

**ROSAT and XMM-Newton counterparts  
using Nway--An accurate Bayesian  
algorithm to pair sources simultaneously  
between N catalogs.**

Mara Salvato, J. Buchner, T. Budavari, T. Dwelly, A. Merloni,  
M. Brusa, A. Rau, S. Fotopoulou, K. Nandra

<http://adsabs.harvard.edu/abs/2018MNRAS.473.4937S>

<https://github.com/JohannesBuchner/nway>

## Structure of the talk:

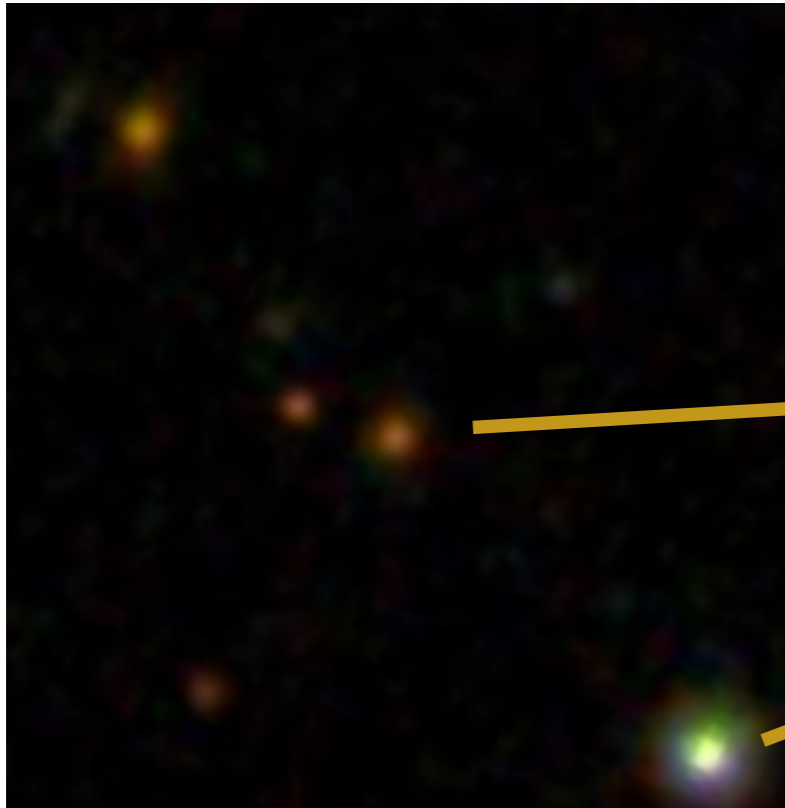
- Why multi-wavelength cross-matches are important ?
- Where is the problem, in particular for matches with X-ray ?
- The difficulties with the old solution ?
- New solution !
- Application and results
- Outlook to future applications

**In Observational Astrophysics, Source  
Characterisation is the starting point**

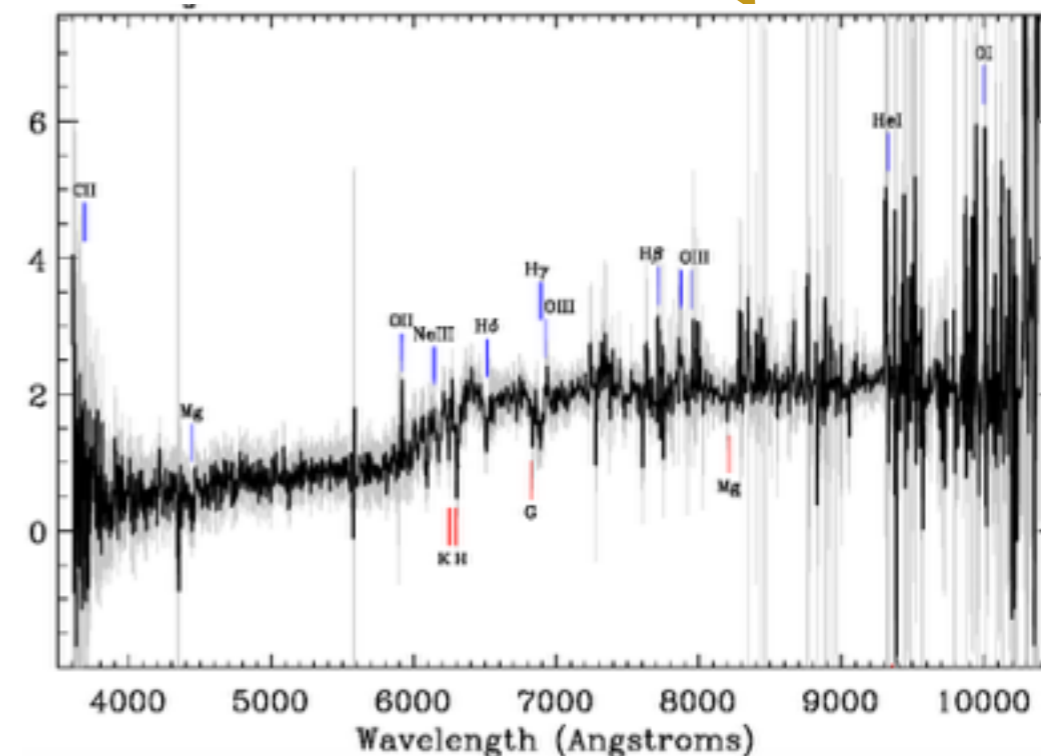
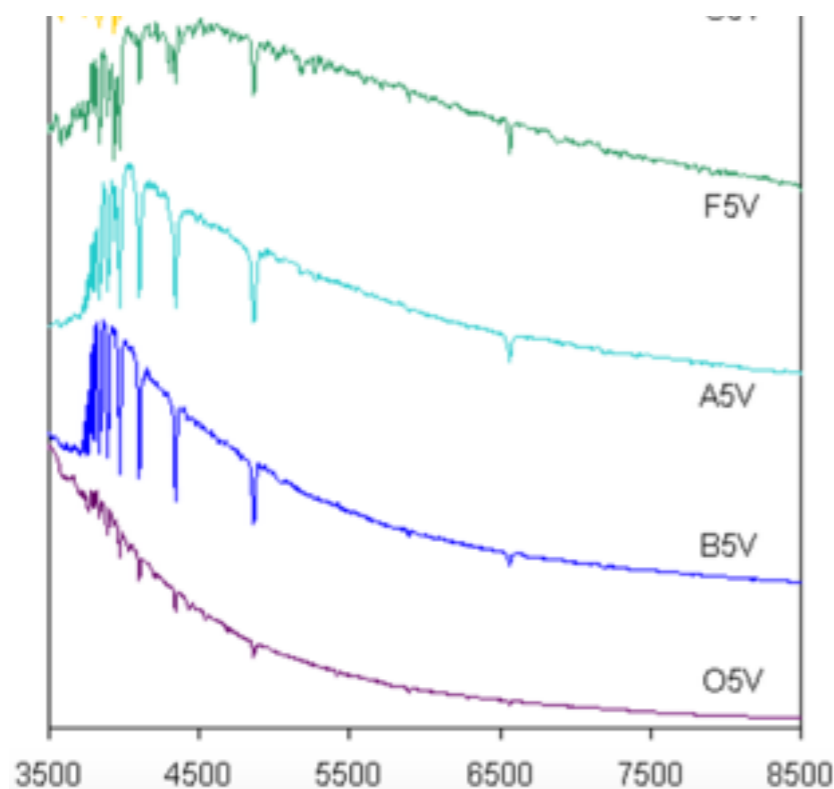
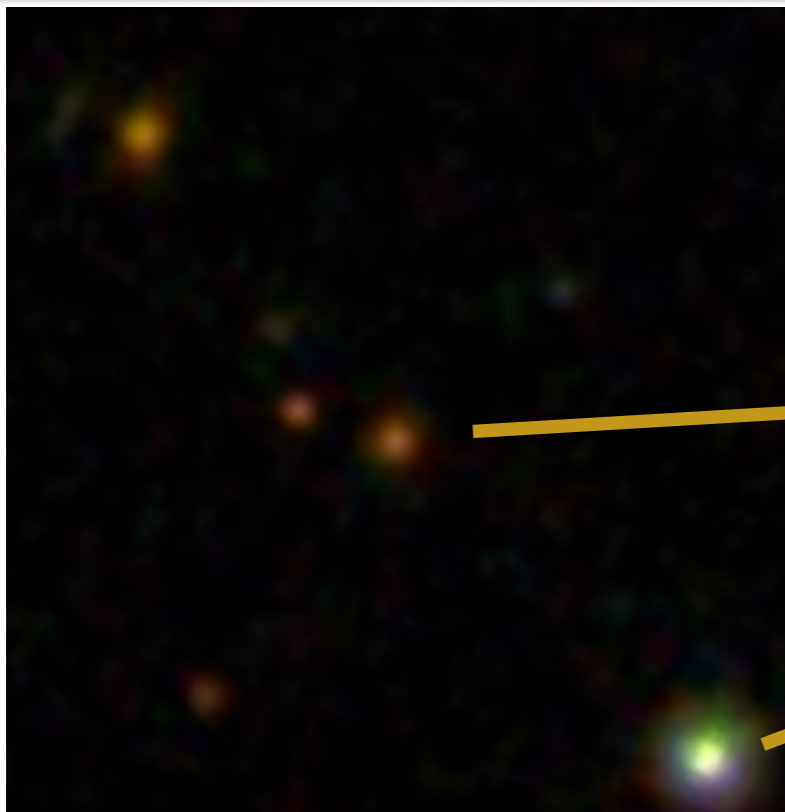
In Observational Astrophysics, Source Characterisation is the starting point



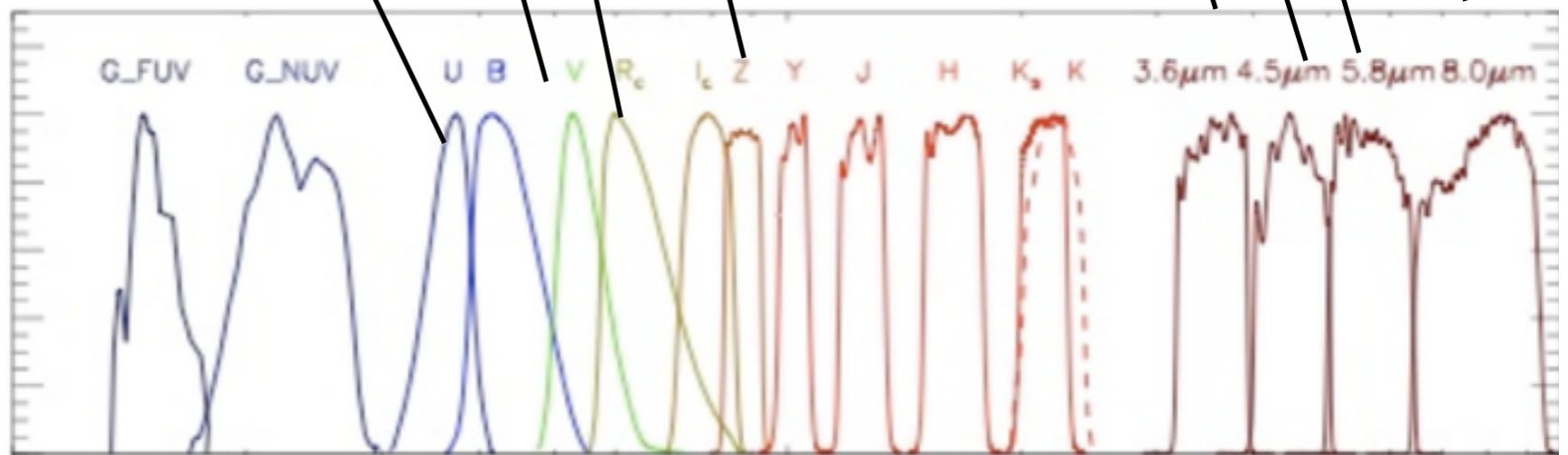
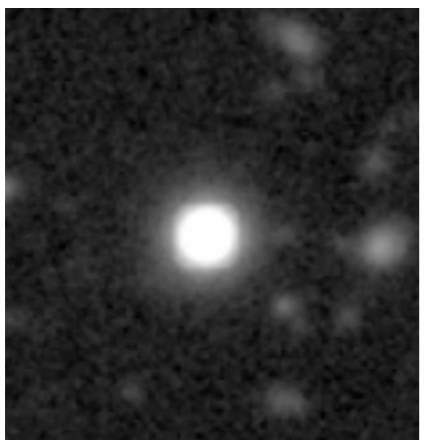
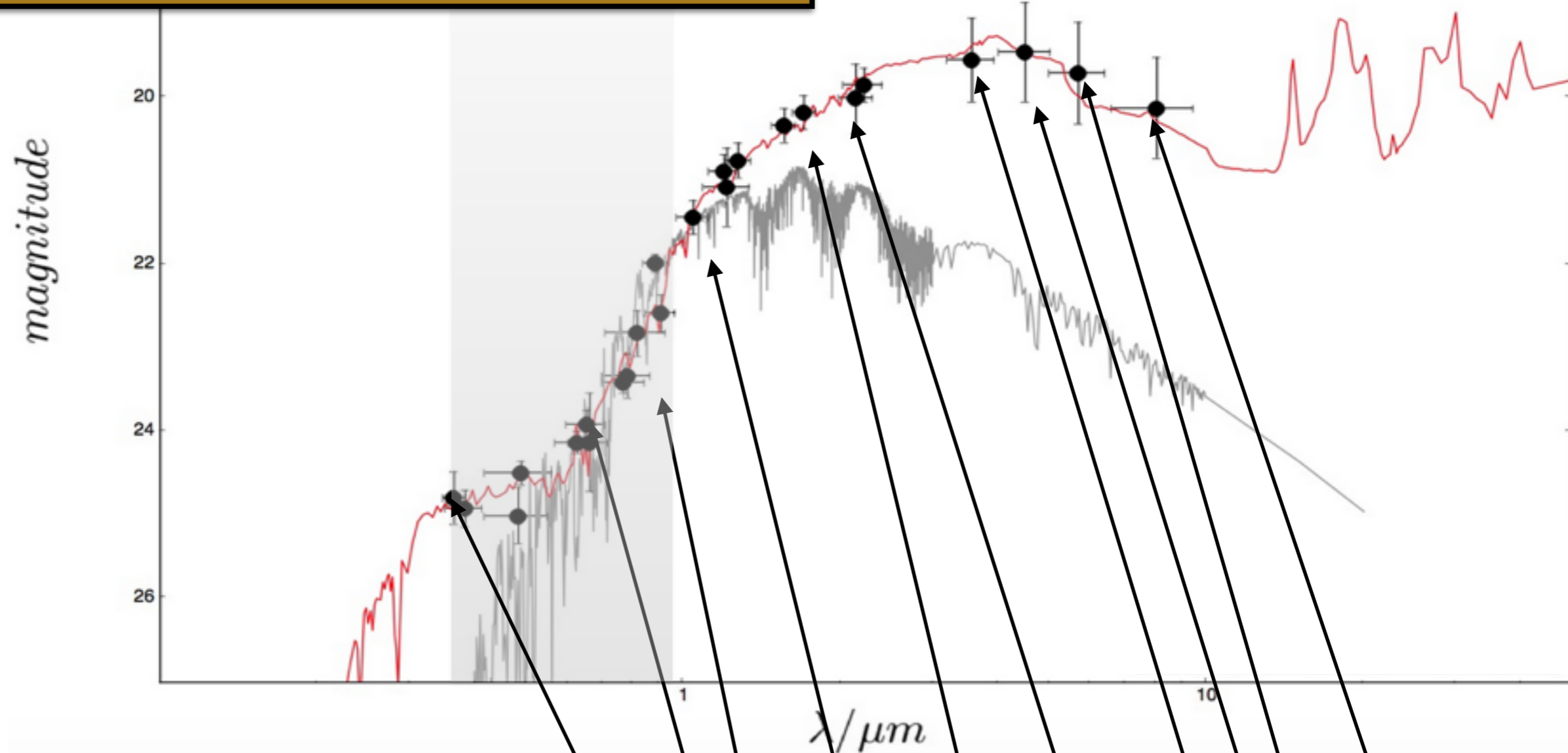
# In Observational Astrophysics, Source Characterisation is the starting point



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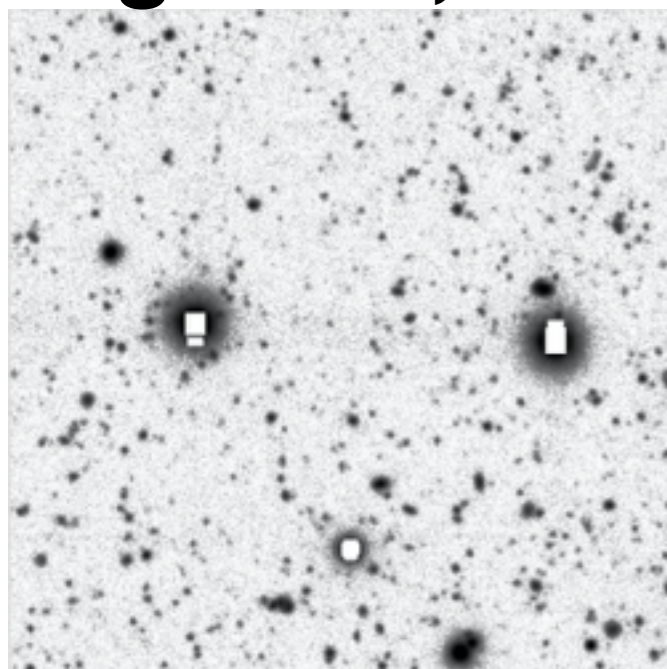


better characterisation using the entire multi- $\lambda$  info

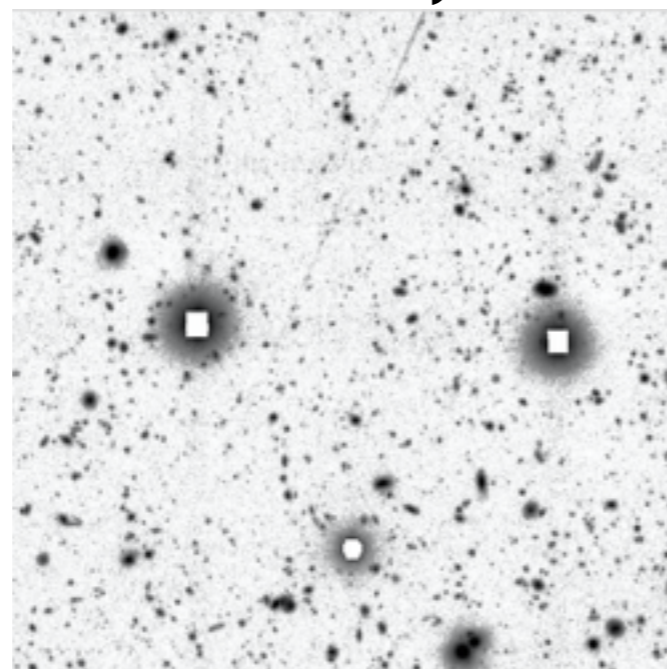


Pairing is easier at optical and near-infrared bands than between X-ray and other bands

**g band, 3'**



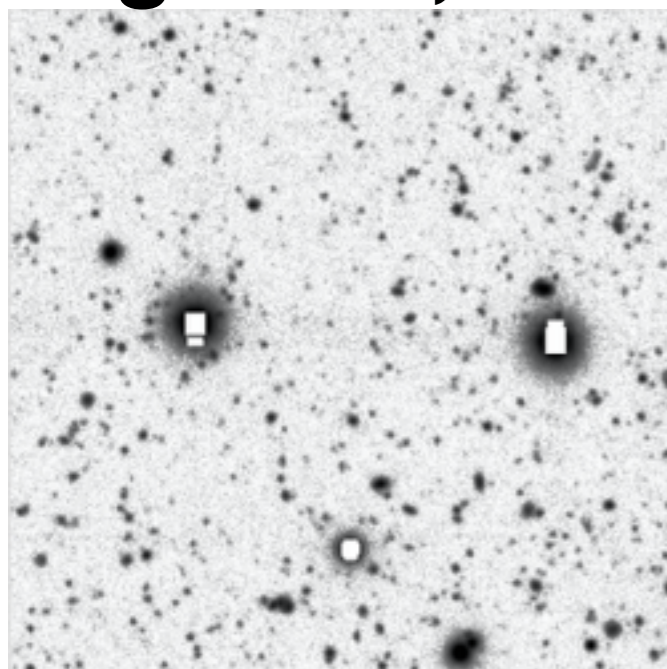
**z band, 3'**



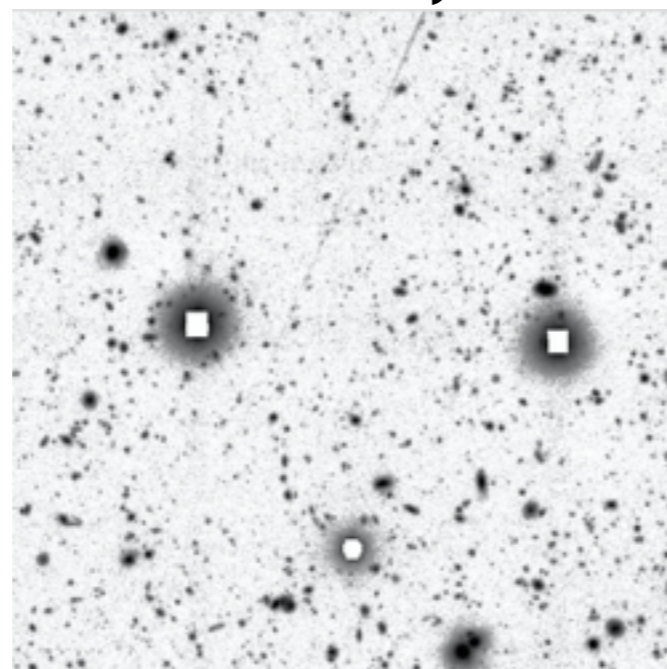


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**z band, 3'**

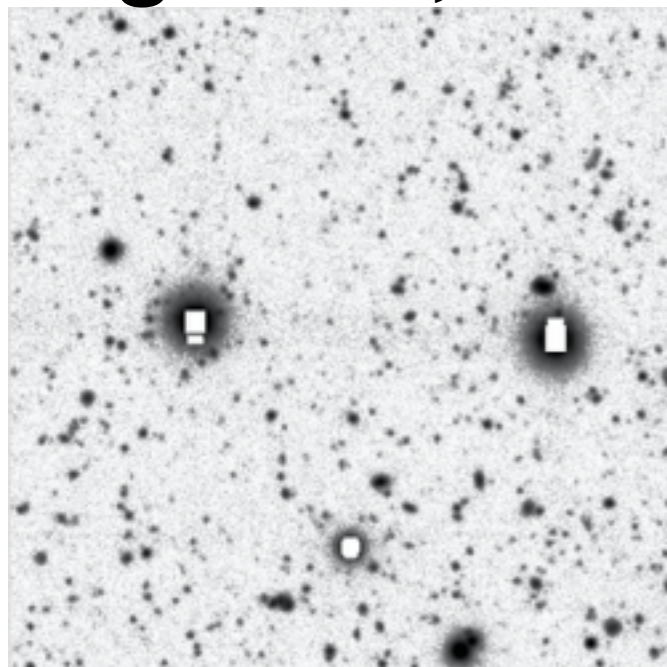


**XMM, 3'**

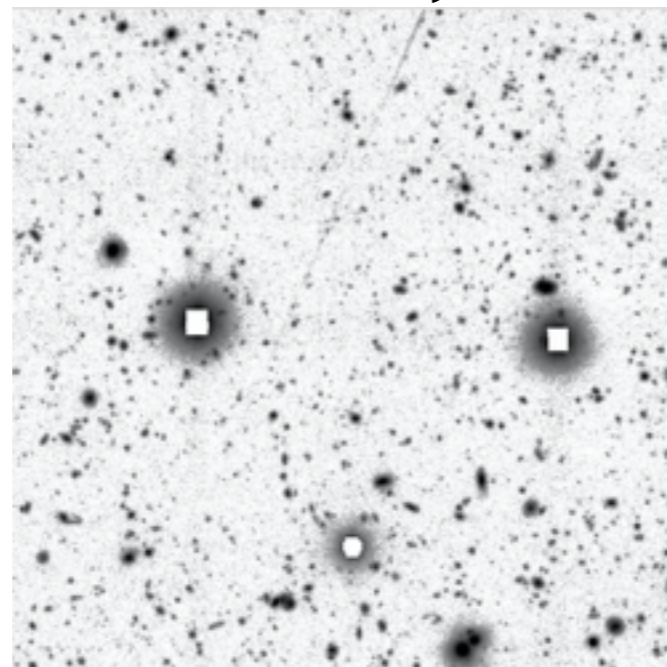


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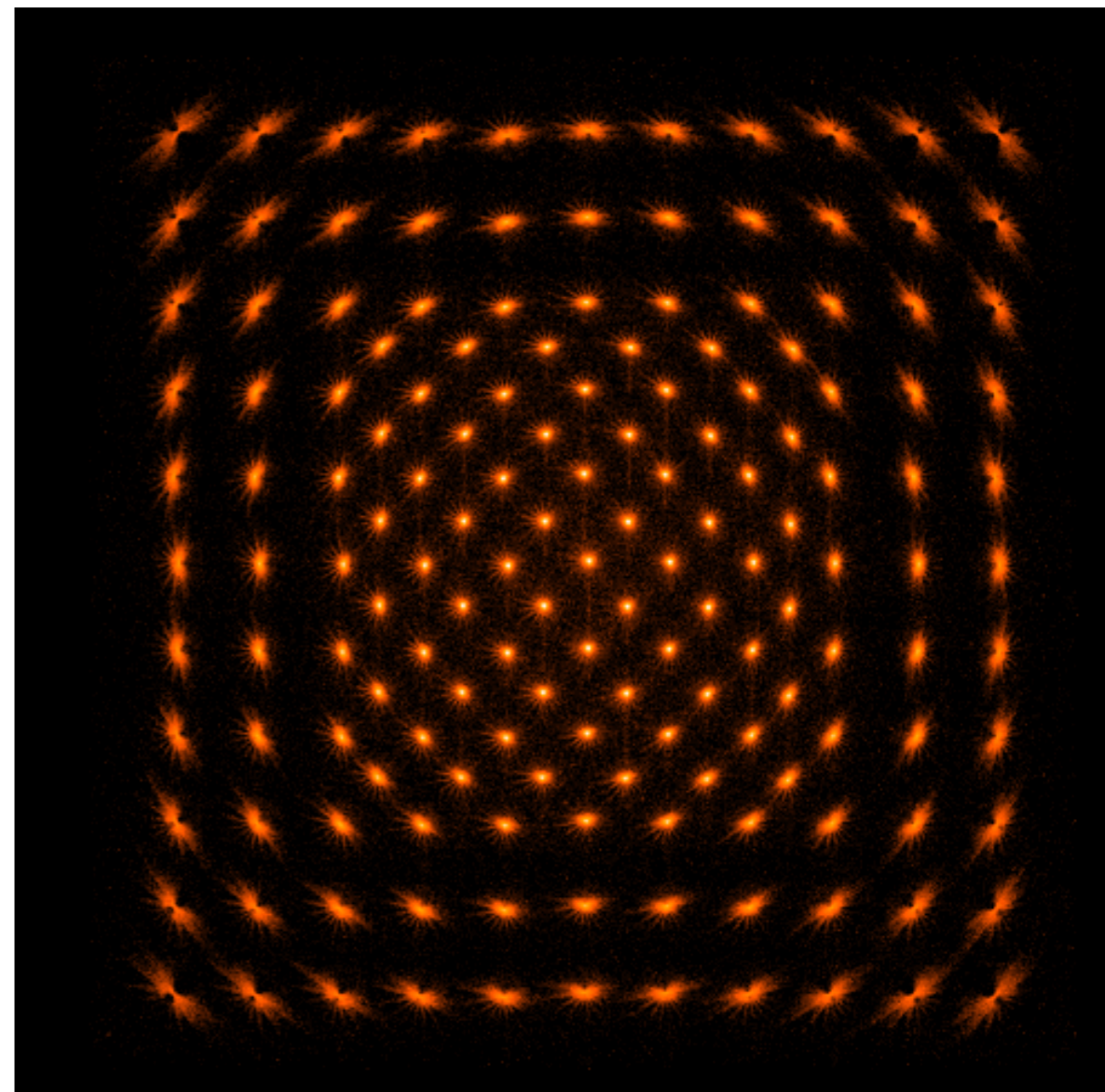
**g band, 3'**



**z band, 3'**



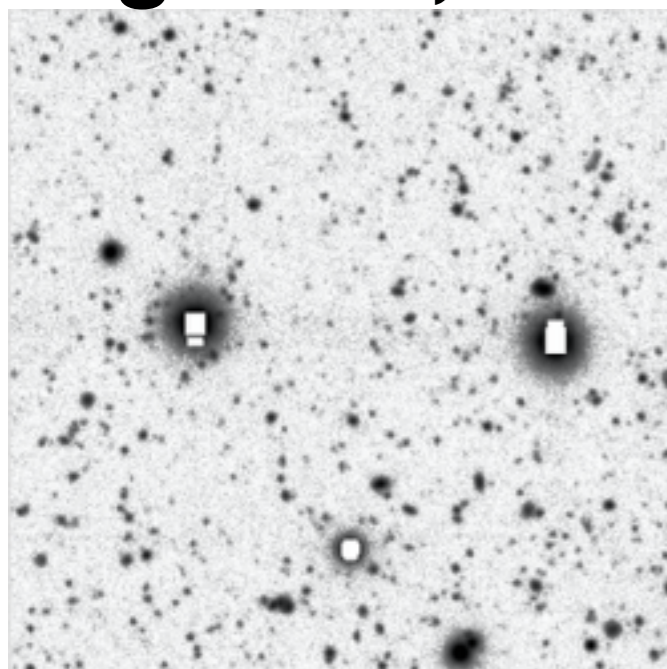
**XMM, 3'**



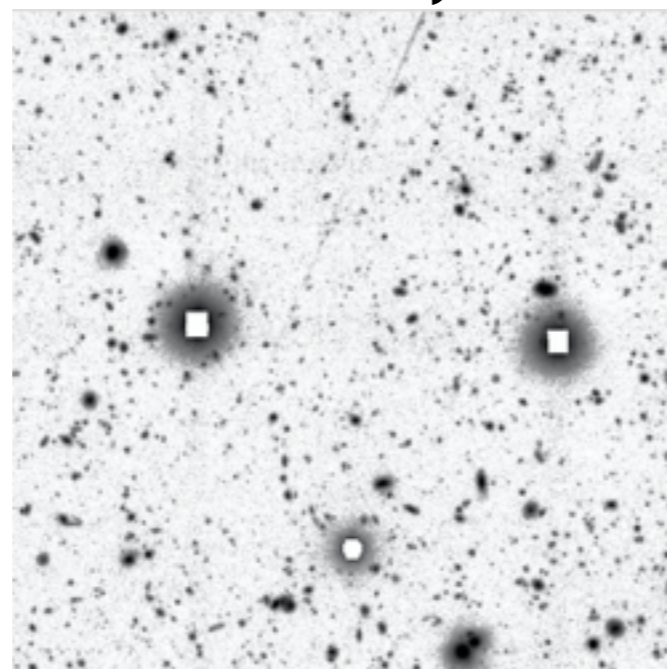
Huge difference in resolution between on/off-axis

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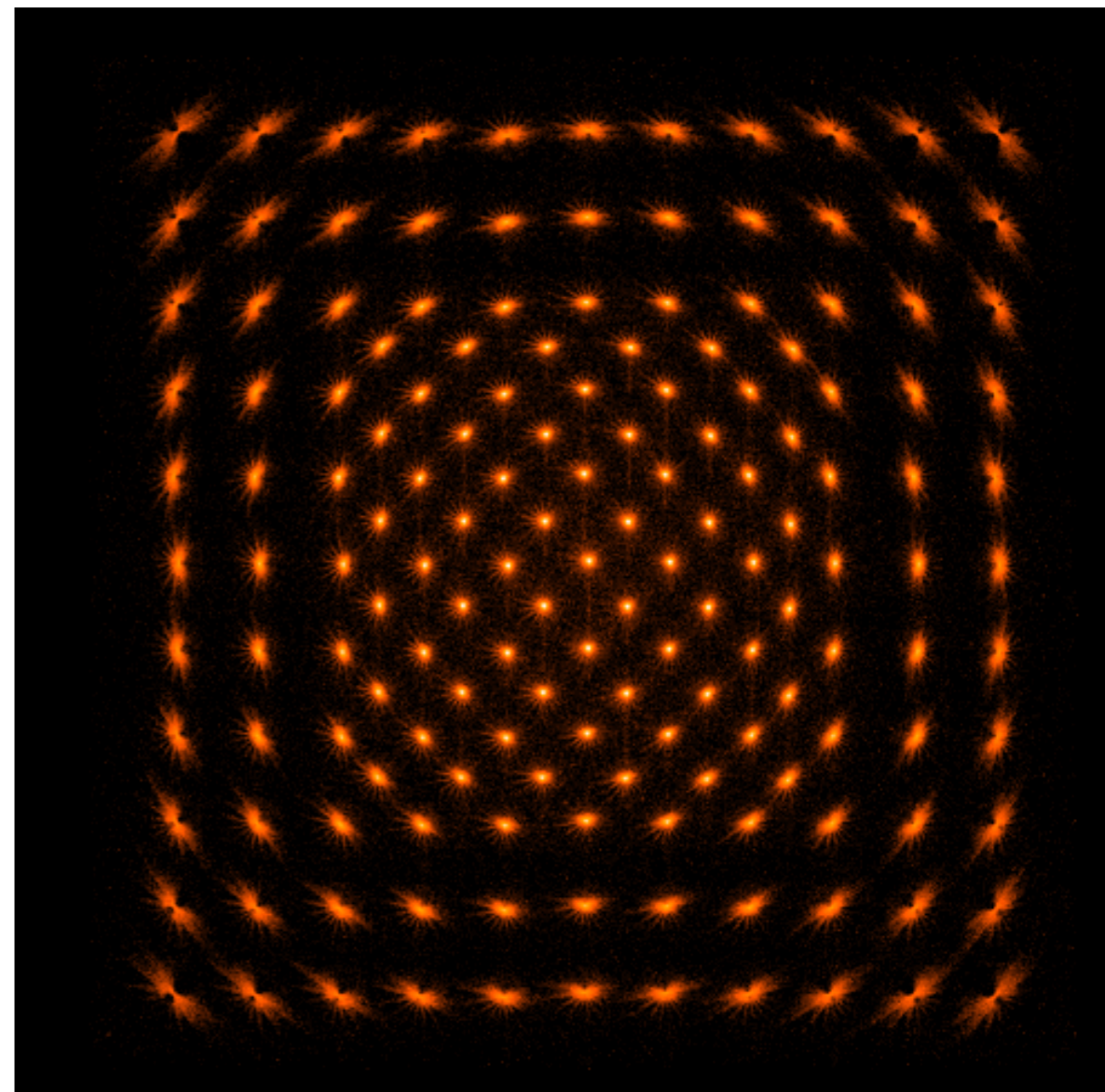
**g band, 3'**



**z band, 3'**



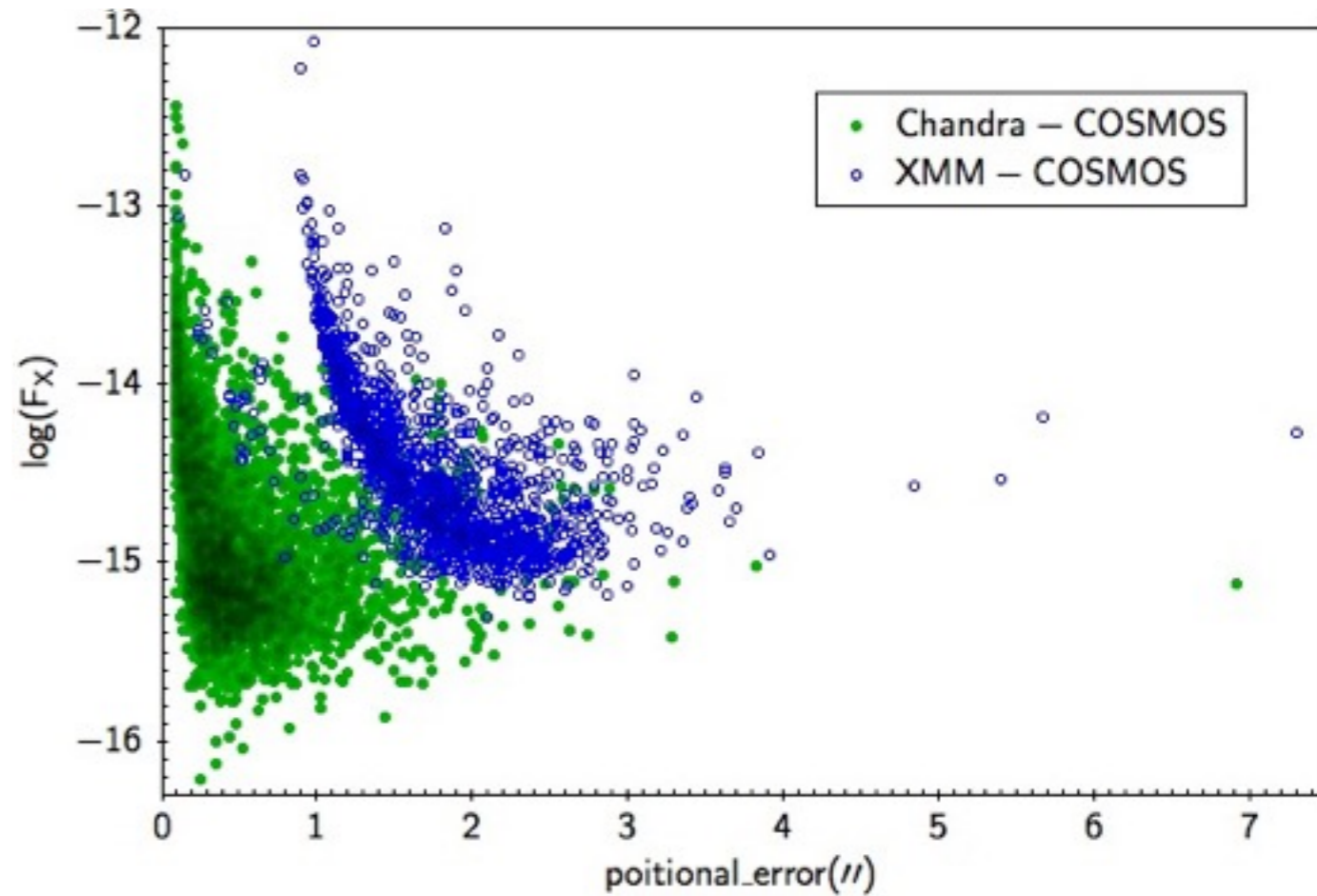
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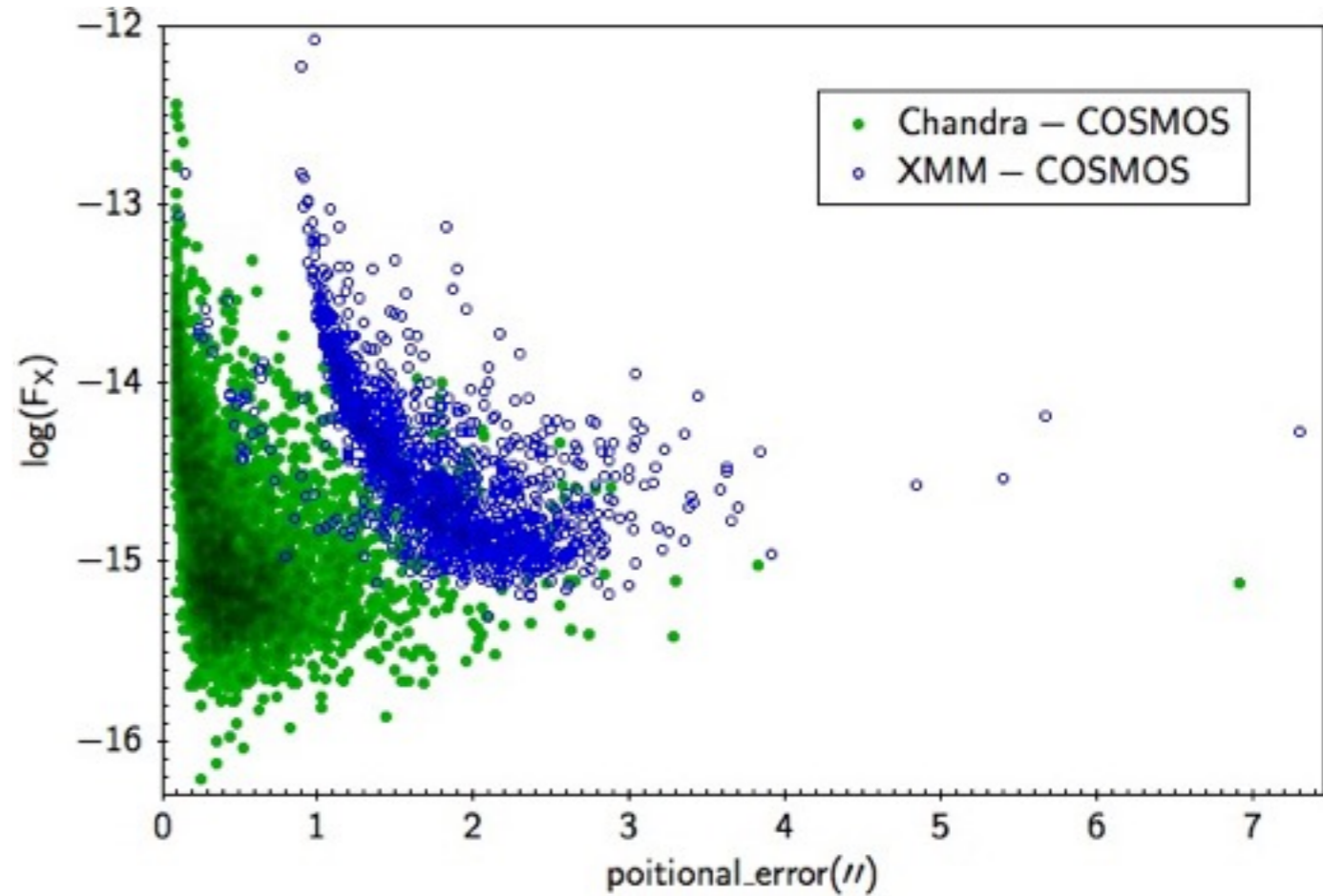
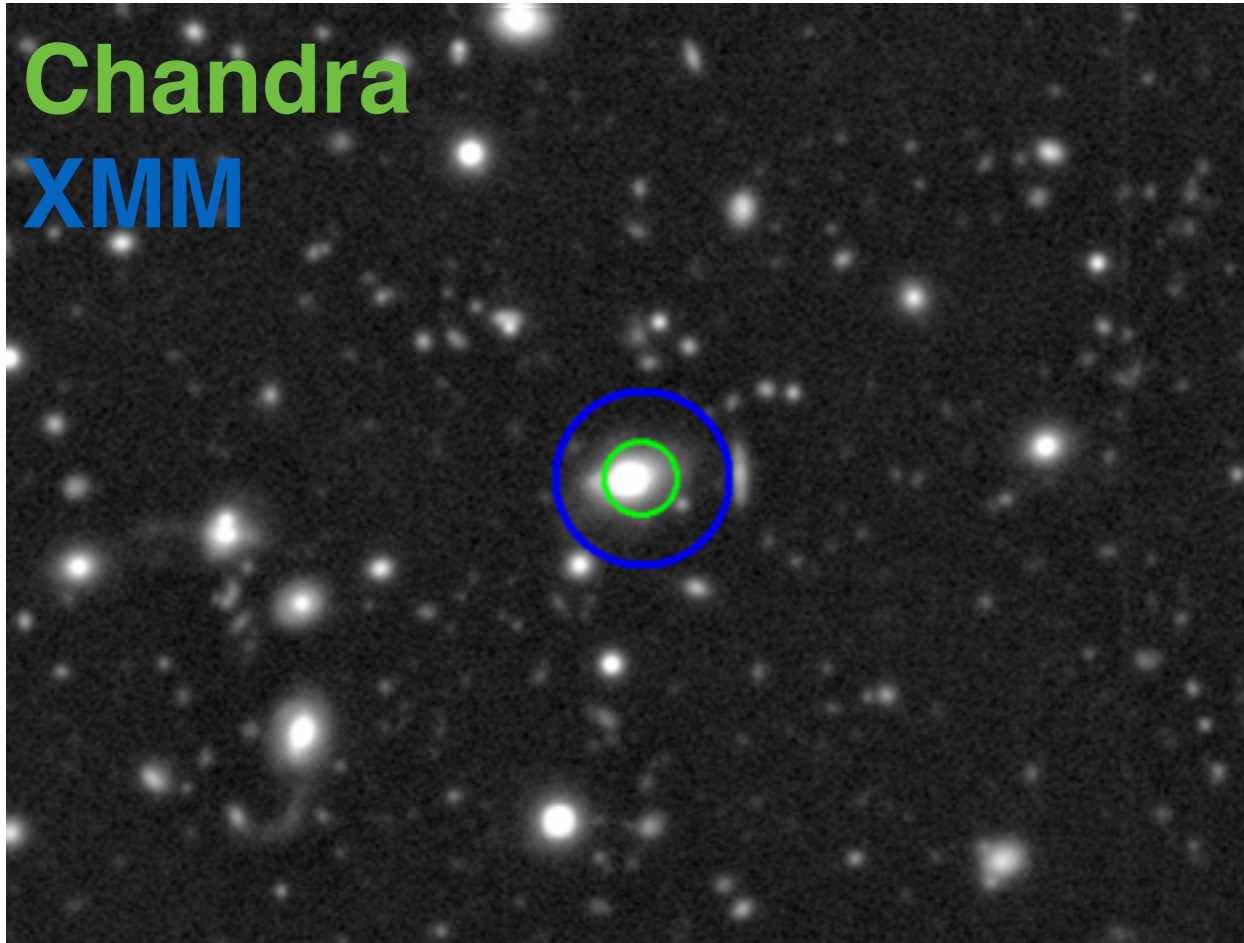
+ few sources per pointing often not sufficient to provide a reliable astrometric solution

# X-ray positional error is flux (and instrument) depending



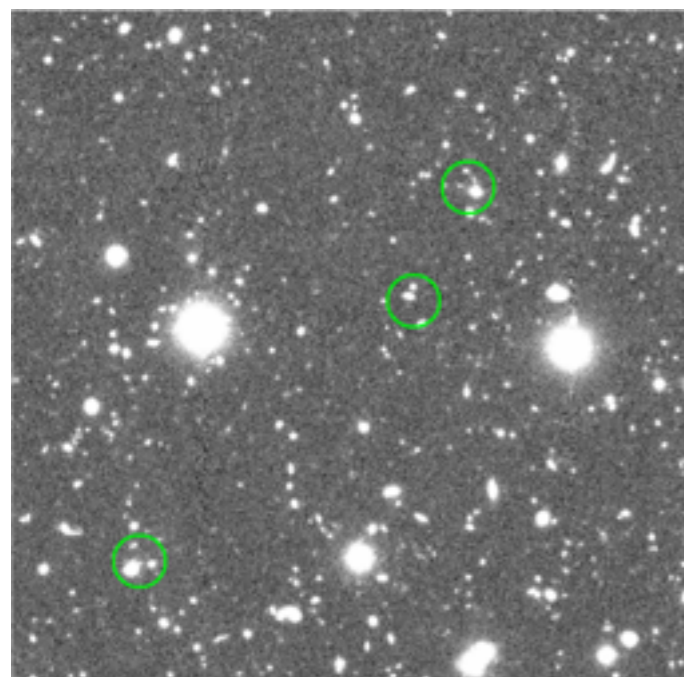
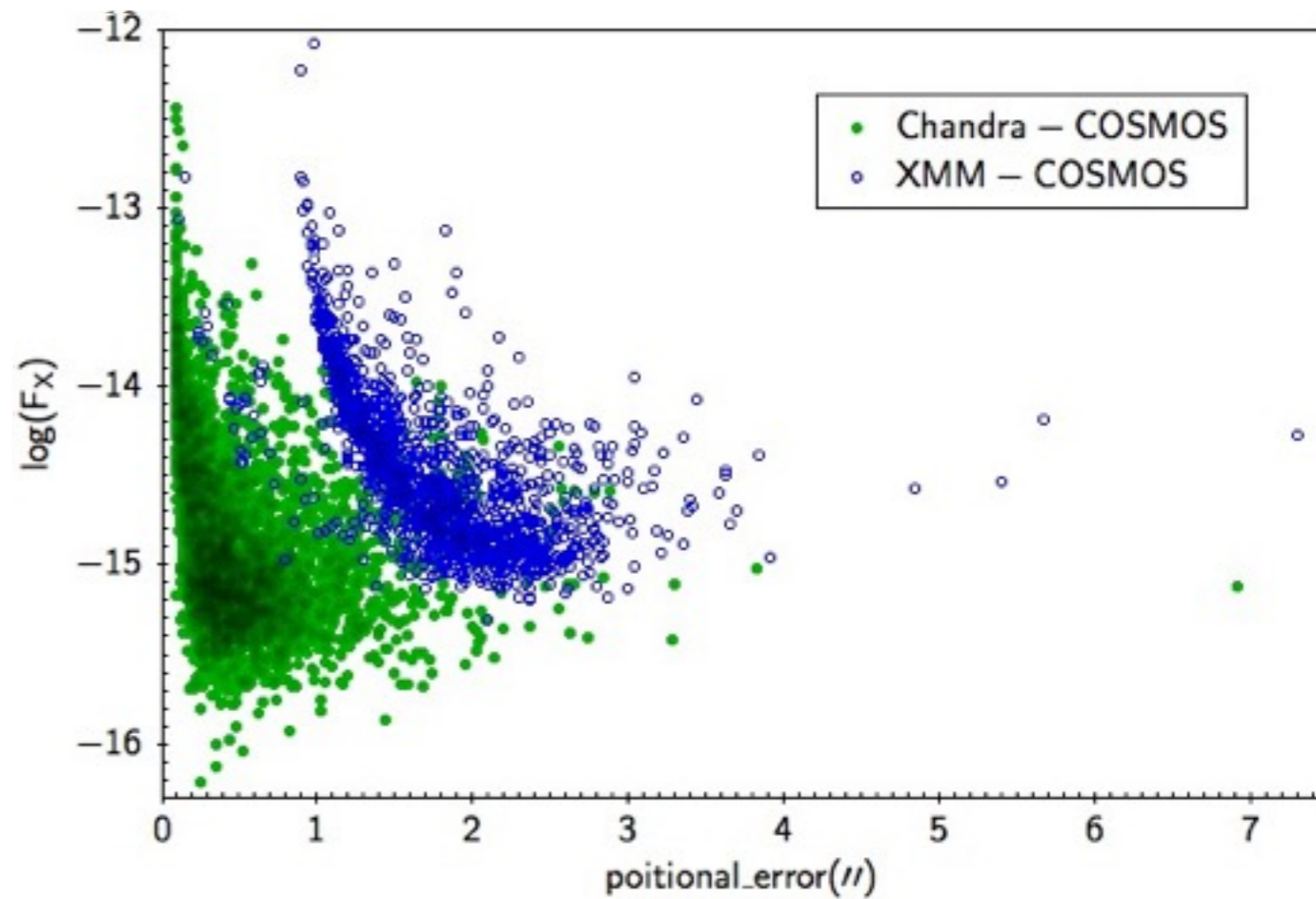
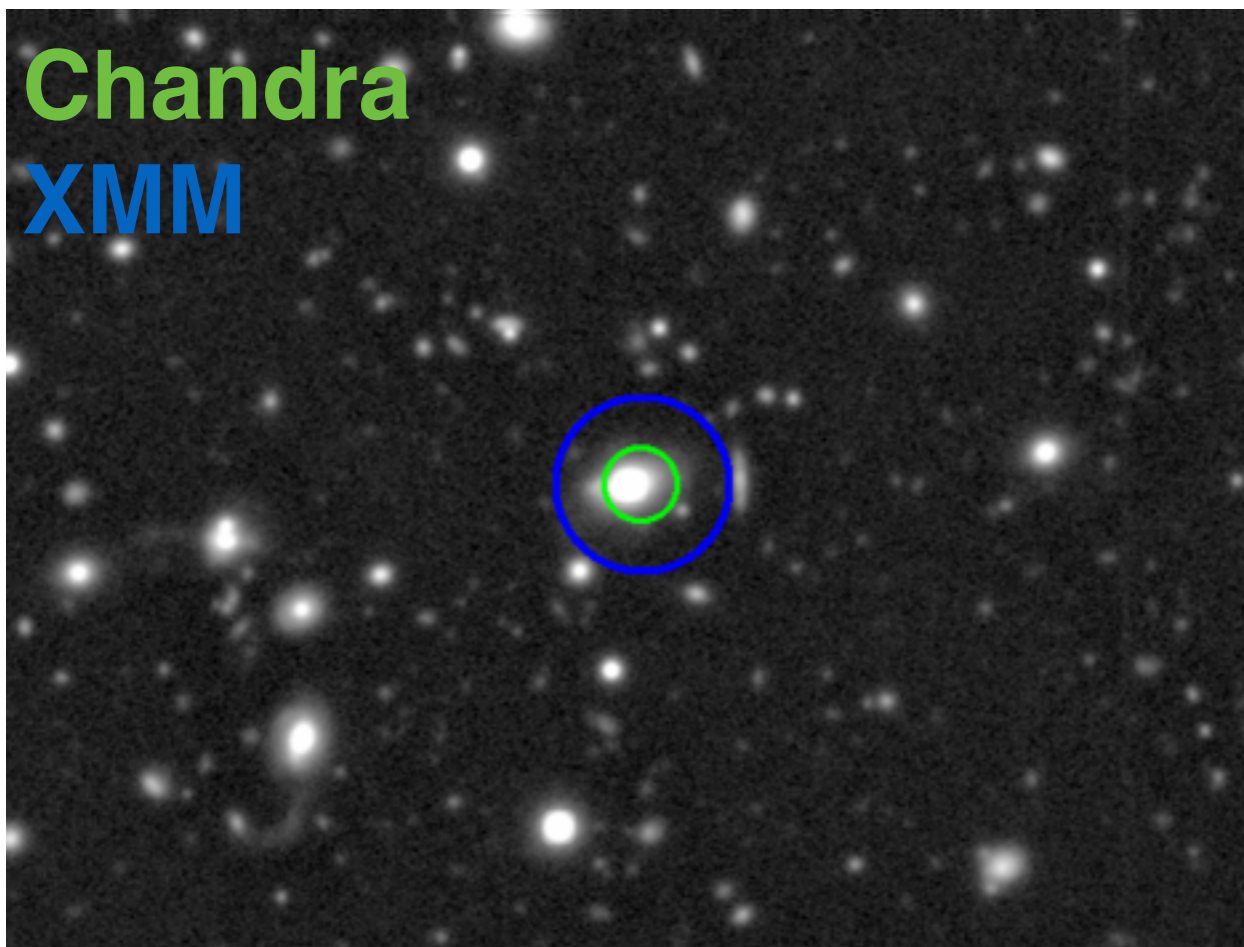
# X-ray positional error is flux (and instrument) depending

Chandra  
XMM



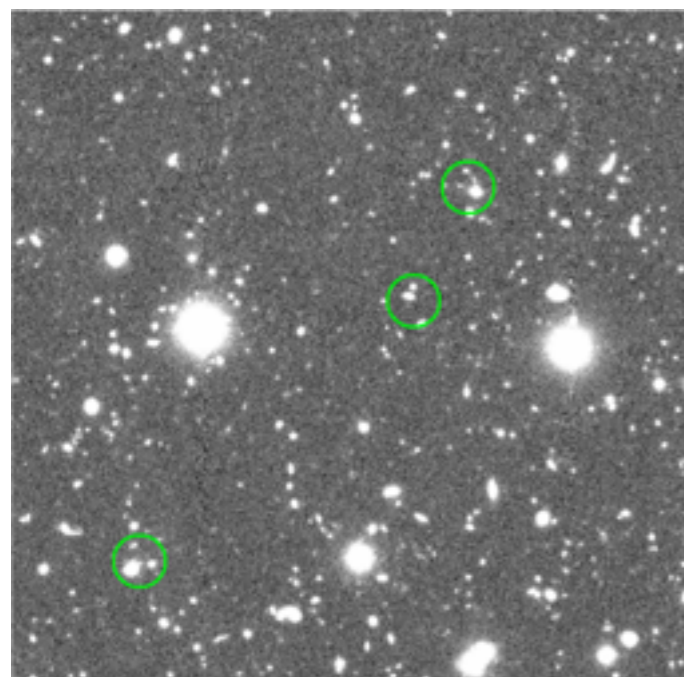
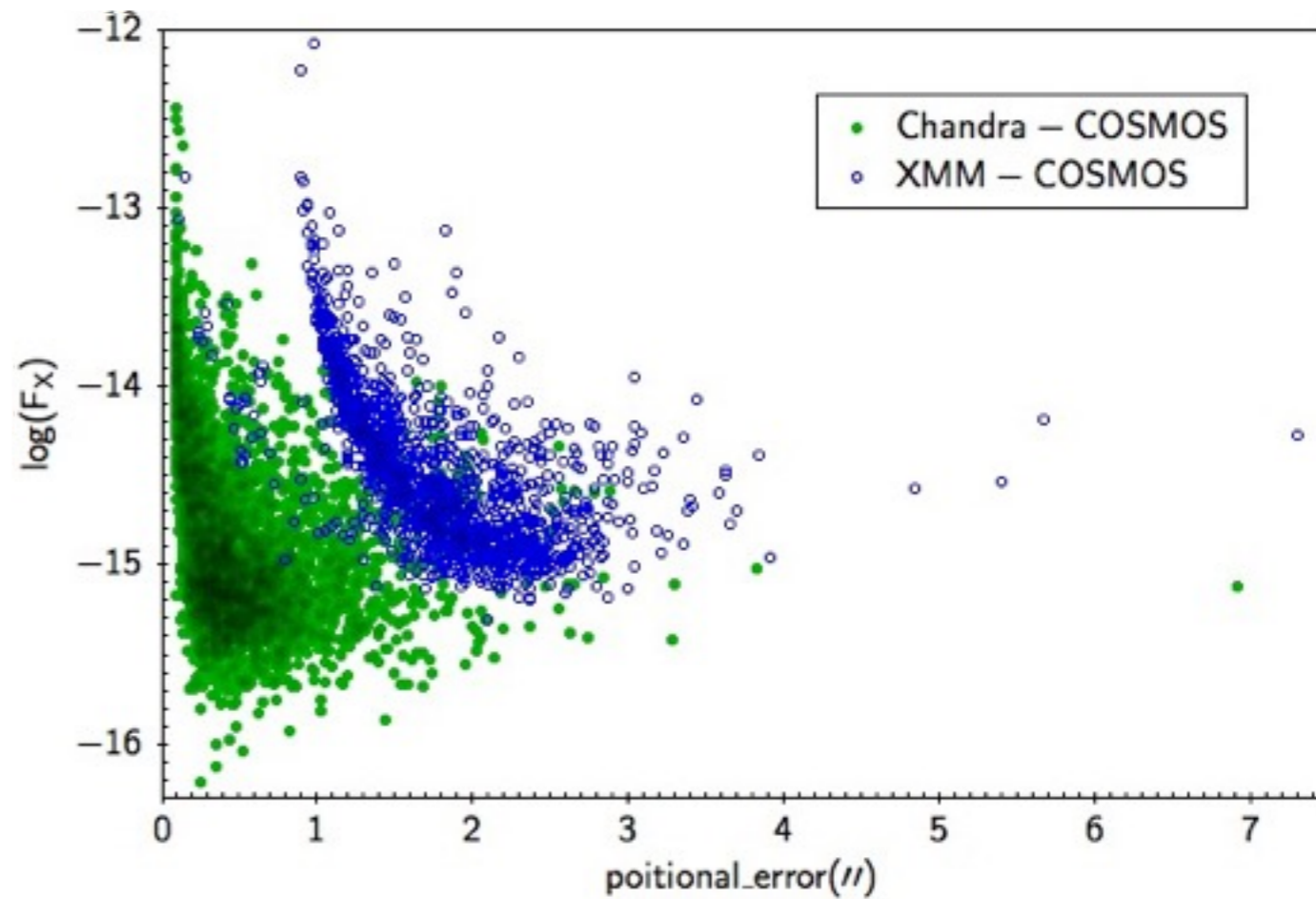
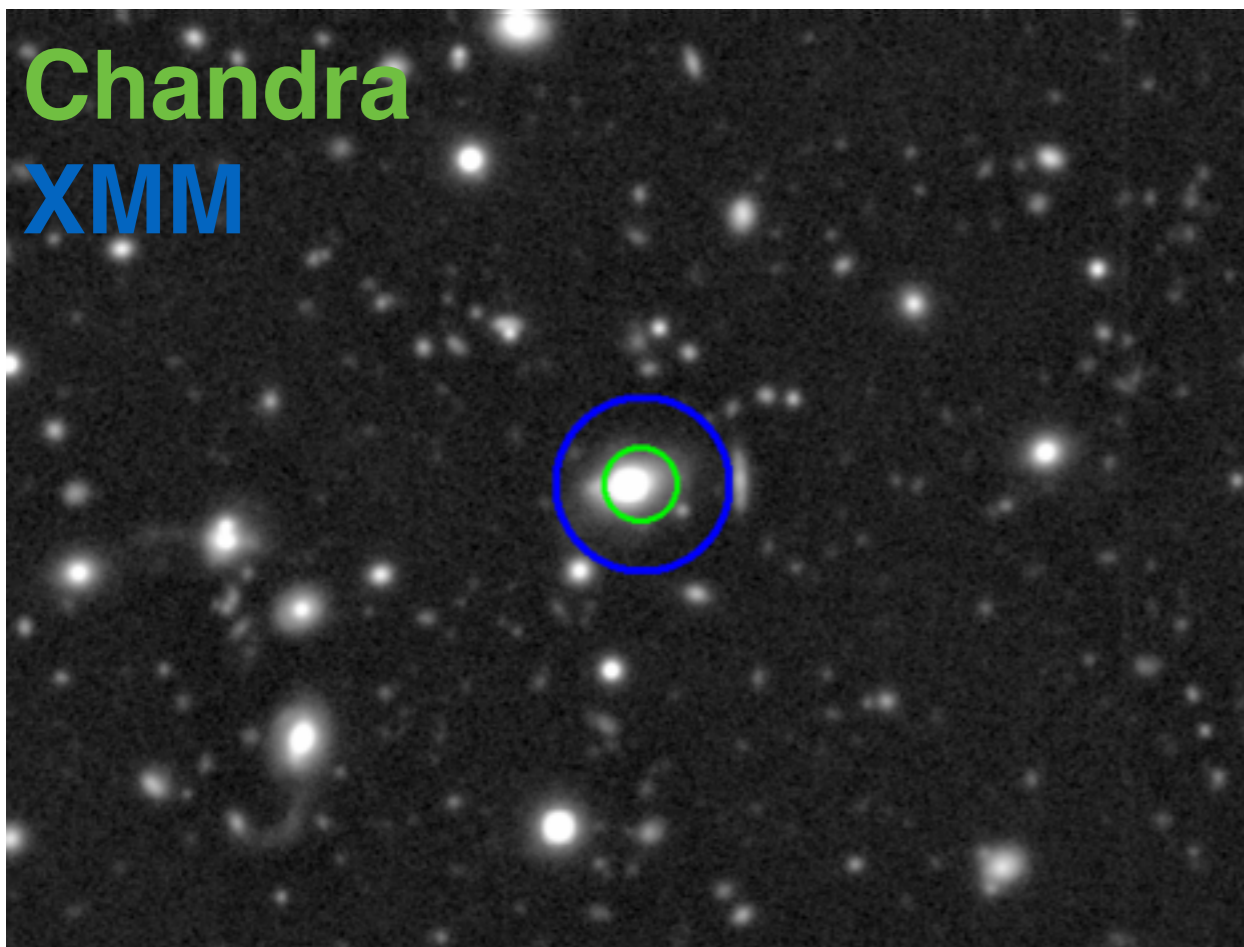
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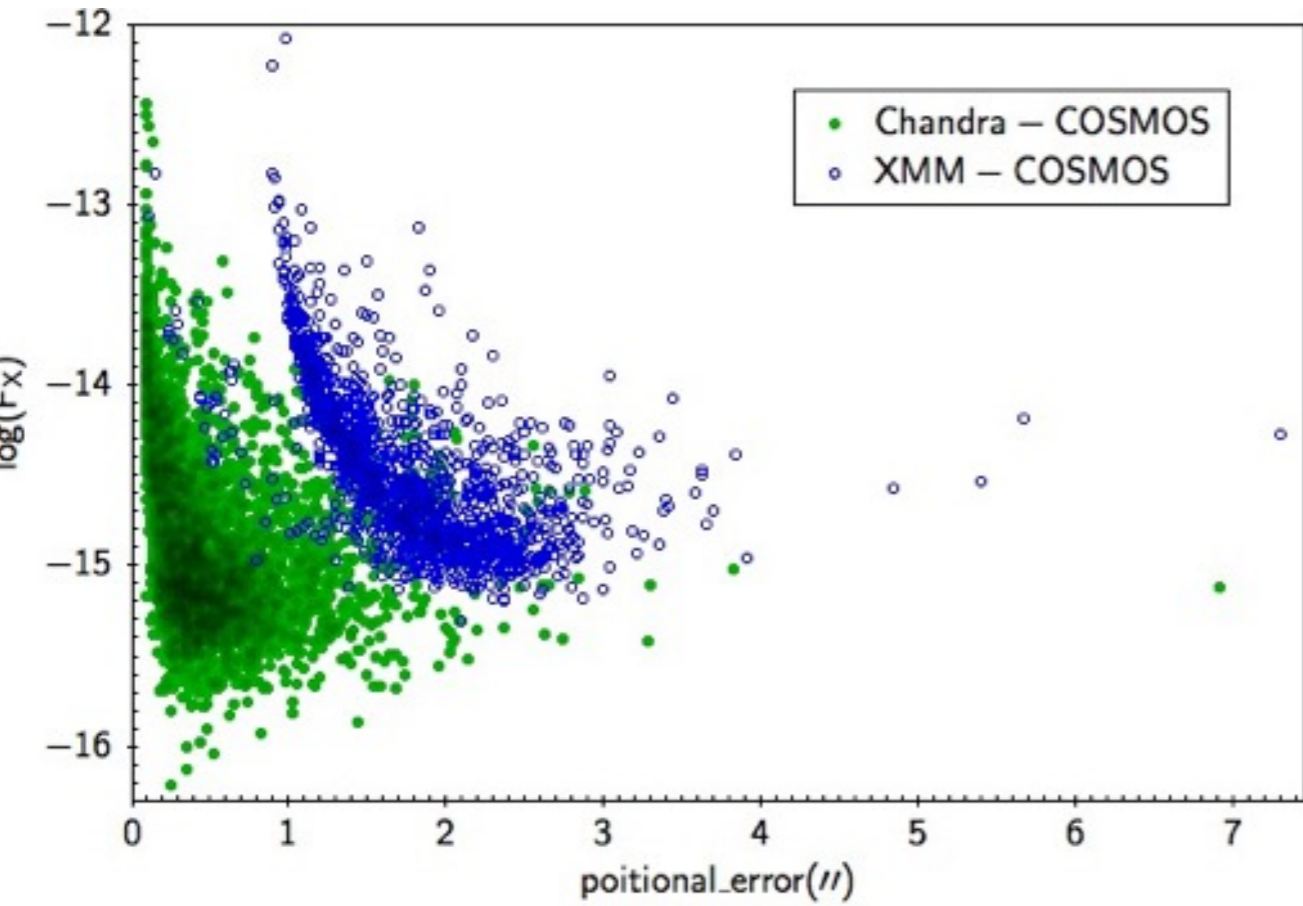
# X-ray positional error is flux (and instrument) depending

Chandra  
XMM



Number of sources depends on depth of images

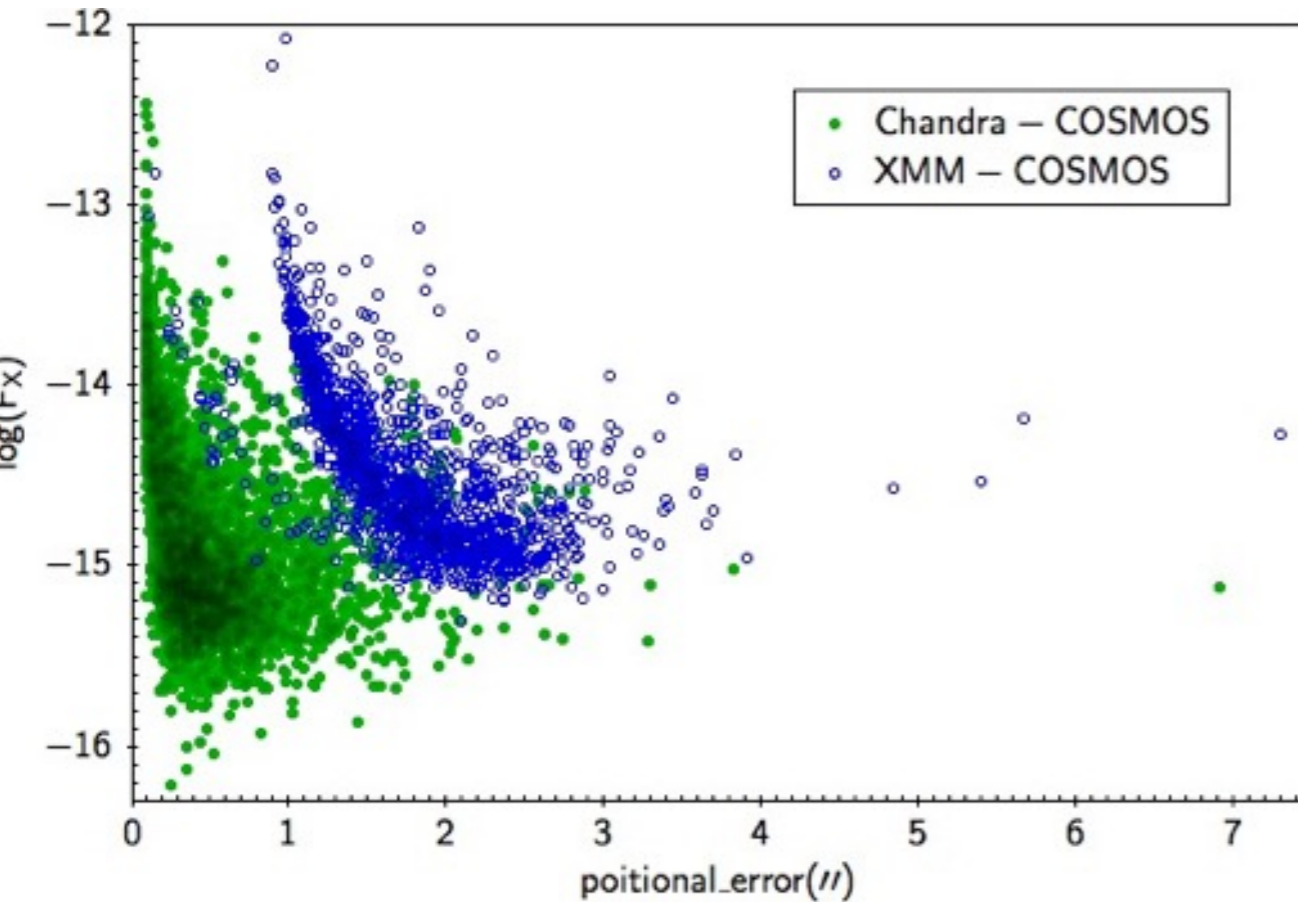
# The problem: ROSAT positional uncertainties are even larger



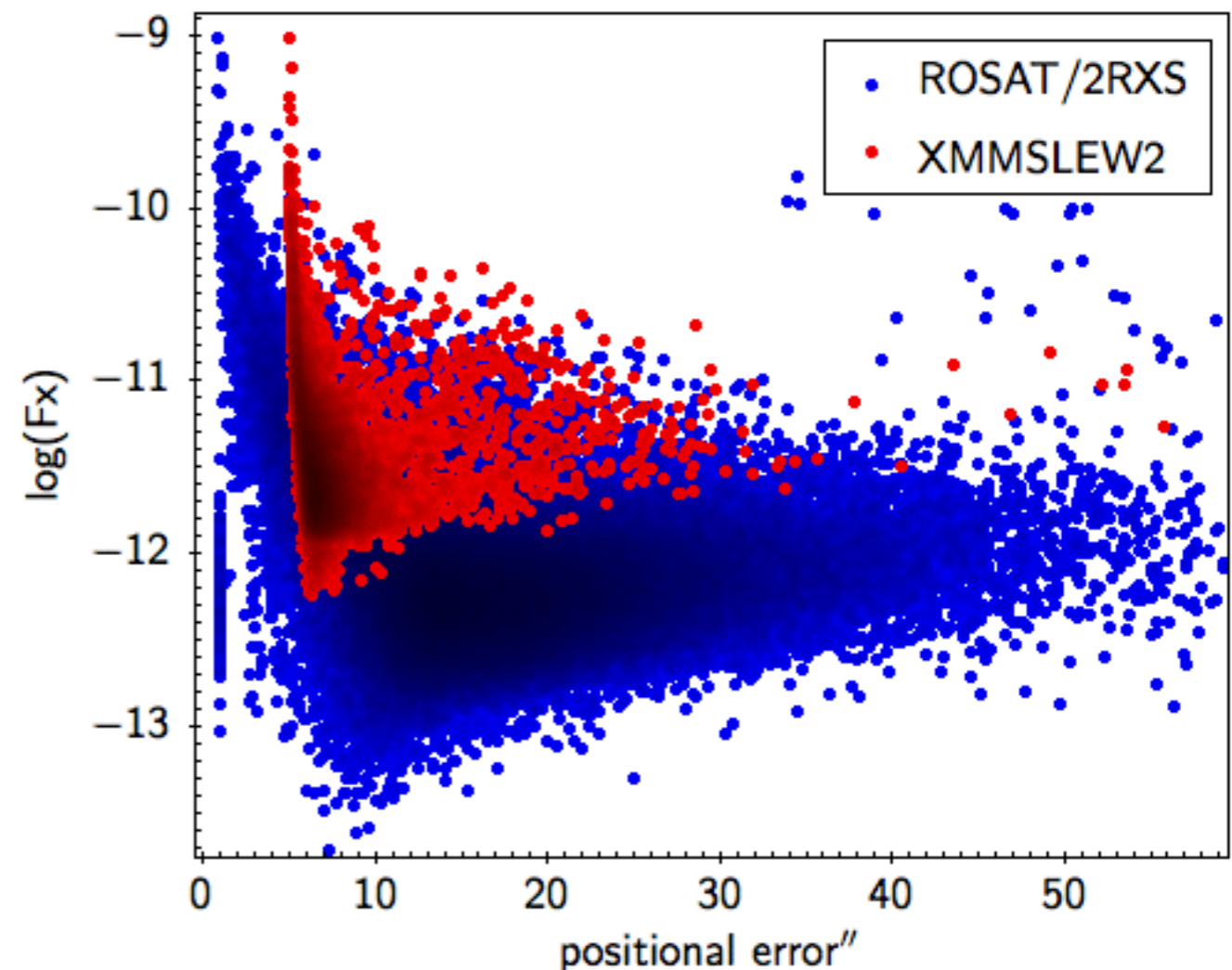
Salvato+ 2018,  
Dwelly+2017



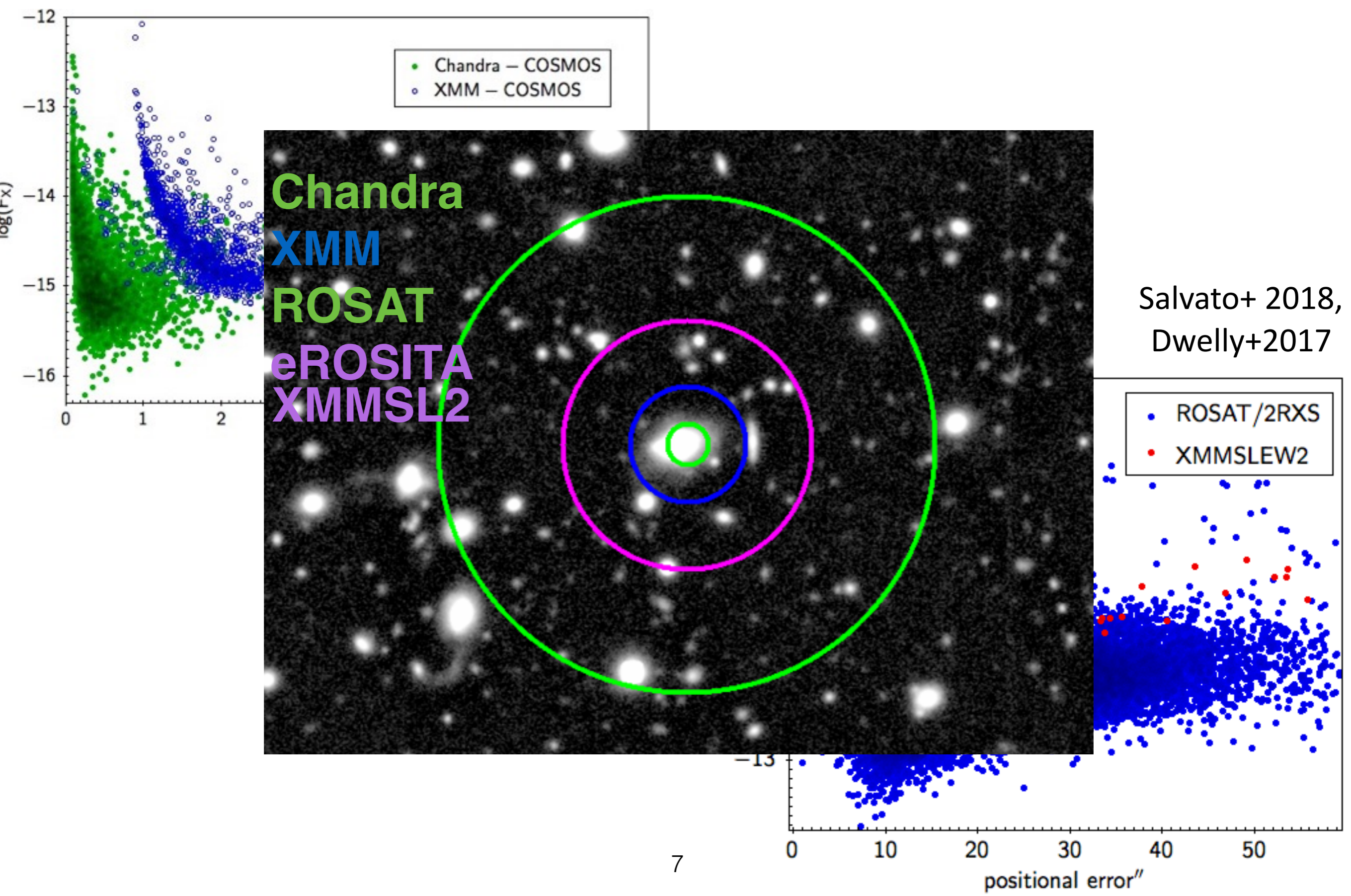
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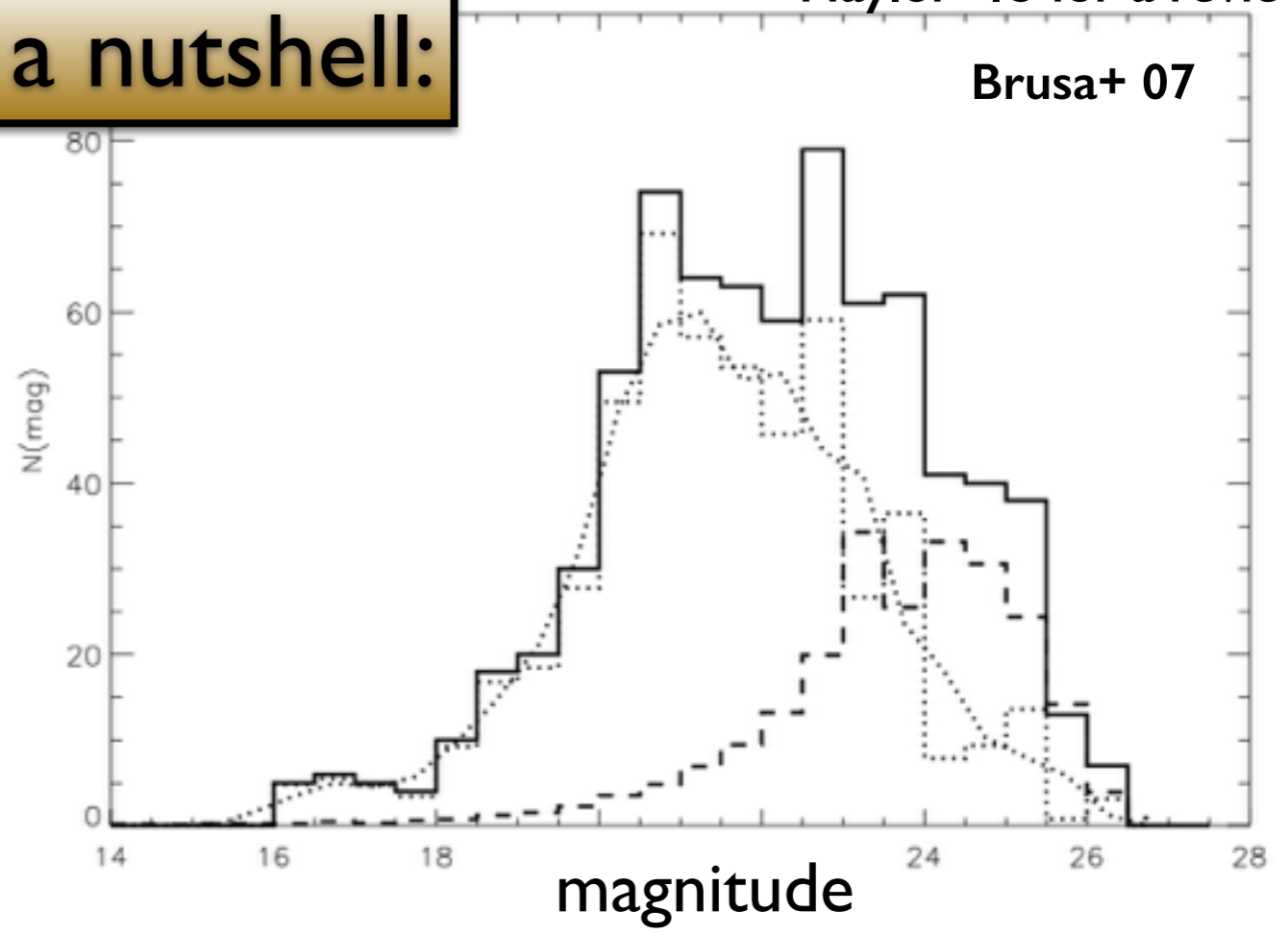
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# Maximum Likelihood (ML) in a nutshell:

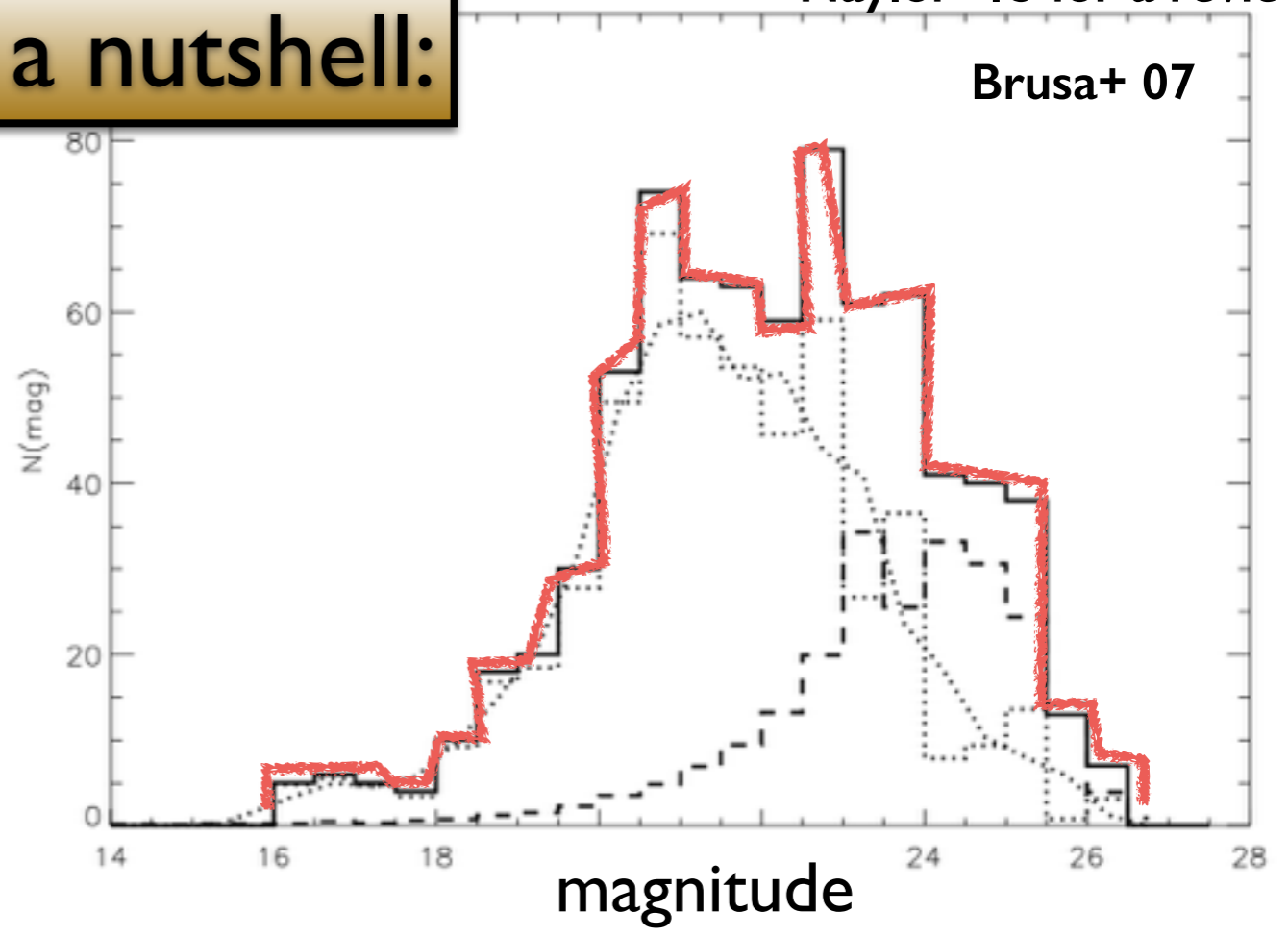
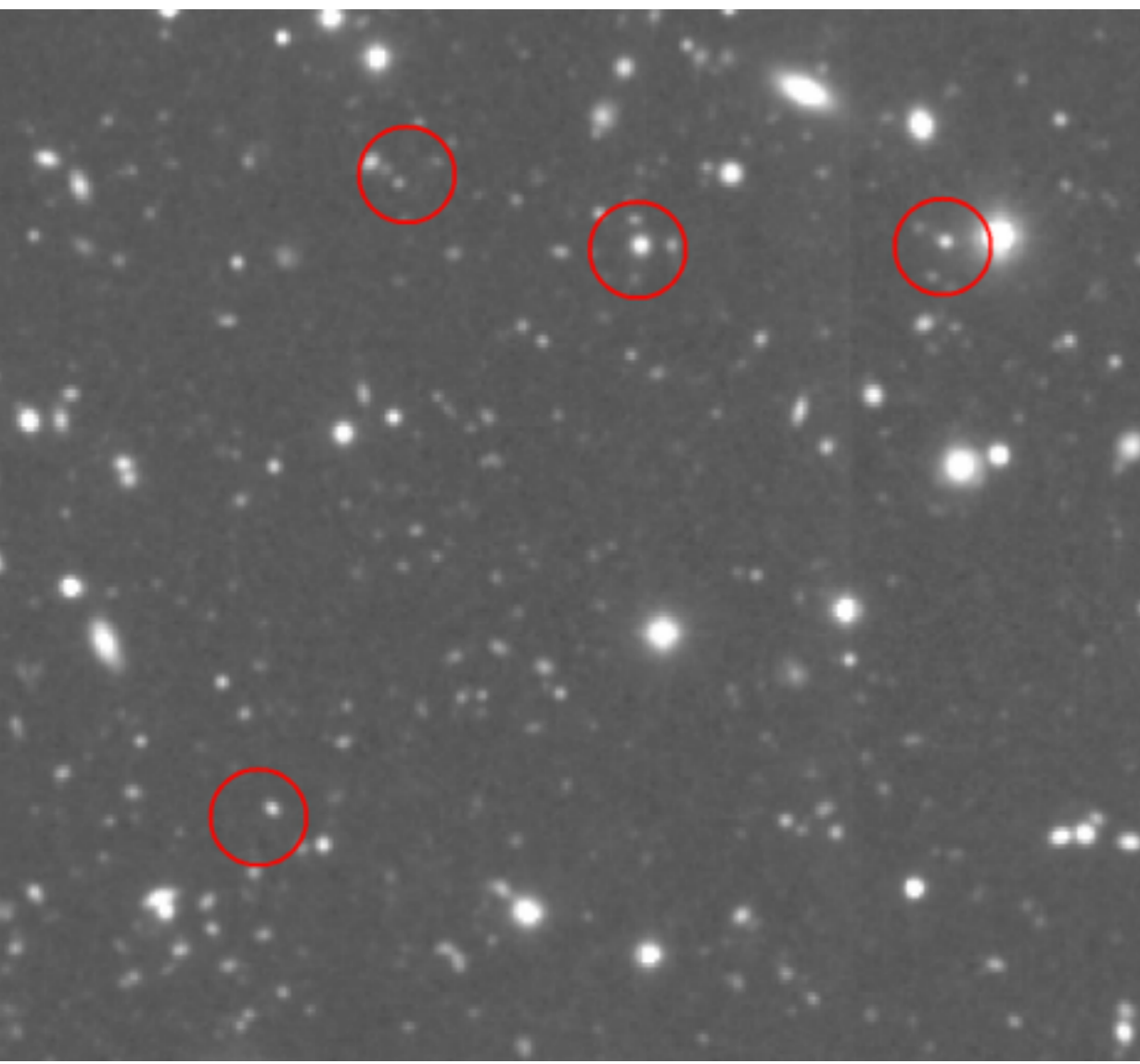
Naylor+13 for a review

Brusa+ 07

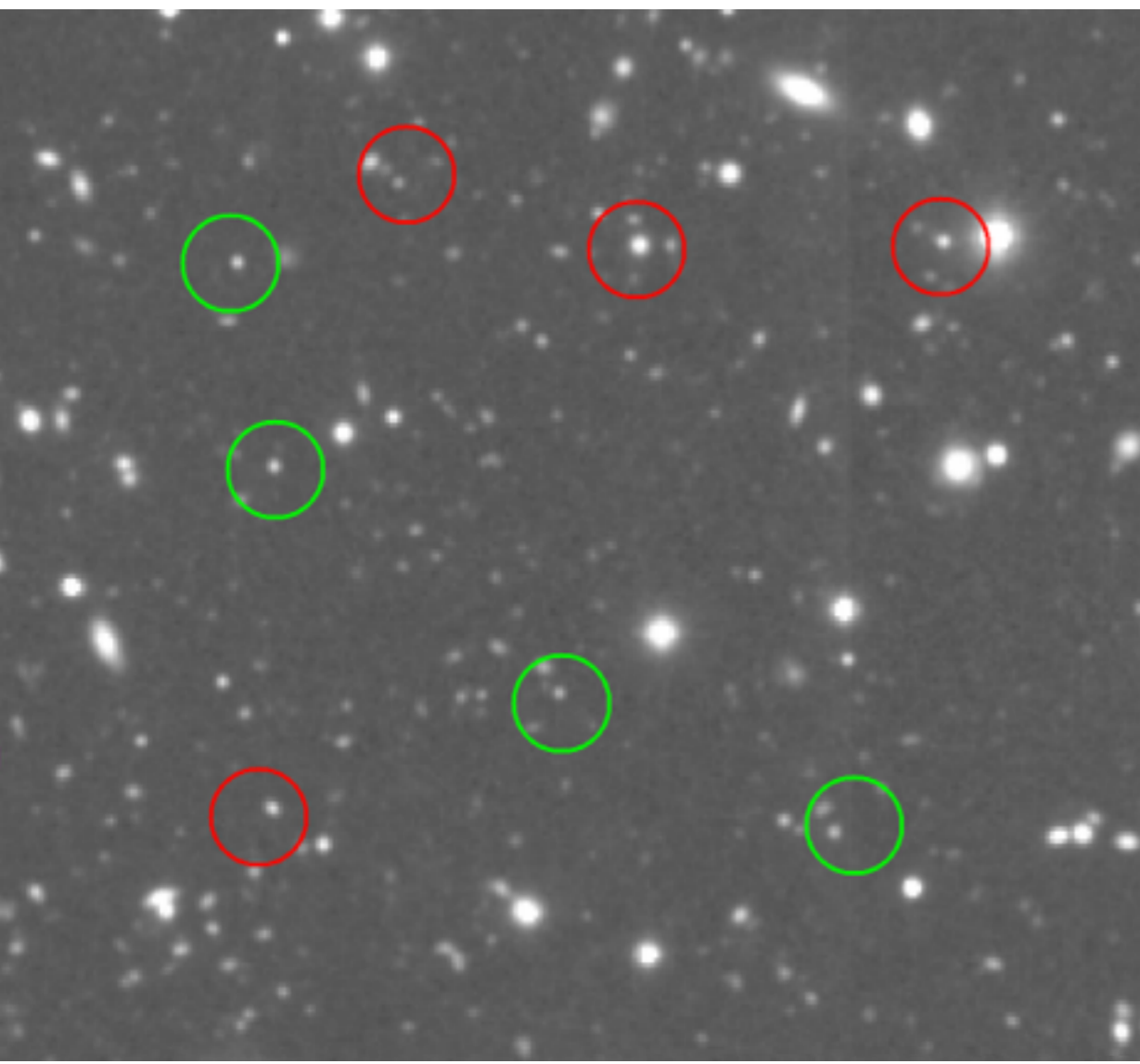


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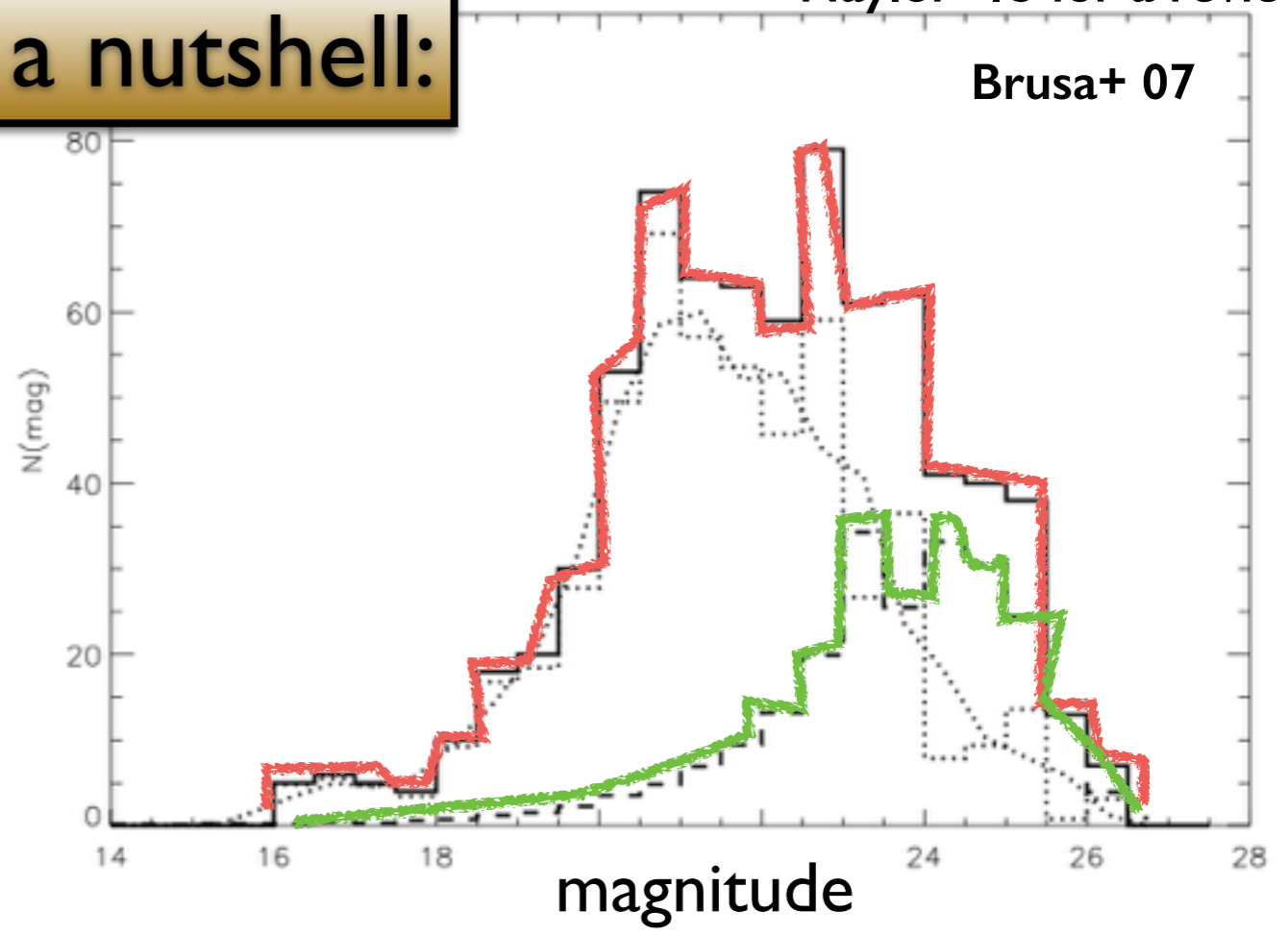


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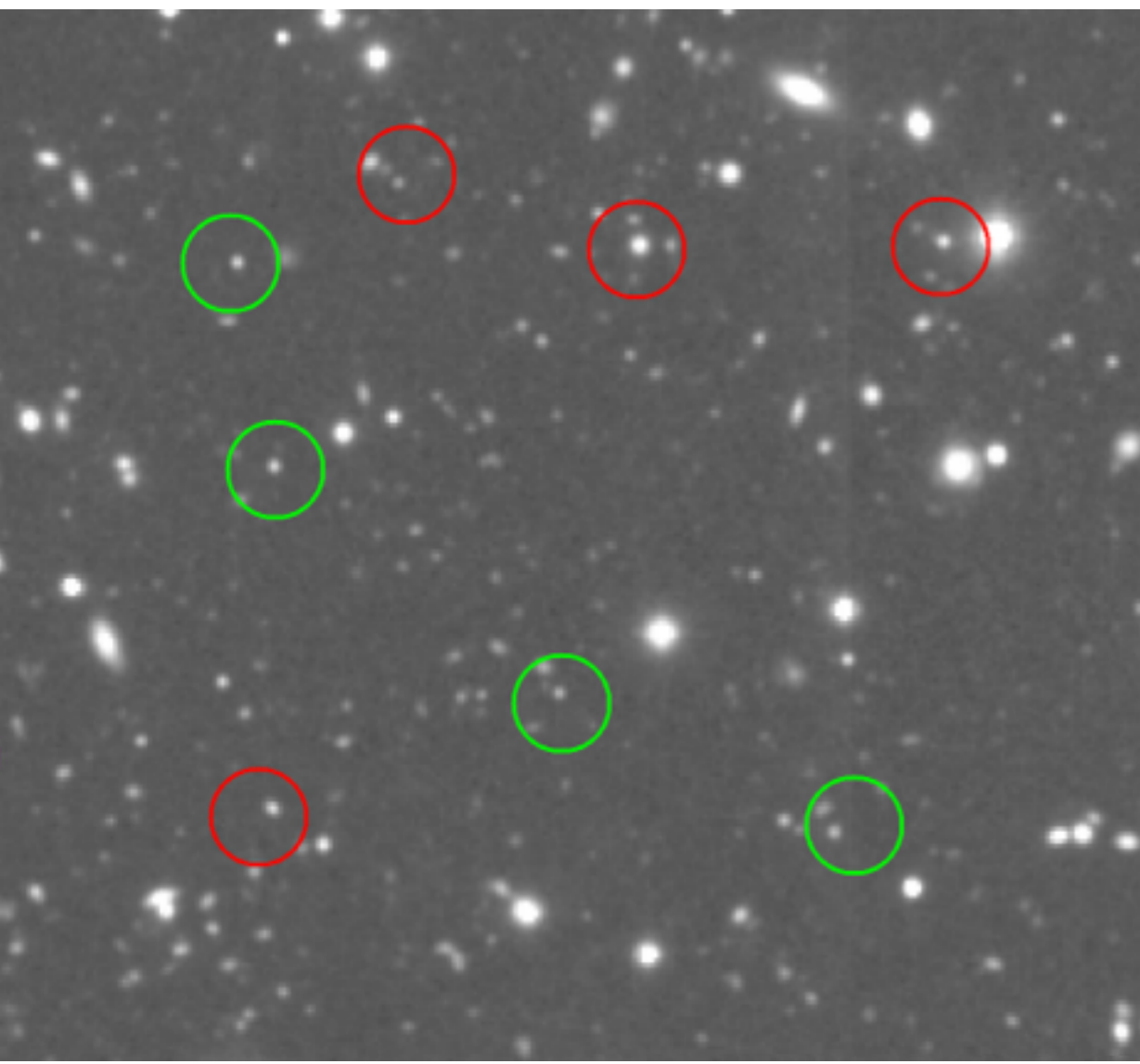


Naylor+13 for a review

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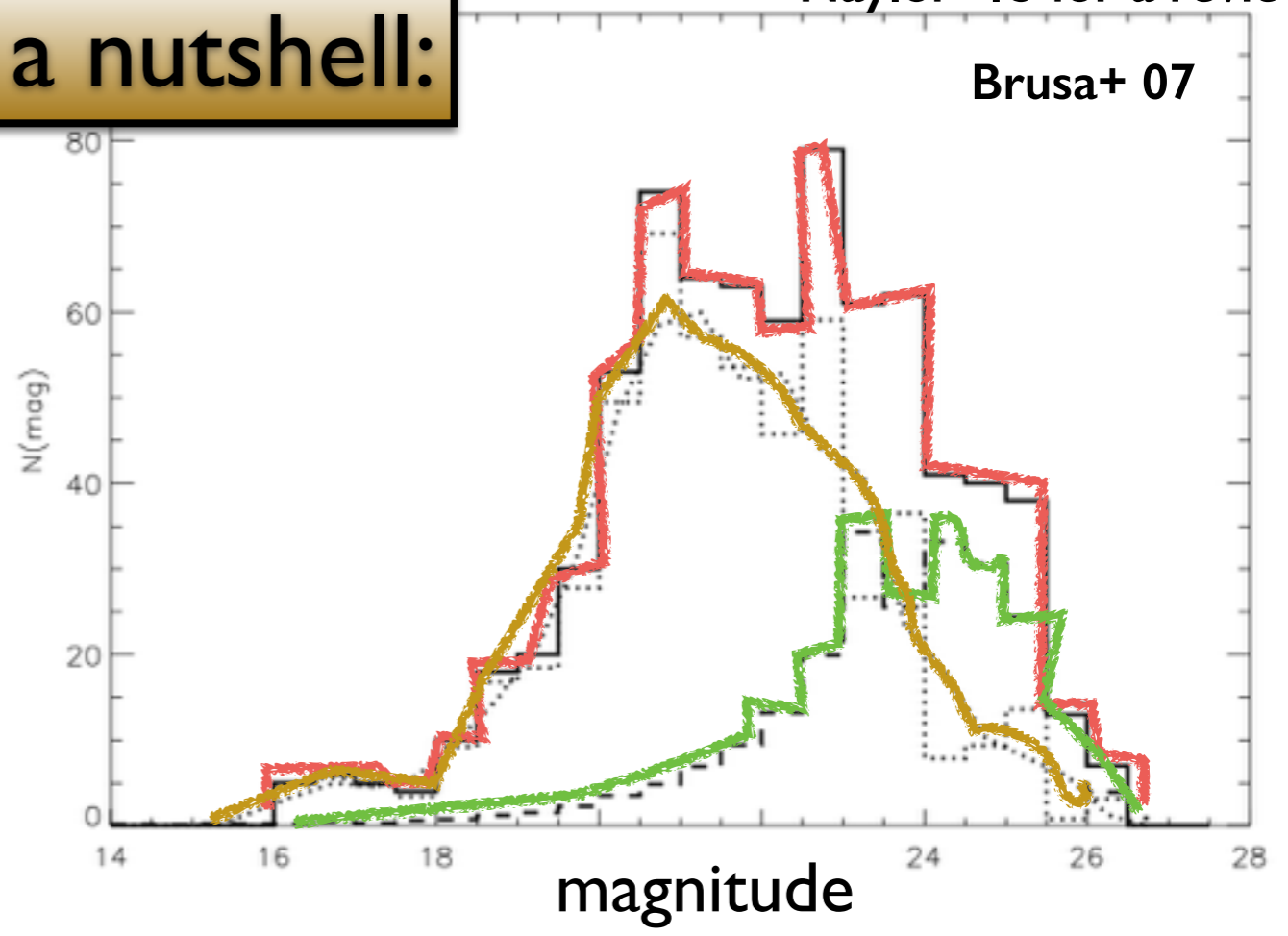


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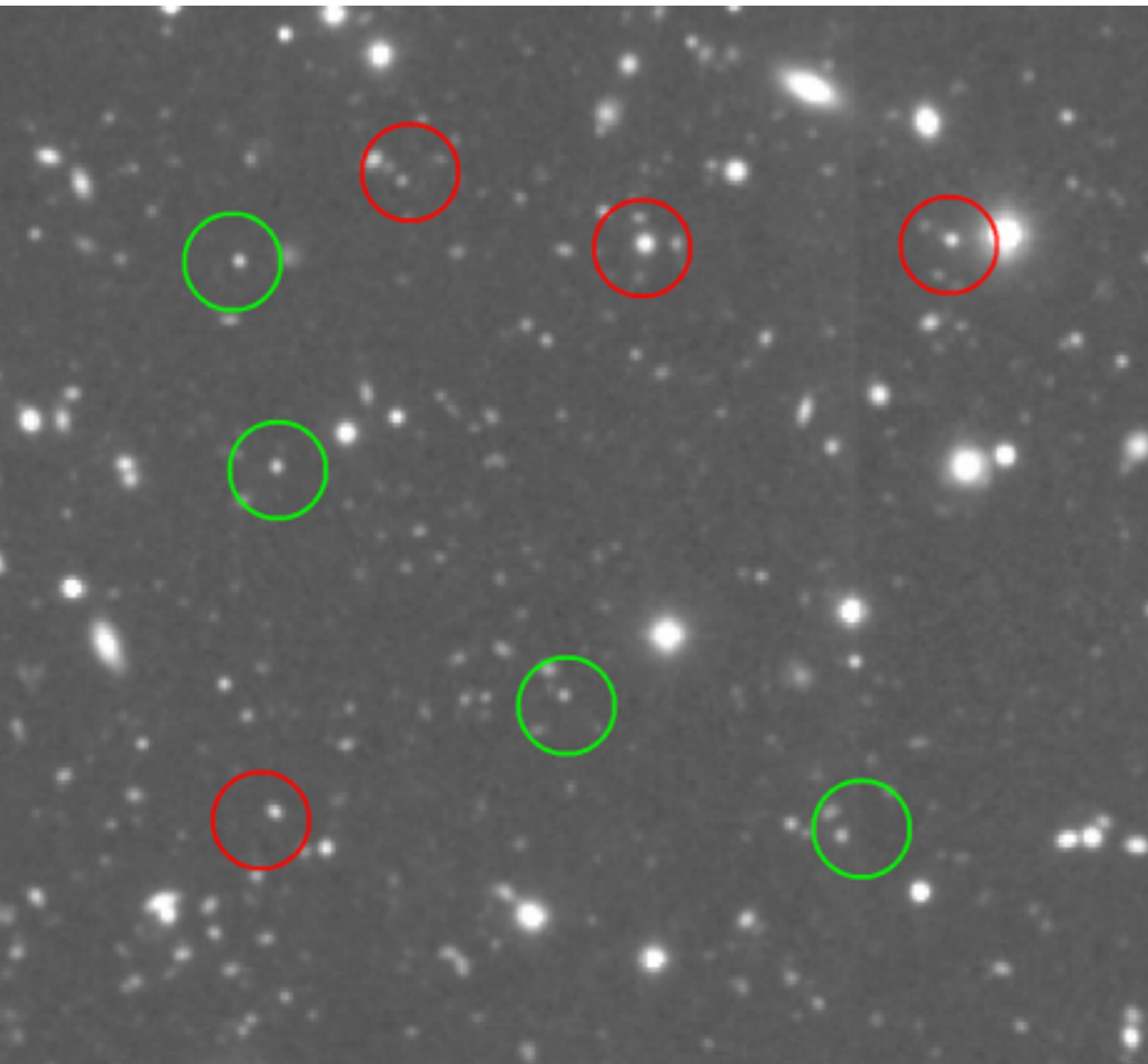


Naylor+13 for a review

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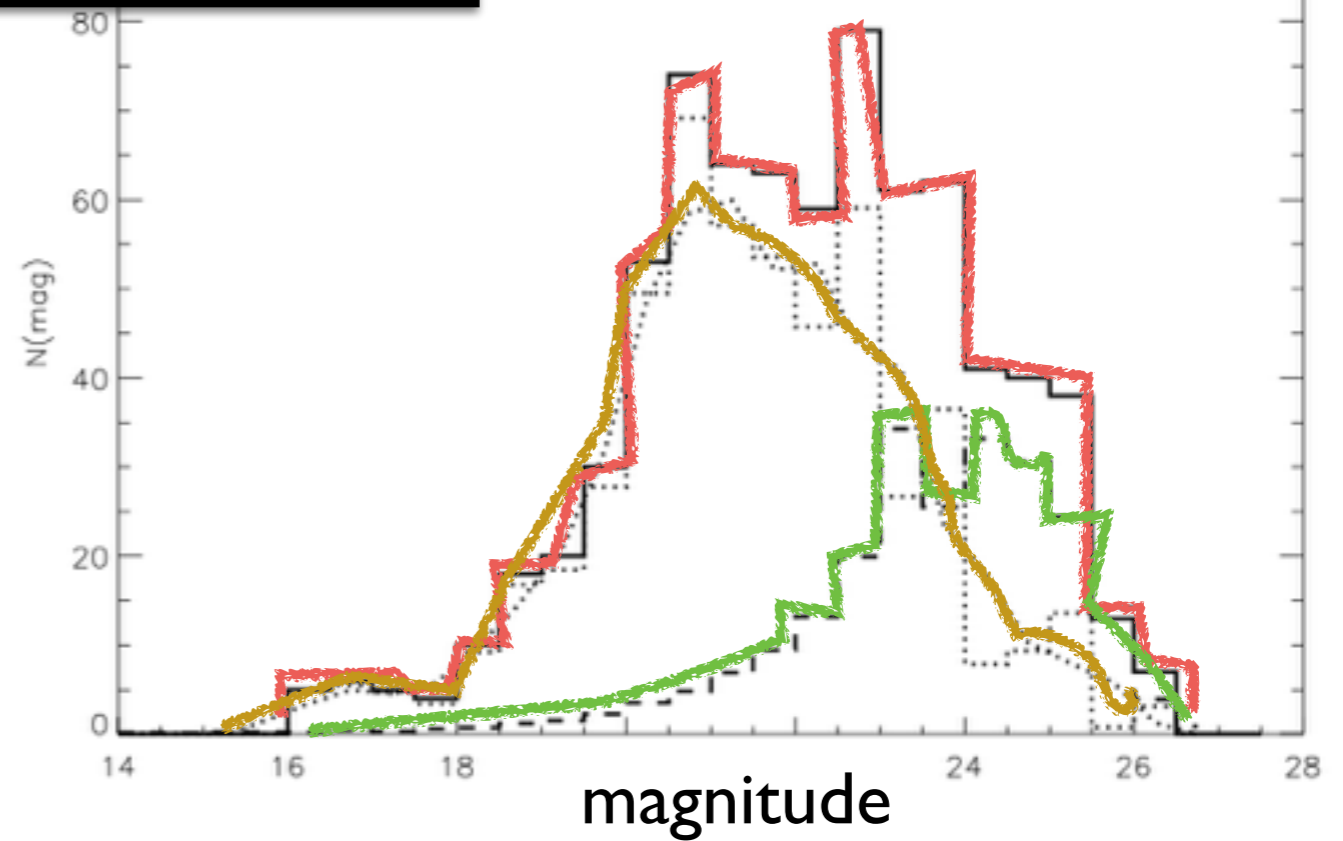


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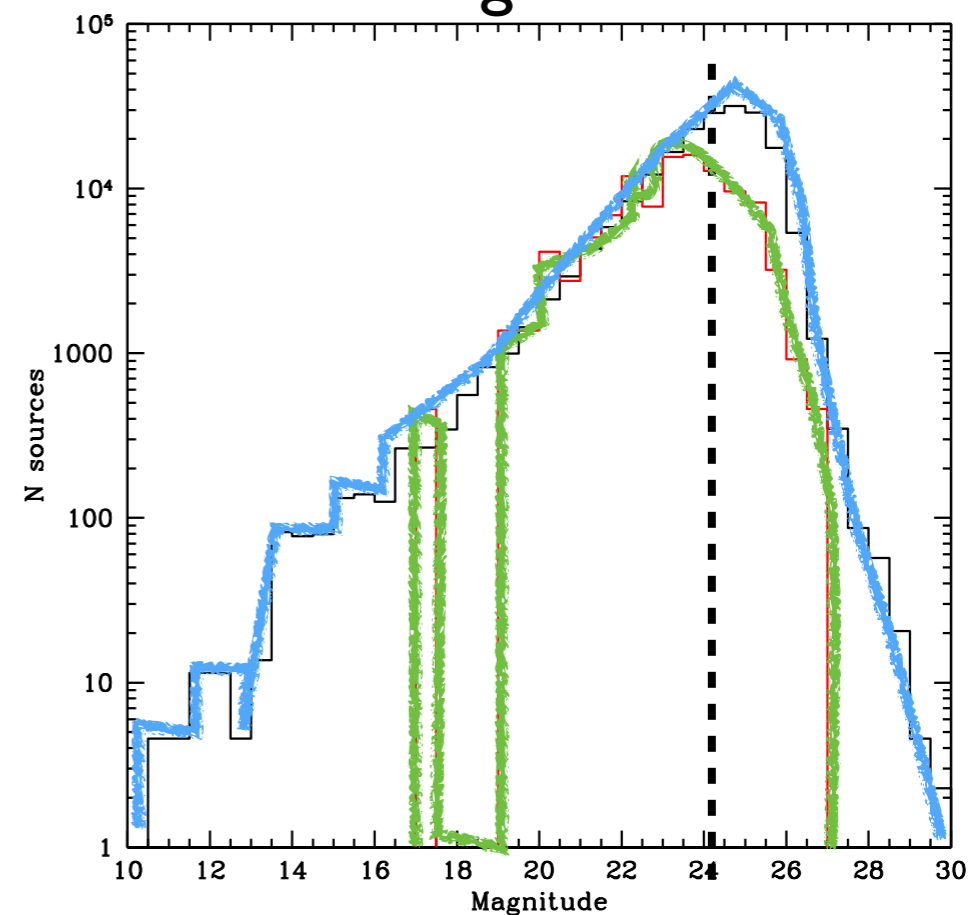
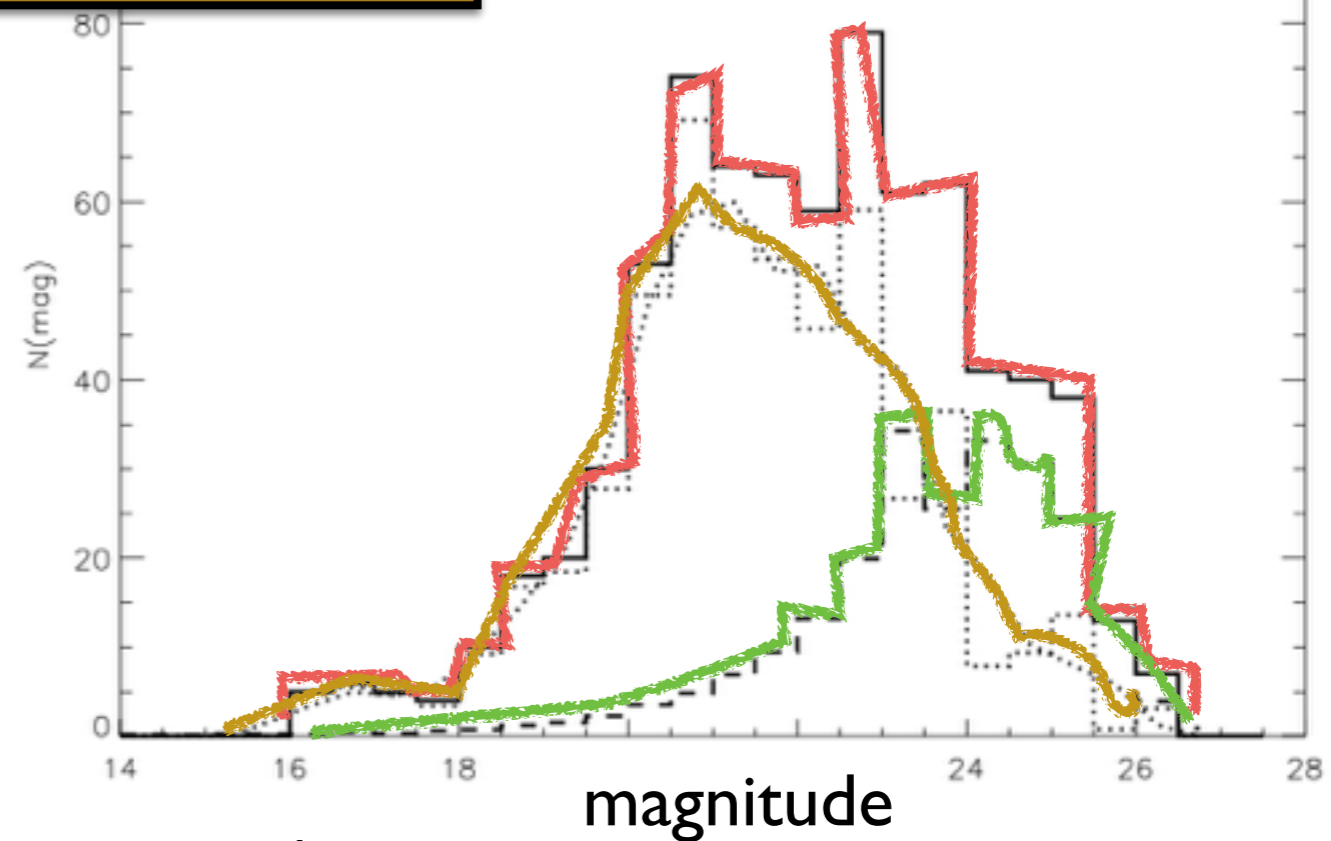
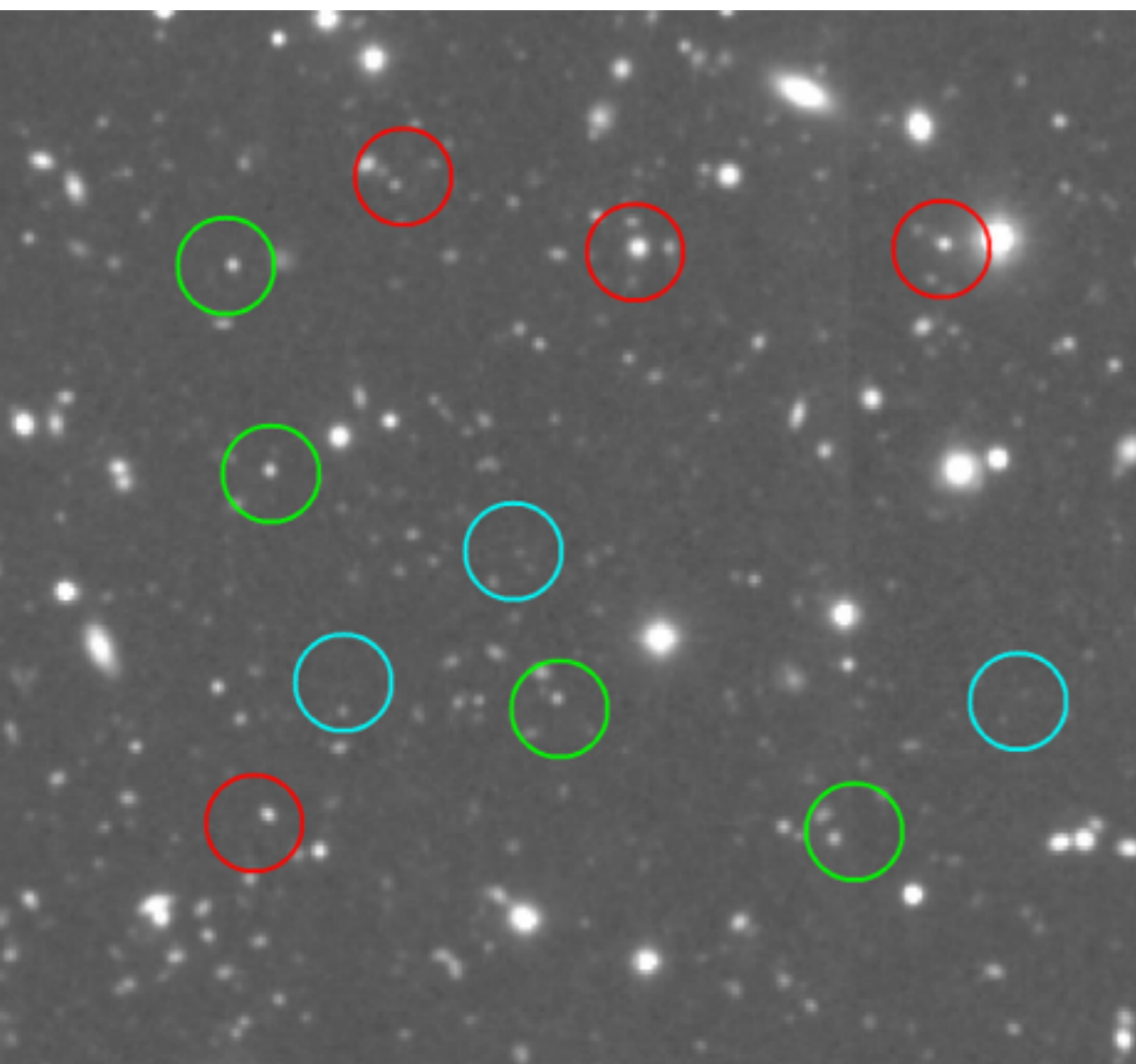
$$LR = q(m) f(r)/n(m)$$

$$R_j = \frac{(LR)_j}{\sum_i (LR)_i + (1 - Q)}$$

# Maximum Likelihood (ML) in a nutshell:

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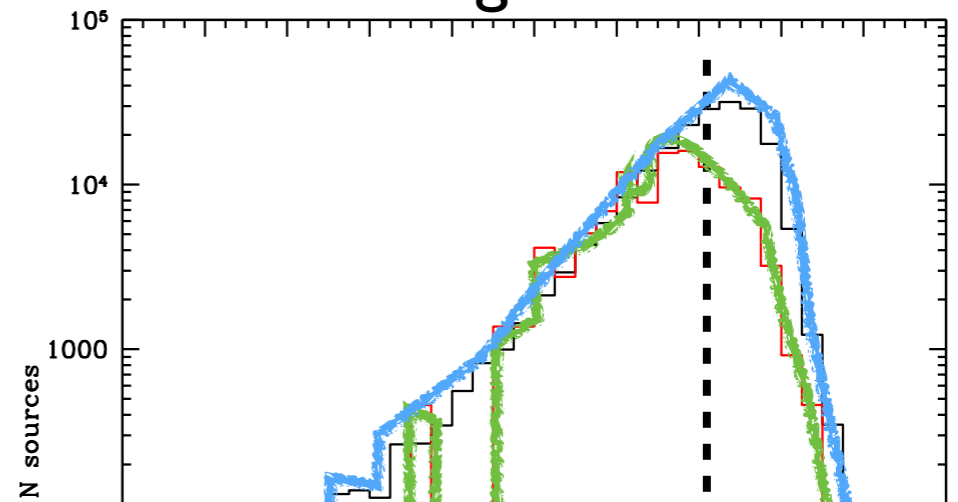
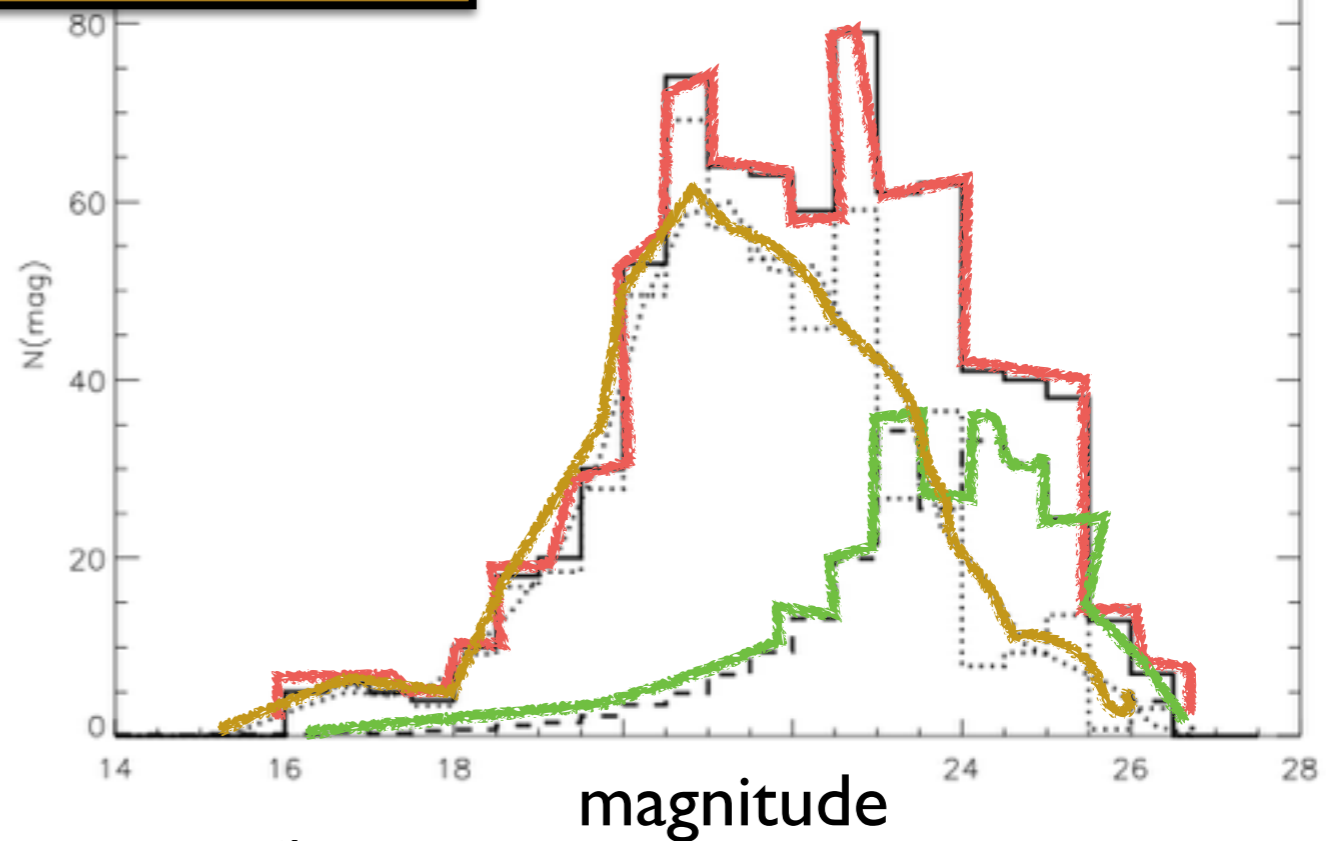
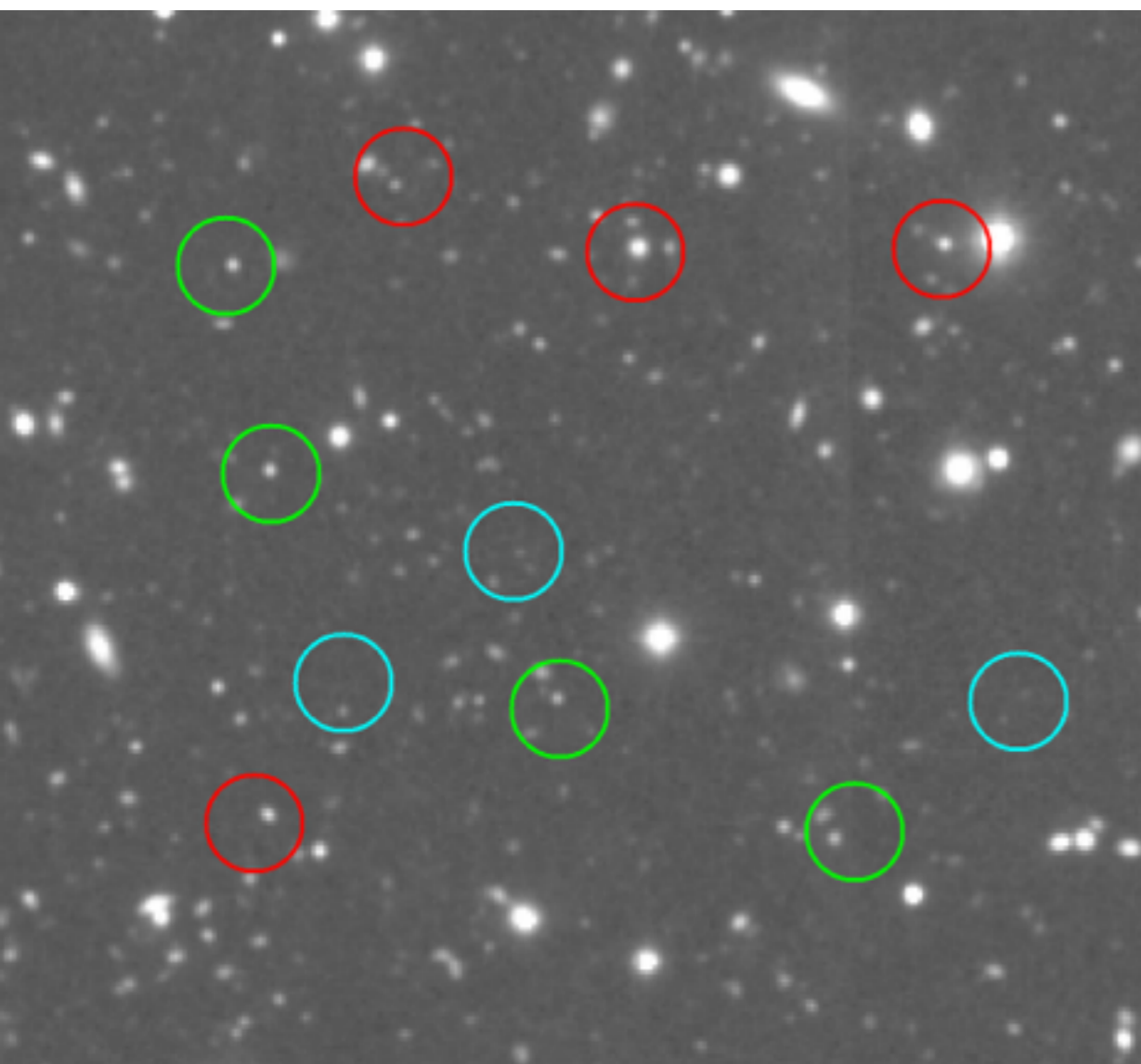
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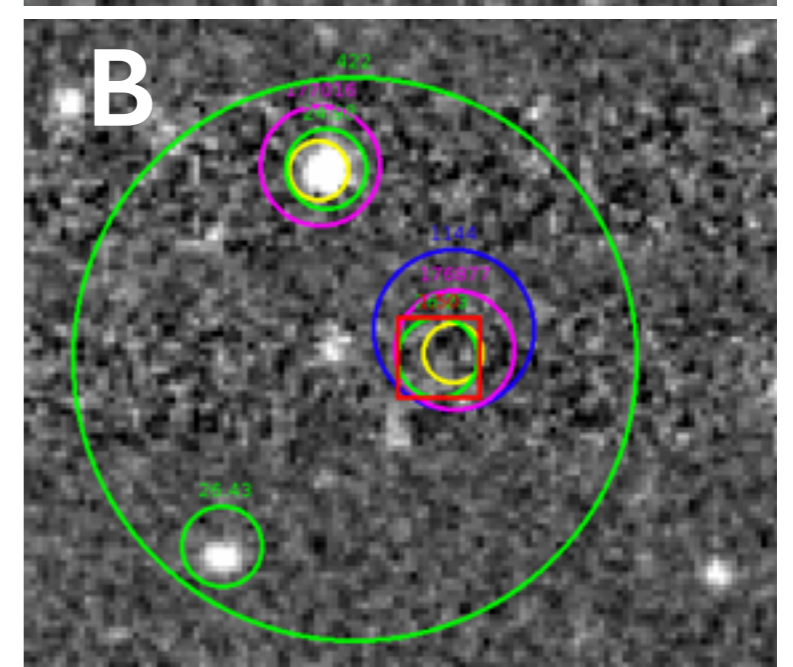
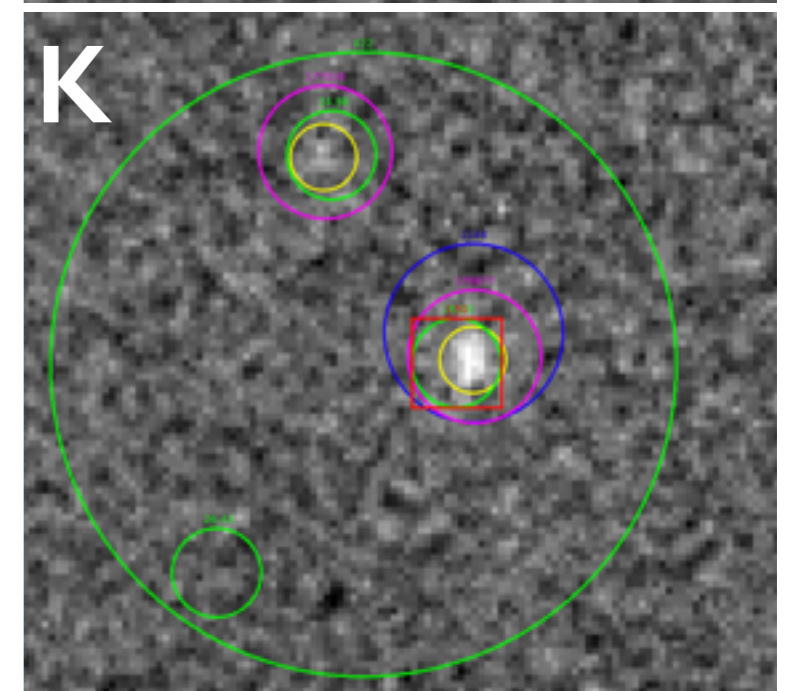
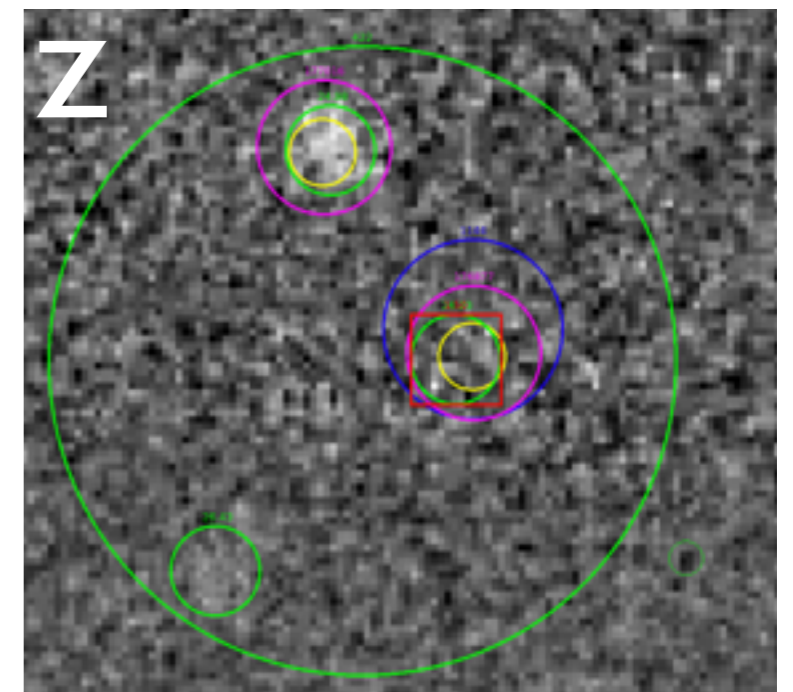
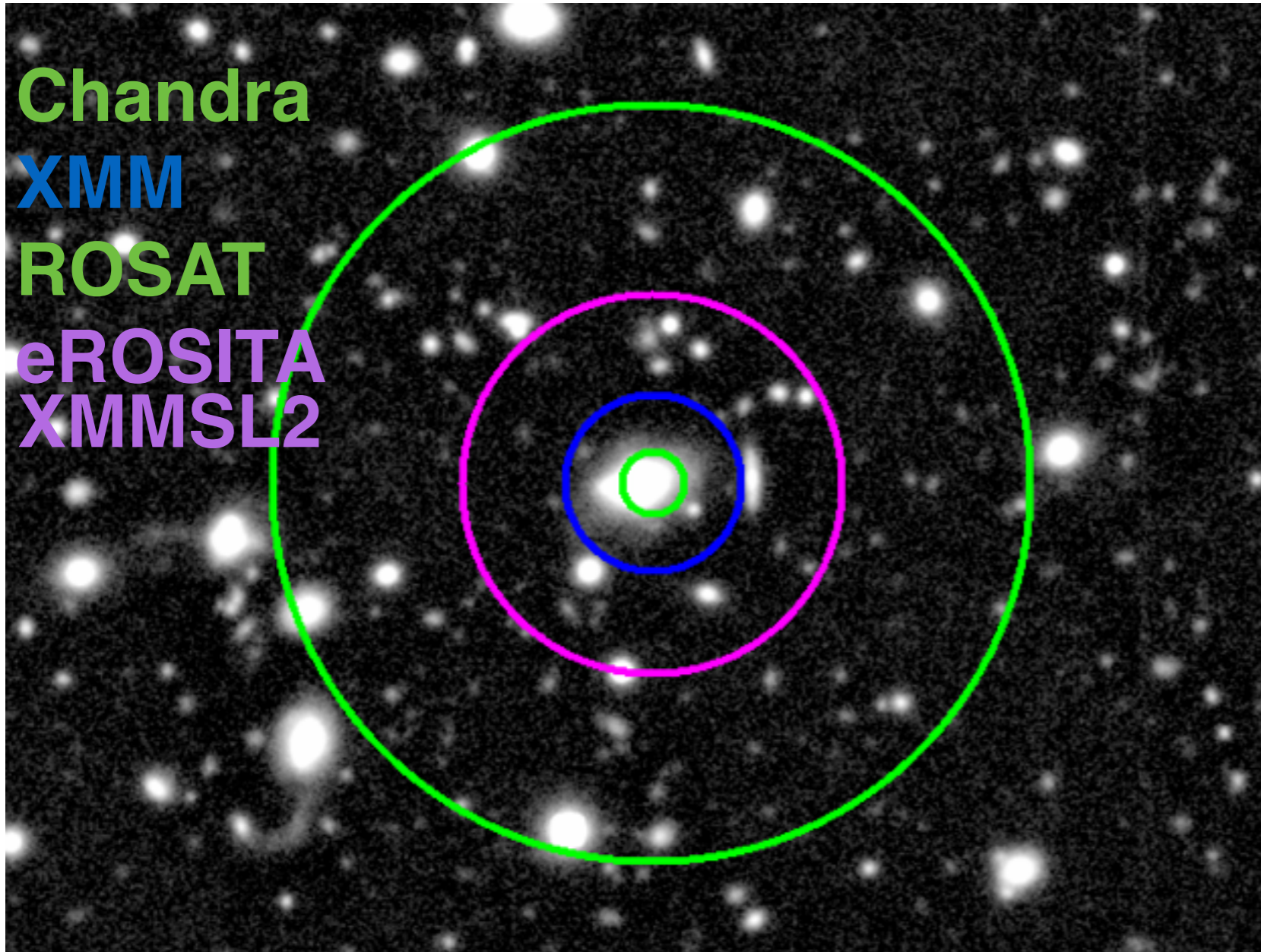


It is data driven:  
problem for small data set

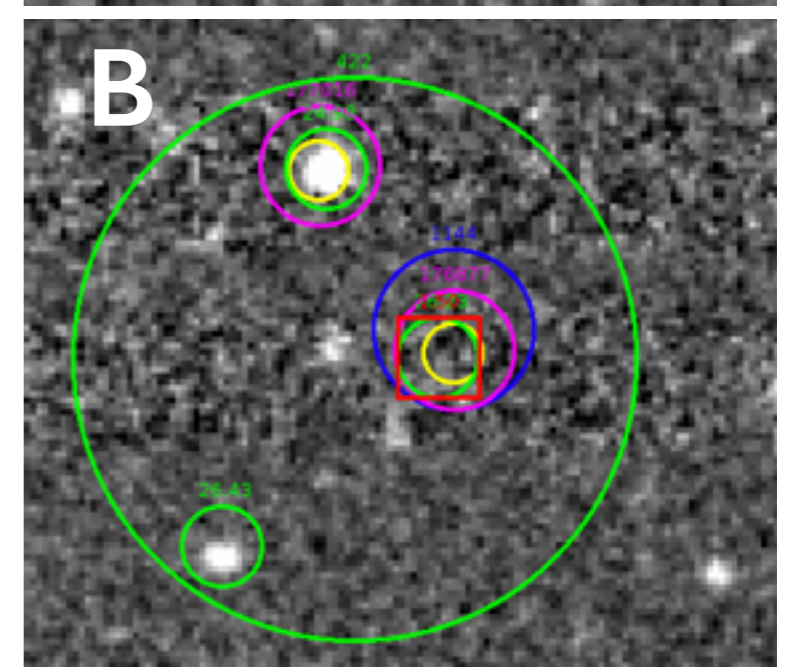
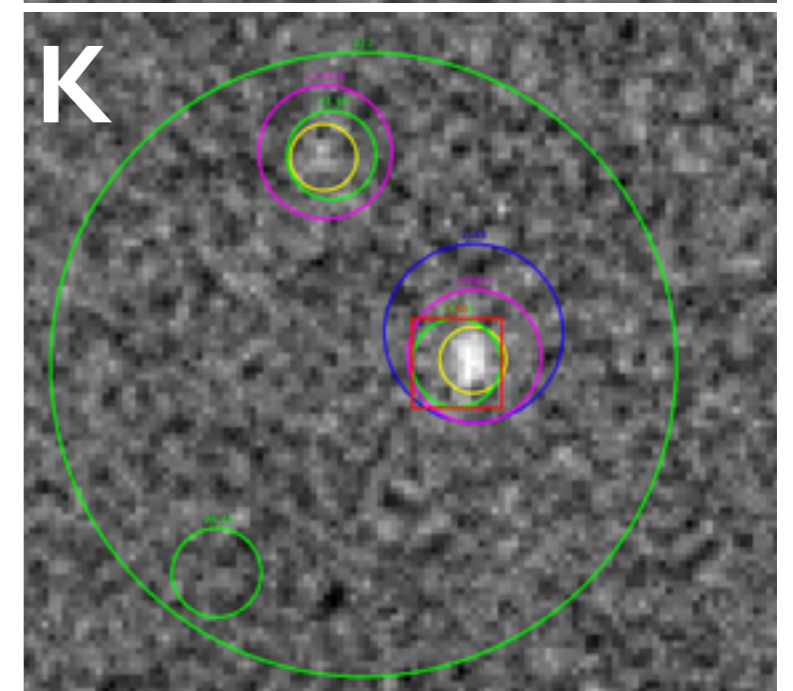
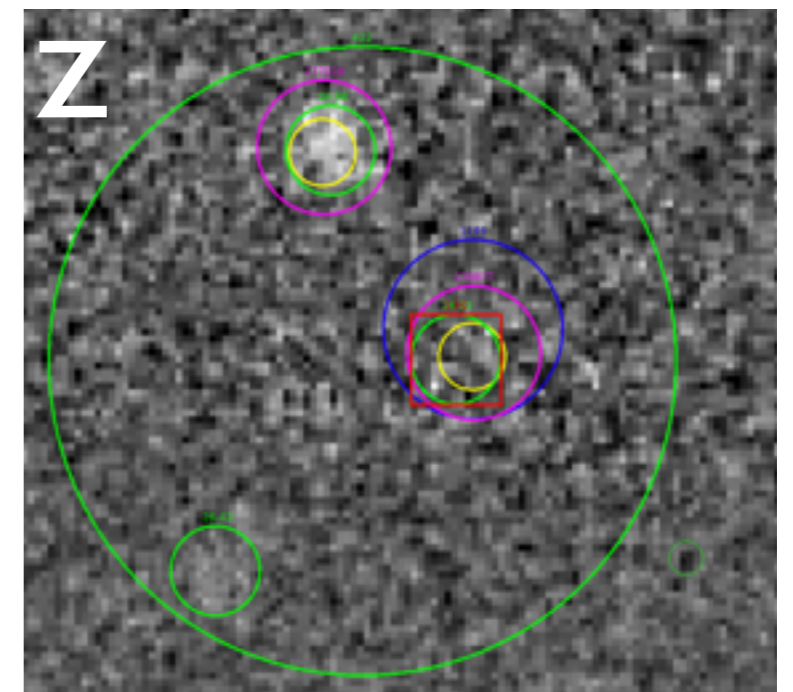
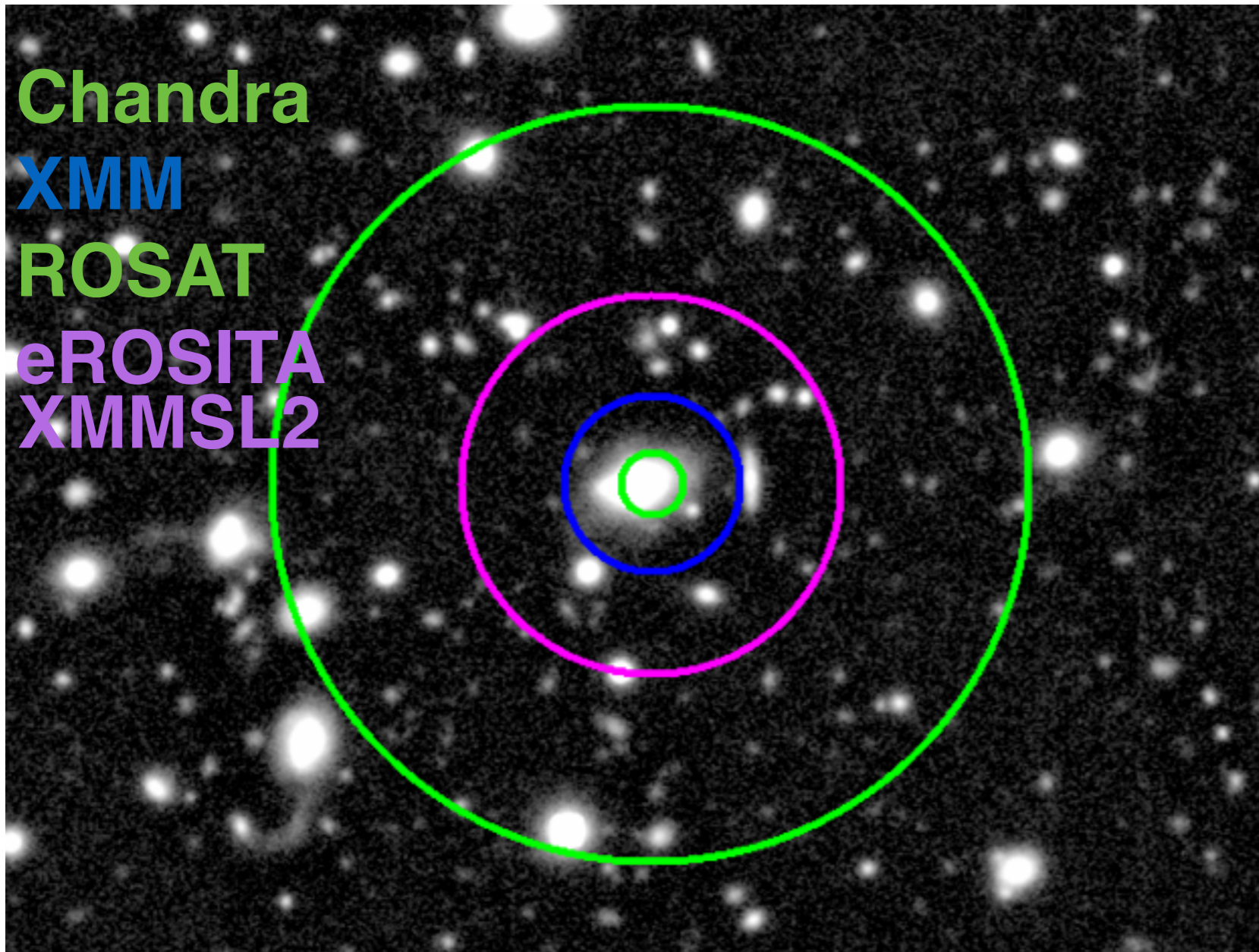
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# X-ray counterparts identification

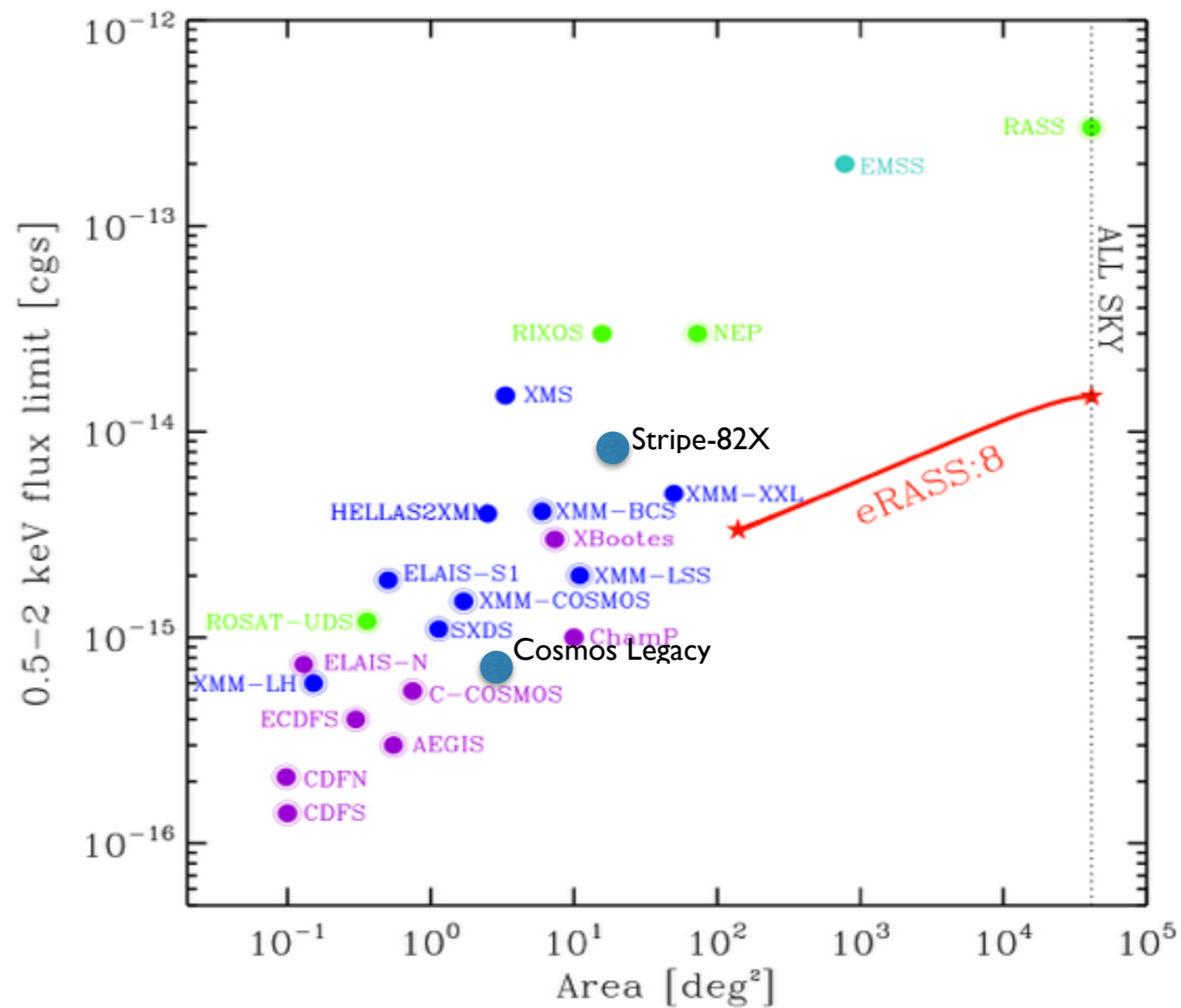
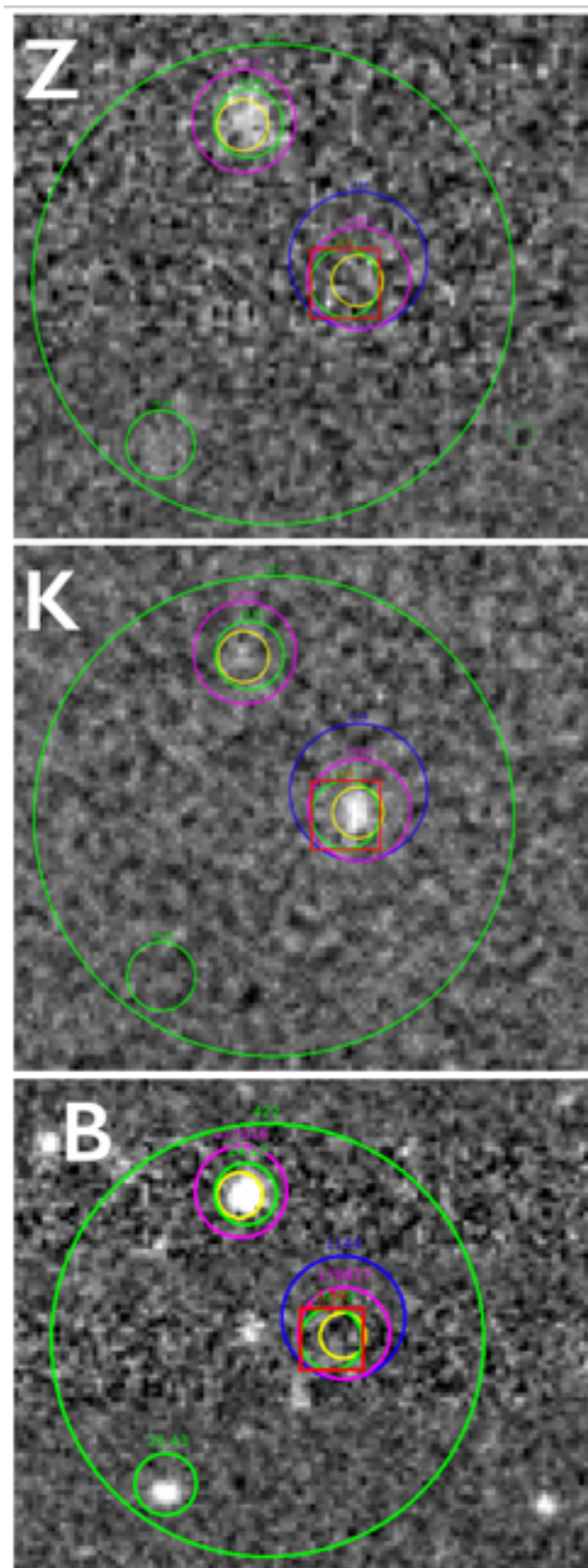


# X-ray counterparts identification



Because of variety of SEDs and large redshift range, one band only, even if deep, is not sufficient

# Use ALL what we know, simultaneously

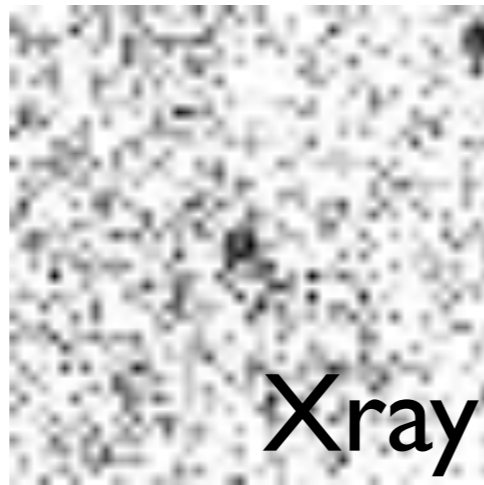


## NWAY (NOT in a nutshell...)

starting from the basics,  
adding informations, step by step

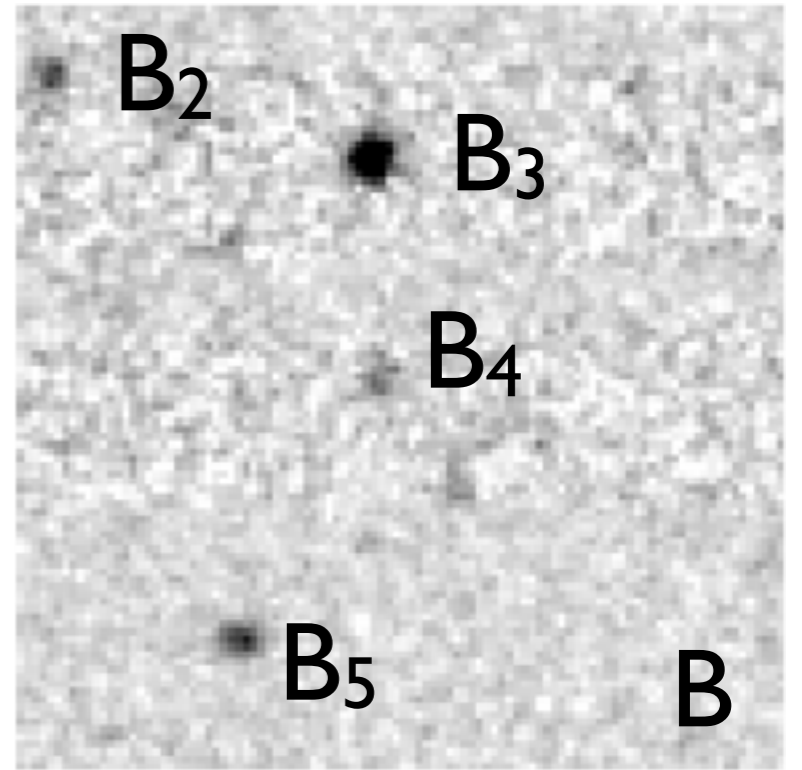
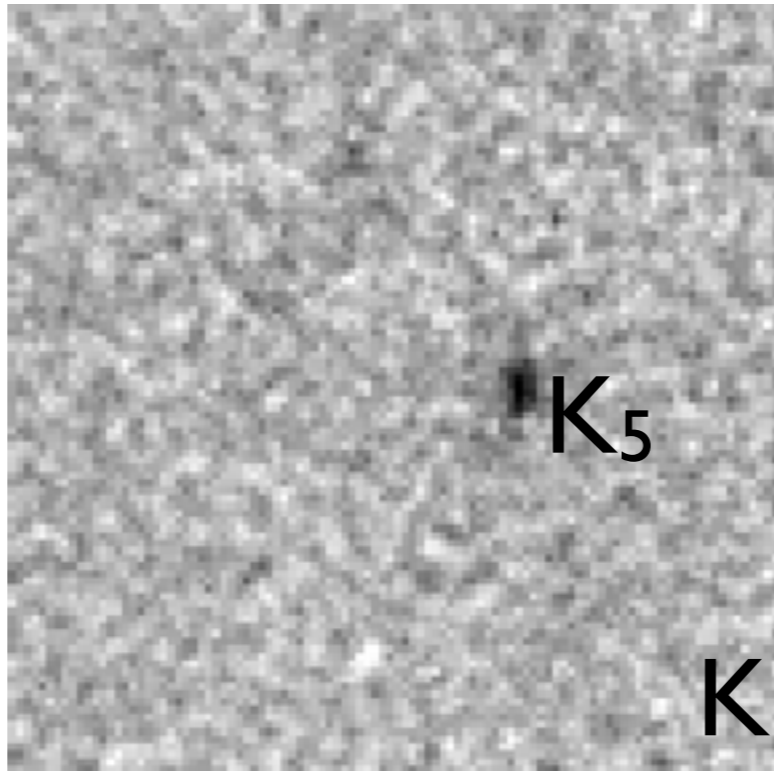
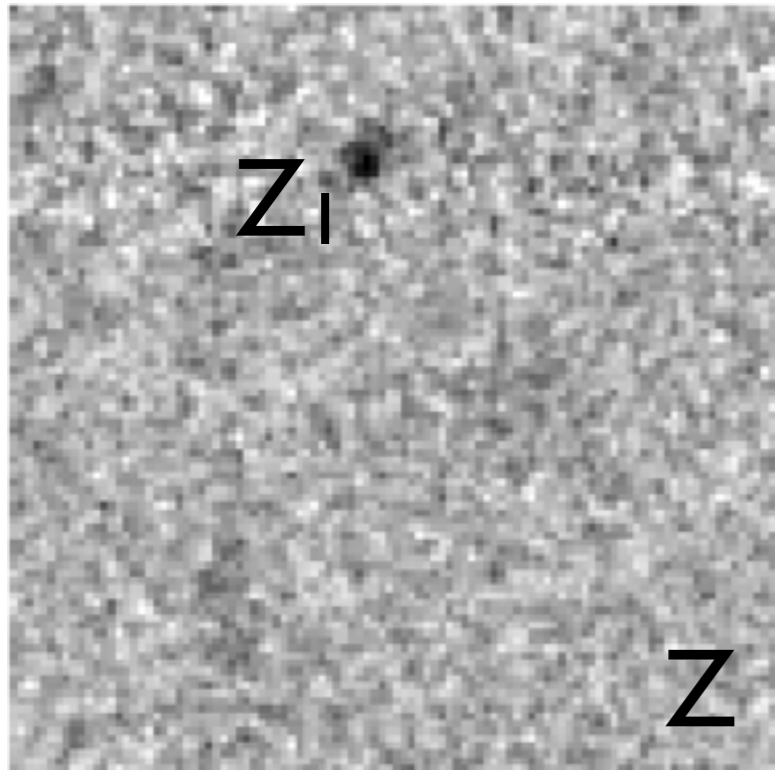
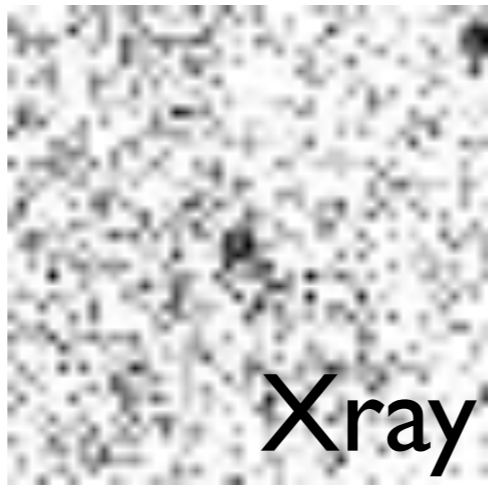
# Inputs

$X_i$   $RA_i$   $Dec_i$   $\sigma_i$



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$X_I$   $RA_I$   $Dec_I$   $\sigma_I$



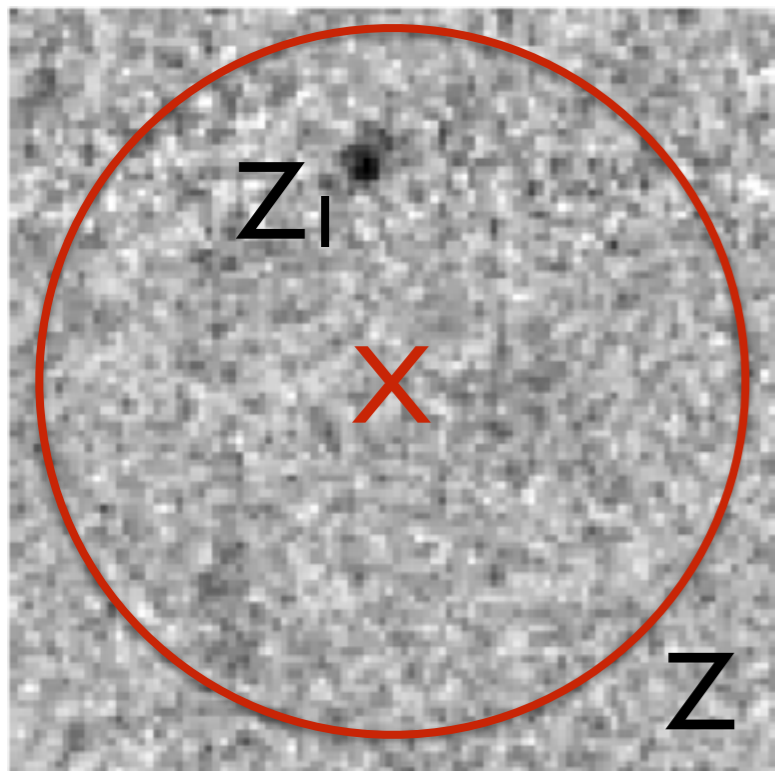
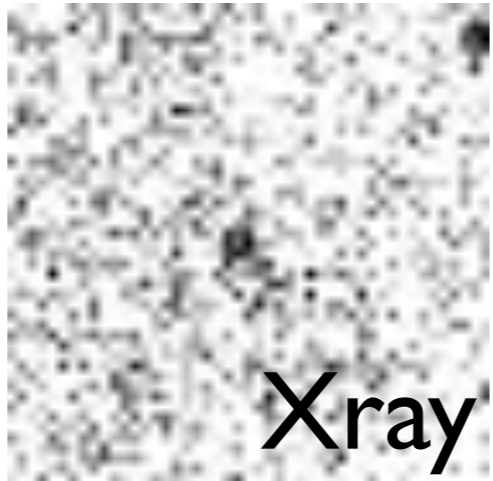
$Z_I$   $RA_{Z_I}$   $Dec_{Z_I}$   $\sigma_{Z_I}$   $mag_{Z_I}$

$K_5$   $RA_{K_5}$   $Dec_{K_5}$   $\sigma_{K_5}$   $mag_{K_5}$

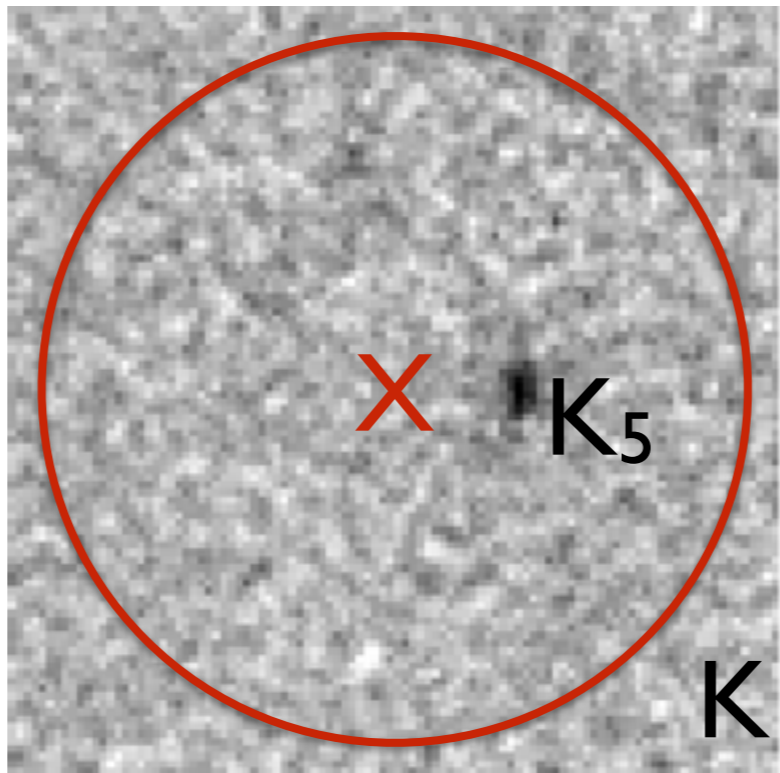
$B_2$   $RA_{B_2}$   $Dec_{B_2}$   $\sigma_{B_2}$   $mag_{B_2}$   
 $B_3$   $RA_{B_3}$   $Dec_{B_3}$   $\sigma_{B_3}$   $mag_{B_3}$   
 $B_4$   $RA_{B_4}$   $Dec_{B_4}$   $\sigma_{B_4}$   $mag_{B_4}$   
 $B_5$   $RA_{B_5}$   $Dec_{B_5}$   $\sigma_{B_5}$   $mag_{B_5}$

# Inputs

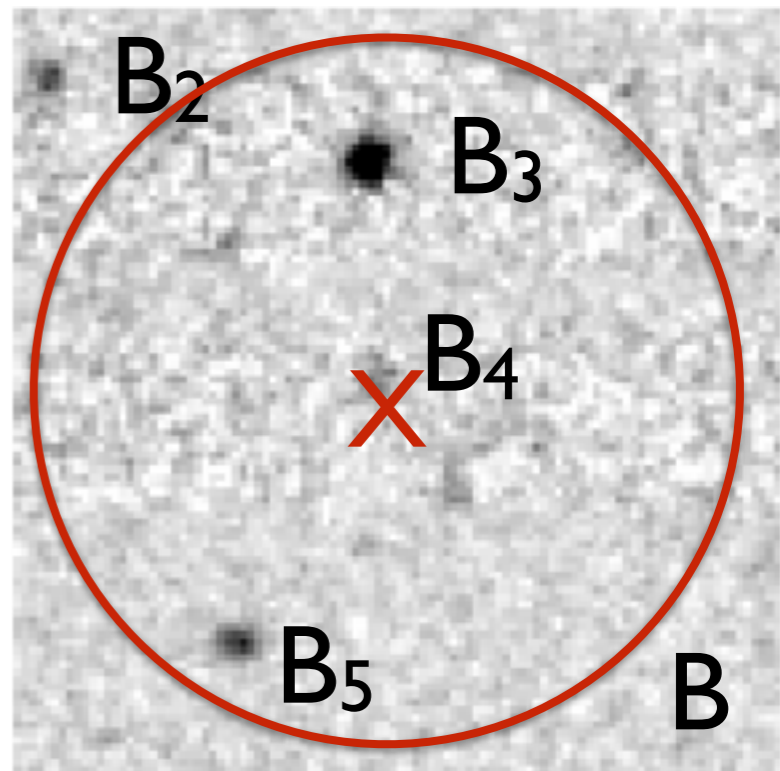
$X_I$   $RA_I$   $Dec_I$   $\sigma_I$



$Z_I$   $RA_{Z_I}$   $Dec_{Z_I}$   $\sigma_{Z_I}$   $mag_{Z_I}$



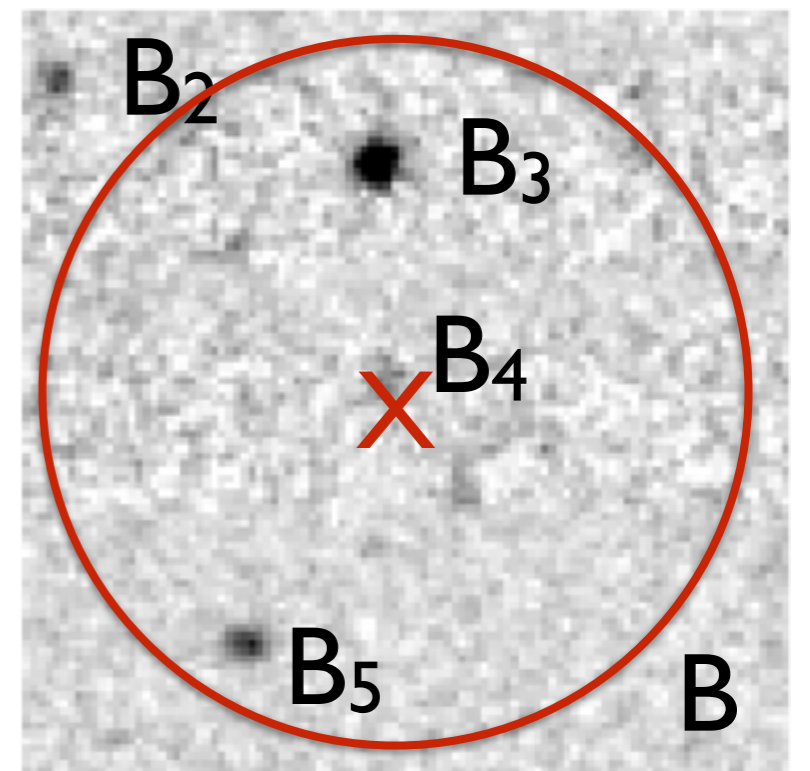
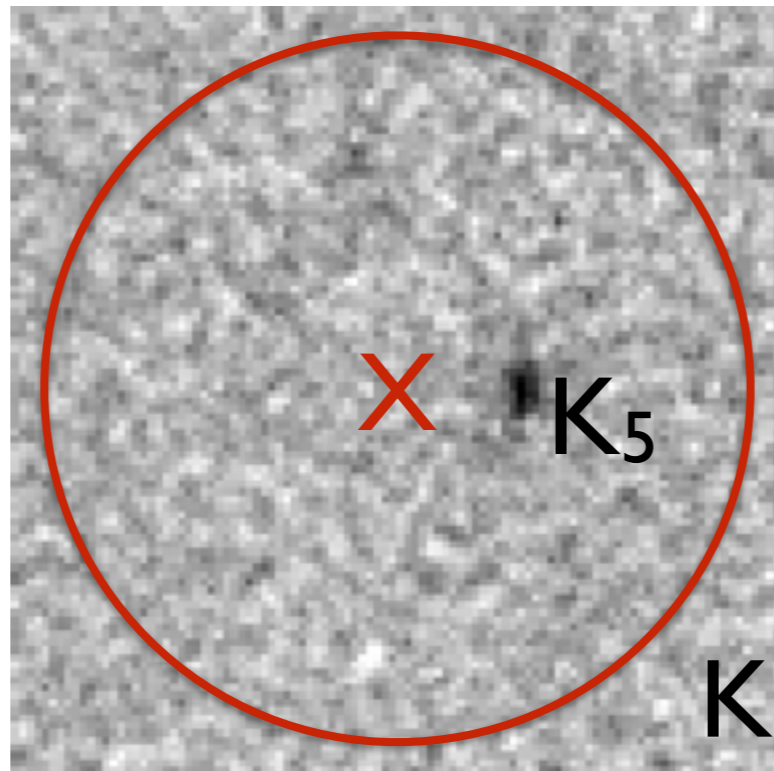
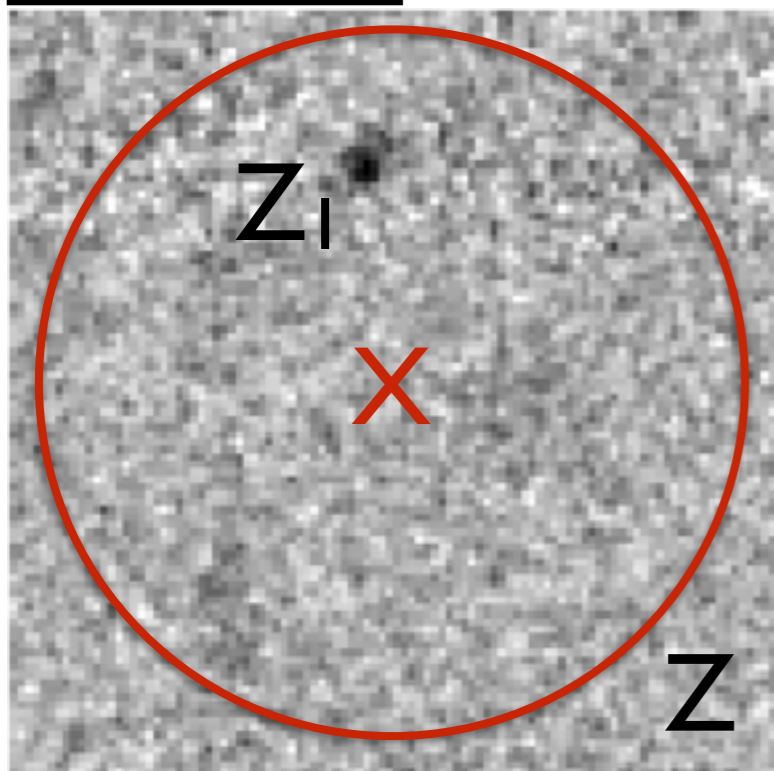
$K_5$   $RA_{K_5}$   $Dec_{K_5}$   $\sigma_{K_5}$   $mag_{K_5}$



$B_2$   $RA_{B_2}$   $Dec_{B_2}$   $\sigma_{B_2}$   $mag_{B_2}$   
 $B_3$   $RA_{B_3}$   $Dec_{B_3}$   $\sigma_{B_3}$   $mag_{B_3}$   
 $B_4$   $RA_{B_4}$   $Dec_{B_4}$   $\sigma_{B_4}$   $mag_{B_4}$   
 $B_5$   $RA_{B_5}$   $Dec_{B_5}$   $\sigma_{B_5}$   $mag_{B_5}$



# output



X cat. entry	Z cat. entry	K cat. entry	B cat. entry	P (X has a ctp)	P (this is the correct ctp)
1	1	—	3	0.8	0.4
1	—	5	—	0.8	0.9
1	—	—	4	0.8	0.1
1	—	—	5	0.8	0.2
2	...	...	...	...	...
2	...	...	...	...	...

**(i) Matching of N catalogues simultaneously.**

**(ii) Computation of all combinatorially possible matches**

catalogues X, Z, K, B with  $N_x, N_z, N_k, N_B$  sources  
and density  $v_i = N_i / \Omega_i$

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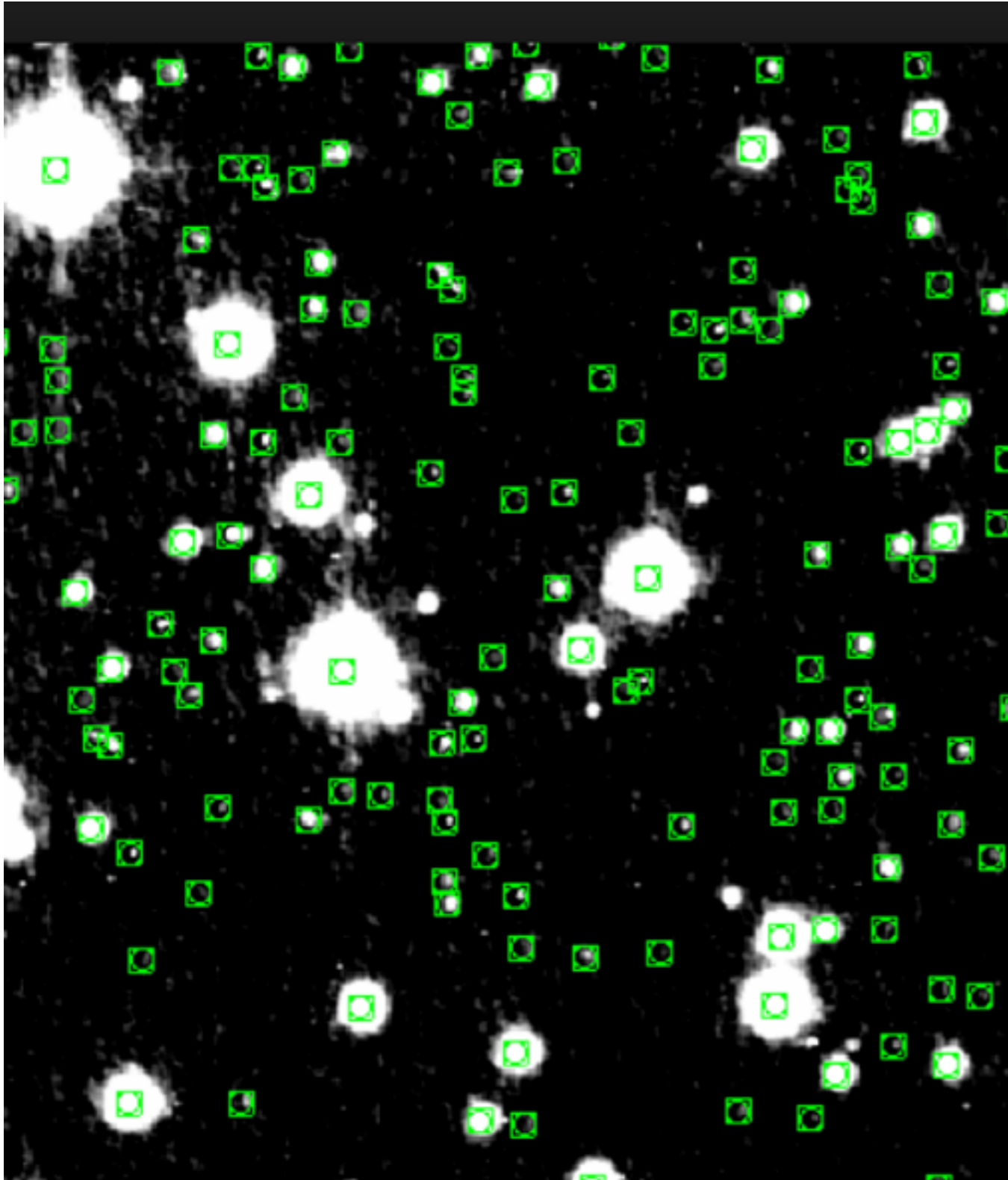
catalogues X, Z, K, B with  $N_x, N_z, N_k, N_B$  sources  
and density  $\nu_i = N_i / \Omega_i$

If each catalogue covers the same area with some respective, homogeneous source density, the probability of one X-ray source to be associated with k (physically unrelated) objects is provide by combinatorial analysis:

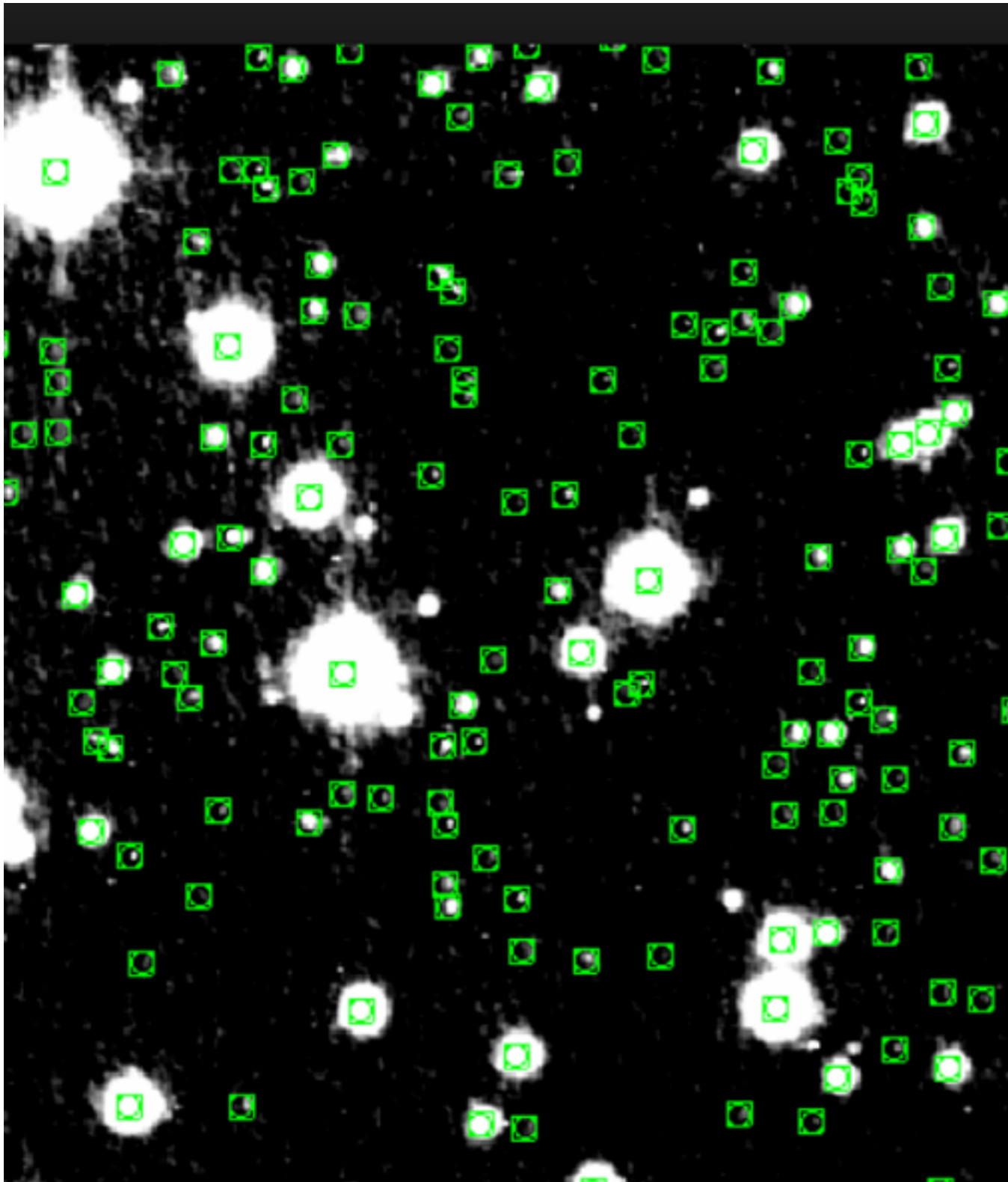
prior P of an  
association

$$P(H) = N_1 / \prod_{i=1}^k N_i = 1 / \prod_{i=2}^k N_i = 1 / \prod_{i=2}^k \nu_i \Omega_i. \quad (B1)$$

# Reality: sources are missing from the catalogs



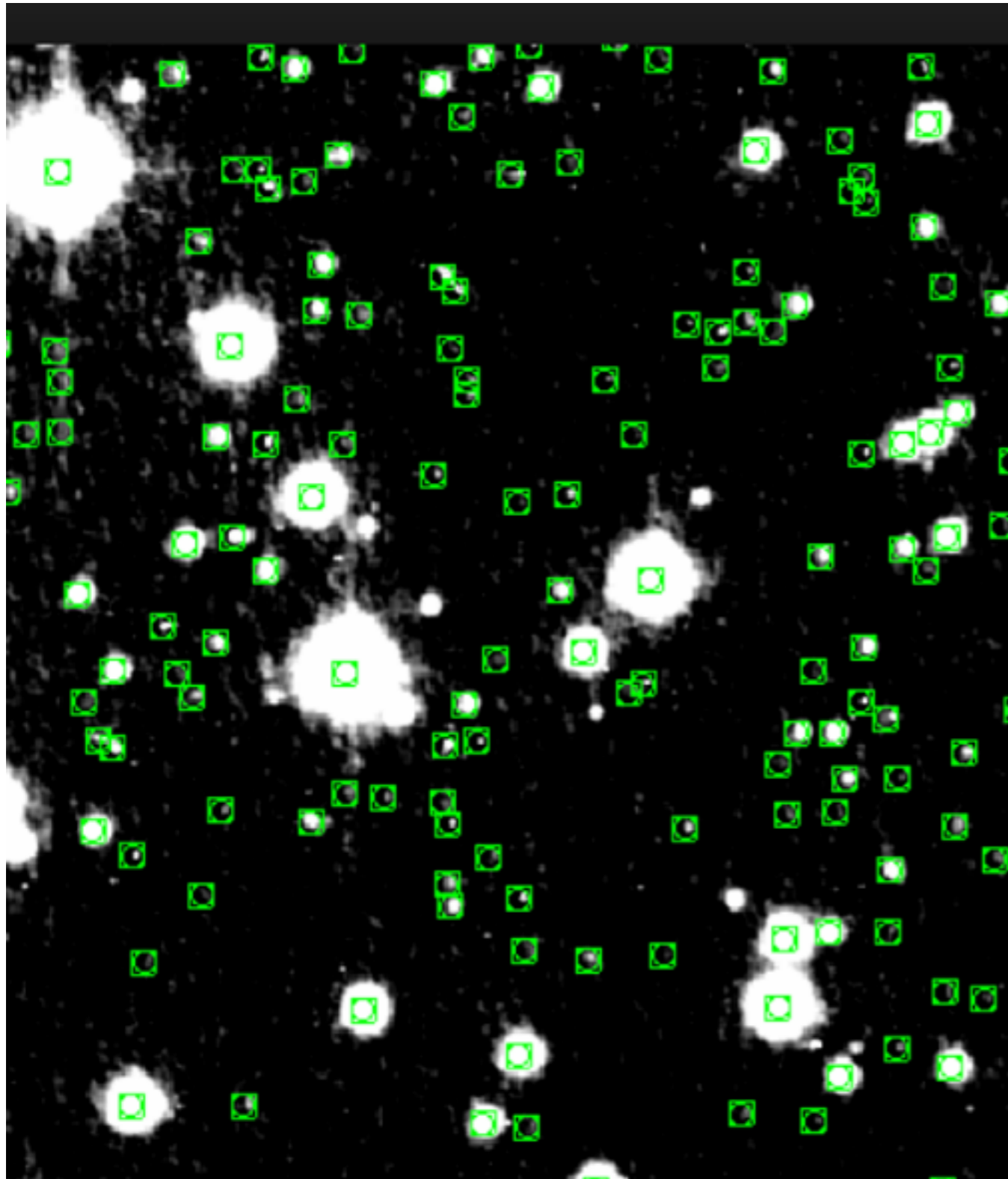
# Reality: sources are missing from the catalogs



prior  
completeness  
factor

$$P(H) = c / \prod_{i=2}^k \nu_i \Omega_{1i}.$$

# Reality: sources are missing from the catalogs



prior  
completeness  
factor

$$P(H) = c / \prod_{i=2}^k \nu_i \Omega_1.$$

“c” gained by experience with associations done with high resolution instruments (e.g. Chandra) and deep ancillary data

**(iii) Computation of the probability of each possible match, using distances, errors and number densities.**

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Budavari&Szalai 2008



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Budavari&Szalai 2008

For a source  $X_1$  in catalogue X  
and a source  $Z_1$  in a catalog Z

$$P(D|X_1 \text{ ass. } Z_1) = \frac{2}{(\sigma_{X_1}^2 + \sigma_{Z_1}^2)} \exp\left[-\frac{Dist_{X_1 Z_1}^2}{2(\sigma_{X_1}^2 + \sigma_{Z_1}^2)}\right]$$

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For a source  $X_1$  in catalogue X and a source  $Z_1$  in a catalog Z and  $K_1$  in catalog K

$$P(D|X_1 \text{ ass. } Z_1 \text{ ass. } K_1) = \frac{4 \exp\left\{-\frac{\sigma_{K_1}^2 Dist_{X_1 Z_1}^2 + \sigma_{X_1}^2 Dist_{Z_1 K_1}^2 + \sigma_{Z_1}^2 Dist_{K_1 X_1}^2}{2(\sigma_{X_1}^2 \sigma_{Z_1}^2 + \sigma_{Z_1}^2 \sigma_{K_1}^2 + \sigma_{K_1}^2 \sigma_{X_1}^2)}\right\}}{\sigma_{X_1}^2 \sigma_{Z_1}^2 + \sigma_{Z_1}^2 \sigma_{K_1}^2 + \sigma_{K_1}^2 \sigma_{X_1}^2}$$

**(iii) Computation of the probability of each possible match, using distances, errors and number densities.**

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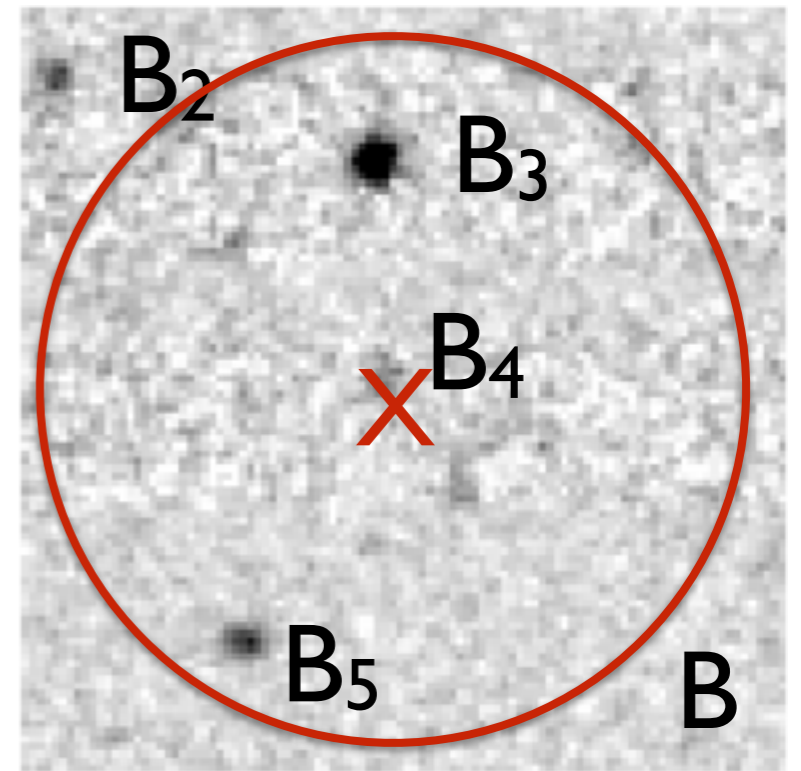
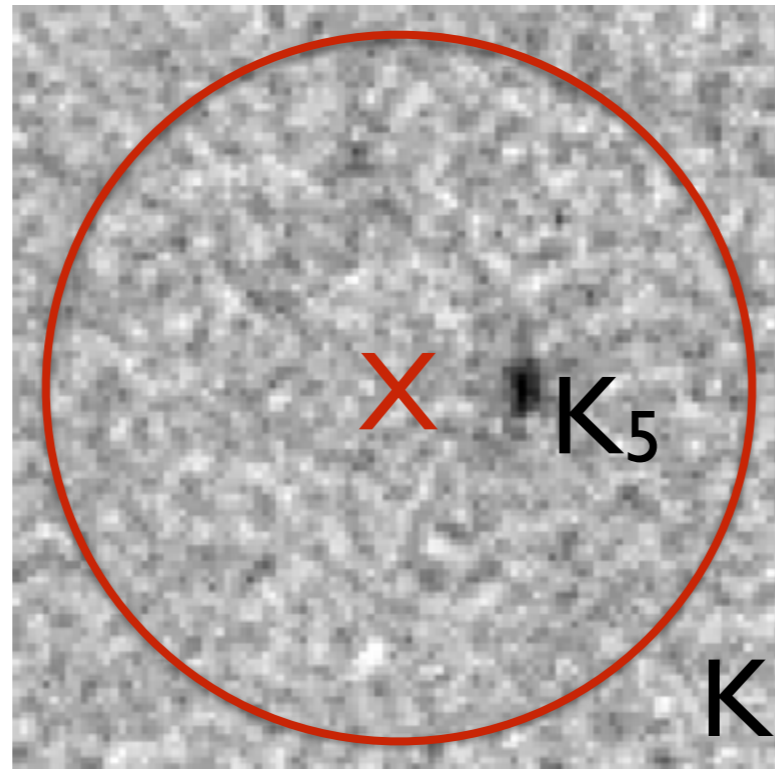
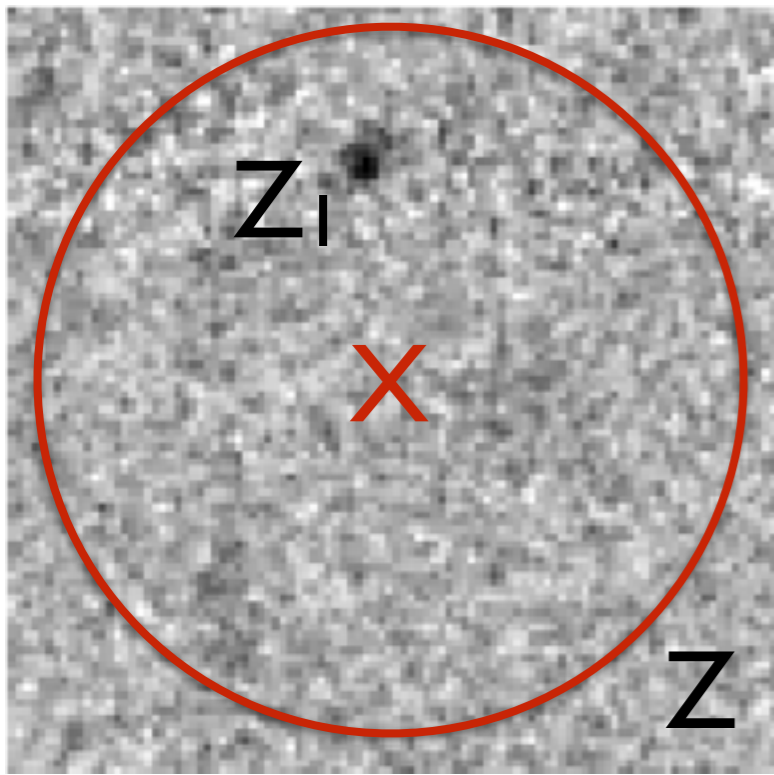
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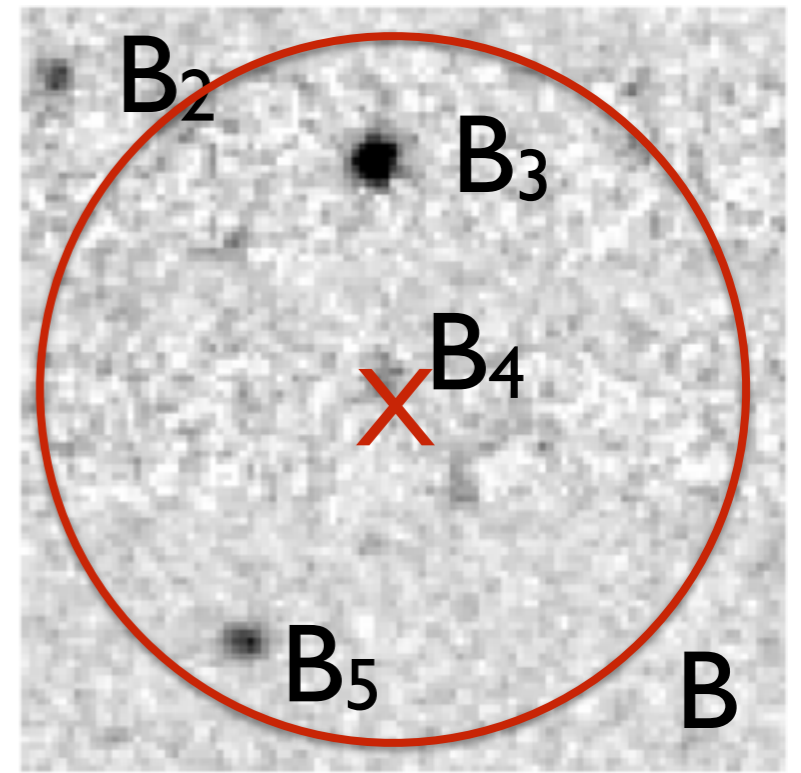
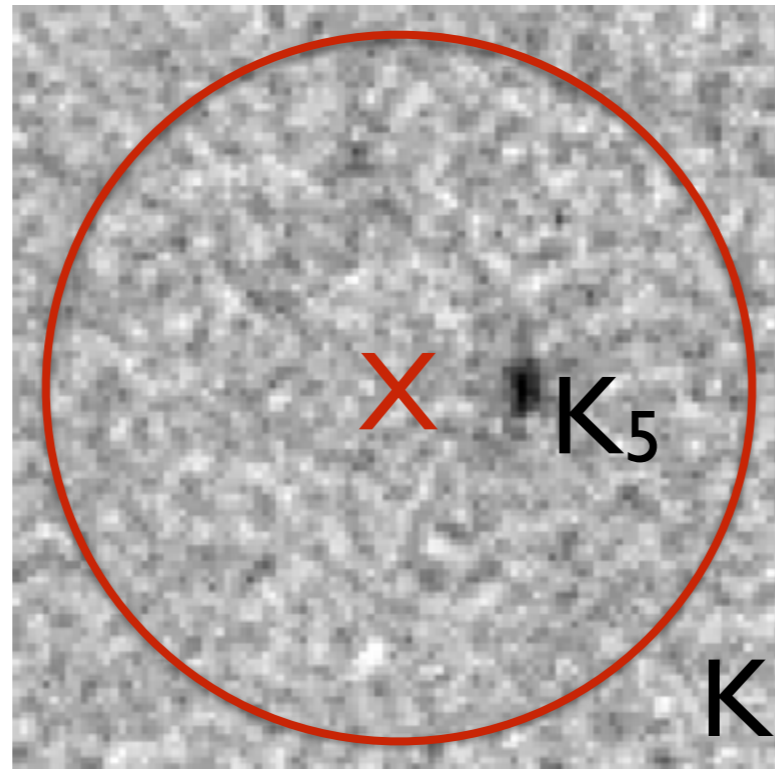
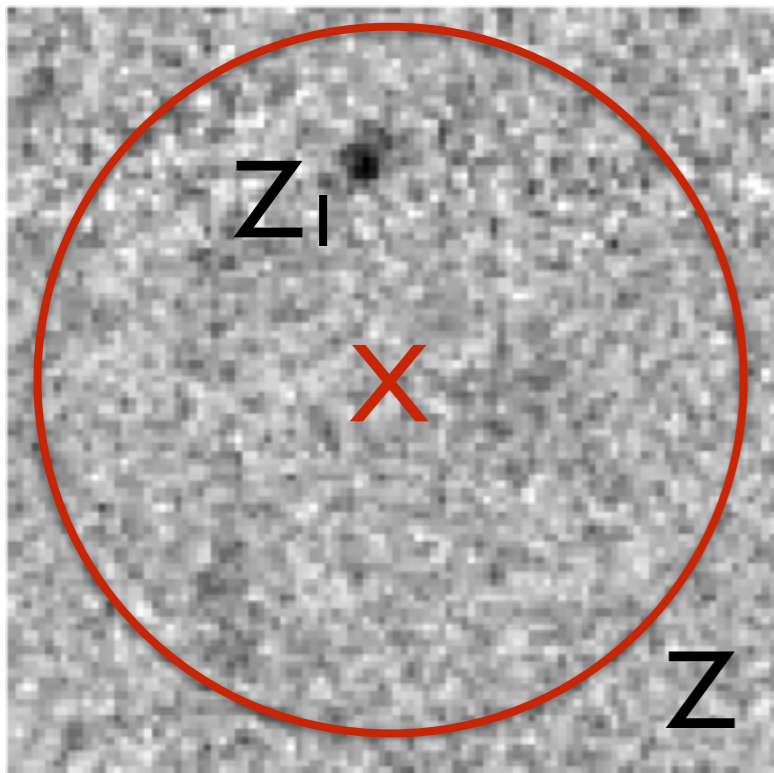
Compact form

$$P(D|H) = 2^{k-1} \frac{\prod \sigma_i^{-2}}{\sum \sigma_i^{-2}} \exp\left\{-\frac{\sum_{i<j} Dist_{ij}^2 \sigma_j^{-2} \sigma_i^{-2}}{2 \sum \sigma_i^{-2}}\right\}$$

RESULTS I

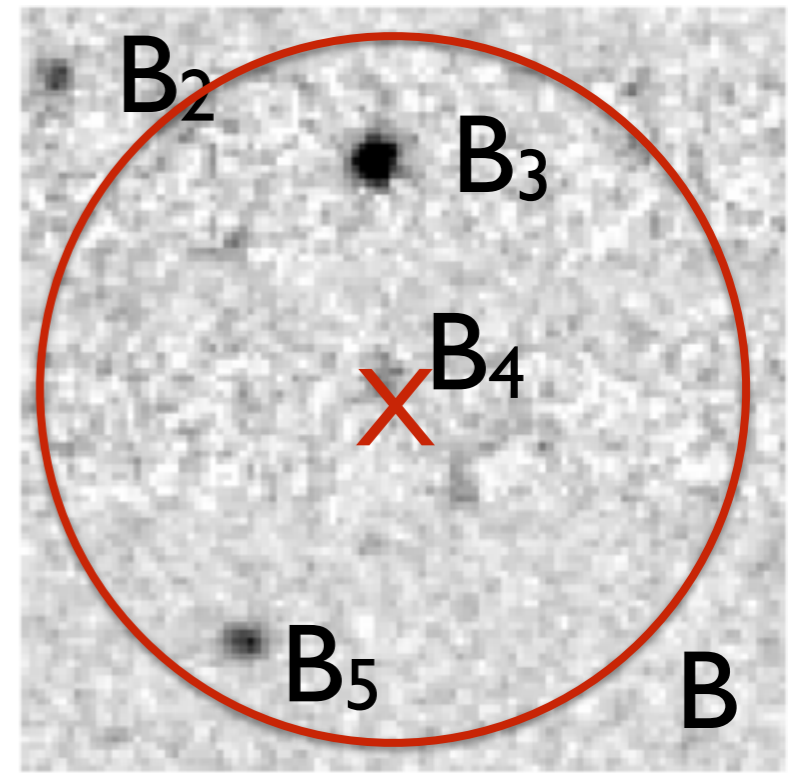
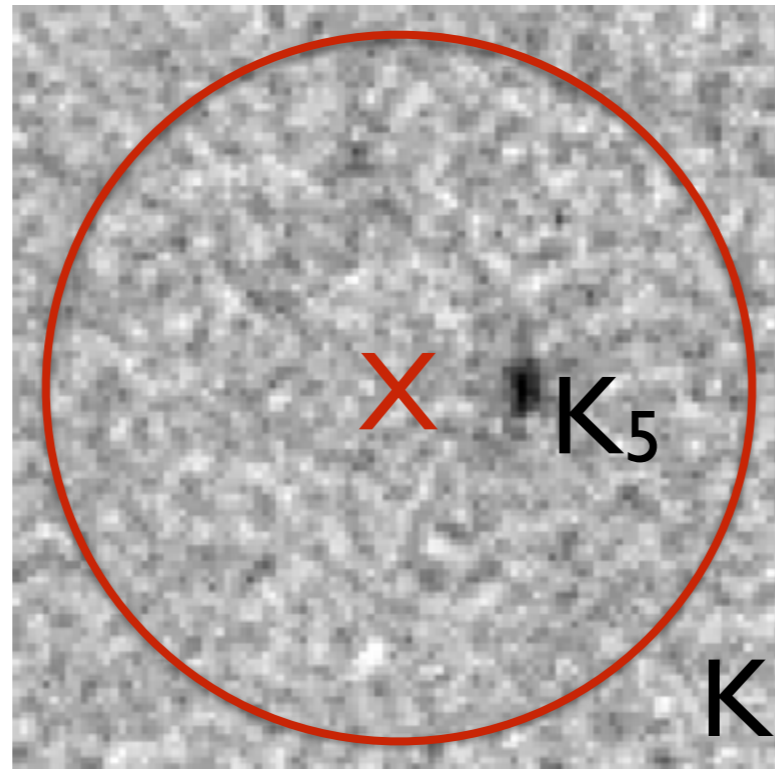
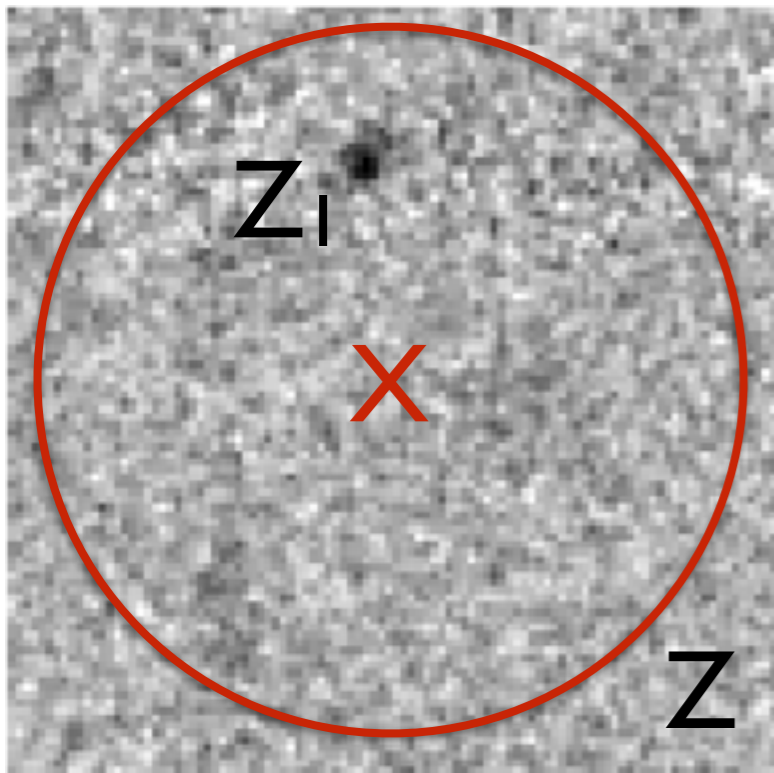


RESULTS I



$$P(H|D) \propto P(H) \times P(D|H).$$

RESULTS I



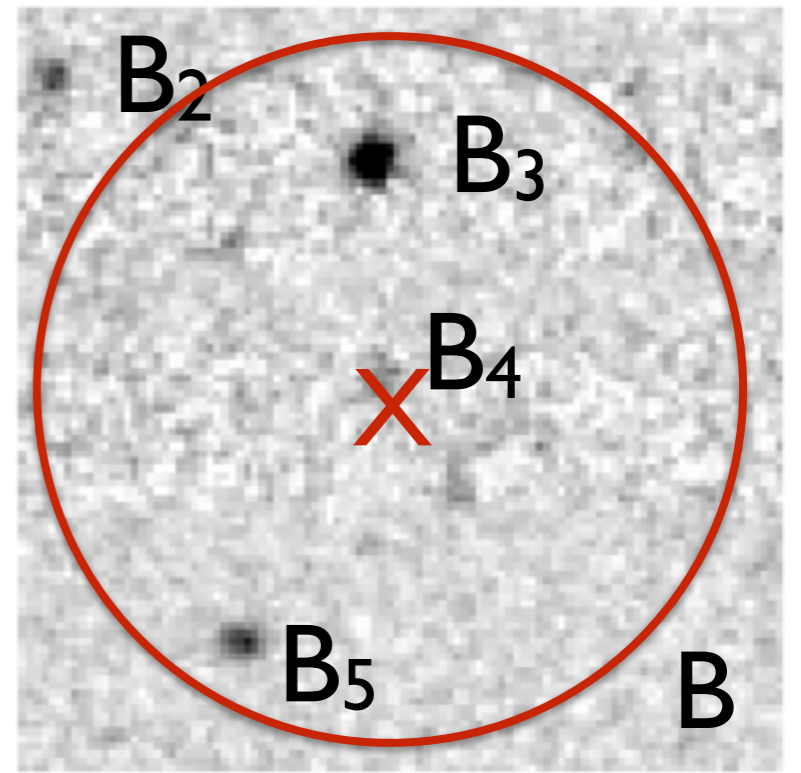
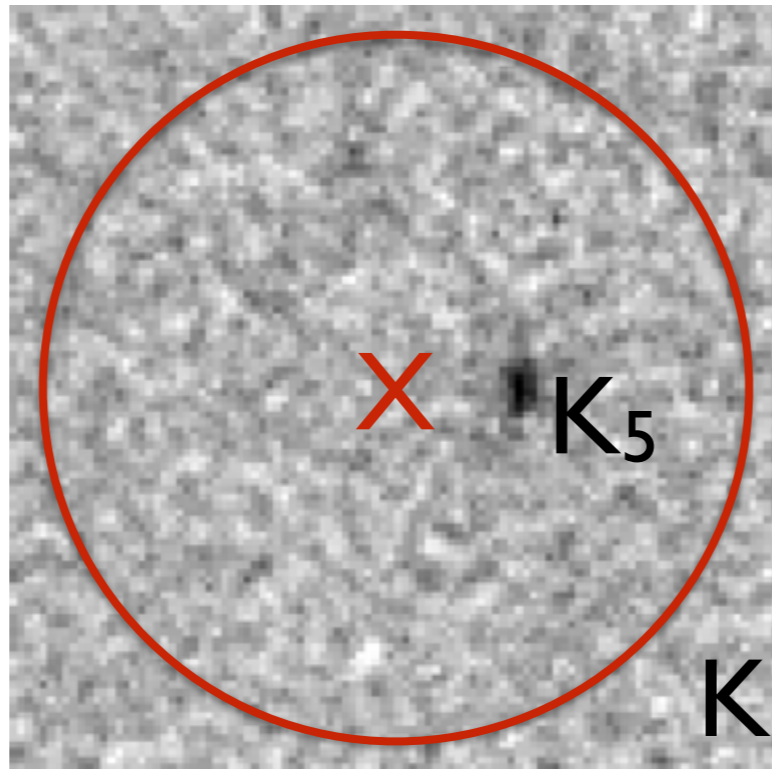
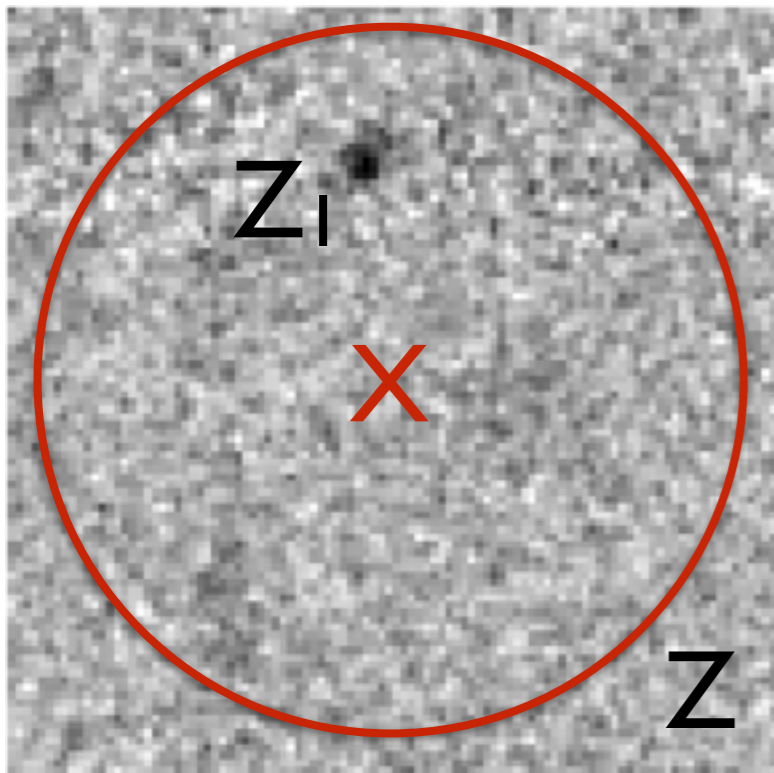
prior

posterior prob.

$$P(H|D) \propto P(H) \times P(D|H).$$

likelihood

**RESULTS I**



prior

posterior prob.

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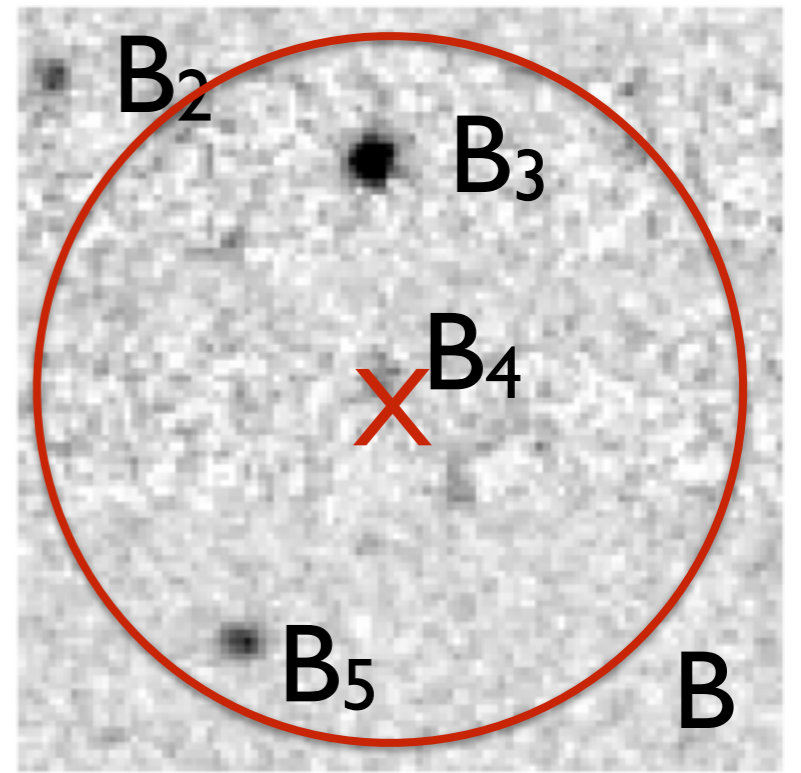
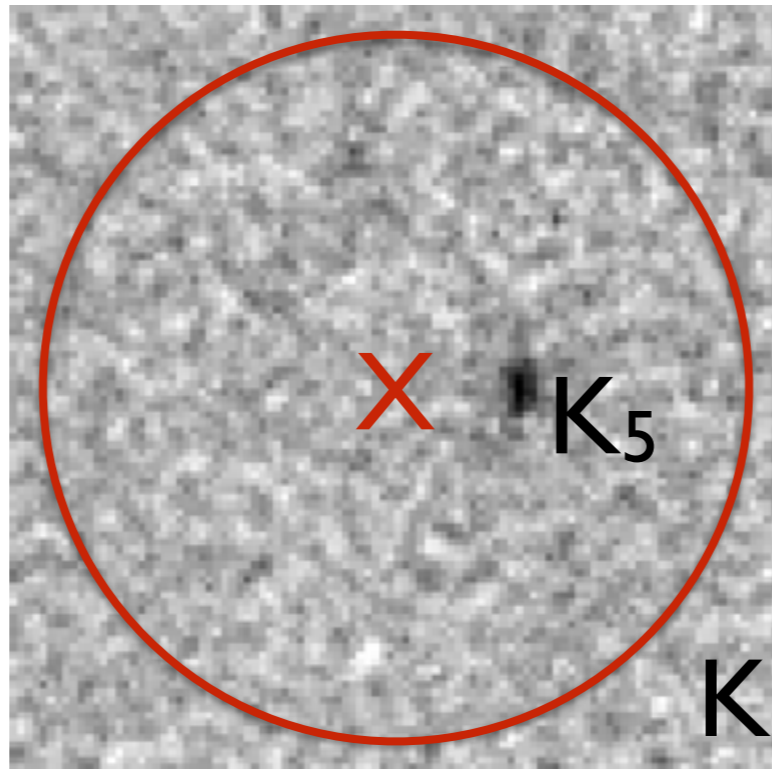
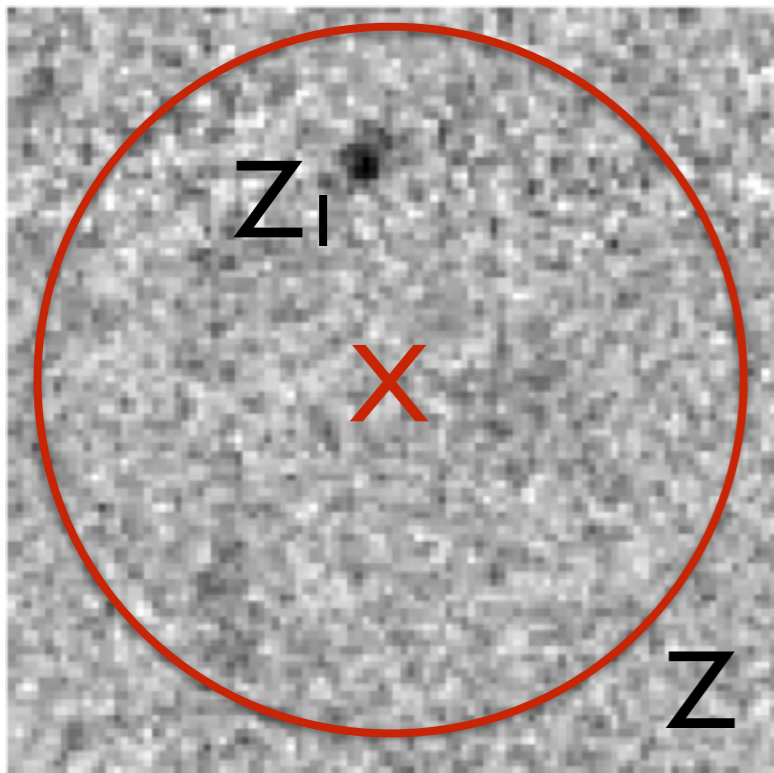
likelihood

The posterior probability that X<sub>1</sub> has any counterpart **p\_any**

$$p\_any = 1 - (P(H_0|D) / \sum_i P(H_i|D))$$

$$P(H_0|D) \propto P(H_0)P(D|H_0) = c \times 1$$

**RESULTS I**



prior

posterior prob.

$$P(H|D) \propto P(H) \times P(D|H).$$

likelihood

The posterior probability that  $X_1$  has any counterpart  $p_{any}$

$$p_{any} = 1 - (P(H_0|D) / \sum_i P(H_i|D))$$

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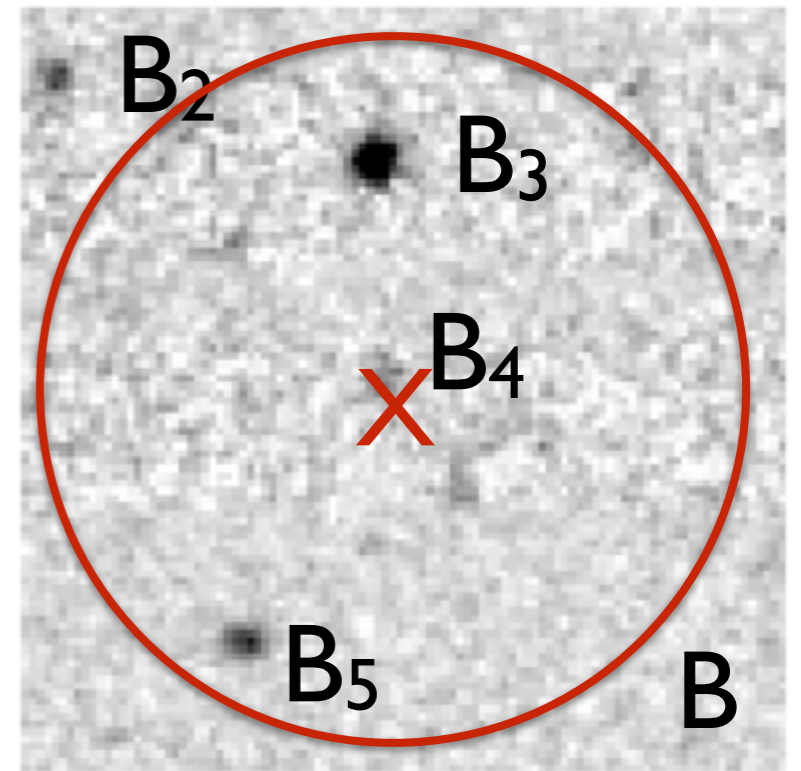
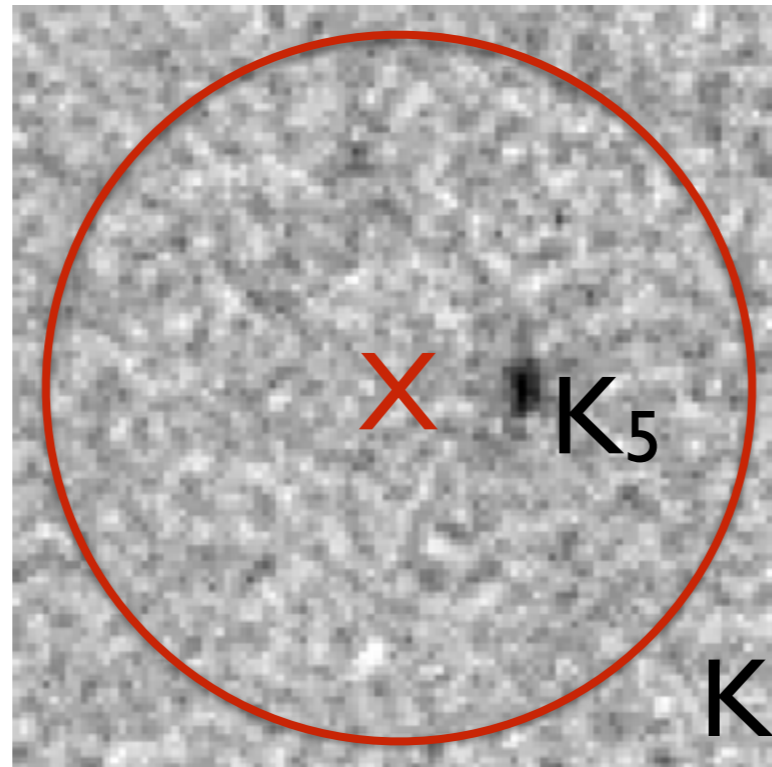
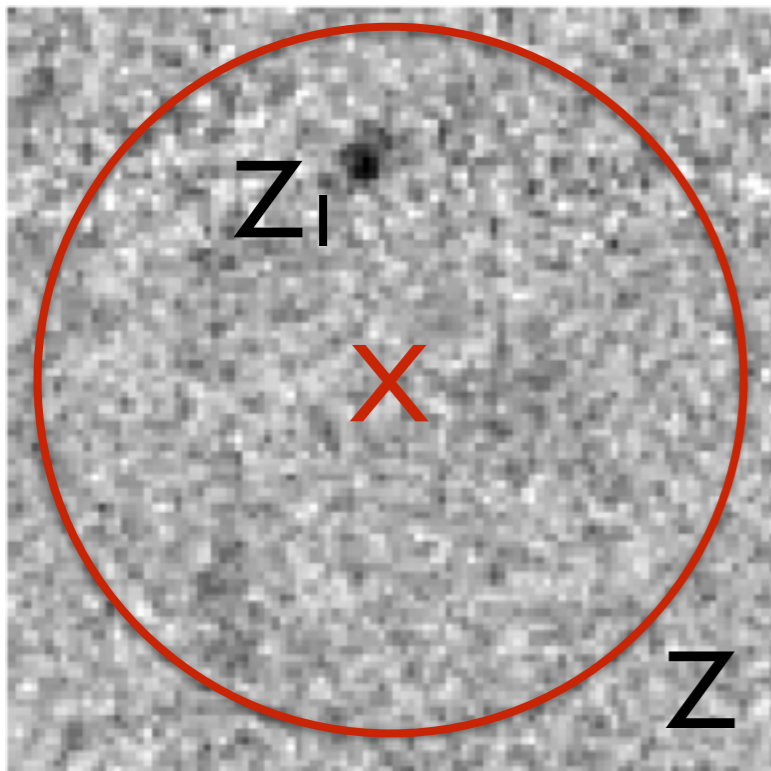
The relative posterior probability for an association,  $p_i$

$$p_i = P(H_i|D) / \sum_{i>0} P(H_i|D)$$



# RESULTS I

See also Pineau et al 2017



prior

posterior prob.

$$P(H|D) \propto P(H) \times P(D|H)$$

likelihood

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(vi) Refining the match probabilities by incorporating magnitude distribution, colors, magnitude&colors or other information about the sources of interest ...

ML  
(Sutherland&Saunders 92)

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$$LR = \frac{q(m)}{n(m)} \times f(r)$$

$$P(D|H) = P(D_\phi|H) \times P(D_m|H) \quad (\text{B11})$$

$$= P(D_\phi|H) \times \frac{\bar{q}(m)}{\bar{n}(m)}, \quad (\text{B12})$$

probability that a correct ctp to a X-ray source or a generic field source has magnitude  $m$ , respectively.

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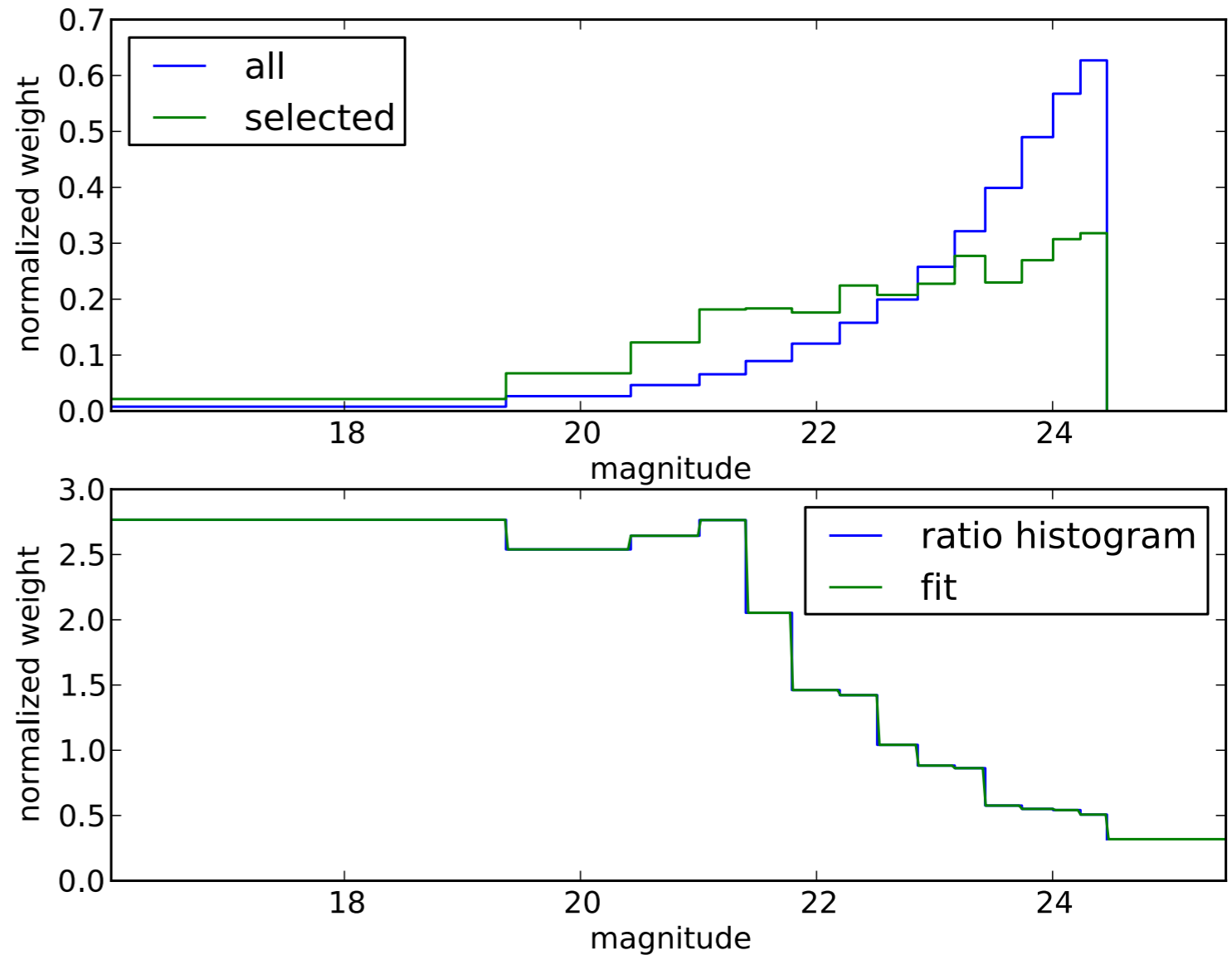
for many catalogs: 
$$P(D_m|H) = \prod \frac{\int_m \bar{q}(m) p(m|D_m) dm}{\int_m \bar{n}(m) p(m|D_m) dm} \quad (\text{B13})$$

Gaussian error distribution with mean  $m$  and standard deviation  $\sigma_m$

(vii) ...either with an external information or internal calibration

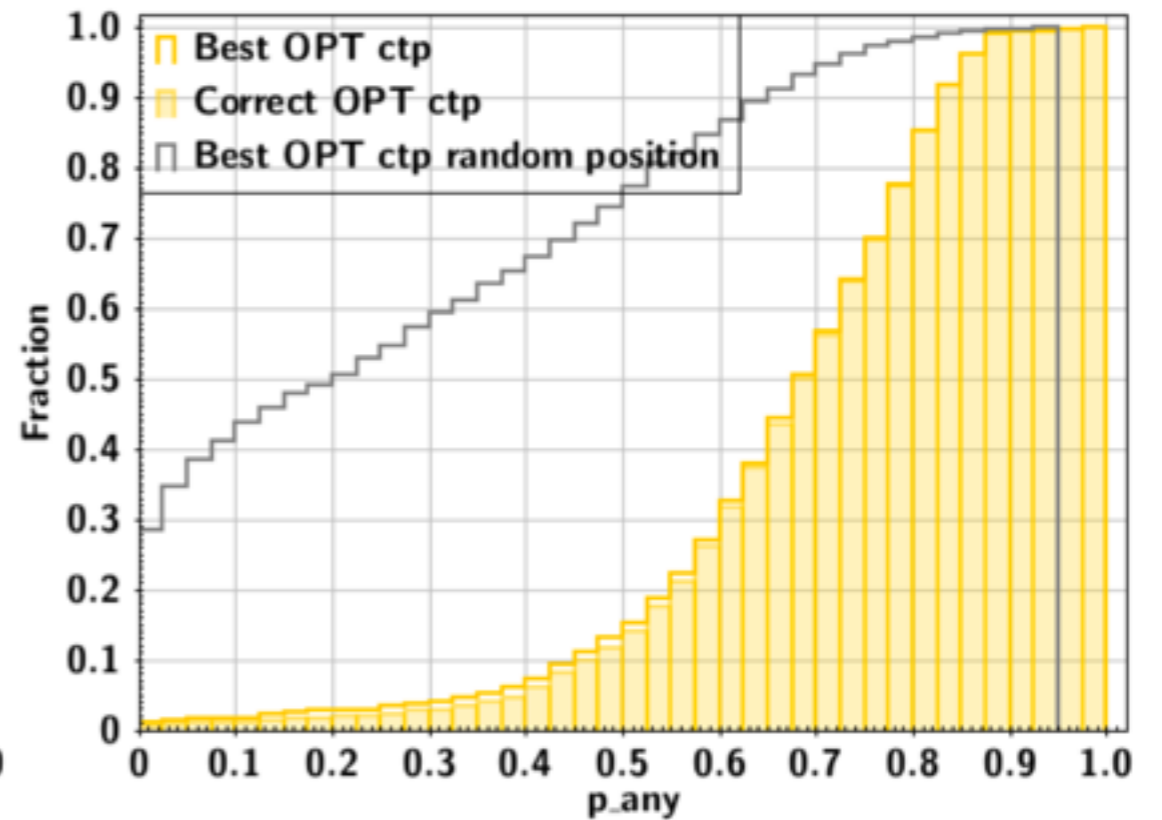
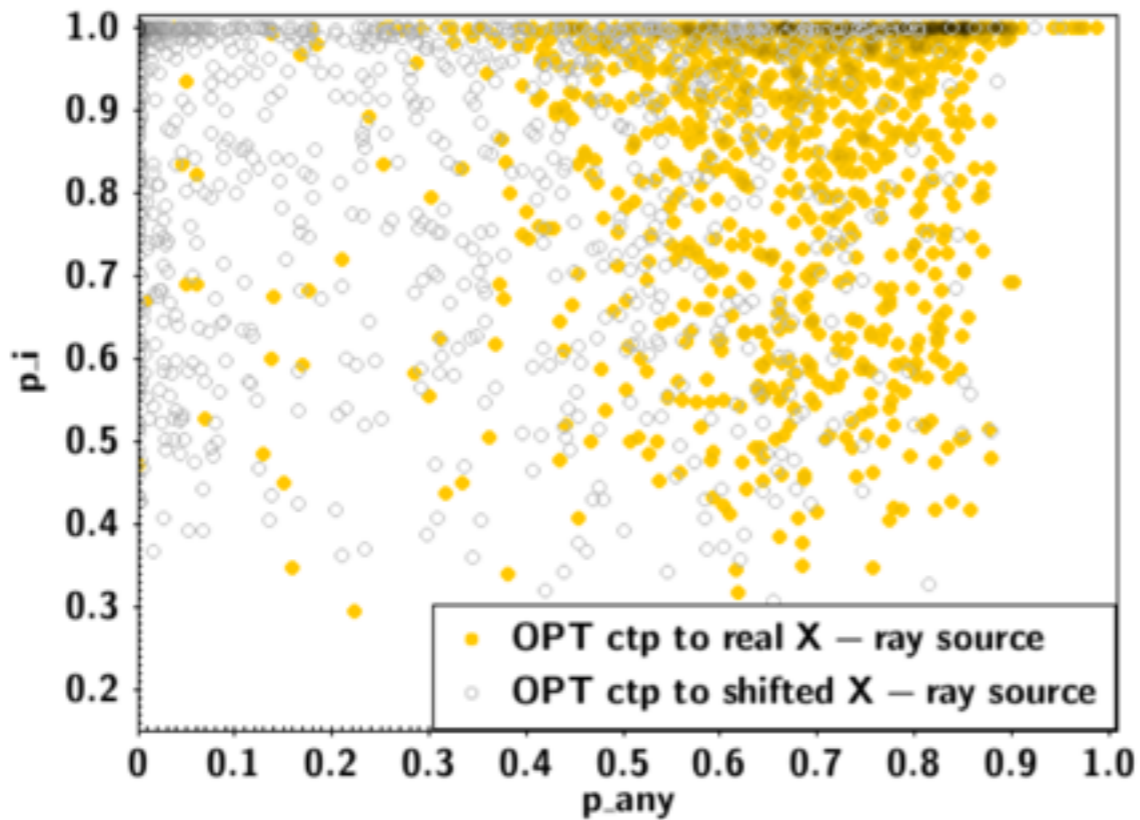
## internal calibration

using the sources with  $p_{\text{any}_\Phi} > 0.9$  and  $p_{\text{any}_\Phi} < 0.1$  for creating the likelihood



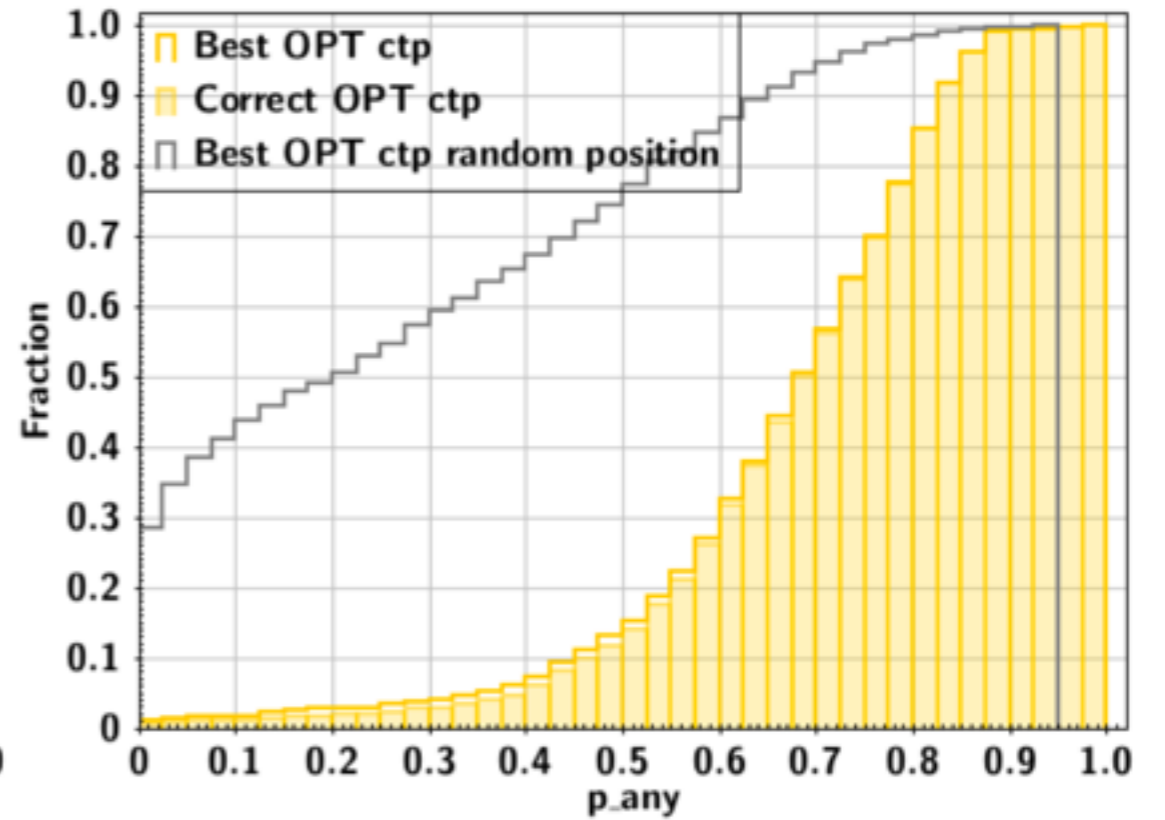
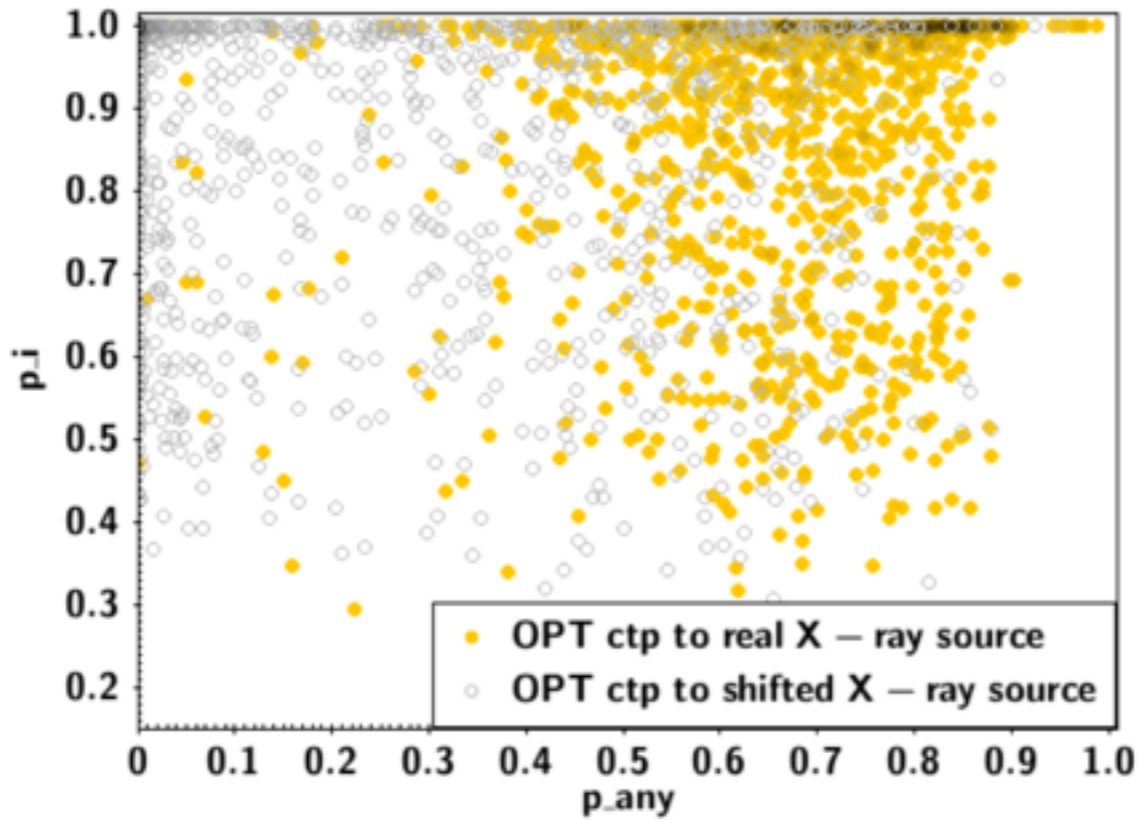
# Example of result from internal calibration: XMM-COSMOS

X+OPT

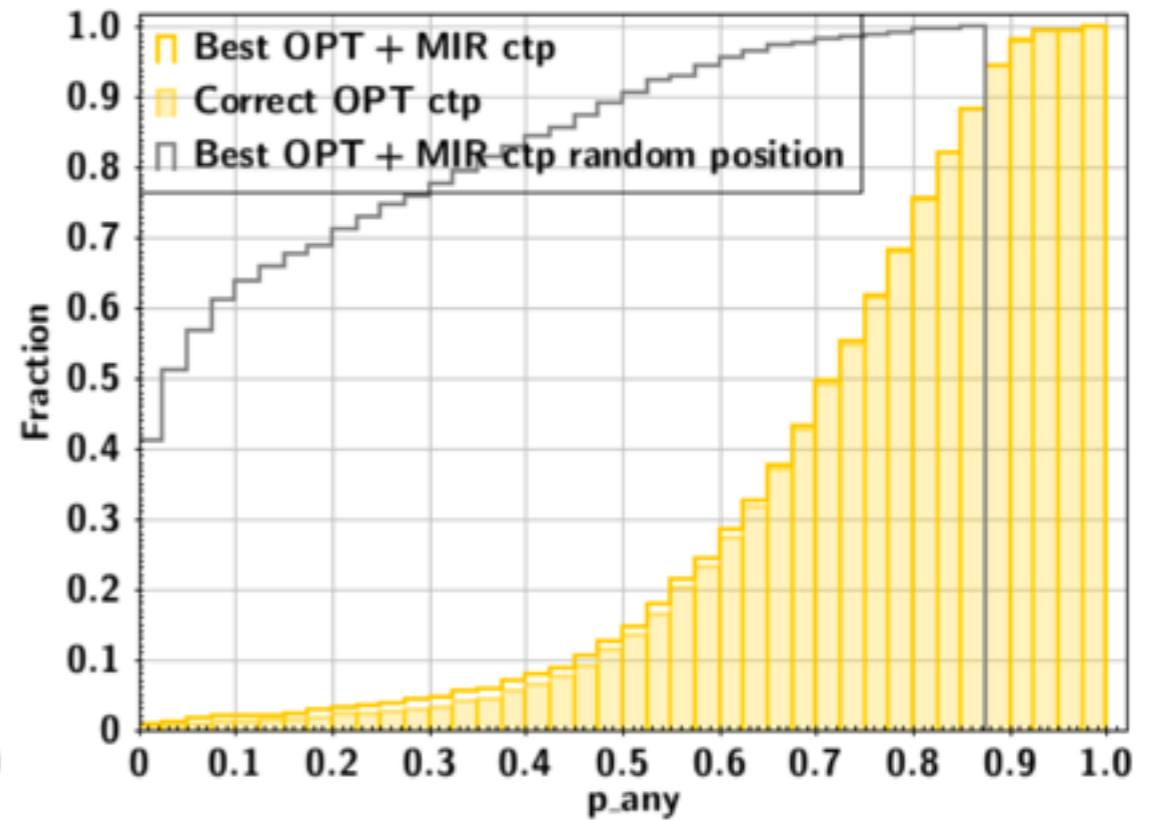
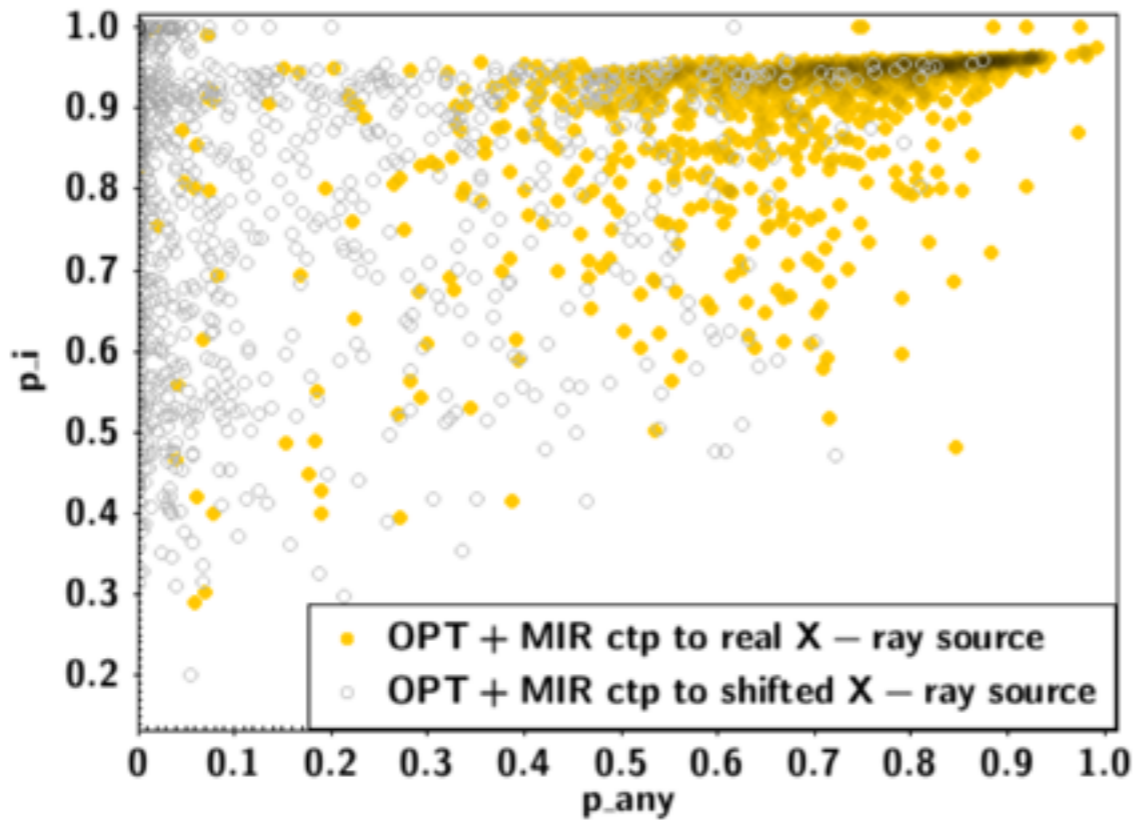


# Example of result from internal calibration: XMM-COSMOS

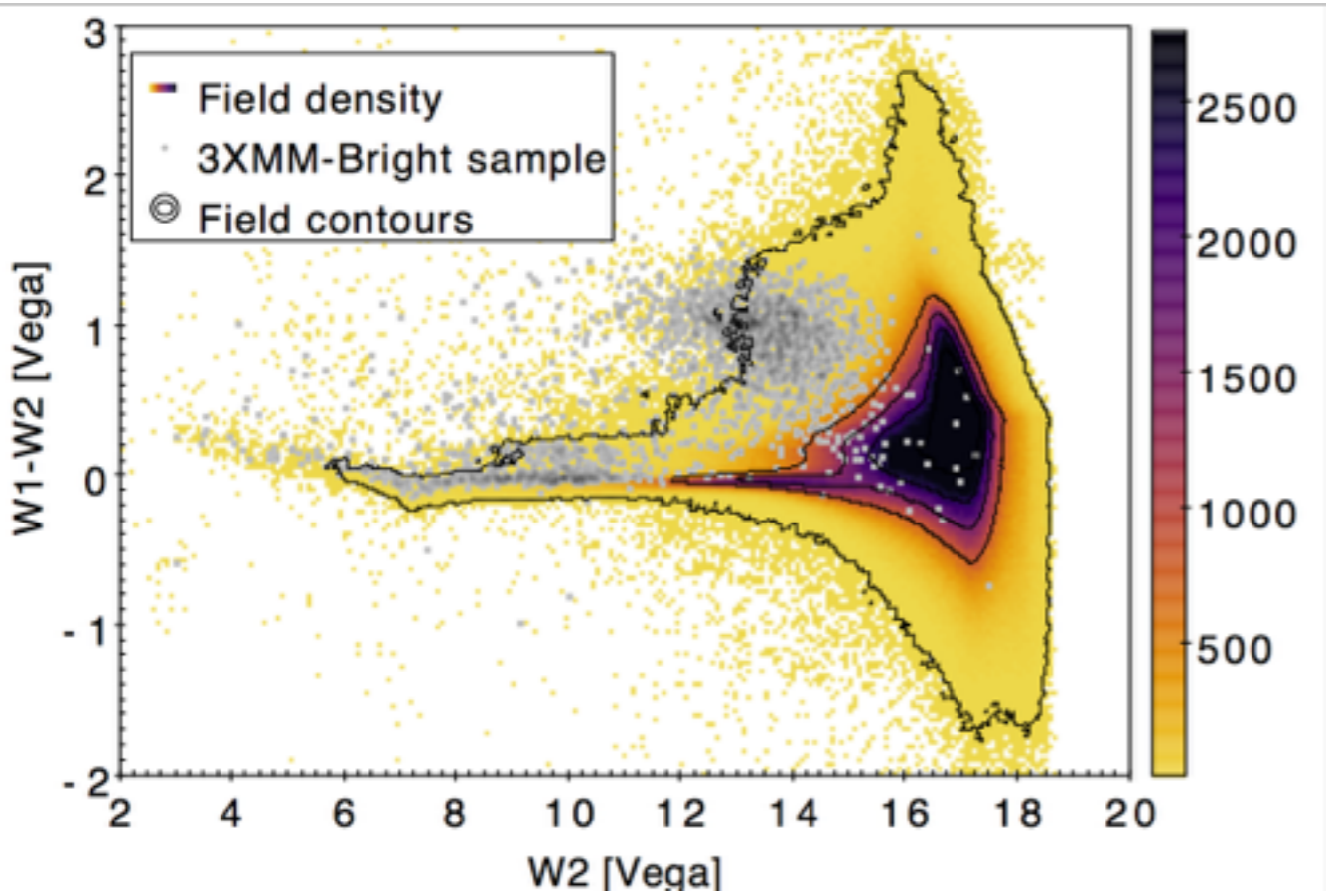
X+OPT



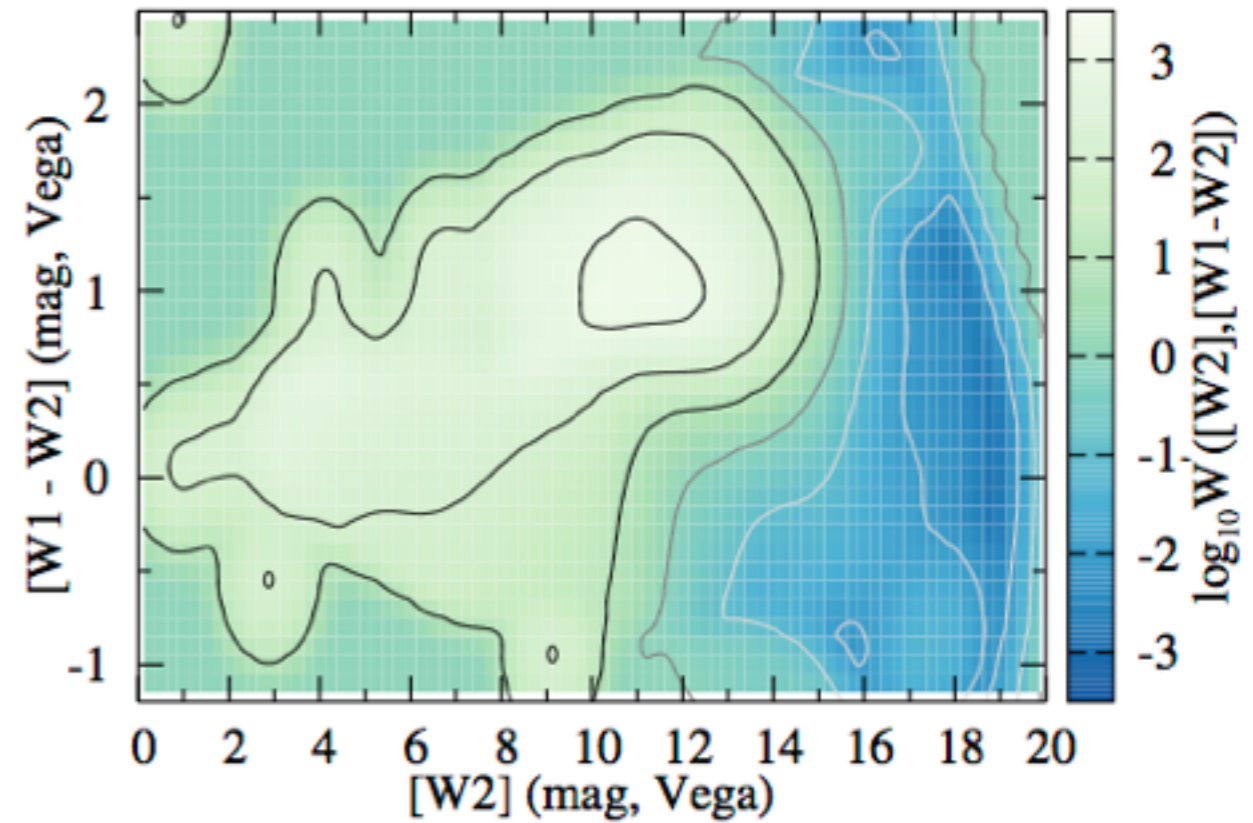
X+  
OPT+  
MIR



# Actual application to ROSAT and XMM-SLEW2



the data used for the prior



The actual likelihood



# Counterparts comparison

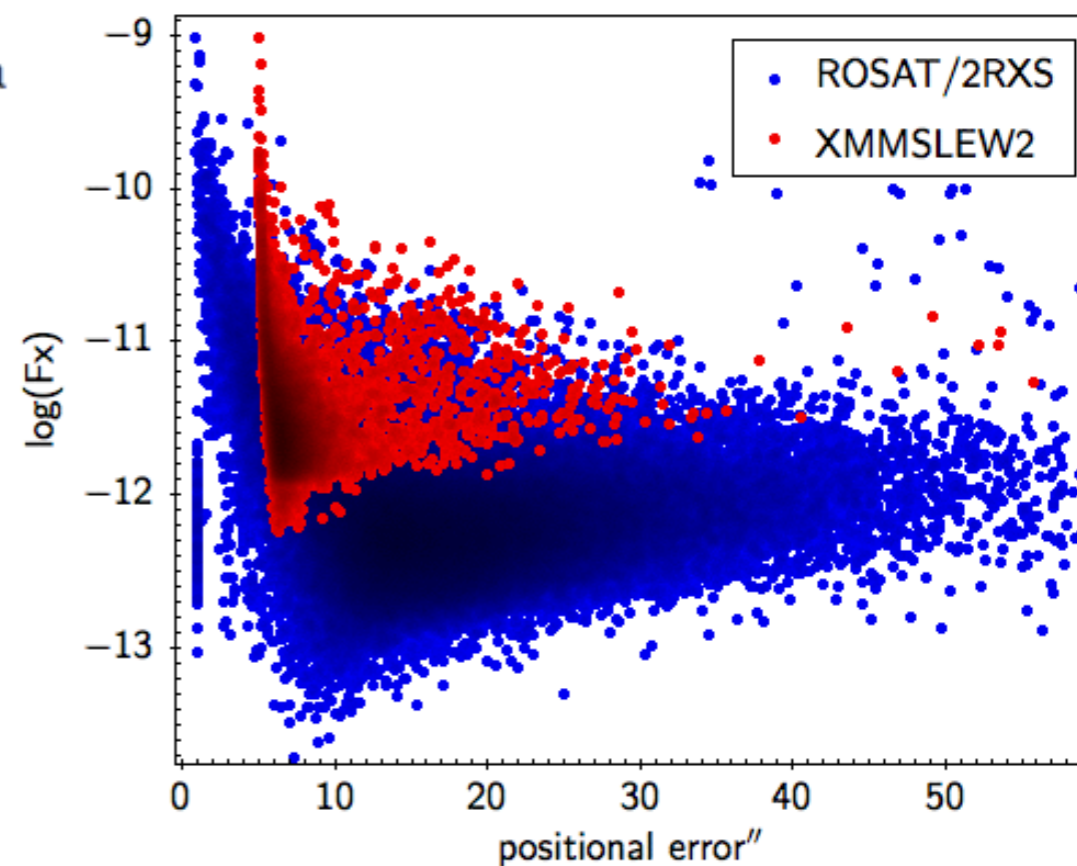
**Table 1.** XMMSL2 vs 2RXS AllWISE association for sources in common

XMMSL2-2RXS		Sources in common N	Identical Best AllWISE ctp. %
Sep. arcsec	Mean Sep. arcsec		
$\leq 5$	3.2	1145	98.5
$\leq 10$	6.1	3559	98.5
$\leq 30$	12.4	8202	95.7
$\leq 60$	15.9	9330	91.6

# Counterparts comparison

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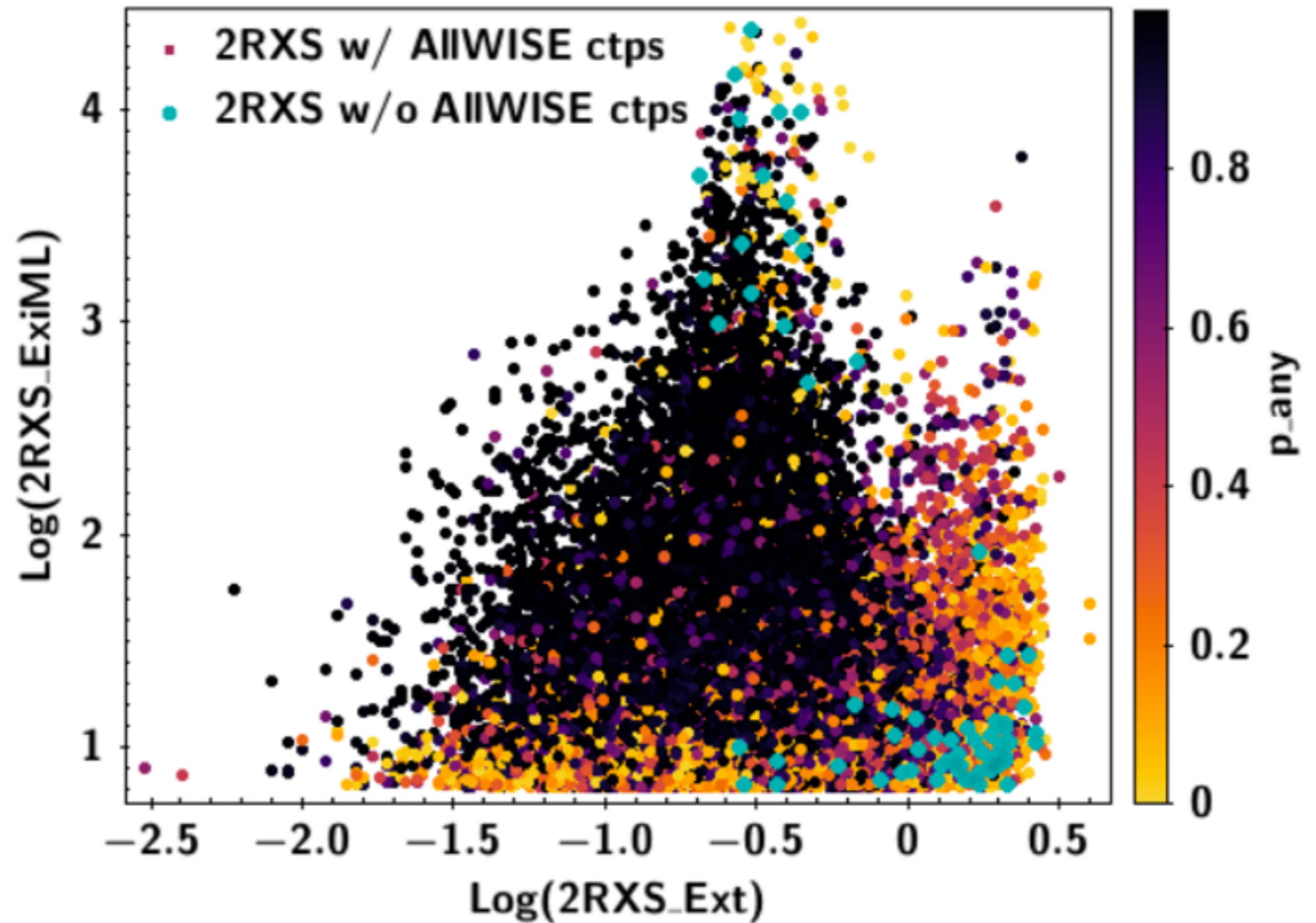
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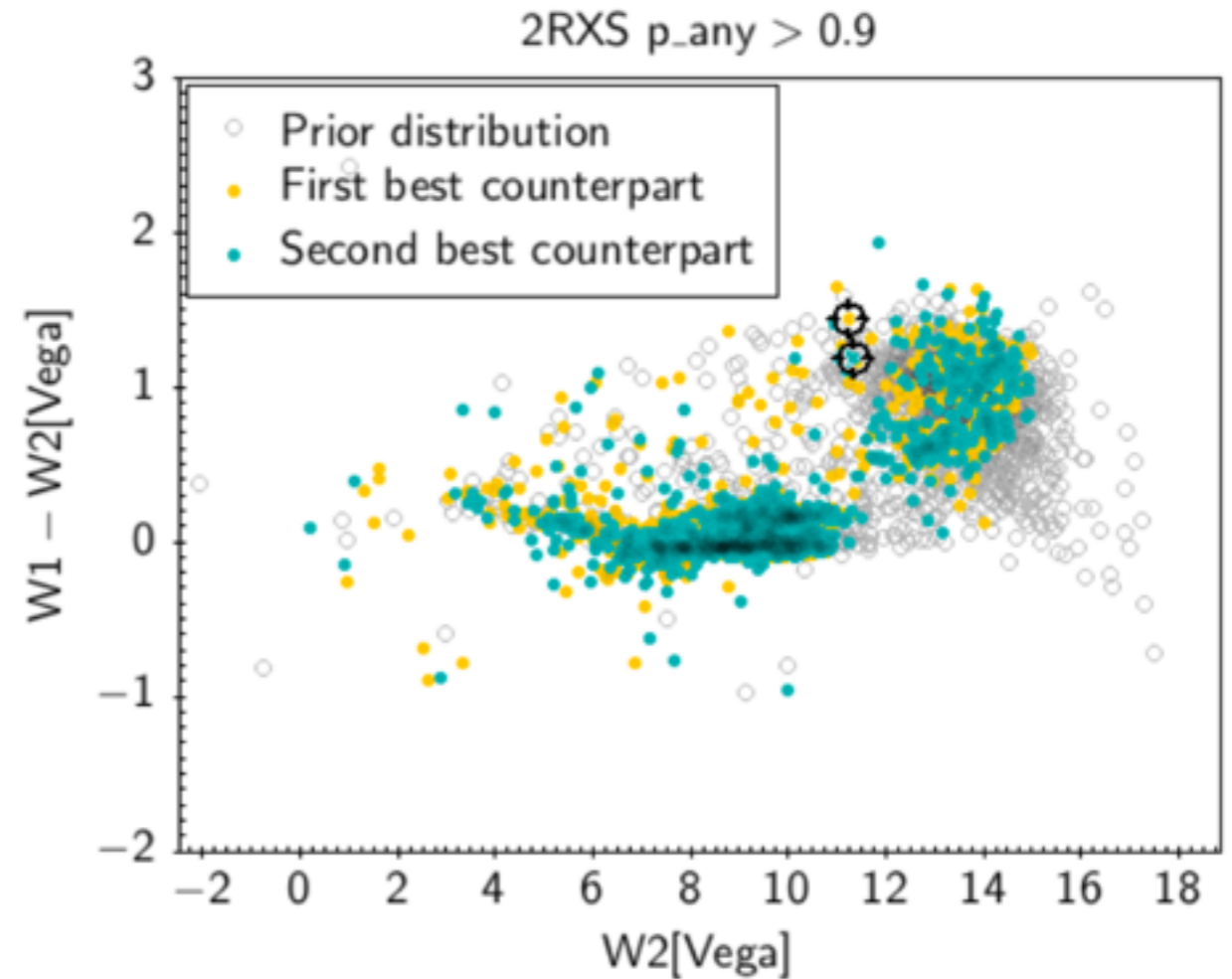
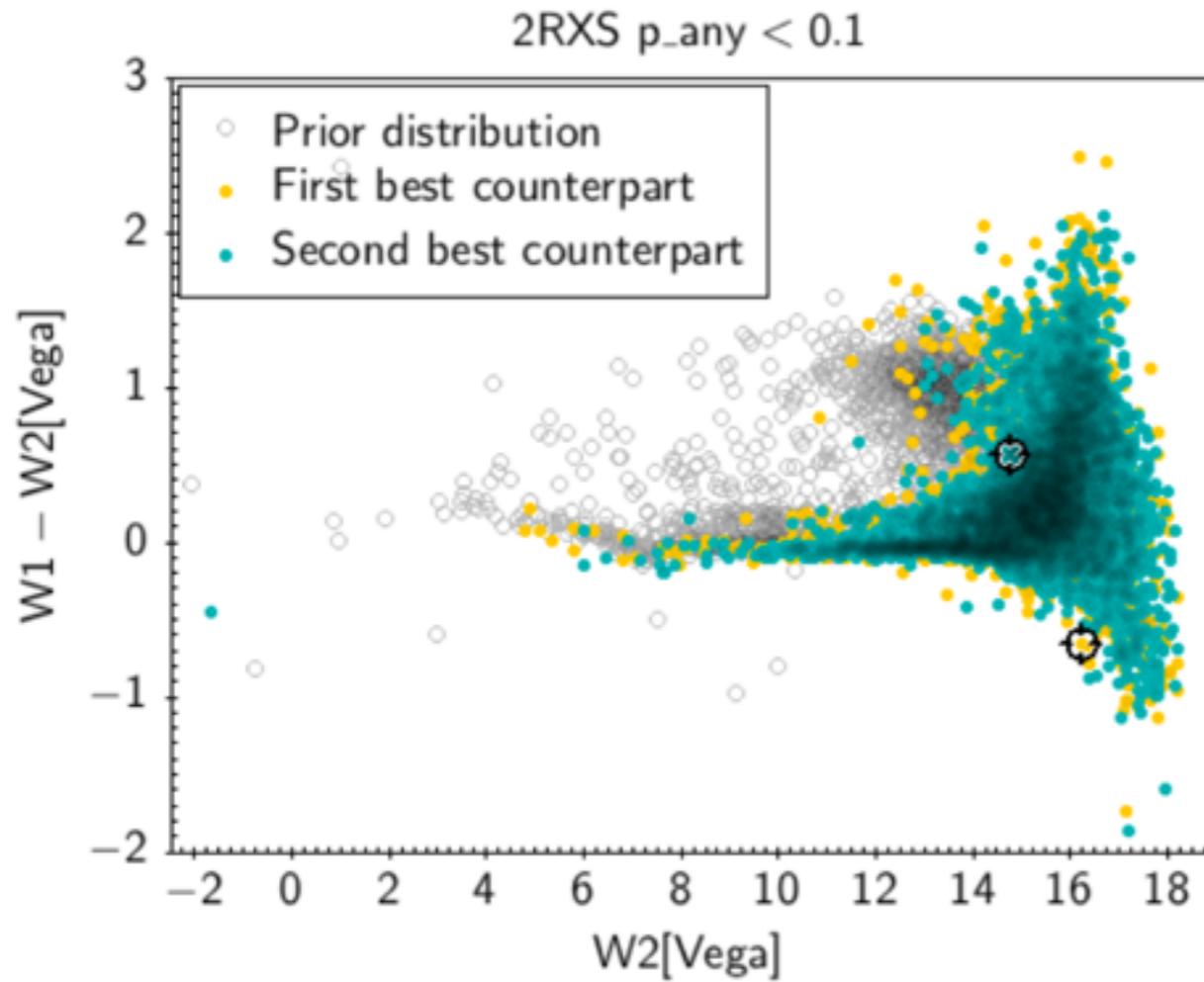
**what p\_any tells us :**

**1) is the X-ray source real?**

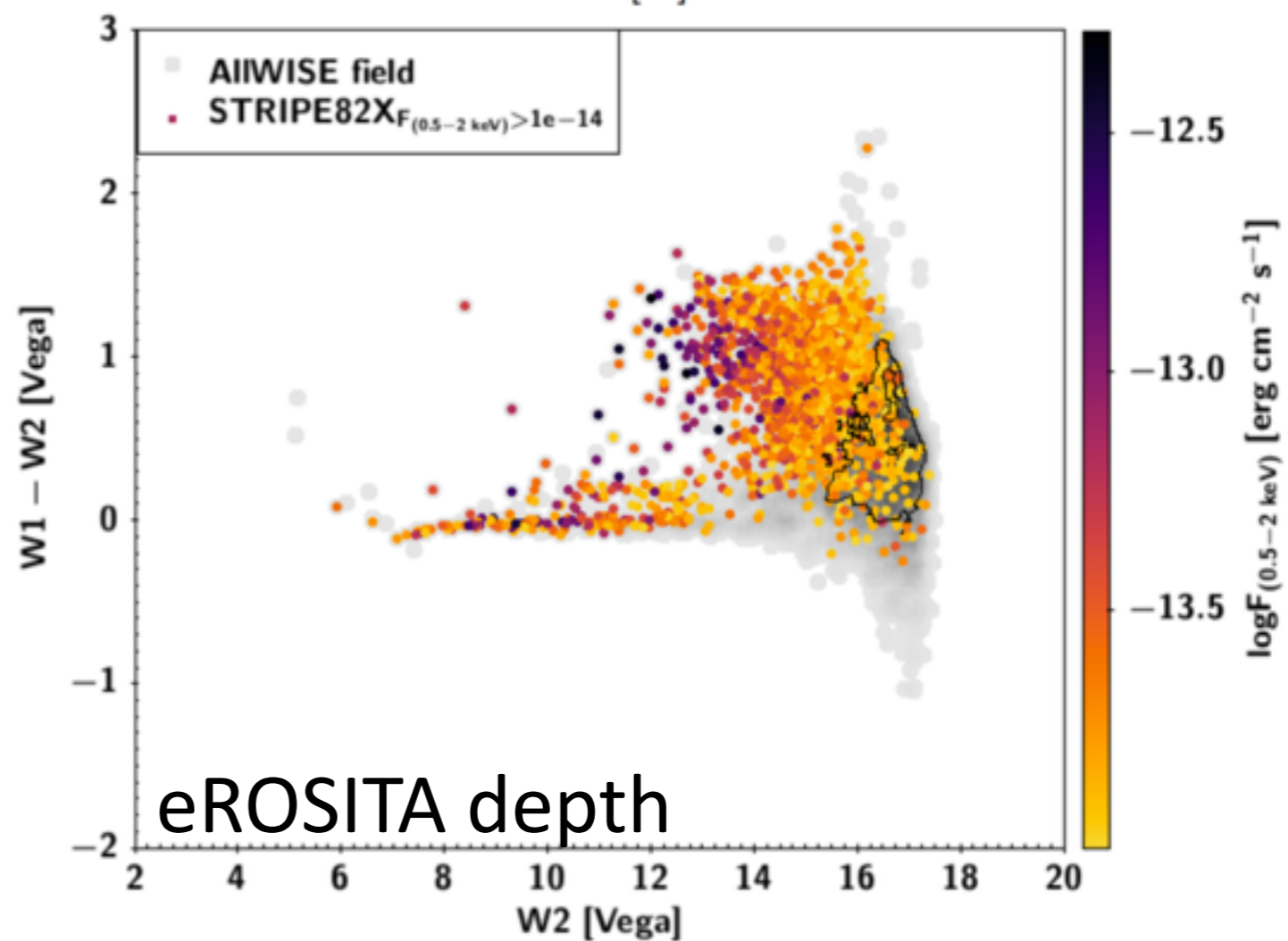
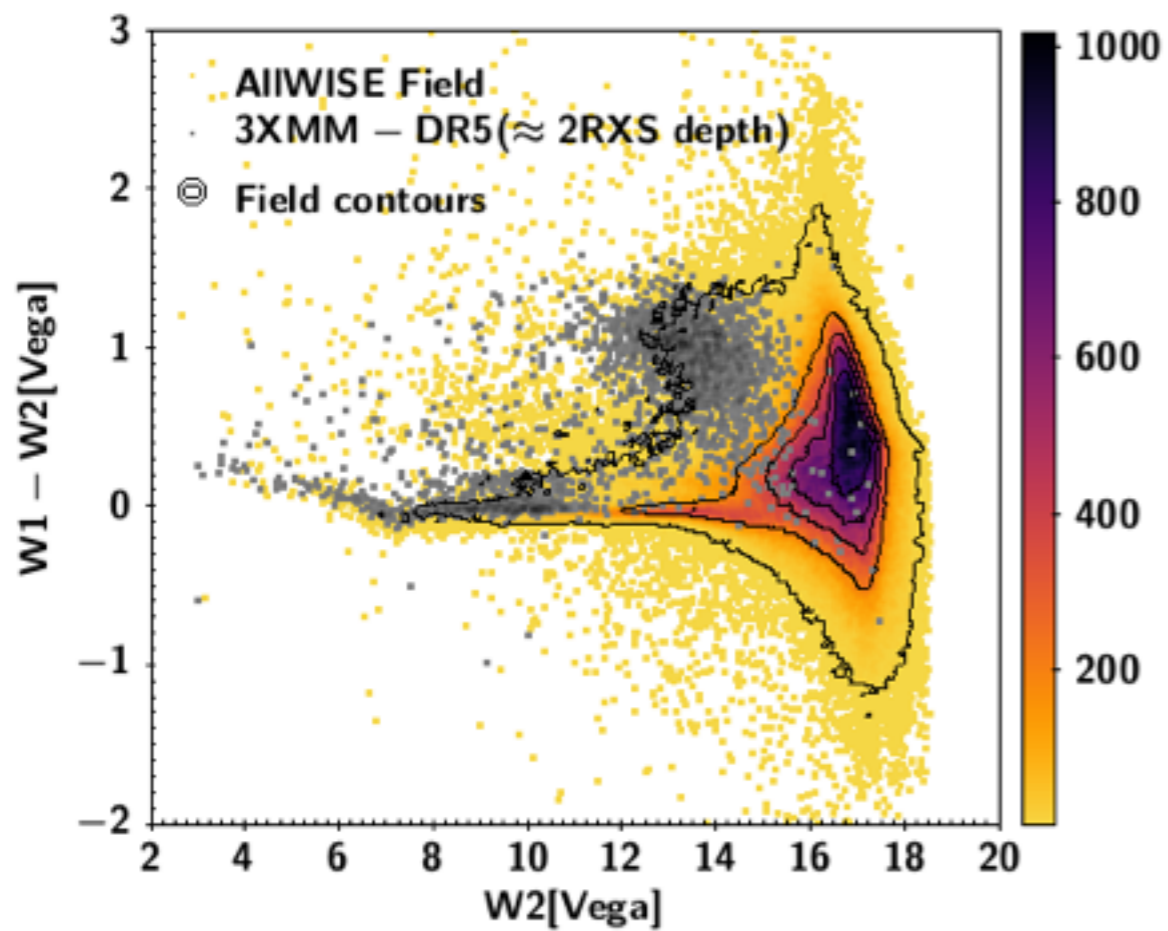
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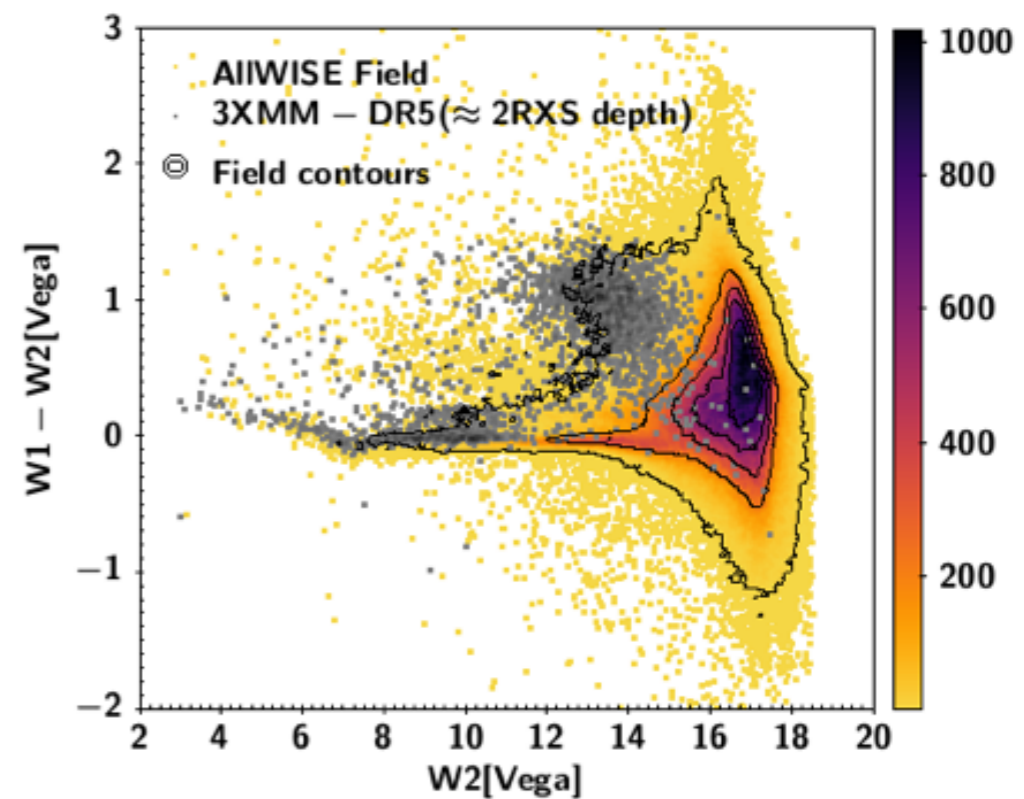
what  $p_{\text{any}}$  tells us :  
1) is there a deblending issue?



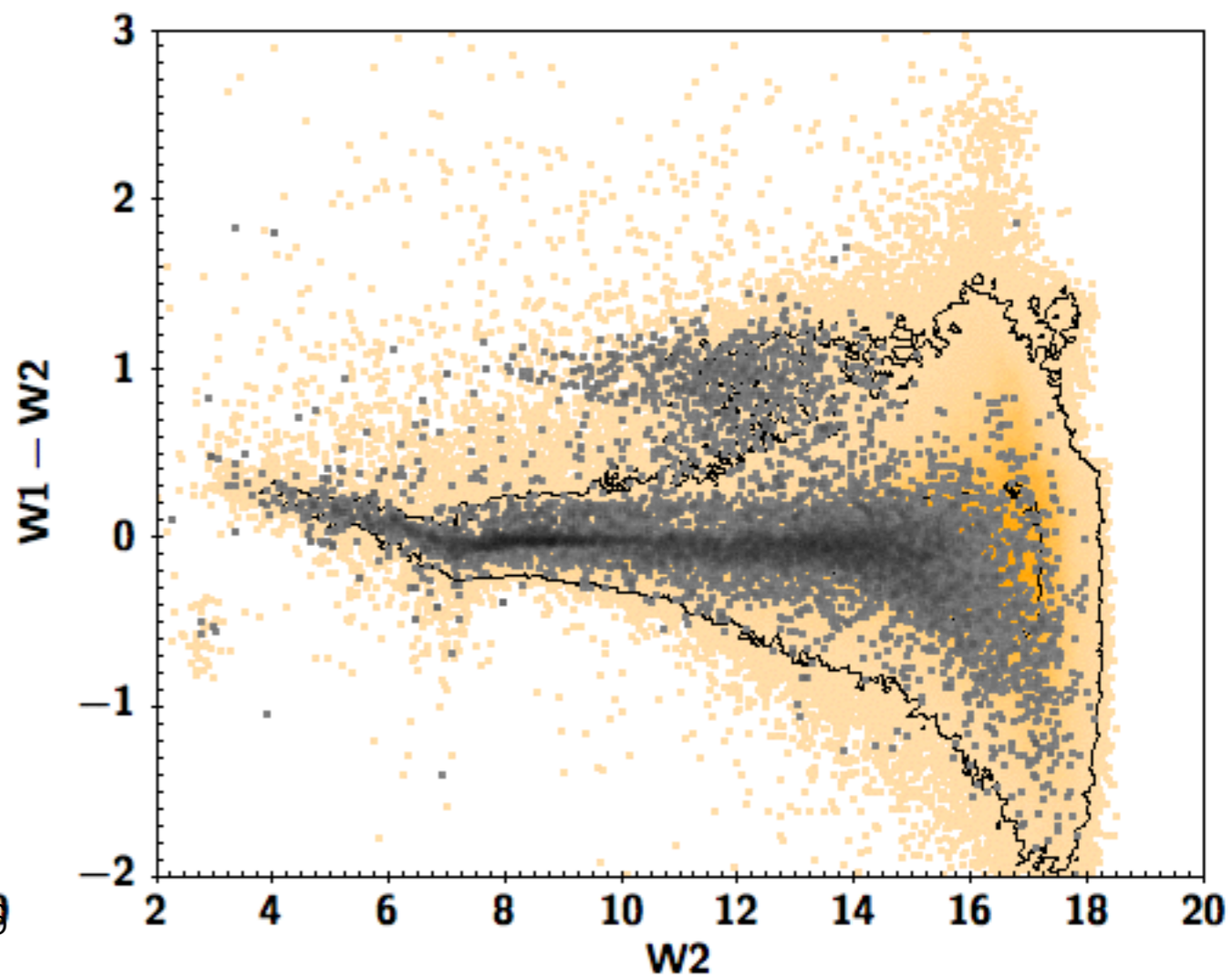
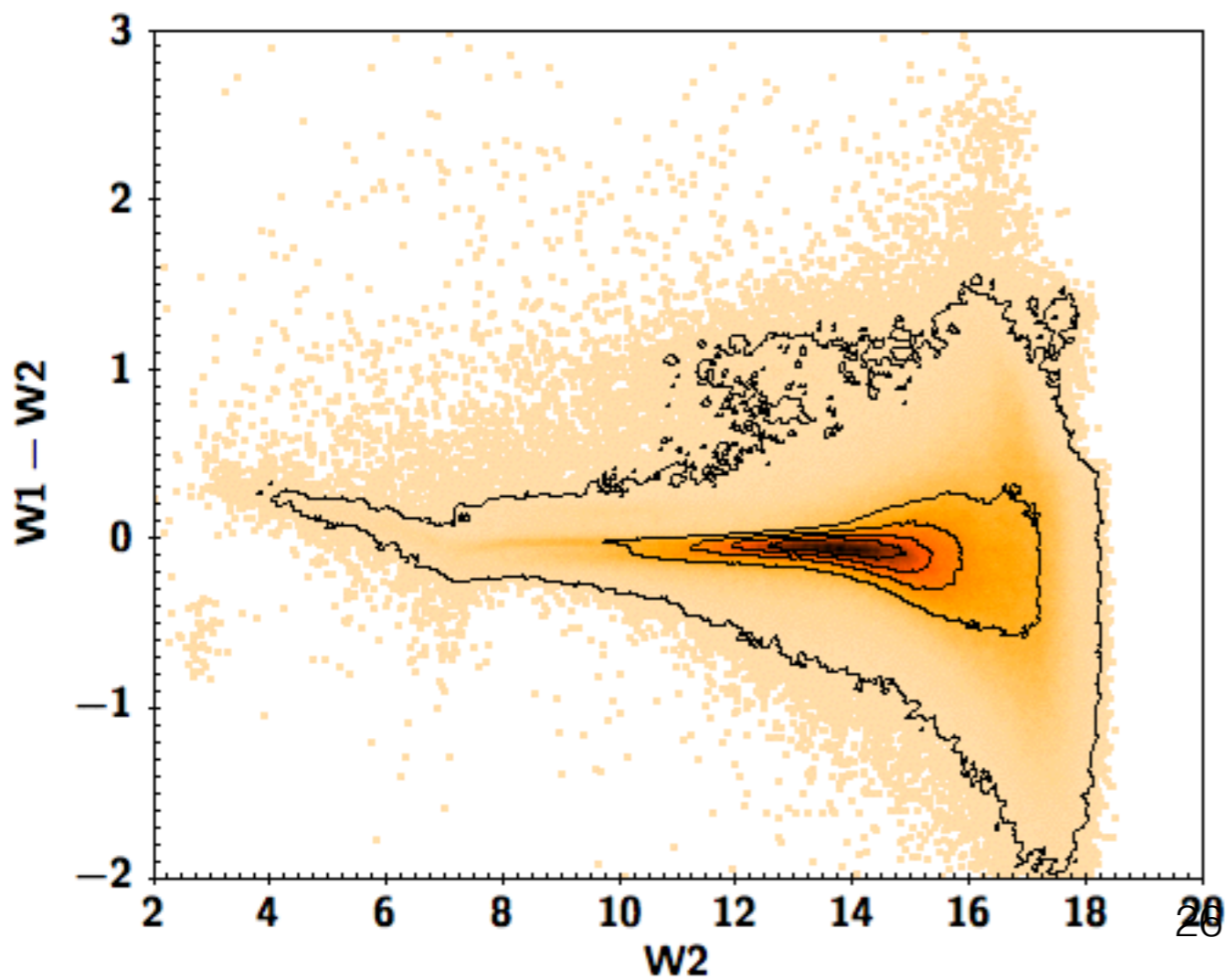
# Prior must be proper



Prior must be proper!



ROSAT in the galactic plane



# Summary

- NWAY provides the mean to use as much as you already know for finding the right ctp of X-ray/radio/mid-infrared/etc sources
- Choose carefull your prior: it will effect your results
- Know well your input catalog of sources
- For each of the sources within a certain radius we provide the probability to be the right ctp. It is up to the user to decide the threshold (compromise of purity and completeness).
- <https://github.com/JohannesBuchner/nway>  
Nway is released together with a manual, data for training and examples of applications.