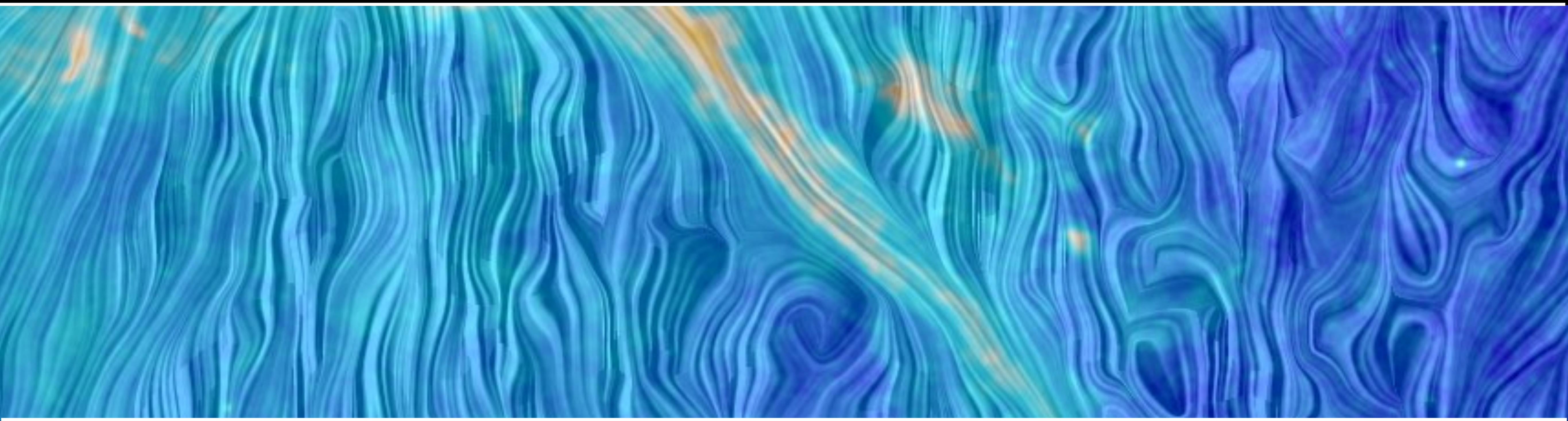
MOMENT EXPANSION

of the Galactic dust cross-spectra SED

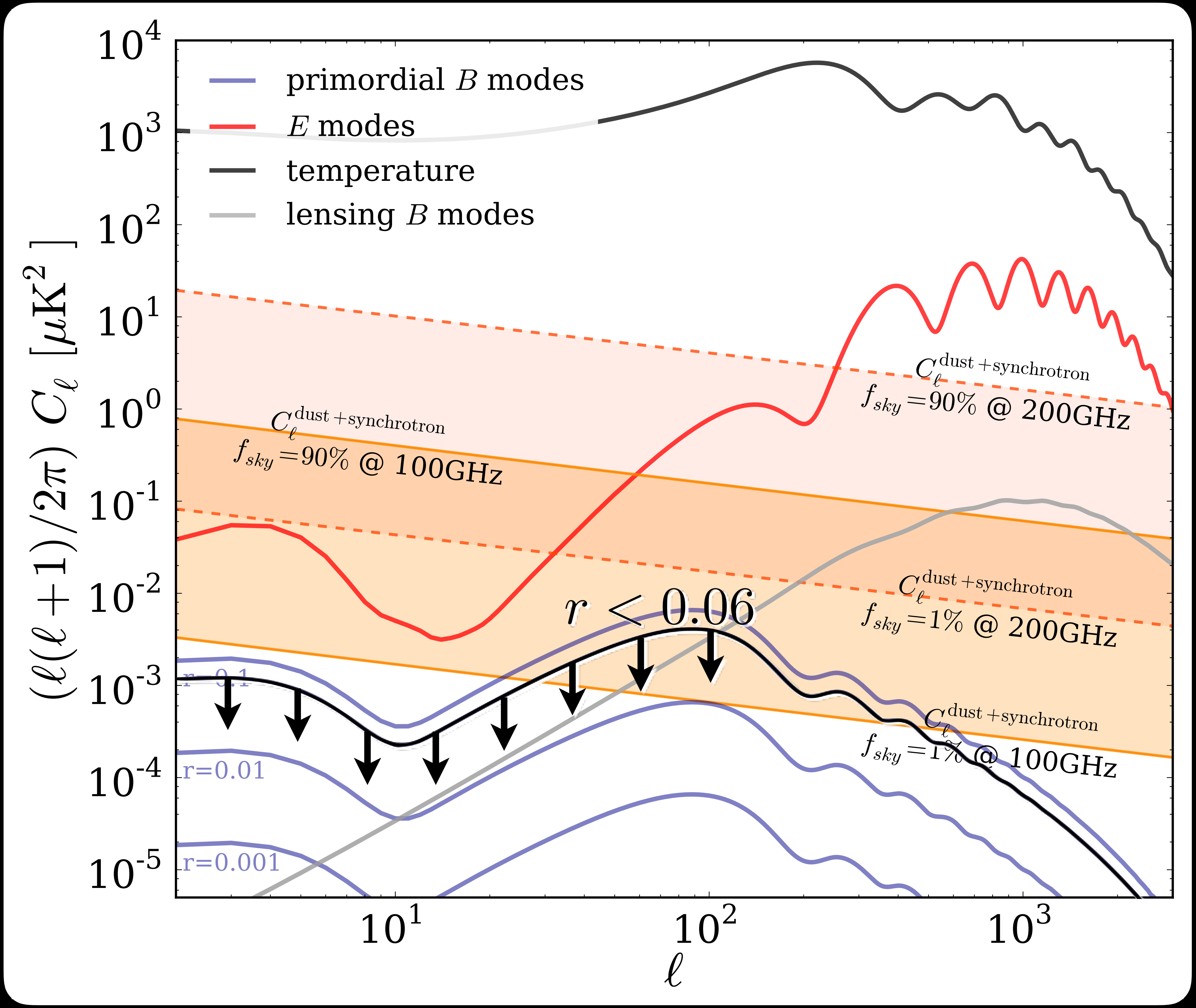


Jonathan Aumont

IRAP — Toulouse, France

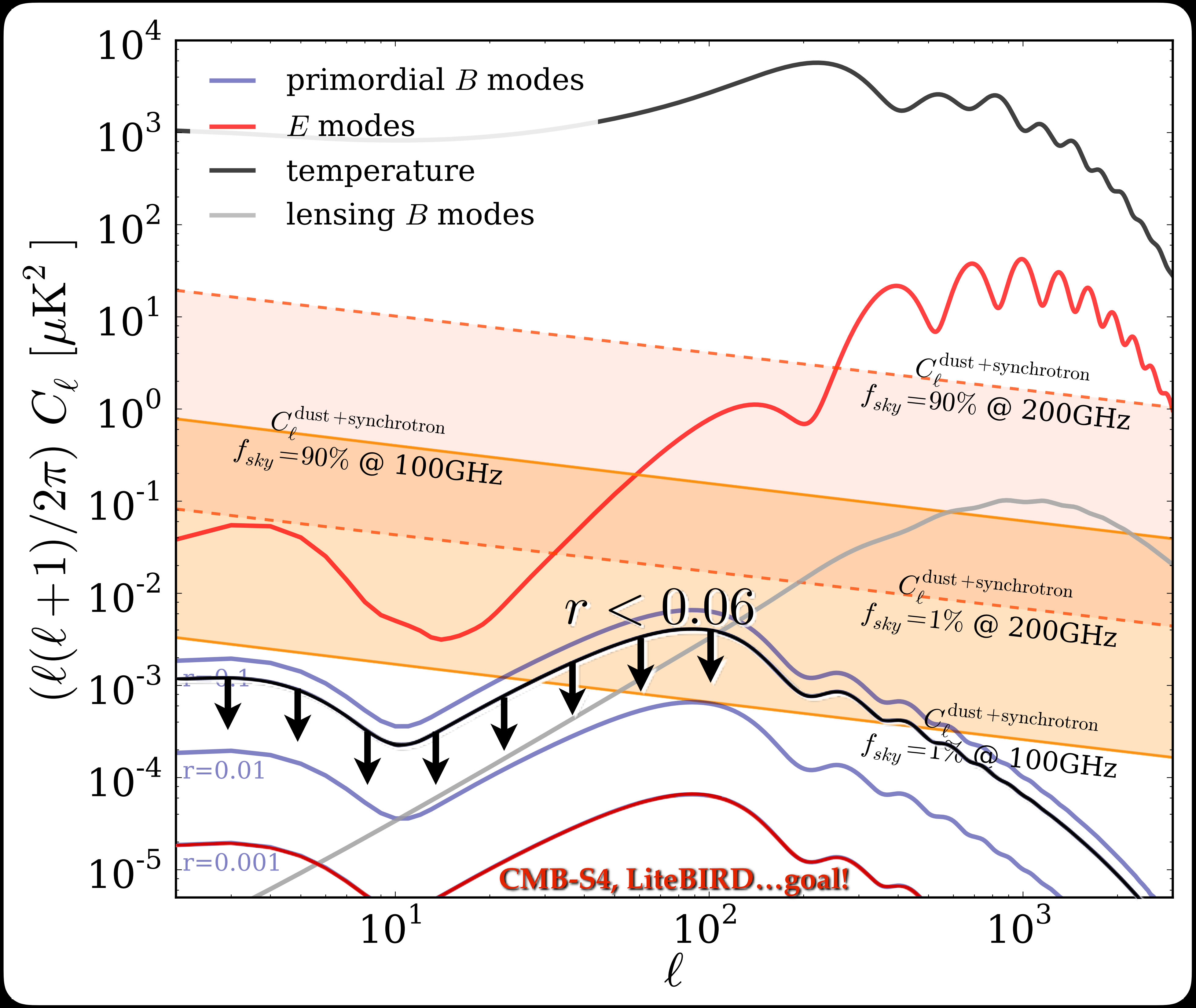
on behalf of A. Mangilli, A. Rotti, F. Boulanger, J. Chluba,
T. Ghosh & L. Montier

INTRODUCTION — The quest for CMB *B*-modes



[CMB-S4 Science Book, 2016]
[BICEP/Keck Collaborations, 2018]

INTRODUCTION — The quest for CMB *B*-modes



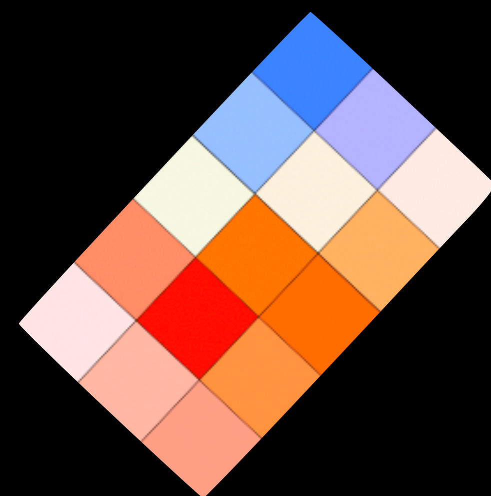
[CMB-S4 Science Book, 2016]
[BICEP/Keck Collaborations, 2018]

The challenge is huge: foregrounds have to be understood (and removed) with an exquisite accuracy

DUST — Dust SED spatial variations

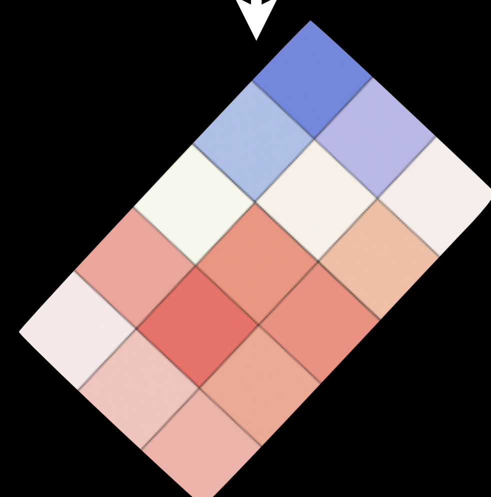
Sky

$\nu_{\mathrm{high-frequency}}$

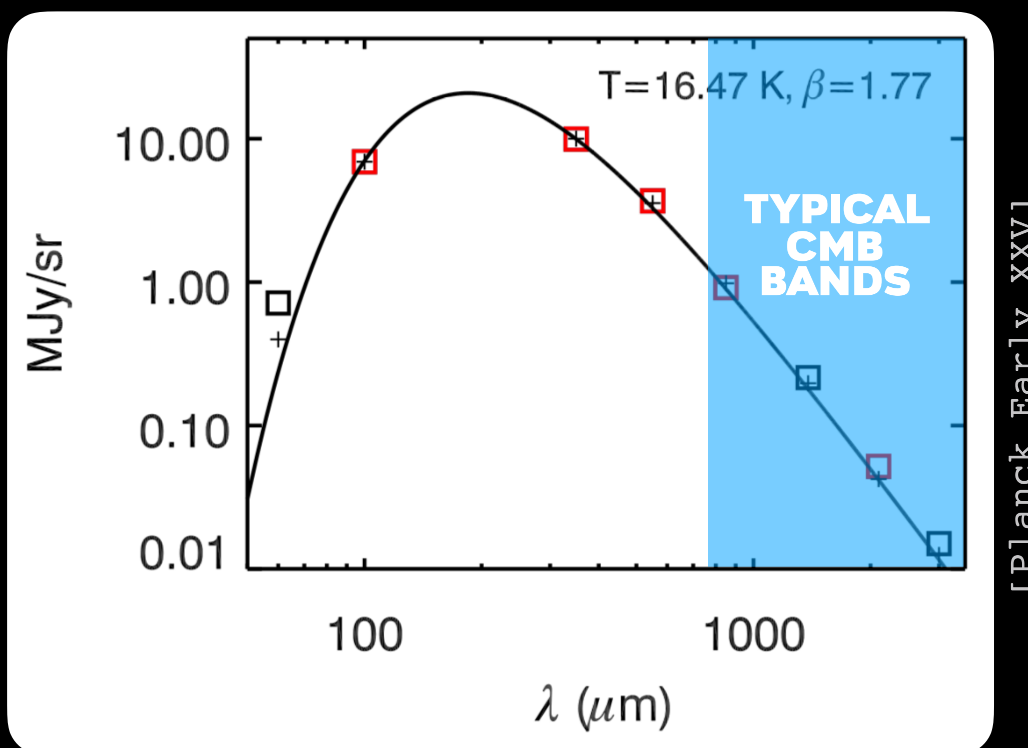


$$I_\nu \propto \nu^\beta B_\nu(T)$$

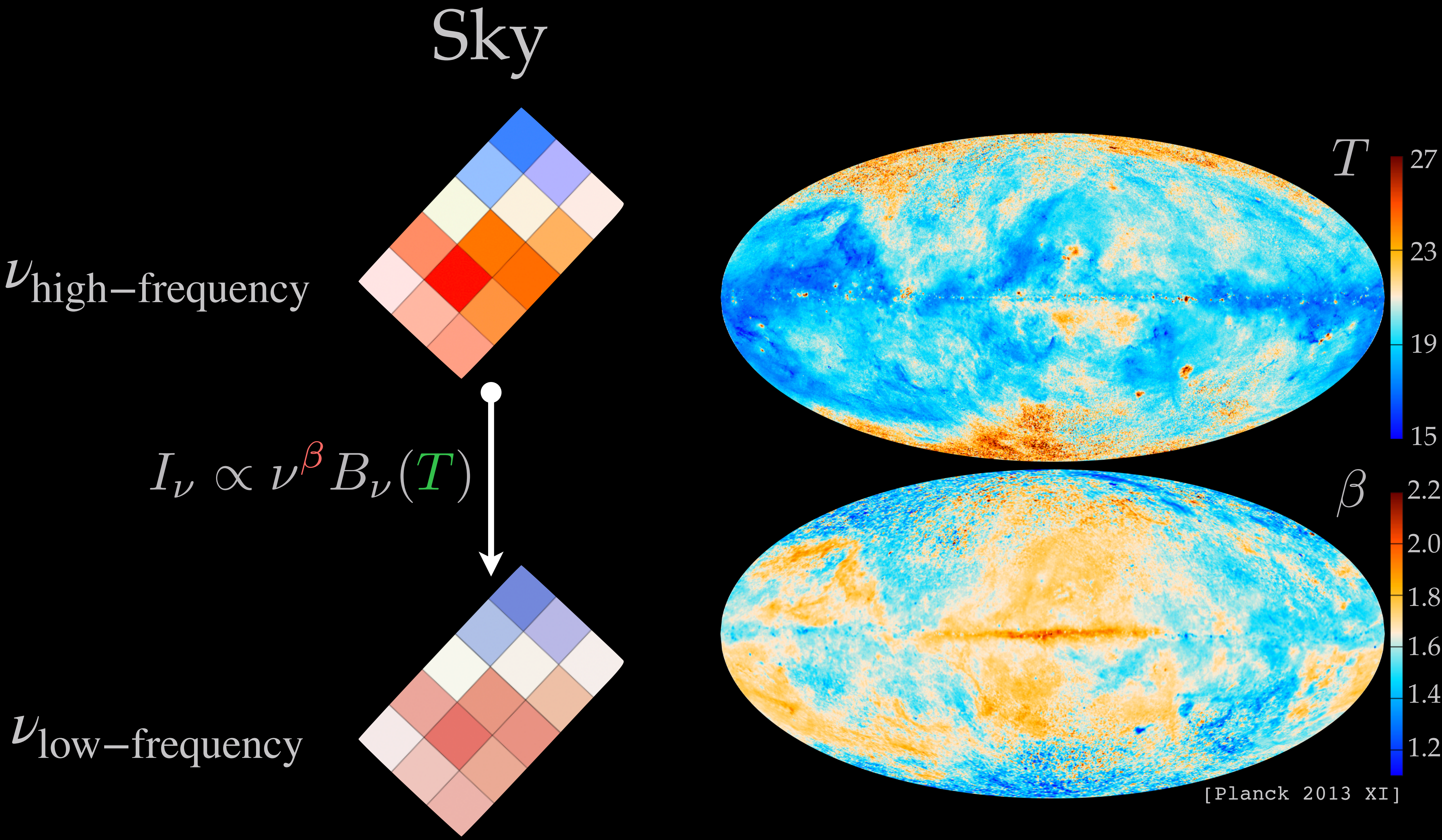
$\nu_{\mathrm{low-frequency}}$



Modified black body (MBB)



DUST — Dust SED spatial variations



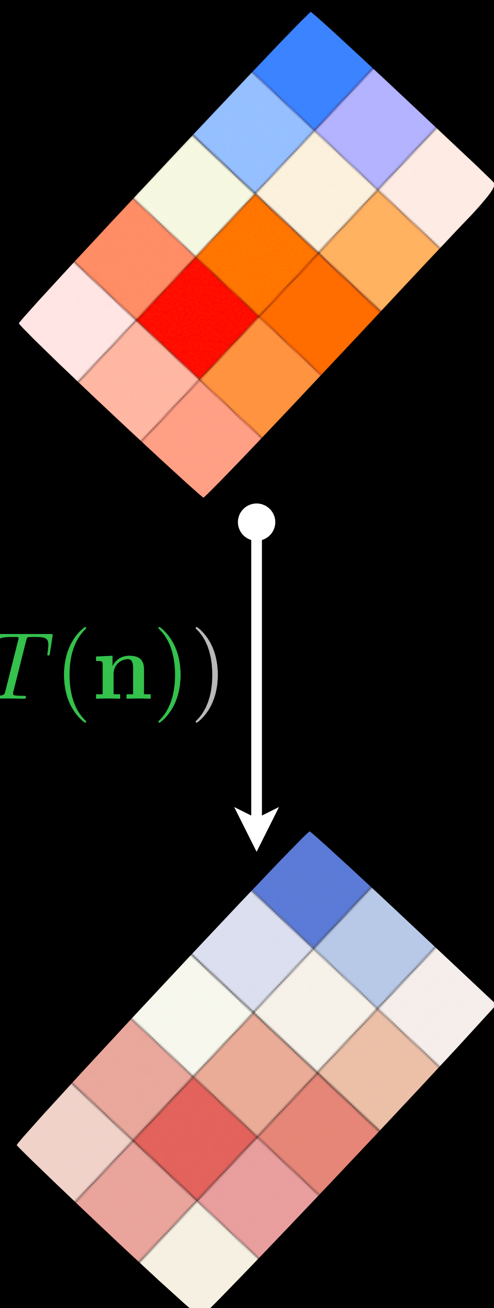
DUST — Dust SED spatial variations

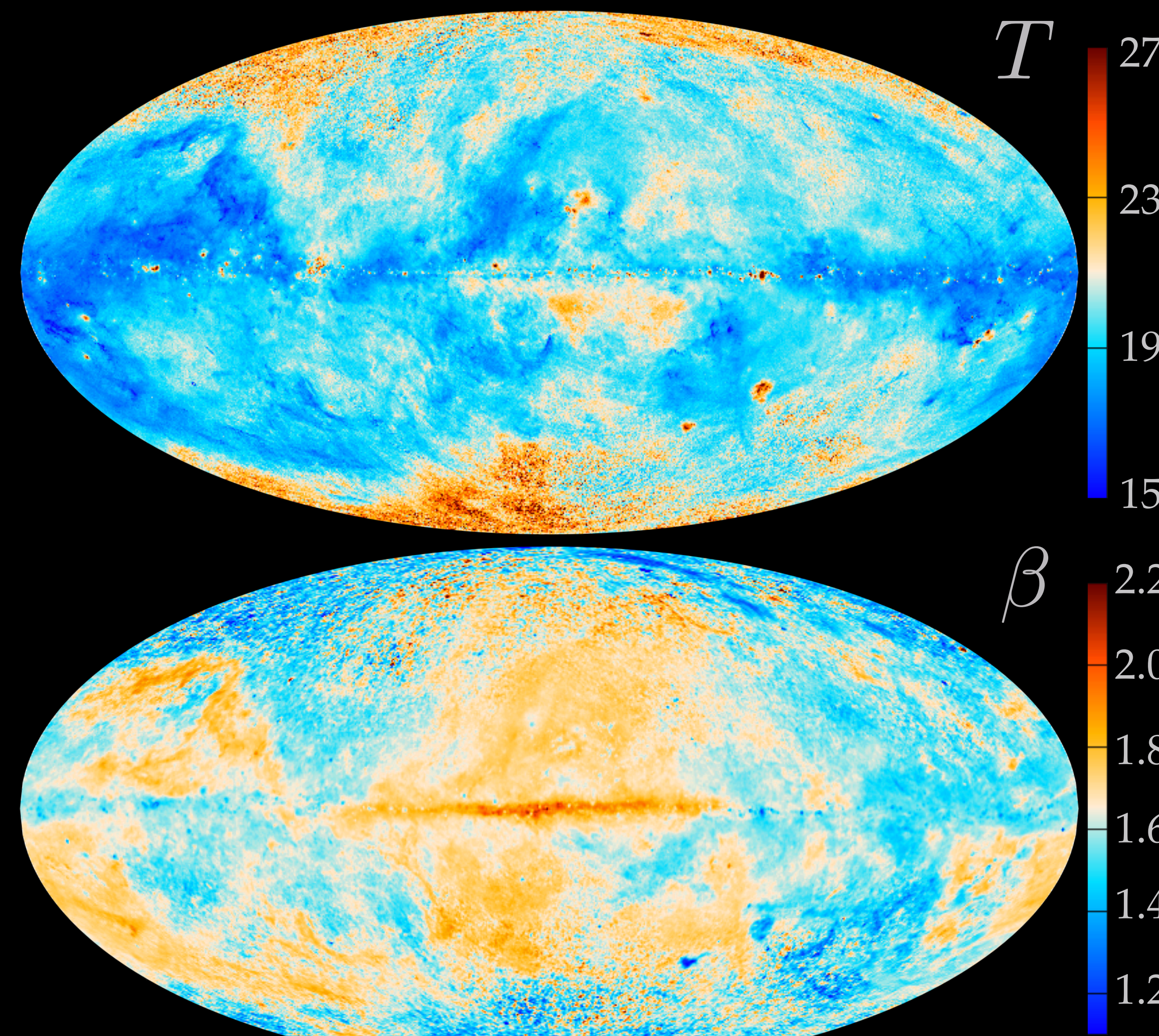
Sky

$\nu_{\mathrm{high-frequency}}$

$I_\nu \propto \nu^{\beta(\mathbf{n})} B_\nu(T(\mathbf{n}))$

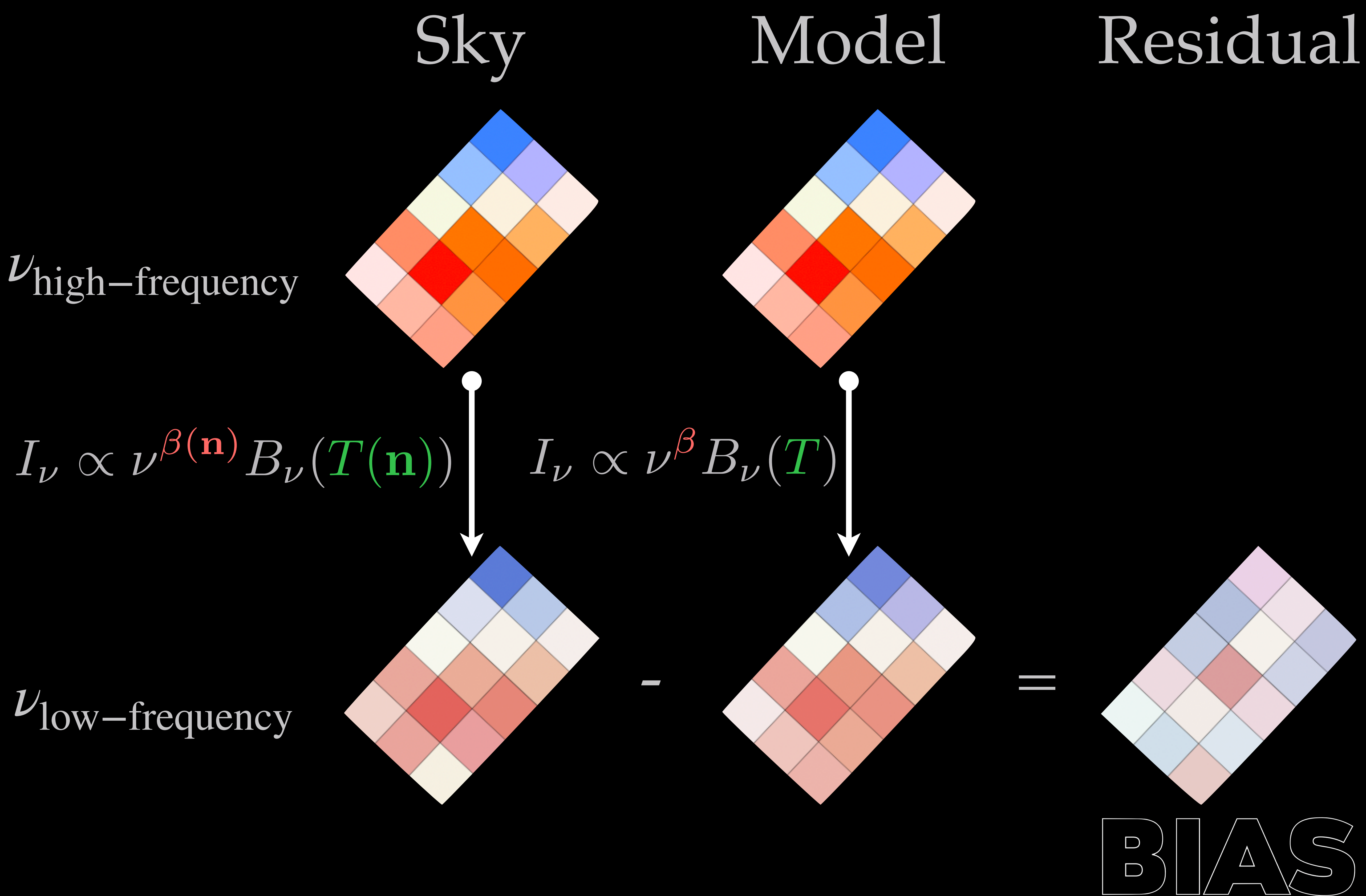
$\nu_{\mathrm{low-frequency}}$





[Planck 2013 XI]

DUST — Dust SED spatial variations



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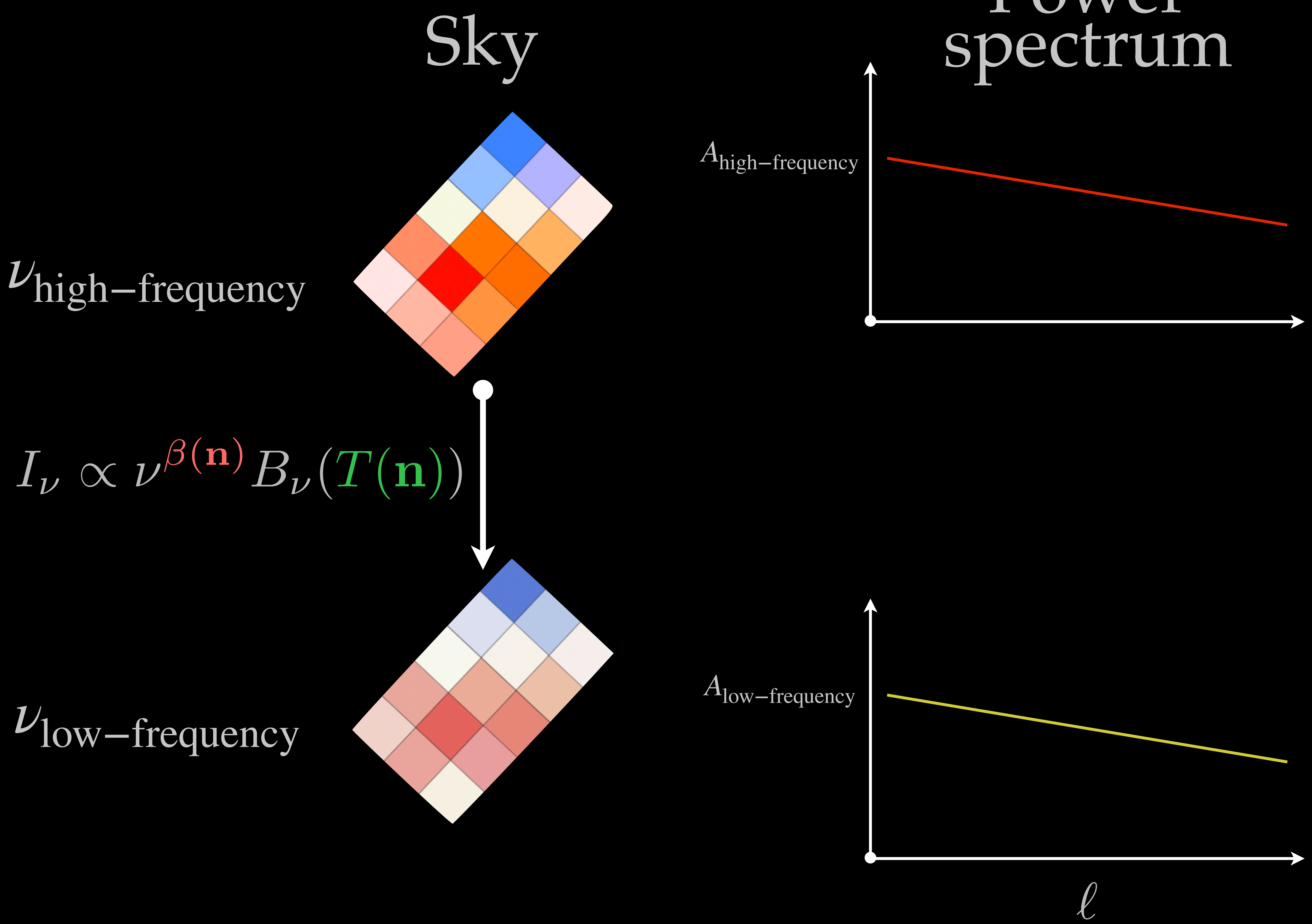

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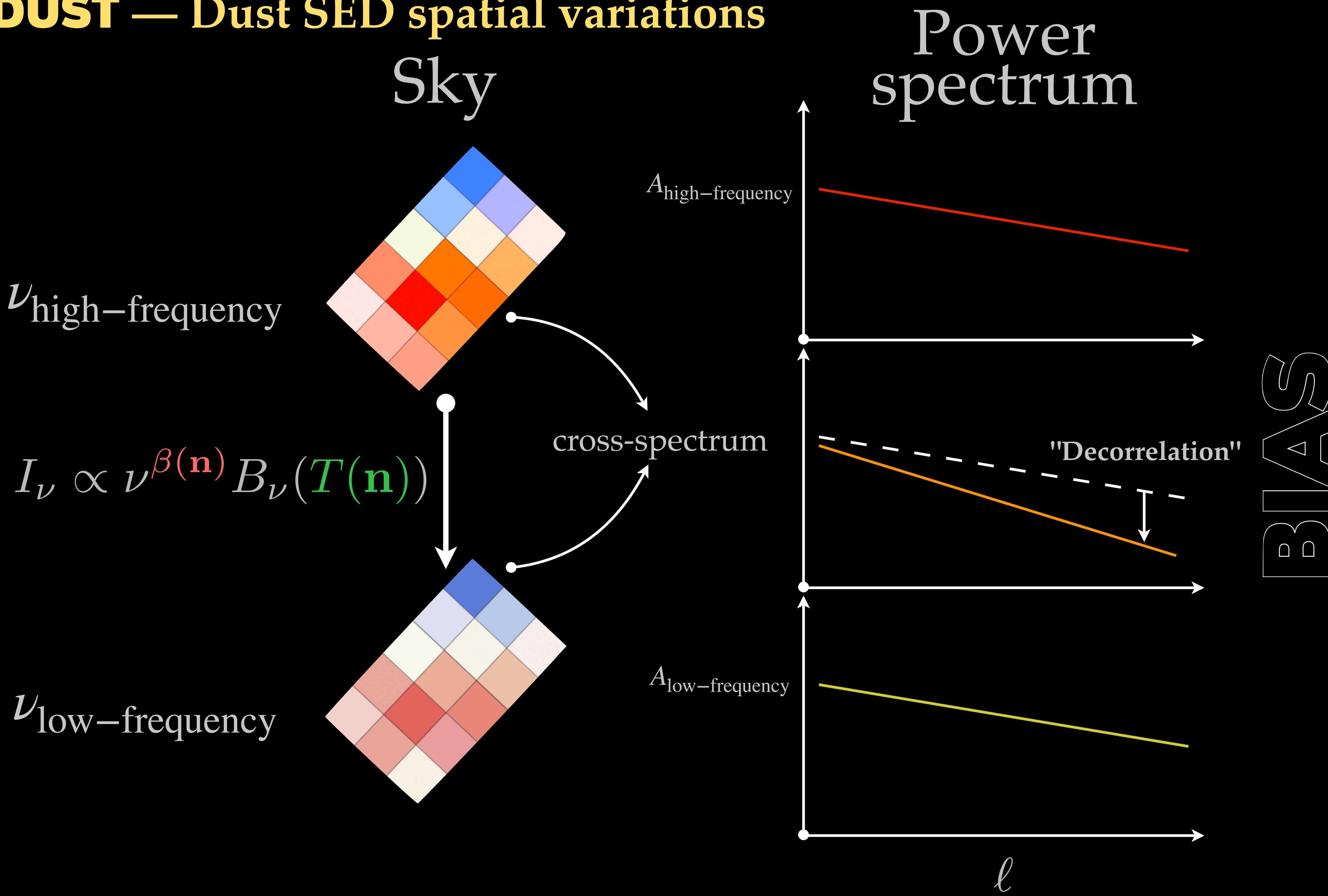
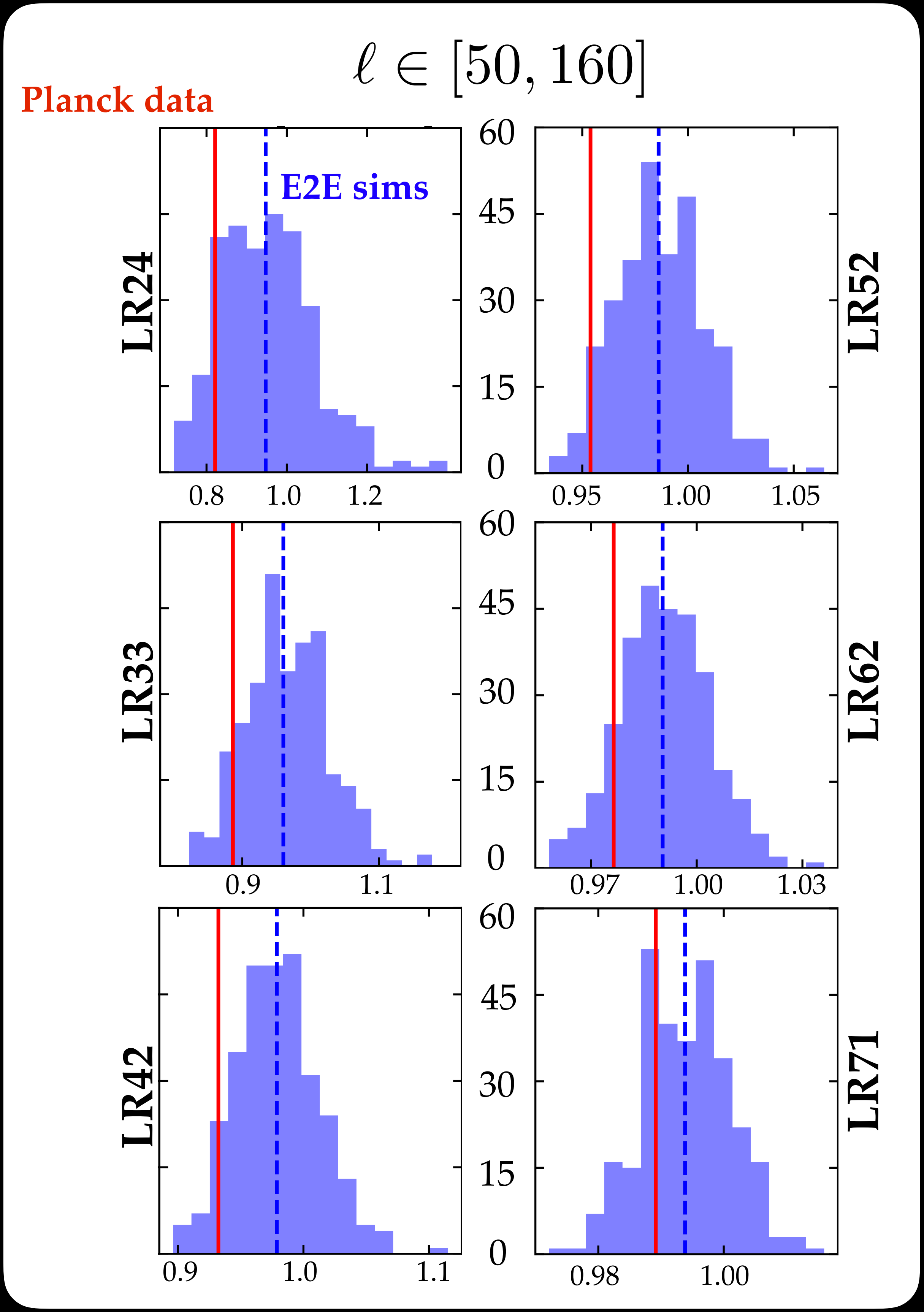


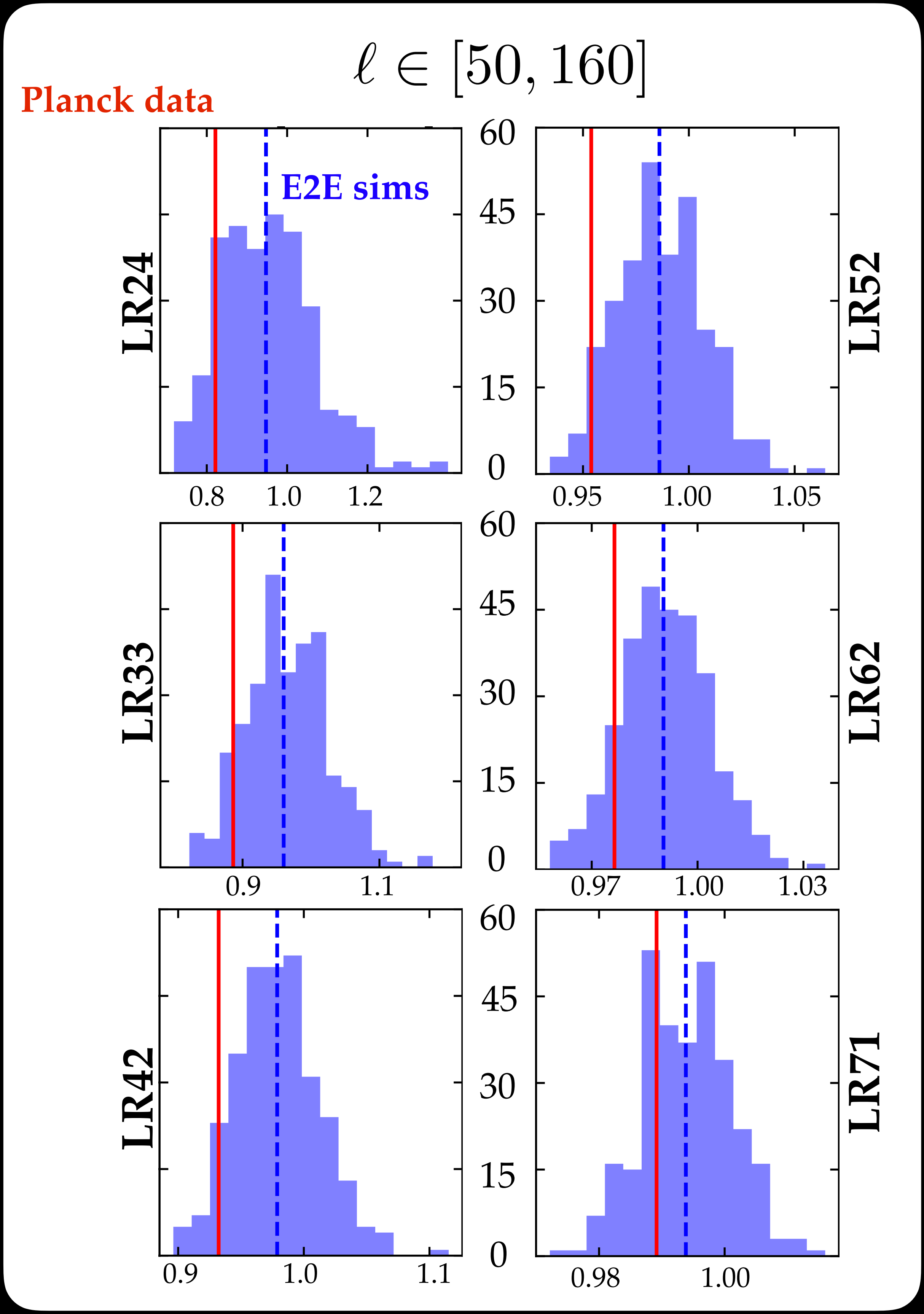
DUST — *B*-mode decorrelation as constrained by Planck



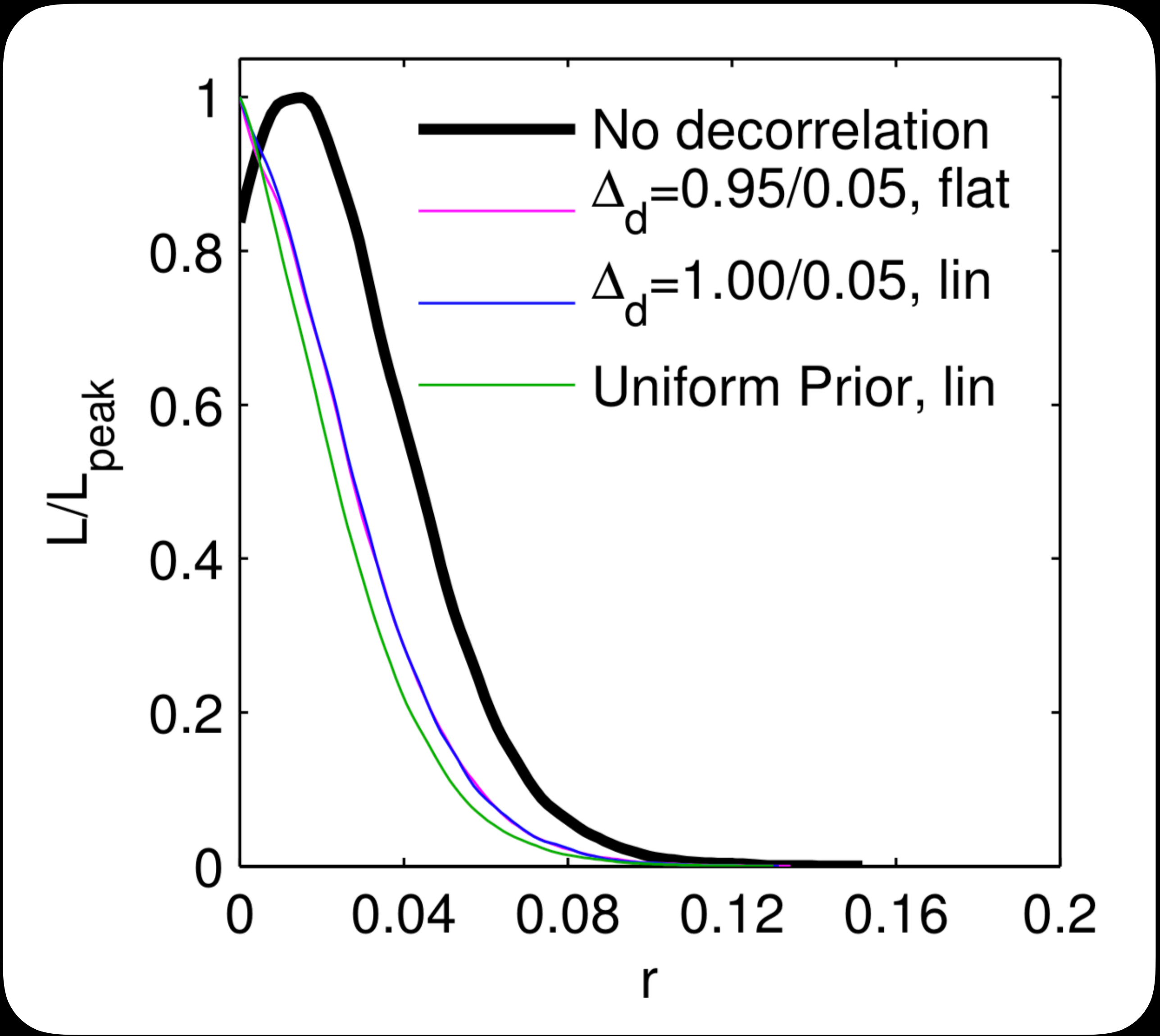
[Planck 2018 XI]

$$\mathcal{R}_\ell^{BB} \equiv \frac{C_\ell^{BB}(217 \times 353)}{\sqrt{C_\ell^{BB}(217 \times 217) C_\ell^{BB}(353 \times 353)}}$$

DUST — Decorrelation as constrained by Planck and BICEP

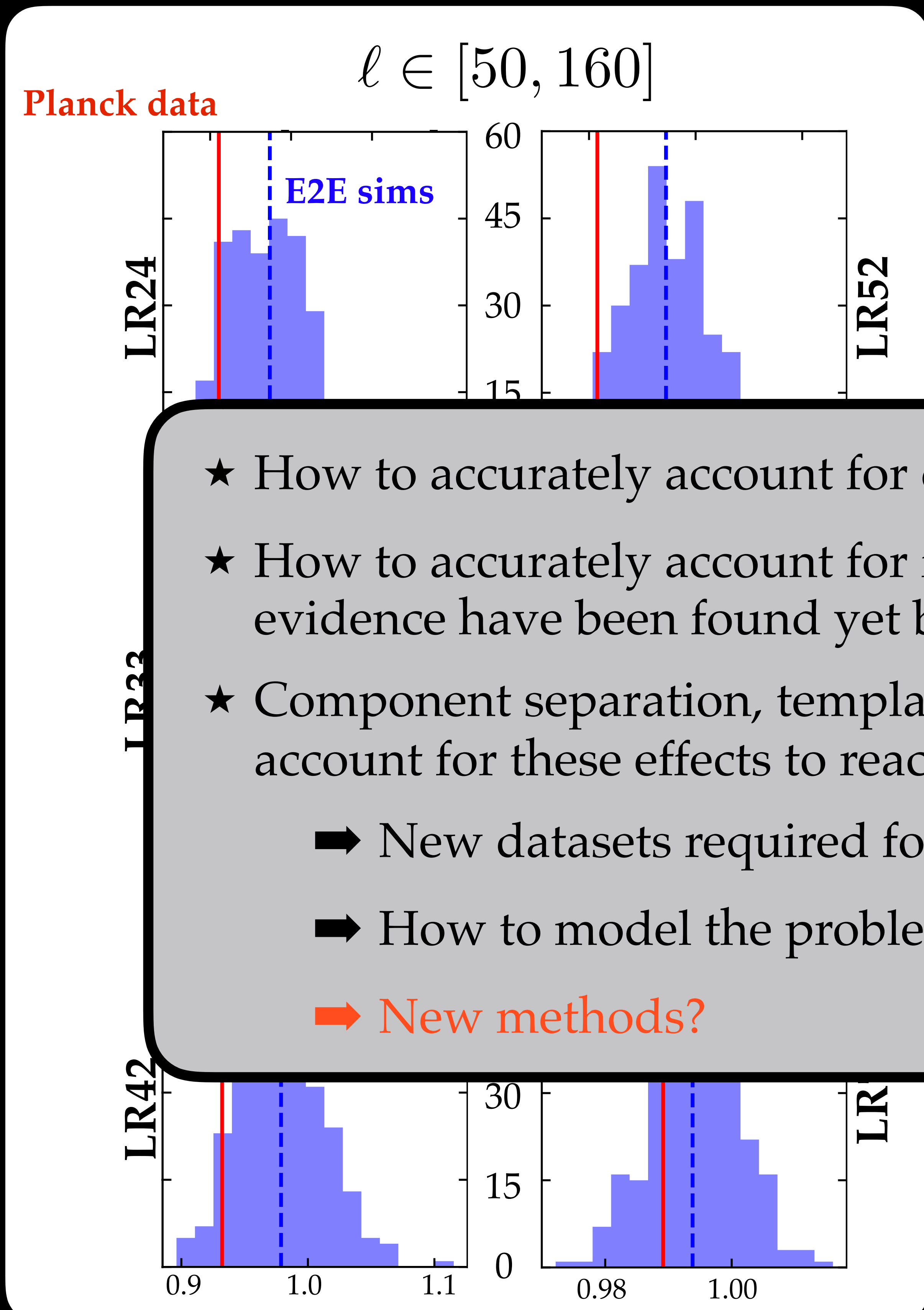


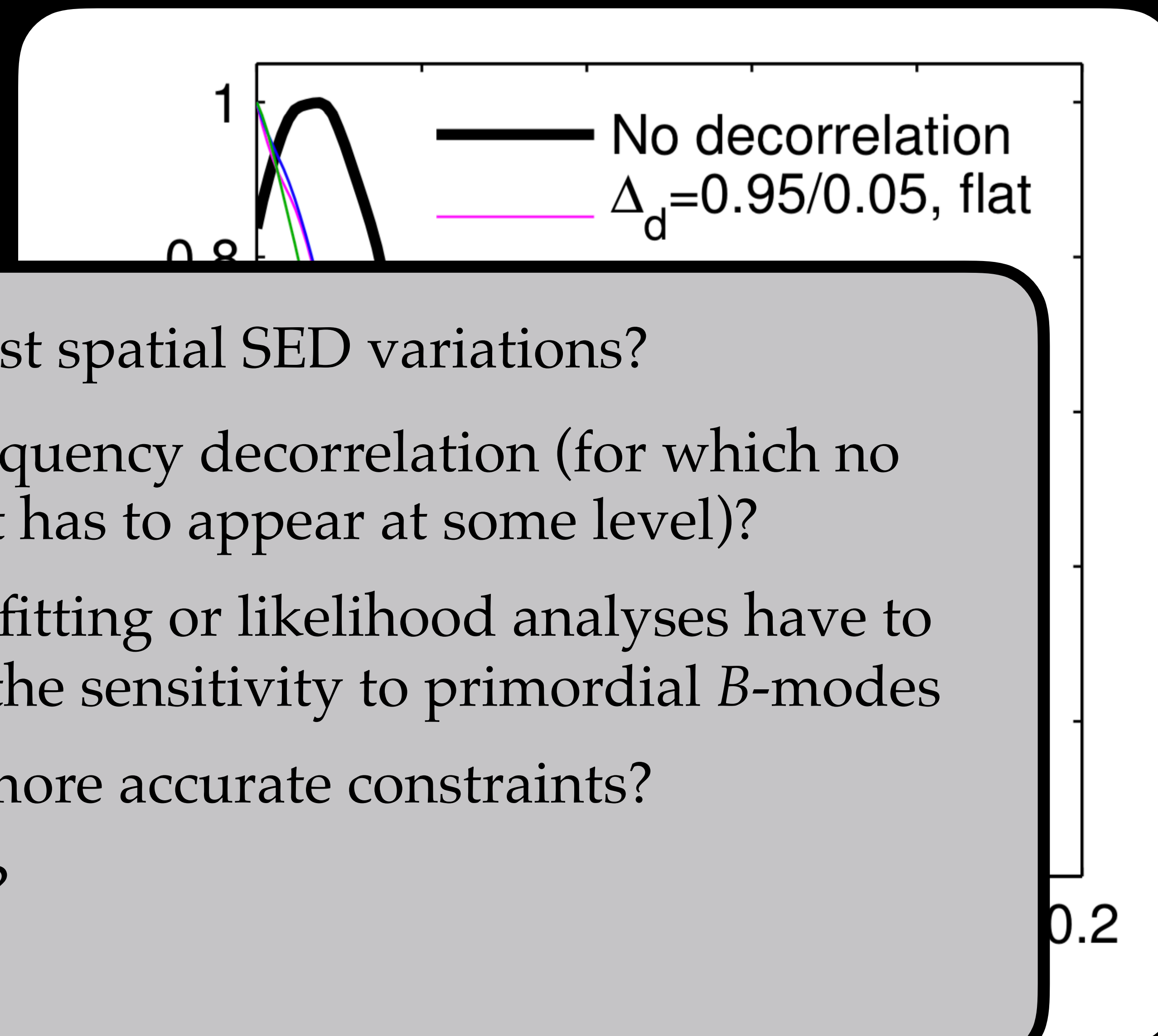
[Planck 2018 XI]



[BICEP/Keck X 2018]

DUST — Decorrelation as constrained by Planck and BICEP

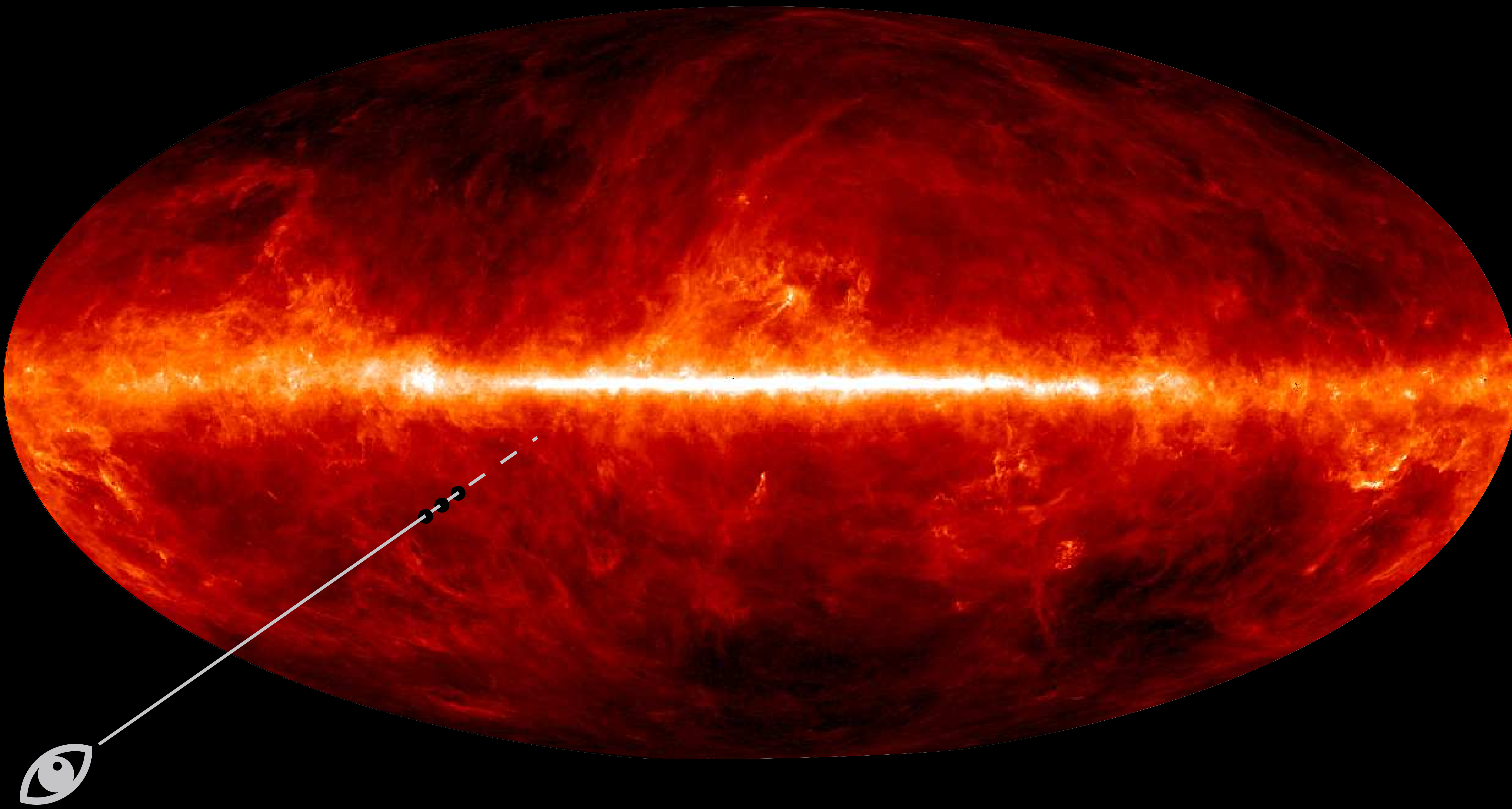




[BICEP/Keck x 2018]

- ★ How to accurately account for dust spatial SED variations?
- ★ How to accurately account for frequency decorrelation (for which no evidence have been found yet but has to appear at some level)?
- ★ Component separation, template fitting or likelihood analyses have to account for these effects to reach the sensitivity to primordial *B*-modes
  - ➡ New datasets required for more accurate constraints?
  - ➡ How to model the problem?
  - ➡ New methods?

DUST — Averaging SEDs

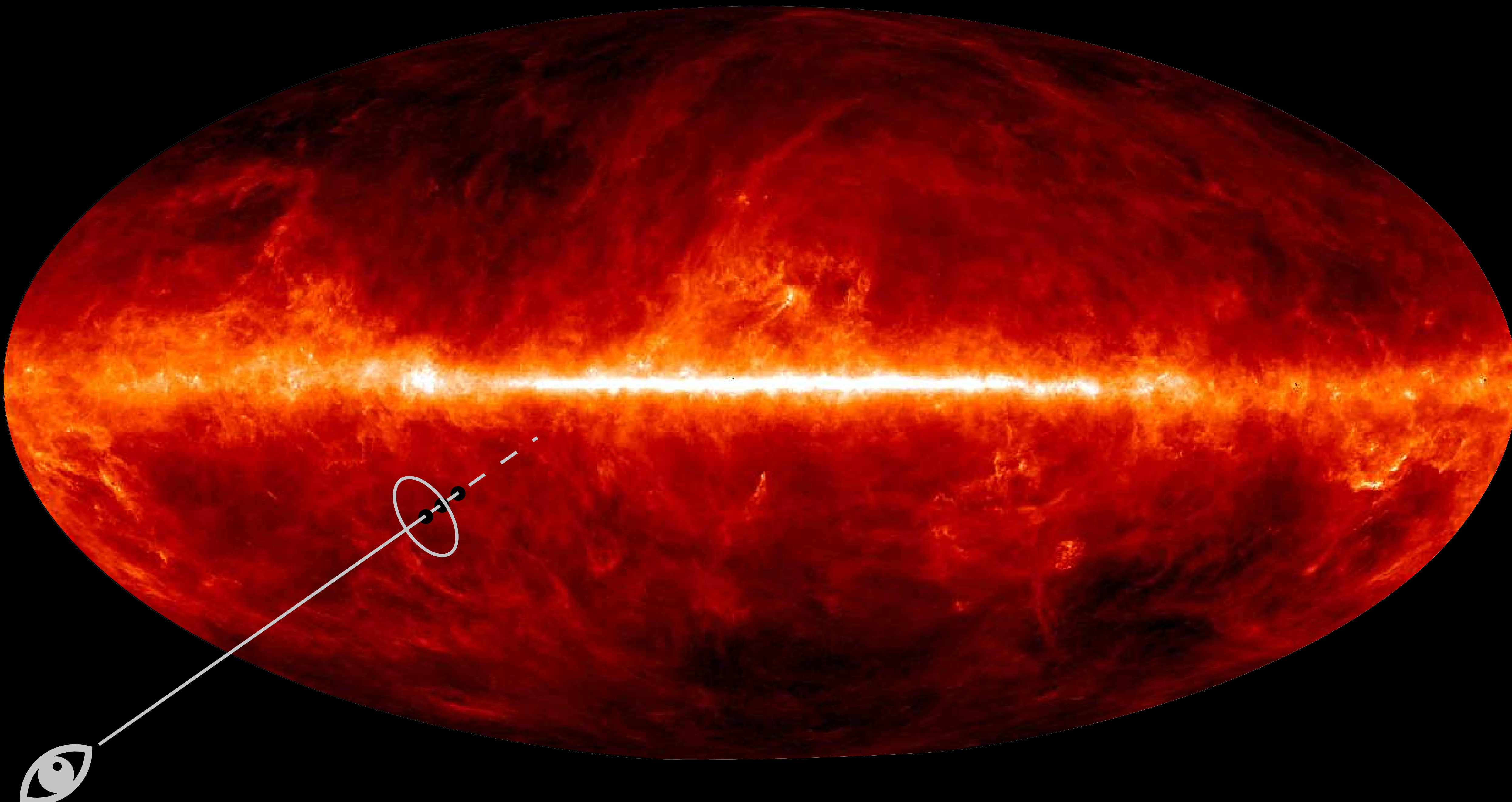


[Planck 2018 IV]

Modified black-body SED in *every* volume element

- ⋆ Line-of-sight average (*always there*!)

DUST — Averaging SEDs

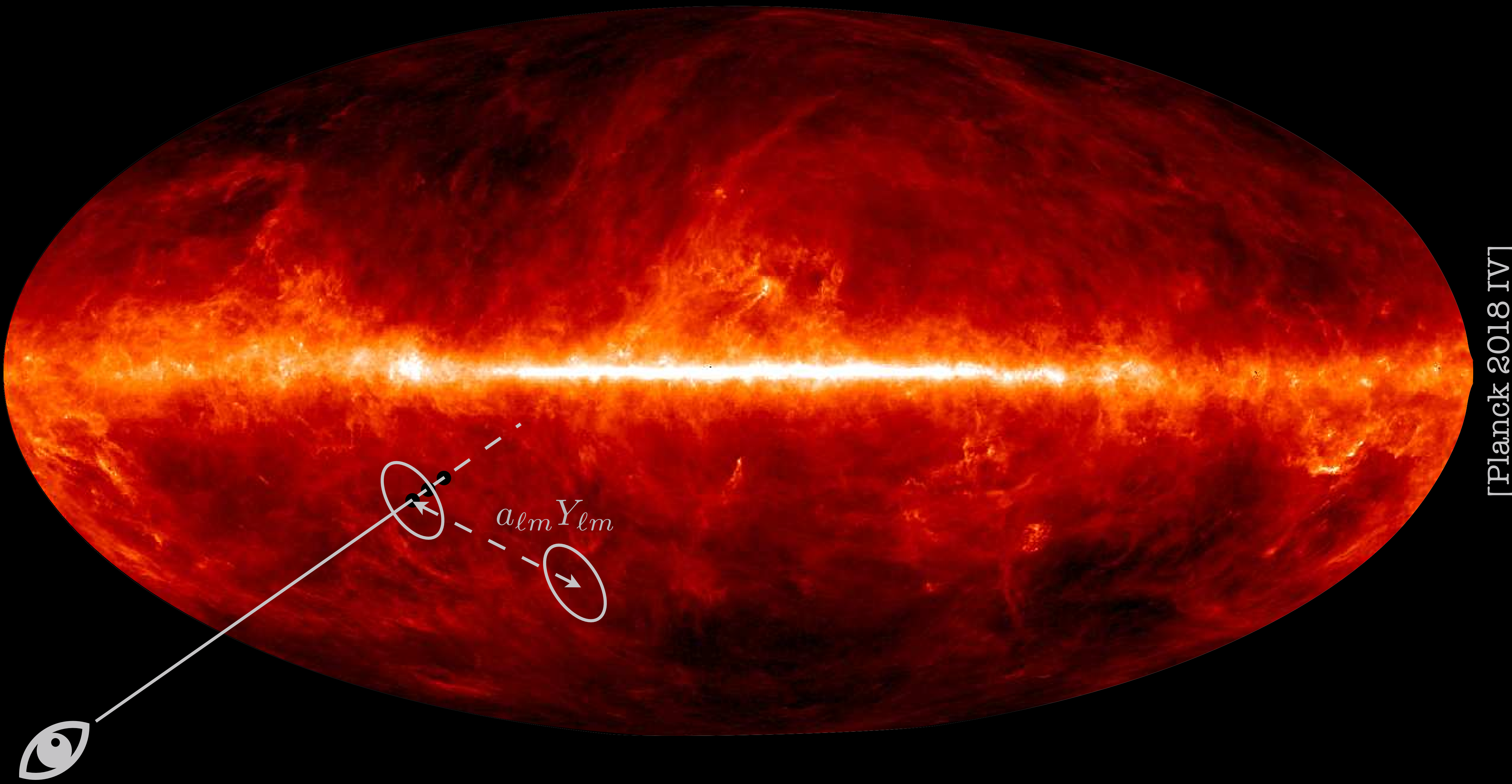


[Planck 2018 IV]

Modified black-body SED in *every* volume element

- ★ Line-of-sight average (*always there*!)
- ★ Experimental beam and frequency average

DUST — Averaging SEDs



Modified black-body SED in *every* volume element

- ⋆ Line-of-sight average (*always there*!)
- ⋆ Experimental beam and frequency average
- ⋆ Map operations average (e.g., spherical harmonic expansion)

DUST — Averaging SEDs

$$I_\nu\left(p\left(\vec{n}_i\right)\right) + I_\nu\left(p\left(\vec{n}_j\right)\right) + \cdots$$

Line-of-sight, beam or spherical harmonics average →

- ⋆ Not anymore accurately described by the fondamental SED
- ⋆ Can produce a significant scale (ℓ) dependence of the SED [Chluba et al., 2017]

DUST — Moment expansion

$$\left.\begin{matrix} I_\nu\left(p\left(\vec{n}_i\right)\right) \\ + \\ I_\nu\left(p\left(\vec{n}_j\right)\right) \\ + \\ \cdots \end{matrix}\right\}$$

Line-of-sight, beam or spherical harmonics average

Moment expansion of the SED

$$\langle I_\nu(\mathbf{p})\rangle = I_\nu(\bar{\mathbf{p}}) + \sum_i \omega_i \partial_{p_i} I_\nu(\bar{\mathbf{p}}) + \frac{1}{2}\sum_{ij} \omega_{ij} \partial_{p_i}\partial_{p_j} I_\nu(\bar{\mathbf{p}}) + \frac{1}{6}\sum_{ijk} \omega_{ijk} \partial_{p_i}\partial_{p_j}\partial_{p_k} I_\nu(\bar{\mathbf{p}}) + \ldots$$

[Chluba et al., 2017]

X-SPECTRA MOMENT EXPANSION — Formalism

MAP DOMAIN

modified black-body spectrum
no SED variations

dust map

$$I_{\mathrm{D}}(\nu, \hat{\mathbf{n}}) = \frac{I_\nu(T_0, \beta_0)}{I_{\nu_0}(T_0, \beta_0)} A_{\mathrm{D}}(\hat{\mathbf{n}})$$

X-SPECTRA MOMENT EXPANSION — Formalism

MAP DOMAIN

$$I_{\rm D}(\nu, \hat{\mathbf{n}}) \neq \frac{I_\nu(T_0, \beta_0)}{I_{\nu_0}(T_0, \beta_0)} A_{\rm D}(\hat{\mathbf{n}})$$

$$I_{\rm D}(\nu, \hat{\mathbf{n}}) \simeq \frac{I_\nu(T_0, \beta_0)}{I_{\nu_0}(T_0, \beta_0)} \left[A_{\rm D}(\hat{\mathbf{n}}) + \omega_1(\hat{\mathbf{n}}) \ln \frac{\nu}{\nu_0} + \frac{1}{2}\omega_2(\hat{\mathbf{n}}) \ln^2 \frac{\nu}{\nu_0} + \frac{1}{6}\omega_3(\hat{\mathbf{n}}) \ln^3 \frac{\nu}{\nu_0} + \dots \right]$$

[Chluba et al., 2017]

X-SPECTRA MOMENT EXPANSION — Formalism

SPHERICAL HARMONICS DOMAIN

ℓ-dependent β

$$(I_\mathrm{D}(\nu))_{\ell m} \simeq \frac{I_\nu(T_0, \overline{\beta_0(\ell)})}{I_{\nu_0}(T_0, \beta_0(\ell))}\Big[(A_\mathrm{D})_{\ell m} + (\omega_1)_{\ell m} \ln\frac{\nu}{\nu_0} + \frac{1}{2}(\omega_2)_{\ell m}\ln^2\frac{\nu}{\nu_0} + \frac{1}{6}(\omega_3)_{\ell m}\ln^3\frac{\nu}{\nu_0} + \dots\Big]$$

[Mangilli et al., 2019, to be submitted]

X-SPECTRA MOMENT EXPANSION — Formalism

POWER SPECTRUM DOMAIN

Order 0 (MBB)

$$\mathcal{D}_\ell(\nu_1 \times \nu_2) = \frac{I_{\nu_1}(T_0, \beta_0(\ell)) I_{\nu_2}(T_0, \beta_0(\ell))}{I_{\nu_0}(T_0, \beta_0(\ell))^2} \Big\{ \mathcal{D}_\ell^{A_D A_D}$$

Order 1

$$+ \left[\ln \frac{\nu_1}{\nu_0} + \ln \frac{\nu_2}{\nu_0} \right] \mathcal{D}_\ell^{A_D \omega_1}$$

Leading order term ~ bias on , $\beta_0(\ell)$ in the limit of many moments

$$+ \left[\ln \frac{\nu_1}{\nu_0} \ln \frac{\nu_2}{\nu_0} \right] \mathcal{D}_\ell^{\omega_1 \omega_1}$$

Order 2

$$+ \frac{1}{2} \left[\ln^2 \frac{\nu_1}{\nu_0} + \ln^2 \frac{\nu_2}{\nu_0} \right] \mathcal{D}_\ell^{A_D \omega_2}$$

$$+ \frac{1}{2} \left[\ln \frac{\nu_1}{\nu_0} \ln^2 \frac{\nu_2}{\nu_0} + \ln \frac{\nu_2}{\nu_0} \ln^2 \frac{\nu_1}{\nu_0} \right] \mathcal{D}_\ell^{\omega_1 \omega_2}$$

$$+ \frac{1}{4} \left[\ln^2 \frac{\nu_1}{\nu_0} \ln^2 \frac{\nu_2}{\nu_0} \right] \mathcal{D}_\ell^{\omega_2 \omega_2}$$

Order 3

$$+ \frac{1}{6} \left[\ln^3 \frac{\nu_1}{\nu_0} + \ln^3 \frac{\nu_2}{\nu_0} \right] \mathcal{D}_\ell^{A_D \omega_3}$$

$$+ \frac{1}{6} \left[\ln \frac{\nu_1}{\nu_0} \ln^3 \frac{\nu_2}{\nu_0} + \ln \frac{\nu_2}{\nu_0} \ln^3 \frac{\nu_1}{\nu_0} \right] \mathcal{D}_\ell^{\omega_1 \omega_3}$$

$$+ \frac{1}{12} \left[\ln^2 \frac{\nu_1}{\nu_0} \ln^3 \frac{\nu_2}{\nu_0} + \ln^2 \frac{\nu_2}{\nu_0} \ln^3 \frac{\nu_1}{\nu_0} \right] \mathcal{D}_\ell^{\omega_2 \omega_3}$$

$$+ \frac{1}{36} \left[\ln^3 \frac{\nu_1}{\nu_0} \ln^3 \frac{\nu_2}{\nu_0} \right] \mathcal{D}_\ell^{\omega_3 \omega_3}$$

$$+ \ldots \Big\}$$

[Mangilli et al., 2019, to be submitted]

X-SPECTRA MOMENT EXPANSION — Formalism

Order 0 (MBB)

$$\mathcal{D}_\ell(\nu_1 \times \nu_2) = \frac{I_{\nu_1}(T_0, \beta_0(\ell)) I_{\nu_2}(T_0, \beta_0(\ell))}{I_{\nu_0}(T_0, \beta_0(\ell))^2} \Big\{ \mathcal{D}_\ell^{A_D A_D}$$

Order 1

$$+ \left[\ln \frac{\nu_1}{\nu_0} + \ln \frac{\nu_2}{\nu_0} \right] \mathcal{D}_\ell^{A_D \omega_1}$$

$$+ \left[\ln \frac{\nu_1}{\nu_0} \ln \frac{\nu_2}{\nu_0} \right] \mathcal{D}_\ell^{\omega_1 \omega_1}$$

$$+ \frac{1}{2} \left[\ln^2 \frac{\nu_1}{\nu_0} + \ln^2 \frac{\nu_2}{\nu_0} \right] \mathcal{D}_\ell^{A_D \omega_2}$$

Leading order term ~ bias on , $\beta_0(\ell)$ in the limit of many moments

- ★ New approach that allow to describe, at the angular power spectrum level, the dust SED distortions due to line-of-sight, beam and map operations averages
- ★ Naturally describes, in a model-independent way, the effect of spatial variations of the dust SED, additional dust components and in a more general way of dust frequency decorrelation!

$$+ \frac{1}{12} \left[\ln^2 \frac{\nu_1}{\nu_0} \ln^3 \frac{\nu_2}{\nu_0} + \ln^2 \frac{\nu_2}{\nu_0} \ln^3 \frac{\nu_1}{\nu_0} \right] \mathcal{D}_\ell^{\omega_2 \omega_3}$$

$$+ \frac{1}{36} \left[\ln^3 \frac{\nu_1}{\nu_0} \ln^3 \frac{\nu_2}{\nu_0} \right] \mathcal{D}_\ell^{\omega_3 \omega_3}$$

$$+ \dots \Big\}$$

POWER SPECTRUM DOMAIN

[Mangilli et al., 2019, to be submitted]

X-SPECTRA MOMENT EXPANSION — Our work

- ★ Extension of the [Chluba et al., 2017] formalism to the cross-spectra of the dust emission
- ★ As working in the Rayleigh-Jeans regime of the dust spectra, we choose to expand the dust SED in β
  - ➡ B-mode analysis usually a cross-spectra analysis — accurate description of the SED is required at the cross-spectra level
  - ➡ Increase SNR with respect to map-based, no noise bias, systematic effects mitigation

[Mangilli et al., 2019, to be submitted]

X-SPECTRA MOMENT EXPANSION — Our work

- ★ Extension of the [Chluba et al., 2017] formalism to the cross-spectra of the dust emission
- ★ As working in the Rayleigh-Jeans regime of the dust spectra, we choose to expand the dust SED in β
  - ➡ *B*-mode analysis usually a cross-spectra analysis — accurate description of the SED is required at the cross-spectra level
  - ➡ No noise bias, systematic effects mitigation
- ★ Application to the Planck intensity data and comparison to simulations
  - ➡ Assess the dimensionality of the problem with high SNR data
  - ➡ Representative of future *B*-mode analysis in terms of signal-to-noise and frequency coverage
  - ➡ In polarization we will be sensitive to the intensity spectral variations — emphasized by magnetic field and grain properties effects

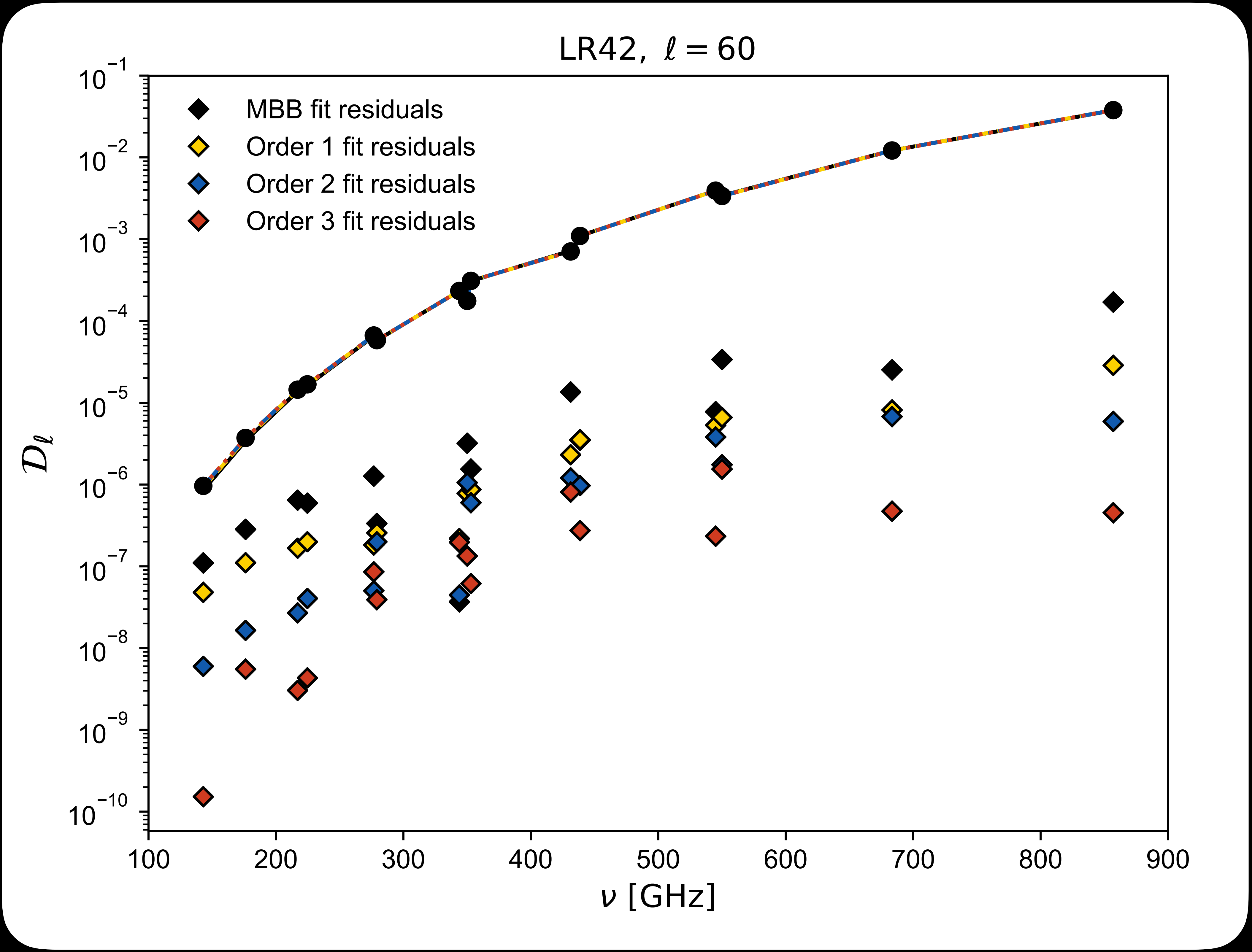
[Mangilli et al., 2019, to be submitted]

X-SPECTRA MOMENT EXPANSION — Implementation

- ★ Intensity cross-spectra from 143, 217, 353, 545 & 857 GHz half-mission maps
- ★ 15 cross-spectra, allow fits up to 3rd order
- ★ Cross-spectra computation with XPOL [Tristram et al. 2005], 15 ℓ-bins from 25 to 300
- ★ LR42 sky mask, CO masked
- ★ 3-step fit:
  - ➡ step 1: order 0, T_0 fixed, fit $\beta_0(\ell)$
  - ➡ step 2: fix $\beta_0(\ell)$ and fit higher orders moments
  - ➡ step 3: fix $\beta_0(\ell)+\Delta\beta_0(\ell)$ and fit higher orders moments
- ★ 5 intensity (simulated and real) data sets:
  - ➡ SIM1: dust emission without any spectral index variations (T_0 & $\beta_0 =$ csts)
  - ➡ SIM2: dust with random (Gaussian) spectral index variations ($\beta(\mathbf{n})$)
  - ➡ SIM3: dust with GNILC spectral index and temperature variations ($\beta(\mathbf{n})$, $T(\mathbf{n})$)
  - ➡ SIM4: SIM3 + CIB
  - ➡ PR3: actual Planck 2018 intensity data
- ★ 300 simulations with the same sky but varying Planck E2E noise + systematics simulations
- ★ Error bars and covariances computed from these 300 simulations
- ★ Mean noise+systematic cross-spectra subtracted
- ★ Dust intensity template from GNILC, CIB from PSM

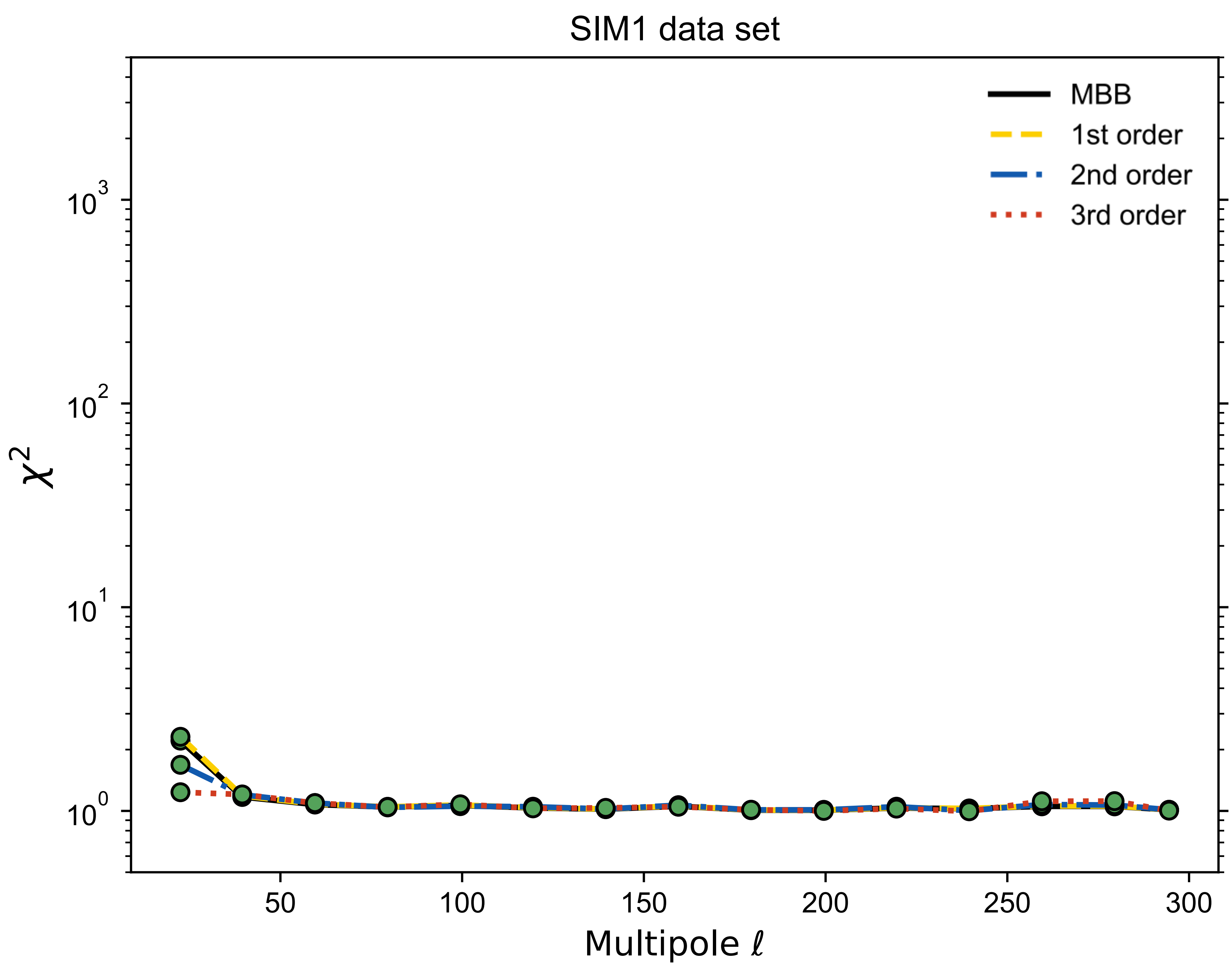
[Mangilli et al., 2019, to be submitted]

X-SPECTRA MOMENT EXPANSION — Cross-spectra SED fit



[Mangilli et al., 2019, to be submitted]

X-SPECTRA MOMENT EXPANSION — Goodness of fit



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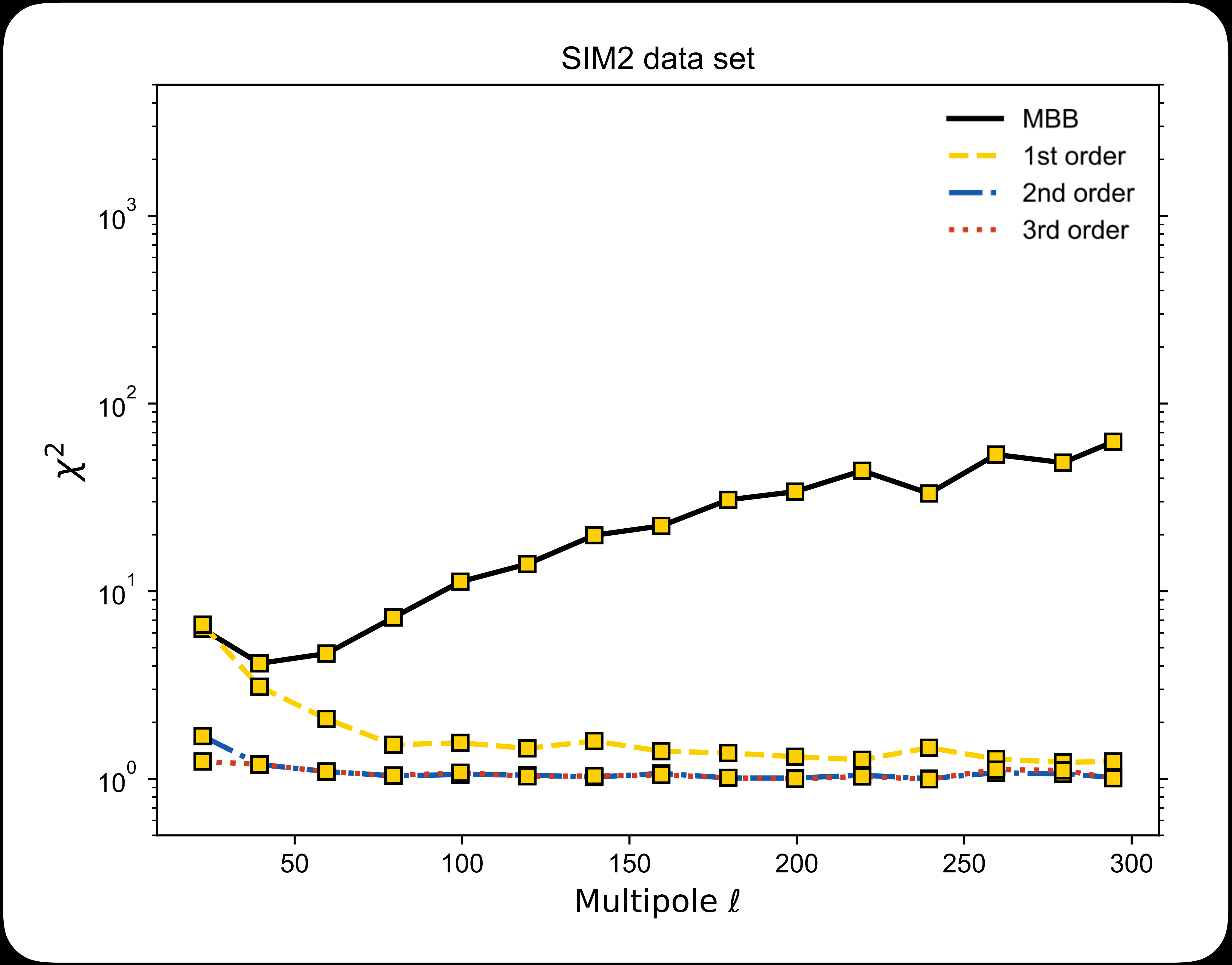

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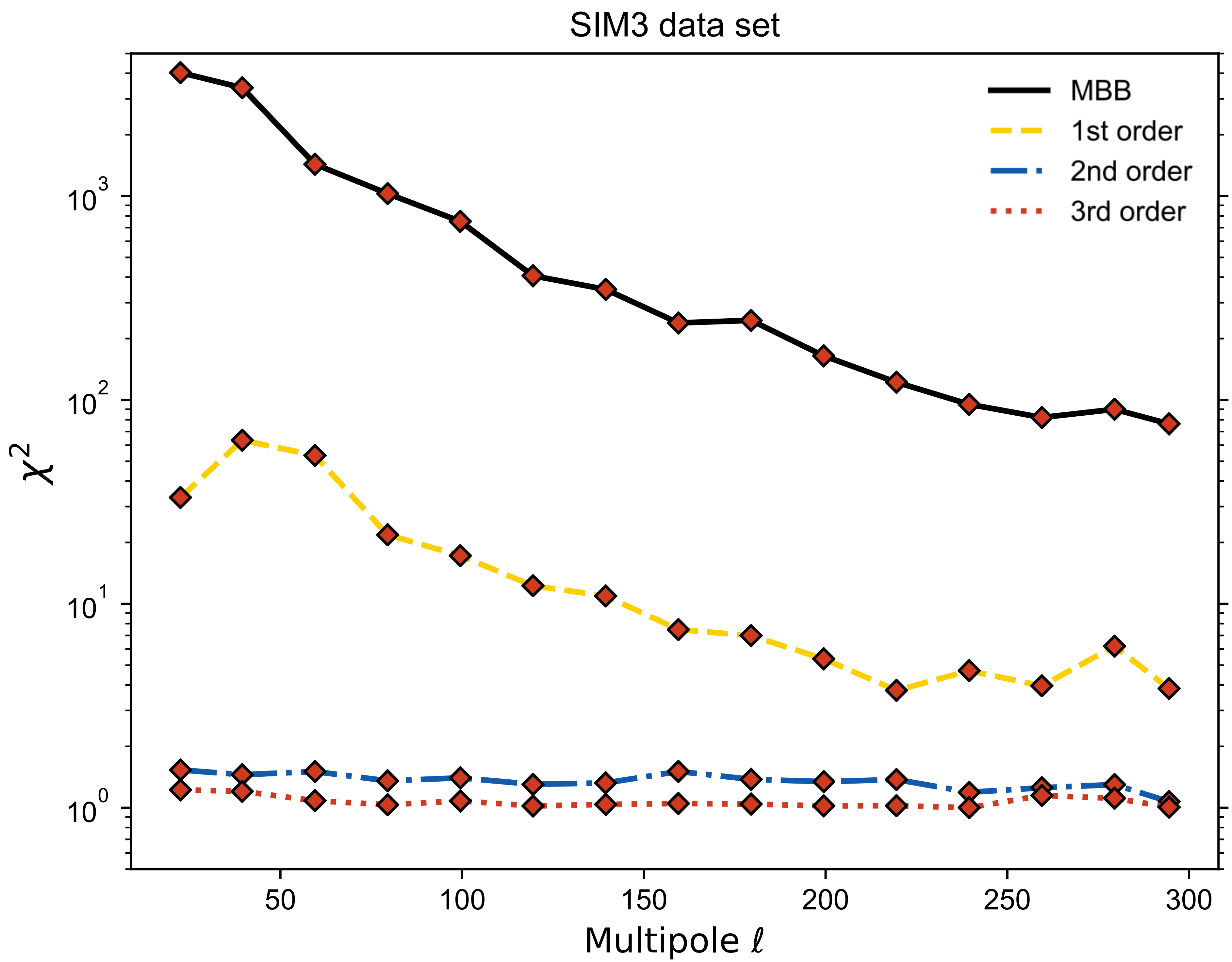

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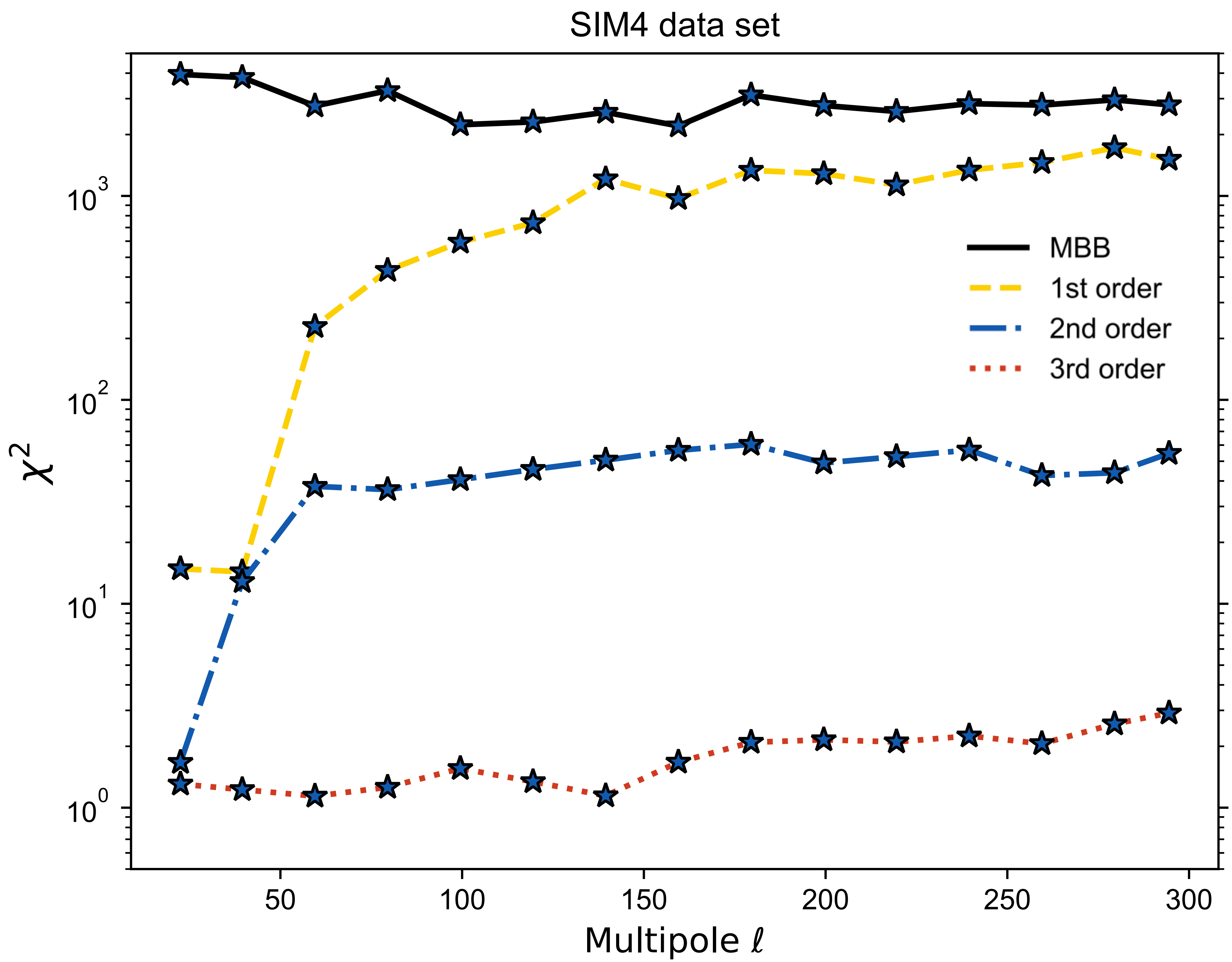

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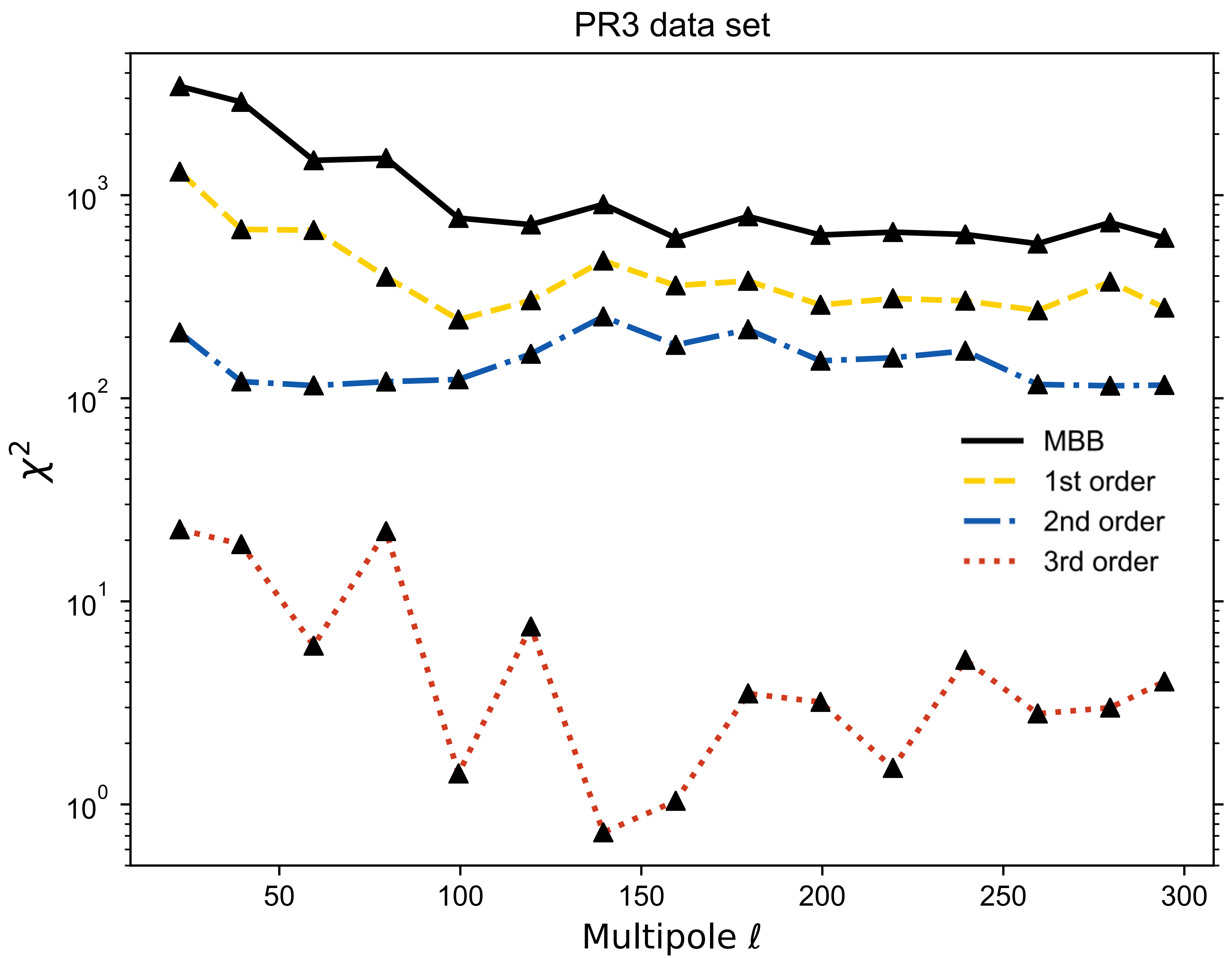
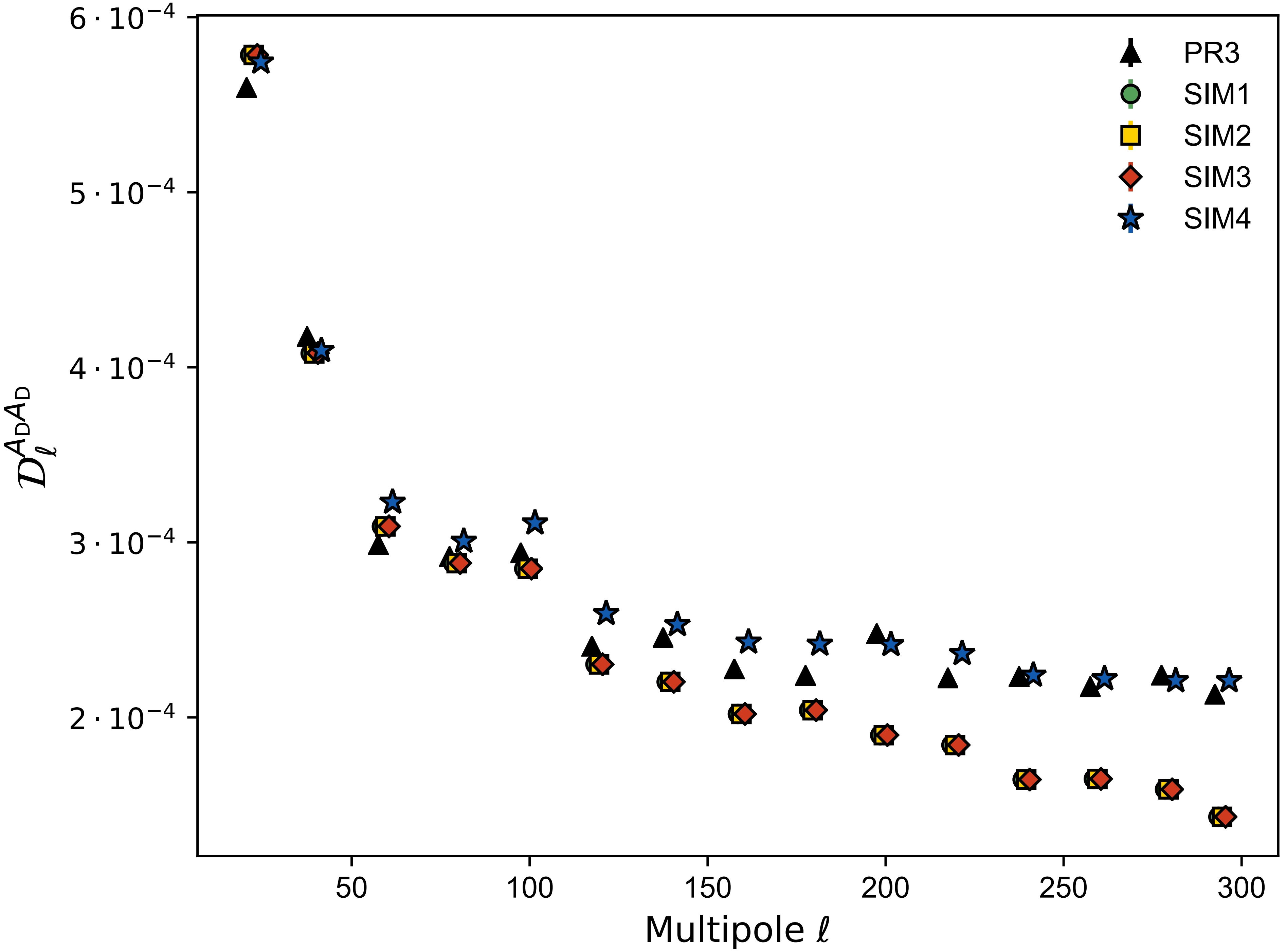


[Mangilli et al., 2019, to be submitted]

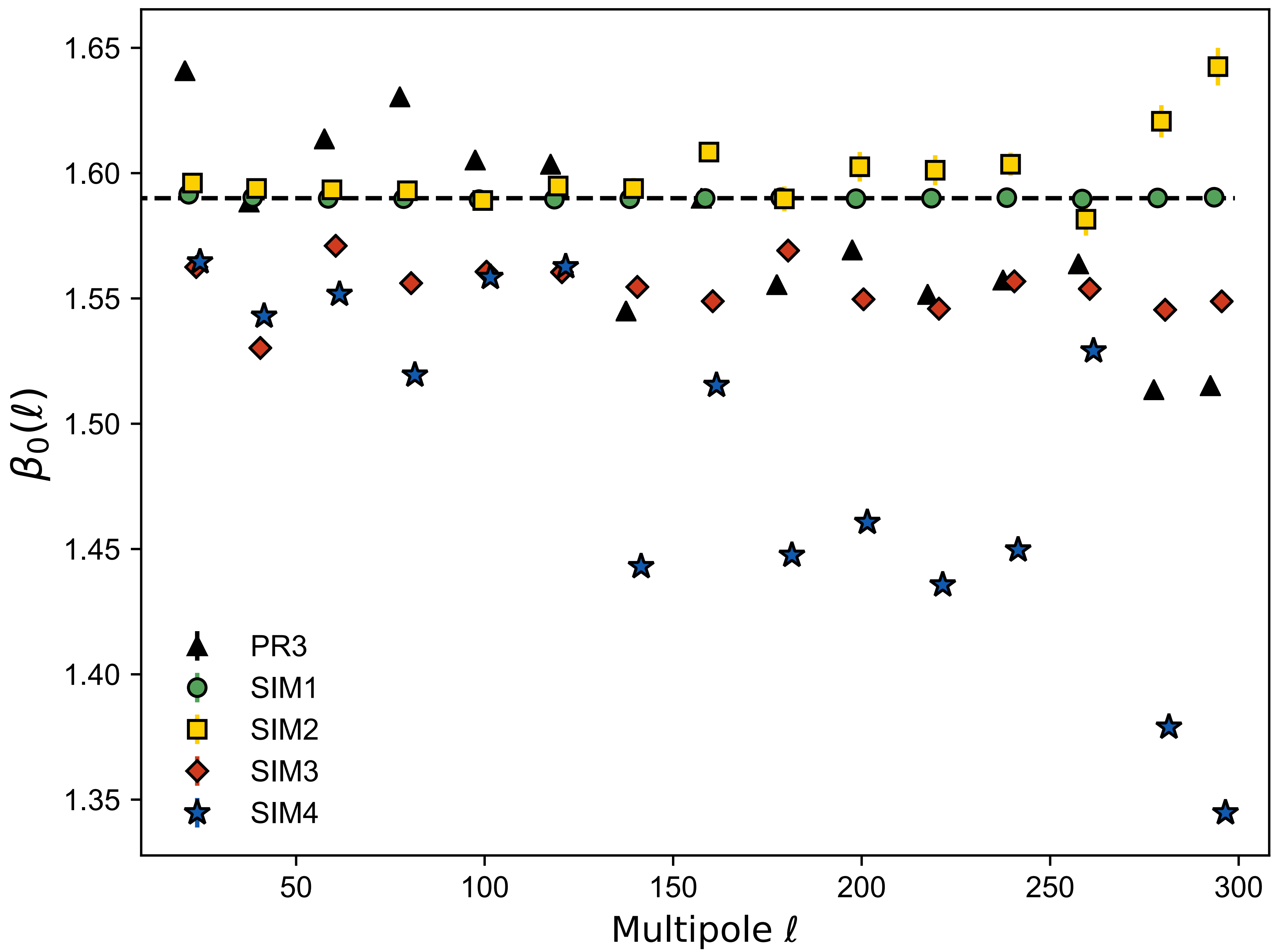
X-SPECTRA MOMENT EXPANSION — Dust amplitude



[Mangilli et al., 2019, to be submitted]

X-SPECTRA MOMENT EXPANSION — Dust spectral index

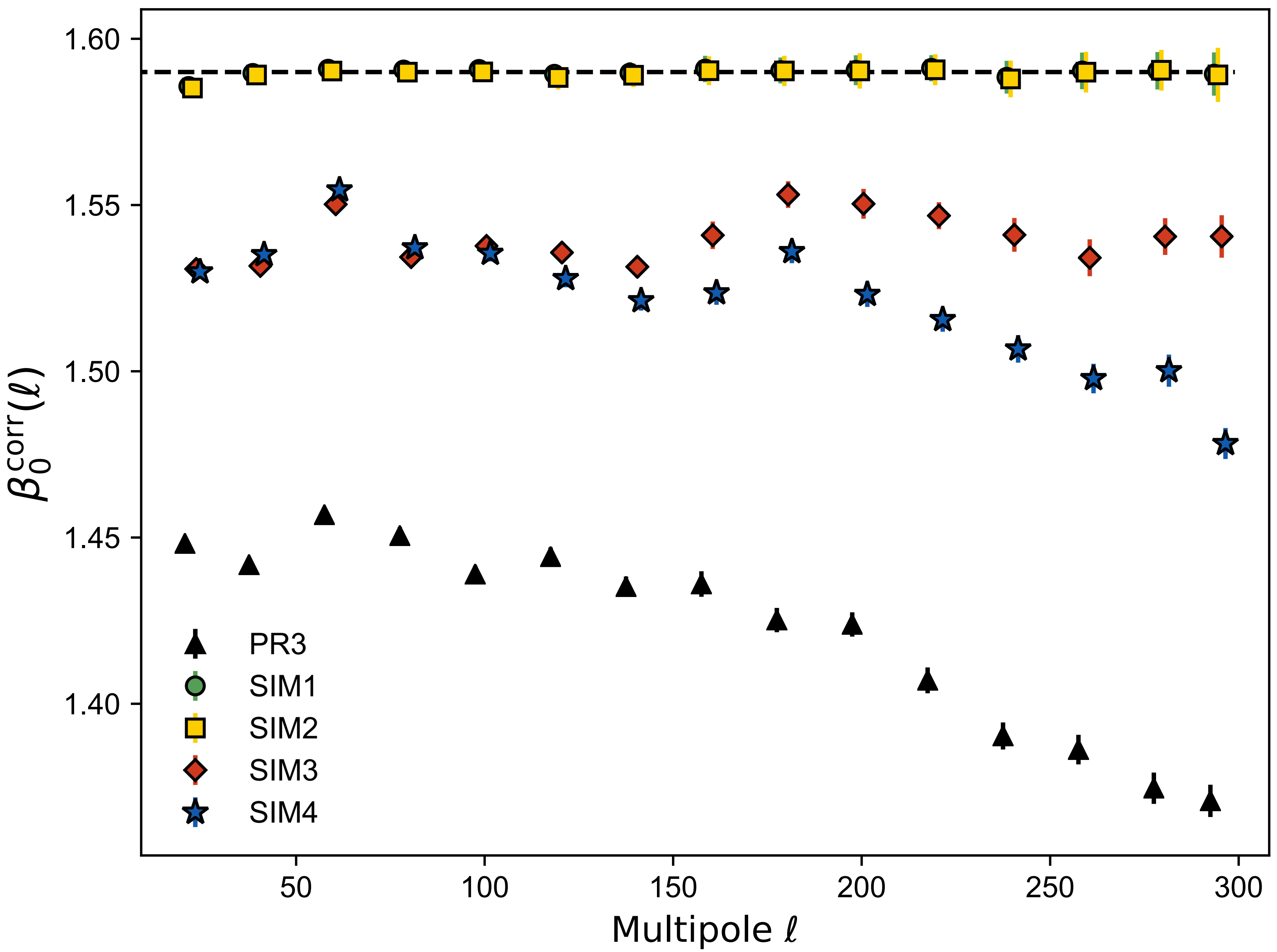
STEP-1



[Mangilli et al., 2019, to be submitted]

X-SPECTRA MOMENT EXPANSION — Dust spectral index

STEP-3

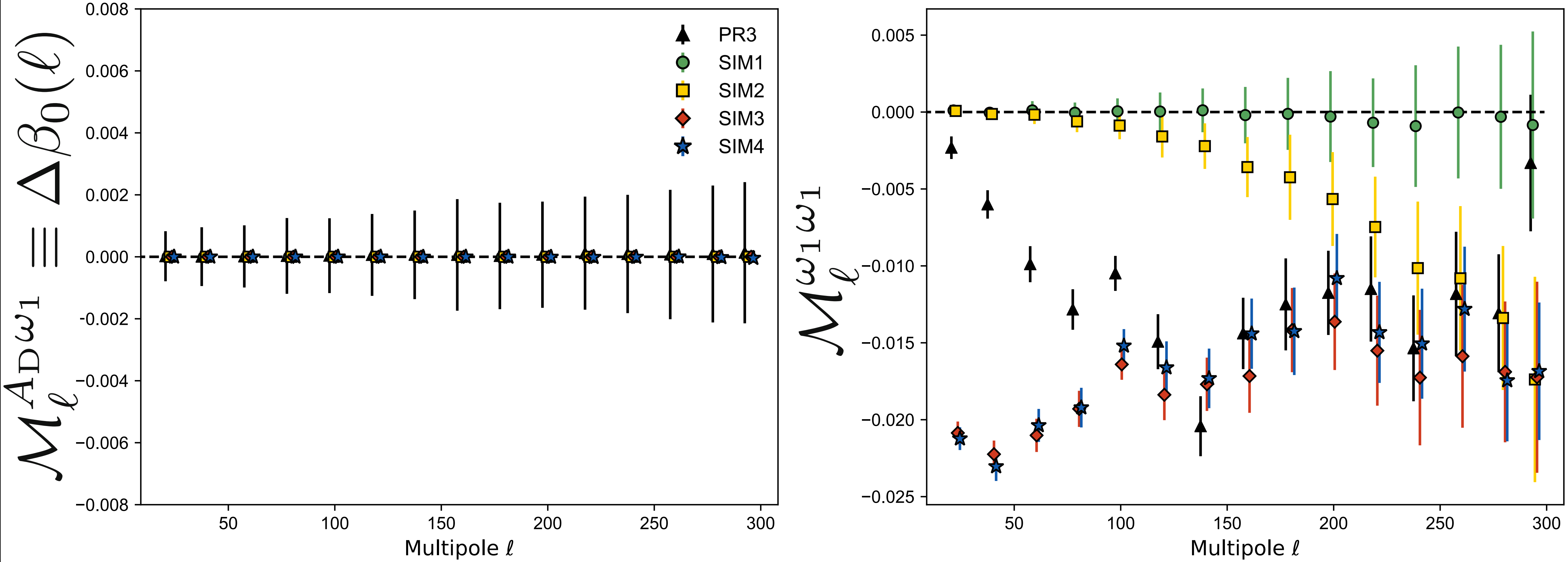


[Mangilli et al., 2019, to be submitted]

X-SPECTRA MOMENT EXPANSION — Moment functions

$$\mathcal{M}_\ell^{ab}(\nu_i,\nu_j) = c^{ab}(\nu_i,\nu_j,\nu_0)\,\mathcal{D}_\ell^{ab}/\mathcal{D}_\ell^{A_\mathrm{D}A_\mathrm{D}}$$

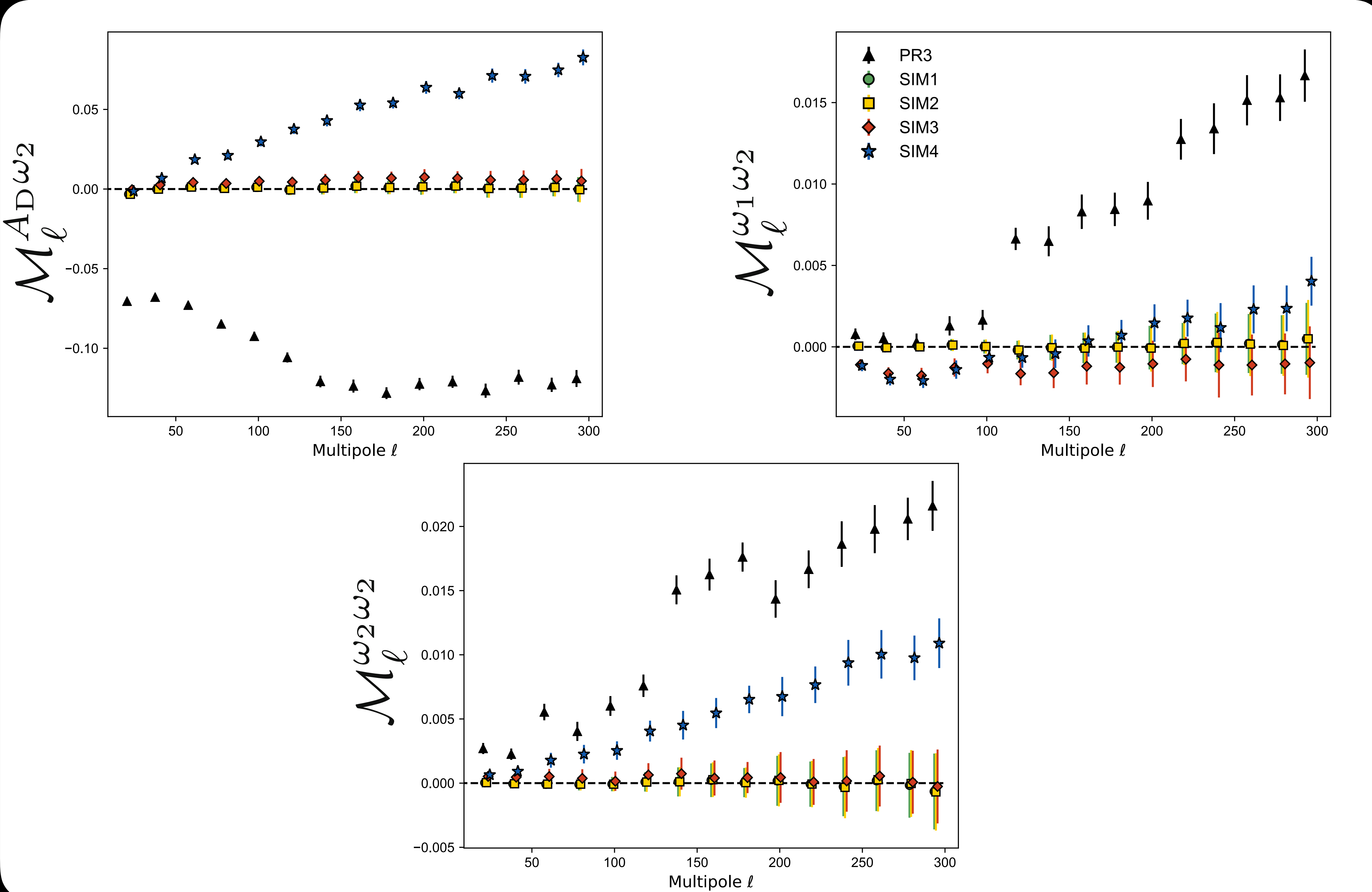
$$\{a,b\} \in \{A_\mathrm{D},\omega_1,\omega_2,\omega_3\}$$



[Mangilli et al., 2019, to be submitted]

"ORDER 1"

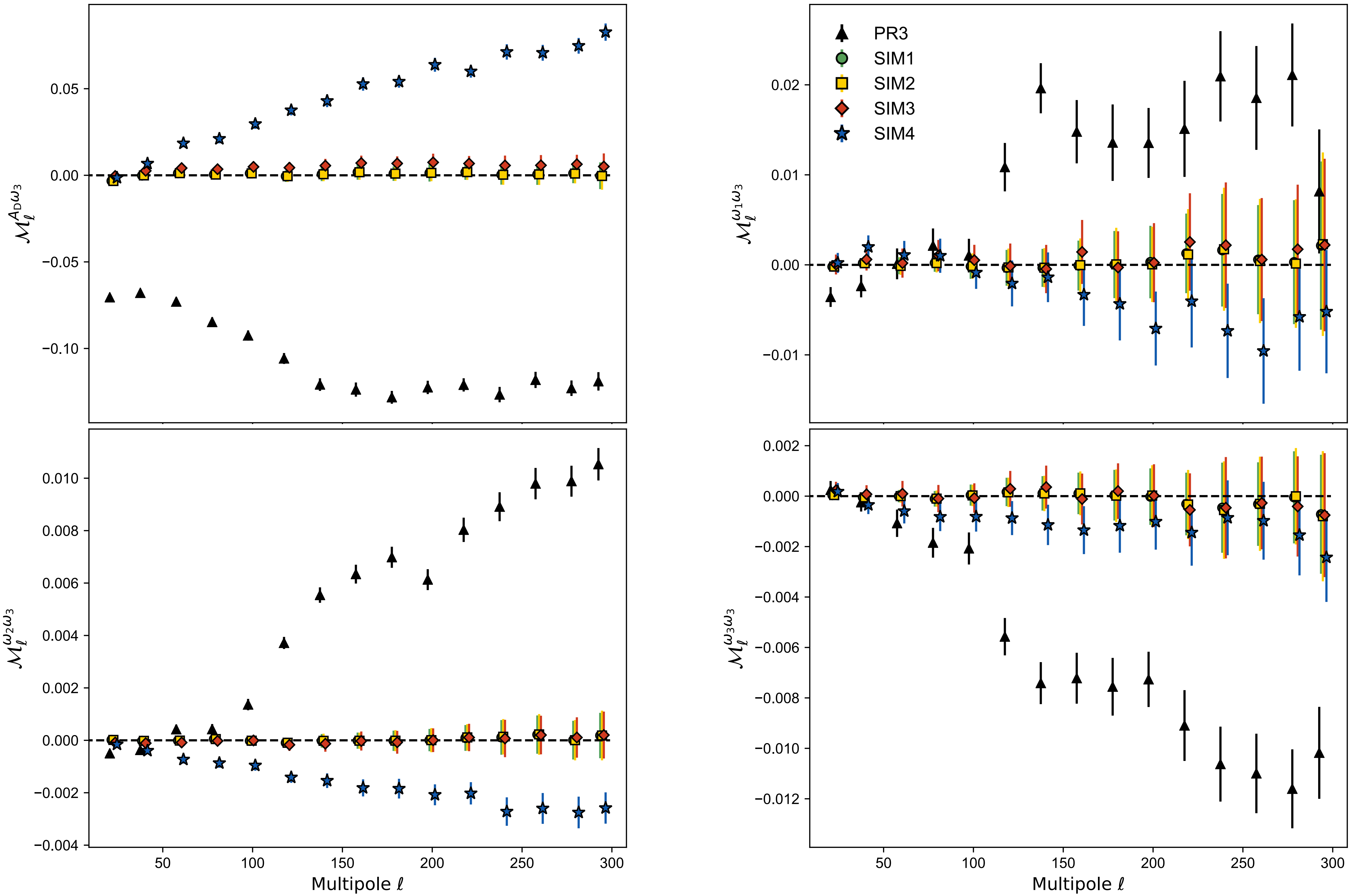
X-SPECTRA MOMENT EXPANSION — Moment functions



[Mangilli et al., 2019, to be submitted]

"ORDER 2"

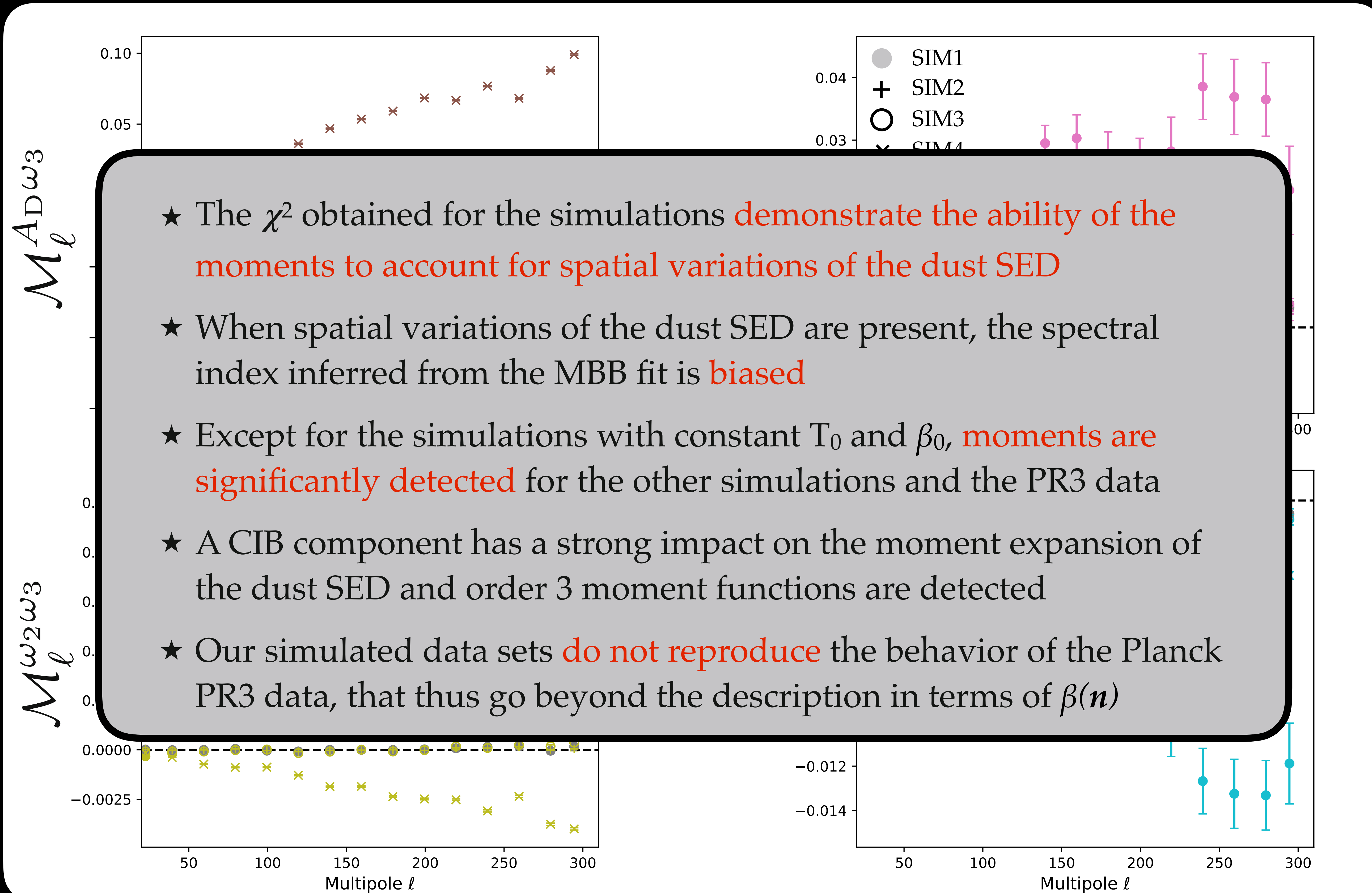
X-SPECTRA MOMENT EXPANSION — Moment functions



[Mangilli et al., 2019, to be submitted]

"ORDER 3"

X-SPECTRA MOMENT EXPANSION — Moment functions



- ★ The χ^2 obtained for the simulations demonstrate the ability of the moments to account for spatial variations of the dust SED
- ★ When spatial variations of the dust SED are present, the spectral index inferred from the MBB fit is biased
- ★ Except for the simulations with constant T_0 and β_0, moments are significantly detected for the other simulations and the PR3 data
- ★ A CIB component has a strong impact on the moment expansion of the dust SED and order 3 moment functions are detected
- ★ Our simulated data sets do not reproduce the behavior of the Planck PR3 data, that thus go beyond the description in terms of $\beta(\boldsymbol{n})$

[Mangilli et al., 2019, to be submitted]

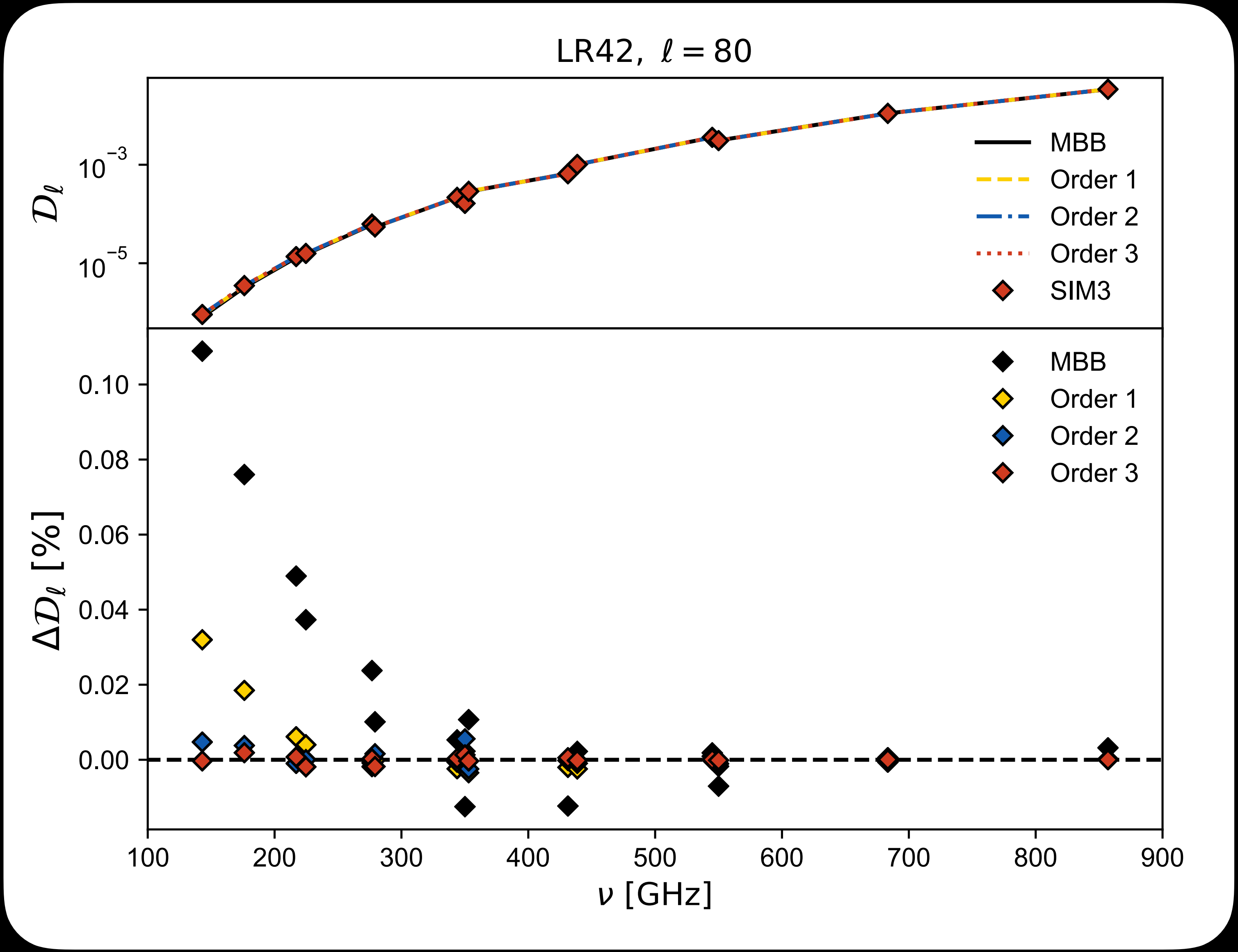
ORDER 3

X-SPECTRA MOMENT EXPANSION — Residual *B*-modes

★ No reason for SED distortions in intensity to be absent in polarisation

★ We look at SIM3 because actual data is *at least* that complex

★ We look at the relative residual at 143 GHz and convert it to r_D, at $\ell = 80$

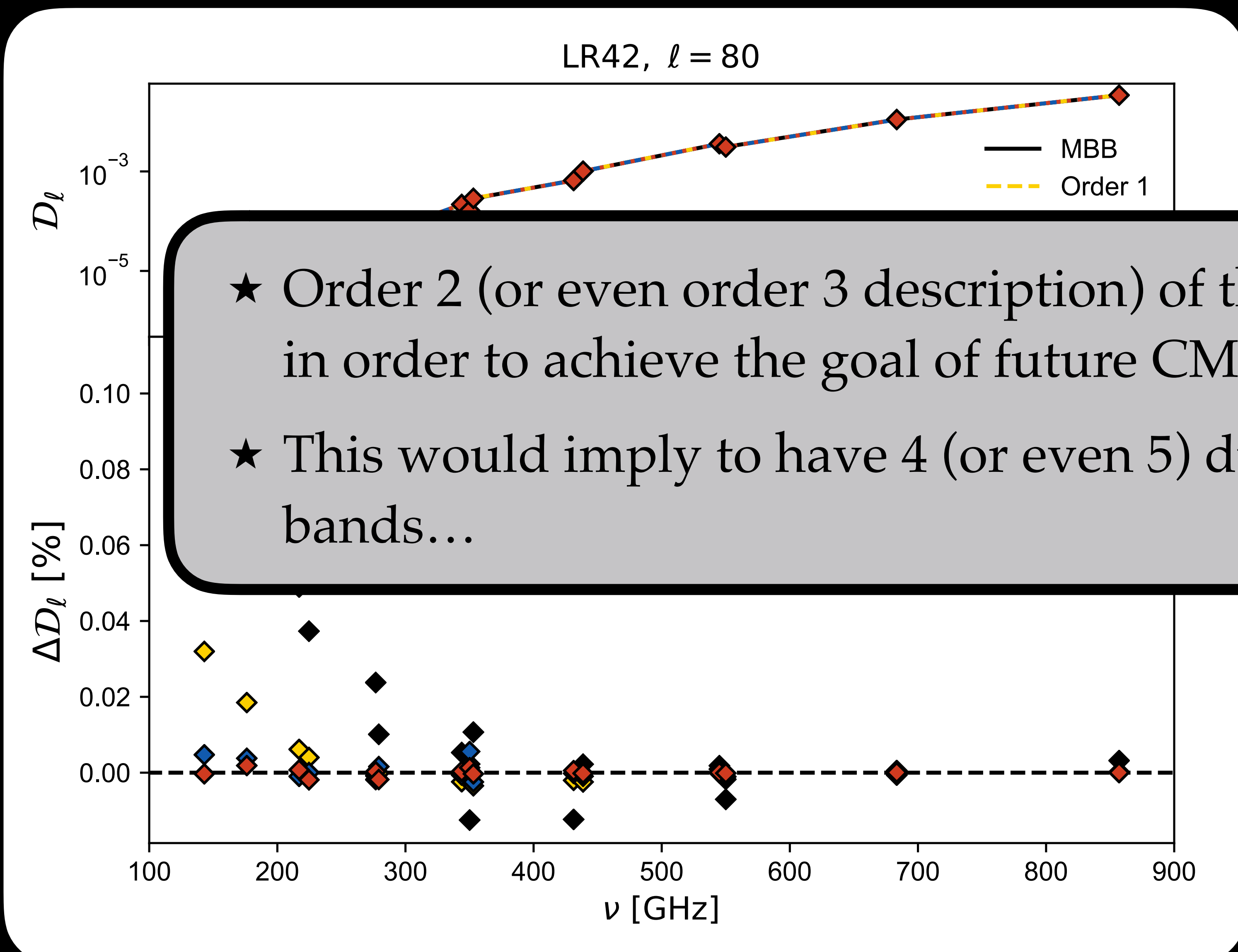


	r_D LR42	r_D "BICEP" region
r_D total	1.8	0.11
MBB (order 0) residual	0.2	10^{-2}
Order 1 residual	0.07	4×10^{-3}
Order 2 residual	0.009	5×10^{-4}
Order 3 residual	6×10^{-4}	3×10^{-5}

[Mangilli et al., 2019, to be submitted]

X-SPECTRA MOMENT EXPANSION — Residual B-modes

- ★ No reason for SED distortions in intensity to be absent in polarisation
- ★ We look at SIM3 because actual data is *at least* that complex
- ★ We look at the relative residual at 143 GHz and convert it to r_{D}



- ★ Order 2 (or even order 3 description) of the dust SED might be required in order to achieve the goal of future CMB B-mode experiments
- ★ This would imply to have 4 (or even 5) dust-dedicated frequency bands…

	LR42	"BICEP" region
Order 2 residual	0.009	5×10^{-4}
Order 3 residual	6×10^{-4}	3×10^{-5}

[Mangilli et al., 2019, to be submitted]

X-SPECTRA MOMENT EXPANSION — Summary

- ★ We extend the formalism of [Chluba et al., 2017] to the dust angular cross-power spectra
- ★ We apply this formalism to the Planck intensity data and simulations
- ★ We detect for the first time dust SED distortions in the Planck data
- ★ Transposing our results to CMB *B*-modes, we find that ignoring the dust SED distortions, or trying to model them with a single decorrelation parameter, could lead to analysis biases larger than the targeted sensitivity for the next generation of *B*-mode experiments (e.g. LiteBIRD)

- ★ Moment expansion of the dust SED is a general parametrization of additional features of the underlying distribution of physical parameters
- ★ Very few *a priori* assumptions, captures the spectral and spatial variations of the SED in an "agnostic" way
- ★ It is a promising tool to model the dust component at a level of precision that will allow the measurement CMB primordial *B*-modes
  - ➡ [Mangilli et al., 2019] will be submitted this week: stay tuned!

X-SPECTRA MOMENT EXPANSION — Further steps

- ★ The moments decomposition could be used as a quantitative metric to obtain simulations that better match the data
- ★ Development of the formalism for polarized cross-spectra
- ★ Assess the impact on the recovered parameters (e.g., r) when using component separation not accounting for moments, for next generation experiment sensitivities (e.g., LiteBIRD, CMB-S4, etc…)
- ★ Optimize the frequency bands design for future CMB experiments
- ★ Implement component separation methods accounting for moment expansion of the dust (synchrotron?) moments