BASIC KNOWLEDGE ABOUT NON-THERMAL PROCESSES

VALENTINA VACCA

- BASICS FACTS
  - COSMIC-RAY PROTONS, COSMIC-RAY ELECTRONS, B-FIELDS IN MILKY WAY, GALAXIES, AND GALAXY CLUSTERS
- COSMIC-RAY ACCELERATION
  - FERMI I
  - FERMI II
  - HADRONIC INTERACTIONS
- COSMIC-RAY LOSSES
  - LEAKY BOX MODEL
  - SYNCHROTRON LOSSES
  - INVERSE COMPTON LOSSES
  - BREMSSTRAHLUNG LOSSES
  - COULOMB LOSSES
  - HADRONIC INTERACTIONS
  - ADIABATIC LOSSES
- COSMIC-RAY AND MAGNETIC FIELD OBSERVABLES
  - SYNCHROTRON EMISSION
  - INVERSE COMPTON EMISSION

REFERENCES

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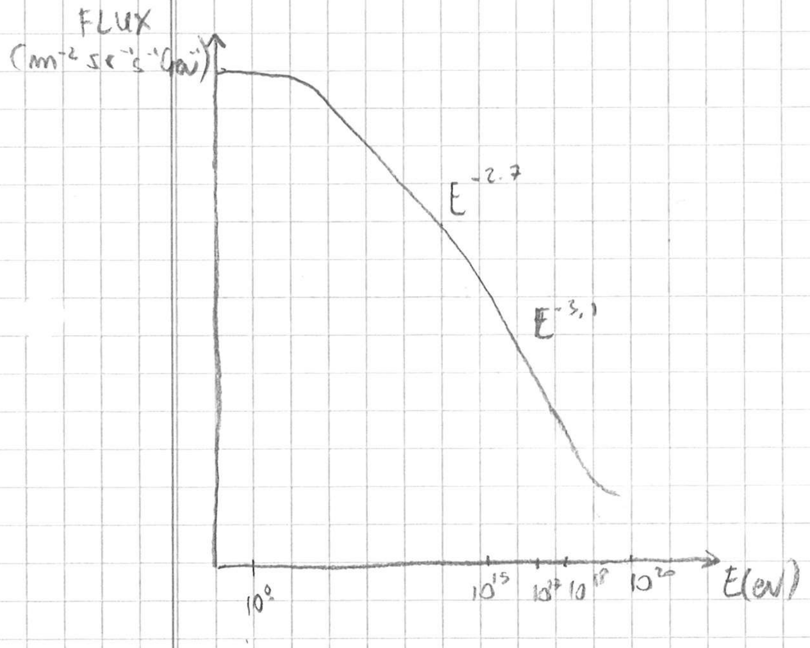
OVERVIEW

COSMIC-RAYS: HIGH ENERGY PARTICLES $10^{6}-10^{20}$ eV

$\sim 99\%$ NUCLEI : 90% p, 9% α-PARTICLES, AND 1% NUCLEI OF HEAVIER ELEMENTS

$\sim \ \%$ e



FIRST KNEE $\sim 4 \cdot 10^{15}$ eV

SECOND KNEE $\sim 4 \cdot 10^{17}$ eV

ANKLE $\sim 4 \cdot 10^{18}$ eV

ABOVE $5 \cdot 10^{19}$ eV THERE IS A SUPPRESSION OF THE FLUX, THE SO-CALLED GREISEN-ZATSEPIN-KUZMIN (GZK) CUTOFF DUE TO THE INTERACTION OF ULTRA-HIGH-ENERGY COSMIC-RAYS WITH THE COSMOLOGICAL MICROWAVE BACKGROUND.

COSMIC-RAYS WITH ENERGIES $< 10^{15}$ eV ARE BELIEVED TO BE OF GALACTIC ORIGIN, WHILE THOSE WITH ENERGIES $> 10^{19}$ eV ARE BELIEVED TO COME FROM EXTRAGALACTIC SOURCES. PROBABLY, INTERGALACTIC MAGNETIC FIELDS PREVENT LOW-ENERGY COSMIC-RAYS FROM REACHING US. THE SPECTRUM OF COSMIC-RAYS DEPENDS STRONGLY ON THE ACCELERATION MECHANISM AND ON MECHANISMS OF ENERGY LOSSES THAT AFFECT THEIR PROPAGATION.

COSMIC-RAYS OF GALACTIC ORIGIN CAN BE PRODUCED IN SUPERNOVAE REMNANTS, DUE TO SHOCK-PROPAGATION, AS WELL AS BY PULSARS THAT HAVE AN ENERGY RESERVOIR IN FORM OF ROTATIONAL

ENERGY AFTER THE SUPERNOVA EXPLOSION.

COSMIC-RAYS OF EXTRAGALACTIC ORIGIN CAN BE PRODUCED BY AGN.

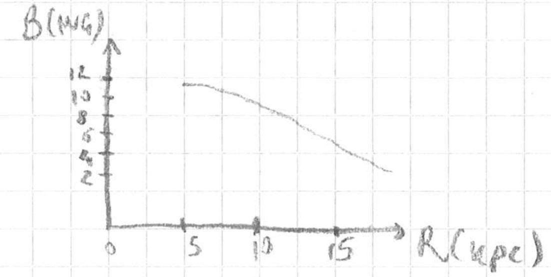
• MAGNETIC FIELDS

a) MILKY WAY

• ZEEMAN MEASUREMENTS OF HI AND MOLECULAR LINES IN GAS CLOUDS

$\sim 6\ \mu G$

• RADIO SYNCHROTRON EMISSION WITH EQUIPARTITION ASSUMPTIONS



$\sim 6\ \mu G$ NEAR THE SUN

$\sim 10\ \mu G$ INNER GALAXY

• ROTATION MEASURE AND DISPERSION MEASURE

$\sim 1-5\ \mu G$

b) OTHER SPIRAL GALAXIES

$\langle B \rangle \sim 10\ \mu G$ FROM EQUIPARTITION ASSUMPTIONS

M31, M33 $\sim 5\ \mu G$

M51, M83 $\sim 15\ \mu G$

$\langle u_B \rangle = \langle u_{CR} \rangle \simeq 10^{-11} \div 10^{-12}\ erg\,cm^{-3}$

HIGHER STRENGTHS HAVE BEEN OBSERVED IN STAR-BURST GALAXIES

$\sim 50-100\ \mu G$

c) RADIO GALAXIES

$\sim \mu G$

d) GALAXY CLUSTERS

$\sim \mu G$ FROM EQUIPARTITION ASSUMPTIONS

$\lesssim \mu G$ SYNCHROTRON + INVERSE COMPTON EMISSION

$\sim 10\ \mu G$ ROTATION MEASURE, COLD FRONTS

RADIAL DECREASE OF THE MAGNETIC FIELD STRENGTH

DIFFUSION - LOSS EQUATION

$$f(x, y, z, p, t) = \frac{d^4 N}{dx\,dy\,dz\,dp}$$

WHERE:

$$p = \frac{P}{mc} \quad \text{DIMENSIONLESS PARTICLE MOMENTUM}$$

ASSUMPTION: QUASI ISOTROPIC COSMIC-RAY DISTRIBUTION DUE TO RAPID PITCH ANGLE SCATTERING

A ONE-DIMENSIONAL SPECTRUM IS THEREFORE A GOOD DESCRIPTION.

ADDITIONALLY WE ASSUME A POWER-LAW SPECTRUM FOR THE PARTICLES:

$$f(x, y, z, p, t) = C(x, y, z, t)\, p^{-\delta}$$

THE DIFFUSION - LOSS EQUATION CAN BE WRITTEN AS FOLLOWS

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial x_i}(v_i f) + \frac{\partial}{\partial p}(\dot{p} f) = \frac{\partial}{\partial x_i}\left(D_{ij} \frac{\partial}{\partial x_j} f\right) + Q - \frac{f}{\tau_{loss}}$$

$\frac{\partial}{\partial x_i}(v_i f)$ CONVECTION TERM, WITH v BEING THE TRANSPORT VELOCITY OF THE GAS PLUS STREAMING VELOCITY OF COSMIC-RAYS WITH RESPECT TO THE GAS.

$\frac{\partial(\dot{p} f)}{\partial p}$ MOMENTUM LOSS / GAIN TERM

$\frac{\partial}{\partial x_i}\left(D_{ij} \frac{\partial}{\partial x_j} f\right)$ DIFFUSION TERM, WITH D DIFFUSION COEFFICIENT

Q SOURCE TERM

$-\frac{f}{\tau_{loss}}$ TERM DESCRIBING LOSSES OF PARTICLES DUE TO COLLISIONS AND DECAYS. A RADIATIVE LOSS OF SPECIES i MIGHT BE A SOURCE OF SPECIES k.

COSMIC-RAY ACCELERATION

PARTICLES IN ASTROPHYSICAL ENVIRONMENTS CAN REACH ENERGIES MUCH HIGHER THAN THE THERMAL ENERGY. SITES WHERE PARTICLES CAN BE ACCELERATED ARE E.G. PULSAR MAGNETOSPHERES, SUPERNOVAE, ACTIVE GALACTIC NUCLEI, AND EXTENDED RADIO SOURCES.
WE ASSUME A POWER-LAW MOMENTUM SPECTRUM:

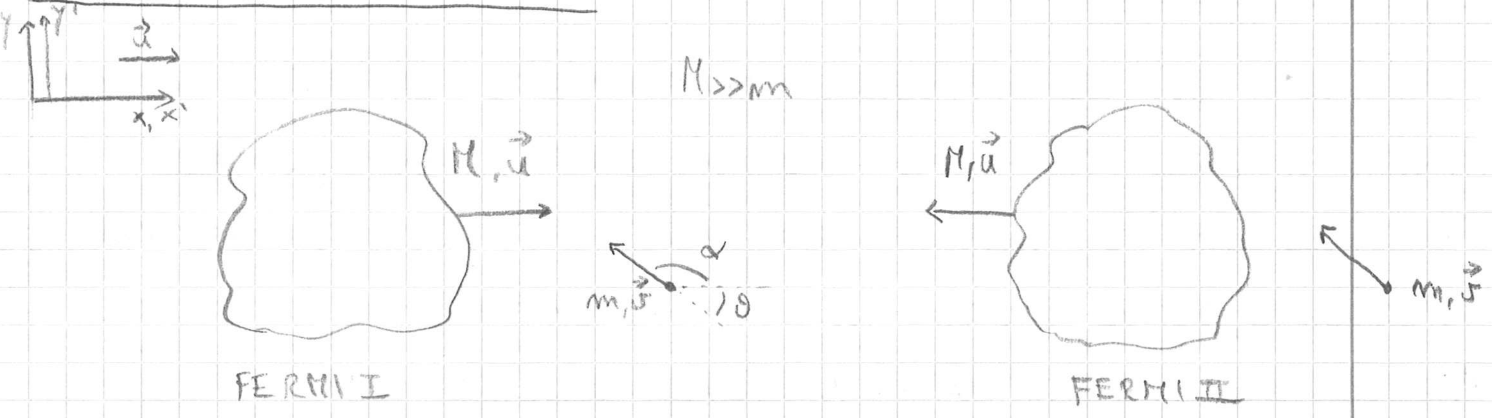
$$f(p)\,dp \propto p^{-\delta}\,dp \qquad 2 \leq \delta \lesssim 3$$

ACCELERATION MECHANISMS CAN BE DISTINGUISHED IN:

- STOCHASTIC PROCESSES, THEY CONSIST OF A HIGH NUMBER OF MICRO-PROCESSES, DURING ONE OF THEM THE PARTICLE MAY GAIN OR LOSE ENERGY, BUT ON AVERAGE THE GAINED ENERGY IS LARGER THAN THE LOST ENERGY. THE EFFICIENCY OF THESE PROCESSES IS LOW;
- SYSTEMATIC PROCESSES, THEY ARE HIGHLY EFFICIENT AND CAN TAKE PLACE DURING SHOCK WAVE DIFFUSION. ACTUALLY THEY ARE STOCHASTIC PROCESSES AS WELL.

ORIGINALLY, THE FERMI MECHANISM WAS PROPOSED BY FERMI (1949) AS A STOCHASTIC PROCESS DESCRIBING THE ACCELERATION OF PARTICLES THROUGH THE COLLISION WITH INTERSTELLAR CLOUDS. CHARGED PARTICLES ARE REFLECTED WHEN THEY HIT MAGNETIC FIELD IRREGULARITIES. REGIONS WITH STRONG MAGNETIC FIELDS CORRESPOND TO REGIONS WITH HIGH DENSITY OF PARTICLES, THAT IS TO INTERSTELLAR (OR INTERGALACTIC) CLOUDS.
FOR THE SAKE OF SIMPLICITY IN THE FOLLOWING WE CONSIDER ELASTIC COLLISIONS.

- FERMI SECOND ORDER



$$\cos\vartheta = -\cos\alpha$$

$\cos\alpha > 0$ REAR-ON COLLISION (LOSS OF ENERGY)

$\cos\alpha < 0$ HEAD-ON COLLISION (GAIN OF ENERGY)

SINCE $M >> m$, THE CLOUD REST FRAME AND THE BARYCENTRIC COORDINATE SYSTEM ARE EQUIVALENT. IN THIS REFERENCE FRAME THE ENERGY AND THE MOMENTUM OF THE PARTICLE ARE:

$$E' = \gamma_u (E - \vec{u}\cdot\vec{P}) = \gamma_u (E + uP\cos\vartheta)$$

$$\vec{P}' = \gamma_u \left(\vec{P} - \vec{u}\frac{E}{c^2}\right) \qquad P'_x = (P\cos\vartheta)' = -\gamma_u \left(P\cos\vartheta + u\frac{E}{c^2}\right)$$

WHERE

$$\gamma_u = \left(1 - \frac{u^2}{c^2}\right)^{-1/2}$$

IN THE COLLISION THE ENERGY IS CONSERVED, THEREFORE, IN THE OBSERVER REFERENCE FRAME, THE ENERGY AFTER THE COLLISION READS:

$$E'' = \gamma_u (E' + \vec{u}\cdot\vec{P}') = \gamma_u (E' + uP_x')$$

SINCE $P'_{x\,(AFTER)} = -P'_{x\,(BEFORE)}$ AND $\frac{P_x}{E} = \frac{\gamma m v \cos\vartheta}{\gamma m c^2}$

FOLLOWS

$$E'' = \gamma_u \left(E' + u P'_{x\,(\text{AFTER COLLISION})} \right) = \gamma_u \left(E' - u P'_x \right) = \gamma_u^2 E \left(1 + \frac{2uP}{E}\cos\vartheta + \frac{u^2}{c^2} \right)$$

EXPANDING TO THE SECOND ORDER IN $\frac{u}{c}$:

$$\frac{\Delta E}{E} = \frac{E'' - E}{E} \approx 2\frac{uv}{c^2}\cos\theta + 2\frac{u^2}{c^2}$$

BECAUSE OF SCATTERING WITH E.G. MAGNETIC FIELD IRREGULARITIES, THE PITCH ANGLE CAN BE ASSUMED TO HAVE AN ISOTROPIC DISTRIBUTION.

BY AVERAGING OVER θ WE CAN THEREFORE EVALUATE THE MEAN INCREASE IN ENERGY.

THE PROBABILITY OF HEAD-ON COLLISIONS ($\propto v + u\cos\vartheta$) IS SLIGHTLY LARGER THAN THE PROBABILITY OF REAR-ON COLLISIONS ($\propto v - u\cos\vartheta$).

SINCE $P(\theta)\,d\theta \propto \sin\theta\,d\theta$, FOLLOWS:

$$\left\langle \frac{\Delta E}{E} \right\rangle = \frac{\int_0^\pi (v + u\cos\vartheta)\left(2\frac{uv}{c^2}\cos\vartheta + 2\frac{u^2}{c^2}\right)\sin\vartheta\,d\vartheta}{\int_0^\pi (v + u\cos\vartheta)\sin\vartheta\,d\vartheta} = \frac{8}{3}\frac{u^2}{c^2}$$

NOTE: THIS IS NOT A RELATIVISTIC EFFECT!!!

IN THIS PROCESS THERE IS A NET GAIN OF ENERGY BUT ONLY IN <u>SECOND ORDER</u> IN $\frac{u}{c}$. THE GAIN IN ENERGY IS SMALL AND THE EFFICIENCY OF THE PROCESS IS LOW.

$$\left\langle \frac{\Delta p}{p} \right\rangle \approx \left\langle \frac{\Delta E}{E} \right\rangle = \frac{8}{3}\frac{u^2}{c^2}$$

IF THE MEAN FREE PATH BETWEEN TWO COLLISIONS IS ℓ, THE AVERAGE TIME BETWEEN COLLISIONS IS $\tau \sim \frac{\ell}{c}$. THE TYPICAL RATE OF MOMENTUM INCREASE IS CONSEQUENTLY:

$$\frac{dp}{dt} = \frac{\Delta p}{\tau} \approx \frac{8}{3}\frac{u^2}{c^2}\,p\,\frac{c}{\ell} = \alpha p$$

IF THE PARTICLE REMAINS IN THE ACCELERATING REGION A CHARACTERISTIC TIME τ_{esc}, IF WE NEGLECT DIFFUSION, CONVECTION, AND ASSUME

THERE ARE NO SOURCES ABOVE SOME p_{inj}, THE STEADY-STATE SOLUTION $\frac{\partial f}{\partial t} = 0$ OF THE DIFFUSION LOSS EQUATION IS

$$-\frac{d}{dp}\left[\alpha p f(p)\right] - \frac{f(p)}{\tau_{esc}} = 0$$

$$-\alpha f(p) - \alpha p \frac{df(p)}{dp} - \frac{f(p)}{\tau_{esc}} = 0$$

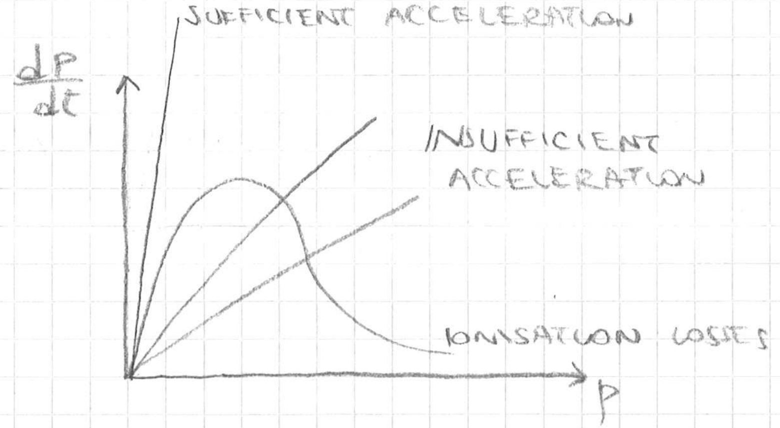
$$\frac{df(p)}{dp} = -\frac{f(p)}{\alpha p}\left(\alpha + \frac{1}{\tau_{esc}}\right)$$

$$\frac{df(p)}{f(p)} = -\left(1 + \frac{1}{\alpha \tau_{esc}}\right)\frac{dp}{p}$$

$$f(p) \propto p^{-\left(1 + \frac{1}{\alpha \tau_{esc}}\right)}$$

THE FERMI PROCESS RESULTS IN A POWER-LAW SPECTRUM. $u \ll c$ AND THE COLLISIONS ARE RARE ($\ell \sim 1\,pc$ IN GALACTIC ENVIRONMENTS $\rightarrow$ 1 COLLISION / YEAR, $\ell \sim 100\,kpc$ IN CLUSTERS $\rightarrow$ 1 COLLISION / 10^6 YEARS). THIS PROCESS CANNOT CAUSE A SIGNIFICANT INCREASE IN THE PARTICLE MOMENTUM. MOREOVER, LOSSES (E.G. IONISATION LOSSES) HAVE NOT BEEN TAKEN INTO ACCOUNT.

FIG. 21.2 LONGAIR (1981)



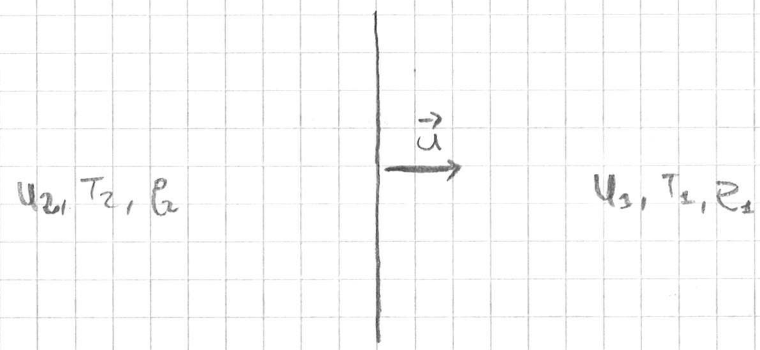
CONSEQUENTLY, THE ACCELERATION MECHANISM CAN BE EFFECTIVE EITHER IF THE INITIAL MOMENTUM OF THE PARTICLE IS HIGHER

THAN THAT CORRESPONDING TO THE MAXIMUM MOMENTUM LOSS RATE OR IF THE ACCELERATION PROCESS DOMINATES, OVER THE MOMENTUM LOSSES.

IN THIS DERIVATION WE ARE NEGLECTING THE STATISTICAL NATURE OF THE PROCESS. A PROPER DERIVATION TAKING THIS INTO ACCOUNT SHOULD USE A FOKKER-PLANCK EQUATION (BLANDFORD AND EICHER 1987).

ACCELERATION IN PRESENCE OF STRONG SHOCK-WAVES

JONES, F. C. (1994)



SECOND ORDER FERMI PROCESSES CAUSE GAINS IN MOMENTUM $\sim \frac{u^2}{c^2}$ BECAUSE OF REAR-ON COLLISIONS. PROCESSES CONSISTING ONLY OF HEAD-ON COLLISIONS ARE MUCH MORE EFFICIENT. IN THIS CASE THE RELATIVE GAIN IN MOMENTUM IS $\sim \frac{u}{c}$, THEREFORE THEY ARE CALLED FIRST ORDER FERMI PROCESSES. THESE PROCESSES AS WELL RESULT IN A POWER-LAW SPECTRUM FOR ACCELERATED PARTICLES.

LET'S CONSIDER THE PROPAGATION OF A SHOCK-WAVE WITH SPEED $\vec{u}$ AND ASSUME THE PRESENCE OF A FLUX OF PARTICLES BOTH IN FRONT AND BEHIND THE SHOCK FRONT. THE SPEED OF THE SHOCK IS $u \ll c$ AND THE THICKNESS OF THE SHOCK IS MUCH SMALLER THAN THE GYRORADIUS OF THE THERMAL PARTICLES.

TURBULENCE BEHIND THE SHOCK FRONT AND IRREGULARITIES IN FRONT OF IT, SCATTER PARTICLES IN SUCH A WAY THAT THE DISTRIBUTION OF THEIR VELOCITIES BECOMES ISOTROPIC WITH RESPECT TO THE FLOW ON THE TWO SIDES OF THE SHOCK FRONT. EVERY TIME A PARTICLE

CROSSES THE SHOCK, ITS MOMENTUM ALWAYS INCREASES, BEING THE INCREMENT IN MOMENTUM THE SAME IN BOTH DIRECTIONS, SINCE THE COLLISIONS ARE ALWAYS HEAD-ON COLLISIONS.

AT EACH COLLISION THE PARTICLE GAINS AN INCREMENT OF MOMENTUM PROPORTIONAL TO ITS MOMENTUM.

AFTER N CYCLES (ONE CYCLE INCLUDES TWO CROSSINGS OF THE SHOCK), THE PARTICLE MOMENTUM WILL BE

$$p(N) = p_0 \prod_{i=1}^{N} \left(1 + \left\langle \frac{\delta p}{p} \right\rangle_i \right)$$

$\left\langle \frac{\delta p}{p} \right\rangle_i$ FLUX-AVERAGED RELATIVE MOMENTUM INCREMENT ON THE i-th cycle.

DURING EACH CYCLE THERE IS A PROBABILITY ε_i ($\ll 1$) THAT THE PARTICLE WILL BE SWEPT DOWNSTREAM FROM THE SHOCK AND IS THEREFORE LOST.

PROBABILITY THAT THE PARTICLE UNDERGO N CYCLES:

$$\mathcal{P}(N) = \prod_{i=1}^{N} (1 - \varepsilon_i)$$

$$\ln \left[\frac{p(N)}{p_0} \right] = \sum_{i=1}^{N} \ln \left(1 + \left\langle \frac{\delta p}{p} \right\rangle_i \right) \approx \sum_{i=1}^{N} \left\langle \frac{\delta p}{p} \right\rangle_i$$

$$\ln \left[\mathcal{P}(N) \right] = \sum_{i=1}^{N} \ln (1 - \varepsilon_i) \approx - \sum_{i=1}^{N} \varepsilon_i$$

$$\frac{\ln \mathcal{P}(N)}{\ln \left[\frac{p(N)}{p_0} \right]} = \frac{- \sum_{i=1}^{N} \varepsilon_i}{\sum_{i=1}^{N} \left\langle \frac{\delta p}{p} \right\rangle_i} = - F(N)$$

$$\mathcal{P}(p) = \left(\frac{p}{p_0} \right)^{-F(N)}$$

INTEGRAL SPECTRUM, PROBABILITY THAT A PARTICLE HAS A MOMENTUM $\geq p$

IF F DEPENDS ON N THE SPECTRUM IS NOT A POWER-LAW.

$\tau_i \rightarrow$ TIME FOR ONE CYCLE

$$\frac{\sum_{i=1}^{N} \langle \frac{\delta p}{p} \rangle_i}{\sum_{i=1}^{N} \tau_i} = \frac{1}{p}\frac{dp}{dt} = \alpha(N)$$

$$\frac{\sum_{i=1}^{N} \varepsilon_i}{\sum_{i=1}^{N} \tau_i} = \frac{1}{T(N)}$$

THEREFORE:

$$F(N) = \frac{\sum_{i=1}^{N} \varepsilon_i}{\sum_{i=1}^{N} \langle \frac{\delta p}{p} \rangle_i} = \frac{\sum_{i=1}^{N} \varepsilon_i / \sum_{i=1}^{N} \tau_i}{\sum_{i=1}^{N} \langle \frac{\delta p}{p} \rangle_i / \sum_{i=1}^{N} \tau_i}$$

$$f(p) \propto \left(\frac{p}{p_0}\right)^{-\left(1 + \frac{1}{\alpha(N)T(N)}\right)}$$ <u>DIFFERENTIAL SPECTRUM</u>

WE HAVE TO FIND OUT IF $\alpha(N)T(N)$ DEPENDS ON N.

FOR A PARTICLE BOUND TO A FLOWING PLASMA BY SCATTERING / ELECTROMAGNETIC FORCES:

$$\dot{p} = -\frac{p}{3}\vec{\nabla}\cdot\vec{u}$$

$\vec{u} \rightarrow$ FLOW VELOCITY

CONSIDERING THE NORMAL OF A PLANE SHOCK (ASSUMED TO BE INFINITE) ALONG THE X-AXIS

$$\dot{p} = -\frac{p}{3}\frac{\partial u}{\partial x}$$

$$\delta p = \int_{u_1}^{u_2} \dot{p}\,dt = -\frac{1}{3}\int_{u_1}^{u_2} p\,\frac{du}{dx}\,\frac{dx}{v_x} = \frac{1}{3}\frac{p}{v_x}(u_1 - u_2)$$

u_1: UPSTREAM FLOW SPEED

u_2: DOWNSTREAM FLOW SPEED

$$\left\langle \frac{1}{v_x} \right\rangle = \frac{2}{v}$$ UNDER THE ASSUMPTION OF ISOTROPY

$$\left\langle \frac{\delta p}{p} \right\rangle_i = \frac{4}{3} \frac{(u_1 - u_2)}{v_i}$$ TO TAKE INTO ACCOUNT THAT DURING ONE CYCLE THE SHOCK IS CROSSED TWO TIMES

THEREFORE IT IS A FIRST ORDER PROCESS.

$$\frac{\left| \int_{-v}^{-u} (u + v_x)\, dv_x \right|}{\int_{-v}^{v} dv_x} = \frac{(v-u)^2}{4v}$$ LEFT TO RIGHT FLUX

$$\frac{\left| \int_{-u}^{v} (u + v_x)\, dv_x \right|}{\int_{-v}^{v} dv_x} = \frac{(v+u)^2}{4v}$$ RIGHT TO LEFT FLUX

PROBABILITY OF RETURN:

$$P(i) = \frac{(v_i - u)^2}{(v_i + u)^2} = \left(\frac{1 - u/v_i}{1 + u/v_i} \right)^2 \approx 1 - \frac{4u}{v_i}$$

$v_i \rightarrow$ PARTICLE VELOCITY ON THE i-th CYCLE

$$F(N) = \frac{4 u_2 \sum_{i=1}^{N} \frac{1}{v_i}}{\frac{4}{3} (u_1 - u_2) \sum_{i=1}^{N} \frac{1}{v_i}} = \frac{3 u_2}{u_1 - u_2} = \frac{3}{r-1}$$

$r \equiv u_1 / u_2$ COMPRESSION RATIO

F DOE NOT DEPEND ON N AND THE SPECTRUM IS THEREFORE A POWER-LAW.

IF $r \approx 4$ (AS IN MANY ASTROPHYSICAL SHOCKS), $F = 1$ AND

$$f(p) \propto \left(\frac{p}{p_0} \right)^{-2}$$

AS TYPICALLY OBSERVED.

HADRONIC INTERACTIONS

THE INTERACTION OF COSMIC-RAY PROTONS WITH THERMAL GAS IN THE ISM AND IN THE ICM CAUSES THE PRODUCTION OF NEUTRAL AND CHARGED PIONS IF THE COSMIC-RAY PROTON MOMENTA IS LARGER THAN THE THRESHOLD $q_{thr} \, m_p c^2 = 0.78 \, GeV$:

$$p_{CR} + p_{THERMAL} \rightarrow 2p + \pi^{\pm}_{0}$$

AFTER A LIFETIME $\tau = 9 \cdot 10^{-17} s$ PIONS DECAY IN GAMMA RAY PHOTONS AND SECONDARY ELECTRONS:

$$\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu} / \bar{\nu}_{\mu} \rightarrow e^{\pm} + \nu_e / \bar{\nu}_e + \nu_{\mu} / \bar{\nu}_{\mu}$$

$$\pi^{0} \rightarrow 2\gamma$$

IN STRONG AND ELECTRO-WEAK INTERACTIONS THE NUMBER OF BARYONS IS CONSERVED. THEREFORE:

$$\frac{dm_{CR}}{dt} = 0 \qquad m_{CR} = \text{CONSTANT}$$

THE DECAY PRODUCTS (GAMMA RAYS AND SECONDARY ELECTRONS) ALLOW TO TRACE THE COSMIC-RAY PROTONS ABOVE THE KINEMATIC THRESHOLD q_{thr}.

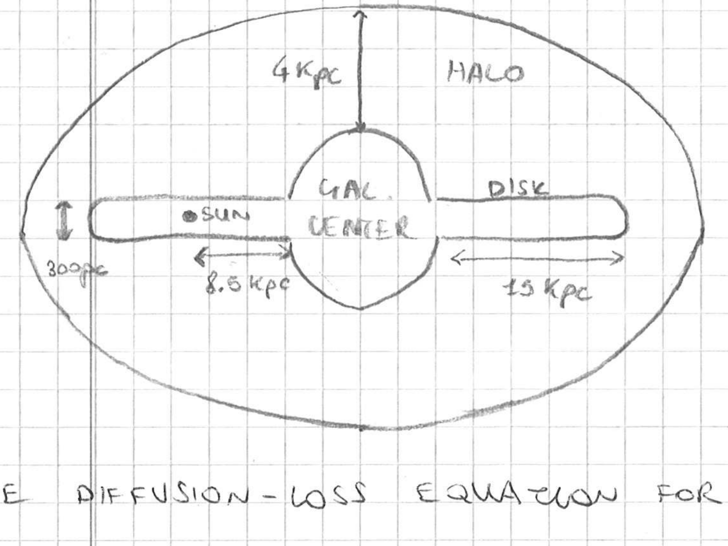
LEAKY - BOX MODEL

LONGAIR (1981), GAISSER (1990)

OBSERVATIONS OF SECONDARY TO PRIMARY RATIOS OF FRAGMENTATION REACTIONS INDICATE THAT COSMIC RAYS WITH $E \sim GeV$ TRAVERSE $G \sim 5\, g/cm^2$ OF MATTER IN OUR GALAXY. IF WE ASSUME AN AVERAGE DENSITY FOR THE INTERSTELLAR MEDIUM OF ~ 1 PARTICLE $/cm^3$ AND THAT HYDROGEN ATOMS DOMINATE, WE INFER A CONFINEMENT TIME $\tau_c = \frac{G}{\rho c} \sim 3 \cdot 10^6 yr$ FOR PARTICLES TRAVELLING AT THE SPEED OF LIGHT.

IF WE CONSIDER THE RADIUS OF OUR GALAXY ~ 15 Kpc, THE TIME REQUIRED FOR ESCAPING IS $\tau \sim 5 \cdot 10^4 yr$ FOR PARTICLES MOVING AT THE SPEED OF LIGHT. THEREFORE, COSMIC-RAYS DO NOT FREELY PROPAGATE IN THE GALAXY.

A SIMPLIFIED RAPRESENTATION IS GIVEN BY THE "LEAKY-BOX" MODEL. COSMIC-RAYS ARE ASSUMED TO FREELY PROPAGATE IN A CONFINEMENT VOLUME AND TO BE STRONGLY REFLECTED AT THE BOUNDARIES WITH A SMALL, BUT FINITE, PROBABILITY TO ESCAPE τ_{esc}^{-1}.



THE DIFFUSION-LOSS EQUATION FOR PARTICLES THAT FREELY PROPAGATE WITH ESCAPE PROBABILITY τ_{esc}^{-1}, IN ABSENCE OF MOMENTUM LOSSES, CONVECTION, COLLISIONS, AND FOR A DELTA-FUNCTION INJECTION $Q(p,t) = f_0(p)\,\delta(t)$, RESULTS IN:

$$f(p,t) = f_0(p) \, EXP(-\tau/\tau_{esc})$$

WHERE τ_{esc} IS THE AVERAGE TIME SPENT IN THE CONFINEMENT VOLUME BY A PARTICLE.

LET'S CONSIDER SOLUTIONS AT EQUILIBRIUM: WE NEGLECT CONVECTION, MOMENTUM LOSSES AND GAINS, AND FRAGMENTATION. WE ASSUME THAT THE SYSTEM IS IN A STEADY-STATE:

$$\frac{\partial f(p)}{\partial t} = 0$$

IF THE MEAN TIME BETWEEN TWO FRAGMENTATION EVENTS IS τ, THE PROBABILITY THAT A PARTICLE IS DESTROYED DURING A FRAGMENTATION REACTION IS τ^{-1}. THEREFORE, FROM THE DIFFUSION-LOSS EQUATION FOLLOWS:

$$\frac{f(p)}{\tau_{esc}} = Q(p) - \frac{f(p)}{\tau}$$

$$f(p) = \frac{Q(p)\,\tau_{esc}}{\left(1 + \tau_{esc}/\tau\right)}$$

HERE WE ASSUMED THAT THE LIFETIME OF THE PARTICLE $\tau_L \gg \tau$, τ_{esc}, I.E. WE ARE CONSIDERING STABLE PARTICLES. OTHERWISE, WE SHOULD TAKE IT INTO ACCOUNT IN THE DIFFUSION-LOSS EQUATION.

IF $\tau_{esc} \ll \tau$:

$$f(p) \sim Q(p)\,\tau_{esc}$$

FROM A SPERIMENTAL FIT: $\tau_{esc} \propto p^{-\gamma}$

SINCE $f(p) \propto p^{-\delta} \rightarrow Q(p) \propto p^{-(\delta-\gamma)}$

ON THE CONTRARY, IF $\tau \ll \tau_{esc}$, THE SPECTRUM REFLECTS THE SOURCE SPECTRUM:

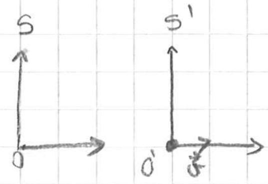
$$f(p) \sim Q(p)\,\tau$$

RELEVANT FORMULAS

- ENERGY LOSS

$$\frac{dE}{dt} = \frac{dE'}{dt'}$$

ENERGY LOSS IS A RELATIVISTIC INVARIANT



- LARMOR'S FORMULA

$$\frac{dE}{dt} = \frac{q^2}{6\pi\varepsilon_0 c^3}\ddot{r}^2$$

NON-RELATIVISTIC - CASE

$$\frac{dE}{dt} = \frac{q^2}{6\pi\varepsilon_0 c^3}\gamma^4\left(|\ddot{r}_\perp|^2 + \gamma^2|\ddot{r}_\parallel|^2\right)$$

RELATIVISTIC - CASE

- PARSEVAL THEOREM

$$\int_{-\infty}^{\infty} |\ddot{\vec{r}}(\omega)|^2 d\omega = \int_{-\infty}^{\infty} |\ddot{\vec{r}}(t)|^2 dt$$

WHERE:

$$\ddot{\vec{r}}(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\ddot{\vec{r}}(\omega)\, \mathrm{EXP}(-i\omega t)\, d\omega$$

IT FOLLOWS FOR THE TOTAL EMITTED RADIATION

$$\int_0^{\infty} I(\omega)\, d\omega = 2\int_0^{\infty}\frac{q^2}{6\pi\varepsilon_0 c^3}|\ddot{\vec{r}}(\omega)|^2 d\omega$$

$$I(\omega) = \frac{q^2}{3\pi\varepsilon_0 c^3}|\ddot{\vec{r}}(\omega)|^2$$

TOTAL ENERGY PER UNIT OF BANDWIDTH EMITTED DURING THE PERIOD OF ACCELERATION

MOMENTUM LOSSES

DURING THEIR PROPAGATION COSMIC RAYS SUFFER ENERGY LOSSES THAT CHANGE THE INJECTION MOMENTUM SPECTRUM.
THESE LOSSES ARE DUE TO INTERACTIONS WITH MATTER, RADIATION, AND MAGNETIC FIELDS. TO FIGURE OUT TYPICAL SPECTRA WE NEED TO INVESTIGATE THE MOMENTUM DEPENDENCE OF THESE LOSSES.

WE CAN MENTION DIFFERENT KINDS OF LOSSES:

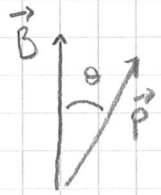
- IONISATION
- BREMSSTRAHLUNG
- ADIABATIC
- SYNCHROTRON
- INVERSE COMPTON
- COULOMB
- HADRONIC

SYNCHROTRON LOSSES

THEY ARE DUE TO THE DECELERATION OF COSMIC-RAYS BY MAGNETIC FIELDS AND ARE THE DOMINANT PROCESS IN HIGH ENERGY ASTROPHYSICS. THIS MECHANISM IS RESPONSIBLE FOR RADIO EMISSION OBSERVED FROM SUPERNOVAE REMNANTS, GALAXIES, GALAXY CLUSTERS, AND OTHER EXTRAGALACTIC RADIO SOURCES.

$$\frac{d}{dt}(\vec{p}) = \frac{q}{m}\vec{\beta}\times\vec{B}$$

$p = \beta\gamma$



$$F_\perp = m\gamma\ddot{r}_\perp$$

$$\frac{dE}{dt} = \frac{q^4}{6\pi\varepsilon_0 c\, m^2}B^2p^2\sin^2\theta$$

ELECTRON LOSSES (PROTON LOSSES IN COMPARISON TO ELECTRON LOSSES ARE NEGLIGIBLE $\propto m_e^2/m_p^2$):

$$\frac{dE}{dt} = \frac{e^4}{6\pi\varepsilon_0 c\, m_e^2}B^2p^2\sin^2\theta$$

SINCE $U_{mag} = \frac{B^2}{2\mu_0}$ AND $c^2 = (\mu_0\varepsilon_0)^{-1}$

AVERAGING OVER THE PITCH ANGLE:

$$\langle(\sin\theta)^2\rangle = 2/3$$

$$\frac{dE}{dt} = \frac{4}{3}\frac{e^4}{6\pi\varepsilon_0^2c^3m_e^2}p^2U_{mag}$$

$$\frac{dE}{dt} = \frac{4}{3}c\sigma_T p^2 U_{mag}$$

WHERE:

$$\sigma_T = \frac{e^4}{6\pi\varepsilon_0^2c^4m_e^2}$$ IS THE THOMSON CROSS-SECTION

SYNCHROTRON RADIATION FROM A POPULATION OF RELATIVISTIC ELECTRONS WITH A POWER-LAW MOMENTUM SPECTRUM.

EMISSIVITY

$$J(\nu) = C_\alpha f_0 B^{\alpha+1} \nu^{-\alpha}$$

LUMINOSITY

$$L(\nu) = V J_\nu = C_\alpha f_0 V B^{\alpha+1} \nu^{-\alpha}$$

WHERE:

$$\alpha = \frac{\delta - 1}{2}$$

AND

$$f(p)\,dp = f_0 p^{-\delta} dp$$

INVERSE COMPTON LOSSES

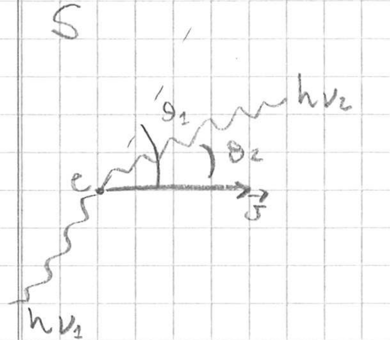
IF AN ELECTRON INTERACTS WITH A PHOTON WHOSE ENERGY IS LOWER THAN THE ELECTRON KINETIC ENERGY:

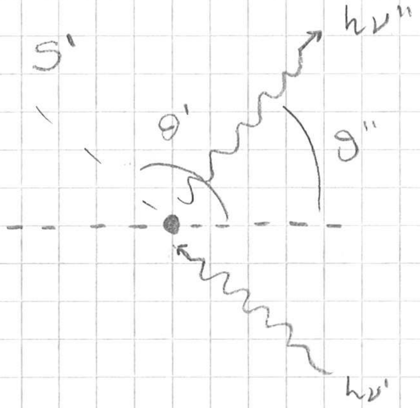
$$\left(\sqrt{1+p^2}-1\right)mc^2 >> h\nu$$

THE RESULT IS THAT ENERGY IS TRANSFERRED FROM THE ELECTRON TO THE PHOTON. THIS MECHANISM PRODUCES RADIATION IN THE X- AND γ-RAY BANDS.

S: OBSERVER REST FRAME

S': ELECTRON REST FRAME



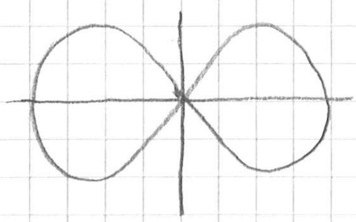


$$S': \quad \nu' = \nu_1 \gamma \left(1-\beta\cos\vartheta_1\right)$$

$\nu'' = \nu'$ THE ELECTRON IS AT REST IN S' THEREFORE THE INTERACTION IS A THOMSON DIFFUSION

$$S: \quad \nu_2 = \nu''\gamma\left(1+\beta\cos\vartheta''\right) = \nu_1\gamma^2\left(1-\beta\cos\vartheta_1\right)\left(1+\beta\cos\vartheta''\right)$$

IF THE ELECTRON INTERACTS WITH AN ISOTROPIC PHOTON GAS, FOR MOST OF THE PHOTONS $\vartheta_1 = \pi/2$ AND IN S' THE THOMSON DIFFUSION FOLLOWS A LARMOR DIPOLE WITH THE MOST OF THE POWER EMITTED ON LARGE ANGLES $\rightarrow \cos\vartheta'' \sim 0$. CONSEQUENTLY:



$$\nu_2 \simeq \gamma^2 \nu_1$$

- RADIATED POWER

SINCE IN S' WE HAVE A THOMSON DIFFUSION, WE CAN USE THE LARMOR FORMULA:

$$a = \frac{eE_0}{m_e}\cos\omega t$$

$$\frac{dE}{dt\,d\Omega} = \frac{1}{16\pi^2\varepsilon_0}\frac{e^4 E_0^2}{m_e^2 c^3}\sin^2\theta$$

ENERGY DENSITY OF PHOTONS IN S': $u_f' = \frac{\varepsilon_0 E_0^2}{2}$.

INTEGRATING OVER THE SOLID ANGLE:

$$\frac{dE}{dt} = \frac{1}{6\pi\varepsilon_0^2}\frac{e^4}{m_e^2 c^3}u_f' = c\sigma_T u_f'$$

$$u_f' = n_\gamma' \cdot \varepsilon_f' = u_f \gamma^2 (1-\beta\cos\theta)^2$$

AVERAGING OVER ALL THE ANGLES:

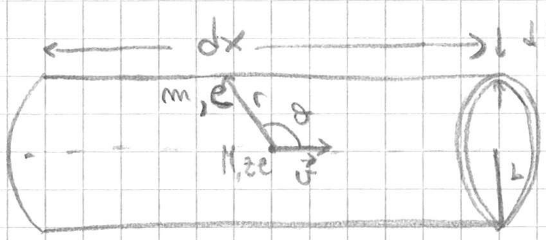
$$\left\langle \frac{dE}{dt} \right\rangle = c\sigma_T u_f \gamma^2 \left(1 + \frac{1}{3}\beta^2\right)$$

THE ENERGY LOSS IS GIVEN BY THE DIFFERENCE OF THE RADIATED ENERGY AND THE PHOTON ENERGY BEFORE THE SCATTERING:

$$\frac{dE}{dt} = \left\langle \frac{d\varepsilon}{dt} \right\rangle - c\sigma_T u_f = \frac{4}{3}c\sigma_T u_f p^2$$

IONISATION LOSSES

COLLISION OF A PARTICLE WITH A STATIONARY ELECTRON.



b = COLLISION PARAMETER

MOMENTUM IMPULSE $\int F\,dt$

- NON-RELATIVISTIC CASE

BY SYMMETRY, THE FORCES PARALLEL TO THE LINE OF FLIGHT OF THE PARTICLE CANCEL OUT

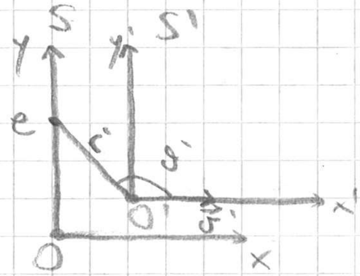
$$F_\perp = \frac{ze^2}{4\pi\varepsilon_0 r^2}\sin\vartheta \qquad dt = \frac{dx}{v} \qquad \sin\vartheta = \frac{b}{r}$$

$$\Delta P_\perp = \int_{-\infty}^{\infty} F_\perp\,dt = \int_{-\infty}^{\infty} \frac{ze^2}{4\pi\varepsilon_0 r^3}\frac{b}{v}\,dx = \int_{-\infty}^{\infty}\frac{ze^2}{4\pi\varepsilon_0}\frac{b}{v}\frac{1}{(b^2+x^2)^{3/2}}\,dx$$

SINCE : $\displaystyle\int \frac{dy}{(a^2+y^2)^{3/2}} = \frac{y}{a^2(a^2+y^2)^{1/2}}$

$$\Delta P_\perp = \frac{ze^2}{2\pi\varepsilon_0 v b}$$

- RELATIVISTIC CASE



WHEN THE PARTICLE IS AT THE DISTANCE OF CLOSEST APPROACH $t = t' = 0$ AND $x = x' = 0$

IN S' THE ELECTRIC FIELD E OF THE PARTICLE IS SPHERICALLY SYMMETRIC ABOUT THE ORIGIN O':

$$E'_{\parallel} = \frac{ze}{4\pi\varepsilon_0 r'^2}\cos\theta' = \frac{ze}{4\pi\varepsilon_0}\frac{x'}{r'^3}$$

$$E'_\perp = \frac{ze}{4\pi\varepsilon_0 r'^2}\sin\theta' = \frac{ze}{4\pi\varepsilon_0}\frac{b}{r'^3}$$

FROM THE LORENTZ TRANSFORMATIONS:

$$t' = \gamma\left(t - \beta x/c\right)$$

$x = 0$ FOR THE ELECTRON $\Rightarrow$ $t' = \gamma t$

$$E'_\parallel = \frac{ze(\gamma v t)}{4\pi\varepsilon_0\left[b^2 + (\gamma v t)^2\right]^{3/2}}$$

$$E'_\perp = \frac{zeb}{4\pi\varepsilon_0\left[b^2 + (\gamma v t)^2\right]^{3/2}}$$

TRANSFORMING BACK THE FIELD IN S $(B'_\parallel = B'_\perp = B'_z = 0)$:

$$E_\parallel = \frac{\gamma z e v t}{4\pi\varepsilon_0\left[b^2 + (\gamma v t)^2\right]^{3/2}}$$

$$E_\perp = \frac{\gamma z e b}{4\pi\varepsilon_0\left[b^2 + (\gamma v t)^2\right]^{3/2}}$$

SINCE $E_\parallel$ IS SYMMETRIC ABOUT $t=0$ $\Delta P_\parallel = 0$, WHILE:

$$\Delta P_\perp = \int F_\perp\, dt = \int e E_\perp\, dt = \frac{ze^2\gamma b}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{dt}{\left[b^2 + (\gamma v t)^2\right]^{3/2}}$$

IF WE WRITE $x = \frac{\gamma v t}{b}$

$$\Delta P_\perp = \frac{ze^2}{2\pi\varepsilon_0 v b}$$

KINETIC ENERGY TRANSFERRED TO THE ELECTRON:

$$\frac{(\Delta P_\perp)^2}{2m} = \frac{z^2 e^4}{8\pi^2\varepsilon_0^2 b^2 v^2 m}$$

THE AVERAGE ENERGY PER UNIT TIME CAN BE DERIVED BY MULTIPLYING THIS ENERGY BY $2\pi b\, db\, n_e v$ (TOTAL NUMBER OF ELECTRONS ENCOUNTERED PER UNIT TIME). IT FOLLOWS THAT THE

ENERGY LOSS IS:

$$-\frac{dE}{dt} = \int_{b_{min}}^{b_{max}} 2\pi b \, db \, m_e v \frac{z^2 e^4}{8\pi^2 \varepsilon_0^2 b^2 v^2 m}$$

$$\frac{(\Delta p_\perp)^2}{2m} \propto b^{-2} \Rightarrow \frac{dE}{db} = -\frac{2E}{b}$$

$$-\frac{dE}{dt} = \frac{z^2 e^4 m_e}{4\pi \varepsilon_0^2 v m} \int_{E_{min}}^{E_{max}} \frac{dE}{E}$$

TO GET APPROXIMATED EXTIMATIONS OF E_{MIN} AND E_{MAX}, WE CAN DO THE FOLLOWING CONSIDERATIONS

E_{MIN}

IF AN INELASTIC COLLISION TAKES PLACE

$$E_{MIN} = I_0$$

WHERE I_0 IS THE AVERAGE IONIZATION ENERGY.

E_{MAX}

FROM KINEMATICS THE MAXIMUM ENERGY THAT CAN BE TRANSFERRED IN S IS:

$$E_{MAX} = \frac{2p^2 m c^2}{b(\gamma)} \qquad p = \beta\gamma$$

WHERE

$$b(\gamma) = 1 + 2\gamma \frac{m}{M} + \left(\frac{m}{M}\right)^2$$

THEREFORE:

$$-\frac{dE}{dt} = \frac{z^2 e^4 m_e}{4\pi \varepsilon_0^2 v m} \ln\left(\frac{2p^2 m c^2}{I_0 \, b(\gamma)}\right)$$

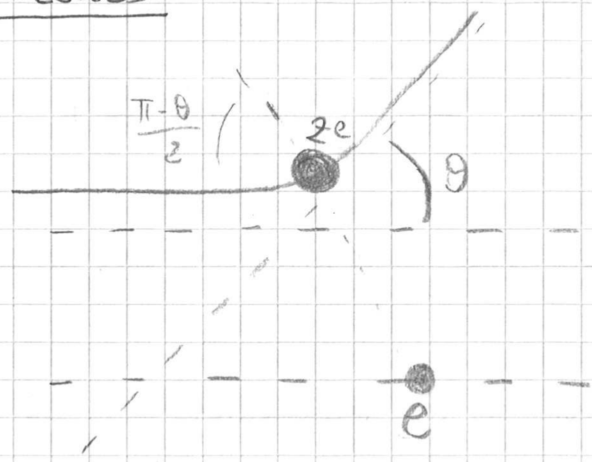
IF WE CONSIDER THE QUANTUM MECHANICAL EFFECTS:

$$-\frac{dE}{dt} = \frac{z^2 e^4}{4\pi\varepsilon_0 v m} \sum_{Z_1} Z_1 n_{Z_1} \left[\ln\left(\frac{2mc^2 p^2}{I_{Z_1} b(\gamma)} \right) - \beta^2 - \delta_{Z_1}/2 \right]$$

WHERE WE REPLACED n_e WITH $\sum_{Z_1} Z_1 n_{Z_1}$, WITH Z_1 BEING THE ELECTRON NUMBER, TO TAKE INTO ACCOUNT DIFFERENT ATOMIC SPECIES.

$\delta_{Z_1}/2$ IS THE DENSITY CORRECTION FACTOR (POLARIZATION OF THE MEDIUM). THE POLARIZATION OF THE MEDIUM TRANSLATES IN A SMALLER UPPER LIMIT TO THE RANGE OF COLLISION PARAMETERS, SEE e.g. ENßLIN ET AL. (2007).

COULOMB LOSSES GOULD (1971)



- RELATIVISTIC CASE

INTERACTION OF A RELATIVISTIC ION AND PLASMA-ELECTRONS:

- ION : ze, M
- ELECTRON: $-e$, m

ENERGY LOSSES BY ELECTRONS DIFFER BECAUSE OF EXCHANGE EFFECTS AND BECAUSE THE ELECTRON CAN LOSE A LARGE FRACTION OF ITS ENERGY IN ONE SCATTERING.

WE NEGLECT THE RECOIL OF THE ION. THIS MEANS THAT THE FOLLOWING IS VALID ONLY FOR $\gamma \ll M/m$.

WE SEPARATE THE INTERACTION IN TWO DOMAINS:

- LARGE-MOMENTUM TRANSFERS
- SMALL-MOMENTUM TRANSFERS

a) LARGE-MOMENTUM TRANSFERS

WE CAN ASSUME THAT THE INTERACTION IS TAKING PLACE WITH INDIVIDUAL ELECTRONS OF THE PLASMA. THEREFORE, THE MAGNETIC MOMENT OF ELECTRONS HAS TO BE TAKEN INTO ACCOUNT.

$$-\frac{dE}{dt} = n_e v \int \Delta E_{LAB}\, d\sigma$$

ΔE_{LAB}: ENERGY TRANSFER FROM THE ION TO THE ELECTRON IN THE LABORATORY FRAME

$d\sigma$: DIFFERENTIAL COULOMB-SCATTERING CROSS-SECTION.

IN TERMS OF PRIMED QUANTITIES IN THE REST FRAME OF THE ION:

$$\frac{d\sigma}{d\Omega'} = \frac{1}{4}\frac{z^2 e^2}{4\pi\varepsilon_0 mc^2}(\beta p)^{-2}\operatorname{cosec}^4\frac{\theta'}{2}\left(1-\beta^2\sin^2\frac{\theta'}{2}\right)$$

SINCE THE RECOIL IS NEGLECTED, THE INTERACTION CAN BE CONSIDERED AS A COULOMB SCATTERING.

THE TERM $\beta^2 \sin^2\frac{\theta'}{2}$ IS DUE TO THE MAGNETIC MOMENT OF THE ELECTRON.

MOMENTUM TRANSFER:

$$q' = 2mc\, p \sin\frac{\theta'}{2}$$

THIS IMPLIES A ENERGY EXCHANGE IN THE LABORATORY FRAME:

$$\Delta E_{LAB} = 2mc^2 p^2 s^2$$

WHERE: $s = \sin\frac{\theta'}{2}$

$s_0 < s < 1$

$s_0 \rightarrow$ BOUNDARY BETWEEN LARGE- AND SMALL-MOMENTUM TRANSFER DOMAINS

$$-\frac{dE}{dt} = \frac{z^2 e^4}{4\pi\varepsilon_0^2 mc^2} N_e v \beta^{-2}\int_{s_0}^{1}\frac{1}{s}\left(1-\beta^2 s^2\right) ds$$

$$-\frac{dE}{dt} = \frac{z^2 e^4}{4\pi\varepsilon_0^2 mc^2} N_e v \beta^{-2}\left[\ln\left(\frac{1}{s_0}\right) - \frac{\beta^2}{2}\right]$$

b) SMALL-MOMENTUM TRANSFERS

THE ION TRANSFERS TO THE ELECTRONS A SMALL AMOUNT OF ENERGY, SO THAT THEY ARE NON-RELATIVISTIC IN THE LABORATORY FRAME.

THE INTERACTION TAKES PLACE WITH A GROUP OF ELECTRONS, BUT SINCE THEY ARE AT GREAT DISTANCES FROM THE ION THEY SUFFER THE SAME MOMENTUM TRANSFER. THEREFORE, WE CAN CONSIDER AN INTERACTION WITH INDIVIDUAL ELECTRONS. THEIR MAGNETIC MOMENT CAN BE NEGLECTED.

IN THE LABORATORY FRAME:

$$\Delta E_{LAB} = \frac{q^2}{2m}$$

WITH:

$$q = 2pMc \sin\frac{\vartheta}{2} \qquad \text{MOMENTUM TRANSFER}$$

$$d\sigma = 8\pi \left(\frac{ze^2}{4\pi\varepsilon_0} \frac{1}{\beta c} \right)^2 \frac{dq}{q^3}$$

$$-\frac{dE}{dt} = \frac{z^2 e^4}{4\pi\varepsilon_0^2} \frac{m_e v}{mc^2} \beta^{-2} \ln\left(\frac{q_0}{q_{MIN}}\right)$$

SINCE THE MOMENTUM TRANSFER IS PERPENDICULAR TO THE RELATIVE VELOCITY OF THE TWO FRAMES OF REFERENCE:

$$q_0 = q_0' = 2mcp S_0$$

q_{MIN} CORRESPONDS TO THE PLASMON ENERGY $\hbar\omega_p$:

$$q_{MIN} = \frac{\omega_p \hbar}{v}$$

SUMMING THE ENERGY LOSS IN THE TWO DOMAINS WE GET THE TOTAL ENERGY LOSS:

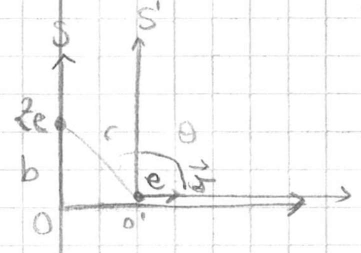
$$-\frac{dE}{dt} = \frac{z^2 e^4}{4\pi\varepsilon_0^2 mc} n_e \beta^{-1} \left[\ln\left(\frac{2mc^2\beta p}{\hbar\omega_p}\right) - \frac{\beta^2}{2} \right]$$

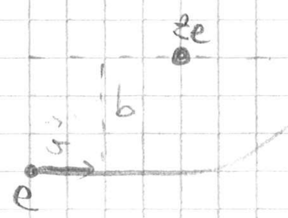
THE NON-RELATIVISTIC FORMULATION LEADS TO THE SAME RESULT APART FROM THE MAGNETIC MOMENT CONTRIBUTION.

BREMSSTRAHLUNG LOSSES

THIS RADIATION IS ASSOCIATED WITH ACCELERATION OF CHARGED PARTICLES IN THE ELECTROSTATIC FIELDS OF ATOMIC-NUCLEI / IONS.

BY SIMMETRY THE FIELD EXPERIENCED BY THE ELECTRON IN ITS REST FRAME IS THE SAME AS IN THE CASE OF IONISATION LOSSES WHERE THE ROLES OF THE TWO PARTICLES WERE INTERCHANGED.





$$a'_x = \dot{v}'_x(t') = \frac{eE_{x'}}{m} = \frac{\gamma Z e^2 v t'}{4\pi\varepsilon_0 m\,[b^2 + (\gamma v t')^2]^{3/2}}$$

$$a'_y = \dot{v}'_y(t') = \frac{eE_{y'}}{m} = \frac{\gamma Z e^2 b}{4\pi\varepsilon_0 m\,[b^2 + (\gamma v t')^2]^{3/2}}$$

WE ARE INTERESTED IN THE RADIATION SPECTRUM:

$$I(\omega') = \frac{e^2}{3\pi\varepsilon_0 c^3}\left[\,|a'_x(\omega')|^2 + |a'_y(\omega')|^2\,\right]$$

WE NEED TO EVALUATE THE FOURIER TRANSFORM OF $a'_x(t')$ AND $a'_y(t')$:

$$a'_x(\omega') = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \frac{\gamma Z e^2 v t'}{4\pi\varepsilon_0 m\,[b^2 + (\gamma v t')^2]^{3/2}}\, \mathrm{EXP}(i\omega t')\,dt'$$

$$a'_y(\omega') = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \frac{\gamma Z e^2 b}{4\pi\varepsilon_0 m\,[b^2 + (\gamma v t')^2]^{3/2}}\, \mathrm{EXP}(i\omega t')\,dt'$$

WE CAN EXPRESS THEM IN TERMS OF THE MODIFIED BESSEL FUNCTIONS OF ORDER ZERO K_0 AND ONE K_1:

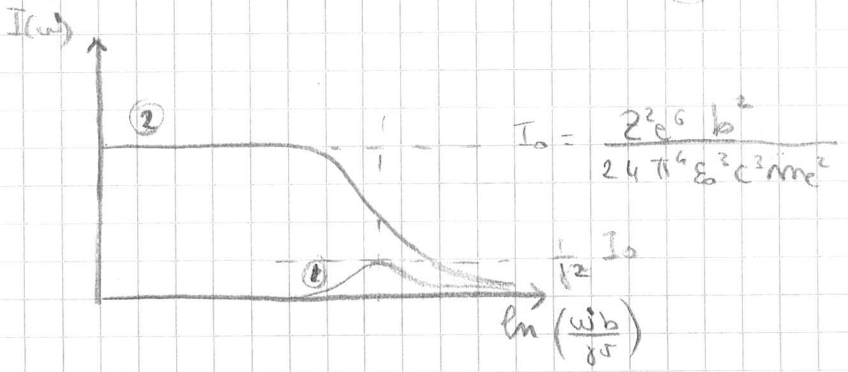
$$a'_x(\omega') = \frac{1}{\sqrt{2\pi}} \frac{Ze^2}{4\pi\varepsilon_0 m} \frac{1}{\gamma b v} 2i \frac{\omega' b}{\gamma v} K_0\left(\frac{\omega' b}{\gamma v}\right)$$

$$a'_y(\omega') = \frac{1}{\sqrt{2\pi}} \frac{Ze^2}{4\pi\varepsilon m} \frac{1}{bv} 2 \frac{\omega' b}{\gamma v} K_1\left(\frac{\omega' b}{\gamma v}\right)$$

FOR THE RADIATION SPECTRUM WE OBTAIN:

$$I(\omega') = \frac{e^2}{3\pi\varepsilon_0 c^3} \left[|a_x(\omega')|^2 + |a_y(\omega')|^2 \right] =$$

$$= \frac{Z^2 e^6}{24\pi^4 \varepsilon_0^3 c^3 m^2} \frac{\omega'^2}{\gamma^2 v^2} \Big[\underbrace{\frac{1}{\gamma^2} K_0^2\left(\frac{\omega' b}{\gamma v}\right)}_{1} + \underbrace{K_1^2\left(\frac{\omega' b}{\gamma v}\right)}_{2} \Big]$$



ASYMPTOTIC LIMITS:

- $\frac{\omega' b}{\gamma v} << 1$ LOW FREQUENCIES

$$K_0\left(\frac{\omega' b}{\gamma v}\right) = -\ln\left(\frac{\omega' b}{\gamma v}\right) \qquad K_1\left(\frac{\omega' b}{\gamma v}\right) = \frac{\gamma v}{\omega' b}$$

- $\frac{\omega' b}{\gamma v} >> 1$ HIGH FREQUENCIES

$$K_0\left(\frac{\omega' b}{\gamma v}\right) = K_1\left(\frac{\omega' b}{\gamma v}\right) = \left(\frac{\pi}{2} \frac{\gamma v}{\omega' b}\right)^{\frac{1}{2}} \mathrm{Exp}\left(-\frac{\omega' b}{\gamma v}\right)$$

THEN THE RADIATION SPECTRUM BECOMES

- $\frac{\omega' b}{\gamma v} >> 1$ HIGH FREQUENCIES

$$I(\omega') = \frac{Z^2 e^6}{48\pi^3 \varepsilon_0^3 c^3 m^2} \frac{\omega'}{\gamma b v} \left(\frac{1}{\gamma^2} + 1\right) \exp\left(-\frac{2\omega' b}{\gamma v}\right)$$

WHEN $\omega \gtrsim \gamma v / b$ A NEGLIGIBLE POWER IS EMITTED.

- $\frac{\omega' b}{\gamma v} \ll 1$ LOW FREQUENCIES

$$I(\omega') = \frac{Z^2 e^6}{24\pi^4 \varepsilon_0^3 c^3 m^2} \cdot \frac{1}{b^2 v^2} \left[1 - \gamma^{-2}\left(\frac{\omega' b}{\gamma v}\right)^2 \ln^2\left(\frac{\omega' b}{\gamma v}\right)\right]$$

WHEN $\frac{\omega' b}{\gamma v} \ll 1$, THE SECOND TERM IN BRACKETS CAN BE NEGLECTED. CONSEQUENTLY:

$$I(\omega') = \frac{Z^2 e^6}{24\pi^6 \varepsilon_0^3 c^3 m^2} \frac{1}{b^2 v^2} = \text{CONST} = I'$$

AT LOW FREQUENCIES THE DURATION OF THE COLLISION IS MUCH SMALLER THAN THE PERIOD OF THE WAVES $\left(\omega' \ll \frac{\gamma v}{b} \sim \frac{1}{\tau}\right)$. THEREFORE THE MOMENTUM IMPULSE CAN BE CONSIDERED A DELTA-FUNCTION AND ITS FOURIER TRANSFORM IS A CONSTANT.

IN THE REST FRAME OF THE ELECTRON THE DENSITY OF NUCLEI IS $n_n' = \gamma n_n$, WHERE n_n IS THE NUCLEI DENSITY IN THE OBSERVER FRAME OF REFERENCE. THEREFORE, THE RADIATION LOSS RATE PER UNIT BANDWIDTH IN THE REST FRAME OF THE ELECTRON:

$$I(\omega') = \int_{b_{min}}^{b'_{max}} 2\pi b' \gamma n_n v I' db' = \frac{Z^2 e^6}{12\pi^3 \varepsilon_0^3 c^3 m^2} \frac{1}{v} \ln\left(\frac{b'_{max}}{b_{min}}\right)$$

- NON-RELATIVISTIC CASE

b_{max}

$$b < \frac{\beta c}{\omega}$$

b_{min}

$$b = \frac{ze^2}{8\pi\varepsilon_0 m p^2 c^2} \quad \text{FOR LOW VELOCITIES}$$

$$b = \frac{\hbar}{2m pc} \quad \text{FOR HIGH VELOCITIES}$$

TOTAL ENERGY LOSS RATE:

$$\omega_{max} = \frac{2\pi}{\tau} \approx \frac{m p^2 c^2}{2\hbar}$$

$$-\frac{dE}{dt} \approx \int_0^{\omega_{max}} \frac{z^2 e^6 n_m}{12\pi^3 \varepsilon_0^3 c^3 m^2 pc} \ln\Lambda \, d\omega$$

$$-\frac{dE}{dt} \approx \frac{z^2 e^6 n_m}{24\pi^3 \varepsilon_0^3 c^2 m \hbar} p \ln\Lambda$$

$$\Lambda = \frac{8\pi\varepsilon_0 m p^3 c^3}{z e^2 \omega} \quad \text{FOR LOW VELOCITIES}$$

$$\Lambda = \frac{2m p^2 c^2}{\hbar\omega} \quad \text{FOR HIGH VELOCITIES}$$

- **RELATIVISTIC CASE**

b_{max}

THE ELECTRON INTERACTS WITH NUCLEI SHIELDED BY THEIR ELECTRON CLOUDS UNLESS THE COLLISION PARAMETER IS VERY SMALL;

BY USING THE FERMI-THOMAS MODEL OF THE ATOM THE ELECTROSTATIC POTENTIAL OF THE NUCLEUS CAN BE APPROXIMATED TO:

$$V(r) = \frac{ze^2}{r} \operatorname{Exp}\left(-\frac{r}{a}\right) \qquad \begin{cases} a = 1.4\, a_0 z^{-1/3} \\ a_0 = 4\pi\varepsilon_0 \hbar^2 / m_e e^2 \end{cases}$$

THEREFORE $b'_{max} = 1.4\, a_0 z^{-1/3}$.

IN THE ULTRARELATIVISTIC CASE THIS RESULTS TO BE SMALLER THAN THE LIMIT GIVEN BY $b \sim \frac{\beta c}{\omega}$

b_{min}

FROM THE UNCERTAINTY PRINCIPLE:

$$b_{min} \approx \frac{\hbar}{m \beta c}$$

PLUGGING IN THESE VALUES WE OBTAIN:

$$I(\omega') = \frac{z^2 e^6 \gamma m_n}{12 \pi^3 \varepsilon^3 c^3 m^2 \beta c} \ln\left(\frac{1.4\, a_0\, m\, \beta c}{z^{1/3}\, \hbar}\right)$$

TOTAL ENERGY LOSS RATE:

$$-\frac{dE}{dt} = \int_0^{\omega_{max}} \frac{z^2 e^6 m_n}{12 \pi^3 \varepsilon_0^3 c^4 m^2 \beta} \ln\left(\frac{1.4\, a_0\, m\, \beta c}{z^{1/3}\, \hbar}\right) d\omega$$

WHERE WE USED $d\omega = \gamma\, d\omega'$.

$$\omega_{max} = \frac{m c^2 p}{\hbar}$$

SINCE $\frac{1.4\, a_0\, m\, \beta c}{z^{\frac{1}{3}}\, \hbar} = \frac{182\, \beta}{z^{1/3}}$

$$-\frac{dE}{dt} = \frac{z^2 e^6 m_n}{12 \pi^3 \varepsilon^3 m\, c^2 \beta \hbar}\, p \ln\left(\frac{182\, \beta}{z^{1/3}}\right)$$

THE FORMULA DERIVED WITH A FULL RELATIVISTIC QUANTUM APPROACH IS INSTEAD (BETHE AND HEITLER FORMULA):

$$-\frac{dE}{dt} = \frac{z(z+1.3) e^6 m_n}{16 \pi^3 \varepsilon_0^3 m\, c^2 \hbar}\, p \left[\ln\left(\frac{183\, \beta}{z^{1/3}}\right) + \frac{1}{8}\right]$$

FOR THE RELATIVISTIC CASE, WHILE IN THE NON-RELATIVISTIC LIMIT THEY DERIVED:

$$-\frac{dE}{dt} = \frac{Z^2 e^6 m_n}{24\pi^3 \varepsilon_0^3 c^2 m \hbar} \, p \, \ln\left[\frac{1 + \left(1 - \frac{2\hbar\omega}{m p^2 c^2}\right)^{\frac{1}{2}}}{1 - \left(1 - \frac{2\hbar\omega}{m p^2 c^2}\right)^{\frac{1}{2}}} \right]$$

ADIABATIC LOSSES

IF THE VOLUME WITHIN WHICH THE PARTICLES ARE CONTAINED EXPANDS, THE PARTICLES LOOSE ENERGY.

FIRST LAW OF THERMODYNAMICS

$$dU = \delta Q - P dV$$

$\left\{\begin{array}{l} P: \text{PRESSURE OF THE GAS} \\ U: \text{INTERNAL ENERGY OF THE GAS} \\ V: \text{VOLUME OF THE GAS} \\ \delta Q: \text{HEAT SUPPLIED BY THE SURROUNDINGS} \end{array}\right.$

IF $\delta Q = 0$

$$dU = -P dV$$

- NON-RELATIVISTIC CASE. FOR A PERFECT GAS:

$$P = nkT \qquad U = \frac{3}{2} nkTV$$

$\left\{\begin{array}{l} n: \text{PARTICLE NUMBER DENSITY} \\ T: \text{TEMPERATURE OF THE GAS} \end{array}\right.$

BY COMBINING

$$dU = nV dE$$

$$dU = -P dV = -nkT\, dV = -\frac{2}{3} n E dV$$

$$\frac{dE}{dt} = -\frac{2}{3} \frac{nE}{N} \frac{dV}{dt}$$

$$\frac{dV}{dt} = (\vec{\nabla} \cdot \vec{v}) V$$ RATE OF EXPANSION OF THE VOLUME V DUE TO THE VELOCITY FIELD $\vec{v}(\vec{r})$

$$\frac{dE}{dt} = -\frac{2}{3} \frac{nE}{N} (\vec{\nabla} \cdot \vec{v}) V$$

$$\frac{dp}{dt} = -\frac{1}{3} c p (\vec{\nabla} \cdot \vec{\beta})$$ $p \rightarrow$ DIMENSIONLESS MOMENTUM

• RELATIVISTIC CASE

$$P = \frac{1}{3} U \qquad U = 3nkTV$$

BY COMBINING:

$$dU = nV\,dE$$

$$dU = -P\,dV = -nkT\,dV = -\frac{1}{3} nE\,dV$$

$$\frac{dE}{dt} = -\frac{1}{3} \frac{nE}{N} \frac{dV}{dt}$$

$$\frac{dE}{dt} = -\frac{1}{3} E \left(\vec{\nabla} \cdot \vec{v} \right)$$

$$\frac{dp}{dt} = -\frac{1}{3} pc \left(\vec{\nabla} \cdot \vec{\beta} \right)$$

THE ENERGY LOSS IS PROPORTIONAL TO THE MOMENTUM OF THE PARTICLES.

HADRONIC LOSSES

COSMIC-RAY PROTONS INTERACTING WITH THERMAL PROTONS PRODUCE $\pi^{\pm}$ AND π^{0} IF THEIR MOMENTUM IS LARGER THAN

$$q_{thr}\, m_p c^2 = 0.78 \text{ GeV}$$

THE TOTAL ENERGY LOSS RATE DOES NOT DEPEND ON THE EXACT MECHANISM THAT TOOK PLACE. IT CAN BE WRITTEN AS:

$$-\frac{dE}{dt} = K c \, n_N \, \sigma_{pp} \, T \, \theta(p - q_{thr})$$

WHERE

$K \simeq \frac{1}{2}$ INELASTIC COLLISION

$n_N = \dfrac{n_e}{1 - 0.5\, X_{He}}$, $X_{He} = 0.24$ DENSITY OF THERMAL PROTONS

$\sigma_{pp} \simeq 32 \left(0.96 + \mathrm{EXP}(4.4 - 2.4\, \alpha_\gamma)\right)$ mbarn, TOTAL PION CROSS-SECTION

$$\alpha_\gamma = \frac{4}{3}\left(\delta - \frac{1}{2}\right)$$

$T = (\sqrt{1+p^2} - 1)\, m_p c^2$ AVERAGE KINETIC ENERGY OF COSMIC-RAY PROTONS

$\theta(p - q_{thr})$ HEAVISIDE STEP FUNCTION

p, m DIMENSIONLESS PARTICLE MOMENTUM AND MASS OF COSMIC-RAY PROTONS

TYPICAL SPECTRA (RELATIVISTIC ELECTRONS)

THE INITIAL INJECTION SPECTRUM CAN SUFFER MODIFICATIONS IN TIME DEPENDING ON DIFFERENT CONCURRENT PROCESSES AND IF THE INJECTION IS CONTINUOUS OR NOT.

THE COSMIC-RAY SPECTRUM SHOULD FOLLOW THE EVOLUTION EQUATION:

$$\frac{\partial f(p,t)}{\partial t} + \frac{\partial}{\partial p}\left(\dot{p}(p)\, f(p,t)\right) = Q(p) - \frac{f(p,t)}{\tau_{esc}}$$

WHERE

$$\dot{p}(p) = \frac{dE}{dt} \cdot \frac{dp}{dE}$$

IF WE CONSIDER A CONFINEMENT TIME $\tau_{esc} \to \infty$, A CONTINUOUS INJECTION OF RELATIVISTIC ELECTRONS WITH AN INJECTION SPECTRUM

$Q(p) = A\, p^{-\delta_{inj}}$

THE STEADY-STATE SPECTRUM IS GIVEN BY:

$$f(p) = \frac{A\, p^{-(\delta_{inj}-1)}}{(\delta_{inj}-1)\, \frac{dE}{dt} \cdot \frac{dp}{dE}}$$

$$\frac{dp}{dE} = (mc^2\beta)^{-1} = (mc^2)^{-1}$$ FOR RELATIVISTIC ELECTRONS

$\frac{dE}{dt} = c_1 + c_2 p + c_3 p^2$ DESCRIBES THE FOLLOWING ENERGY-LOSSES:

c_1: IONISATION AND COULOMB LOSSES

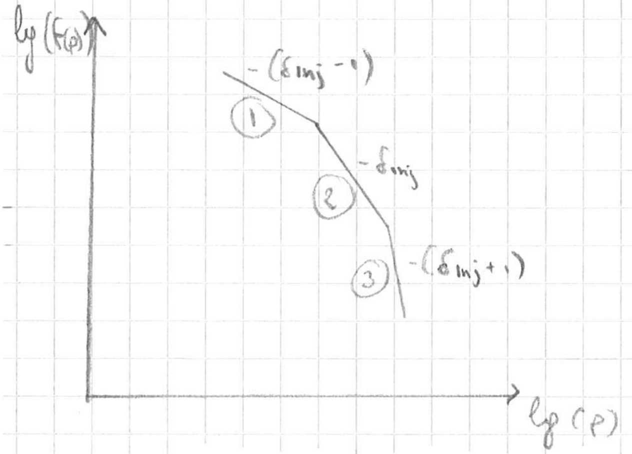
c_2: BREMSSTRAHLUNG AND ADIABATIC LOSSES

c_3: SYNCHROTRON AND INVERSE COMPTON LOSSES

CONSEQUENTLY:

1. $f(p) \propto p^{-(\delta_{inj}-1)}$ IF IONISATION AND COULOMB LOSSES DOMINATE;
2. $f(p) \propto p^{-\delta_{inj}}$ IF ADIABATIC AND BREMSSTRAHLUNG LOSSES DOMINATE;
3. $f(p) \propto p^{-(\delta_{inj}+1)}$ IF SYNCHROTRON AND INVERSE COMPTON LOSSES

DOMINATE.



SOLUTIONS FOR SYNCHROTRON AND INVERSE COMPTON LOSSES

1) ELECTRONS INJECTED ONLY AT $t_0 = 0$

$$-\frac{dp}{dt} = k p^2$$

$$-\frac{dp}{p^2} = k\, dt$$

$$-\int_{p_0}^{p} \frac{dp}{p^2} = k \int_{t_0=0}^{t} dt$$

$$\frac{1}{p} - \frac{1}{p_0} = k t$$

$$\frac{p}{p_b} = \frac{p_0/p_b}{1 + p_0/p_b} \qquad \text{WHERE} \quad p_b = (kt)^{-1}$$

WE CANNOT HAVE PARTICLES WITH MOMENTUM $p > p_b$ (BREAK).
THE SOLUTION OF THE DIFFUSION-LOSS EQUATION CAN BE COMPUTED WITH THE FOLLOWING CONSIDERATIONS.
IF NEW PARTICLES ARE NOT INJECTED, THEIR NUMBER IS CONSERVED:

$$f(p)\, dp = f(p_0)\, dp_0$$

$$f(p) = f(p_0) \frac{dp_0}{dp} = A\, p_0^{-\delta_{inj}} \frac{dp_0}{dp}$$

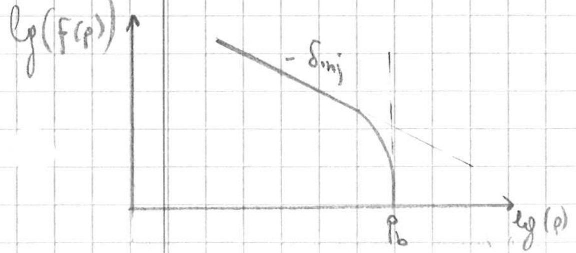
SINCE

$$\frac{1}{p} - \frac{1}{p_0} = \frac{1}{p_b} \rightarrow p_0 = \frac{p}{1 - p/p_b}$$

$$\frac{dp_0}{dp} = \frac{1}{1 - p/p_b} + \frac{p/p_b}{(1 - p/p_b)^2} = \frac{1}{\left(1 - \frac{p}{p_b}\right)^2}$$

$$f(p) = A\, p_0^{-\delta_{inj}} \frac{dp_0}{dp} = A\, p^{-\delta_{inj}} \left(1 - \frac{p}{p_b}\right)^{\delta_{inj} - 2}$$

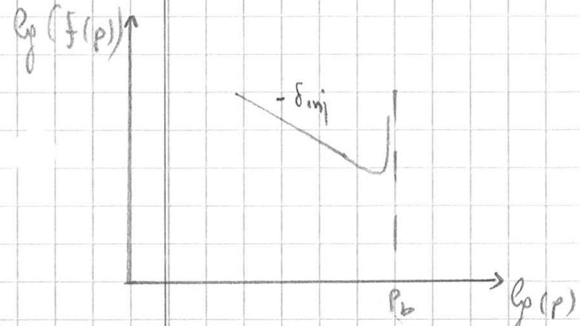
$\delta_{inj} > 2$



$p \rightarrow p_b \quad f(p) \rightarrow 0$

$p \ll p_b \quad f(p) \approx A\, p^{-\delta_{inj}}$

$\delta_{inj} < 2$



$p \rightarrow p_b \quad f(p) \rightarrow \infty$

$p \ll p_b \quad f(p) \simeq A\, p^{-\delta_{inj}}$

2) CONTINUOUS INJECTION

LET'S ASSUME TO CONTINUOUSLY PROVIDE PARTICLES WITH THE SAME INJECTION SPECTRUM AS BEFORE. SINCE THE INJECTION IS CONTINUOUS THE NUMBER OF PARTICLES IS NOT CONSTANT ANYMORE:

$$Q(p,t) = A\, p^{-\delta_{inj}}$$

THEREFORE

$$\frac{df}{dt} = A\, p^{-\delta_{inj}} \left(1 - \frac{p}{p_b}\right)^{\delta_{inj}-2}$$

THE NUMBER OF PARTICLES AT A GIVEN TIME t IS:

$$f(p) = A \int_{\tau=0}^{\tau=t} p^{-\delta_{inj}} \left(1 - \frac{p}{p_b}\right)^{\delta_{inj}-2} d\tau$$

$$p_b = (k\tau)^{-1}$$

$$1 - \frac{p}{p_b} = 1 - k\tau p$$

BY USING THE SUBSTITUTION:

$$1 - k\tau p = z \qquad dz = -kp\, d\tau$$

$$f(p) = A \int_{1}^{1-kpt} p^{-\delta_{inj}} z^{\delta_{inj}-2} \frac{dz}{-kp}$$

$$f(p) = A \frac{p^{-\delta_{inj}-1}}{k} \int_{1-kpt}^{1} z^{\delta_{inj}-2} dz$$

a) $p < p_b = (kt)^{-1}$

$$f(p) = \frac{A}{k(\delta-1)} p^{-\delta_{inj}-1} \left[1 - \left(1 - \frac{p}{p_b}\right)^{\delta_{inj}-1}\right]$$

(b) $p \geq p_b = (\kappa t)^{-1}$

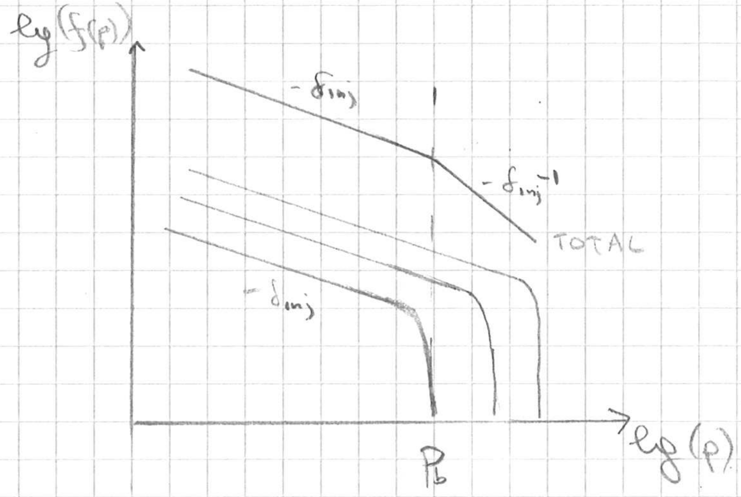
THERE IS A TIME t^* BEYOND WHICH WE CANNOT ANYMORE SEE PARTICLES BECAUSE THEY LOST ALREADY THE MOST OF THEIR MOMENTUM:

$$t^* = (\kappa p)^{-1}$$

$$f(p) = \frac{A p^{-\delta_{inj}-1}}{\kappa} \int_{1-\kappa p t^* = 0}^{1} z^{\delta_{inj}-2}\, dz = \frac{A}{\kappa(\delta-1)} p^{-\delta_{inj}-1}$$

$$f(p) = f_0 p^{-\delta_{inj}-1} \quad \text{WITH} \quad f_0 = \frac{A}{\kappa(\delta-1)}$$

f_0 DOESN'T DEPEND ON TIME. IT MEANS THAT THE INJECTION RATE IS SUCH THAT THE NUMBER OF PARTICLES INJECTED IS THE SAME AS THOSE LOST IN A GIVEN RANGE OF MOMENTUM.



(c) $p \ll p_b = (\kappa t)^{-1}$

$$f(p) \simeq \frac{A}{\kappa(\delta_{inj}-1)} p^{-\delta_{inj}-1} (\delta_{inj}-1) \frac{p}{p_b}$$ BY USING TAYLOR EXPANSION

$$f(p) \simeq \frac{A}{\kappa} p^{-\delta_{inj}} \kappa t$$

$$f(p) \simeq A t p^{-\delta_{inj}}$$

$$f(p) \simeq f_0 p^{-\delta_{inj}} \quad \text{WITH} \quad f_0 = At$$

ELECTRON LIFETIME

IF PARTICLES LOSE ENERGY VIA SYNCHROTRON EMISSION AND INVERSE COMPTON SCATTERING OF COSMIC BACKGROUND PHOTONS

$$-\frac{dE}{dt} = k'(B^2 + B_{CMB}^2)E^2$$

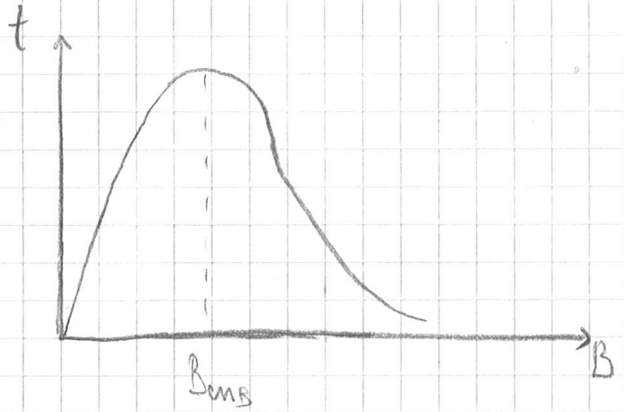
WHERE B_{CMB} IS THE EQUIVALENT MAGNETIC FIELD STRENGTH OF THE CMB

$$t = \frac{E}{\dot{E}} = \frac{1}{k'(B^2 + B_{CMB}^2)E}$$

SINCE $\nu = k'' B E^2$

$$t = \frac{\nu^{-\frac{1}{2}}}{k'} \frac{B^{\frac{1}{2}}}{B^2 + B_{CMB}^2}$$

$$\begin{cases} B << B_{CMB} & t \propto B^{\frac{1}{2}} \\ B >> B_{CMB} & t \propto B^{-3/2} \end{cases}$$



OBSERVABLES : SYNCHROTRON AND INVERSE COMPTON EMISSION

IF THE SYNCHROTRON RADIO AND INVERSE COMPTON X-RAY EMISSION ARE PRODUCED BY THE SAME POPULATION OF RELATIVISTIC ELECTRONS, THEIR FLUXES CAN BE USED TO DERIVE INFORMATION ON THE MAGNETIC FIELD AND ON THE RELATIVISTIC ELECTRONS RESPONSIBLE OF THE EMISSION.

FROM BLUMENTHAL & GLOUD (1970):

$$S_{syn}(\nu_R) = \frac{\phi V}{4\pi D_L^2} \frac{4\pi e^3}{(m_e c^2)^\delta} f_0 B^{\frac{\delta+1}{2}} \left(\frac{3e}{4\pi m_e c}\right)^{\frac{\delta-1}{2}} a(\delta)\, \nu_R^{-\frac{\delta-1}{2}}$$

$$S_{IC}(\nu_x) = \frac{\phi V}{4\pi D_L^2} \frac{8\pi^2 r_0^2}{c^2} h^{-\frac{\delta+3}{2}} f_0 (m_e c^2)^{1-\delta} (kT)^{\frac{\delta+5}{2}} F(\delta)\, \nu_x^{-\frac{\delta-1}{2}}$$

ϕ: FILLING FACTOR

V: VOLUME OF THE SOURCE

D_L: LUMINOSITY DISTANCE OF THE SOURCE

B: MAGNETIC FIELD STRENGTH

r_0: CLASSICAL ELECTRON RADIUS

h: PLANCK CONSTANT

f_0: AMPLITUDE OF THE ELECTRON ENERGY DISTRIBUTION

δ: SPECTRAL INDEX OF THE ELECTRON MOMENTUM DISTRIBUTION

T: RADIATION TEMPERATURE OF THE CMB

$$B \propto \left(\frac{S_{syn}(\nu_R)}{S_{IC}(\nu_x)}\right)^{\frac{2}{\delta+1}} \left(\frac{\nu_R}{\nu_x}\right)^{\frac{\delta-1}{\delta+1}}$$

VOLUME AVERAGED MAGNETIC FIELD

ENERGY EQUIPARTITION

THE TOTAL ENERGY IN A RADIO SOURCE DUE TO THE NON-THERMAL COMPONENTS IS:

$$E_{tot} = E_{PART} + E_B$$

THE ENERGY IN THE FORM OF PARTICLES IS DUE BOTH TO PROTONS AND ELECTRONS:

$$E_{PART} = E_p + E_e = (1+k)E_e \qquad k = \frac{E_p}{E_e}$$

THE ENERGY IN RELATIVISTIC ELECTRONS IS:

$$E_e = N_0 V \int_{E_1}^{E_2} E^{-\delta} E \, dE$$

SINCE:

$$L_\nu = C_\alpha N_0 V B^{\alpha+1} \nu^{-\alpha} \qquad N_0 V = \frac{L_\nu}{C_\alpha B^{\alpha+1} \nu^{-\alpha}}$$

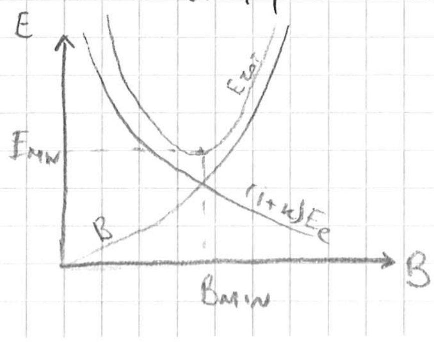
THEREFORE:

$$E_{tot} = (1+k) \frac{L_\nu}{C_\alpha B^{\alpha+1} \nu^{-\alpha}} \int_{E_1}^{E_2} E^{1-\delta} dE + \frac{VB^2}{2\mu_0}$$

$$U_{tot} = \frac{E_{tot}}{V} = (1+k) \frac{J_\nu}{C_\alpha B^{\alpha+1} \nu^{-\alpha}} \cdot \frac{E_2^{2-\delta} - E_1^{2-\delta}}{2-\delta} + \frac{B^2}{2\mu_0}$$

THERE IS NO WAY FROM RADIO OBSERVATIONS ALONE TO DISENTANGLE THE CONTRIBUTION FROM THE FIELD TO THAT DUE TO THE PARTICLES. NEVERTHELESS, WE CAN ASSUME A MINIMUM ENERGY CONDITION:

$$\frac{\partial U_{tot}}{\partial B} = 0$$



$\alpha = 1$

$$(1+k)\frac{J_\nu}{C_\alpha \nu^{-\alpha}}\frac{E_2^{2-\delta}-E_1^{2-\delta}}{2-\delta}(-1-\alpha)B^{-(\alpha+2)}+\frac{B}{\mu_0}=0$$

$$B_{MIN}^{3+\alpha}=\mu_0(1+k)(1+\alpha)\frac{J_\nu}{C_\alpha \nu^{-\alpha}}\frac{E_2^{2-\delta}-E_1^{2-\delta}}{2-\delta}$$

$$B_{MIN}=\left[\mu_0(1+k)(1+\alpha)\frac{J_\nu}{C_\alpha \nu^{-\alpha}}\frac{E_2^{2-\delta}-E_1^{2-\delta}}{2-\delta}\right]^{\frac{1}{3+\alpha}}$$

$$U_B=\frac{B^2}{2\mu_0}=\frac{1}{2\mu_0}\left[\mu_0(1+k)(1+\alpha)\frac{J_\nu}{C_\alpha \nu^{-\alpha}}\frac{E_2^{2-\delta}-E_1^{2-\delta}}{2-\delta}\right]^{\frac{2}{3+\alpha}}$$

$$U_{PART}=(1+k)\frac{J_\nu}{C_\alpha \nu^{-\alpha}}\frac{E_2^{2-\delta}-E_1^{2-\delta}}{2-\delta}\left[\mu_0(1+k)(1+\alpha)\frac{J_\nu}{C_\alpha \nu^{-\alpha}}\frac{E_2^{2-\delta}-E_1^{2-\delta}}{2-\delta}\right]^{-\frac{\alpha+1}{3+\alpha}}$$

$$\frac{U_B}{U_{PART}}=\frac{1}{2\mu_0}\left[(1+k)\frac{J_\nu}{C_\alpha \nu^{-\alpha}}\frac{E_2^{2-\delta}-E_1^{2-\delta}}{2-\delta}\right]^{-1}\left[\mu_0(1+k)(1+\alpha)\frac{J_\nu}{C_\alpha \nu^{-\alpha}}\frac{E_2^{2-\delta}-E_1^{2-\delta}}{2-\delta}\right]$$

$$\frac{U_B}{U_{PART}}=\frac{1}{2\mu_0}\mu_0(1+\alpha)=\frac{1+\alpha}{2}$$

THE MINIMUM ENERGY CONDITION CORRESPOND TO A PERFECT EQUIPARTITION ONLY FOR $\alpha=1$.

GAMMA RAY EMISSION

γ-RAY SOURCE FUNCTION FROM PION-DECAY OF A POWER LAW COSMIC RAY POPULATION (ENßLIN ET AL. 2007, PFROMMER & ENßLIN 2004):

$$S_\gamma(E_\gamma)\,dE_\gamma\,dV \simeq \sigma_{pp}\, c\, M_{GAS}\, \xi^{2-\alpha_\gamma}\, M_{CRP}\, \frac{4}{3\alpha_\gamma}\left(m_{\pi^0}c^2\right)^{-\alpha_\gamma}\left[\left(\frac{2E_\gamma}{m_{\pi^0}c^2}\right)^{\delta_\gamma}+\left(\frac{2E_\gamma}{m_{\pi^0}c^2}\right)^{-\delta_\gamma}\right]^{-\frac{\alpha_\gamma}{\delta_\gamma}} dE_\gamma\, dV$$

$$\delta_\gamma = 0.14\,\alpha_\gamma^{-1.6} + 0.44$$

$$\sigma_{pp} = 32\left(0.96 + e^{4.4-2.4\alpha_\gamma}\right) \text{ mbarn}$$

$$\alpha_\gamma = \frac{4}{3}\left(\delta - \frac{1}{2}\right)$$

GAMMA RAY EMISSION FROM PION-DECAY GIVES INFORMATION ABOUT M_{GAS} AND M_{CRP}