

29.4. Entwicklung zu den Weissen Zwergen

(Hier nur ganz kurzen Abriss)

a) Beobachtungen:

∃ WZ mit H-reicher Hülle (Spektraltyp DA, ca. 75% aller WZ), wobei

$$\underbrace{10^{-16} M_{\odot}}_{\tau \approx 1} \lesssim M_H \lesssim \underbrace{10^{-7} M_{\odot} \dots 10^{-4} M_{\odot}}$$

aus Pulsationstheorie für
ZZ Ceti-Sterne abgeleitet

∃ WZ mit He-reicher Hülle, kein H sichtbar (Spektraltyp DB, ca. 20% aller WZ)

$$\rightarrow M_H \ll 10^{-16} M_{\odot}$$

▶ Dichotomie der chem. Zusammensetzung der Atmosphären ist theoretisch nicht befriedigend verstanden

b) Theorie:

Entwicklungsmodelle, die vom AGB kommen haben bei Max (T_{eff}) noch eine H-reiche Hülle mit $M_H \approx 10^{-4} M_{\odot}$. (Für $M_H < 10^{-4} M_{\odot}$ "erlischt" die H-Schalenquelle).

- ▶ Bei der Abkühlung von Sternen mit $M_H \approx 10^{-4} M_{\odot}$ spielt die Energieerzeugung durch H-Brennen noch eine Rolle (→ z.B. Mazzitelli, I., D'Antona, F.: 1986, Astrophys. J. 308, 706)
- ▶ Zeitskala für Entwicklung zu sehr kleinem L ($\log L/L_{\odot} \lesssim 4.5 \dots 5$) ist stark von der chem. Zusammensetzung und der Masse der H-reichen Hülle abhängig (→ z.B. Mazzitelli & D'Antona, 1986)

ENTWICKLUNG VON STERNEN MITTLERER UND KLEINER MASSE MIT MASSENVERLUST

aus: Schönberner, D.: 1979, Astron. Astrophys., 79, 108

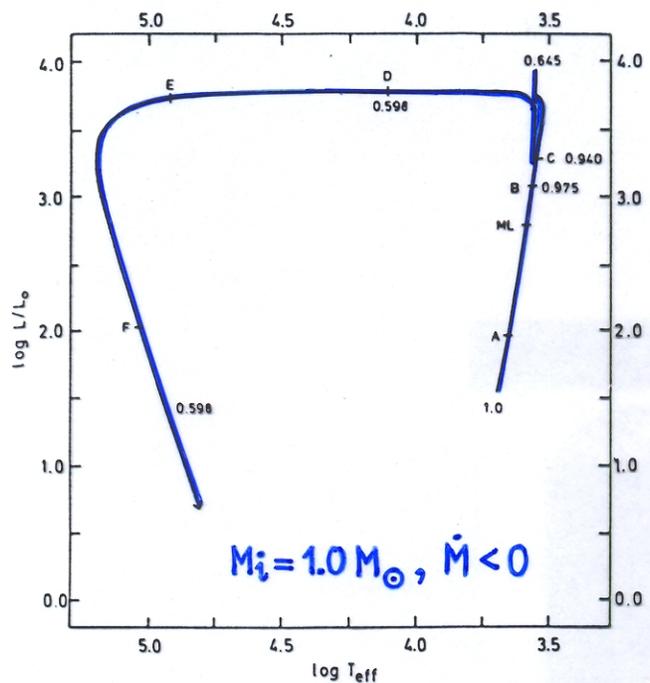


Fig. 1. Evolutionary track of a $1 M_{\odot}$ star (Sequence 1). The numbers give the actual masses in solar units. The meaning of the letters is explained in the text

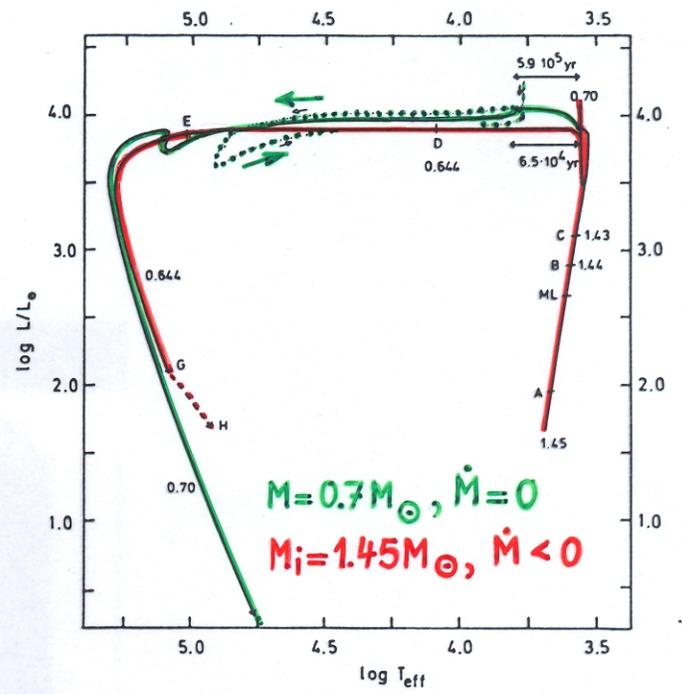


Fig. 2. Evolutionary tracks of stars with $1.45 M_{\odot}$ (Sequence 2) and $0.7 M_{\odot}$ (Sequence 2a). The numbers and symbols have the same meaning as in Fig. 1

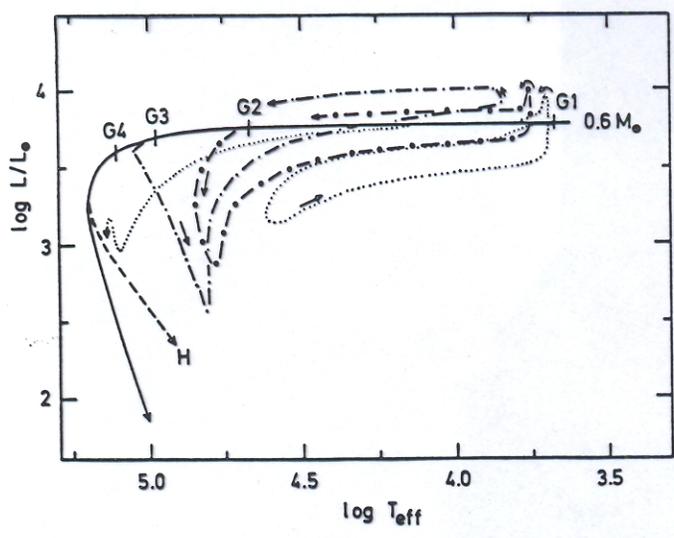


Fig. 7. The same track as in Fig. 1, but with additional loops caused by the occurrence of flash No. 11 at the starting points G1, ..., G4 (see text)

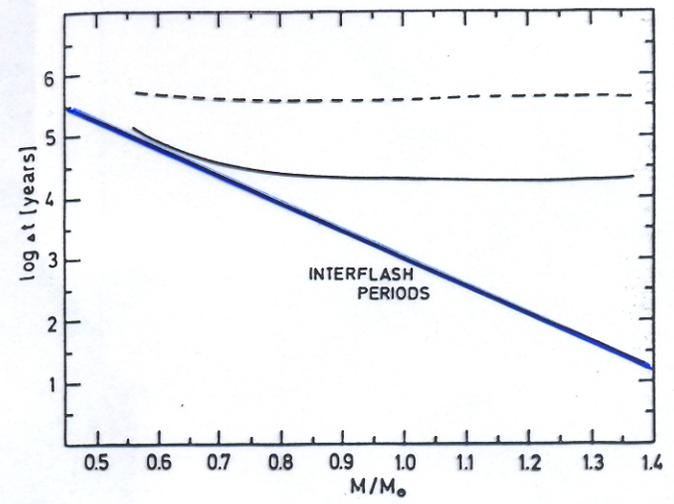
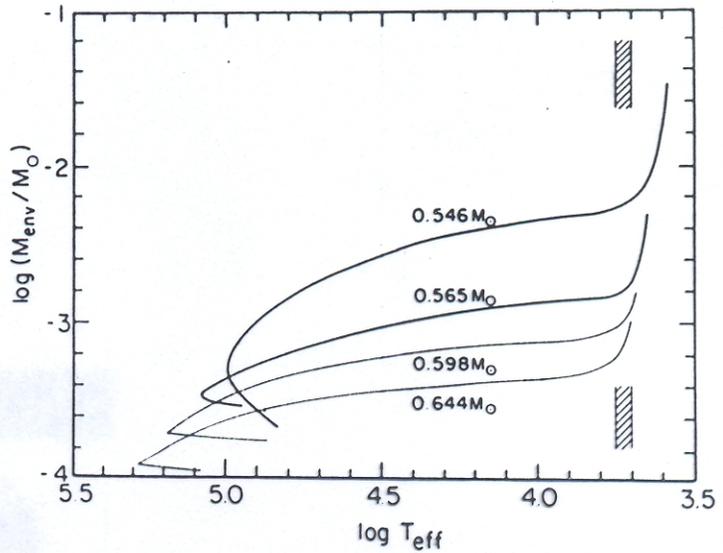
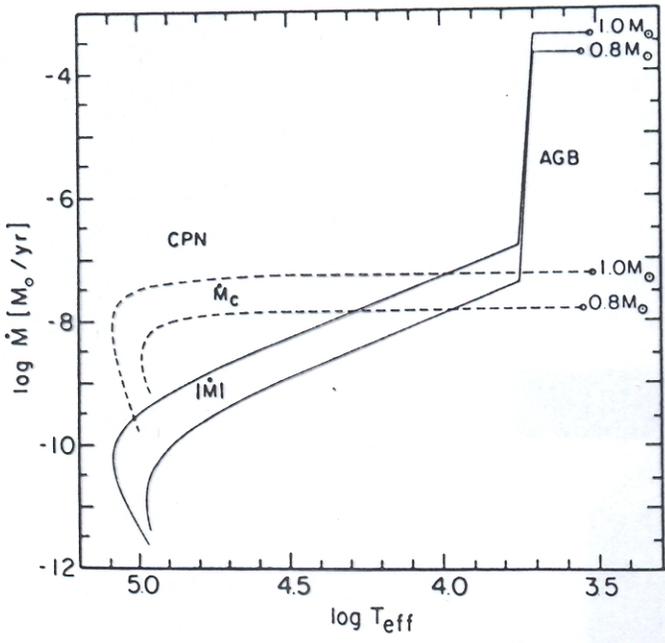


Fig. 8. Times Δt for the evolution across the H.R.-diagram from $\log T_{\text{eff}} = 3.5$ till $\log T_{\text{eff}} = 5.0$ with (—) and without (---) mass loss for different stellar masses after Paczyński (1970), and Paczyński's (1975) core mass-interflash periods. Note that these crossing times are only estimates (see text)

- A = Ende des zentr. He-Brennens
- B : $L_H > L_{He}$
- C : 1. Thermischer Puls
- ML : Beginn des Massenverlusts
- D : $\dot{M}_c/M_c = |\dot{M}/M|$

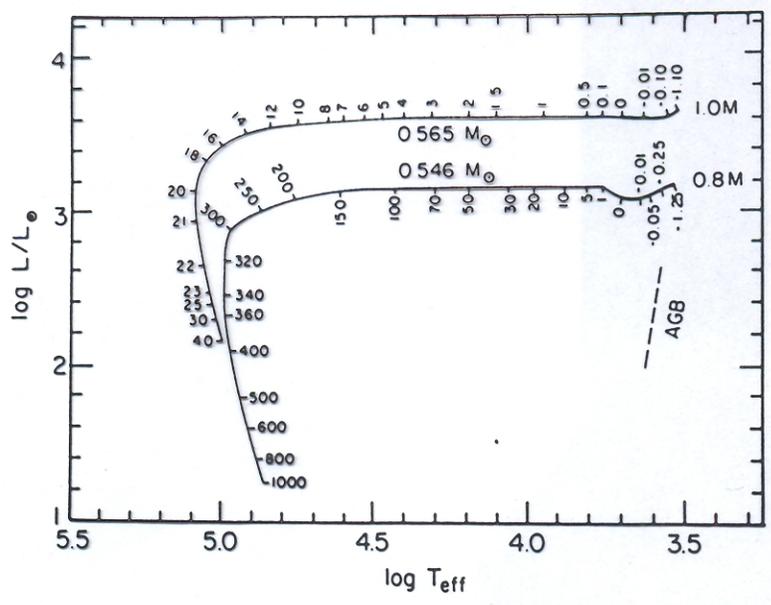
ENTWICKLUNG VON STERNEN MITTLERER UND KLEINER MASSE MIT MASSENVERLUST

aus: Schönberner, D.: 1983, Astrophys. J., 277, 708

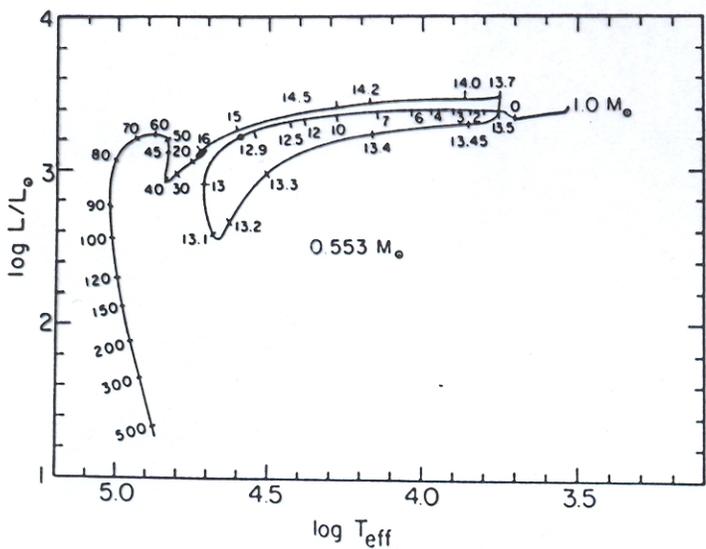


Variation der Hüllenmasse M_{env} mit T_{eff}

Angenommene Massenverlustrate \dot{M} ———
Zunahme der Kernmasse \dot{M}_c - - - - -



Entwicklung im HRD für eine chem. Zus. der Hülle von $X=0.739$, $Y=0.240$ und $Z=0.021$. Die Zahlen entlang der Entwicklungswege geben das Alter in tausend Jahren, gerechnet vom Punkt, wo $\log T_{\text{eff}}=3.7$ ist.



Wie oberes HRD, jedoch für einen Stern mit $M_c=0.553 M_{\odot}$, der auf dem Weg zu hohen Effektivtemperaturen noch einen letzten Helium-Schalenflash erleidet.

aus: Iben, I., Jr.: 1985, Quart. J. Roy. Astron. Soc., 26, 1

Iben, J., Jr., Renzini, A.: 1983, Ann. Rev. Astron. Astrophys., 21, 271

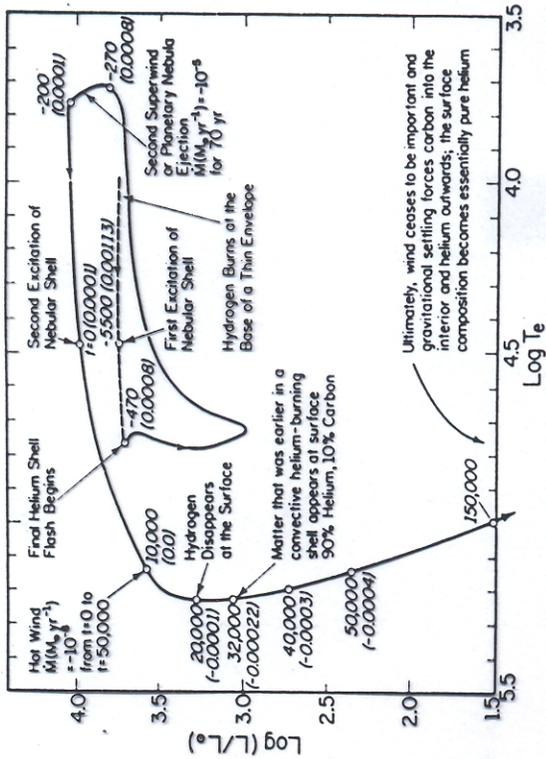


FIG. 17. Evolutionary track of a star that experiences a final helium shell flash while still burning hydrogen at high luminosity. Time, measured from the moment the shell flash begins, is given in years beside each solid circle. The location in mass below the surface of the centre of the hydrogen profile is given next to each time. After first ejecting a nebular shell of material and leaving the asymptotic giant branch, the remnant star is of mass about $0.6 M_{\odot}$. About 5000 yr after first causing the nebular shell to fluoresce, the star experiences a final helium shell flash which takes it back to its original location on the AGB. As a 'born-again' AGB star, it experiences the same instability that led to the ejection of a first nebular shell. The second ejection event is approximated by abstracting mass at a rate of $10^{-5} M_{\odot} \text{ yr}^{-1}$ until the model star, of its own accord, moves rapidly away from the AGB, having lost all but about $10^{-4} M_{\odot}$ of hydrogen-rich material. On reaching a surface temperature of 30 000 K the star again lights up the nebular shell ejected initially and continues to do so for perhaps 30 000 yr, or until the shell has expanded to such an extent that its surface brightness is too low for detectability. During this second nebular excitation phase, it is the energy liberated by helium-burning reactions that is the ultimate cause of excitation. It is known that the hot central stars of planetary nebulae lose mass via a fast stellar wind and it is clear that, if this mass loss continues long enough and at a sufficiently large rate, all vestiges of hydrogen-rich matter must eventually be lost from the star. In constructing the model track presented here, mass loss at (a fairly conservative) rate of $10^{-6} M_{\odot} \text{ yr}^{-1}$ is assumed as the star evolves from $\log T_{\text{e}} \sim 4.5$, $\log L \sim 4.0$ to $\log T_{\text{e}} \sim 5.15$, $\log L \sim 2.3$. At the beginning of this mass loss phase, the centre of the hydrogen profile is located at about $0.001 M_{\odot}$ below the surface and hydrogen extends inward a further $0.001 M_{\odot}$. After approximately 20 000 yr of mass loss, the surface of the star is $0.001 M_{\odot}$ below where the centre of the hydrogen profile was located at the beginning of the mass loss phase (hence -0.001 in parentheses) and there is no more hydrogen to be seen at the surface. In short order, matter that was earlier in a convective helium-burning shell is also exposed and the surface composition becomes roughly 90 per cent helium and 10 per cent carbon (by number). The mass loss rate is expected to decrease with decreasing luminosity (naively one might guess $\dot{M} \propto L$) and eventually the dominant process which influences surface composition is gravitational settling. Carbon diffuses preferentially inward and helium diffuses outward until the surface becomes essentially pure helium. Adapted from Iben, I. (Jr), 1984. *Astrophys. J.*, 277, 333.

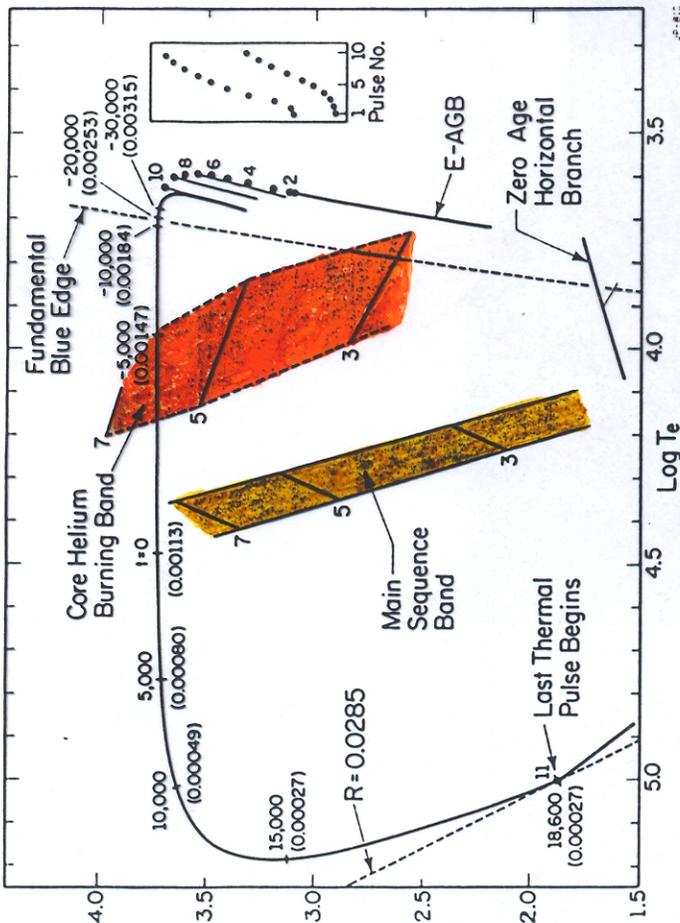
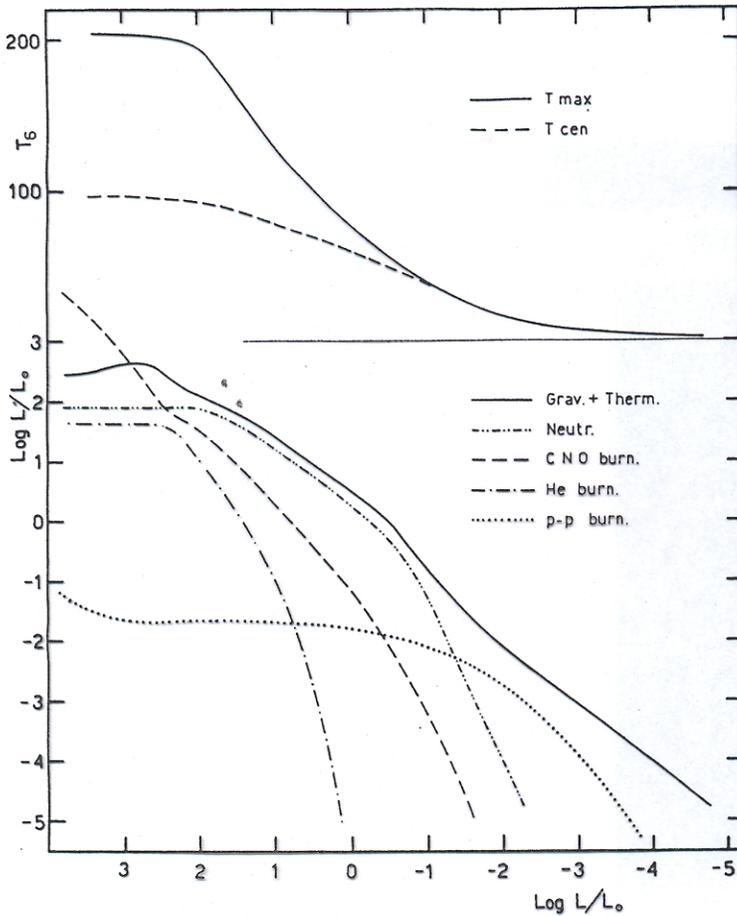


FIG. 16. Evolution of a low mass star during the early and thermally pulsing asymptotic giant branch phases and during the planetary nebula nucleus phase. The evolutionary track might be appropriate for a star of initial mass about $2 M_{\odot}$ which loses about $1.2 M_{\odot}$ during the AGB phase via an ordinary wind and then ejects a nebular shell of mass about $0.2 M_{\odot}$ after developing a C-O core of mass about $0.6 M_{\odot}$. The solid circles along the track mark the onset of a thermal pulse and number). Planetary nebula ejection is assumed to occur after the tenth pulse and the remnant appears at some point (which must at this stage be guessed) to the blue of main AGB branch. Each number in parentheses gives the mass remaining in the hydrogen-rich envelope corresponding to a given location in the HR diagram and the number above the parentheses gives the time in years for the star to evolve to (or from) the point marked $t = 0$, if the rate of evolution is controlled completely by hydrogen burning. In fact, nebular ejection causes the star to traverse a portion of the track much more rapidly than indicated. For example, if nebular ejection leaves a hydrogen-rich shell of mass $0.00147 M_{\odot}$, the stellar remnant appears almost immediately at the point marked (0.00147), traversing in only a few hundred years that portion of the track between, say, the point marked (0.00315) and the point marked (0.00147). Thereafter, the rate of evolution is as marked. Fluorescence of the surrounding nebular shell occurs at point $t = 0$ and is maintained at high luminosity for about 15 000 yr. When the mass in the hydrogen-rich envelope is decreased to about $0.0027 M_{\odot}$, the rate of hydrogen burning drops abruptly and matter above the CO core contracts rapidly. In this particular case (which is expected to be representative of about one out of every 10 cases), the mass in the helium-rich zone below the surface hydrogen layer is sufficiently close to the critical mass for helium ignition in the AGB phase that the heating in the helium zone that is engendered by compression as the star approaches a white dwarf configuration for the first time leads to a final helium shell flash. From Iben, I. (Jr) & Renzini, A., 1983. *Ann. Rev. Astr. Astrophys.*, 21, 271.

ENTWICKLUNG VON STERNEN MITTLERER UND KLEINER MASSE MIT MASSENVERLUST

Entwicklung zu den Weissen Zwergen:

aus: Mazzitelli, I., D'Antona, F.: 1986,
Astrophys. J., 308, 706



Beitrag der verschiedenen Energiequellen bei der Abkühlung eines Weissen Zwergs mit einer Wasserstoff-Schalenquelle und $Z=0.02$. Der Verlauf der Zentral- und der Maximaltemperatur sind ebenfalls dargestellt.

Entwicklungszeiten (in 10^9 Jahren) der verschiedenen Abkühlsequenzen. Man beachte, wie stark die Abkühlzeiten von der chemischen Zusammensetzung der Hülle des Weissen Zwergs abhängig sind.

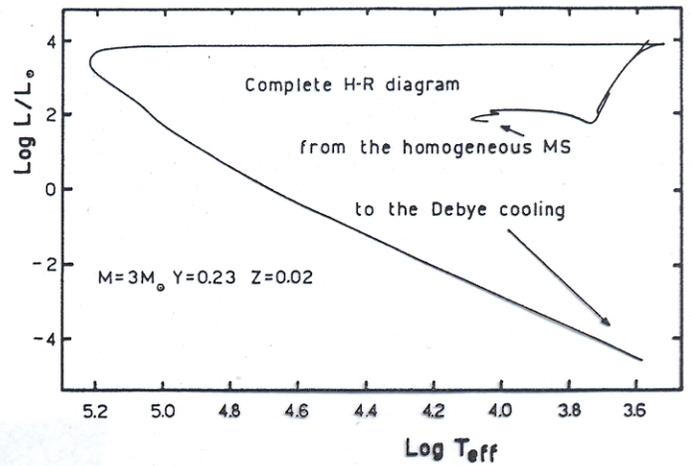


FIG. 1.—Full evolutionary track in the H-R diagram for the $3 M_{\odot}$ star, from the main sequence to the white dwarf stage. The mass is reduced from 3.0 to $0.68 M_{\odot}$ at the top of the asymptotic branch evolution.

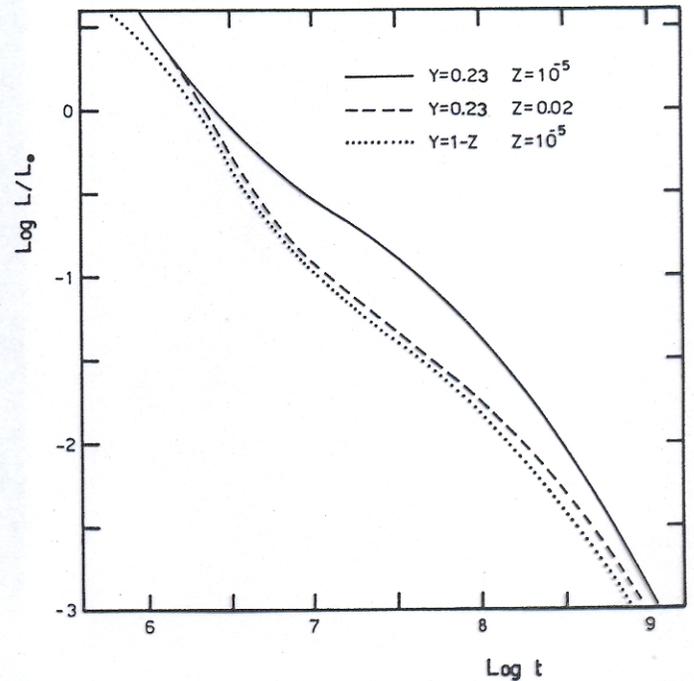
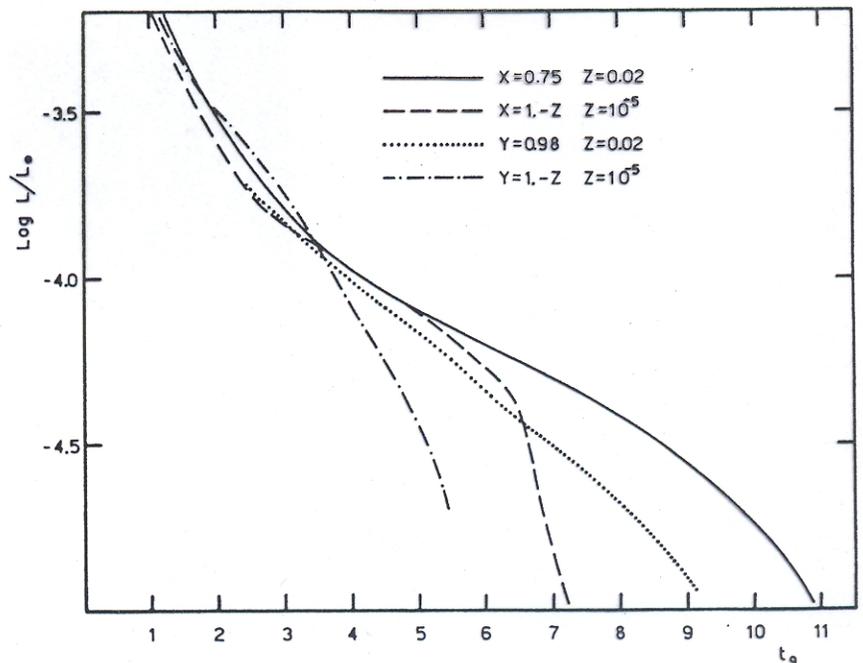


FIG. 9.—Evolutionary times for sequence A (dashed line), B (solid line), and C (dotted line) in the high-luminosity portion of WD evolution. Sequences D and E behave like sequence C. The major difference between sequence B and the others, in the range $-0.5 > \log(L/L_{\odot}) > -2.5$, is due to the large contribution of $p-p$ burning.



29.5. Das Massenspektrum der Weissen Zwerge

Betrachten hier nur C-O-Weisse Zwerge

$$\text{Massenspektrum der WZ: } \frac{d \ln N(M_{WZ})}{dM_{WZ}} = \frac{1}{N(M_{WZ})} \frac{dN(M_{WZ})}{dM_{WZ}}$$

Gegeben:

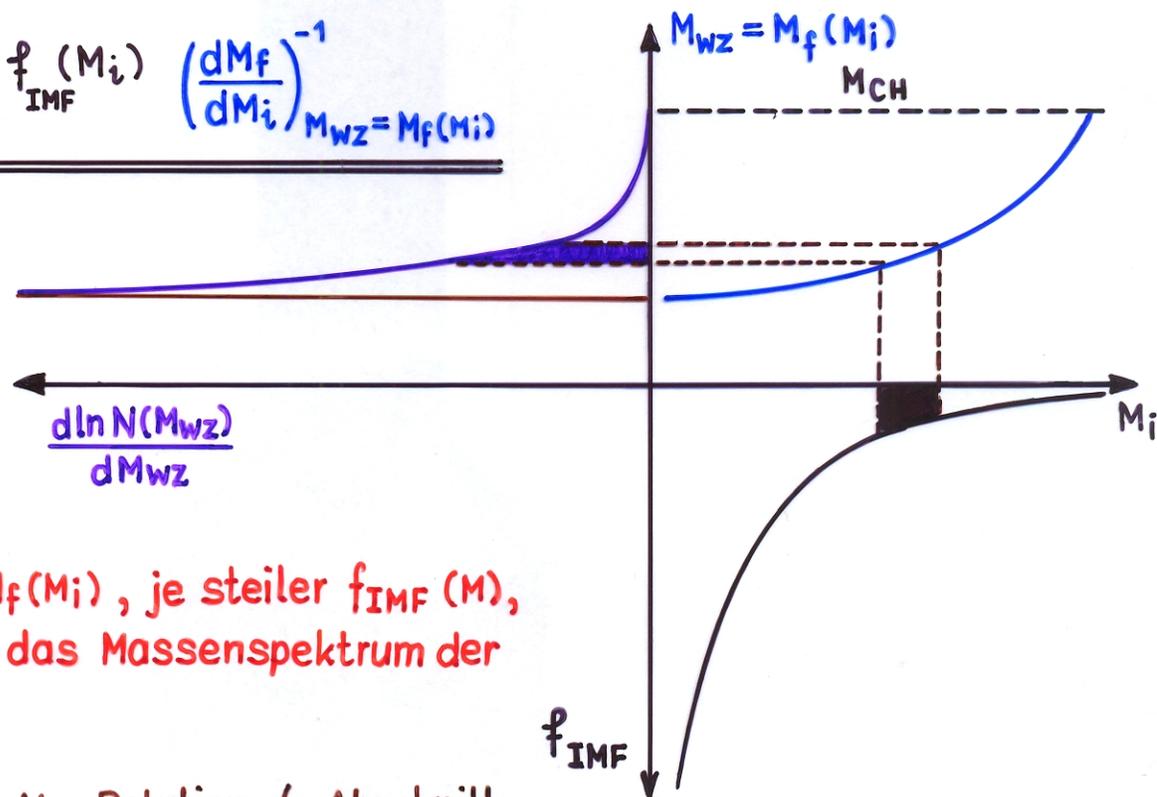
- 1.) die Häufigkeitsverteilung der Sternmassen auf der HR, d.h. die IMF (= initial mass function):

$$f_{\text{IMF}}(M) = \frac{d \ln N(M)}{dM}, \text{ wobei}$$

$$\text{für } 1M_{\odot} \lesssim M \lesssim 10M_{\odot} \quad f_{\text{IMF}}(M) \sim M^{-2.35} \quad (\text{Salpeter, 1955})$$

- 2.) die $M_i - M_f$ -Beziehung für WZ : $M_{WZ} = M_f(M_i)$

$$\rightarrow \frac{d \ln N(M_{WZ})}{dM_{WZ}} = f_{\text{IMF}}(M_i) \left(\frac{dM_f}{dM_i} \right)^{-1}_{M_{WZ}=M_f(M_i)}$$



- ▶ Je flacher $M_f(M_i)$, je steiler $f_{\text{IMF}}(M)$, desto enger das Massenspektrum der WZ.
- ▶ Empirische $M_i - M_f$ -Relation (\rightarrow Abschnitt 26.2) + Salpeter IMF liefert sehr enges Massenspektrum mit

$$\langle M_{WZ} \rangle \approx 0.6 M_{\odot}, \quad \sigma_M \approx 0.1 M_{\odot}$$

- ▶ Beobachtungen: $\langle M_{WZ} \rangle = 0.58 M_{\odot}, \sigma_M \approx 0.1 M_{\odot}$ (Weidemann und Koester, 1983)
- $\langle M_{ZPN} \rangle = 0.58 M_{\odot}, \sigma_M \approx 0.03 M_{\odot}$ (Schönberner, 1981)

29.6. Massenverlust auf dem 1. Riesenast

Wissen bereits, \exists Indizien für Massenverlust vor dem zentralen Heliumbrennen (Sterne auf dem Horizontalast (Pop. II), sdO- und sdB-Sterne, vereinzelt ZPN, ...)

Wissen nicht (genau), ob Massenverlust durch $\left\{ \begin{array}{l} \text{Sternwind auf dem} \\ \text{1. Riesenast} \\ \text{Helium-Flash} \end{array} \right.$

Machen hier eine einfache Abschätzung:

Massenverlust auf dem 1. Riesenast durch einen stellaren Wind

(Vorgehen analog zu Abschnitt 26.2.)

► Betrachten Sterne vor dem He-Flash, $M_i \lesssim 2.5 M_\odot$

Auf dem 1. Riesenast: \exists entartetes He-Zentralgebiet, Masse M_c ,
H-Schalenquelle, keine Thermischen Pulse

\exists Kernmasse - Leuchtkraft - Beziehung (\rightarrow Schalenquellen - Homologie,
 \rightarrow Abschnitt 23.4)

$$\underline{L = L(M_c) = L_\odot \left(\frac{M_c}{M_\odot} \right)^\nu}$$

Hatten schon früher: $L = X_\odot Q_H \dot{M}_c$: nukleare Energieerzeugung

Erhalten durch Integration $M_c(t)$ und $L(t)$:

$$\underline{M_c(t) = \left\{ [M_c(t=0)]^{1-\nu} + \frac{(1-\nu)L_\odot}{X_\odot Q_H M_\odot^\nu} t \right\}^{\frac{1}{1-\nu}}}$$

$$\underline{L(t) = L_\odot \left\{ \left(\frac{M_c(t=0)}{M_\odot} \right)^{1-\nu} + \frac{(1-\nu)L_\odot}{X_\odot Q_H M_\odot^\nu} t \right\}^{\frac{\nu}{1-\nu}}}$$

Machen nun den gleichen Ansatz für \dot{M} wie früher (26.2):

$$\dot{M} = -k_R \frac{LR}{GM}, \quad k_R = 1.23 \cdot 10^{-5} \eta_R, \quad (\text{Reimers 1975}), \quad \eta_R \approx 0.3 \dots 3$$

und die gleiche Annahme über die HL: $T_{\text{eff}} = \text{const.}$

\rightarrow Radius $R(t)$:

$$R(t) = (4\pi\sigma)^{-1/2} L(t)^{1/2} T_{\text{eff}}^{-2}$$

Können nun $M(t)$ berechnen:

$$M\dot{M} = \frac{1}{2} \frac{d}{dt} (M^2) = - \frac{k_R}{G(4\pi\sigma)^{1/2} T_{\text{eff}}^2} L(t)^{3/2} = - \frac{k_R L_o^{3/2}}{G(4\pi\sigma)^{1/2} T_{\text{eff}}^2} \left\{ \left(\frac{M_{c,0}}{M_o} \right)^{1-\nu} + \frac{(1-\nu) L_o}{X_o Q_H M_o} t \right\}^{3/2}$$

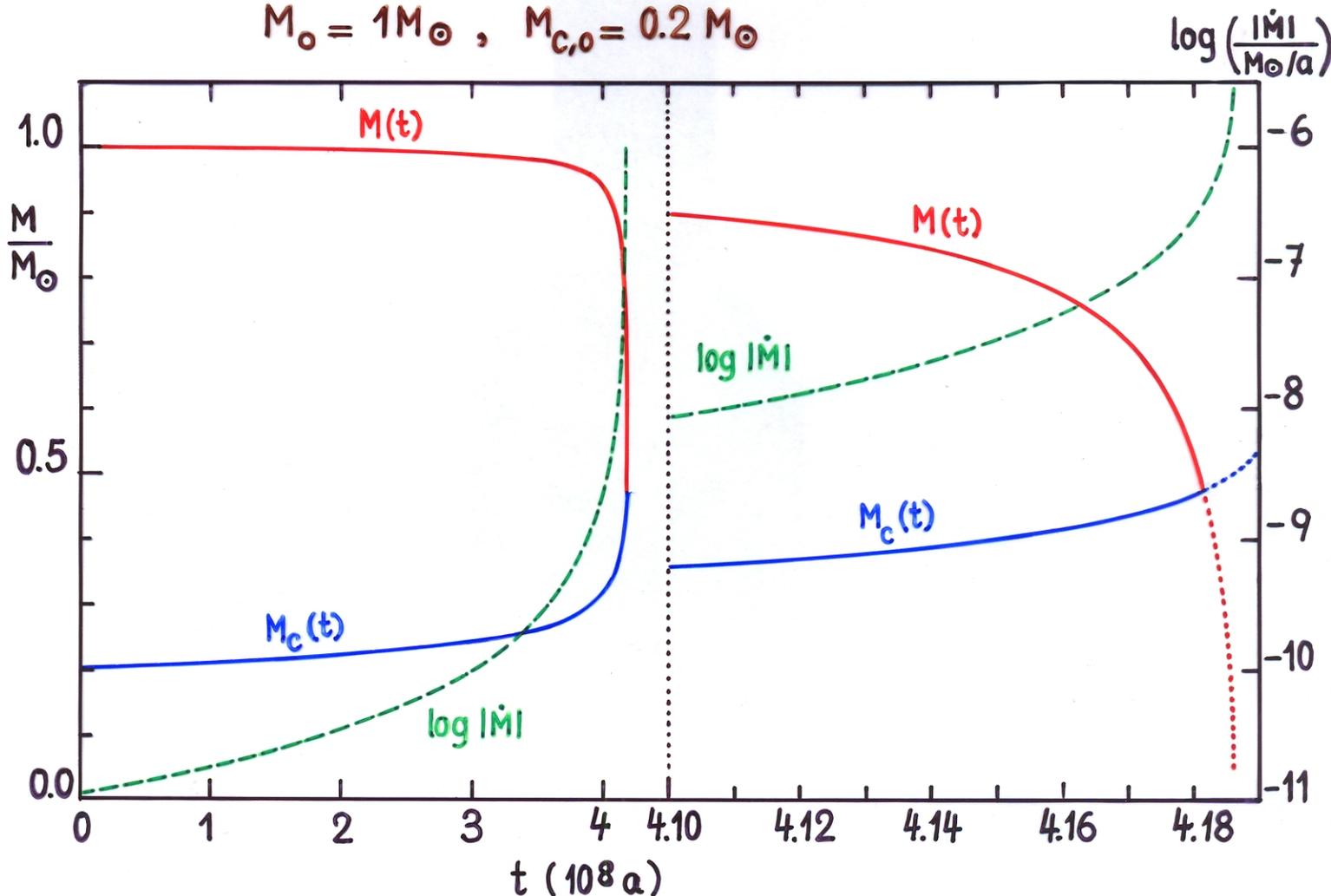
$$M^2 + \text{const.} = - \frac{4 L_o^{1/2} X_o Q_H k M_o}{(4\pi\sigma)^{1/2} G T_{\text{eff}}^2 (\nu+2)} \left\{ \left(\frac{M_{c,0}}{M_o} \right)^{1-\nu} + \frac{(1-\nu) L_o}{X_o Q_H M_o} t \right\}^{\frac{\nu+2}{2}}$$

Sei $M(t=0) = M_o$, so folgt:

$$\left(\frac{M(t)}{M_o} \right)^2 = \left(\frac{M_o}{M_o} \right)^2 - \frac{4 L_o^{1/2} X_o Q_H k R}{(4\pi\sigma)^{1/2} G T_{\text{eff}}^2 (\nu+2) M_o} \left\{ \left(\frac{M_c(t)}{M_o} \right)^{\frac{\nu+2}{2}} - \left(\frac{M_{c,0}}{M_o} \right)^{\frac{\nu+2}{2}} \right\}$$

Beispiel: $L_o = 10^6 L_\odot$, $\nu = 7.5$ (gute Approximation im Bereich $0.2 M_\odot \lesssim M_c \lesssim 0.5 M_\odot$), $k_R = 1.23 \cdot 10^{-5} \eta_R$, $\eta_R = 1$ (zu gross!), $X_o = 0.75$, $Q_H = 10^{18.8} \text{ erg/g}$, $\log T_{\text{eff}} (\text{K}) = 3.6$

$$M_o = 1 M_\odot, \quad M_{c,0} = 0.2 M_\odot$$



Maximale Kernmasse: M_c wächst, solange $M_H = M - M_c > 0$

In erster Näherung gilt daher für $M_{c,f}$: $M(t_f) = M_c(t_f) = M_{c,f}$

$$\underline{\underline{\left(\frac{M_0}{M_\odot}\right)^2 = \left(\frac{M_{c,f}}{M_\odot}\right)^2 + \frac{4L_\odot^{1/2} \chi_\odot Q_H k_R}{(4\pi\sigma)^{1/2} G T_{\text{eff}}^2 (\gamma+2) M_\odot} \left\{ \left(\frac{M_{c,f}}{M_\odot}\right)^{\frac{\gamma+2}{2}} - \left(\frac{M_{c,0}}{M_\odot}\right)^{\frac{\gamma+2}{2}} \right\}}}$$

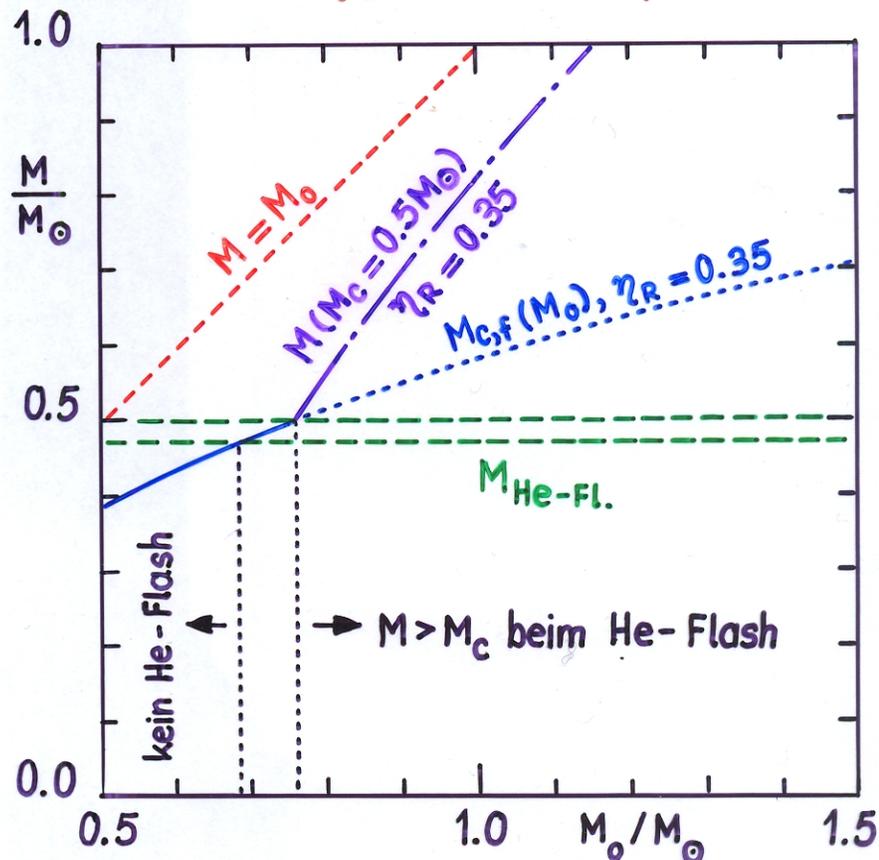
→ theoretische $M_i - M_f$ -Beziehung für He-WZ, die durch Windverluste auf dem 1. Riesenast entstehen.

▶ Nebenbedingung: $M_{c,f} < M_{\text{He-Fl.}} \approx 0.47 - 0.50 M_\odot$

Beispiel: (gleiche Parameter wie oben, jedoch $\eta_R = 0.35$ und $M_{c,0} = 0.15 M_\odot$)

Folgerungen:

- wenn $\eta_R \approx 0.35$, so erreichen Sterne mit $M_0 \lesssim 0.70 - 0.75 M_\odot$ den He-Flash nicht mehr. (N.B. die nukleare Zeitskala für solche Sterne ist grösser als das gegenwärtige Weltalter.)
- Pop. II Sterne: Wegen geringerem Z ist $L(M_c)$ und damit $-\dot{M}$ viel kleiner als für Pop. I Sterne. → Nur wenn $M_0 \lesssim 0.60 - 0.65 M_\odot$ wird der Helium-Flash nicht mehr erreicht.



▶ Für Pop. II Sterne mit $Z \approx 10^{-4}$ reicht der Massenverlust mit $\eta_R \approx 0.35$ nicht aus, um die Lage der Horizontalast-Sterne mit $M_0 \approx 0.80 M_\odot$ zu erklären. → Zusätzlicher Massenverlust während des He-Flashes wahrscheinlich.