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Is stellar granulation turbulence?

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Abstract. We show that power spectra of granulation images or velocity fields cannot be compared meaningfully with spectra from theoretical models based on turbulent cascades. The small scale power in these images is due almost entirely to the sharp edges between granules and intergranular lanes, not to turbulence in the usual sense. This is demonstrated with a number of experiments with result from numerical simulations, and with simpler synthetic data with power spectra similar to that of granulation. The reason for the seemingly laminar behavior of the granulation flow, in spite of the high Reynolds numbers involved, is the influence of stratification on the local ratio of turbulence to bulk flow. The rapid expansion of upflows, their deep origin and near-adiabatic stratification lead to low levels of turbulence in the overturning fluid at the surface. Higher levels of turbulence are expected in the converging flows near downdrafts, but mostly at scales that are below current observational resolution limits, and contributing relatively little to the total convective flux and to spectral line broadening.

Key words: convection – Sun: granulation – Stars: atmospheres

1. Introduction

Observers of solar granulation have often remarked on the great similarity of its well-ordered geometric appearance, flow pattern, and evolution with that of laboratory convection at low Reynolds number. Most of these observations were made at times when the the prevailing theoretical framework required the observed flows to be called 'fiercely turbulent' rather than well-ordered. This framework consisted (simplifying somewhat) of the paradigm that a high Reynolds number is equivalent to turbulence, which is equivalent to a description in terms of a local hierarchy of scales, with velocities obeying Kolmogorov scaling with a power spectrum slope of $-5/3$. This strong theoretical preference explains the focus, in many interpretations of observational data, on using power spectra of single granulation images: the premise is that one has statistically steady turbulence, so that the power spectrum of a single image (over a sufficiently large area) contains the essence of the turbulent cascade. The power spectrum of a single image, however, is a very strong reduction of the information contained in even that same image, let alone the observable time evolution of the flow patterns. How strong, we demonstrate in the following by comparison of power spectra of various images, including explicitly non-turbulent ones. From this we conclude that the power spectrum of a granulation image, or its comparison with spectra of theoretical turbulence, can not be used as evidence for or against such models. Such a comparison is useful only if the turbulent nature of the flow (as defined in the sense of the models with which it is compared) has already been established by independent means.

Are local turbulence models applicable to solar granulation? There are observational and theoretical arguments against this idea. Observationally, the flow as observed in a sequence of images contains vastly more information than a power spectrum. If this information shows the flow to be more similar to ordered large scale flows observed elsewhere in non-turbulent situations, it must be questioned that an average power spectrum contains enough evidence to invalidate this similarity. For example, the power seen at short length scales may arise from the edges between granules and intergranular lanes, i.e. from the geometric properties of single large scale structures rather than from a hierarchy of structures on different scales.

Theoretically, the turbulence models with which power spectra are compared are essentially local ones, that is they assume the existence af a *large* length scale on which the flow (and the forcing) is statistically homogeneous, compared with the range of length scales over which the spectrum is measured. In the case of granulation, such a length scale does not exist. In fact, the situation is exactly the opposite. The atmosphere is fundamentally inhomogeneous on scale of the order of the pressure scale height, 150 km. This is *smaller* than *any* of the lengths scales that can be reliably observed with existing telescopes! There is no reason to expect that any model patterned after the usual quasi-homogeneous turbulence ideas is applicable, since their basic assumption is violated in the extreme.

This does not mean that theoretical understanding is not available; it just means one has to look elsewhere to find it. Numerical simulations and laboratory convection at high Rayleigh numbers has given us a very detailed picture how the observed

Send offprint requests to: H.C. Spruit organized flows come about, and why they do not look like homogeneous turbulence.

2. Interpreting the power spectrum of granulation

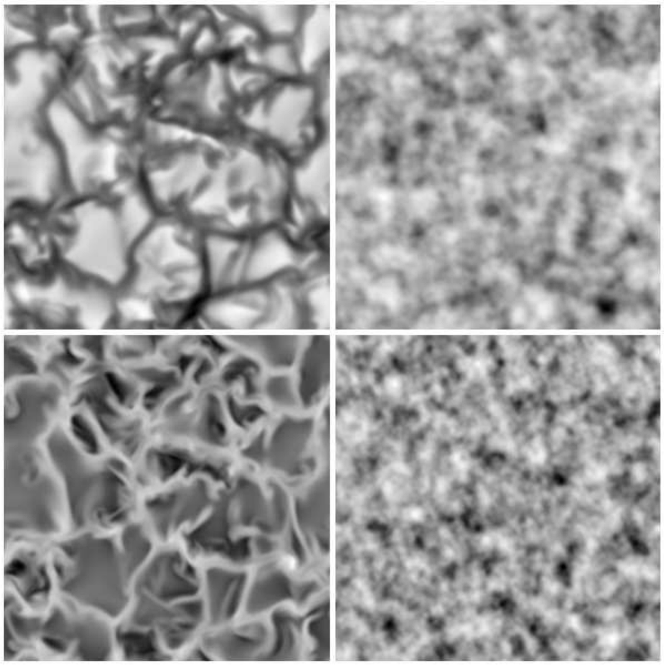

Fig. 1. Left column: continuum image (top) and vertical velocity (bottom) of granulation from a $253^2 \times 163$ numerical simulation (Nordlund Stein 1997). The scale of the images is 6 Mm. Right column: Images obtained from the left column by randomizing the phases of the Fourier components. The small scale power due to the sharp edges of the granules now appears in the form of cloud-like small scale structure.

Figure 1 shows a granulation image and the corresponding velocity field taken from a numerical simulation. The resolution of this simulation is 253×253 points in the horizontal directions and 163 in depth. Further details of this computation are given elsewhere (Stein & Nordlund 1997). The power spectrum of this image is shown in Fig. 2. Its shape is very similar to that of observed solar granulation (Espagnet et al. 1993, their Fig. 6). This reflects the more basic fact that the image itself, appropriately smeared to mimic seeing and finite instrumental resolution, is nearly indistinguishable from observed granulation (Nordlund & Stein 1996).

By choosing an appropriate range of wave numbers, a slope of -5/3 can be fitted through this spectrum, but since it is rather curved, any other slope between about 0 and -6 can be made to fit equally well, depending on the wave number range chosen.

By fitting a spectral shape from a theory of turbulence to the observations, the implication is made that the observations show turbulence. Inspection of the flow, however, (whether one takes observed or simulated granulation) does not give a turbulent impression at all. The development of granules looks very much like laminar convection as seen at low Rayleigh number in the laboratory. The cells have an organized flow pattern that lasts for their life time, clearly defined edges, and relatively little substructure. If the power at small scales seen in the spectrum is interpreted as due to turbulence, where, then, is this turbulence located in the images?

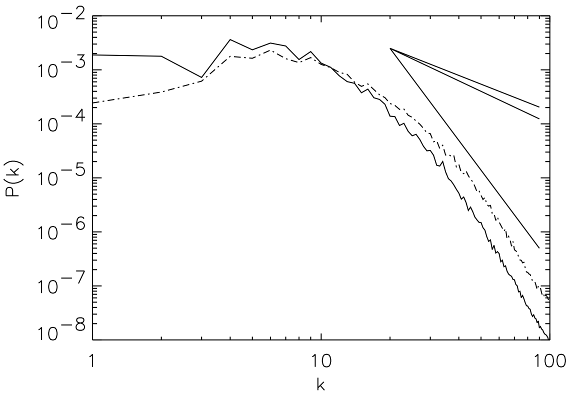

Fig. 2. The average one-dimensional power spectra of the intensity (solid) and velocity (dashed) images of Fig. 1. Lines with slopes of -5/3, -2 and -17/3 are shown for comparison. The numerical diffusion that was used in the simulations is expected to not influence the spectrum significantly below wavenumbers of about $k_{\max}/5$,where k_{max} is the maximum (Nyquist) wave number.

The answer is that most of the small scale power in granulation images is not due to turbulence but simply reflects the sharpness of the edges between granules and intergranular lanes. This applies to the intensity images as well as the vertical velocity field. To demonstrate this, we present a few experiments with images from numerical simulations, and from some much simpler synthetic data.

One way to demonstrate that the small scale power in granulation is due mainly to edges is to compare it with an image that has the same power spectrum, but a different realization of the Fourier phases. This is shown in Fig. 1. Here, we have taken the intensity and velocity images of Fig. 1, Fourier transformed them, added random phase shifts between 0 and 2π to the Fourier components, and transformed back to real space. Though the resulting images have *exactly* the same power spectrum as the original, it is immediately evident that they bear no resemblance at all to granulation. There is a large amount of structure in the form of fuzzy clouds or blobs on small scales that is not seen in granulation. It has an organization of smaller scale structures inside larger scales, such as one might perhaps expect from a canonical turbulent cascade picture. In the granulation image, the organization is very different: the structures are much larger, and have very much sharper edges. It is the power in these edges that dominates the spectrum at large wave numbers, rather than turbulence.

A single edge in the interval $(-a, a)$:

$$f(x) = \begin{array}{ll} 1 & (x < 0) \\ 0 & (x > 0) \end{array} \tag{1}$$

has the power spectrum

$$P(k) = \hat{f}\hat{f}^* = \frac{2}{k^2}[1 - \cos(ak)]. \tag{2}$$

Apart from the $1-\cos(ak)$ modulation which depends on the window used, the spectrum is a power law with slope -2. A granulation image contains many edges, in somewhat random positions. One would therefore expect a granulation image to have a spectrum with a slope near -2 at high wave numbers. Given the fluctuations in an observed spectrum, this slope would hardly be distinguishable from -5/3, even if the observed spectrum were close to a power law (which is not the case, as Fig. 2 shows).

A simple one-dimensional model containing only edges is given by the sequence shown in Fig. 3. In this model the length of the 'on' states has a Gaussian distribution with a dispersion of 0.5 times the mean length w (with negative values omitted), the 'off' states have a mean length of $0.2w$ and a dispersion of $0.1w$. The average power spectrum of 100 such sequences is shown in Fig. 4. It shows a bump in the wavenumber range of the mean lengths of the on and off states. At large wavenumbers it has the slope -2 expected from the edges. If these edges are smoothed out a bit, the spectrum becomes steeper at the highest wave numbers, as in the observed spectrum. As in the observed granulation spectrum, a slope of -5/3 can be fitted by selecting an appropriate range of wave numbers.

With this example we have shown a spectrum with a slope close to -5/3, generated by a process that has nothing whatsoever to do with turbulence.

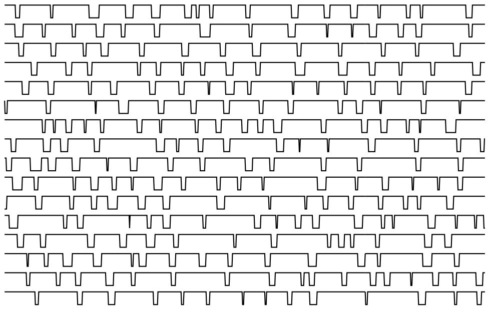

Fig. 3. Sequence of off and on states with Gaussian distribution of widths.

A final example to illustrate the point is given in Fig. 5. Here, we have arranged a pattern of 'intergranular lanes' which will be recognized as being of a non-turbulent nature. The power spectrum of this image (with contrast adjusted to match the rms contrast of granulation), however, is very similar to that of observed or simulated granulation. The reason for this is the same as in the case of granulation: the image is divided into relatively large and uniform bright and dark areas with sharp boundaries. The detailed shape of the spectrum at high k reflects the degree of smoothing of these boundaries.

3. Discussion

The examples shown demonstrate that granulation is not turbulence in the sense of a hierarchy of structures on different length scales. It has an organized structure consisting of large scales with sharp edges. The power at large wave numbers

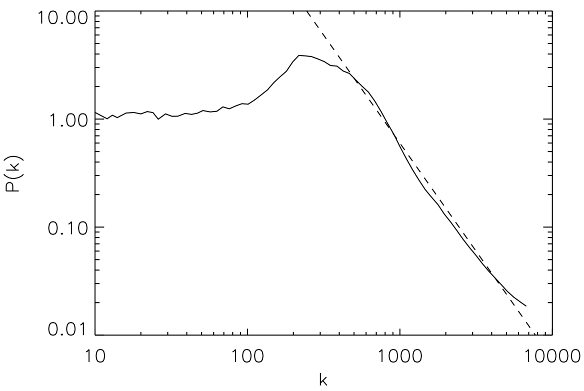

Fig. 4. Power spectrum of the sequence of Fig. 2. Slope of -2 (dashed) shown for comparison

is due mostly to these edges, not to turbulence. This holds both for the intensity and the velocity field. The examples demonstrate the well known fact that *the power spectrum alone does not provide any evidence at all about a turbulent or non-turbulent nature of the process studied.*

At this point, some might take the extreme position that granulation flows, with their very large Reynolds numbers, must be turbulent, and therefore that whatever granulation patterns are seen must be called turbulence. By making it synonymous in this way simply to the presence of a high Reynolds number, the word turbulence loses most of its meaning, however. As we have shown, the further associations usually made almost automatically with turbulence, such as interpretations in terms of local hierarchies of length scales and power spectra, are not meaningful in the case of stellar surface convection.

In our view, the first lesson from the examples given is that the essence of solar granulation has little to do with its power spectrum. The second is that the patterns observed and their time evolution are highly ordered, do not resemble those expected from turbulence, and look much more like laminar convection. Even though flows more closely resembling turbulence are definitely present in numerical simulations, in particular in the intergranular lanes, the overall pattern can not reasonably be called turbulent. Given how different other realizations of the granular power spectrum are (Fig. 1), compared with the actual flows observed in the simulations and on the Sun, it is clear that understanding of the granulation flow is not aided much by inspection of its power spectrum.

Numerical simulations carried out over the past 2 decades, on the other hand, have given us a very detailed understanding of what actually happens in solar granulation. These simulations (Nordlund 1982, 1986, Stein & Nordlund 1989, Nordlund & Stein, 1996, Steffen et al. 1989, Ludwig et al. 1996) show that the observed organized structure of the flows is tied very intimately to the way in which convection transmits the solar energy flux to the surface. Differently organized flows with the same power spectrum would transport energy with a different efficiency. We refer to the work cited above for discussions on the physics involved (for further reviews see Nordlund 86, Spruit at al. 1990, Nordlund & Stein 1996, Spruit 1997). We discuss here only the question of why the velocity field in granulation cells appears to contain so little of the substructure

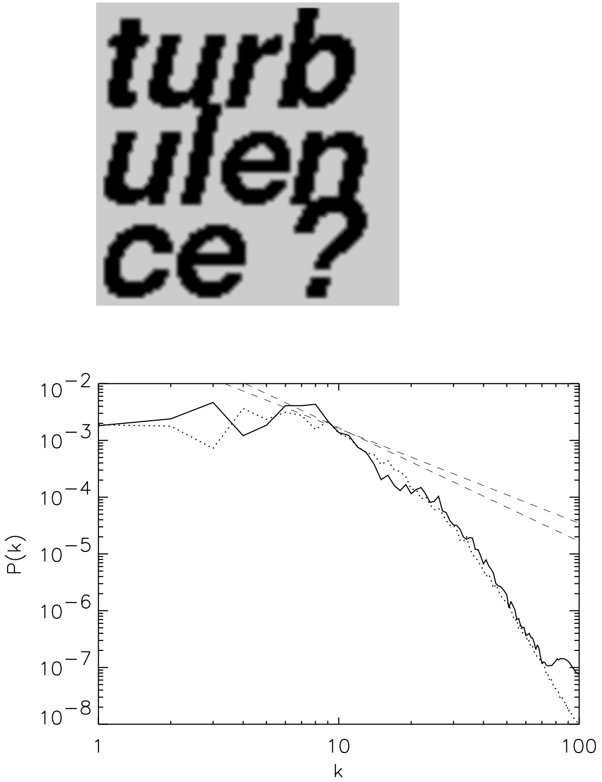

Fig. 5. Top: Pattern of 'intergranular lanes' with slightly defocused edges. bottom: power spectrum of this image (solid line). Dashed lines: slopes of -5/3 and -2. For comparison the granulation spectrum from fig 2 is shown (dotted)

expected from a turbulent flow.

3.1. Why so little turbulence?

It is important to first emphasize that our discussion does not question the validity of Kolmogorov scaling of high Reynolds number flows under homogeneous and isotropic conditions; the possibility of small deviations from the -5/3 law, and the non-Kolmogorov scaling of higher order structure functions is an entirely different matter (cf. Hunt et al. 1991 and references therein). What we are concerned with is an example of what happens when conditions are neither homogeneous nor isotropic.

Specifically, one may draw the parallel to laboratory experiments with grid-generated turbulence (see, e.g., Tsinober et al. 1992). It is only relative to a frame of reference moving with the fluid, and at some distance away from the grid, that one may expect the flow to obey near-Kolmogorov scaling. In fact, in such a frame of reference one is witnessing decaying turbulence, whereas in the laboratory frame one sees locally stationary conditions, but with the ratio of turbulence to mean flow speed changing with distance away from the source of forcing (the grid).

One could imagine changing the ratio between the turbulence amplitude and the bulk velocity by other means than just looking further downstream. For example, if the fluid was allowed to expand significantly along its path, with a corresponding increase in bulk velocity, the ratio of turbulence amplitude to bulk speed would decrease, both because of the increase in bulk speed and because vorticity would be "diluted". Conversely, if the fluid was compressed and slowed down along its path, the ratio would increase.

We conclude that the ratio between bulk speed and small scale turbulence amplitude depends on the (Lagrangian) history of the fluid; where did it get a significant input of turbulent kinetic energy, and what has happened since along its path. If we now apply this line of reasoning to the strongly stratified fluid motions near the solar surface, it becomes obvious why these layers have a very inhomogeneous ratio of local turbulence intensity to local bulk motion, and why they appear to be so laminar, both observationally and in supercomputer simulations.

The fundamental circumstance that must be appreciated is that the size of granulation cells is an order of magnitude larger than the local density scale height. (The thin layer at the optical surface where the density profile has a minor kink is of little importance; over larger scales the density drops exponentially.) Mass conservation thus forces a rapid expansion of ascending flows, and a rapid contraction of descending flows. Thus, even if small scale forcing was distributed homogeneously (which it is not), the ratio of turbulence to bulk motion would be much smaller in the expanding upflows than in the contracting downflows.

To be more specific: turbulence that is visible at the surface inside a granular upflow must either be due to advection of small scale flows from below, or must be generated at the surface in the time interval between first appearance at the surface and disappearance by advection into the intergranular downflows.

The first possibility depends on how much turbulence is injected into the upflows initially, and the amount of expansion since that injection. Statistically, most of the ascending fluid at any one level originates from levels many scale heights below (cf. Fig. 6 in Spruit et al. 1990 — subsequent studies at increasing numerical resolution have shown this to be a robust result; cf. `http://www.astro.ku.dk/~aake/convection/movies/`). The entropy fluctuations at those levels are tiny in comparison, so for all practical purposes the ascending flow may be considered to be isentropic, and thus locally convectively stable. Thus, no "new" turbulence is generated in the ascending fluid; it is effectively a case of decaying *and rapidly expanding* turbulence. We conclude that the ascending fluid that reaches the surface carries only insignificant amounts of small scale turbulence with it from below.

This leaves the second possibility, of turbulence generated in situ at the surface. This effect is limited because of two factors. First, the vertical velocity gradient in the upflows is low because the upper surface is free (as opposed to no-slip in laboratory experiments). In addition, prospective turbulence to be generated by this shear takes a time at least equal to the inverse of the velocity gradient to develop. The available time is limited, however, by the travel time of the flow to an intergranular lane, where the flow disappears below the observed level.

Considering now the downflows, virtually all the arguments are reversed, and lead us to expect much larger ratios of turbulence to bulk motion in the descending fluid. The down-

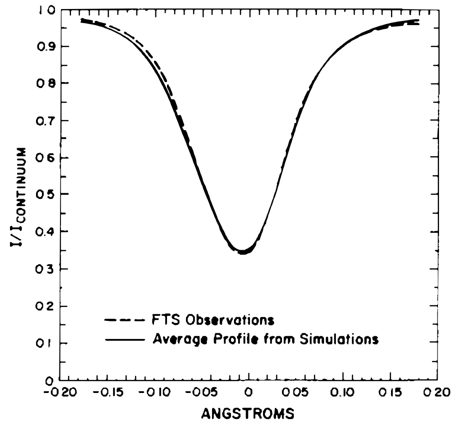

Fig. 6. Line profile from radiative hydrodynamic simulations at 63^3 resolution compared with the observed profile (Fe I λ6302.5). About half of the width of the line, and all of its asymmetry and wavelength shift result from the details of the velocity and temperature fields. The agreement shows that already at modest numerical resolution the simulations account for almost all velocities contributing to the line shape. From Lites et al. (1989).

flows have a strong downward buoyancy (compared with the weaker upward buoyancy and flow speed in the upflows), and are continuously undergoing mixing with overturning ascending (and hence isentropic) flow. The local variations in entropy and hence buoyancy constitute a continuous source of small scale kinetic energy, in addition to the advection and concentration of velocity fluctuations from above. Thus the local source of small scale turbulence is much stronger in the downflows.

The strong downdrafts are engulfed in the overturning and expanding upflows, and are themselves rapidly contracting, as a consequence of mass conservation. This keeps the "loss" of turbulence from the downflow (reinjection into the upflow) at a low level, and also amplifies the turbulence because of the contraction. Thus, not only is most of the turbulence generated by the downflows carried along with the downflow, it is also continuously regenerated and amplified.

For these reasons, in a strongly stratified fluid one should in general expect a much higher ratio of turbulence to bulk speed in downflows than in upflows. Or, conversely, since the turbulence amplitude can hardly be expected to exceed the bulk downward velocity in the downflows, turbulence must be much weaker than the bulk speed in the upflows. Upflows will thus appear to be nearly laminar, irrespective of the high Reynolds number.

3.2. Surface effects

In addition to the general effects of stratification, which reduce the level of turbulence in upflows, there are particular surface effects that further reduce the visibility of turbulence in granulation observations.

As the nearly laminar fluid in the upflows reach the surface, turns over and converges into the downdrafts, it traverses the stably stratified layers above the optical surface. Here, any remaining small scale velocities must develop temperature fluctuations that are anticorrelated with the velocity, and thus their amplitude is damped further.

The convergence of these laminar flows into the intergranular lanes and vertices corresponds to a localized source of turbulent kinetic energy. But there is also a strong sink: the fluid rapidly descends below the visible surface, and we see only the level of turbulence that has time to develop in the mean time.

This downdraft turbulence is seen in high-resolution simulations, but it is not clear that any of it has yet been observed on the Sun, since the expected horizontal scales are comparable to or smaller than current resolution limits. In any case, it apparently does not contribute much to power spectra, since the high wavenumber end of the spectrum, where it would show up, is overwhelmed by the power due to the edges of granules. At intermediate numerical resolution, too low to resolve intergranular turbulence, the average profiles of spectral lines already agree very closely with the observed ones, as illustrated in Fig. 6 (from Lites et al. 1989). This shows that intergranular turbulence, including all length scales that are not resolved by the observations or the simulations, is not a significant contributor to the observed line broadening.

4. Summary and Concluding Remarks

The stratification effects discussed in the previous section make understandable why granules appear to be essentially laminar flows, in spite of the huge Reynolds numbers: the level of turbulence to bulk flow speed is much smaller than unity in the convective upflows, because of their rapid expansion and near adiabaticity.

In laboratory convection (e.g. Castaing et al. 1989) and numerical simulations thereof (Kerr 1996), the upflows are much more turbulent because they contain small scale vorticity generated at the lower boundary of the box. In stellar envelope convection, this effect is negligible because of the strong density stratification. Any vorticity generated at the lower boundary of the solar convection zone is expanded by a factor 10^6 (by volume) by the time it reaches the surface, and is correspondingly feeble. In addition, the flow velocities available for generating turbulence at the lower boundary themselves are lower because of the high gas density.

Finally, let us return briefly to the relevance, or lack thereof, of turbulence models to stellar convection: When attempts are made to apply analytical models of turbulence to stellar convection, one of the main purposes of the game is to obtain "improved" estimates of the convective flux (often expressed as a *local* function of the stellar structure; c.f. Canuto & Mazitelli 1991, 1992). But the bulk of the convective flux is transported by motions whose scales exceed the local scale height with an order of magnitude. As discussed above, these motions violate all of the assumptions that are typically made in the turbulence models:

1. On the scale of energy transport, the motions are strongly anisotropic.
2. On sufficiently small scales, where the motions may be assumed to be more nearly isotropic, the level of turbulence is instead strongly inhomogeneous; intense in or near downflows, and feeble in upflows.
3. The scale of variation of background properties is small compared to the energy carrying scales.
4. Radiative transfer effects are crucial in the very layers where the details of the convective transport matter the most: the thin superadiabatic region at the surface.

One may choose to disregard the fact that all of these fundamental properties are in conflict with the assumptions behind a theory, and formally adopt a local theoretical estimate of the convective flux to compute one-dimensional average models. As long as the description of the physics is so rudimentary, however, such a procedure should be regarded essentially as a somewhat elaborate fitting formula.

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