Cosmological Radiative Transfer

Ali Rahmati (MPA)

Outline:

- Introduction: on the importance of radiative transfer on cosmological scales

- Two Important cases I: reionization

- Two Important cases II: Ultra Violet Background Radiation

- Numerical Radiative Transfer Methods
  - moment-based methods
  - ray-tracing
  - Monte-Carlo approach
  - Hybrid: TRAPHIC

- almost all the data we collect in order to study the universe is through detecting electro-magnetic radiation (for now - but high energy particles), therefore we need to know what is the relation between what we see (radiation we receive) and what is causing it (original radiation emitted from sources)

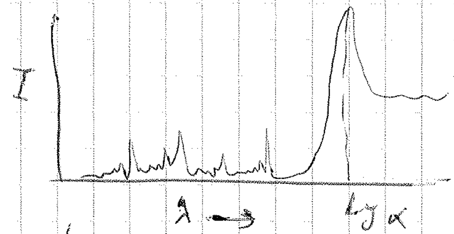
- radiation changes the physical state of baryons (energy exchange). To study the energy exchange between matter and light we need radiative transfer calculations

and many more!

Hydrogen Reionization

~ 0.4 Myr after the Big Bang, gas was cold enough that hydrogen atoms formed

several observational evidences indicate that the IGM (Inter-Galactic Medium) was highly ionized by $z \sim 6$



e.g.

* Gunn-Peterson trough
* Thomson scattering optical depth

$$\tau_{es} = \int n_e(z)\, \sigma_T \left(c\, \frac{dt}{dz} \right) dz$$

$\sigma_T \rightarrow 6.65 \times 10^{-25}\ cm^2$ Thomson scattering cross-section

- Note that τ_{es} is only a measure of the total probability of scattering and does not constrain the history of reionization independently!

$\Downarrow$

Something (first stars, galaxies, BH's, etc.) should have caused the universe to become / remain highly ionized at $z \lesssim 6$.

Outstanding questions about reionization

- the beginning of it
- the end of it
- how exactly it happend
- main sources of ionizing photons

  ⋮

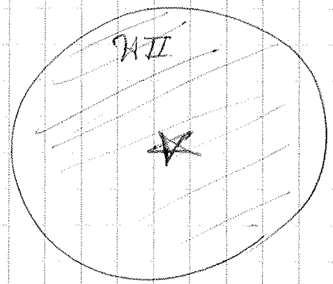
Observations are hard to obtain and even harder to interpret (e.g., 21 cm) and therefore highly model-dependent

$$\Downarrow$$

Theoretical understanding of plausible scenarios is essential.

Although there is not much to do analytically, it is useful to see a few simple examples to see the limited power of analytic arguments for this complex problem, and also to develope some intuition about the key players in radiative transfer

Cosmological Strömgren problem



Steady state solution $\equiv$ ionization balances recombinati.

$\Rightarrow$ # recombination = # ionization

$\Rightarrow$ totall recombination rate = ionization rate

source photon production rate $\dot{Q}$

$\alpha_B \times n_e \times n_p \times V \rightarrow$ proper volume

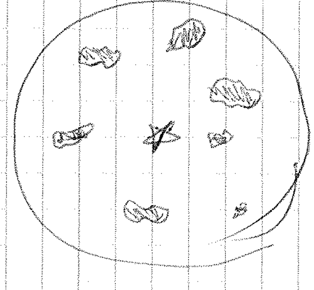
case B recombination: ionizing photons from recombination are absorbed on-the-spot

$\alpha_B(T)$

$\alpha_B = 2.6 \times 10^{-13}\ \mathrm{cm}^3\ \mathrm{s}^{-1}$ at $T = 10^4$ K

HII region $\Rightarrow$ $n_e \approx n_p \approx n_H$

$$\alpha_B \bar{n}_H^2 V = \dot{Q} = \frac{dN_\gamma}{dt}$$



$\Rightarrow \langle n_H^2 \rangle \neq \langle n_H \rangle^2 \rightarrow$ clumpy universe!

$\Rightarrow \dfrac{\langle n_H^2 \rangle}{\langle n_H \rangle^2} \equiv C \rightarrow$ clumping factor

$$\Rightarrow \quad \alpha_B \, \bar{n}_H^2 \, C \, V = \frac{dN_\gamma}{dt}$$

$$\Rightarrow \quad \bar{n}_H^\circ \frac{dV_{comoving}}{dt} = \frac{dN_\gamma}{dt} - \alpha_B \frac{C}{a^3} \bar{n}_H^{\circ 2} V_{comoving}$$

$$\Rightarrow \quad \frac{dV}{dt} = \frac{1}{\bar{n}_H^\circ} \frac{dN_\gamma}{dt} - \alpha_B \frac{C}{a^3} \bar{n}_H^\circ V$$

$$\Rightarrow \quad V(t) = \int_{t_i}^{t} \frac{1}{\bar{n}_H^\circ} \frac{dN_\gamma}{dt'} e^{F(t',t)} dt',$$

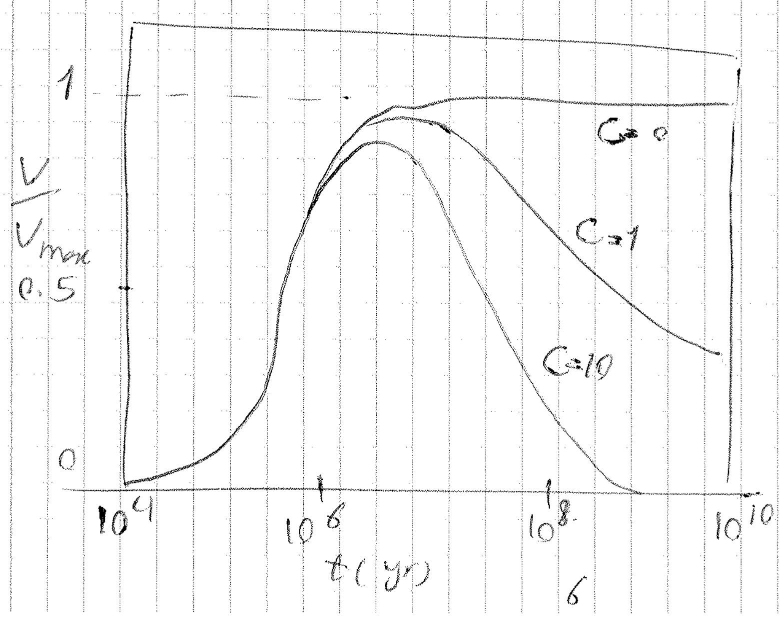
$$F(t',t) = -\alpha_B \bar{n}_H^\circ \int_{t'}^{t} \frac{C(t'')}{a^3(t'')} dt''$$

assuming the source turns on at $t = t_i$

$F(t',t)$ can be further simplified if we assume C is constant and we have $H(t) \simeq 2/3t$

$$F(t',t) = -\frac{2}{3} \frac{\alpha_B \bar{n}_H^\circ}{\sqrt{\Omega_m} H_0} C \left[f(t') - f(t) \right]$$

where $f(t) = a(t)^{-\frac{3}{2}} \xrightarrow[\Omega_m + \Omega_\Lambda = 1]{\Lambda CBM} \boxed{f(t) = \sqrt{\frac{1}{a^3} + \frac{1-\Omega_m}{\Omega_m}}}$

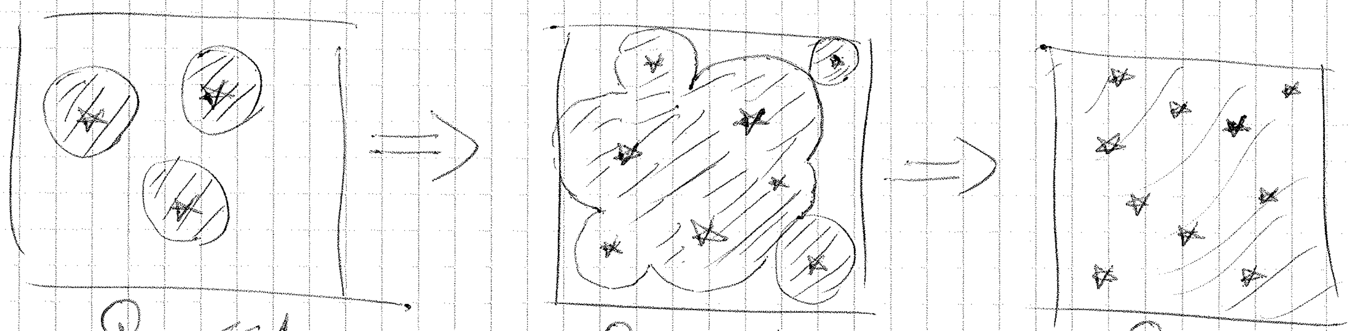


see:

Shapiro & Giroux 87

Barkana & Loeb 01

reionization starts from individual HII regions around first sources of ionization and as those HII regions grow and overlap, it approaches its final stages:



$Q_{HII} \ll 1$ $\qquad$ $Q_{HII} < 1$ $\qquad$ $Q_{HII} \sim 1$

Q_{HII} : the volume filling factor of ionized Hydroge

The simplest reionization formalizm:

$$\frac{dQ_{HII}}{dt} = \frac{\dot{n}_\gamma}{\bar{n}_H} - \frac{Q_{HII}}{\bar{t}_{rec}}$$

$$\bar{t}_{rec} = \frac{1}{\text{recombination rate per atom}} = \frac{1}{\alpha_B \, \bar{n}_H^0 \, C \, (1+z)^3}$$

$$\approx 1 \, \text{Gyr} \left(\frac{C_{HII}}{3}\right)^{-1} \left(\frac{T}{2\times10^4 K}\right)^{0.7} \left(\frac{1+z}{7}\right)^{-3}$$

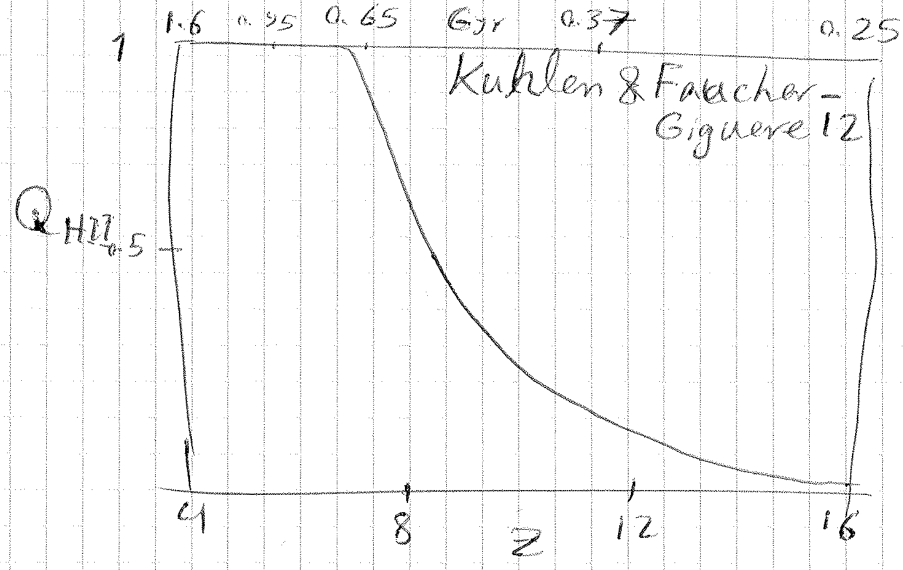
$\dot{n}_\gamma \rightarrow$ emissivity (photons per sec per cm^3)

one way to estimate this is from galaxy luminosity func

$$\dot{n}_\gamma = \int_{M_{lim}}^{\infty} dM_{UV}\, \phi(M_{UV})\, \gamma_{ion}(M_{UV})\, f_{esc}$$

UV magnitude   UVLF   ionizing luminosity

$f_{esc}(z) \longrightarrow$ escape fraction



$\dot{n}_\gamma \sim 10^{50}\ s^{-1}\, cMpc^{-3}$ in $z: [6-2]$

to be consistent with Thomson scattering optical depth

10 Myr time scale

also it can be defined as the ionizing efficiency, number of ionizing photons per atom:

$$\zeta = 30 \left(\frac{N_\gamma}{4{,}000}\right)\left(\frac{f_{esc}}{0.15}\right)\left(\frac{f_*}{0.05}\right)\left(\frac{f_b}{1}\right)$$

# of ionizing photons per stellar baryon (from SPS models)

fraction of galactic baryon in stars

fraction of baryons in galaxies

Simple arguments like what was shown so far are useful to estimate the impact of different underlying parameters e.g., f_{esc} on observable outcomes like the CMB Thomson optical depth. However, having too many uncertain assumptions and parameters is a serious limitation. Also they do not provide much information about the geometry of reionization etc. Therefore we have to resort to cosmological radiative transfer simulations.

Problems for the simple approach:

A) Recombination:

- Case A vs. Case B recombination

  accurate treatment requires RT calculation
  i.e., optical depth calculation
  recombination radiation (Raicevic 14)

- clumping factor:

  using simulations to calibrate $\Rightarrow$ $C \sim 30$ (Mellema 06)

  problem: photoheating $\Rightarrow$ $C \sim 3$ (Pawlik 09)

  the main issue here is that the structure of the absorbing gas is not independent from the ionizing radiation field!

B) neglecting additional absorption within HII regions:

Lyman limit systems (optically thick to the ionizing radiation)

$N_{HI} \gtrsim 10^{18}\ cm^{-2}$

the can chang substantially the morphology of reionization by imposing a limit to the size of isolated HII regions

(e.g. Furlanetto & Oh 05, Alvarez & Abel 12)

LLSs are relatively well studied at $z \lesssim 5$ but their properties are not fully constrande at higher redshifts

durring reionization the nature and aboundance of LLSs could chang significantly

To study them, high resolution, radiative transfer simulations is required.

5) ionization efficiency (neglecting other sources: X-ray)

$$\xi = N_\gamma \, f_{esc} \, f_* \, f_b$$

uncertain due metallicity dependence
stellar population synthesis

only empirical relations based on local observation

unknown! → dependent on halo mass
feedback
reionization

f_{esc} is hard to constrain both observationally and theoretically (simulations)

sensitive to almost all aspects of baryonic physics
detailed calculation requires extremely high resolution which is only forseeable for small galaxies at high redshifts

$$\dot{n}_\gamma = \int_{M_{lim}}^{\infty} dM_{UV} \, \Phi(M_{UV}) \, \gamma_{ion}(M_{UV}) \, f_{esc}$$

faint end problem + observational uncertainties

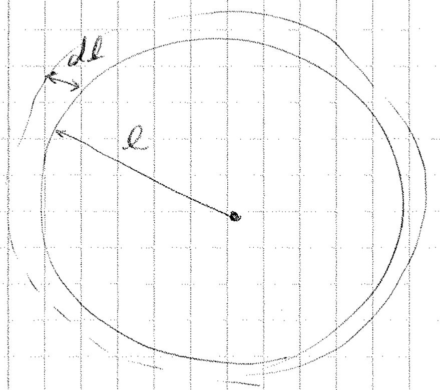
SED properties + dust correction etc.

Simulations can tackle some of the challenges but still hard to do

- should be coupled to RT (expensive)
- large scales should be resolved (100 Mpc)
- high resolution is requiered ($\sim 10^5 M_\odot$)

Uniform UltraViolet (ionizing) Background Radiation (UVB)

After the reionization of the Universe (Hydrogen) IGM becomes highly transparent to hydrogen ionizing photons, which allows ionizing photons to travel easily to large cosmological distances.
as a result, at any given point in the Universe large number of sources are visible and they can contribute to the intensity of ionizing radiation
A uniform distribution of sources then naturally results in a uniform radiation field.



u : energy per cm^3 per frequency

$u = \frac{4\pi}{c} J_\nu \rightarrow$ mean intensity

lets assume single frequency

$$dU = \frac{4\pi \cancel{l^2}\, u\, dl}{\cancel{l^2}} \quad \text{assuming no absorption}$$

$$\Rightarrow \quad U = 4\pi u l \Big|_0^{L_{max}}$$

Olbers' paradox : the sky is dark at night

$\Rightarrow$ L_{max} should be finite $\neq$ eternal static infinite Universe in the presence of absorption:

$$dU = \frac{4\pi \cancel{\ell^2} d\ell\, u}{\cancel{\ell^2}} e^{-\frac{\ell}{\lambda}}$$

λ : mean-free-path i.e. the distance at which the intensity of radiation drops to e^{-1} of its original value

$$\Rightarrow \quad U = 4\pi u \lambda\, e^{-\frac{\ell}{\lambda}} \Big|_{L_{max}}^{0}$$

$L_{max} \rightarrow \infty \quad \Rightarrow \quad \boxed{U = 4\pi u \lambda}$

$\Rightarrow$ if one knows the average emissivity of the Universe as a function of time and frequency, and also knows the mean-free-path as a function of time and frequency, then it is possible to calculate the average intensity of radiation as a function of time and frequency

This is the main idea behind UVB models such as Haadt & Madau

The main equation for calculating the background radiation intensity:

$$J_{\nu_0}(z_0) = \frac{c}{4\pi} \int_{z_0}^{\infty} \frac{dt}{dz}\, dz\, \frac{(1+z_0)^3}{(1+z)^3}\, e_\nu(z)\, e^{-\bar{\tau}}$$

$$\nu = \nu_0 (1+z) / (1+z_0)$$

$$dt/dz = \frac{1}{H\,(1+z)}$$

$\bar{\tau}$ is the effective absorption optical depth of the IGM

e_ν is the proper volume emissivity

Continuum absorption:

$$\bar{\tau}_c(\nu_0, z_0, z) = \int_{z_0}^{z} dz' \int_0^{\infty} dN_{HI}\, f(N_{HI}, z')\,(1 - e^{-\tau_c})$$

where τ_c is

$$\tau_c(\nu') = N_{HI}\,\sigma_{HI}(\nu') + N_{HeI}\,\sigma_{HeI}(\nu') + N_{HeII}\,\sigma_{HeII}(\nu')$$

with $\nu' = \nu_0 (1+z') / (1+z_0)$

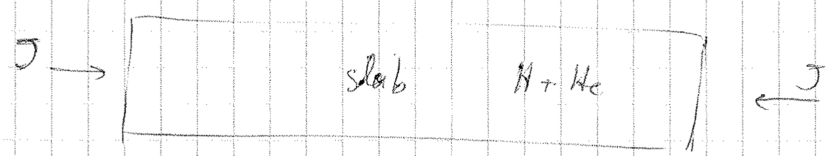
and $f(N_{HI}, z')$ is the distribution of absorbers in redshift and columndensity

$f(N_{HI}, z)$ can be constrained based on observations of HI in absorption by counting the incidence rate of absorbers along the lines of sight of background quasars

However, radiative transfer calculation is needed to calculate N_{HeI} and N_{HeII} for a given N_{HI}.

Note that this is an itterative procedure

$$\begin{array}{c} N_{HeI}(N_{HI}, z) \\ \text{and} \\ N_{HeII}(N_{HI}, z) \end{array} \rightleftarrows J(z)$$



+

resonant (line) absorption and recombination emission + Lyman series cascades

$J(z, \nu)$ but we still need to know the emissivity!

The main sources of radiation are galaxies and quasars.

– Star-forming galaxies:

Luminosity function of galaxies

$\Phi(L, z) \rightarrow$

+

SED from stellar population synthesis

+ dust reddening

$\Downarrow$

average star-formation history of the Universe + dust

together with SPS models gives the effective photon production rate (emissivity) and dust reddening at different epochs

dust extinction below 1 Ryd and f_{esc} above 1 Ryd

e.g., $\langle f_{esc} \rangle = 1.8 \times 10^{-4} (1+z)^{3.4}$

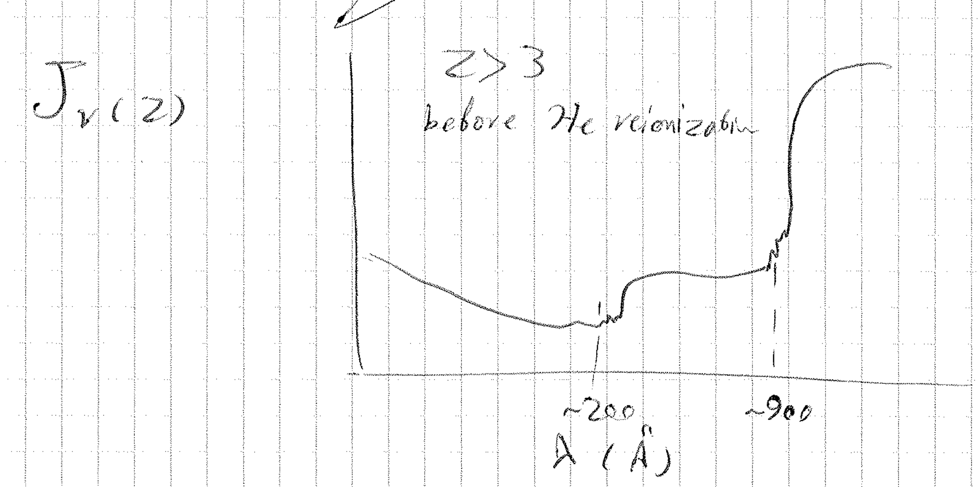
Haardt & Madau 12

- AGN emissivity:

QSO luminosity function

+

some empirical assumptions about their SED

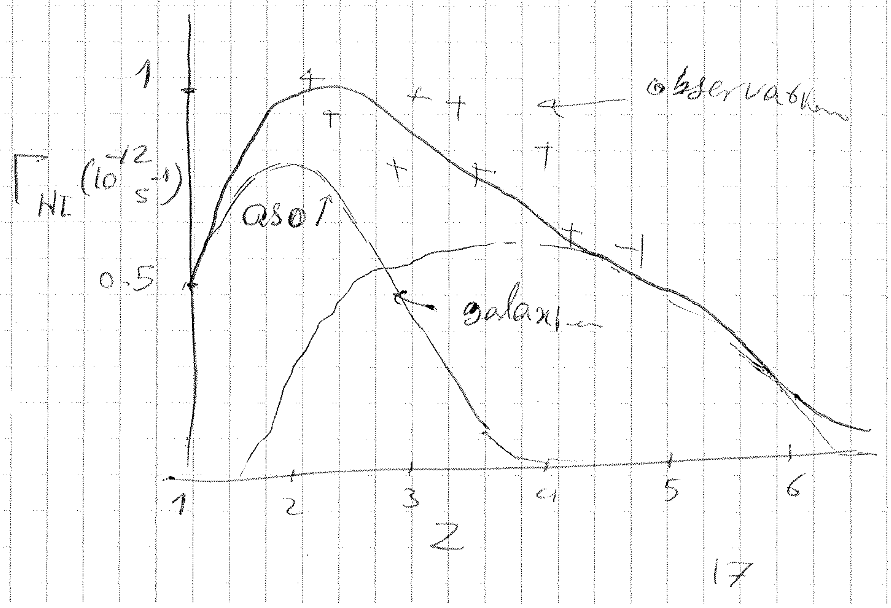


a useful quantity to measure (also for comparison with observation)

Photoionization Rate

$$\Gamma_{HI} = \int d\nu \, \frac{4\pi J_\nu}{h\nu} \, \sigma_{HI}(\nu)$$

Hydrogen absorption cross section



Observational constrants on the UVB are consistent with the simple models and imply

$$\Gamma_{HI} \sim 10^{-12} \; s^{-1}$$ at a wide range of redshifts

⇒ IGM which contains most of the mass in the universe is highly ionized

also, this ionizing background radiation heat up the gas or slows down its cooling. Due to the importance of those processes in the evolution of gas and galaxy formation, cosmological simulation usually adapt the UVB based on simple models. However there are two major problems:

- self-shielding at high densities is usually ignored
- spatial and temporal variations are neglected

fully investigating the impact of those issuse on ionization state of gas requires radiative transfer simulations.

in the following we investigate them a bit more.

Self-shielding starts when the column-density of gas is high enough to increase the optical depth significantly

e.g., $\tau = N_{HI} \, \bar{\sigma}_{HI} \gtrsim 1$

$\Rightarrow$ the column density at which self-shielding becomes important is

$$N_{HI,\,S\text{-}Sh} \sim \frac{1}{\bar{\sigma}_{HI}}$$

effective $\bar{\sigma}_{HI}$ depends on the shape of the ionizing background (radiation hardness)

for Lyman limit radiation ($\lambda = 912\,\mathring{A}$)

$$\sigma_{912} = 6.3 \times 10^{-18}\ cm^2$$

$$\Rightarrow N_{H,\,SSh} \sim 10^{17}\ cm^{-2} \quad (LLSs)$$

it is also possible to estimate size and density of systems for which self-shielding become important:

assuming close to hydrostatic equilibrium

$\Rightarrow$ the scale length is simillar to the Jeans scale

$$\Rightarrow L_J = \underbrace{c_s}_{\text{sound speed}} \times \underbrace{\frac{1}{\sqrt{\rho G}}}_{\text{gravitational time scale for colapse}} = 0.5\,kpc\ n_H^{-\frac{1}{2}}\, T_4^{\frac{1}{2}} \left(\frac{f_g}{0.16}\right)^{\frac{1}{2}}$$

Schaye 2001

moreover if we assume a highly ionized gas in equilibrium with UVB ionization:

$$\frac{n_{HI}}{n_H} \approx 0.5 \; n_H \; T_4^{-0.76} \; \Gamma_{12}^{-1} \qquad \text{(Schaye 2001)}$$

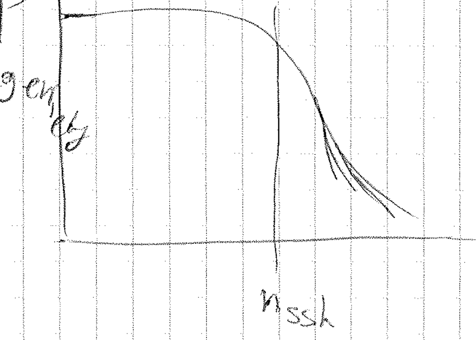
$\Rightarrow$ N_{HI} for systems with Jeans length and in ionization equilibrium would be:

$$N_{HI} = L_J \times n_{HI} \Rightarrow n_{HI} = \frac{N_{HI}}{L_J}$$

using those equations we can calculate the density which corresponds to the self shielding column density:

$$\boxed{n_{H,ssh} \sim 3 \times 10^{-3} \, cm^{-3} \left(\frac{\sigma_{HI}}{6.3 \times 10^{-18} cm^2} \right)^{-\frac{2}{3}} \times \Gamma_{12}^{\frac{2}{3}}}$$

Although this result is in good agreement with RT cosmological simulations, in practice the transition between optically thin and highly neutral gas happens more smoothly:



- RT effects (shadowing, inhomogeneity of absorbers ...
- recombination radiation

Inhomogeniety of the UVB

Depending on the strength of the ionizing source, there is a region around it where the intensity of the local radiation is more important than the UVB

- for QSOs this is called the proximity effect and could be ~~at~~ the orther of ~1-10 Mpc

  (this is actually a signature that could be used to put constraints of the amplitude of the UVB at different redshifts)

- for star-forming galaxies this "proximity effect" could be affecting scales up to ~10-100 kpc

Also, due to time-dependent quasar and star-burst activities in galaxies, the radiation field within proximity zones is varying with time.

That could change the cooling/heating of gas and interpretation of observed metal line absorption

(see e.g., Oppenheimer & Schaye 2013)

Note that the impact of local radiation in star-forming galaxies works against the self-shielding and a simple argument may suggest that those two effects cancel out each other (at least at low redshifts)

∴ Starting from Kennicutt-Schmidt star-formation law we have (Rahmati et al. 2013b)

$$\dot{\Sigma}_{*} \simeq 1.5 \times 10^{-4}\, M_{\odot}\, yr^{-1}\, kpc^{-2} \left(\frac{\Sigma_{gas}}{1\, M_{\odot}\, pc^{-2}} \right)^{1.4}$$

and we know from stellar population synthesis models that for a constant star-forming region the rate of hydrogen ionizing photons production rate is

$$\dot{Q}_{\gamma} \sim 2 \times 10^{53}\, s^{-1} \left(\frac{SFR}{1\, M_{\odot}\, y^{-1}} \right)$$

(within a time-scale of 10-20 Myr)

$$\Gamma_{*} = \frac{\dot{\Sigma}_{\gamma}}{N_H} = \frac{\dot{Q}_{\gamma} \times \dot{\Sigma}_{*}}{N_H} \rightarrow \boxed{\text{assuming negligible escape fraction}}$$

and $$N_H \simeq 9.4 \times 10^{19}\, cm^{-2} \left(\frac{\Sigma_{gas}}{1\, M_{\odot}\, pc^{-2}} \right) \left(\frac{X}{0.75} \right)$$

$$\Rightarrow \quad \Gamma_{*} \sim 8.5 \times 10^{-14}\, s^{-1} \left(\frac{N_H}{10^{21}\, cm^{-2}} \right)^{0.4}$$

(Rahmati et al. 13b)

Γ_{UVB} for $z \lesssim 1$ is $\sim 10^{-13}\, s^{-1}$ which is similar to Γ_{*} calculated above and does not scale strongly with N_H

using the Jeans-scale argument again we can even calculate the average stellar photoionization rate as a function of gas density.

$$N_H \sim 2.8 \times 10^{21}\ \mathrm{cm}^{-2} \left(\frac{n_H}{1\ \mathrm{cm}^{-3}}\right)^{\frac{1}{2}} T_4^{\frac{1}{2}} \left(\frac{f_g}{f_{th}}\right)^{\frac{1}{2}}$$

where $P_{th} = f_{th} P_{tot}$ → the fraction of thermal pressure in the gas

$$\Rightarrow \Gamma_* \sim \boxed{1.3 \times 10^{-13}\ \mathrm{s}^{-1}} \left(\frac{n}{1\ \mathrm{cm}^{-3}}\right)^{0.2} \left(\frac{f_g}{f_{th}}\right)^{0.2} T_4^{0.2}$$

This may suggest that models that assume the UVB is constant independent of density (i.e., ignoring the self-shielding effect) are not completely wrong!

But note that this argument works only to get the average photoionization rate on Kpc scales and ignors huge variations due to inhomogeneous distribution of stars inside galaxies

---

Bottom line: simple UVB models only give a good answer on larg scales and far from any strong ionizing source but most actions are happening inside and close to sources

⟶ Actuall RT calculation is needed

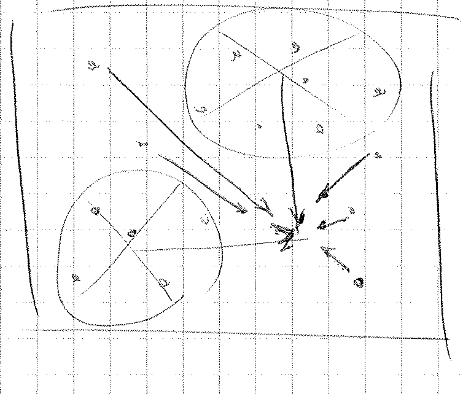
Numerical Radiative Transfer

Although many physical processes that regulate the formation and evolution of galaxies and the IGM are very sensitive to radiation, RT is usually omited from calculations/simulations

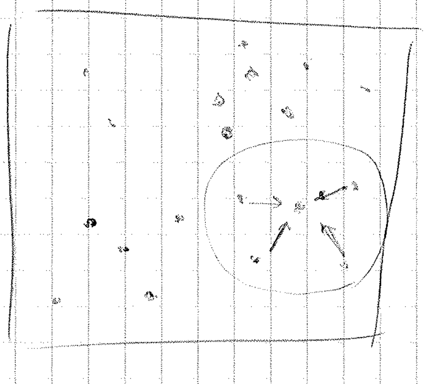
The main reason for this is high dimensionality of the RT governing equation which makes it hard to simplify and also very expensive to calculate numerically

Why RT is expensive?:

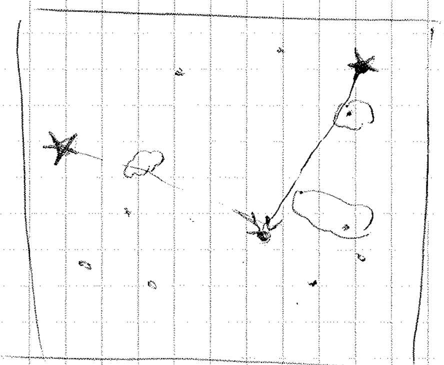
to answer this question, we can compare RT calculation with gravity and hydro calculation



Gravity: only depends on distances and masses
One can reduce the number of calculation by assuming far away objects can be represented by single objects



Hydrodynamics: Only properties of nearby resolution elements are important (i.e. short characteristic forces)



for radiation field calculation one needs to know the distribution of sources.
In the absence of absorption radiation field calculations would have been like gravity. However, due to absorption, one not only needs to know where the sources are, but also what is happening to the light coming from those sources! (i.e., grouping far away sources does not work)

On top of that, radiation is multifrequency and therefore very expensive (absorption is frequency dependent)

On top of that, we have scattering, re-radiation, redshifting (both cosmological and due to velocity differences) etc.

Cosmological Radiative Transfer Equation in the expanding universe (Gnedin & Ostriker 1997)

Define $F(t, x^i, p^k)$ to be the photon distribution function in comoving coordinates, x^i and comoving momentum

$$p^k = a \frac{h\nu}{c} n^k$$

where $a(t)$ is the scale factor, h is the Planck constant and n^k is the unit vector in the direction of photon propagation

$$\Rightarrow \quad N_\gamma = \int F(t, x^i, p^k)\, d^3x\, d^3p$$

is the total number of photons in the universe. In the absence of absorption and emission N_γ is constant. Using the phase space continuity equation we have:

$$\frac{\partial F}{\partial t} + \frac{\partial}{\partial x^i}(\dot{x}^i F) + \frac{\partial}{\partial p^k}(\dot{p}^k F) = \text{Source} - \text{Sink}$$

lets use specific intensity instead of photon distribution function. $J_\nu\, d\nu\, d\Omega\, dA\, dt$ is the energy of photons with frequencies between ν and $\nu + d\nu$ passing through the area dA in the time interval dt in the solid angle $d\Omega$ around n^k. We also know $a^3 d^3x = c\,dt\,dA$ and $d^3p = p^2 dp\, d\Omega$ which gives:

$$J_\nu = h\nu F \frac{d^3x\, d^3p}{d\nu\, d\Omega\, dA\, dt} = \frac{h^4\nu^3}{c^2} F$$

after doing the algebra we get

$$\frac{d J_\nu}{dt} + \frac{\partial}{\partial x^i}(\dot{x}^i J_\nu) - H\left(\nu \frac{\partial J_\nu}{\partial \nu} - 3 J_\nu\right) = \underbrace{-k_\nu J_\nu}_{\text{sink term}} + \underbrace{S_\nu}_{\text{source term}}$$

Even if we constrain ourself in a singl frequency, solving this equation with 6 independant variable is computationally too expensive!

3 spatial
2 angular + ν } 7 parameters
+ time

$\Downarrow$

simplifying assumptions are needed to make the numerical solutions tractable

Some of the main methods are

- moment-based method
- ray-tracing methods
- ~~[illegible]~~ Monte-carlo techingues
- hybrid methods: TRAPHIC

Replacing multi-frequency nature of radiation with Gray approximation

Motivation: Numerical calculation of large number of frequencies in RT is very expensive and in some cases not necessary!

Main idea: replacing a range of frequencies with a single intermediate frequency:

$$\boxed{\ \nu\ }_{\;\nu_1 \longleftrightarrow \nu_2} \longrightarrow \boxed{\ \nu\ }_{\;\bar{\nu}}$$

recal the photoionization rate:

$$\Gamma = \int_{\nu_1}^{\nu_2} 4\pi \frac{J_\nu}{h\nu} \sigma \, d\nu$$

$$= \frac{4\pi\bar{\sigma}}{h} \int_{\nu_1}^{\nu_2} \frac{J_\nu}{\nu} \, d\nu$$

frequency independent average cross-section

$$\Rightarrow \boxed{\bar{\sigma} = \frac{\int_{\nu_1}^{\nu_2} \frac{J_\nu \sigma}{\nu} \, d\nu}{\int_{\nu_1}^{\nu_2} \frac{J_\nu}{\nu} \, d\nu}}$$

Problem: different frequencies have different mean-free-paths which is not captured by a singe frequency approximation. This gives too sharp I-fronts and wrong heating in the self-shielded region

Moment-based Methods

e.g., OTVET (Gnedin & Abel 01)

Aubert & Teyssier 08, Petkova & Springel 09

Finlator et al. 09, Krumholz et al. 07, Rosdahl et al. 13

etc...

Main idea: reducing the dimension of general RT equation by eliminating the two angular dimensions of the RT equation and treating the radiation field as a fluid.

Advantages: – it is fast

– differential equations that should be solved are similar to other hydro equations and therefore similar numerical techniques can be used.

disadvantages: – the directionality of radiation is largely lost and radiation propagates diffusively! This is not a big issue when the optical depth is high but becomes a problem in optically thin regimes (e.g., it does not capture shadows accurately)

– Courant stability condition: radiation should not propagate through more than one volume element in each time step $\Longrightarrow$ one needs to use reduced speed of light (fine for optically thick regimes but wrong for optically thin regimes)

One example: OTVET (Gnedin & Abel 01)
Optically Thin Variable Eddington Tensor:

the specific intensity, J_ν, in the expanding universe is given by:

$$\frac{\partial J_\nu}{\partial t} + \frac{\partial}{\partial x^i}(\dot{x}^i J_\nu) - H\left(\nu \frac{\partial J_\nu}{\partial \nu} - 3 J_\nu\right) = -k_\nu J_\nu + S_\nu$$

$$\dot{x}^i = c\frac{n^i}{a}$$

extinction   source

n^i is the unit vector showing photo propagation directi

we define the mean specific intensity:

$$\bar{J}_\nu(t) \equiv \langle J_\nu(t, \vec{x}, \vec{n})\rangle$$ where $\langle\,\rangle$ is averaging over position and direction

$$\langle J_\nu(\vec{x}, \vec{n})\rangle = \lim_{V\to\infty} \frac{1}{4\pi V}\int_V d^3x \int d\Omega\, J_\nu(\vec{x}, \vec{n})$$

$$\Rightarrow \frac{\partial \bar{J}_\nu}{\partial t} - H\left(\nu \frac{\partial \bar{J}_\nu}{\partial \nu} - 3\bar{J}_\nu\right) = -\bar{k}_\nu \bar{J}_\nu + \bar{S}_\nu$$

where $\bar{S}_\nu \equiv \langle S_\nu\rangle$ and $\bar{k}_\nu \equiv \dfrac{\langle k_\nu J_\nu\rangle}{\bar{J}_\nu}$

then we can define relative specific intensity, $f_\nu(t, \vec{x}, \vec{n})$, which is the difference between the local value of the specific intensity and the mean:

$$J_\nu \equiv f_\nu \bar{J}_\nu \Rightarrow \langle f_\nu\rangle = 1$$

then one can assume that the universe does not expand substantially over the time photons need to travel the scale of interest, and $v/c \ll 1$

$$\left(\text{i.e., } \frac{\partial}{\partial \nu} = 0 \text{ and } \frac{v}{c} = 0\right)$$

and write: (see: Gnedin & Abel 01)

$$\frac{a}{c}\frac{\partial f_\nu}{\partial t} + n^i \frac{\partial f_\nu}{\partial x^i} = -\hat{k}_\nu f_\nu + \psi_\nu$$

where $k_\nu \equiv \frac{a}{c} K_\nu$, $\hat{k}_\nu \equiv k_\nu - \bar{k}_\nu + \frac{\bar{S}_\nu}{\bar{J}_\nu}\frac{a}{c}$

and $\psi_\nu \equiv \frac{a}{c}\frac{S_\nu}{\bar{J}_\nu}$

Then we define the moments of the transfer equation:

radiation energy density: $E_\nu(t, \vec{x}) \equiv \frac{1}{4\pi}\int d\Omega\, f_\nu(t, \vec{x}, \vec{n})$

flux F_ν^i : $F_\nu^i(t, \vec{x}) \equiv \frac{1}{4\pi}\int d\Omega\, n^i f_\nu(t, \vec{x}, \vec{n})$

and the Eddington tensor h_ν^{ij}

$$E_\nu(t, \vec{x})\, h_\nu^{ij}(t, \vec{x}) \equiv \frac{1}{4\pi}\int d\Omega\, n^i n^j f_\nu(t, \vec{x}, \vec{n})$$

then one can show the following equation hold:

$$\frac{a}{c}\frac{\partial E_\nu}{\partial t} + \frac{\partial F_\nu^i}{\partial x^i} = -\hat{k}_\nu E_\nu + \psi_\nu \rightarrow \text{Conservation of the number of photons}$$

and

$$\frac{a}{c}\frac{\partial F_\nu^j}{\partial t} + \frac{\partial}{\partial x^i} E_\nu h_\nu^{ij} = -\hat{k}_\nu F_\nu^j \rightarrow \text{flux conservation}$$

One need to close those two PDEs with specifying the Eddington tensor, h_ν^{ij}

The choice of the Eddington tensor

Eddingto tensor gives the distribution of sources as seen by observers at different locations

$$h_\nu^{ij} = \frac{P^{ij}}{\mathrm{Tr}\, P^{ij}}$$

where $$P_\nu^{ij} = \int d^3x_1\, \rho_*(\vec{x}_1)\, e^{-\tau_\nu(\vec{x},\vec{x}_1)} \frac{(x^i - x_1^i)(x^j - x_1^j)}{(\vec{x} - \vec{x}_1)^4}$$

where $\tau_\nu(\vec{x}, \vec{x}_1)$ is the optical depth between points $\vec{x}$ and $\vec{x}_1$. ρ_* is ~~the~~ mass density of sources

assuming negligible absorption on large scales simplifies this equation significantly (i.e., optically thin limit)

$$\Rightarrow \quad P_\nu^{ij} = \int d\vec{x}_1^3\, \rho_*(\vec{x}_1) \frac{(x^i - x_1^i)(x^j - x_1^j)}{(\vec{x} - \vec{x}_1)^4}$$

for alternative choices see Aubert & Teyssier 10

Jiang, Store & Davis 12

also see Flux-limited Diffusion (optically thick = diffusion along the direction of decreasing energy) etc

e.g., Krumholz et al 07, Reynolds et al. 09

Ray tracing RT (usually on grid)

e.g., Abel & Wandelt 02, Razoumov & Scott 99
Mellema et al. 06, Wise & Abel 11

Main idea: Connecting all resolution elements and every source in the simulation with rays and calculating how much radiation can be absorbed / transmitted

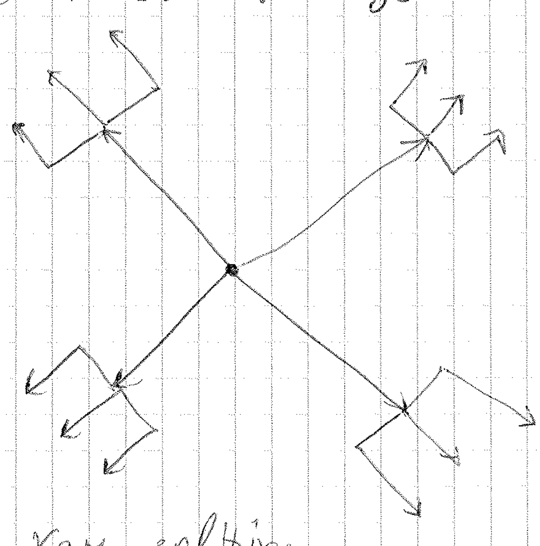
Advantages: The most natural way of doing RT which is very similar to how photons propagate in reality

In the limit of infinite number of rays, it provides exact solution!

Disadvantages: It is very expensive where multiple sources exist $\sim N_{sources} \times N_{cells}$

however there are different tricks to reduce the computational costs: $\Longrightarrow$ using a grid, finite number of rays

problem: finite number of rays $\Longrightarrow$ low angular resolution and missing cells far from surces



solution: ray spliting

Monte Carlo RT (usually on grid)

e.g., Ciardi et al. 01, Maselli et al. 03
Cantalupo & Porciani 11, Jonsson 06

Main idea: Similar to ray tracing but by propagating photon packets that are sampling appropriate probability distribution functions (angular distribution, SED, etc.)

Advantages: almost the same as ray tracing
+ it can tackle scattering easily

disadvantage: ~~ + could be a bit noisy

TRAnsport of PHotons In Cones (SPH) (and grid)

TRAPHIC: Pawlik & Schaye 08, 11

Main idea: a fusion of ray-tracing and Monte-Carlo RT for SPH which has a computational time independent from number of sources.

Advantages: - it uses the full resolution available through SPH simulation (hence adaptive)

- it preserves the directionality of radiation field (unlike moment-based methods)

- it does not scale with the number of sources (unlike ray-tracing and Monte-Carlo)

- Photon transport with the speed of light (unlike moment based methods)

disadvantages:

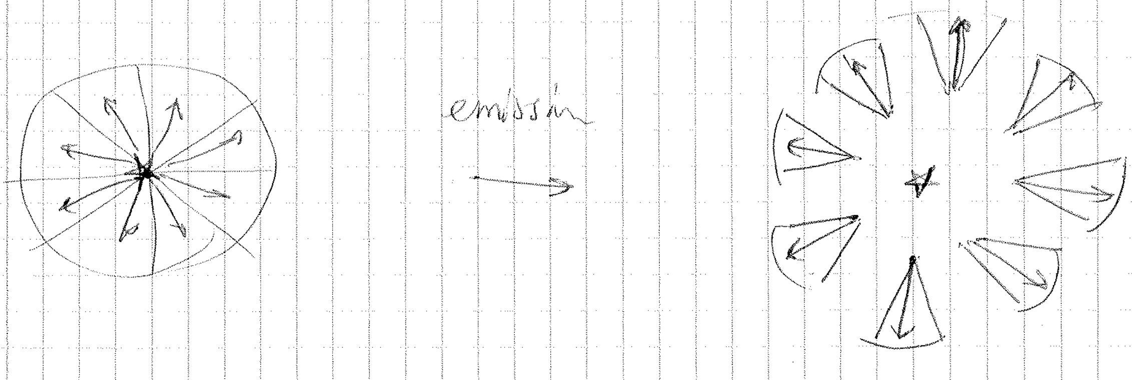
- could be a bit noisy (sampling noise)

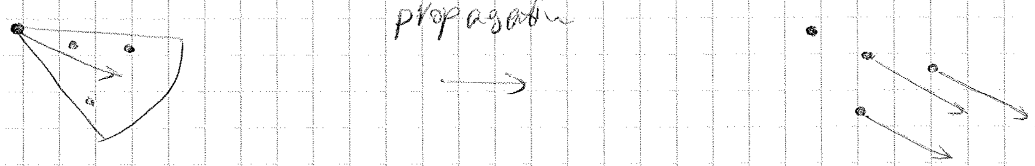
How it works:

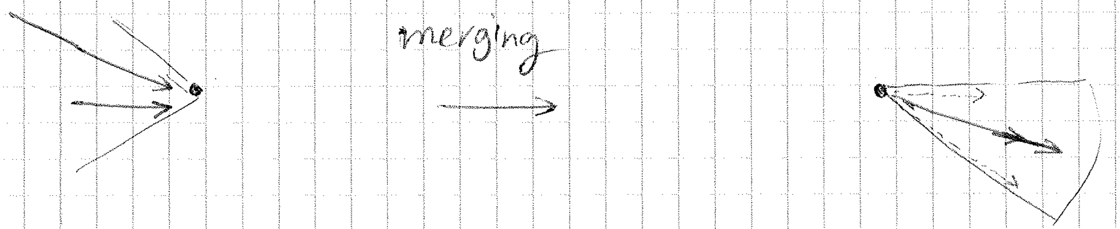
- emission from source in cones

- transmission to neighbors within cones

- photon packets with close enough propagation direction are merged into one photon packet with an intermediate direction







and photons are absorbe as they travel

note: descretizig the emission direction into finite number of cones introduces sampling issues

⇒ emission directions are randomly rotated in each time step (or less frequently)