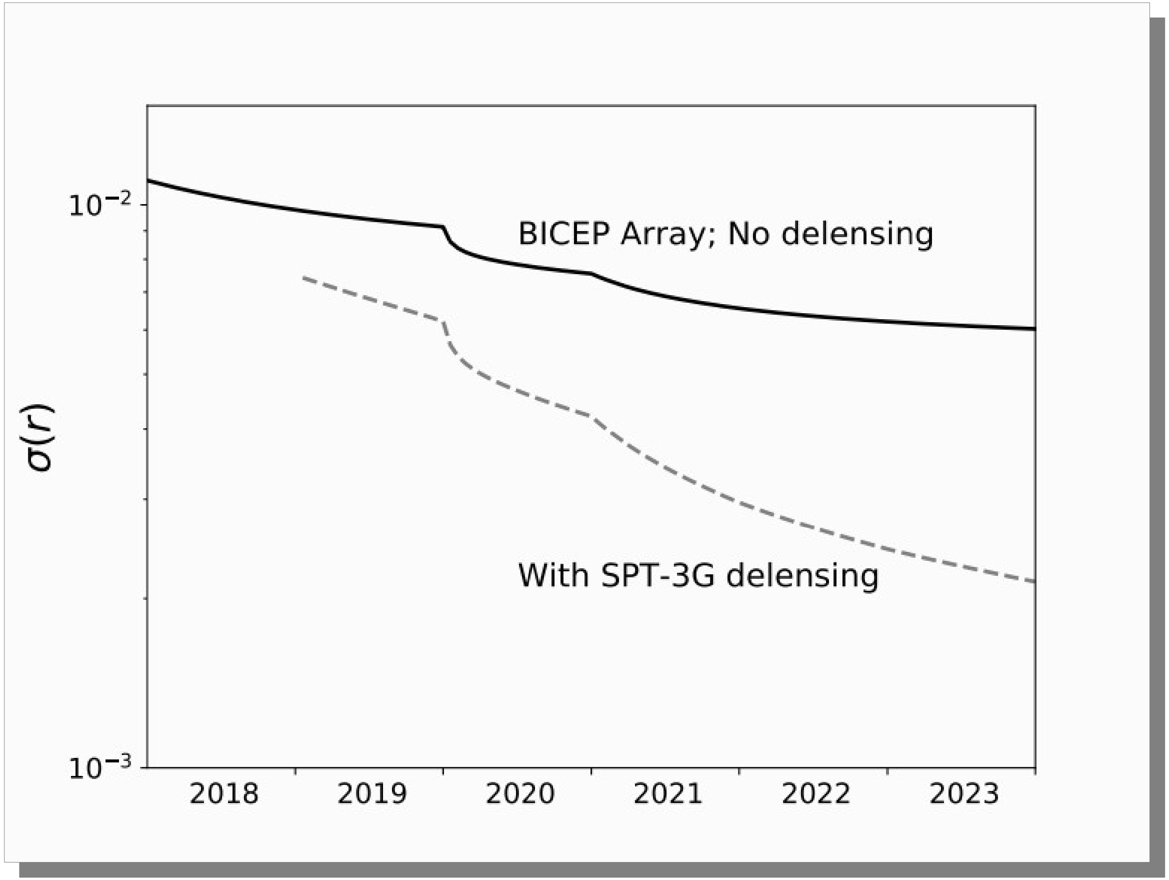
With: SPT Collaboration, Ethan Anderes, Ben Wandelt

B Modes From Space – MPA Garching – Dec 18, 2019

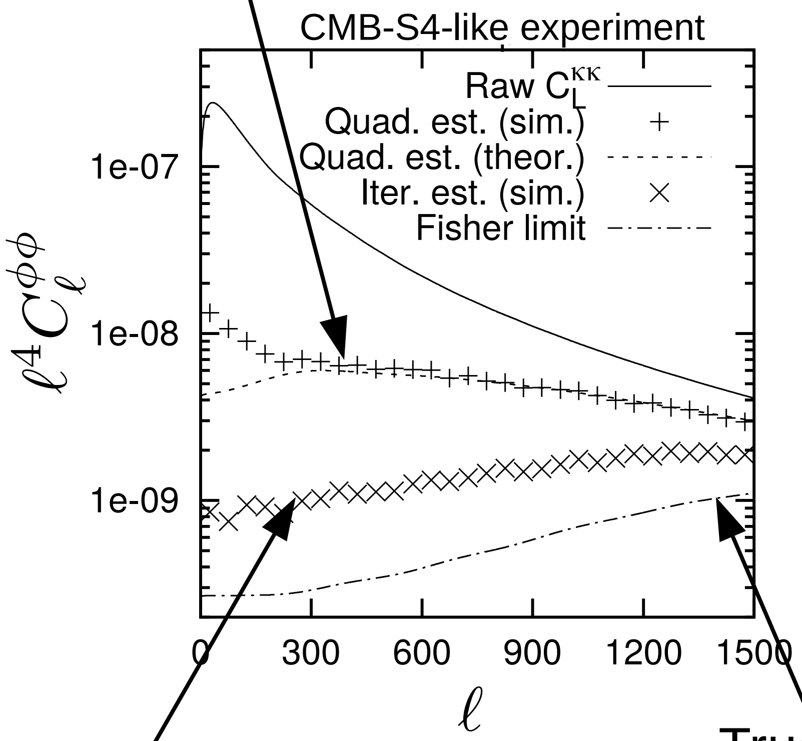
Outline

- The quadratic estimate (QE) and beyond
- Discuss different approaches and present our Bayesian method
- Preview of some results from the South Pole Telescope
- Future extensions

The need for delensing



Quadratic estimate noise spectrum

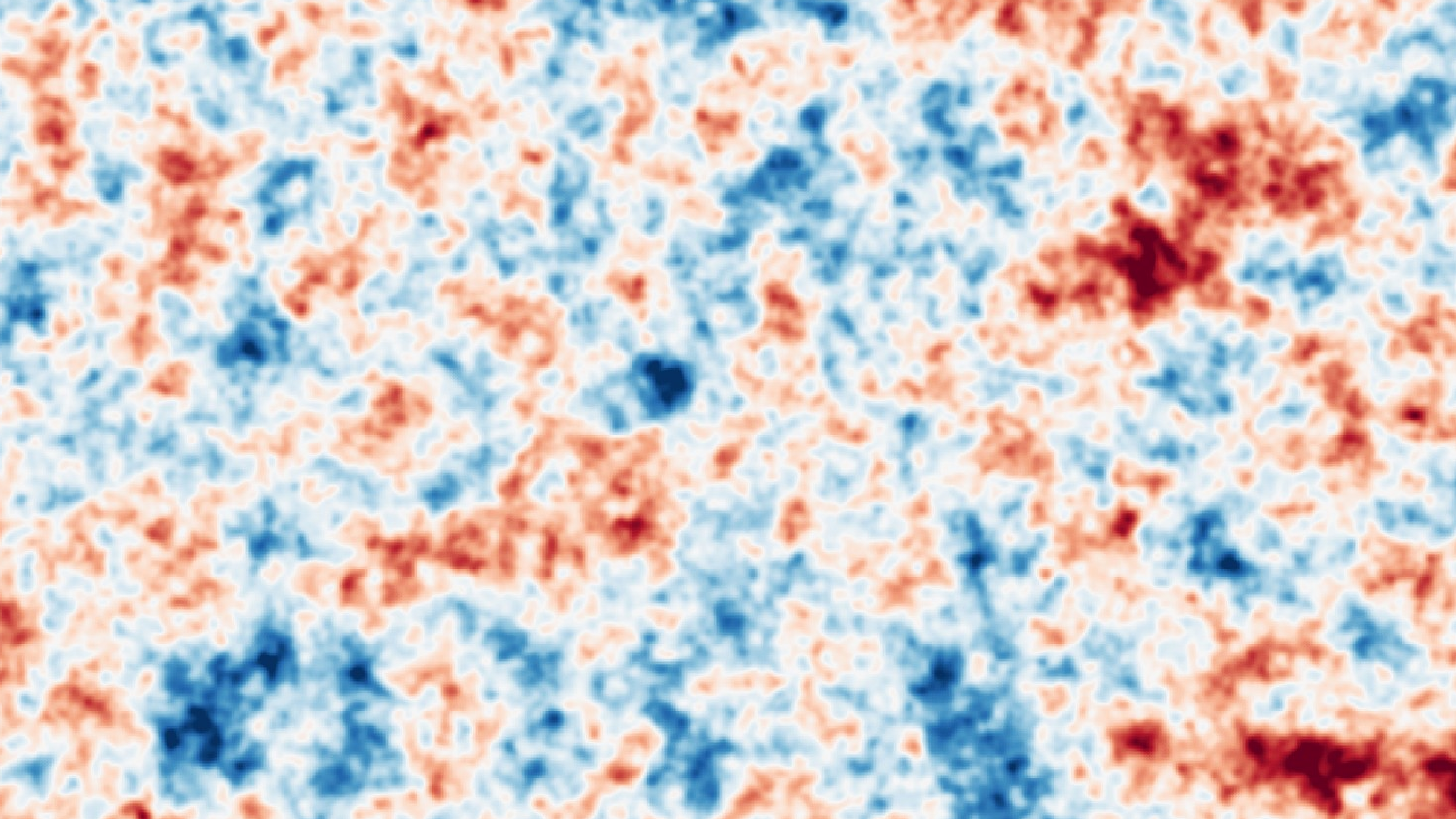


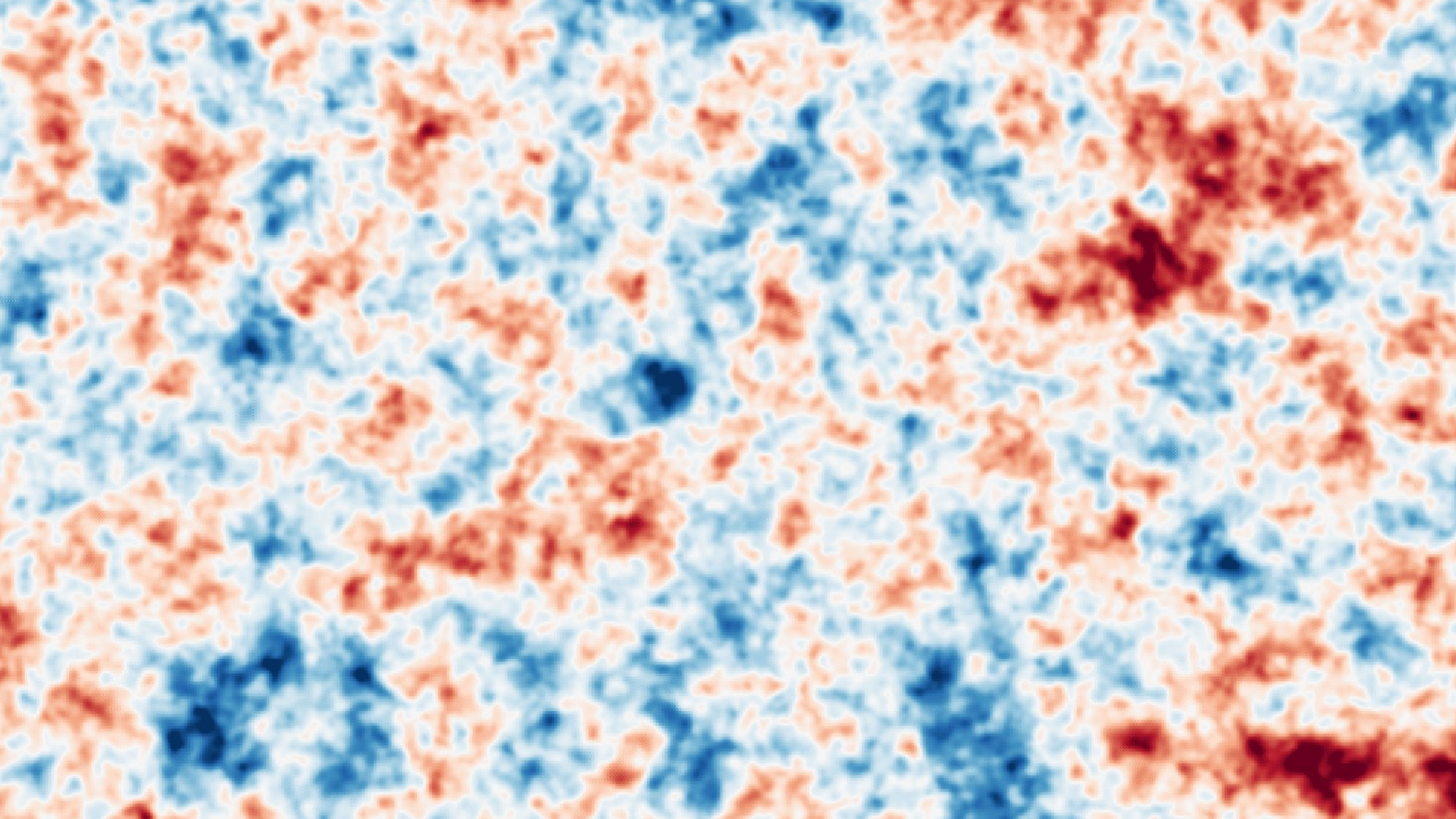
Forecast for iterating

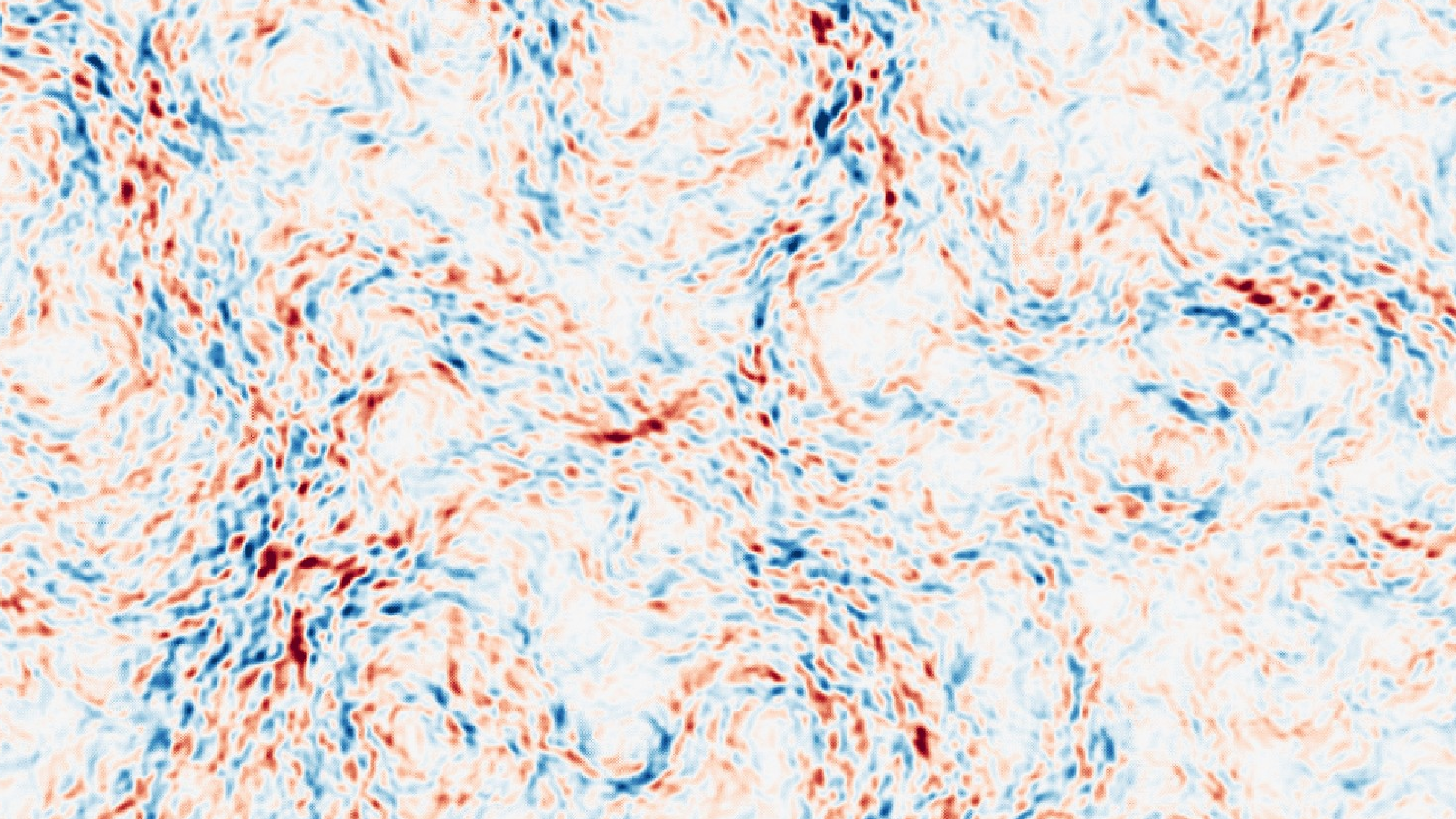
True Fisher forecast

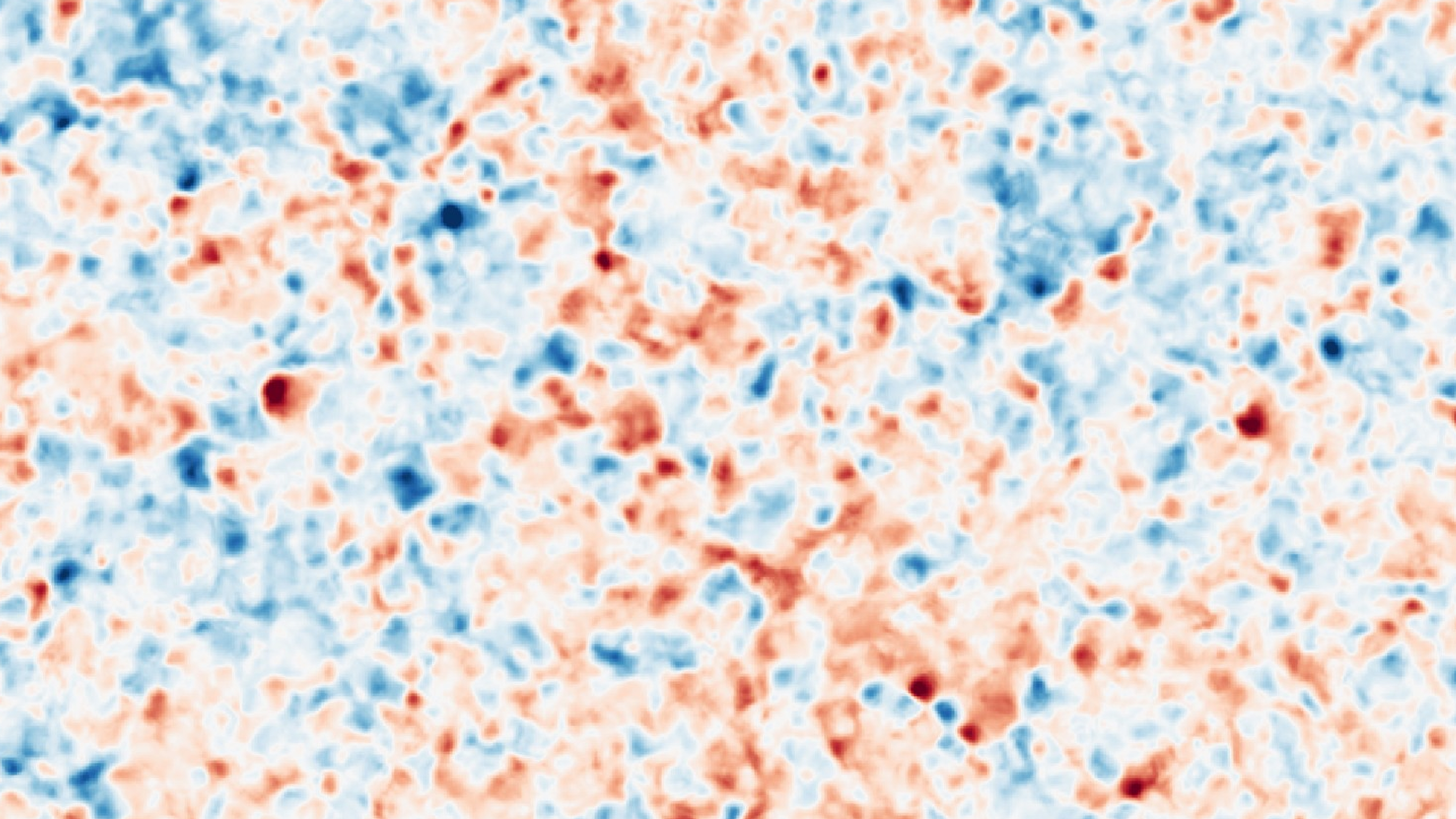
Hirata & Seljak (2003)

- These are forecasts or highly simplified analyses
- How do we do this to real data?









Isotropic Gaussian random fields:

$$\langle f(\vec{\ell}) f(\vec{\ell}') \rangle = \mathbb{C}_f(\ell)\delta(\vec{\ell} - \vec{\ell}')$$

$f \equiv (T, E, B)$

CMB “fields”

Lensed fields:

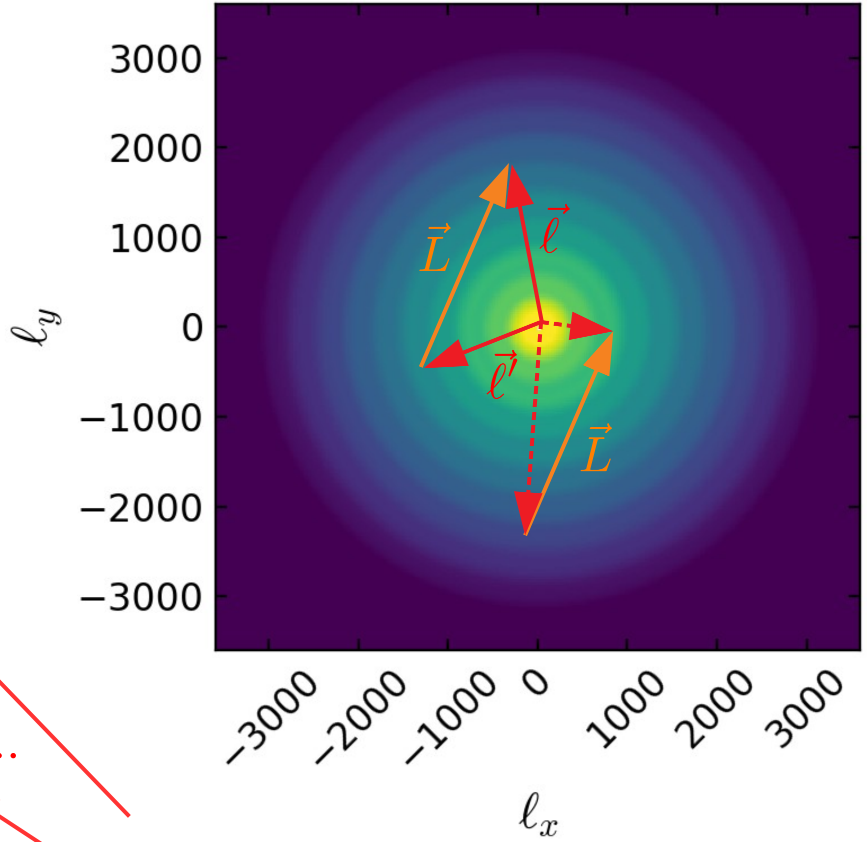
$$\langle \tilde{f}(\vec{\ell}) \tilde{f}(\vec{\ell} + \vec{L}) \rangle \sim \phi(\vec{L}) \;\; + \phi^2 + \ldots$$

Quadratic estimate:

$$\hat{\phi}_{\rm QE}(\vec{L}) = \sum_{\vec{\ell}} d(\vec{\ell}) d(\vec{\ell} + \vec{L}) \quad \sum ddd\ldots$$

The data

$\mathbb{C}_f(\ell)$ CMB 2D power-spectrum



Quadratic estimator is suboptimal because it misses this information

Towards optimality

- **DeepCMB** (Caldeira et al. 2018)
  - Achieves noise levels comparable to the iterative-forecast
  - Challenges in how to quantify uncertainty
- **Gradient inversion** (Horowitz et al. 2018, Hadzhiyska et al. 2018)
  - Simple, but only optimal in the asymptotic limit of small scales
- **Optimal filtering** (Mirmelstein et al. 2019)
  - A way to more optimally filter a QE ϕ map before taking its power spectrum
  - May be useful mainly in the short term
- **Iterative QE**
  - Does not actually exist
- **Bayesian methods...**

Bayesian Lensing

Cosmological parameters

Data model:

$$d = \mathbb{L}(\phi) f + n$$

Priors:

$$f \sim \text{Gaussian}(0, \mathbb{C}_f(\theta)) \qquad n \sim \text{Gaussian}(0, \mathbb{C}_n)$$

$$\phi \sim \text{Gaussian}(0, \mathbb{C}_\phi(\theta)) \qquad \theta \sim \text{Uniform}$$

"Joint" posterior (MM,Anderes,Wandelt 2018):

$$\mathcal{P}(f, \phi, \theta \,|\, d) = \exp\left\{-\frac{(d - \mathbb{L}(\phi) f)^2}{2\mathbb{C}_n}\right\} \exp\left\{-\frac{f^2}{2\mathbb{C}_f(\theta)}\right\} \exp\left\{-\frac{\phi^2}{2\mathbb{C}_\phi(\theta)}\right\}$$

Reparametrizations are crucial

LenseFlow allows lensing gradients

"Marginal" posterior (Hirata&Seljak 2003; Polarbear et al. 2019):

$$\mathcal{P}(\phi, \theta \,|\, d) = \exp\left\{-\frac{d^2}{2\mathbb{C}_d}\right\} \exp\left\{-\frac{\phi^2}{2\mathbb{C}_\phi(\theta)}\right\} / \det \mathbb{C}_d^{1/2}$$

Killer, computationally

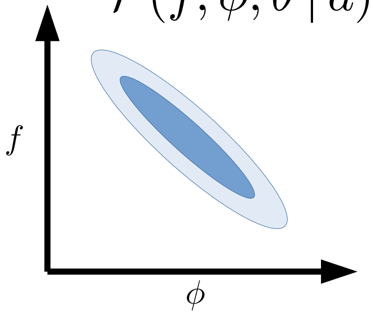
Any MAP estimate is biased & θ-dependent

where $\mathbb{C}_d(\phi, \theta) \equiv \mathbb{L}(\phi)\mathbb{C}_f(\theta)\mathbb{L}(\phi)^\dagger + \mathbb{C}_n$

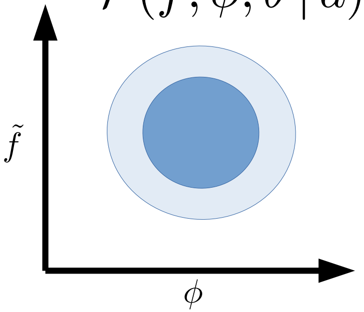
Notation: $x^2/\mathbb{C} \equiv x^\dagger \mathbb{C}^{-1} x$

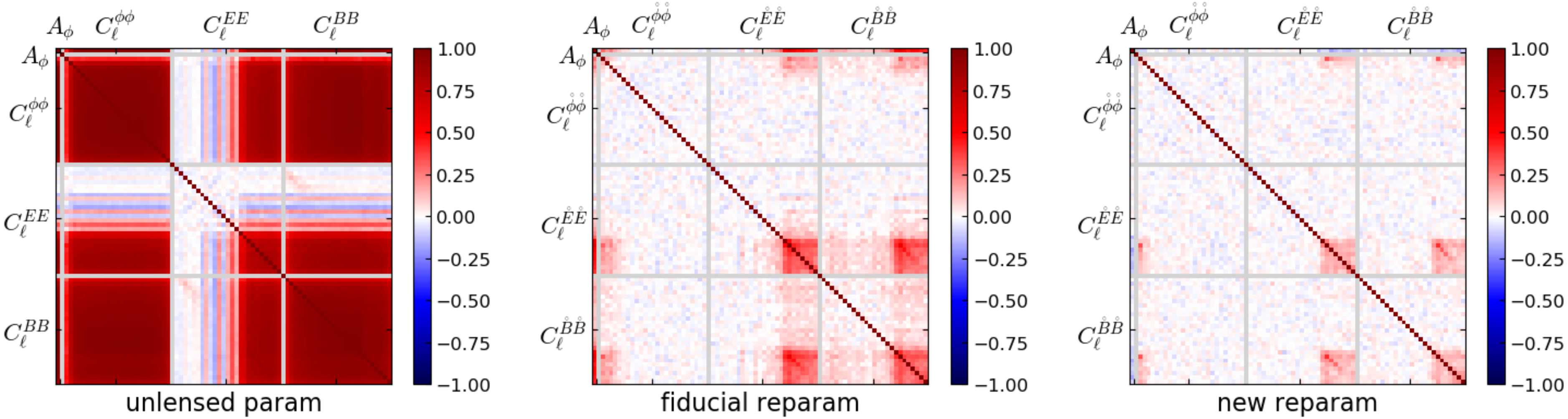
The posterior is extremely degenerate / NG

$$\mathcal{P}(f, \phi, \theta \,|\, d)$$

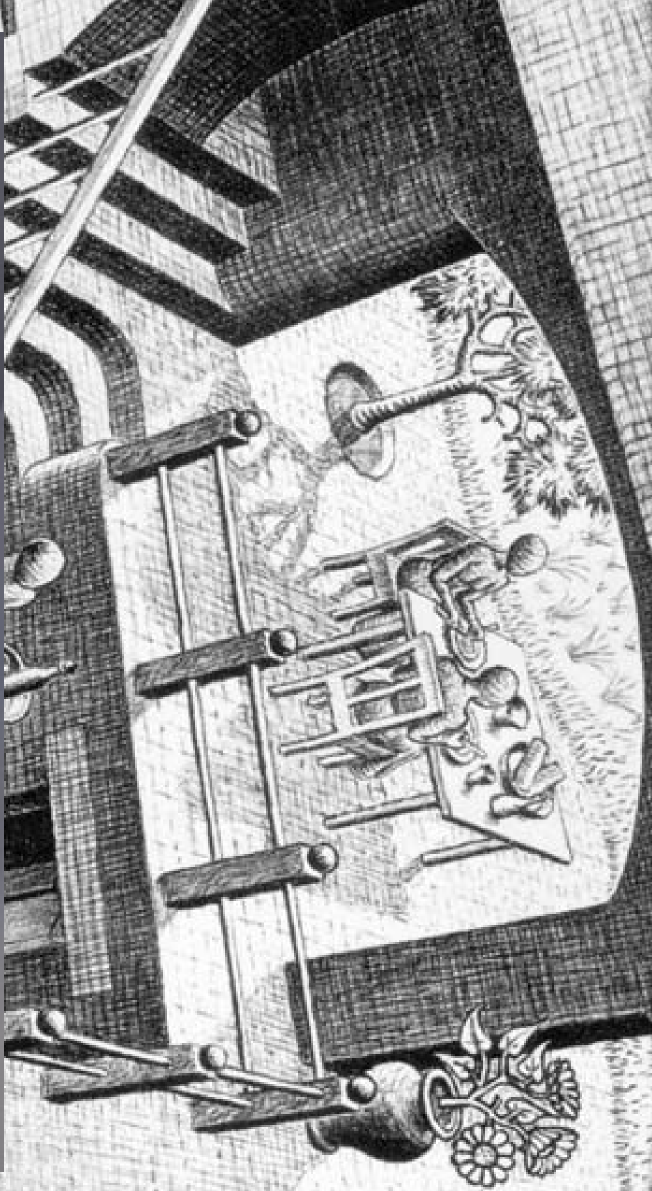


$$\mathcal{P}(\tilde{f}, \phi, \theta \,|\, d) = \mathcal{P}(f = \mathbb{L}(\phi)^{-1}\tilde{f}, \phi, \theta \,|\, d) \times \left|\det \mathbb{L}(\phi)\right|^{-1}$$





B-mode from Space



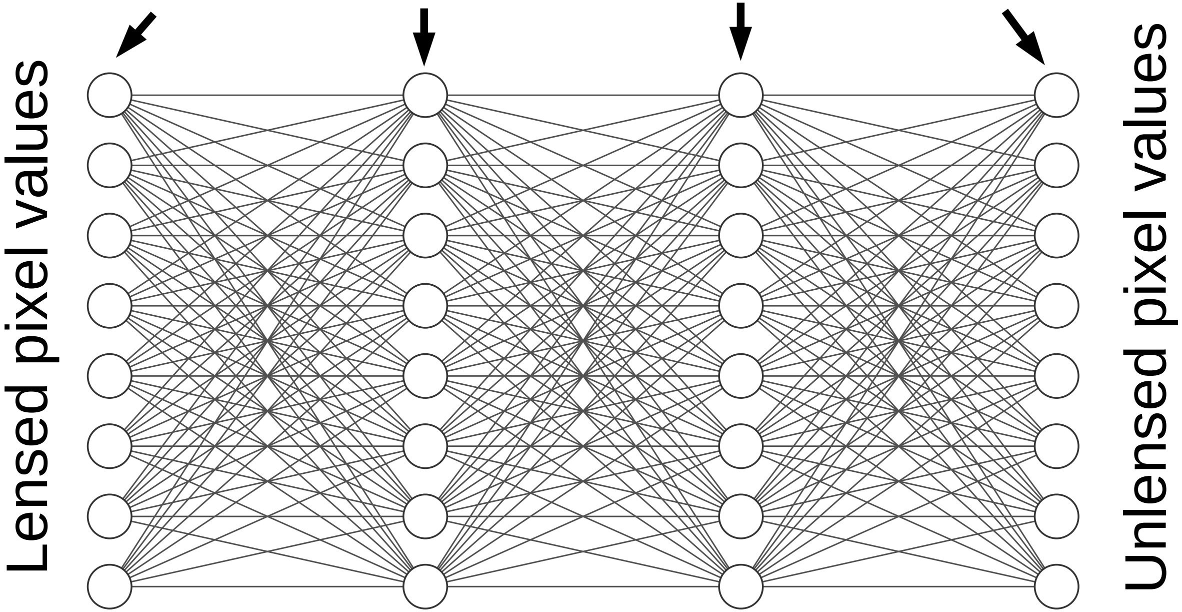
Traditional lensing:

$$\tilde{f}(x) = f(x + \nabla\phi) \approx f(x) + \nabla f(x)\nabla\phi(x) + ...$$

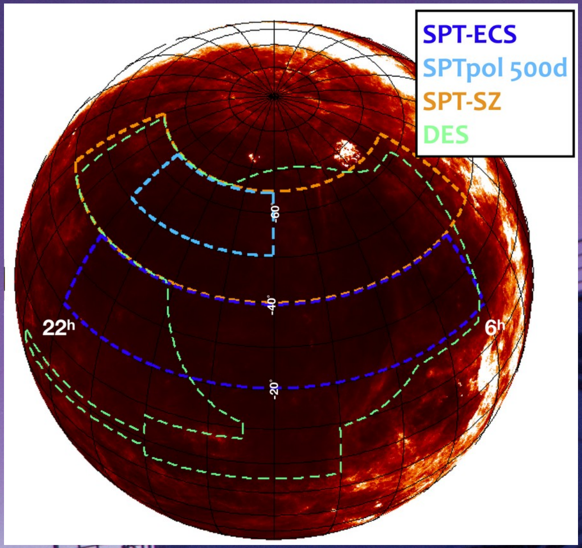
LenseFlow:

$$f_t(x) = f(x + t\nabla\phi) \qquad \frac{df_t(x)}{dt} = \underbrace{\nabla\phi(x)\cdot\left[\mathbb{1} + t\nabla\nabla\phi(x)\right]^{-1}\cdot\nabla}_{\mathbb{V}_t} f_t(x)$$

$$f_{t=1} = \left[\mathbb{1} + \Delta t\,\mathbb{V}_{t_n}\right] ... \left[\mathbb{1} + \Delta t\mathbb{V}_{t_1}\right] f_{t=0}$$

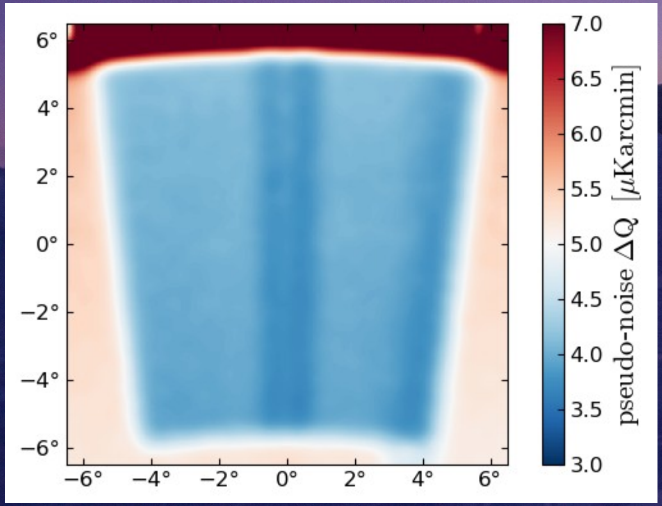


Upcoming South Pole Telescope Analysis

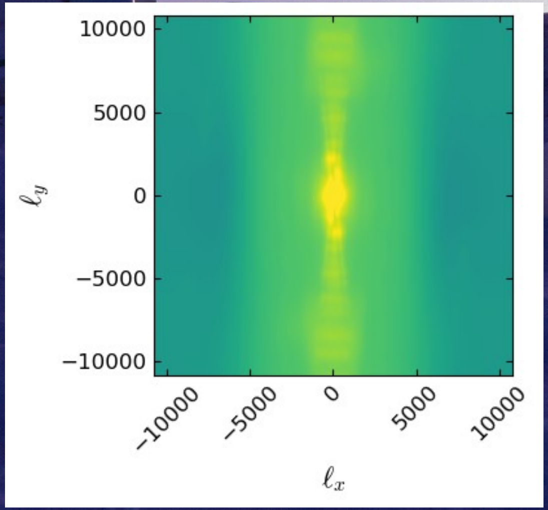


Deepest $100 deg^2$ polarization measurements to-date at the angular scales most relevant for lensing.

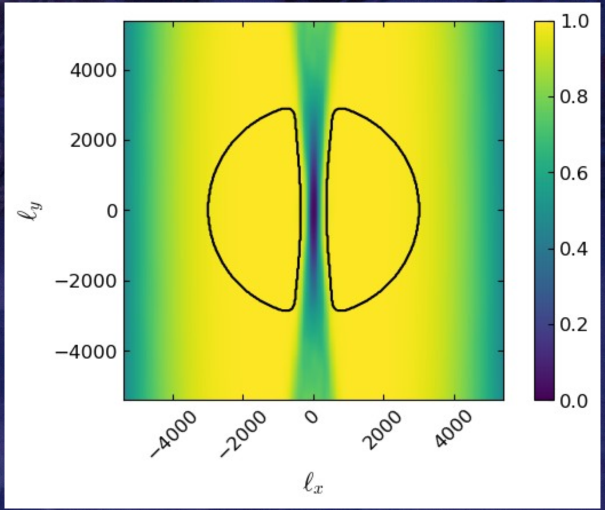
Spatially varying noise



Non-white noise



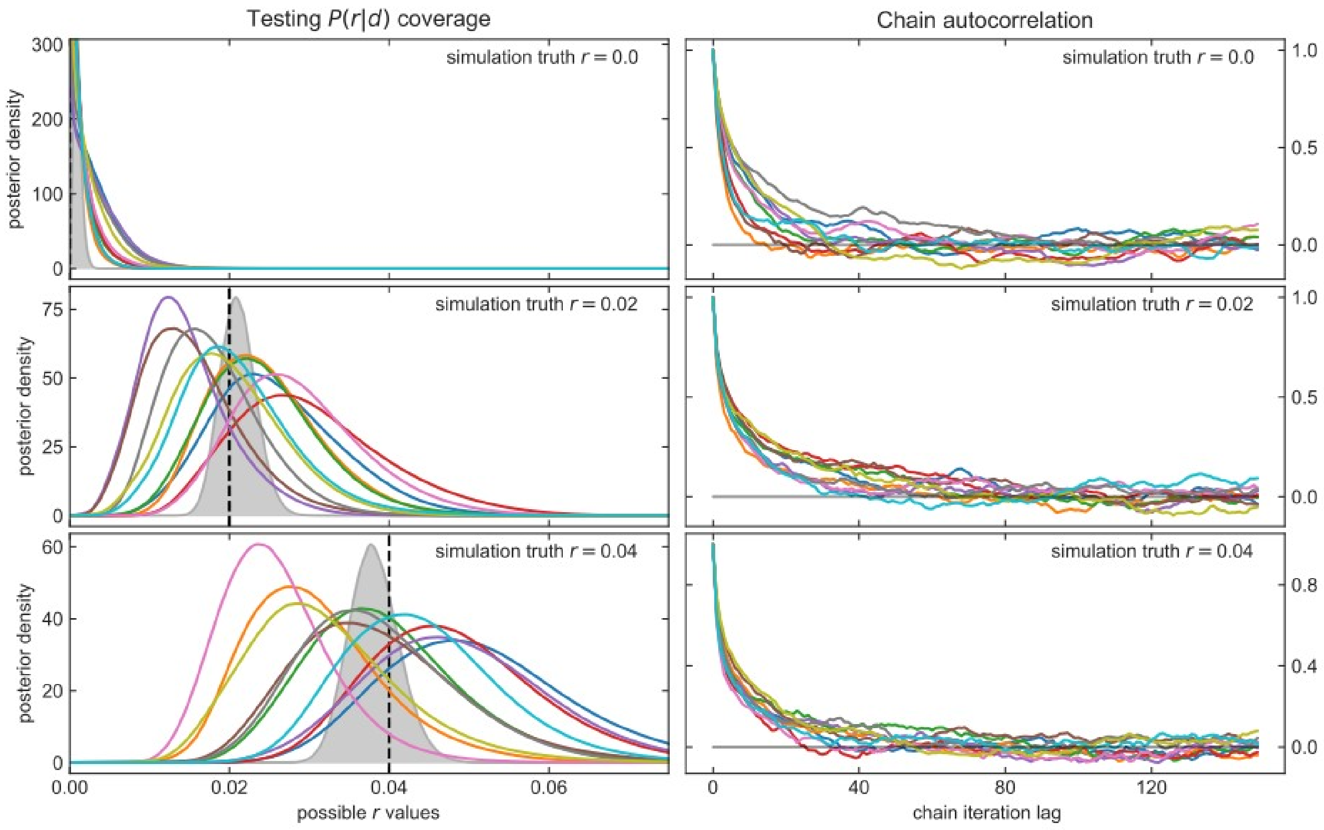
Anisotropic x-fer func



(preliminary slide removed)

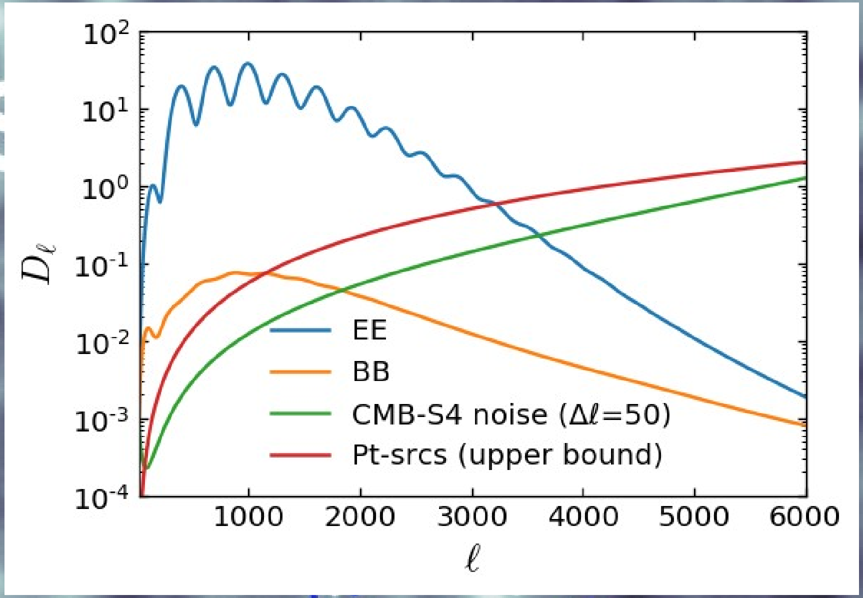
400deg^2 of CMB-S4-like observations

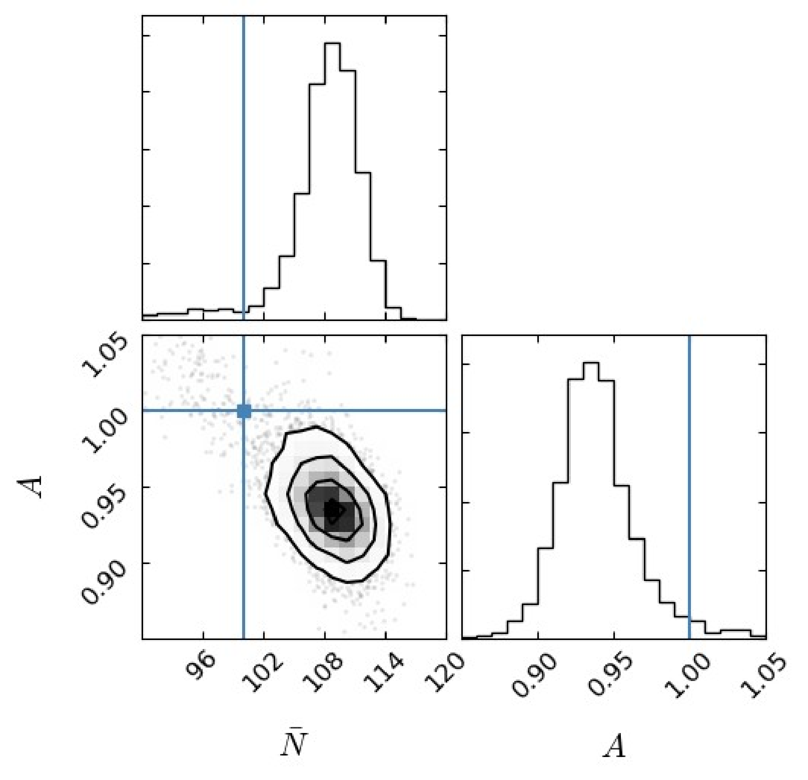
We can sample *r*, check coverage, and compute exact Fisher information



MM, Anderes, Wandelt, in prep

What else will we need to include in future CMB lensing analyses?





For point sources in the 1-halo regime:

$$\mathcal{P}\left(S_{\rm pix}\right) = \int_{-\infty}^{\infty} dt \exp\left\{ itS_{\rm pix} + \int_0^{S_{\rm cut}} dS \frac{d\bar{N}}{dS\,d\Omega_{\rm pix}} \left[\exp(itS) - 1\right] \right\}$$

1°

work with Emmanuel Schaan

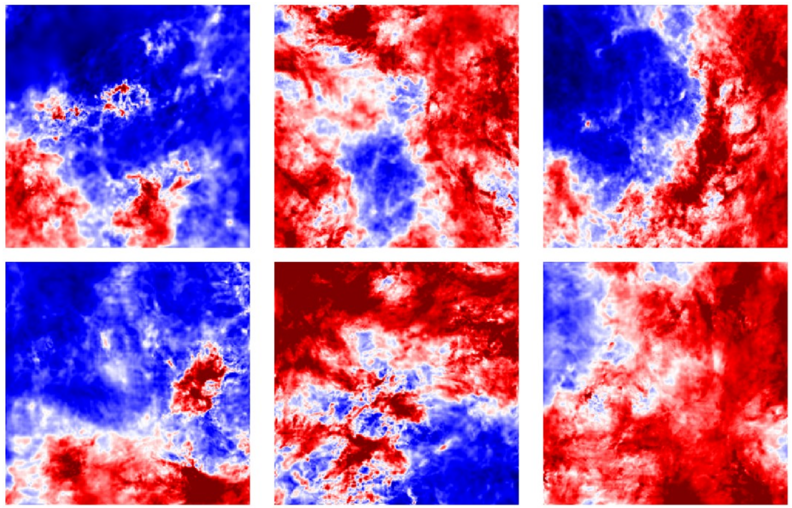
Galactic Dust GANs

Aylor et al. 2019

Generative neural network

$$\mathrm{Gaussian}(0, \mathbb{I}_{64})$$

Prior distribution



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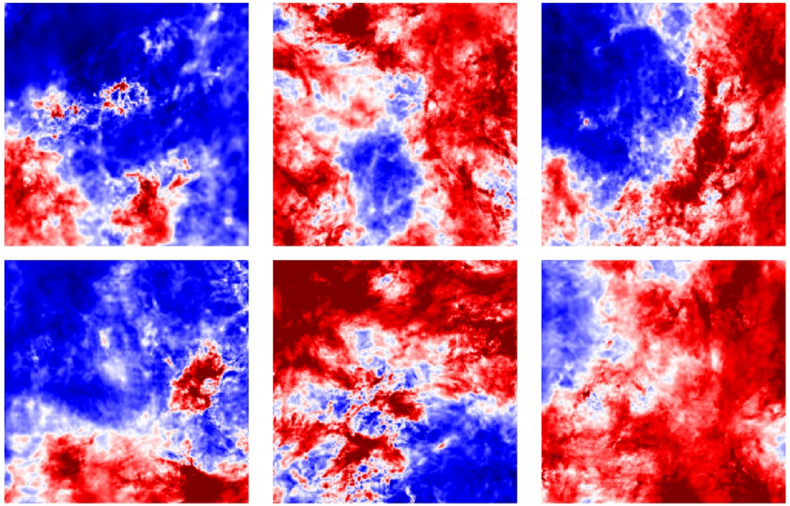
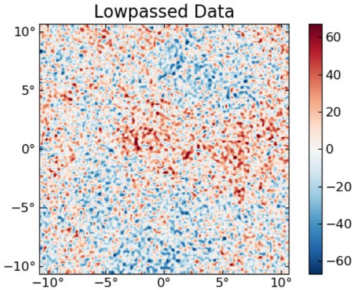
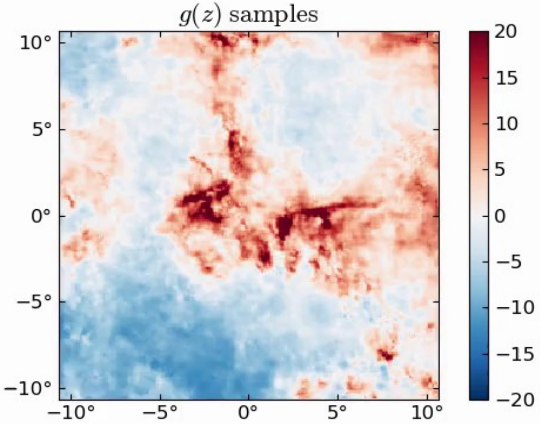
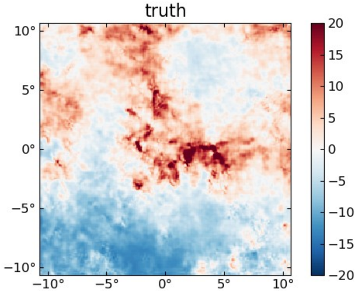








Conclusions

- Through the 2020s, all lensing analyses will eventually go beyond the QE
- The Bayesian solutions seems a promising way forward
  - (Bayesian ≠ sampling though)
- It can currently be used as a benchmark of any other methods
- We are working towards using this for the South Pole Observatory and eventually CMB-S4 and LiteBIRD

cosmicmar.com/CMBLensing.jl

Install

```
git clone https://github.com/marius311/CMBLensing.jl.git
cd CMBLensing.jl
docker-compose pull
docker-compose up
```

Use

```
lnP(0,f,ϕ,(r=0.03,),ds)
gradient(ϕ->lnP(0,f,ϕ,(r=0.03,),ds), ϕ)
```

CMBLensing.jl

Search docs

CMBLensing.jl

- Documentation
- Installation

Lensing a flat-sky map

The Lensing Posterior

MAP estimation

Calling from Python

Field Basics

API

CMBLensing.jl

docs stable | launch binder | build passing

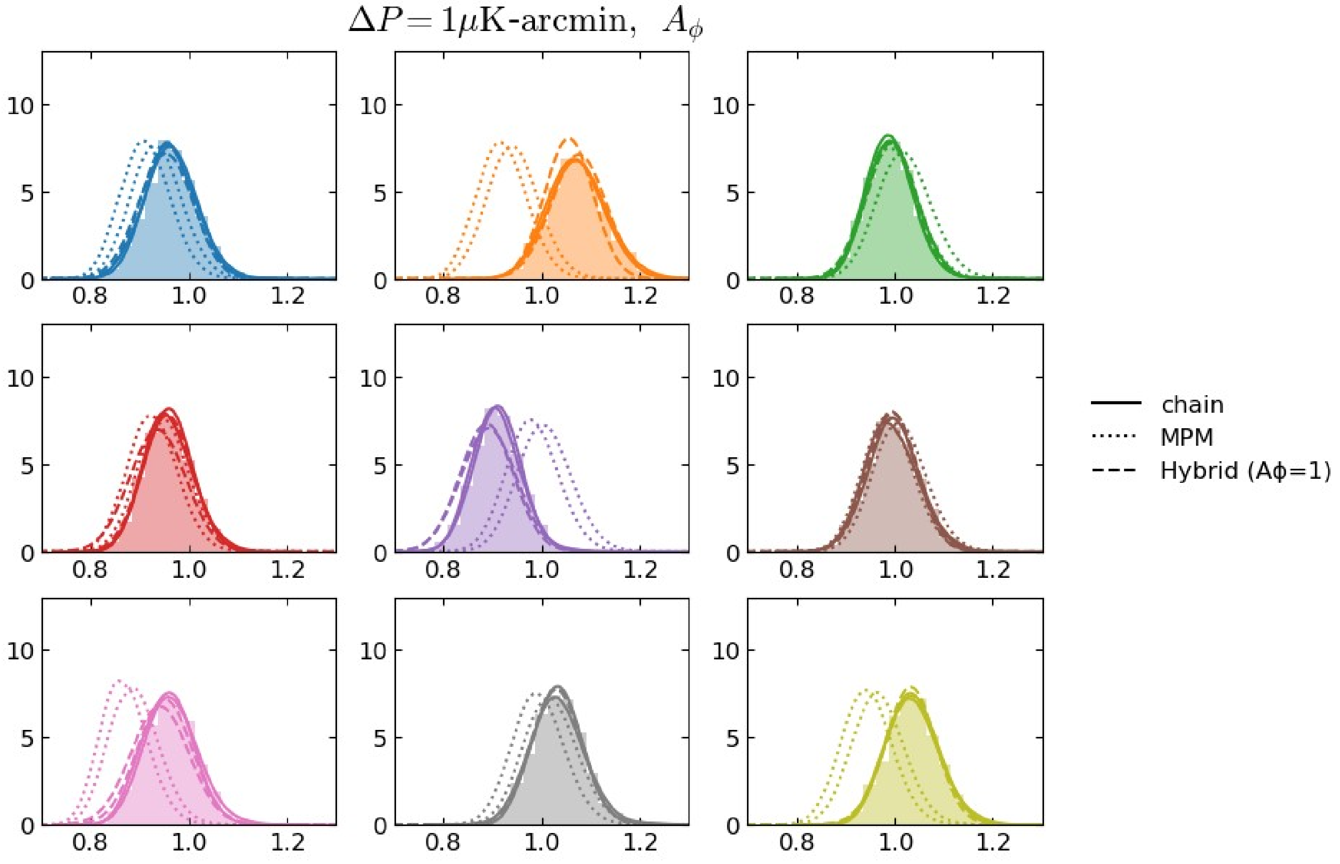
CMBLensing.jl is a next-generation tool for analysis of the lensed Cosmic Microwave Background. It is written in Julia and transparently callable from Python.

At its heart, CMBLensing.jl maximizes or samples the Bayesian posterior for the CMB lensing problem. It also contains tools to quickly manipulate and process CMB maps, set up modified posteriors, and take gradients using automatic differentation.

Highlights

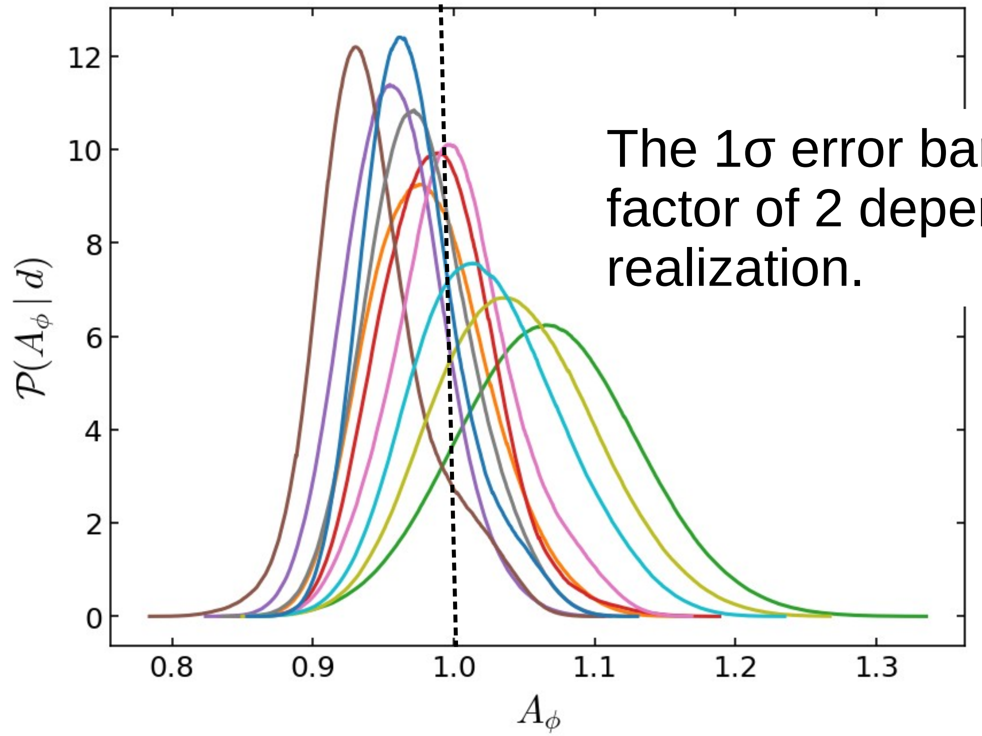
- Fully Nvidia GPU compatible (speedups over CPU are currently 3x-10x, depending on the problem size and hardware).

Problems with point estimates and the marginal posterior



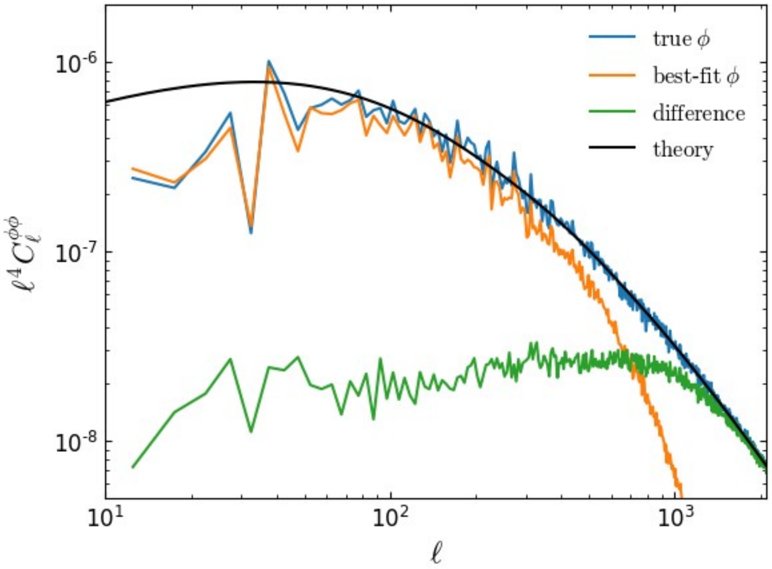
MM & Seljak, in prep

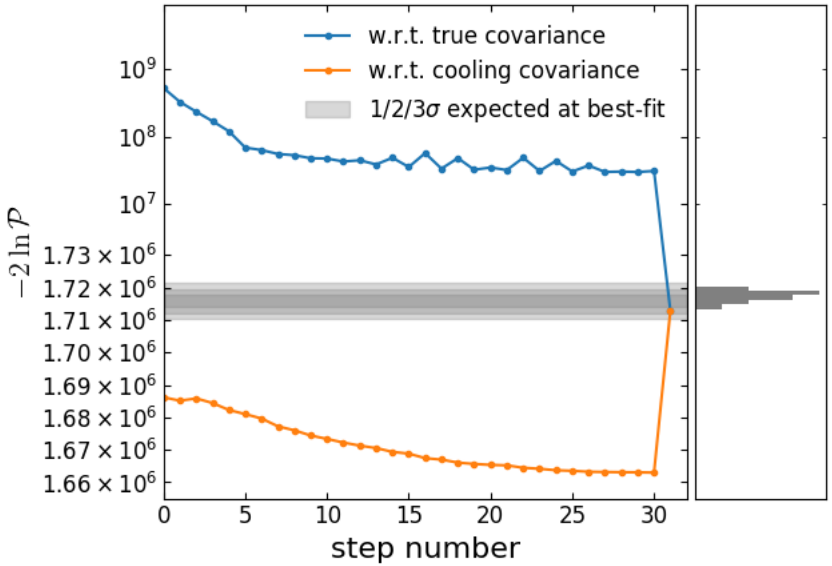
Different simulated realizations of data:



The 1σ error bar varies by a factor of 2 depending on the data realization.

- Any frequentist analysis would have missed this
- May explain why we are not reaching the Fisher limit


true ϕ
best-fit ϕ
difference
theory
$\ell^4 C_\ell^{\phi\phi}$
ℓ
10^{-6}
10^{-7}
10^{-8}
10^1
10^2
10^3


w.r.t. true covariance
w.r.t. cooling covariance
1/2/3σ expected at best-fit
$-2\ln\mathcal{P}$
step number
10^9
10^8
10^7
1.73×10^6
1.72×10^6
1.71×10^6
1.70×10^6
1.69×10^6
1.68×10^6
1.67×10^6
1.66×10^6
0
5
10
15
20
25
30