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Exercise sheet 1

Exercise 1 - 1

Happy Birthday!

A number of persons, k, meet. Assume that the probability of a person to have his/her birthday is the same for every day of the year. Assume further that the number of days per year is always 365.

a) How high is the probability that the birthday of at least q of these people is on the first of January (2 Points)?

b) How high is the probability of at least two persons in the room having their birthday on the same day (1 Point)?

c) For which k is this probability larger than 50% (1 Point)?

Exercise 1 - 2

Weather in Markovia

You are traveling to the beautiful country of Markovia. Your travel guide tells you that the weather w_i in Markovia on a particular day i is sunny, $w_i = \mathrm{s}$, for 80% of all days or it is cloudy, $w_i = \mathrm{c}$, for 20% of all days. There are no other weather conditions in Markovia and the weather changes only during nights. The probability for a weather change is 10% if it is sunny,

$$P(w_{i+1} = \mathrm{c}|w_i = \mathrm{s}) = 0.1, \tag{1}$$

and 40% if it is cloudy,

$$P(w_{i+1} = \mathrm{s}|w_i = \mathrm{c}) = 0.4, \tag{2}$$

irrespective of what it has been on earlier days, $P(w_{i+1}|w_i,\, w_{i-1},\, w_{i-2},\, \ldots) = P(w_{i+1}|w_i)$.

a) You arrive on a sunny day, $w_i = \mathrm{s}$, in Markovia. Calculate the probability that it was cloudy there the day before, $P(w_{i-1} = \mathrm{c}|w_i = \mathrm{s})$ Hint: Use Bayes-Theorem. (2 points).

b) What is the total probability for a weather change $P(w_{i+1} \neq w_i)$ in Markovia during an arbitrary night (1 point)?

c) The Markovian weather forecast for some day i predicts a sunshine probability of

$$p_i = P(w_i = \mathrm{s}\,|\mathrm{forecast}). \tag{3}$$

What is the sunshine probability there for the following day,

$$p_{i+1} = P(w_{i+1} = \mathrm{s}\,|\mathrm{forecast})? \tag{4}$$

(1 point)

d) Verify or correct the travel guide's statement on the frequency of 80% sunny and 20% cloudy days in Markovia (1 point).

<u>Hint</u>: Your result of question **c)** might be useful for this.

e) Implement an algorithm to simulate the weather in Markovia and verify your results numerically (optional).

Exercise 1 - 3

Weak Syllogism

Given three statements A, B, and C, label the statement "$A \Rightarrow BC$", i.e. "if A is true, B and C are true" with I. Show

a) $P(A|BI) \geq P(A|I)$ (1 point).

b) $P(A|B+C, I) \geq P(A|I)$.
Note that a comma binds the arguments of a probability function as a logical "and", i.e., $P(A|B+C, I) = P(A|(B+C)I)$. (1 point)

Exercise 1 - 4

The end is near!
In this exercise, we calculate the probability that the apocalypse will happen in your lifetime. Let n denote the total amount of humans that will ever live.

a) Take one random human. Numbering all humans from first to last by the time they were born, what is the probability of him being the m-th human (with $m \in \mathbb{N}$) given that there will be a total of n humans ever to live? (1 point)

(1 point)

b) Calculate the posterior probability for n given that the random human of a) is the m-th human. For this, take a uniform prior

$$P(n) = \begin{cases} \frac{1}{x} \text{ for } n < x \\ 0 \text{ otherwise.} \end{cases} \tag{5}$$

Hint: You can approximate sums as integrals at your convenience. (4 points)

c) Discuss why a prior is necessary for this problem. (1 point)

d) You are approximately the $1,12 \cdot 10^{10}$-th human (assuming you were born in 1994). If you grow to be 90 years old, then at the time of your death an approximate total of $2,0 \cdot 10^{10}$ humans will have been born according to today's extrapolations. What is the probability of the apocalypse happening in your lifetime, i.e. $P(n < 2,0 \cdot 10^{10} | m = 1,12 \cdot 10^{10})$ assuming $x = 10^{13}$ for the prior? (2 points)

e) Discuss how the choice of prior affected the above outcome. What happens for $x = 10^{20}$? How can one choose an appropriate prior? (2 points)

Exercise 1 - 5

Brothers
Assume here, for the sake of simplicity, that children are either boys or girls with equal probability and no prior correlations between genders or birthdays of different childs exist. A person has two children. What is the probability that these are brothers?

a) In case of **no further data**? (1 point)

b) In case **the first born is a boy**? (1 point)

c) In case **one of them is a boy**? (2 points)

d) In case **one of them is a boy born on a Monday**? (3 points)

e) In case (d), but **weeks have n days**? (2 points)

f) Examine the cases $n = 1$ and $n = \infty$ and explain why some of the probabilities from (a)-(c) are reproduced! What happens in between these extremes? (1 point)

<u>Hint</u>: You may use the notation $b_i =$ "child i is a boy" and $\overline{b_i} =$ "child i is not a boy" with $i \in \{1, 2\}$ for stating the gender of the two children, $m_i =$ "child i is born on a Monday" , and $d_{(\mathrm{j})}$ with $j \in \{$a,... e$\}$ for the data given in bold in questions (a)-(e)!

This exercise sheet will be discussed during the exercises.
Group 01, Wednesday 18:00 - 20:00, Theresienstr. 37, A 449,
Group 02, Thursday, 10:00 - 12:00, Theresienstr. 37, A 249,

https://wwwmpa.mpa-garching.mpg.de/ ensslin/lectures/lectures.html