PHYSICS OF GALAXY CLUSTERS

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Motivation

Q. Why Galaxy Clusters?

A. Not too hard (c.f. galaxy formation), but still non-trivial.
Potentially powerful probe of cosmology & astrophysics.

Q. Why "Physics"?

A. because the use of galaxy clusters as a cosmological probe is limited by poorly understood cluster astrophysics.

Outline

(1) Interaction of Matter & Radiation
- Sunyaev - Zel'dovich Effect
- X-ray emission

(2) Cluster Structure
- Collisionless Dynamics of Dark Matter
- Hydrodynamics

(3) Application to Cosmology

(4) Beyond Hydrodynamics

(1) Interaction of Matter & Radiation

1.1. Sunyaev - Zel'dovich Effect

- $T_e \sim \frac{GM}{R} \sim 10^8\,{}^\circ K \sim 10\,keV$

  $\frac{k_B T_e}{m_e c^2} \sim \frac{10\,keV}{500\,keV} \sim \frac{1}{50}$ {Non-relativistic thermal particles}

- $\frac{h\nu_{CMB}}{m_e c^2} \ll 1 \Rightarrow \varepsilon' \approx \varepsilon$

  $\sigma^{KN}_{(\nu)} \xrightarrow[\varepsilon' \approx \varepsilon]{} \sigma_T$ {Thompson scattering limit}

- $\tau \sim n_e \sigma_T R$

  $\sim (10^{-3} cm^{-3})(6 \times 10^{-25} cm^2)(3 \times 10^{24} cm)$

  $\sim 10^{-3}$ {Optically thin}

Radiative transfer of CMB photons scattering off of thermal electrons in galaxy clusters is SIMPLE!

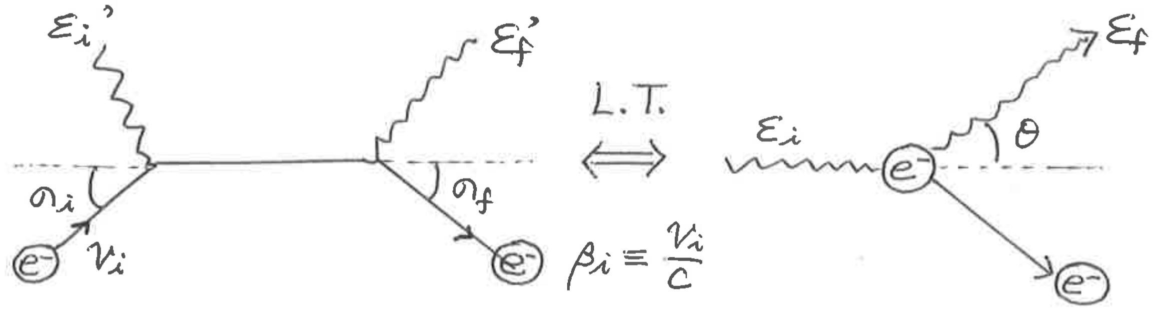
REFERENCES :

Sunyaev & Zel'dovich, 1980

Birkinshaw, 1999, Phys. Rep.

Rybicki & Lightman, 1979

1.2. Compton Scattering



Observer's Frame　　　　　Electron's Rest Frame

$$\frac{\mathcal{E}_f'}{\mathcal{E}_i'} = \frac{1 - \beta_i \cos\alpha_i}{1 - \beta_i \cos\alpha_f + \frac{\mathcal{E}_i}{\gamma m_e c^2}(1-\cos\theta)} \qquad \frac{\mathcal{E}_f}{\mathcal{E}_i} = \frac{1}{1 + \frac{\mathcal{E}_i}{m_e c^2}(1-\cos\theta)} \lesssim 1$$

$$\xrightarrow[\beta_i \ll 1]{} 1 - \beta_i \cos\alpha_i + \beta_i \cos\alpha_f + \beta_i^2 \cos^2\alpha_f - \frac{\mathcal{E}_i}{m_e c^2}$$

	$\cos\theta$	
$\theta = 0$	1	no scattering
$\theta = 90°$	0	$\mathcal{E}_f < \mathcal{E}_i$
$\theta = 180°$	-1	$\mathcal{E}_f \ll \mathcal{E}_i$

Averaging over angles :

$$\left\langle \frac{\mathcal{E}_f'}{\mathcal{E}_i'} \right\rangle \approx 1 + \frac{1}{3}\beta_i^2 - \frac{\mathcal{E}_i}{m_e c^2}$$

For a thermal distribution of velocities

$$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} k_B T_e$$

$$\beta_i^2 \approx \frac{3 k_B T_e}{m_e c^2}$$

$$\left\langle \frac{\Delta\mathcal{E}}{\mathcal{E}_i} \right\rangle \approx \frac{kT_e - \mathcal{E}_i}{m_e c^2}$$ for a single scattering event

If $\mathcal{E}_i \ll k_B T$, γ gains energy $\Rightarrow$ "Inverse Compton Scattering"

If $\mathcal{E}_i \gg k_B T$, γ loses energy $\Rightarrow$ "Compton Scattering"

More proper treatment of the radiative transfer gives the Kampaneets Eq., which describes repeated scattering of photons by non-relativistic electrons :

$$\left\langle \frac{\Delta \varepsilon}{\varepsilon_i} \right\rangle = \frac{4k_B T_e - \varepsilon_i}{m_e c^2} \sim \frac{1}{10}$$ for N.R. electrons in thermal equilibrium

↑

Scattering is almost elastic.

For multiple scattering events,

$$\left\langle \frac{\Delta \varepsilon}{\varepsilon_i} \right\rangle_t = \frac{4k_B T_e - \varepsilon_i}{m_e c^2} \tau$$

Key Results :

(1) Spectral distortion has an analytical form in the N.R. limit.

$$\Delta I_{SZE} = g(x)\, I_0\, y$$

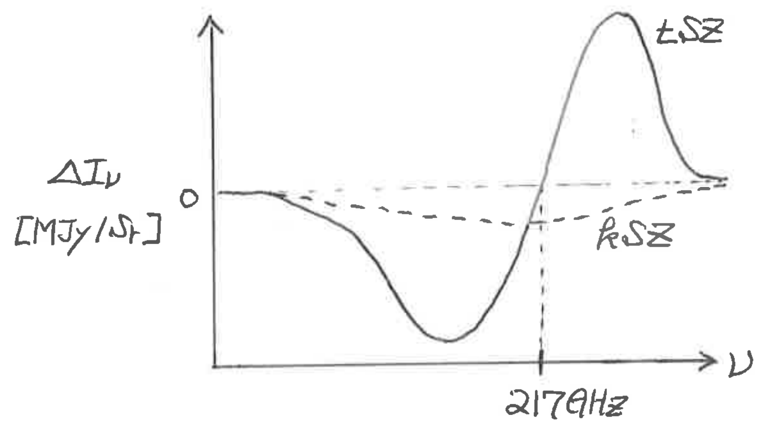
Relativistic Correction accurate to ≪1% (SZPack: Chluba+12)

$$g(x) = \frac{x^4 e^x}{(e^x-1)^2}\left(x\frac{e^x+1}{e^x-1} - 4\right)\left[1+\delta_{REL}(x, T_e)\right] \rightarrow -2$$ in R.J+N.R limit

(2) The amplitude of the SZE is proportional to the Compton-y parameter :

$$y = \frac{k_B \sigma_T}{m_e c^2}\int n_e T_e \, dl$$ ···· Integrated Pressure along the l.o.s.

Thermal SZE



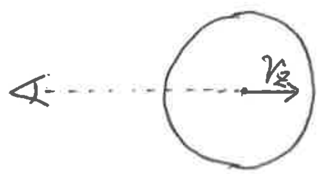
$$\frac{\Delta I}{I_0} \approx -2y = -2\left(\frac{k_B T_e}{m_e c^2}\right)\tau \approx -2(2\times10^{-2})(10^{-3}) \approx -4\times10^{-5}$$

↑ in RJ + N.R limit

Kinematic SZE

$$\frac{\Delta I}{I_0} \approx \frac{v}{c}\tau \approx \left(\frac{300\,\mathrm{km/s}}{3\times10^5\,\mathrm{km/s}}\right)(10^{-3}) \approx 10^{-6}$$

CMB Frame



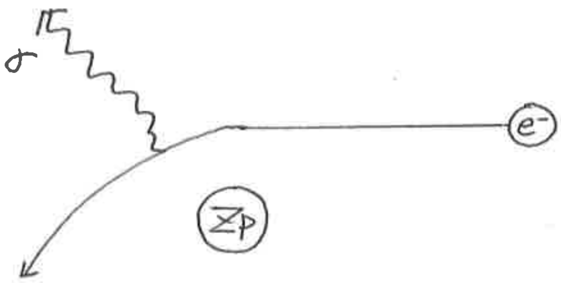
Cluster Frame

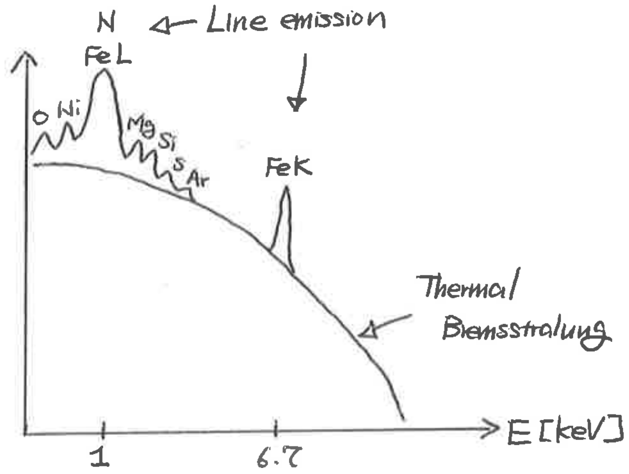
1. Incident CMB γs are isotropic ⟶ 2. CMB γs look anisotropic

↓

4. Scattered CMB γs are anisotropic ⟵ 3. IC scattering re-isotropize the radiation slightly

1.3 : X-ray emission





$$\varepsilon_\nu^{ff} \equiv \frac{dW}{dV\,dt\,d\nu} \propto Z_i^2 n_e n_i T^{-1/2} e^{-\frac{h\nu}{k_B T}} \underset{\text{Gaunt Factor}}{\bar{g}_{ff}}$$

- <u>X-ray surface Brightness</u>

$$S_X \propto \int dl \,(n_e n_p \Lambda_{ep} + n_e n_{He} \Lambda_{eHe})$$

$$\chi \equiv \frac{n_{He}}{n_H} \quad \text{.... He-to-H ratio}$$

$$\propto n_p^2 (1+2X)(1+4X)\Lambda_{ep}$$

Note : $\Lambda_{eHe} = 4\Lambda_{ep}$ & $n_e = n_p + 2n_{He} = n_p(1+2X)$.

Since S_X is observed & fixed,

$$n_p \propto \frac{1}{\sqrt{(1+2X)(1+4X)}}$$

$$\rho_{gas} \propto n_p + 4n_{He} \propto \left(\frac{1+4X}{1+2X}\right)^{1/2}$$

Global properties :

$$M_{gas} = \int \rho_{gas}\, dV$$

$$M_{TOT} \approx M_{HSE} = \frac{-r^2}{G\rho}\frac{dP}{dr} \propto \frac{rT_e}{\mu}\frac{d\log\rho_{gas}}{d\log r}$$

$$f_{gas} \equiv \frac{M_{gas}}{M_{TOT}}$$

- **Mean Molecular Weight** : $\mu \equiv \frac{\bar{m}}{m_p}$ .... Average mass per particle in units of H atom mass (m_p)

- To relate n and ρ, $n = \frac{\rho}{\bar{m}}$

$$\bar{m} = \frac{\sum_j n_j m_j + n_e m_e}{\sum_j n_j + n_e} \approx \frac{\sum_j n_j m_j}{\sum_j n_j + n_e}$$

$m_j \approx A_j m_p$

$\uparrow$ # of ps + ns

- For fully ionized gas,

$n_e \rightarrow \sum_j n_j Z_j$ (Z_j: Atomic #)

In general case, must determine n_e from solution of Saha Equation.

- Mean Molecular Weight :

$$\mu = \frac{\bar{m}}{m_p} = \frac{\sum_j n_j A_j}{\sum_j n_j + n_e} \longrightarrow \frac{\sum_j n_j A_j}{\sum_j n_j (1+Z_j)}$$

Assume $\frac{(1+Z_j)}{A_j} \approx \frac{1}{2}$ for each metal

Define $n_j \equiv \frac{\rho}{m_p} \frac{X_j}{A_j}$ X_j = mass fraction of specie i where $\sum_j X_j = 1$

$$\mu = \left[\sum_j \frac{X_j}{A_j}(1+Z_j) \right]^{-1} \approx \left[2X + \frac{3Y}{4} + \frac{Z}{2} \right]^{-1}$$

- For the primordial abundance ($X=0.75$ & $Y=0.25$)

$\mu = 0.59$ $x = \frac{(Y/4)}{X} = 0.083$

- If x is enhanced by a factor of 2 (e.g., by He sedimentation see 4.3 for more discussion)

$\rho_{gas} \downarrow$ by 5%

$M_{TOT} \uparrow$ by 12%

(2) Cluster Structure

Composition:

- Dark Matter 85%
- Baryons 15%
  - Gas ~12%
  - Stars ~3%

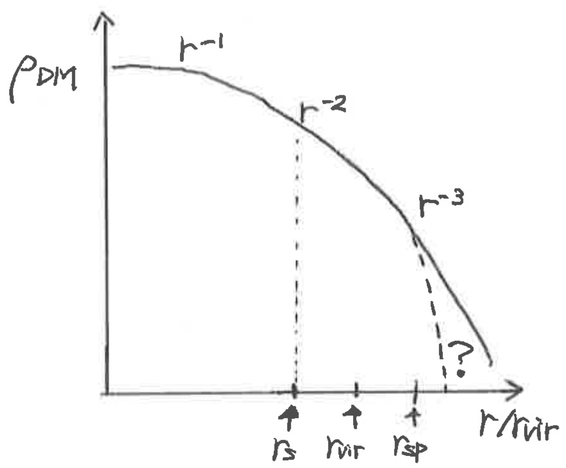
2.1: Collisionless Dynamics of Dark Matter

- Collisionless Boltzmann Eq.

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \dot{x}_i \frac{\partial f}{\partial x_i} + \dot{u}_i \frac{\partial f}{\partial u_i} = 0$$

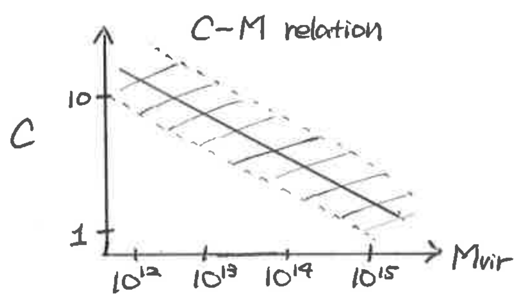
- $f(\vec{x}, \vec{u}, t) \equiv \lim_{\delta V \to 0^+} \frac{\delta N}{\delta V}$ .... Distribution Function
- $f(\vec{u}) = n \left(\frac{m}{2\pi k_B T}\right) \exp\left[-\frac{m(\vec{u}-\vec{u}_0)^2}{2k_B T}\right]$ .... Maxwell–Boltzmann D.F. is the equilibrium solution of the Boltzmann Eq.

- N-body Simulations



$$\rho_{DM}(x) = \frac{\rho_s}{x(1+x)^2} \quad \text{(NFW 1997)}$$

$$x \equiv \frac{r}{r_s} \qquad c \equiv \frac{r_{vir}}{r_s}$$



$$M(<x) = 4\pi \rho_s r_s^3 \left[\ln(1+x) - \frac{x}{1+x}\right]$$

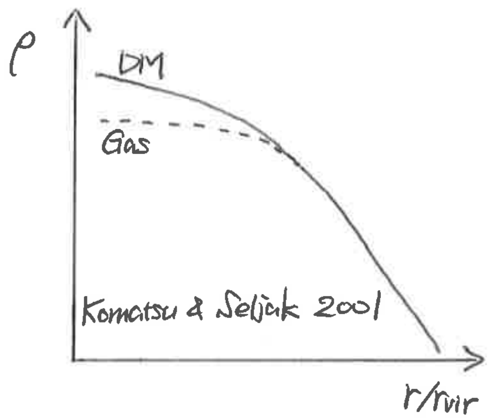
2.2 : Hydrodynamics

$$\frac{df}{dt} = \mathbb{C} \leftarrow \text{Coulomb collision for gas particles}$$

- Conservation Equations

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0 & \cdots\text{ Continuity Eq.} \\ \dfrac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\dfrac{1}{\rho}\vec{\nabla}P + \dfrac{\vec{F}}{m} + \dfrac{\mu}{\rho}\nabla^2\vec{v} & \cdots\text{ Navier-Stokes Eq.} \\ \rho\left(\dfrac{\partial \mathcal{E}}{\partial t} + \vec{v}\cdot\vec{\nabla}\right) - \vec{\nabla}\cdot(K\vec{\nabla}T) + P\vec{\nabla}\cdot\vec{v} = 0 & \cdots\text{ Energy Eq.} \end{cases}$$

- Gas in hydrostatic equilibrium in the NFW potential.



Assume :

(1) HSE $\Rightarrow \dfrac{dP}{dr} = -\dfrac{GM}{r^2}\rho$

(2) Gas traces DM at large r

(3) $P \propto \rho^{\Gamma}$ $\quad \Gamma = \text{const.}$

Good assumptions for $P_{TOT} = P_{th} + \underline{\underline{P_{nt}}}$?

$\uparrow$ Gas motions, $\vec{B}$, CRs

(3) Application to Cosmolosy.

REFERENCES

Carlstrom et al. 2002, ARAA

Allen et al. 2011, ARAA

3.1: SZ + X-ray Hubble constant

$$\begin{cases} \Delta I_{SZE} \propto \int n_e T_e \, dl \propto n_e T_e L \\ S_X \propto \int n_e^2 \Lambda_X(T_e, z) \, dl \propto n_e^2 \Lambda_X L \end{cases} \qquad L = D_A \theta_{\parallel}$$

$$\Rightarrow \quad D_A \propto \frac{(\Delta I_{SZE})^2}{S_X} \frac{\Lambda_X(T_e, z)}{T_e^2 \theta_\perp}$$

$$\boxed{H_0 = 77 \pm 5 \pm \underline{\underline{10}} \ \mathrm{km/s/Mpc}}$$ (Bonamente et al. 2006)

for 38 clusters with $(\Omega_m, \Omega_\Lambda) = (0.3, 0.7)$

Systematics :

(1) Gas clumping : $C \equiv \frac{\langle n^2 \rangle}{\langle n \rangle^2} > 1$?

(2) Asphericity : $\theta_\parallel = \theta_\perp$?

3.2 : Cluster Baryon Fraction ("Relaxed Clusters")

$$f_{b,CL} \equiv \frac{M_b}{M_{TOT}} \approx \frac{\Omega_b}{\Omega_M} \Rightarrow \boxed{\Omega_M \lesssim \Omega_b \left(\frac{M_{TOT}}{M_b}\right)}$$

(White et al. 1993)

$$\underbrace{\frac{f_{b,CL}}{(\Omega_b/\Omega_M)}}_{\text{SIM}} = \frac{\overset{70\%}{M_{gas}} + \overset{10\%}{M_{gal}} + \overset{\lesssim 5\%}{M_{ICS}} + \cdots}{M_{TOT}\,(\Omega_b/\Omega_M)}$$

SIM NEED $M_{HSE} \approx 0.9 M_{TOT}$.

92-100% → 80% unlikely? (Missing Baryon Problem in Clusters)

AGN feedback?

Evaporation? but $\vec{B}$?

3.3 : f_{gas} evolution ("Relaxed Clusters")

Assume : $f_{gas} \equiv \dfrac{M_{gas}}{M_{TOT}} = \text{const.}$

$$\left\{\begin{array}{ll} \text{SZE}: & \Delta I_{SZE} \propto n_{gas} T_e D_A \theta_{11} \\ \text{X-ray}: & S_X \propto n_{gas}^2 \Lambda_X D_A \theta_{11} \end{array}\right. \Rightarrow \left\{\begin{array}{l} n_{gas} \propto D_A^{-1} \\ n_{gas} \propto D_A^{-1/2} \end{array}\right.$$

$$f_{gas} \propto \frac{n_{gas} R^3}{R} \propto n_{gas} R^2 \propto \left\{\begin{array}{ll} D_A & \cdots \text{SZE} \\ D_A^{3/2} & \cdots \text{X-ray} \end{array}\right.$$

$\boxed{w_0 = -1.05 \pm 0.29}$ (Allen et al. 2008)

Systematics : Evolution in the baryon depletion factor ($f_{gas}/(\Omega_b/\Omega_M)$).

3.4. : Cluster Counts

$$\frac{dN}{dM}(>M_{lim}, z) = \int d\Omega \int_{\underline{M_{lim}}}^{\infty} dM \frac{dn}{dM}(M, z) \frac{dV}{d\Omega dz}$$

KEY : Observable - Mass relation

Scatter

$$T \propto \frac{GM}{R} \propto M^{2/3} \quad \leftarrow \quad 20\%$$

$$Y_{SZ} \propto M_{gas} T \propto f_{gas} M^{5/3}$$

$$Y_X \propto M_{gas} T_X$$

8%

SZE : ACT, SPT, Planck

X-ray : ROSAT + Chandra / XMM-Newton , eROSITA

Optical : SDSS, DES, LSST, Euclid

3.5 : SZ power spectrum

$$C_\ell = g_\nu^2 \int_0^{z_{max}} dz \frac{dV}{dz} \int_{M_{min}}^{M_{max}} dM \frac{dn(M,z)}{dM} |\tilde{y}_\ell(M,z)|^2$$

2D F.T. of
the projected Compton - y

$$C_\ell \propto \sigma_8^7 (\Omega_b h)^2$$ (Komatsu & Seljak 2002)

Systematics : Gas pressure profile in Groups & Clusters.

Alternative Approaches : Bispectrum (higher order moments) & Cross-correlation.

(4) Beyond Hydrodynamics

So far, we have assumed that the fully ionized plasma (Consisting of H^+, He^{++}, e^-) in the ICM can be regarded as a "single" fluid in local thermodynamic equilibrium (LTE).

Q. How valid are these assumptions?

Q. Are we solving the right set of equations?

A. Valid only if we consider phenomena with

(1) length scale $\gg \lambda_D$ (Debye length) $\Rightarrow$ charge neutrality

(2) time scale $\ll \nu_c^{-1}$ (Typical particle collision timescale) $\Rightarrow$ LTE.

Q. What are λ_D and ν_c?

A. Let's take a look ...

※ Still ignoring

- Heating & Cooling $\Rightarrow$ Important in cluster cores
- $\vec{B}$-field, Cosmic rays $\Rightarrow$ Dynamically unimportant, but may affect the transport process.

REFERENCE :

The Physics of Fluids & Plasma : An Introduction for Astrophysicists
by Arnab Rai Choudhuri

4.1 : Debye Shielding (Debye & Hückel 1923)

Important concept for understanding how good is the assumption of charge neutrality is.

Let us consider the charge separation by introducing a charge Q inside the plasma.

If n_e, n_i = # densities of electrons & ions (assumed singly ionized), then the charge density at a point in space is given by $(n_i - n_e)e$.

The electrostatic potential (ϕ) is given by the Poisson Eq.

$$\nabla^2 \phi = -4\pi (n_i - n_e) e$$

If the plasma is in thermodynamic equilibrium & n = # density of ions or electrons far away from the charge Q, then we expect

$$n_i = n \exp\left(-\frac{e\phi}{k_B T}\right) \qquad n_e = n \exp\left(\frac{e\phi}{k_B T}\right)$$

Substituting in the Poisson Eq

$$\nabla^2 \phi = 4\pi n e \left[\exp\left(\frac{e\phi}{k_B T}\right) - \exp\left(-\frac{e\phi}{k_B T}\right) \right]$$

- Expanding in Tylor series & neglecting terms quadratic or higher order in $\left(\frac{e\phi}{k_B T}\right)$

$$\nabla^2 \phi = \frac{\phi}{\lambda_D} \qquad \boxed{\lambda_D \equiv \left(\frac{k_B T}{8\pi n e^2}\right)^{1/2}} \cdots\cdots \text{Debye length}$$

- The potential around the charge Q satisfies

$$\phi = Q \frac{\exp(-r/\lambda_D)}{r}$$

Conclusions

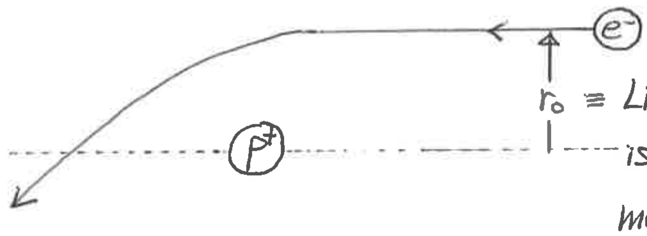
- It thus appears that the effect of the charge is screened beyond a distance λ_D.
- A plasma can therefore be considered charge-neutral when distances larger than the Debye length are considered.

For the ICM,

$$\lambda = 15\,\text{km} \left(\frac{T}{10^8\,{}^\circ\text{K}}\right)^{1/2} \left(\frac{n}{10^{-3}\,\text{cm}^{-3}}\right)^{-1/2} \qquad \text{Very small!}$$

4.2: Collisions in a fully ionized plasma

$u \equiv$ Typical relative velocity b/w two particles



$r_0 \equiv$ Limiting impact parameter for which the deflection is sufficiently large to make the change in momentum of a particle comparable to the original momentum of the particle.

$\frac{r_0}{u} =$ (Time during which the particles are close enough to make the interaction strongest)

Since the strongest interaction is about $\frac{e^2}{r_0^2}$, the impulse is of the order ("Impulse Approximation"):

$$\frac{e^2}{r_0^2} \cdot \frac{r_0}{u} \approx \frac{e^2}{r_0 u} \approx \Delta p$$ .... Change in momentum produced by the impulse.

We want it (Δp) to be of order p or $m_e u$ for the limiting impact parameter r_0:

$$\frac{e^2}{r_0 u} \approx m_e u \quad \Rightarrow \quad \boxed{r_0 \approx \frac{e^2}{m_e u^2}}$$

Only when the impact parameter is less than r_0, the deflection is large enough for the event to be counted as a "collision".

Can therefore take πr_0^2 as the collision cross-section.

A particle moving with a velocity u undergoes collisions in unit time with these particles which lie within a cylinder of volume $\pi r_0^2 u$.

If n is the particle # density, the collision frequency is

$$\nu_c \approx \pi r_0^2 n u \approx \frac{\pi n e^4}{m_e^2 u^3}$$

Writing $u \approx \left(\frac{k_B T}{m_e}\right)^{1/2}$ for the typical thermal velocities,

$$\nu_c \approx \frac{\pi n e^4}{m_e^{1/2} (k_B T)^{3/2}}$$

The typical collision timescale is given by ν_c^{-1}.

More rigorous derivation gives the equilibration time of two particles:

$$t_{eq} = \frac{3 m_1 m_2}{8 (2\pi)^{1/2} n_2 Z_1^2 Z_2^2 e^4 \ln\Lambda} \left(\frac{kT_1}{m_1} + \frac{kT_2}{m_2} \right)^{3/2} \quad \text{(Spitzer 1962)}$$

For the fully ionized ICM (including H^+, He^{++}, e^-)

$$t_{ei} \cong 6.3 \times 10^8 \text{yrs} \frac{(T_e / 10^7 {}^\circ K)^{3/2}}{(n_i / 10^{-5} cm^{-3})(\ln\Lambda / 40)}$$

Note: $t_{ei} \sim \left(\frac{m_i}{m_e}\right)^{1/2} t_{ii} \sim \left(\frac{m_i}{m_e}\right) t_{ee}$

$\boxed{\text{Conclusions}}$

Electrons & ions are in thermodynamic equilibrium in galaxy clusters, except in the low-density region in cluster outskirts.

Example : Non-equilibrium electrons in cluster outskirts

When an e-i plasma passes through the accretion shock around the cluster, most of the kinetic energy goes into heating the heavier ions, causing $T_i \gg T_e$,

After the shock, e & ion slowly "equilibrate" through Coulomb interactions, each converging to the mean gas temperature

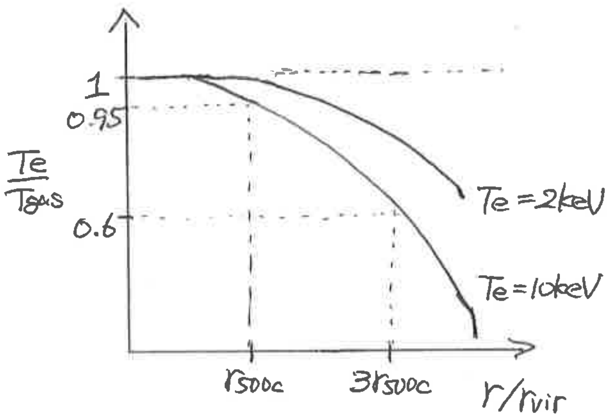
$$T_{gas} = \frac{(n_e T_e + n_i T_i)}{n_e + n_i}$$

over the e-i time

$$t_{ei} \cong 6.3 \times 10^8 \, yr \, \frac{(T_e / 10^7 \, {}^\circ K)^{3/2}}{(n_i / 10^{-5} cm^{-3})(\ln\Lambda / 40)}$$ for the fully ionized ICM (including H^+, He^{++}, e^-)

The time evolution of T_e is given by

$$\frac{dT_e}{dt} = \frac{T_i - T_e}{t_{ei}} - \underbrace{(\gamma - 1) T_e (\vec{\nabla} \cdot \vec{v})}_{\text{adiabatic compression \& heating}}$$



Strong T-dependence & dynamical state.

Rudd & Nagai 2009
Avestruz et al. 2015

Impact on

(1) $M_{HSE} = \frac{-r k T_e}{G \mu m_p} \left(\frac{d\log \rho}{d\log r} + \frac{d\log T_e}{d\log r} \right)$

(2) T & P-profiles + scaling relation in cluster outskirts

4.3 : Particle Diffusion for a multicomponent fluid

Q. What about He ?

A. Heavier ions sink under the influence of gravity (He sedimentation).

Each species s obeys

$$\begin{cases} \dfrac{\partial n_s}{\partial t} + \dfrac{1}{r^2}\dfrac{\partial (r^2 n_s u_s)}{\partial r} = 0 & \cdots (1) \quad \text{(Burger 1969)} \\ \dfrac{\partial P_s}{\partial r} + n_s A_s m_p g - n_s Z_s e \underbrace{\vec{E}}_{\text{Induced } \vec{E}\text{-field}} = \sum_t \underbrace{K_{st}}_{\text{Drag force due to collisions with surrounding particles}} (w_t - w_s) & \cdots (2) \end{cases}$$

Species s has	
mass	$A_s m_p$
charge	$Z_s e$
density	n_s
partial pressure	P_s
velocity	v_s

Ignored small terms

(1) $\dfrac{du_s}{dt} = 0$ ···· Inertial term

(2) The shear stresses (viscosity) due to collisions among the same species

(3) The coupling of thermal & particle diffusions

Resistance Coefficient

$$\boxed{K_{st}^{B=0} = \frac{4\sqrt{2\pi}}{3} \frac{e^4 Z_s^2 Z_t^2 \mu_{st}^{1/2}}{(k_B T)^{3/2}} n_s n_t \ln \Lambda_{st}}$$

for the unmagnetized, fully ionized plasma (Capman & Cowling 1952)

The center of mass of a fluid element moves with a velocity

$$u = \frac{\sum_s n_s A_s u_s}{\sum_s n_s A_s}$$

The differential (or diffusion) velocity between species s and the fluid element is $w_s = u_s - u$.

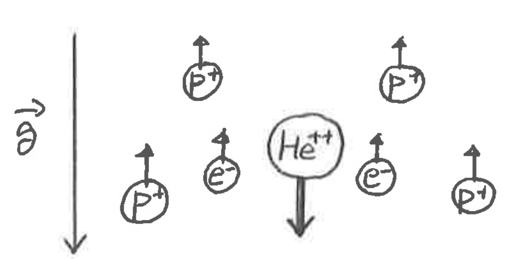
These diffusion velocities satisfy mass & charge conservation:

$$\sum_s A_s n_s w_s = 0 \quad \cdots (3)$$

$$\sum_s Z_s n_s w_s = 0 \quad \cdots (4)$$

Note: the summations include both ions & electrons.

To satisfy these conservation laws:



For each sinking He^{++} nuclei, there are $4p^+ + 2e^-$ that float up.

Note that the sedimentation destroys HSE since the redistribution of particles introduces a temporal change in the total gas pressure.

But, the HSE can be restored quickly. This equilibrium restoring acquires a net inflow with a mean velocity u;

$$\frac{du}{dt} = -\frac{1}{\rho_{gas}} \frac{dP_{gas}}{dr} - g \quad \cdots (5)$$

where $P_{gas} = \sum_s n_s k_B T$

$\rho_{gas} = \sum_s n_s A_s m_u$

Drift Velocity

Consider a drift velocity of a trace He particle in a background of H (i.e., $n_p \gg n_e$ & $w_p = 0$).

In this limiting case,

$$\left\{\begin{array}{ll} \dfrac{\partial P_p}{\partial r} + n_p A_p m_p g - n_p Z_p e E = 0 & \cdots (6) \\ \dfrac{\partial P_{He}}{\partial r} + n_{He} A_{He} m_p g - n_{He} Z_{He} e E = -K_{Hep} w_{He} & \cdots (7) \\ \dfrac{\partial P}{\partial r} = -\rho_{gas} g \Rightarrow \dfrac{\partial P_p}{\partial r} = -\mu m_p n_p g & \cdots (8) \end{array}\right.$$

← Essentially no drag force on the dominant H component

↑ Again, H dominant

From Eq 6 + 8, $(A_p - \mu) m_p g = Z_p e E$

$$\Rightarrow \quad eE = \left(\frac{A_p - \mu}{Z_p}\right) m_p g = 0.5 m_p g \quad \text{for H plasma}$$

Substituting in Eq 7,

$$\Rightarrow \quad w_{He} = (Z_{He} eE - A_{He} m_p g) n_{He} / K_{pHe}$$
$$= (0.5 Z_{He} - A_{He}) m_p g \, n_{He} / K_{pHe}$$
$$= -3 m_p g \, n_{He} / K_{pHe}$$

$$\boxed{w_{He} \approx -80\,\mathrm{km/s} \left(\frac{T}{10\,\mathrm{keV}}\right)^{3/2} \left(\frac{g}{10^{-7.5}\,\mathrm{cm/s^2}}\right) \left(\frac{n_p}{10^{-3}\,\mathrm{cm^{-3}}}\right)^{-1}}$$

Slow, but non-neglible

Conclusions

(1) Typical sedimentation timescale in clusters is generally longer than the Hubbe time.

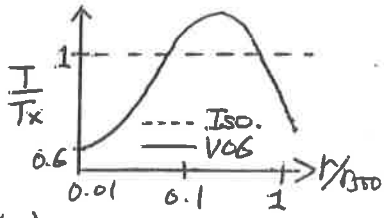
(2) The equilibrium distributions to the Burgers Eq. are therefore not applicable for clusters. Need a full time-dependent calculation!

Full time-dependent calculation

Assume the initial cluster model ; i.e., gas in HSE
with the NFW potential & two types of temperature profile

(1) isothermal

(2) observed T-profile (Vikhlinin et al. 2006)

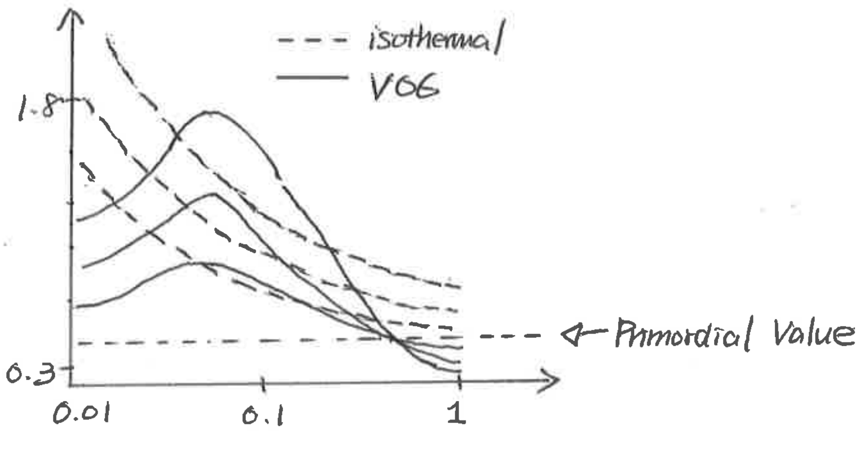


Solve the set of conservation equations (Eq 1-5).

At each time step,

(1) solve Eq 2-4 to obtain w_s (diffusion velocities)
for H, He, e^- as well as E (electric field)

(2) using these w_s & E, update the abundance of each species
by solving Eq 1+5 together.



Peng & Nagai 2009

Questions

- Turbulent mixing ?
- Situations similar to the Earth's atmosphere or stellar interior ?