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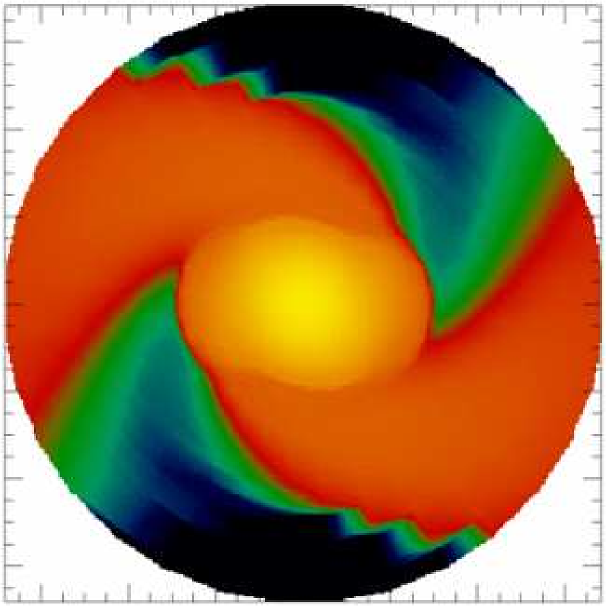

Harald Dimmelmeier

# Gravitational Waves from Supernova Core Collapse:

# Old Hopes and New Results

Presented work in collaboration with

J. Novak (LUTH, Meudon); J.A. Font and J.M. Ibáñez (Universidad de Valencia)
H.-T. Janka, A. Marek, E. Müller, B. Müller, and B. Zink (MPA Garching)
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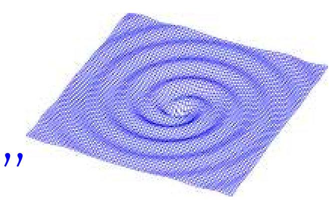
C. Ott (AEI Golm); I. Hawke (University of Southampton, U.K.); E. Seidel (Louisiana State University, U.S.A.)

http://www.mpa-garching.mpg.de/rel_hydro/

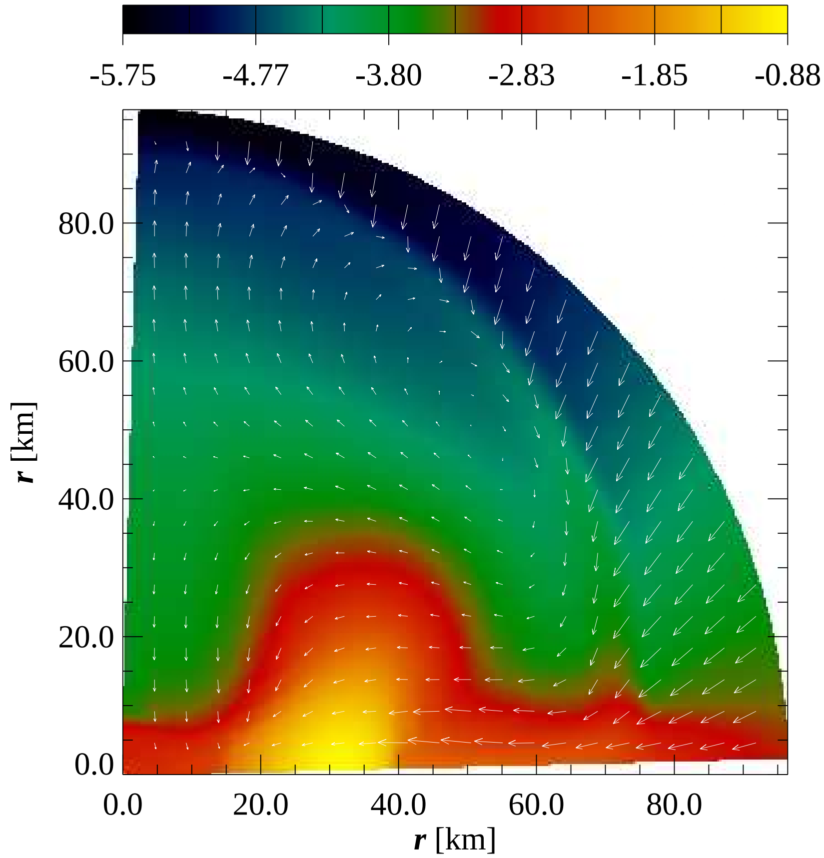
Suggested reading:
Review article by Kim New, "Gravitational Waves from Gravitational Collapse"
http://relativity.livingreviews.org/Articles/lrr-2003-2/



DFG SFB Transregio 7 "Gravitational Wave Astronomy"

# Outline of the Talk

- Astrophysical Context.
- Gravitational Waves.
- Models of Core Collapse.
- Relativistic Equations.
- Numerical Methods.
- Tests and Applications.
- Summary.



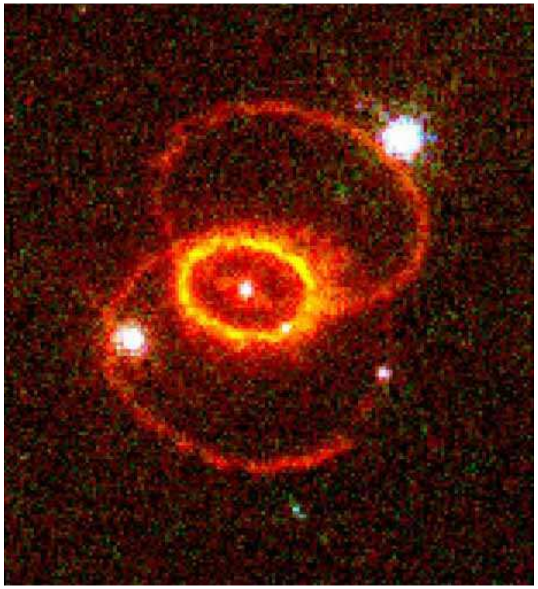
# Astrophysical Sources of Gravitational Waves

Want to sort out role of gravitational waves from core collapse.
$\Longrightarrow$ Compile list of possible astrophysical sources:

- Coalescence and merger of supermassive black holes
  (evidence for galaxy mergers and black holes in galaxies).
- Compact objects swallowed by supermassive black holes (dubbed bothrometry by Phinney).
- Coalescence and merger of solar-mass-type binary black holes
  (estimates for event rates go up).
- Coalescence and merger of binary neutron stars (possible delayed collapse to black hole).
- White dwarf binaries (guaranteed source for LISA detector).
- Cosmic gravitational wave background (remnant from Big Bang, no detectors so far).
- Oscillations of neutron stars (from ringdown, secular or dynamical instabilities).
- Collapse of supermassive stars to black hole
  (does this scenario exist?).
- Collapse of Population III stars.
- Finally: Collapse of core in massive star to neutron star
  (supernova core collapse, several emission mechanisms).

Zoo of possible scenarios with various signal strengths and frequencies!

# The Standard Model for Supernova Core Collapse



After decades of observation and computer simulations:

Standard scenario for supernova core collapse has emerged (still uncertain and debated in details!).

- Subsequent nuclear burning in massive star yields shell structure.
- Iron core with $M \sim 1.4\,M_\odot$ and $R \sim 1000$ km develops in center.
- Equation of state: Relativistic degenerate fermion gas, $\gamma = 4/3$.
- Instability due to photo-disintegration and electron capture.
  $\Longrightarrow$ Evolution becomes strongly dynamic!

- Collapse to nuclear matter densities in $T \sim 100$ ms (collapse proceeds nearly adiabatically, neutrinos are trapped).
- At nuclear matter density EoS stiffens, collapse is halted (core bounce, formation of prompt shock, shock stalls).
- Proto-neutron star forms, deleptonization of matter.
- Proto-neutron cools and shrinks, emits energy as neutrinos (gravitational binding energy was stored as internal energy).
- Stalled shock revived by energy deposition from neutrinos.
- Delayed shock propagates out and disrupts envelope of star.

# The Relevance of Gravitational Waves From Core Collapse

Shock breaks through stellar surface: See supernova explosion as EM waves (light curve).

This happens hours after core collapse!
$\Longrightarrow$ Light curve is only "echo" of supernova driving engine
(can get only indirect information about core collapse).

But: Can use two complementary methods to look at "heart" of supernova:

- Neutrinos: Seconds after collapse.
  Tradeoff: Flux decays with square of distance! $\Longrightarrow$ Only for galactic supernovae.

- Gravitational waves:
  Decouple from matter directly after generation.
  $\Longrightarrow$ Synchronicity with core collapse!

  Advantage: Amplitude decays linearly with distance!
  $\Longrightarrow$ Extragalactic supernovae detectable in future.

New gravitational wave detectors become operational (LIGO, GEO600, TAMA300, VIRGO, ACIGA).

80 years after prediction by Einstein, gravitational waves are coming into range for detection.

# The Quadrupole Formula for Gravitational Waves

What is emission mechanism of gravitational waves?
Obvious answer: Coherent motion of central massive core (during core bounce).

For gravitational wave strain $h$ we have (in Newtonian quadrupole approximation):

$$h_{ij} = \frac{2}{R}\ddot{Q}_{ij},$$

with mass quadrupole moment

$$Q_{ij} = \int dV \rho \left( x_i x_j - \frac{1}{3}\delta_{ij} r^2 \right).$$

Wave strain $h$ is relative amplitude of spacetime distortion.

This formulation is good approximation for waves from supernova core collapse!

# The Problems with Detecting Gravitational Waves

Supernova core collapse is very energetic event:
$E_{\text{bind}} \sim 3 \times 10^{53}$ erg (substancial fraction of $M_{\odot} c^2$).

Still: Deformation of spacetime and thus signal amplitude is very small!
(In most promising scenarios like black hole merger: $E_{\text{GW}} \sim 10^{-2} E_{\text{bind}}$.)

Also upper limit for core collapse in idealized calculation (Saenz and Shapiro, 1978).
With more realistic models: $E_{\text{GW}} \lesssim 10^{-6} E_{\text{bind}}$.
$\Longrightarrow$ Old hope for large signal amplitude destroyed...

For detection we face two problems:

- Relative signal strength only $h \sim 10^{-20}$ (for galactic supernova).
- Burst signal from bounce is very complex.

$\Longrightarrow$ Signal analysis is like search for needle in a haystack!

David Shoemaker (LIGO Collaboration):

"Numerical Relativity Simulations are badly needed!"

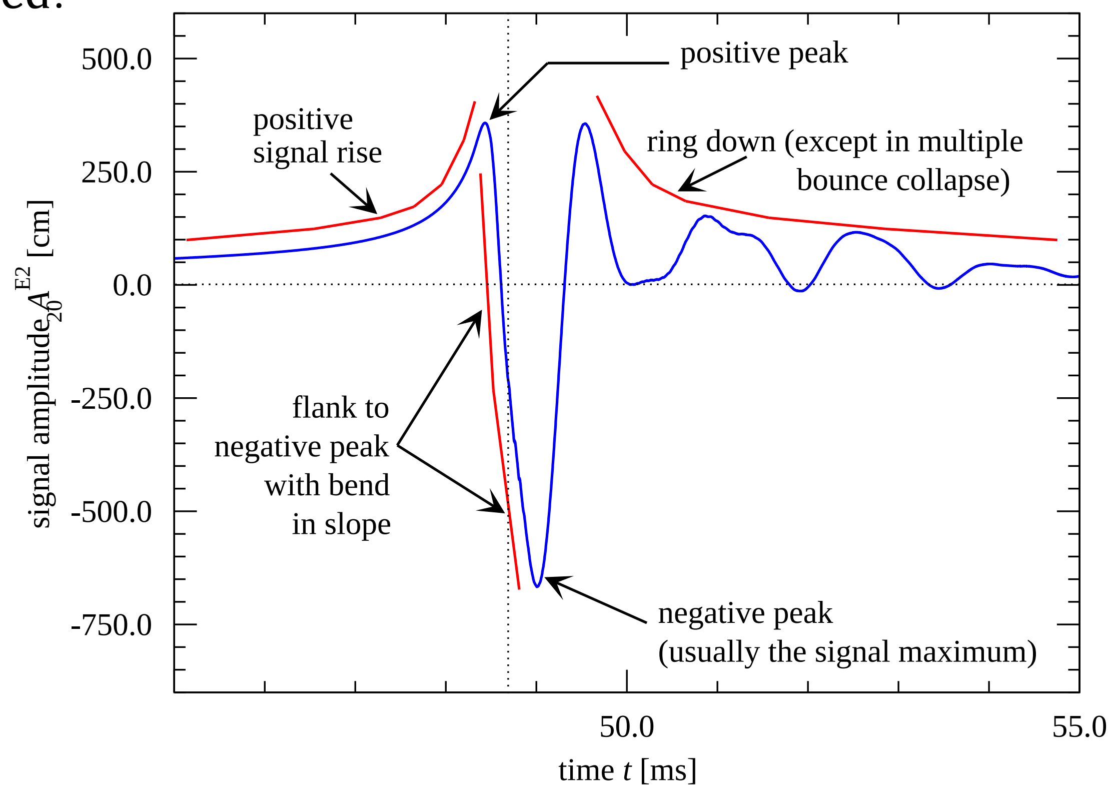
# Benefits from Detecting Gravitational Waves from Supernova Core Collapse

Clear prediction of this signal is hampered by two main points:

- Lots of "dirty" microphysics
  (as compared to clean vacuum situation of e.g. black holes mergers).
- Uncertainty about some of physical aspects involved
  (e.g. EoS above nuclear matter densities, rotation state in core of progenitor star).

But once signal from core collapse is detected:

Can constrain physical models from that ("signal inversion")!



# Physical Complexity of the Core Collapse Scenario

Now look in detail at numerical simulations of core collapse.

General remark about simulations of supernova core collapse:
Can safely neglect back-reaction of gravitational waves on dynamics.

But in principle: Need many complicated ingredients for numerical simulations:

- Gravitational physics: General relativity, as $2M/R \sim 0.3$!
- Initial models: Stellar evolution (still no consistent treatment of rotation!).
- Equation of state and nucleosynthesis: Particle and nuclear physics.
- Correct treatment of shock front: Hydrodynamics.
- Influence of neutrinos (crucial!): Boltzmann transport.
- Rotation or nonspherical instabilities: Multidimensions.
- Magnetic fields: Magneto-hydrodynamics (probably negligible).

Bottomline: Problem is extremely complex!

Promising strategy:

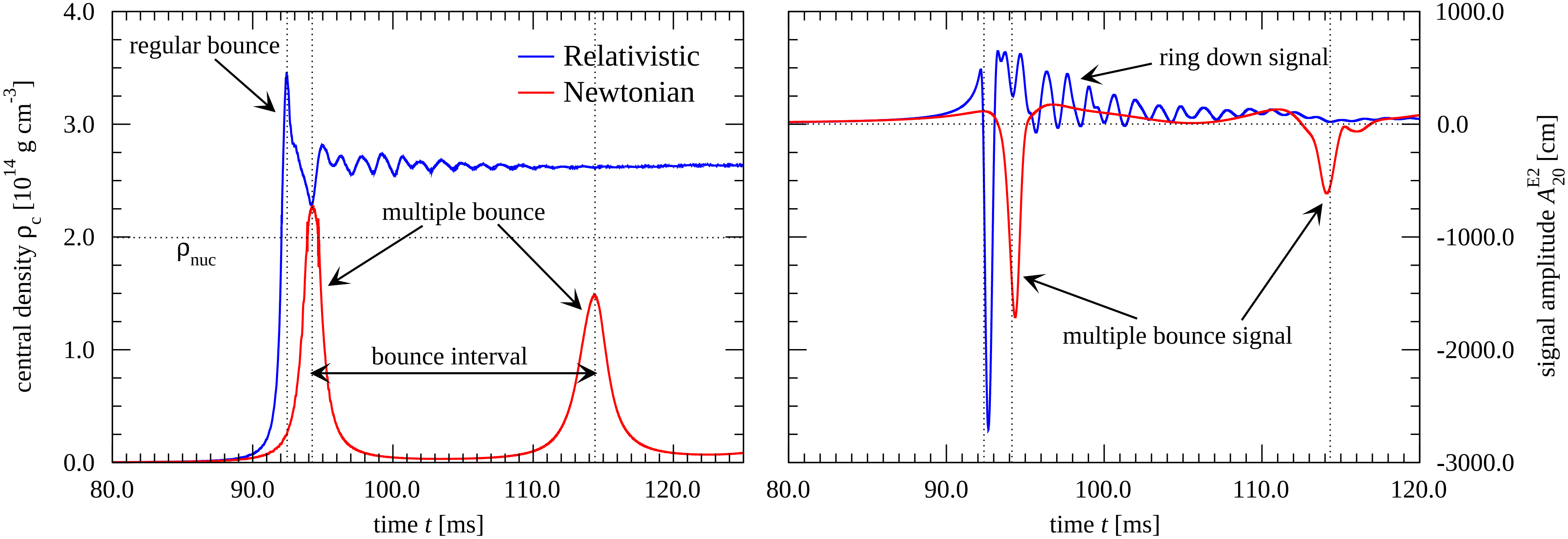
- First split problem into several stages (with various time and length scales).
- Then neglect all unimportant aspects (plus make further simplifications if necessary).

# The Influcence of Relativistic Gravity on the Collapse Dynamics

Inclusion of relativistic effects result primarily in deeper effective potential.
$\Longrightarrow$ Higher densities during bounce, proto-neutron star more compact.

In rotational core collapse, collapse type can change

- from standard single bounce with instantaneous formation of proto-neutron star
- to multiple centrifucal bounce and re-expansion (possibly at subnuclear matter density).



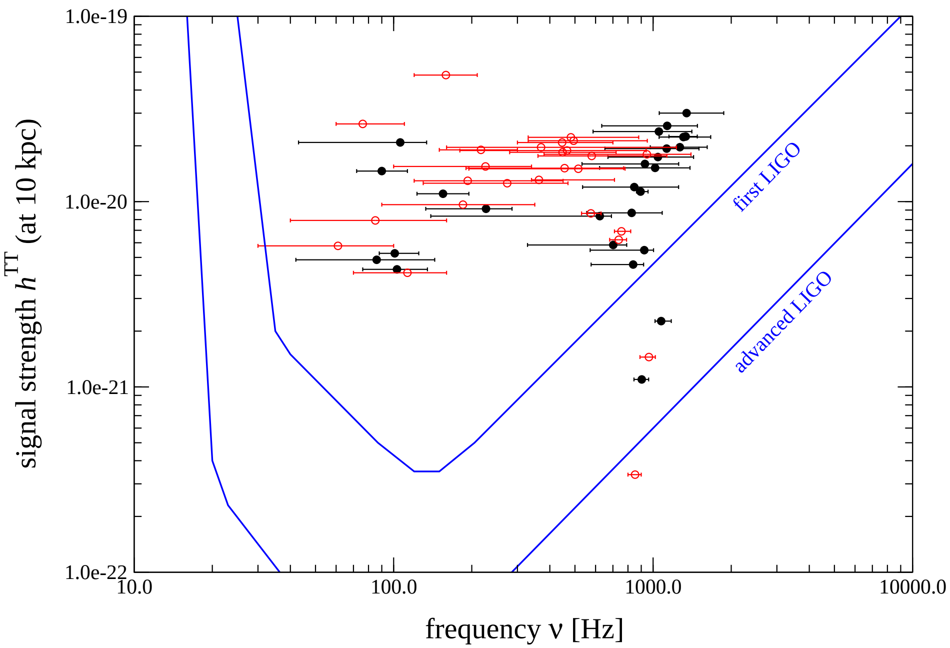

Simulations: Dimmelmeier, Font, Müller, 2002
(Simple polytropic initial models with parametrized rotation, ideal fluid hybrid equation of state, conformal flatness approximation of Einstein equations).

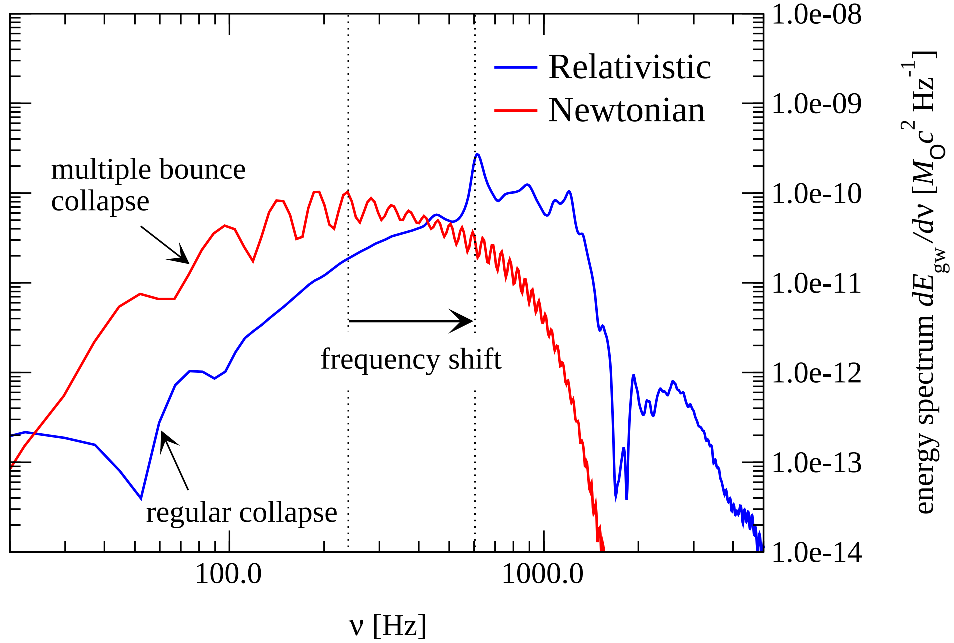
# The Influcence of Relativistic Gravity on the Gravitational Waveforms

With this simple EoS and parametrized rotation we find:

Multiple bounces suppressed!

Also get shift to higher frequencies in signal.




No change in bulk of models due to relativity.
⟹ In principle burst signal from Galactic supernova detectable!

Waveforms from these simulations are collected in a "waveform catalogue".
⟹ These are used as templates in filters for detector data analysis.

# The Importance of Multidimensional Simulations: Neutron Star Kicks

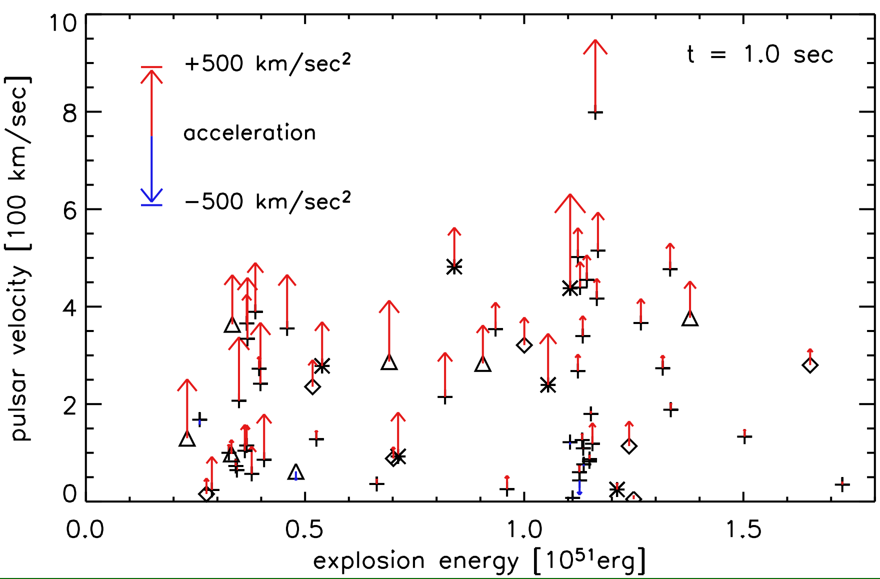
Two opening remarks:

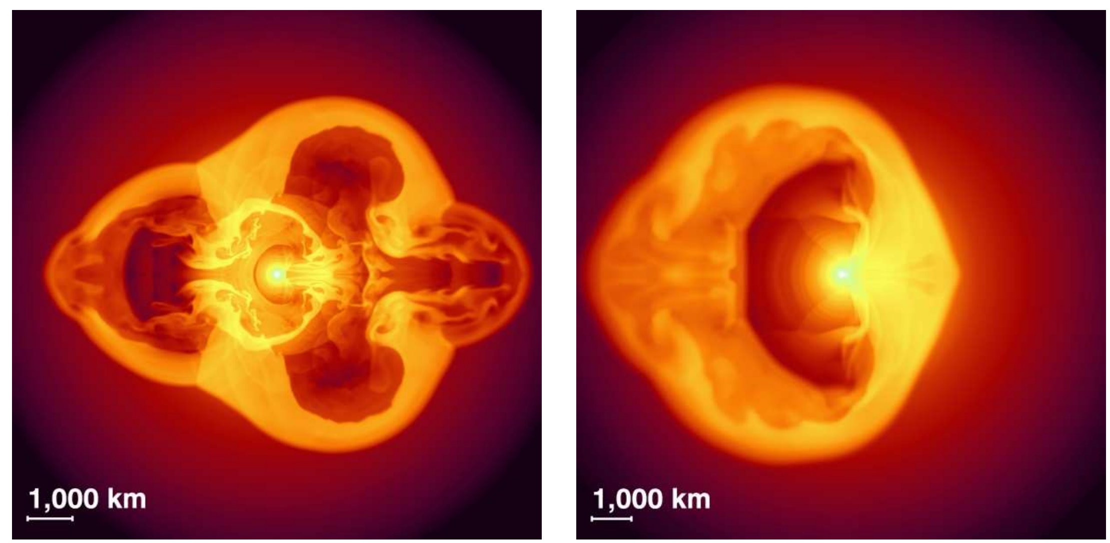
- Link to gravitational waves: No emission in spherical symmetry (Birkhoff theorem)!
- Concerning rotation: Lack of consistent multidimensional stellar evolution simulations.
  $\Longrightarrow$ Rotation state (strength and distribution of angular momentum) of core uncertain!

Even in spherical model: Anisotropic instabilities can develop!

Example: Hydrodynamic $l = 1$ instability.

Simulations: Scheck et al., 2004
(Newtonian gravity, sophisticated microphysics and initial models).

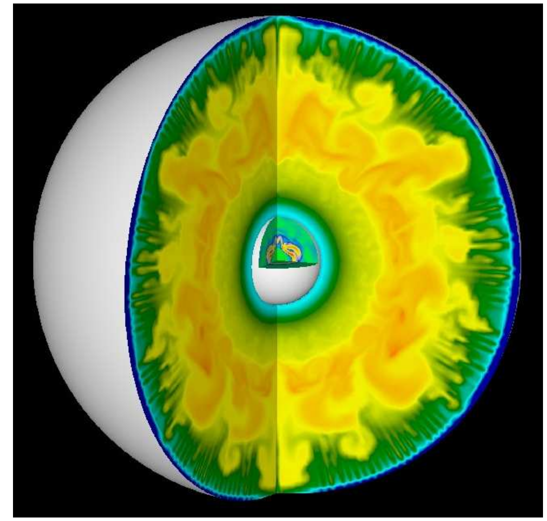



Recoil of neutron star due to this instability could explain “neutron star kicks”!

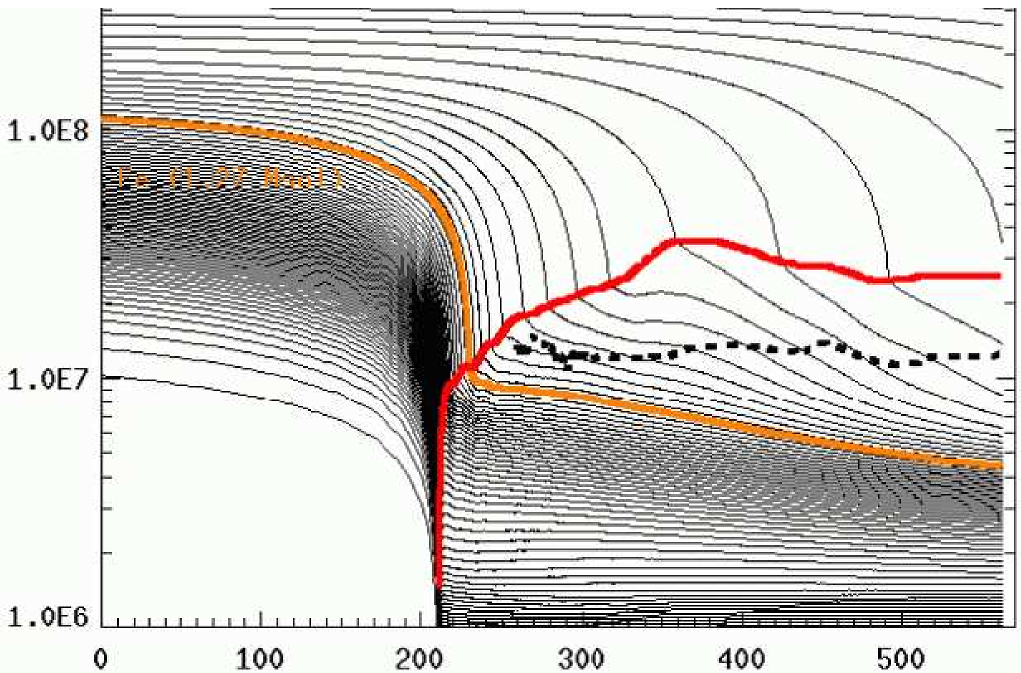
This mechanism could emit gravitational waves.

## The Importance of Multidimensional Simulations: Convective Instabilities



Two more examples for anisotropic instabilities
(also for spherical initial models):

- Ledoux instability leads to convection in neutron star (boiling).
  $\Longrightarrow$ Anisotropy in neutrino emission,
  increase in neutrino flux and absorption.
- Rayleigh–Taylor instability in envelope as shock propagates.
  $\Longrightarrow$ Locally faster propagation of shock front,
  clumping of material from nucleosynthesis.




These convective instabilities may be crucial for reviving stalled shock!

So far we don't get robust explosions!
$\Longrightarrow$ Another old hope turned sour. . .

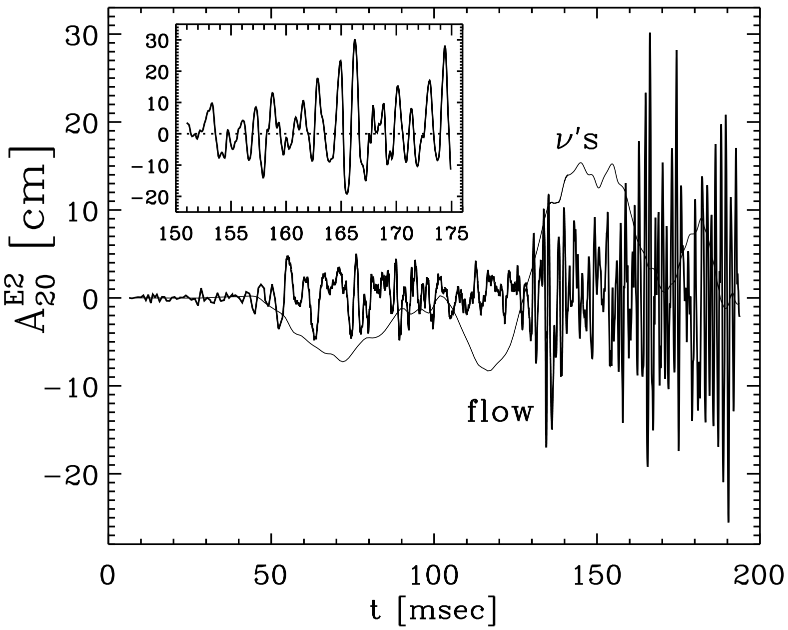
Simulations:
Keil, Janka, Müller, 1996;
Janka and Müller, 1996;
Rampp and Janka, 2000
(with simple neutrino treatment
or Boltzmann transport).

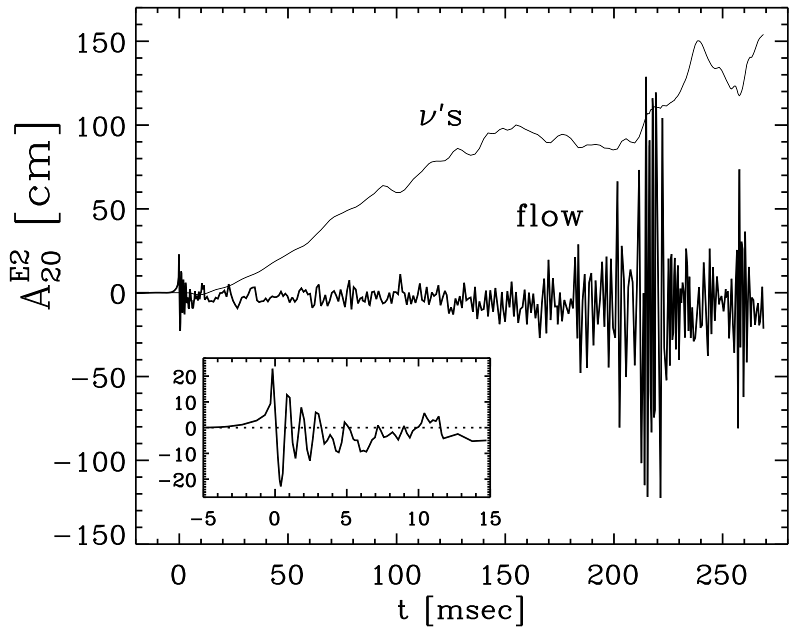
## Gravitational Waves from Convective Instabilities

Additional to (possible) burst signal: Signal from convection!

Gravitational wave emission from both

- convective boiling of neutron star (usual "rest mass" quadrupole moment), and
- anisotropic neutrino emission/absorption (neutrino "energy mass" quadrupole moment).



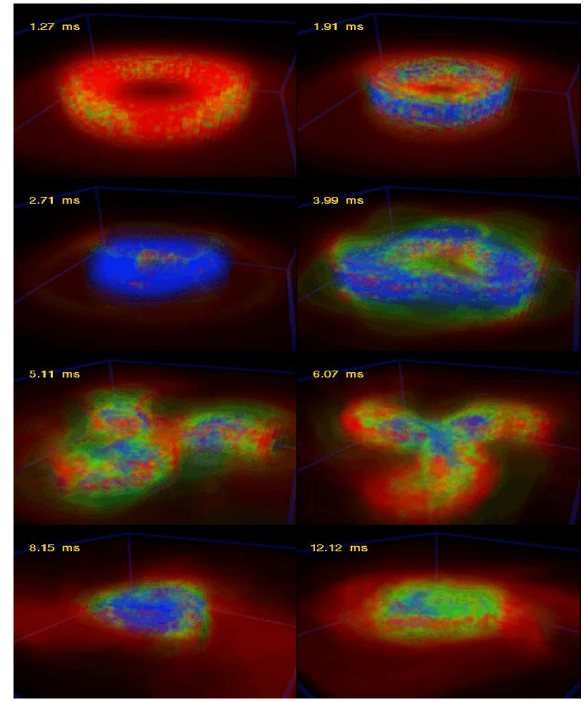




Various signals from core bounce, convection, and neutrinos can have comparable amplitudes!

Long emission time scale can yield relatively high energy for continuous signal!

Simulations: Müller et al., 2004 (Newtonian gravity, latest microphysics and initial models).

# Triaxial Instabilities in Rotating Neutron Stars




For rapidly rotating self-gravitating fluids:
There are numerous instabilities triggered by triaxial perturbations.

They can be classified according to their growth time scales:

- Dynamic instabilities (e.g. bar modes from f-mode instability).
- Secular instabilities (e.g. r-modes from CFS instability).

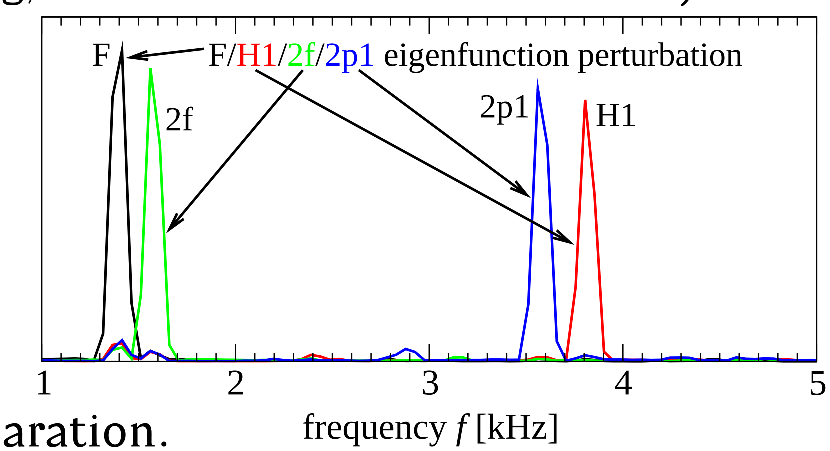
Oscillations can also originate from core collapse ringdown.

Parameters for frequency dependence, criteria for growth window:

- Rotation rate (slow or rapid) and profile (uniform or differential).
- Temperature profile (hot for newborn proto-neutron star, cold for old neutron star).
- Damping mechanisms (viscosity, nonlinear coupling, influence of ambient medium).

In numerical simulations:

Can also excite single eigenmodes by eigenfunction recycling!




Simulations: Rampp, Müller, Ruffert, 1998;
Dimmelmeier, Font, Stergioulas, in preparation.

# Various Approaches to Simulations of Supernova Core Collapse

Now focus on numerical methods: How can core collapse supernovae be simulated?
Remember: We have to solve dynamic equations for a self-gravitating fluid.

Historically, supernova core collapse simulations focus on correct treatment of microphysics
$\Longrightarrow$ Neglect general relativity, use Newtonian gravity (possibly with corrections).

Since few years: Multidimensional general relativistic hydrodynamic simulations feasible
(typically derived from Cartesian black hole vacuum codes, with simple hydro added).

Advantages:

- Consistent relativistic formulation.
- Gravitational wave emission arises naturally.

Difficulties:

- Coordinates usually not adopted to geometry of core collapse.
- Mesh refinement (in core collapse: radial contraction scale $\sim 100$!).
- Issue of gauge freedom in relativity (separate invariants from "coordinate" effects).
- Nonuniqueness of formulating metric equations (crucial issue!).

Now let's construct such a relativistic code with simple EoS using standard techniques!

# The Relativistic Field Equations

Task: Extract equations for hydrodynamics and spacetime metric from Einstein equations.

Einstein field equations of general relativity + Bianchi identities
↓
divergence equations for energy momentum tensor (equations of motion)

$$G^{\mu\nu} = 8\pi T^{\mu\nu} \quad \Longrightarrow \quad \nabla_\nu T^{\mu\nu} = 0,$$

$$\uparrow$$

$$\nabla_\nu G^{\mu\nu} = 0$$

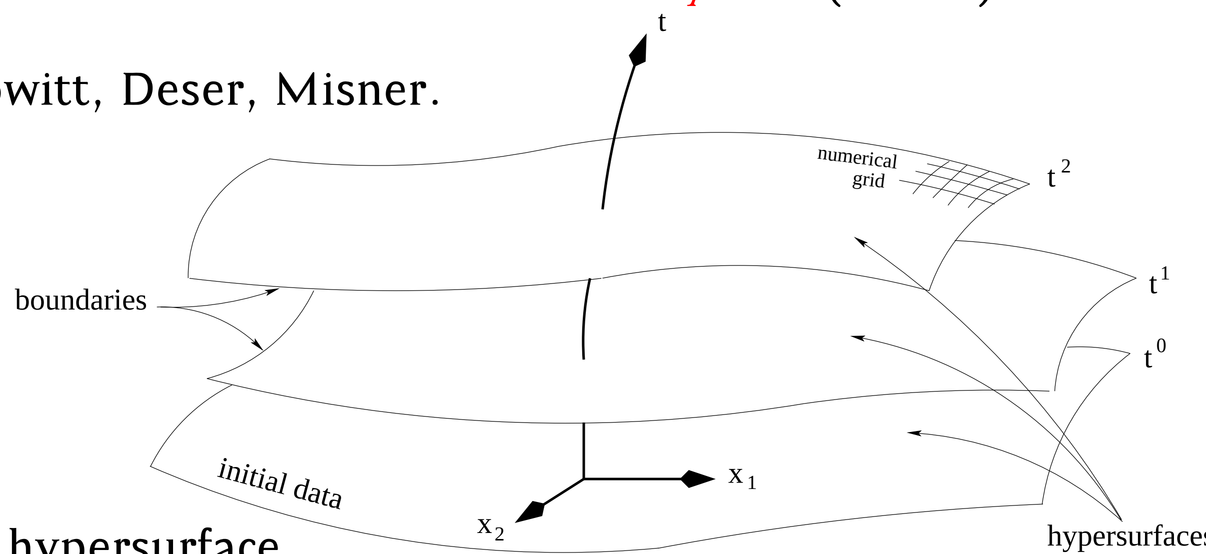
with Einstein tensor $G^{\mu\nu}$ (spacetime curvature) and energy momentum tensor $T^{\mu\nu}$ (matter).

This yields evolution equations for matter.

# The Issue of Coordinate Choice

We want to do numerical physics. $\Longrightarrow$ Choose a suitable coordinate system (metric).

We use ADM $3+1$ formalism by Arnowitt, Deser, Misner.
(This choice is not unique!)




$\Longrightarrow$ Split spacetime into a foliation of 3-dimensional hypersurfaces.

This defines a Cauchy problem:

- Solve initial value problem on one hypersurface.
- Evolve initial data in time with given boundary conditions.

We know: Spacetime concept in general relativity glues together space and time.
But for numerical convenience: Split spacetime again into space and time!

These components constitute metric (concepts from differential geometry):

- Lapse function $\alpha$: Local distance between hypersurfaces.
- Shift vector $\beta^i$: Coordinate shift between hypersurfaces.
- 3-metric $\gamma_{ij}$: Curvature of hypersurfaces.

# The ADM 3 + 1 Spacetime Split

Interval $ds^2$ between two events $\hat{x}^\mu$ and $x^\mu$ in spacetime is given by Pythagoras' theorem:

$$ds^2 = \begin{pmatrix} \text{Proper distance within} \\ \text{the hypersurface} \end{pmatrix}^2 - \begin{pmatrix} \text{Proper time between} \\ \text{the hypersurfaces} \end{pmatrix}^2.$$

ADM line element can thus be written as

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt).$$

Then metric assumes following form:

$$g_{\mu\nu} = \left(\begin{array}{c|ccc} -\alpha^2 + \beta_i\beta^i & \beta_1 & \beta_2 & \beta_3 \\ \hline \beta_1 & & & \\ \beta_2 & & \gamma_{ij} & \\ \beta_3 & & & \end{array}\right).$$

Metric has 10 components, which are not all physically indepentent.

Use gauge freedom to fix some components and to adapt metric to specific evolution situations (compare to Maxwell equations in electrodynamics).

## The Relativistic Conservation Equations

Now apply ADM $3+1$ formalism to matter equations.

We define a set of conserved hydrodynamic quantities:

$$D = \rho W, \qquad S_i = \rho h W^2 v_i, \qquad \tau = \rho h W^2 - P - D,$$

with Lorentz factor $W$, enthalpy $h$, and 3-velocity $v_i$.

Relativistic equations of motion for an ideal fluid
↓
System of hyperbolic conservation equations

$$\frac{1}{\sqrt{-g}}\left(\frac{\partial\sqrt{\gamma}\rho W}{\partial t} + \frac{\partial\sqrt{-g}\,\rho W\hat{v}^i}{\partial x^i}\right) = 0,$$

$$\frac{1}{\sqrt{-g}}\left(\frac{\partial\sqrt{\gamma}\rho h W^2 v^j}{\partial t} + \frac{\partial\sqrt{-g}\,(\rho h W^2 v_j \hat{v}^i + P\delta_j^i)}{\partial x^i}\right) = T^{\mu\nu}\left(\frac{\partial g_{\nu j}}{\partial x^\mu} - \Gamma^{\delta}_{\mu\nu} g_{\delta j}\right),$$

$$\frac{1}{\sqrt{-g}}\left(\frac{\partial\sqrt{\gamma}(\rho h W^2 - P - \rho W)}{\partial t} + \frac{\partial\sqrt{-g}\,((\rho h W^2 - \rho W - P)\hat{v}^i + P v^i)}{\partial x^i}\right) = \alpha\left(T^{\mu 0}\frac{\partial \ln\alpha}{\partial x^\mu} - T^{\mu\nu}\Gamma^{0}_{\mu\nu}\right),$$

where $\Gamma^{\lambda}_{\mu\nu}$ are Christoffel symbols, $g = \det g_{\mu\nu}$, $\gamma = \det \gamma_{ij}$, and $\hat{v}^i = v^i - \beta^i/\alpha$.

These equations are general relativistic extension to hydro equations in Newtonian gravity.

# High-Resolution Shock-Capturing Methods

Straigtforward way for solving hydrodynamic equations:
Simply use finite difference methods with artificial viscosity.

Problem: Conservation of physically conserved quantities violated, problems with shocks!

Better strategy: Exploit their hyperbolic and conservative form:

$$\frac{1}{\sqrt{-g}}\left[\frac{\partial\sqrt{\gamma}F^0}{\partial t}+\frac{\partial\sqrt{-g}F^i}{\partial x^i}\right]=S,$$

with vectors of conserved quantities $F^0$, fluxes $F^i$ and sources $S$.

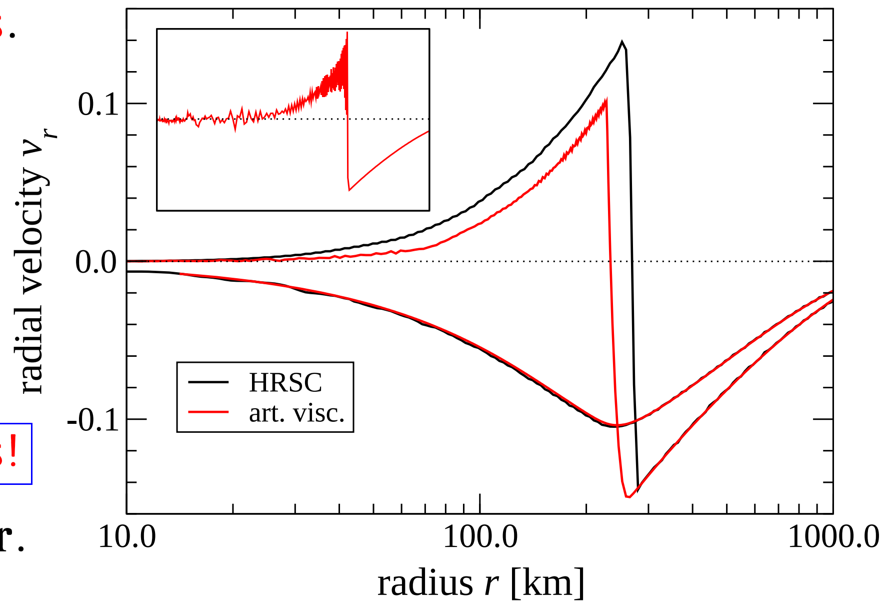
Modern recipe for such equations: High-resolution shock-capturing (HRSC) methods.
$\Longrightarrow$ Use solution of (approximate) Riemann problems.

This method guarantees:

- Convergence to physical solution of the problem.
- Correct propagation velocities of discontinuities.
- Sharp resolution of discontinuities.

HRSC methods are particularly well suited for shocks!

For core collapse: SPH methods are obviously inferior.



# The ADM Metric Equations

For hydrodynamic equations: With HRSC schemes found perfect method!

Now turn to metric equations.

In ADM metric:

Einstein field equations for spacetime metric
↓
Set of evolution and constraint equations

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + \nabla_i \beta_j + \nabla_j \beta_i, \qquad \text{(3-metric evolution)}$$

$$\partial_t K_{ij} = -\nabla_i \nabla_j \alpha + \alpha(R_{ij} + KK_{ij} - 2K_{im}K_j^{m}) + \beta^m \nabla_m K_{ij} + \qquad \text{(extrinsic curvature evolution)}$$
$$+K_{im}\nabla_j \beta^m + K_{jm}\nabla_i \beta^m - 8\pi T_{ij},$$

$$0 = R + K^2 - K_{ij}K^{ij} - 16\pi\alpha^2 T^{00}, \qquad \text{(Hamiltonian constraint)}$$

$$0 = \nabla_i (K^{ij} - \gamma^{ij} K) - 8\pi S^j, \qquad \text{(momentum constraint)}$$

with Riemann scalar $R$ and extrinsic curvature $K_{ij}$ (note similarity to Maxwell equations).

These metric equations have been workhorse of numerical relativity for decades!

In free evolution:
Once constraints are solved, simply use finite difference methods to evolve metric
(use constraints simply to occasionally monitor accuracy of solution).

# Problems with Numerical Solutions of ADM-based Metric Equations

Recapitulate for ADM equations in standard form:
Split into 10 evolution and 4 constraint equations.

Free evolution: Constraint violating modes can grow during evolution
(compare to div $B = 0$ in Maxwell equations).

If they grow exponentially $\Longrightarrow$ Numerical results become "inaccurate"!

Approach for reformulating Einstein equations
(standard system of ADM metric equation is already "somewhat" hyperbolic):

Emphasize hyperbolicity, evolve all degrees of freedom.

This way of massaging equations to obtain analytically equivalent formulations
with improved numerical properties is an art!

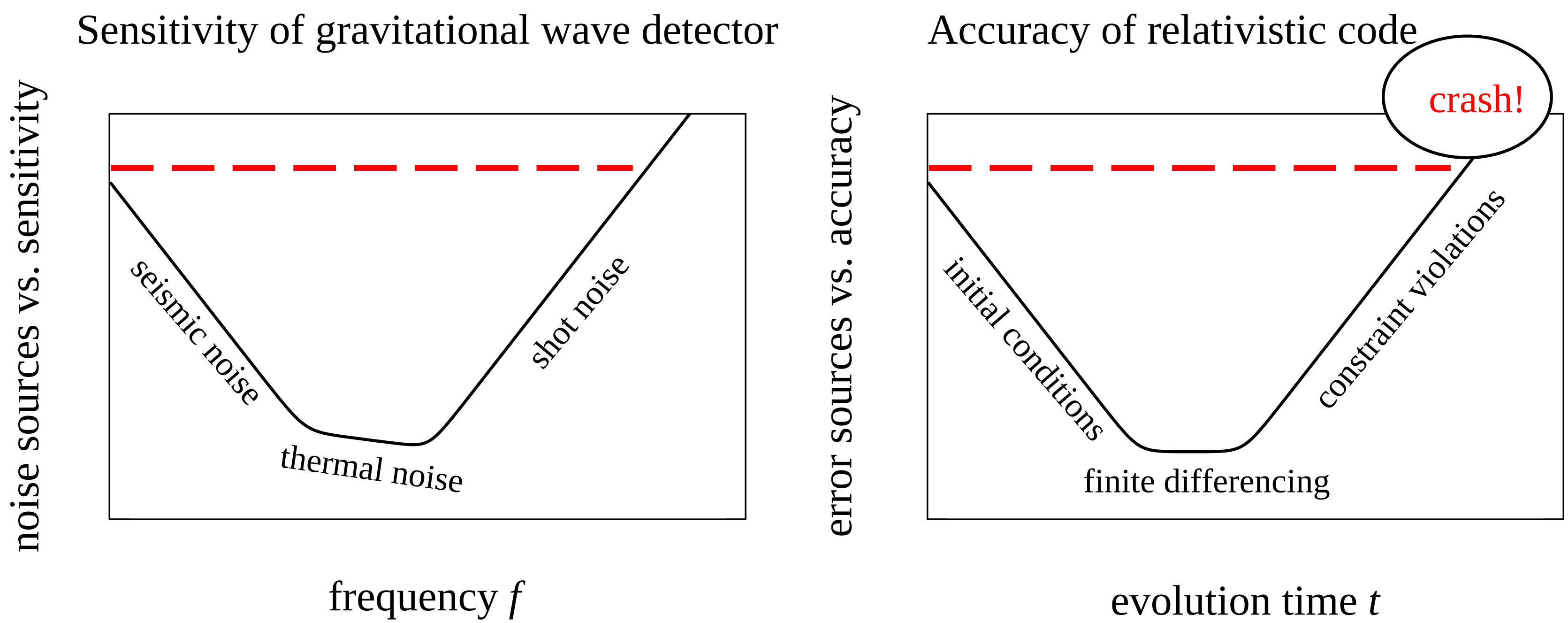
Examples for such recasting of ADM equations: Bona–Massó, ADM-BSSN (NOK), Z4.

# The Window of Stability for General Relativistic Simulations

Sad bottomline for all these reformulations so far:

Can extend evolution time with e.g. BSSN,
but ultimately constraint violations are also present!

Can physical system fit into "time window of stability" (idea for metaphor from D. Pollney)?




New formulations will buy us some time, but in general: Unsatisfactory approach!
$\Longrightarrow$ Next old hope which dies. . .

Possible solution: "Reproject" solution back onto constraint hypersurface (Holst et al., 2004).

# Free Evolution versus Constrained Evolution

Alternative strategy to reformulate Einstein metric equations: Constrained evolution.

Idea:
Most robust solution: Replace some evolution equations by constraints in evolution.
$\Longrightarrow$ Constraint violation impossible (by definition).

Examples:

- ADM constrained evolution.
- Conformal flatness approximation (CFC): No evolutionary degrees of freedom in metric.
- New maximally constrained evolution scheme from Meudon (Bonazzola et al., 2003): Evolve only the two "physical" evolutionary degrees of freedom.

Experience (so far limited) shows: Much more stable!

Crucial disadvantage: Numerical solution of elliptic constraints very expensive.

Possible breakthrough solution: Spectral methods.
$\Longrightarrow$ Very accurate even for small number of collocation points.

Formulations, numerical solvers, and computer power currently become available.

# The Conformally Flat Metric Equations

Now show a constrained evolution scheme a bit more in detail.

Approximate exact three-metric by a conformally flat one: $\gamma_{ij} = \phi^4 \hat{\gamma}_{ij}$.
(This approach is exact for a spherically symmetric spacetime!)

In CFC approximation:

ADM equations for exact metric
↓
System of five coupled elliptic equations for CFC metric

$$\Delta\phi = -2\pi\phi^5 \left(\rho W^2 - P + \frac{K_{ij}K^{ij}}{16\pi}\right),$$

$$\Delta\alpha\phi = 2\pi\alpha\phi^5 \left(\rho h(3W^2 - 2) + 5P + \frac{7K_{ij}K^{ij}}{16\pi}\right),$$

$$\Delta\beta^i = 16\pi\alpha\phi^4 S^i + 2K^{ij}\nabla_j\left(\frac{\alpha}{\phi^6}\right) - \frac{1}{3}\nabla^i\nabla_k\beta^k,$$

where $\nabla$ and $\Delta$ are Nabla and Laplace operator, respectively.

Task: Solve these equations efficiently (particularly in 3d)!

(This is nontrivial and interesting for anyone who needs to solve elliptic equations!)

## Introduction to Spectral Methods

Novel approach in constrained evolution schemes:
Try multidimensional elliptic solver based on spectral methods.

Difference between finite difference and spectral methods:

- Finite difference methods:
  Approximate analytic function by overlapping *local* polynomials of low order.
- Spectral methods:
  Approximate analytic function by *global* smooth functions
  (compare to Fouries expansion).

Functions are complete basis of orthogonal

- Legendre or Chebyshev polynomials in radial direction, and
- spherical harmonics in angular directions.

Crucial advantages related to accuracy:

- For $C^\infty$ function: Errors decreases exponentially with $\hat{n}$.
- Applying linear (differential) operators reduces to simple operations on spectral coefficients.

## Comparison of Spectral Methods with Finite Difference Methods

For typical analytical test functions: Need much less points for accurate representation!

But discontinuities cannot be well represented by expansion methods
(think of Fourier expansion):

- Low number collocation points: No sharp resolution of discontinuity.
- High number of collocation points: Gibbs phenomenon.

$\Longrightarrow$ Not well suited for hydrodynamics (supernova shocks, thin discs).

In such cases: HRSC schemes superior.

But metric is smooth even in presence of matter discontinuities!

Breakthrough concept by Valencia/Meudon groups:
Use HRSC methods for hydrodynamics and spectral methods for metric!

## The “Mariage des Maillage” Approach

As example: Introduce our new collaborative general relativistic hydrodynamics code.

It features:

- 3d grid with spherical polar coordinates (adopted to spheroidal stellar geometry).
- Simple matter model with hybrid ideal gas EoS (will be extended).
- CFC approximation of full Einstein metric equations.
- Spectral methods for solving spacetime metric equations (excellent convergence with few grid points).
- HRSC Riemann-solver methods for hydrodynamics (accurate resolution of shock fronts).


Hybrid methods on separate grids:
“Mariage des Maillage” (grid wedding).

## Testing Multidimensional Relativistic Hydrodynamic Codes

To trust results from numerical simulations, code must be tested:

- Can code reproduce analytic solutions
(like TOV solution, Oppenheimer–Snyder dust collapse, black holes as end states of collapse)?

  Problem: Not many interesting analytic solutions in relativistic hydrodynamics known.

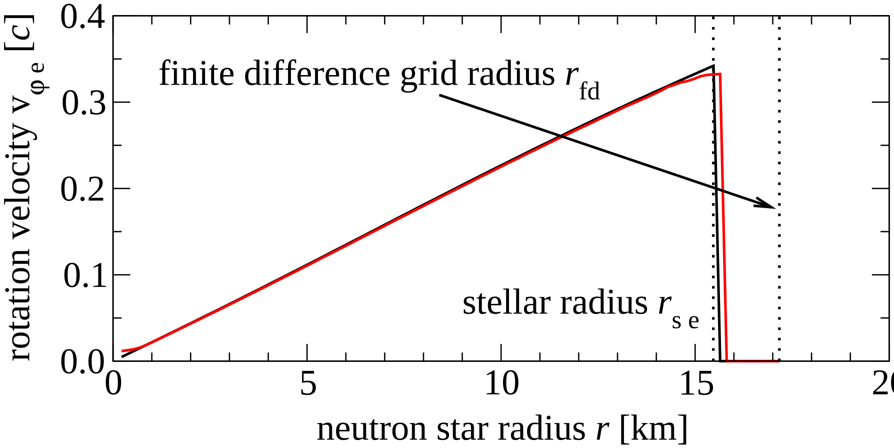
- Can code preserve symmetry of initial conditions (if no physical instabilities are present)?
- Can code keep equilibrium configurations stable (balance is numerically not trivial)?
- Can code recover results from perturbation approaches (like stellar oscillation frequencies)?
- How does code compare to other relativistic hydrodynamic codes (for "standardized" tests)?

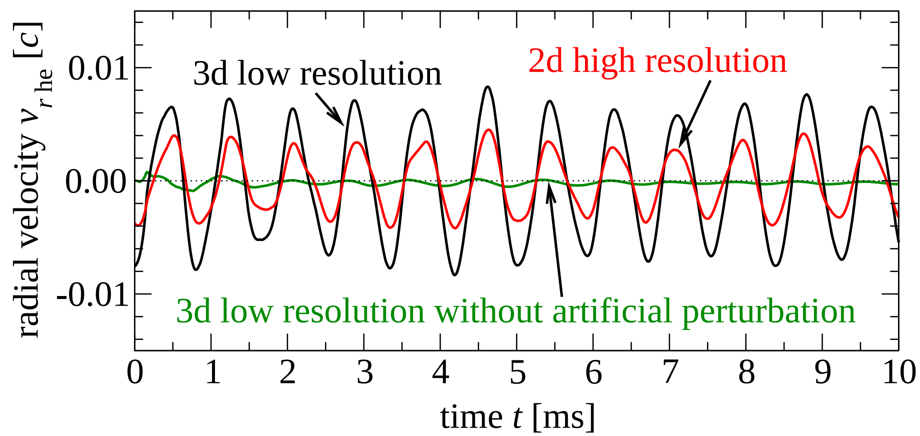
Such a list constitutes a "test suite" to calibrate numerical codes.

# Rotating Neutron Stars in Axisymmetry

Test example: Preservation of rotation velocity profile in rapidly rotating neutron star model.

Simulations: Dimmelmeier, Font, Stergioulas, in preparation
(Relativistic gravity, polytropic EoS).

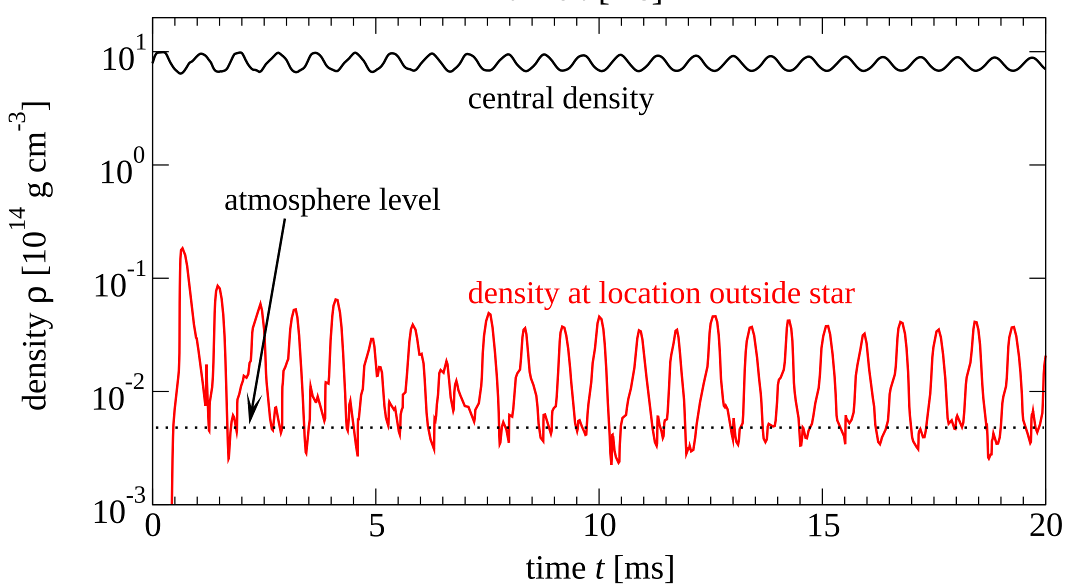



For extremely rapidly rotating neutron stars:
Observe persistent mass shedding.
$\Longrightarrow$ Damping mechanism for oscillations.

Damping time scale is rather long.
Nevertheless: Can use dynamic simulations.

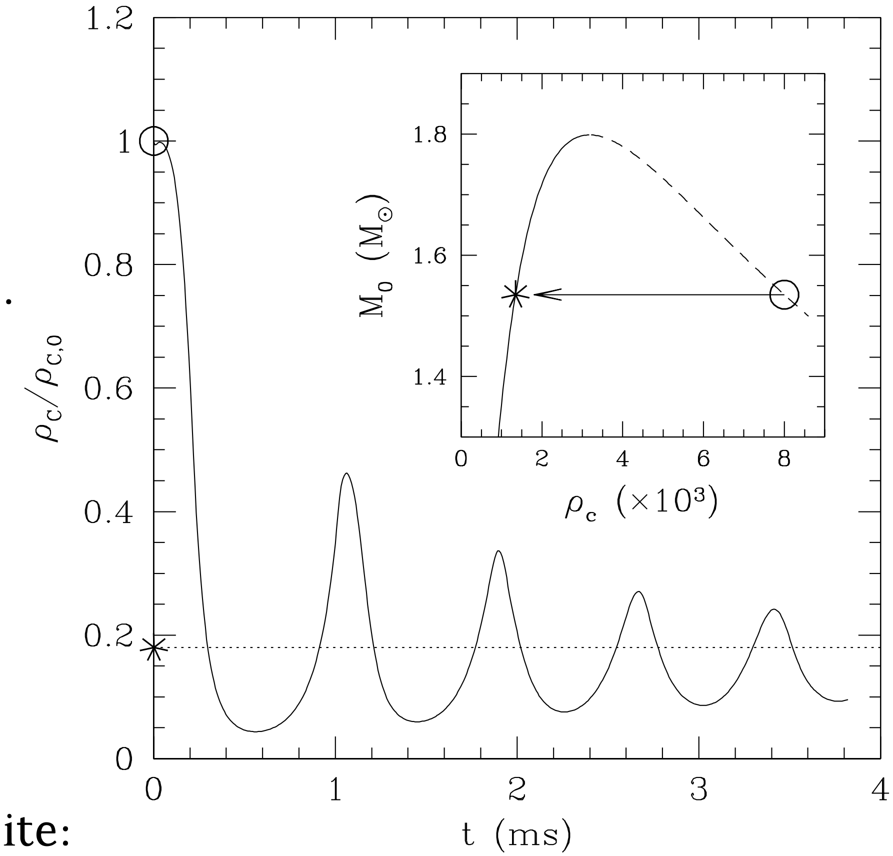
In this setup: Get also gravitational waves!



## The Ladder of Credibility

Another test example:
Migration of spherical neutron star from unstable to stable branch.

Simulation: Baiotti et al., 2004 (Cactus–Whisky code).




This test is very stringent:

- Equilibrium initial state (unstable).
- Equilibrium end state (stable).
- Fully dynamic evolution in between.

Bottomline for test suite:

Code has to prove ability to simulate
static, stationary, and fully dynamic situations.

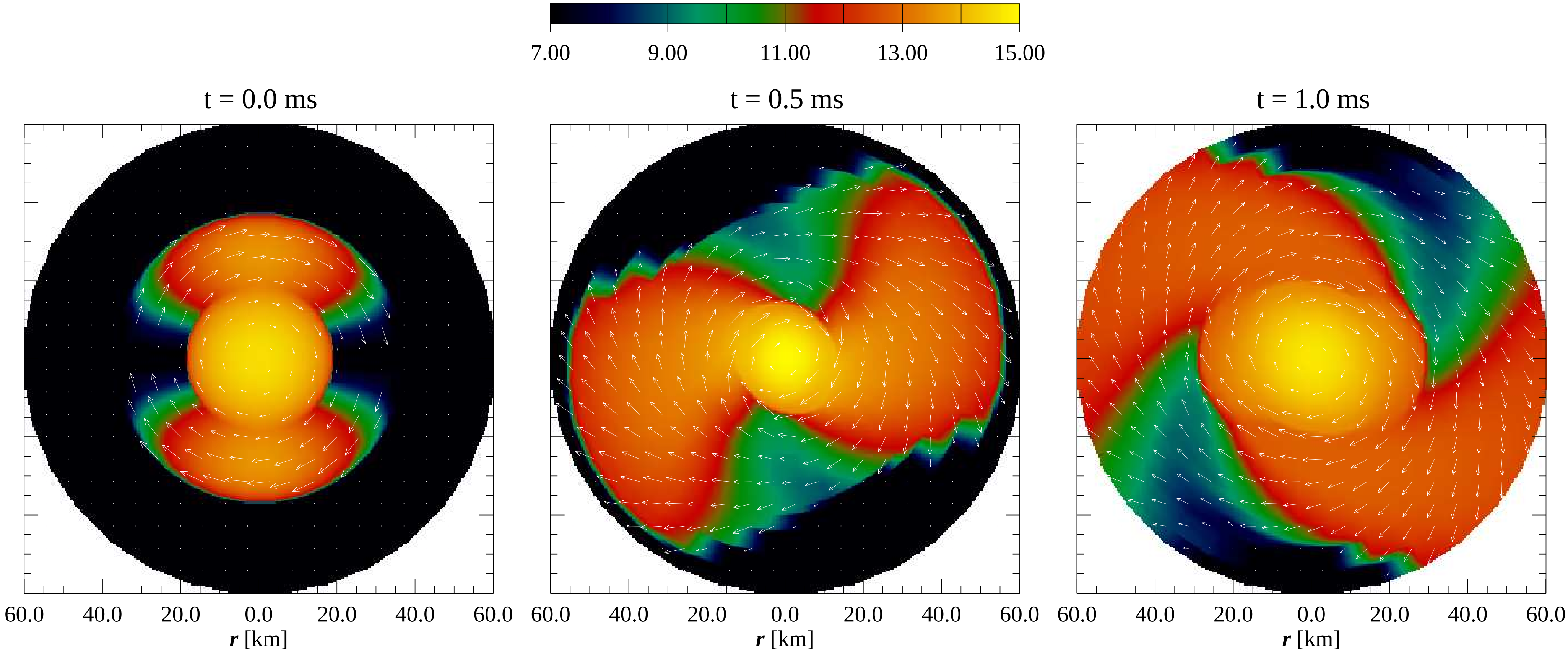
After successful completion of test suite:

Code has climbed “Ladder of Credibility”!

# Application of the Code to New Situations

Now code is ready for simulations in area of intended configurations.

Example setup: Rotating neutron star with strong "bar" perturbation.
Note: This is unphysical setup, code used here as technology demonstrator.



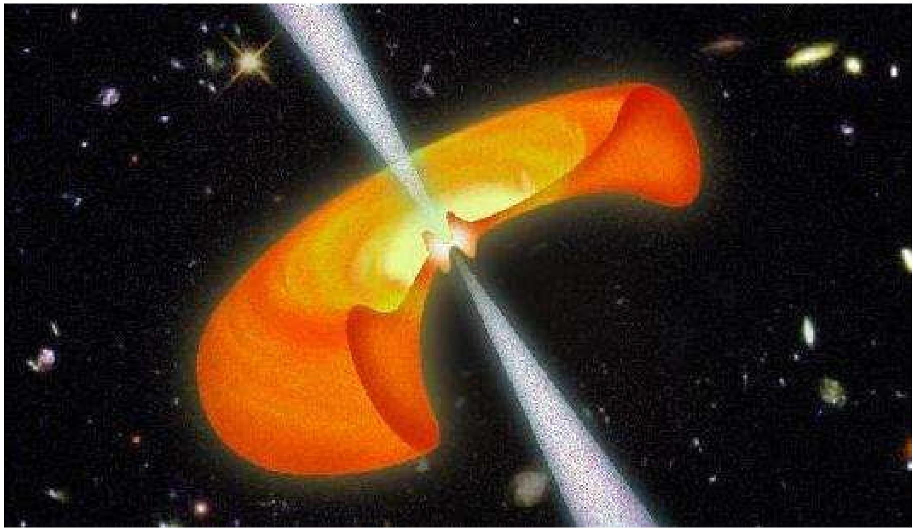

Rotation generates spiral arms!

# The Road to Extreme Spacetimes

With modifications of current codes and new formulations:

New astrophysical scenarios become accessible to numerical simulations!

Will soon be able to simulate extreme spacetimes.



Example: Collapsar scenario for gamma-ray bursts (rotational core collapse to black hole surrounded by matter torus which is accreted).

This is a real challenge for any numerical code:

- Very dynamic and energetic event:
  Need fully dynamic code,
  perturbation approach does not make sense.
- Contains black hole:
  Need excision methods to avoid crash of code when singularity forms.
- Jet produces gamma rays:
  Need special relativity to describe extremely high Lorentz factors in jet.
- Will be source of gravitational waves:
  Newtonian quadrupole approximation will definitely fail (black hole ring down).

# Summary of The State of the Art in Core Collapse Simulations

Numerical simulations of supernova core collapse in relativistic gravity: Tough business!

"Approximation" matrix for multi-dimensional simulations (take it with a grain of salt):

| Microphysics \ Gravity | Newtonian | Modified Newtonian ([redacted] top secret) | Approximate Relativistic (PPN, CFC, CFC+) | Fully Relativistic (ADM, ADM-BSSN, MCE) |
|---|---|---|---|---|
| Ideal gas | ✓ | experimental | ✓ | ongoing work |
| Tabulated EoS | ✓ | ⋮ | ongoing work | experimental |
| Simple neutrino treatment | ✓ | ⋮ | | experimental |
| Boltzmann neutrino transport | ongoing work | ⋮ | | U.S. DoD? |

Hint: Must enter realm of differential geometry at step 3 along "gravity axis".
Interesting idea: Is it possible to improve Newtonian codes to mimick relativistic gravity?

## General Summary and Outlook into the (Hopefully Bright) Future

Summary of talk:

- There is still a lot to explore in field of supernova core collapse (current standard model won't be last word!).
- "Second order" effects may play a decisive role to obtain robust explosion (relativistic gravity, convection, rotation, neutrinos).
- Numerical simulations will pave way to understanding still unresolved issues (even though "Grand Unified Code" does not exist yet and probably never will).
- There has been progress in relativistic numerical hydrodynamic simulations (now know why ADM-based schemes often fail and have a prospective way out).
- In supernova core collapse there are many more wave emission mechanisms than bounce (burst signal need not even be most strong or energetic signal).
- Gravitational wave templates are essential for successful detection (we still hide our physical ignorance in parameter studies, but data analysists are happy).
- When talking about wave templates, we must think hard about many problematic points (convergence, accuracy, uncertain physics, initial conditions, sample statistics, event rates).
- With new formulations, better code, faster computers: New range of applications opens (but faster computer alone won't help).