Mon. Not. R. Astron. Soc. **000**, 1–13 (1997) Printed 27 March 1997 (MN LATEX style file v1.3)

The X-ray spectrum of a disk illuminated by ions

H. C. Spruit

Max-Planck-Institut für Astrophysik, Postfach 1523, D-85740 Garching

Accepted xx Received ; in original form xx

ABSTRACT

The X-ray spectrum from a cool disk embedded in an ion supported torus is computed. A surface layer of the disk is heated by the protons from the torus. The comptonized spectrum produced by this process has a shape that depends only weakly on the incident energy flux and the distance from the accreting compact object. It consists of a 'blue bump' of uncomptonized soft photons and a flat high energy tail, reminiscent of the observed spectra. The slope of the hard tail decreases as the thermalization depth in the cool disk is increased. These results stress the possible importance for 'ion illumination' in disks; further evidence for this process are the Li abundance in the secondaries of low mass X-ray binaries and the 450 keV lines sometimes seen in black-hole transient spectra.

Key words: Black hole physics – accretion – radiative transfer – scattering

1 INTRODUCTION

The X-ray spectra $F(E)$ of AGN as well as galactic black hole candidates (BHC) in their hard states are characterized by a power law of index ≈ 1 and a high energy cutoff $E_{\rm c}$ around 200 keV. Such spectra are well known to be describable by comptonization in an electron scattering layer of optical depth $\tau \approx 0.5$ and temperature $T \sim E_{\rm c}$. One of the classsical problems in X-ray astronomy is to explain why τ and T should have just these values, with little variation between sources. Theoretical arguments can be given that comptonization is in fact the most important interaction between matter and radiation at temperatures of 10–100 keV (e.g. Shakura and Sunyaev 1973), for the inferred radiation energy densities near accreting black holes, but this does not tell us what the the thickness and temperature of the interaction region are.

An optically thick accretion disk would produce spectra peaking at 1 keV and 10–100 eV for BHC and AGN, repectively. The conditions in BHC and AGN allow (at accretion rates well below Eddington) for a second form of accretion, an ion supported advection torus (Shapiro et al. 1976, Liang 1979, Rees et al. 1982, Narayan and Yi 1994, Fabian and Rees 1995, Narayan et al. 1995, 1996). The ions in this flow are near their virial temperature, the electrons much cooler because of their weak interaction with the ions and their strong interaction with the radiation field. Such flows could produce, in principle, the kind of spectrum observed (Narayan et al. 1995, 1996), but again it is not clear which physics restricts the optical depth and temperature of the flow to the observed narrow ranges (cf. Haardt 1997, Maraschi and Haardt 1997).

1.1 Evaporating disks inside tori

Various geometries for the accretion flow near a black hole have been developed, for a review see Collin (1997). One of the possibilities is an ion supported advection torus coexisting with an optically thick accretion disk embedded in it (see fig. 1 in Collin). This possibility is attractive because the spectra of BHC in their high states show evidence of the simultaneous presence of an optically thick, thermal, accretion disk and a hotter component which produces a power law tail at higher photon energies (e.g. Mitsuda et al. 1984, Tanaka 1997).

Theoretically, one would expect exchange of both mass an energy to take place between the disk and the advection torus. Heating of the disk surface by the hot ion supported flow above would lead to an 'evaporation' of the disk surface, feeding mass into the torus. Such evaporation has been studied in detail for the case of disks in Cataclysmic Variable systems by Meyer and Meyer-Hofmeister (1994). In the inner regions of the disk the mass available in the disk is smallest, and the energy budget potentially available for evaporation largest. If the mass flow from disk into torus increases with the energy dissipation rate, and if a steady state develops, one could therefore envisage a structure consisting of three regions: an outer one in which only a geometrically thin optically thick disk is present, inside this a composite region with an evaporating disk inside a hot ion supported advection torus, and inside this a region in which only an ion supported flow exists because all disk mass has evaporated (Meyer and Meyer-Hofmeister 1994).

Depending on details of the processes of mass and energy exchange between disk and torus, the boundaries between these regions may vary. It is not necessary that the structure is steady. The model has, in principle, sufficient ingredients to allow for variability and may perhaps be developed further in the context of the various forms of variability seen in BHC.

1.2 Energy exchange between disk and torus

Energy exchange between disk and torus may be mediated by particles or by radiation. If the interaction is by radiation only, one obtains a model like that of Haardt and Maraschi (1991). To simplify the discussion, assume that the accretion takes place predominantly through the torus (this assumption can easily be relaxed). The radiation produced by the torus illuminates the disk below, which thermalizes it into an approximate blackbody spectrum. These (soft) photons are comptonized in the hot torus. In this model, approximately half the energy comes out as soft radiation and half as comptonized photons. It correctly predicts the slope of the spectrum, but still requires an ad hoc assumption about the temperature of comptonizing region to get the right cutoff energy E_c. For further developments of this model see Haardt (1997).

As second possibility for energetic interaction are the hot protons in the torus with temperature near the virial temperature, $T_\mathrm{p} \approx T_\mathrm{v} \approx 400 r_\mathrm{g}/r$ MeV. At the distance dominating the energy release, $r \approx 7 r_\mathrm{g}$, the protons thus have a temperature around 50 MeV. At this energy, they have a significant penetration depth into the cool disk. They are slowed down mainly by Coulomb interactions with the ensemble of electrons inside their Debye sphere. The 'stopping depth', expressed in terms of the corresponding Thomson optical depth, is

$$\tau_\mathrm{s} \approx \frac{m_\mathrm{p}}{3 m_\mathrm{e} \ln \Lambda} \frac{\beta^4}{\psi - x\psi'}, \tag{1}$$

(e.g. Ryter et al., 1970) where $\beta = v_\mathrm{z}/c \approx (kT_\mathrm{p}/m_\mathrm{p}c^2)^{1/2}$ is the vertical component of the proton velocity, $\theta = kT/m_\mathrm{e}c^2$ is a measure of the temperature of the heated layer, $\ln \Lambda \approx 20$ is a Coulomb logarithm, $x^2 = \beta^2/(2\theta)$, and ψ the error function

$$\psi = \frac{2}{\sqrt{\pi}} \int_0^x e^{-x^2} \mathrm{dx}. \tag{2}$$

This formula holds for nonrelativistic electron temperatures; the relativistic generalization has been given by Stepney (1983) and Stepney and Guilbert (1983). At low electron temperature $kT < m_\mathrm{e}/m_\mathrm{p}\, kT_\mathrm{p}$, x is small and the factor involving the error function can be expanded. This yields

$$\begin{aligned} \tau_\mathrm{s} &\approx \tfrac{m_\mathrm{p}}{m_\mathrm{e} \ln \Lambda} \beta \theta^{3/2}, \\ &\approx \left(\tfrac{kT_\mathrm{p}}{50\mathrm{MeV}}\right)^{1/2} \left(\tfrac{kT}{50\mathrm{keV}}\right)^{3/2}. \end{aligned} \tag{3}$$

2 COMPTONIZATION IN A LAYER HEATED BY PROTONS

2.1 Estimating the depth of the comptonizing layer

Heating by protons yields a Comptonizing layer of thickness equal to the stopping depth $\tau_{\rm s}$. This depth is a function of the electron temperature in the layer, by (1). The electron temperature on the other hand is determined by the heating and cooling processes, so that a consistent calculation of heating and cooling will yield both the electron temperature and the optical depth of the layer. With a simple estimate, we can now show that this will yield $\tau_{\rm s}$ and T in roughly the right range.

The cooling process in the layer is the inverse Compton process, i.e. the energy loss electrons experience as they scatter the soft photons from the cool disk below. We assume that these soft photons are all (or mostly) produced by thermalization of comptonized photons from the heated layer, as in the model of Haardt and Maraschi (1991, hereafter HM). Since approximately half the Comptonized photons escape and the other half illuminates the thermalizing layer, the energy flux in the soft photons at the base of the layer must be about the same as that in the escaping comptonized photons. Such a balance is possible only if the Comptonization is sufficiently strong. In terms of the Compton y-parameter $y \approx 4\theta\tau_{\rm s}$, it requires that $y \approx 1$. If the electron temperature is too low, the energy transfer from the electrons to the soft photons is too low and the layer heats up until $y \approx 1$, and vice versa. Since the y-parameter also determines the slope of the X-ray spectrum, the model yields a fixed spectral slope, which is in the range of the observed values. This is the reason for the success of the HM model. In HM the depth of the layer is a free parameter; in the present model, it is fixed by requiring the stopping depth $\tau_{\rm s}$ to be consistent with the resulting electron temperature T. A simple estimate is obtained by setting $y = 1$, or $\theta = 1/(4\tau_{\rm s})$, and inserting into (1). This yields, assuming again that the protons are near their virial temperature:

$$\tau_{\rm s}^{5/2} = \frac{m_{\rm p}}{8\sqrt{6}\ln\Lambda m_{\rm e}}\left(\frac{r_{\rm g}}{r}\right)^{1/2}, \tag{4}$$

or

$$\tau_{\rm s} \approx 1.3\left(\frac{7r_{\rm g}}{r}\right)^{1/5}, \tag{5}$$

and

$$kT \approx m_{\rm e}c^2/(4\tau_{\rm s}) \approx 60\left(\frac{r}{7r_{\rm g}}\right)^{1/5} \quad \text{keV}. \tag{6}$$

We conclude that proton illumination yields optical depths and temperatures in the right range, with only a weak dependence on the assumed distance from the black hole. Obviously, the estimate is rather crude, and more detailed calculations of the energy transfer from the protons to the electrons, as well as the Comptonization process are needed to test the model. In the following we make a first step in this direction, by means of a radiative transfer calculation.

2.2 Model problem

The aim of the calculation reported below is to compute the electron temperature as a function of depth, together with the emergent photon spectrum from a layer heated by protons who deposit their energy according to (1).

We approximate the heating rate to be distributed uniformly over a layer with depth $\tau_{\rm s}$, where $\tau_{\rm s}$ is computed from the average temperature in this layer using eq. (1). I reality, the heating is somewhat non-uniform because of the dependence of the energy loss rate of the proton on both the proton velocity and the electron temperature. The approximation of a uniform enery input in the layer is deemed sufficient for the present exploratory purpose.

In this layer, we solve the radiative transfer equation iteratively together with the electron temperature $T(\tau)$, and the layer stopping depth $\tau_{\rm s}$ such that the heating is in balance with cooling by comptonization of the soft photons.

The assumptions and simplifications that go into the model are as follows. In the heated layer, the only photon process considered is electron scattering (Comptonization). Below this, we assume that electron scattering continues to be the dominant process down to some depth $\tau_{\rm b}$. At $\tau_{\rm b}$, the downward photons are assumed to be absorbed and their energy reradiated upward as a black body spectrum. Thus, the gradual thermalization with depth by free-free processes is simplified by a step at depth $\tau_{\rm b}$. The value of $\tau_{\rm s}$ is determined from the proton velocity $\beta = v/c$ and the mean electron temperature in the heated layer. Obvious improvements are possible on these simplifications, by explicitly taking into account photon production/destruction process, an by a more accurate treatment of the energy loss of the ions as they penetrate into the disk.

The radiative transport part of the problem is simplified by reducing the angular dependences to a one-stream model: only vertically upward and downward moving photons are considered, and the electron distribution is similarly reduced from a 3-D to a one-dimensional Maxwellian distribution. For the scattering cross section and the electron Maxwellian the relativistic expressions are used. The one-stream simplification is made for programming convenience only: leaving out the full angular dependences leads to very simple expressions. Discretization by a reasonable number of angles would still yield a very modest problem in terms of computing time, and is an obvious next step to improve the calculations.

2.3 Numerical method

The transport equation is of the form (e.g. Rybicki and Lightman, 1976):

$$\frac{\mathrm{dn}}{\mathrm{dz}} = \int \mathrm{d}^3\mathbf{p} \int \mathrm{d}^2\mathbf{\Omega} \tfrac{\mathrm{d}\sigma}{\mathrm{d}\mathbf{\Omega}} [\mathbf{f}_\mathrm{e}(\mathbf{p}')\mathbf{n}(\omega')\times (1+n(\omega)) - \mathbf{f}_\mathrm{e}(\mathbf{p})\mathbf{n}(\omega)(\mathbf{1}+\mathbf{n}(\omega'))], \tag{7}$$

where $n(\omega)$ is the photon occupation number, f_e the electron momentum distribution, $\mathbf{p}$ (respectively $\mathbf{p}'$) the electron momentum, ω (ω') the photon momentum vectors before (after) scattering, and $\mathbf{\Omega}$ the scattering angle. The dependences of $\mathbf{p}'$ and ω' on $(\mathbf{p}, \omega, \mathbf{\Omega})$ follow from the collision kinematics.

This equation is discretized in N_ω logarithmically spaced photon-energy bins. As photon energy scale we use $\omega = h\nu/m_\mathrm{e}c^2$. As depth scale we use the Thomson optical depth τ_T. The number of depth points is fixed, but the grid is stretched such that at each iteration the stopping depth τ_s is located at the same grid point. Half the grid points are used for the heated layer ($0<\tau<\tau_\mathrm{s}$), the other half for the scattering layer below.

If n_i^+ and n_i^- are the occupation numbers of the upward and downward moving photons in bin i, the result is the set of $2N_\omega$ equations

$$\mp\frac{\mathrm{d}}{\mathrm{d}\tau} n_i^\pm = n_i^\pm \sum_j [(B_{ji} - B_{ij})n_j^\mp - B_{ij}] + \sum_j B_{ji} n_j^\mp, \tag{8}$$

where $B_{ij}(T)$ is the scattering cross section integrated over the appropriate frequency and electron momenta, for scattering from bin i into bin j, in units of the Thomson cross section σ_T. At the top boundary there are only outgoing photons:

$$n_i^- = 0 \qquad (\tau = 0), \tag{9}$$

while at the lower boundary the downward photons are thermalized into a blackbody spectrum of upward photons:

$$n_i^+ = n_\mathrm{BB}(\omega_i) = 1/[1-\exp(\omega/\theta_\mathrm{b})] \qquad (\tau = \tau_\mathrm{b}). \tag{10}$$

Here n_BB is the black-body occupation number at temperature θ_b. This temperature follows from the condition that the net energy flux at depth τ_b vanishes:

$$\int \omega^3[n^-(\tau_{\rm b}) - n_{\rm BB}(\omega,\theta_{\rm b})] = 0. \tag{11}$$

This is a result of our assumption that the internal energy production rate in the cool disk can be neglected in comparison with the incident proton energy flux (generalizations are easily made by adding a term corresponding to the internal disspation in the disk).

The condition of energy balance between the assumed heating rate $h(\tau)$ and the Compton cooling by the soft photons is

$$\frac{\rm dF}{\rm dz} = h, \tag{12}$$

where the energy flux F is given by

$$F = \sum_i \omega^3(n_i^+ - n_i^-). \tag{13}$$

The equations (8) are discretized in optical depth by centered first order differences. The resulting set of nonlinear algebraic equations is solved by an iterative process. It turned out that full simultaneous linearization of the transfer equation, the boundary conditions and the energy equation had very poor convergence properties. Instead, an iteration was done in which only the transfer equation and the upper boundary condition were linearized, while the lower bc and the energy equation were dealt with by a modified succesive-substitution process after each iteration of the transfer equation. Convergence, however, was still problematic for large optical depths and for cases where the assumed photon energy range extended too far beyond the cutoff energy. For the cases reported here, where the optical depth is not too large, on the order of 30–100 iterations were required for an accuracy of 10^{-3} in luminosity.

3 RESULTS

The parameters of the problem are the total energy flux F (per unit surface area of the disk), the velocity of the incident protons, and the optical depth $\tau_{\rm b}$ of the thermalizing lower boundary. Assuming the protons to be thermal and virialized, their mean vertical velocity component β_z as a function of the distance r from the compact object is

$$\beta_z = \left(\frac{r_{\rm g}}{6r}\right)^{1/2} \tag{14}$$

The electron temperature as a function of depth is shown in figure 1 for a few values of F for $r/r_{\rm g} = 7$. These fluxes correspond to effective temperatures $\theta_{\rm eff} = (F/\sigma)^{1/4}/(m_{\rm e}c^2)$ of $3\,10^{-4}$, 10^{-3} and $3\,10^{-3}$, r92espectively. The emergent spectrum for these energy fluxes is shown in figure 4, for $r/r_{\rm g} = 7$. The dependence of the spectrum on $r/r_{\rm g}$ at a fixed F

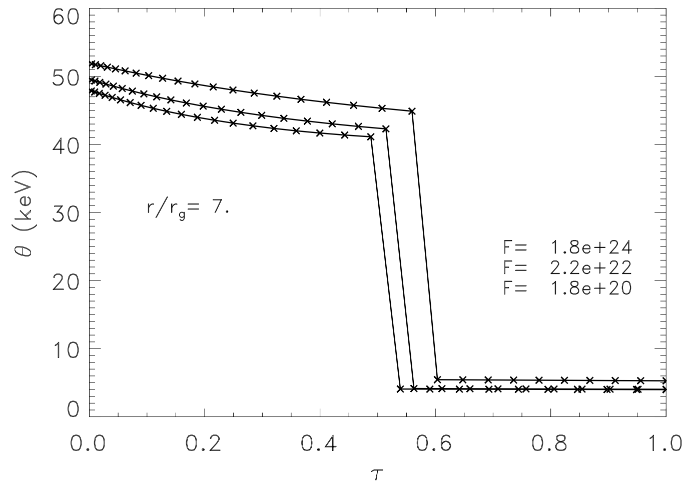

Figure 1. Temperature as a function of optical depth in an electron scattering layer heated by virialized protons incident with energy flux F (erg/cm^2/s), for $r/r_{\rm g} = 7$. Lower boundary (thermalizing layer) is at $\tau = 1$. Crosses mark grid points

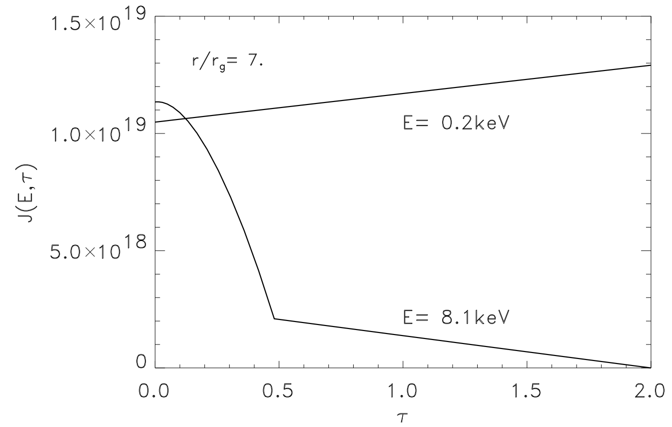

Figure 2. Variation with depth of the mean intensity $J(E,\tau)$ of soft (0.2 keV) and hard (8 keV) photons. Energy flux $F = 1.8\,10^{20}$, $r/r_g = 7$.

is shown in figure 3. The penetration depths are shown in figure 5. All of these cases were computed for $\tau_{\rm b} = 1$.

The jump in temperature at $\tau = \tau_{\rm s}$ is a consequence of the fact that the heating rate jumps at $\tau_{\rm s}$. The photon field is continuous at $\tau_{\rm s}$ as shown in figure 2, as expected because of their diffusion through the scattering layer. In order to balance the energy input, the electrons have to loose enough energy by scattering soft photons. This requires them to be hot where the energy input rate is large, and causes the temperature to follow the heating rate.

The similarity of the spectra, apart from shifts in amplitude and photon energy, is remarkable. The temperature of the heated layer increases only very weakly with increasing

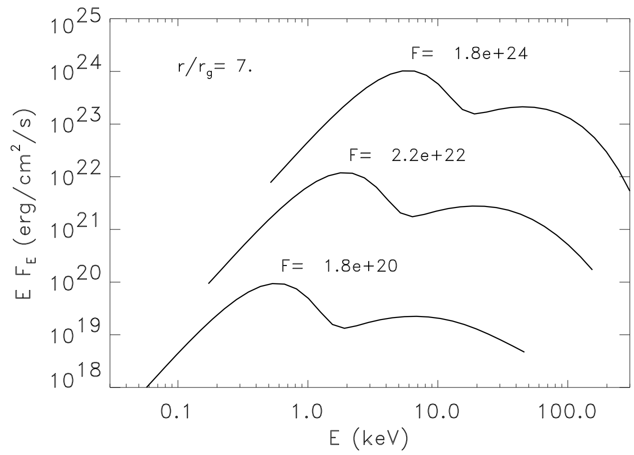

Figure 3. Emergent spectrum of a layer heated by protons at $r = 7r_{\rm g}$ for three values of the energy flux

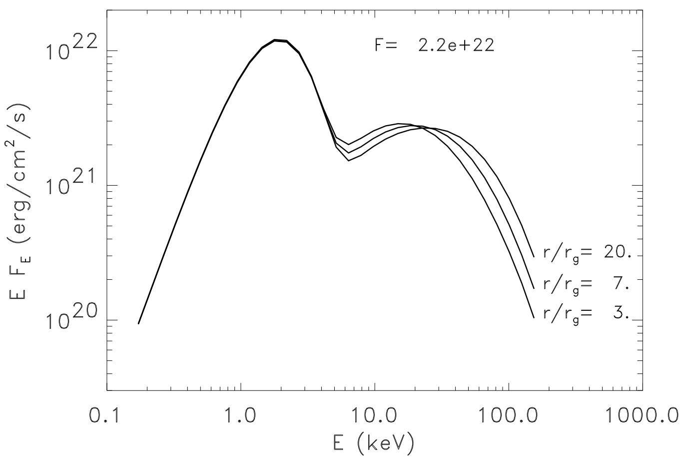

Figure 4. Dependence of the spectrum on distance from the compact object, for a fixed energy flux per cm^2 of disk surface

energy flux. The cutoff energy decreases somewhat with distance from the hole, but again this is dependence is rather weak. On account of modest optical depth of the layer, the spectrum shows a prominent contribution from unscattered soft photons from the reprocessing depth $\tau_{\rm b}$. This peak is smeared out somewhat when the spectra are convolved over distance from the hole, at a given accretion rate. This is shown in figure 6.

The final parameter of the problem is the depth of the thermalizing lower boundary, $\tau_{\rm b}$. As this depth is increased, the thickness of the passive scattering layer between the thermalizing depth and the heated surface layer increases. The fraction of the comptonized photons that is thermalized decreases while the fraction which is reflected by the passive layer increases. This has two effects on the spectrum. First, the strength of the 'blue bump' (consisting of uncomptonized thermal photons from the lower boundary) decreases, secondly

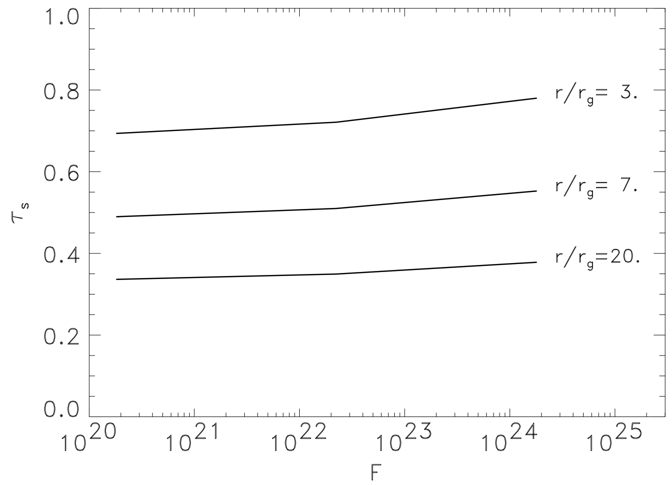

Figure 5. Thomson depth of the heated layer as a function of energy flux and distance from the compact object

the spectrum becomes flatter at the high energy end, because there are fewer soft photons to be upscattered by the hot electrons in the heated layer. This is shown in figure 7. With increasing depth of the thermalizing boundary, the spectrum becomes both smoother and flatter. This can also be estimated quantitatively by an argument along the lines of the HM model. Instead of assuming that all the downward photons are thermalized immediately below the Comptonizing layer, we take into account the scattering albedo $a = 1 - \epsilon$ of the passive scattering layer between the comptonizing layer and the thermalization depth. This albedo depends on the depth of the passive layer, and also on the angular dependences ignored here, but may easily be as high as 0.9 for for optical depths larger than a few. Energy balance then requires that the Comptonization process increases the energy flux of the soft photons by a factor $f = (1 - \epsilon)/\epsilon$. Thus the Compton-$y$ parameter must be of the order $\ln f \approx 2$. This value explains approximately the slope of the spectra seen in figure 7.

4 DISCUSSION

With an admittedly somewhat simplified radiative transfer model I have shown that heating of a cool disk by protons from an ion supported advection torus produces X-ray spectra that are very reminiscent of the hard spectra of accreting black holes. Like the Maraschi and Haardt model (and for the same reason), it yields approximately the right spectral slope, but in addition, it also predicts approximately the right optical depth of the comptonizing layer and the cutoff energy of the spectrum.

The temperature of the heated layer is insensitive to the energy flux, and stays around

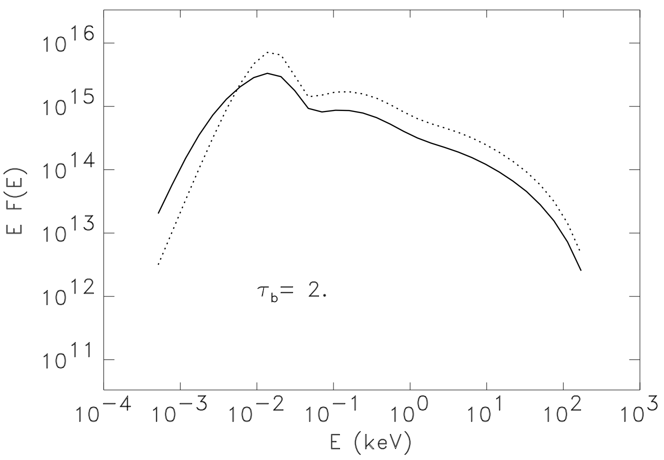

Figure 6. Spectrum integrated over the disk (solid), compared with local spectrum at $r = 7r_g$ (broken).

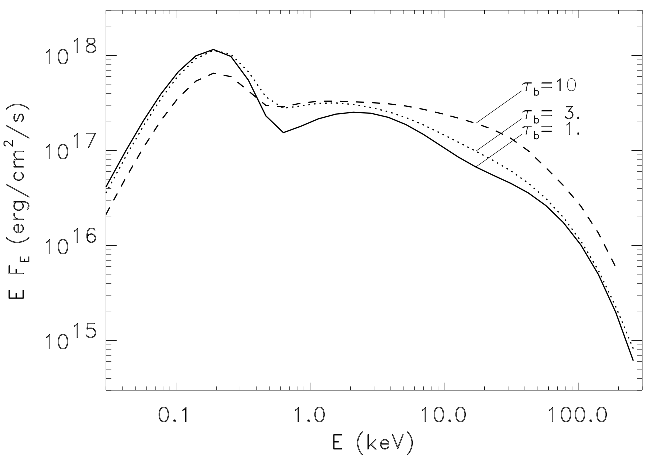

Figure 7. Spectrum as a function of the depth of the thermalizing lower boundary τ_b, for an energy flux of $2.2\,10^{18}$ at $r = 7r_g$

40–60 keV. Instead of getting hotter at high energy flux, the incident energy is spent in upscattering a larger number of soft photons. As in the Maraschi and Haardt model, the reason for this lies in the energy balance condition. In order for the incident energy to be radiated as Comptonized flux, the Compton y-parameter has to be of of the order unity, and the temperature of the layer adjusts accordingly. What is new in the results presented here is that the optical depth of the Comptonizing layer also comes out naturally in the right range, due to the physics of Coulomb interaction between the virialized protons with the electrons in the cool disk. This process also is fairly insensitive to the proton temperature (within the relevant range), so that the emergent spectrum is only a weak function of distance from the accreting object (figure 4).

The agreement of the result with one of the most puzzling features of the hard X-ray spectra of accreting compact objects, viz. the uniformity of the spectral shape, makes it likely that ion illumination plays a major role in the physics of these objects.

4.1 Lithium

An independent observational indication for ion illumination is the observation of high Li abundances in the secondary stars of X-ray binaries (Martín et al. 1992, 1994a, 1995). The energy of virialized protons hitting the cool disk at $r = 7r_{\rm g}$ (where the gravitational energy release peaks for accretion onto a hole) is around 50 MeV, just in the range where Li production by spallation of CNO elements becomes efficient. Since the observed secondaries are of spectral types known to destroy Li on a rather short time scale, a significant continual source of Li is needed. As shown by Martín et al. (1994) the energetics of the accretion process is enough to explain the observed amount of Li on the secondary, if a fraction 10^{-3} of the Li produced in the disk finds its way to the secondary (in the form of a disk wind, for example). Another consequence of ion illumination would be Li and Be production by He nuclei from the virialized flow reacting with He nuclei in the disk. These reactions peak around 50 Mev/nucleon, and are accompanied with emission of $\gamma-$lines at 431 and 478 keV. It is possible that the $\gamma-$lines observed sometimes around this energy (Gilfanov et al. 1991, Sunyaev et al. 1992) are another signature of ion illumination (Martín et al. 1994a,b).

ACKNOWLEDGMENTS

This work was done in the context of Human Capital and Mobility network 'Accretion onto compact objects', CHRX-CT93-0329.

REFERENCES

Collin S., 1997, in Accretion disks: new aspects, eds. E. Meyer-Hofmeister and H.C. Spruit, Lecture Notes in Physics V487, Springer, Heidelberg, in press

Fabian A.C., Rees M.J., 1995, MNRAS 277, 55

Gilfanov M., et al., 1991, SvA Lett. 17(12), 1059

Haardt F., Maraschi L., 1991, ApJ 413, 507

Haardt F., 1997, Mem Soc. Astron. It., in press

Maraschi L., Haardt F., 1996, in Accretion Phenomena and Related Outflows, D. Wickramasinghe, L. Ferrario & G.Bicknell (eds.), IAU Conference, in press

Liang E. P. T., 1979, ApJ 218, 247

Martín E., Rebolo R., Casares J. Charles P.A., 1992, Nature 358, 129

Martín E., Rebolo R., Casares J. Charles P.A., 1994a, ApJ 435, 791

Martín E., Spruit H.C., van Paradijs J., 1994b, A&A 291, L43

Martín E., Casares J. Charles P.A., Rebolo R., 1995, A&A 303, 785

Meyer F. Meyer-Hofmeister E., 1994, A&A 288, 175

Mitsuda K., Inoue H., Nakamura N., Tanaka Y., 1989, Publ. Astron. Soc. Japan 41, 97

Narayan R., Yi I., 1994, ApJ 428, L13

Narayan R., Yi I., 1995, ApJ 452, 710

Narayan R., McClintock J.E., Yi I., 1996, ApJ 457, 821

Rees M.J., Begelman M.C., Blandford R.D., Phinney E.S., 1982, Nature 295, 17

Rybicki G.B., Lightman A., 1979, Radiative Processes in Astrophysics, New York: Wiley

Ryter C., Reeves H., Gradsztajn E., Audouze J., 1970, A&A 8, 389

Shakura, N.I. and Sunyaev, R.A., 1973, A&A 24, 337

Shapiro S.L., Lightman A.P., Eardley D.M., 1976, ApJ 204, 187

Stepney S., 1983, MNRAS 202, 467

Stepney S., and Guilbert P.W., 1983 MNRAS 204, 1269

Sunyaev R.A., et al., 1992, ApJ 389, L75

Tanaka, Y., 1997 in Accretion disks: new aspects, eds. E. Meyer-Hofmeister and H.C. Spruit, Lecture Notes in Physics V487, Springer, Heidelberg, p1, in press

This paper has been produced using the Royal Astronomical Society/Blackwell Science LaTeX style file.