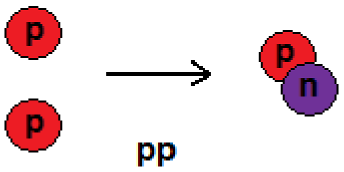
Seminar "Nuclei in the cosmos"
WS 2011/12

Hydrogen burning reactions



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$$\begin{tikzcd}
{}^{12}\mathrm{C} \arrow[r] & {}^{13}\mathrm{N} \arrow[r] & {}^{13}\mathrm{C} \arrow[dd] \\
 & \mathit{CNO} & \\
{}^{15}\mathrm{N} \arrow[uu] & {}^{15}\mathrm{O} \arrow[l] & {}^{14}N \arrow[l]
\end{tikzcd}$$

Part I: pp-chain reactions

Basic reactions:

$$^{1}\mathrm{H} + {}^{1}\mathrm{H} \rightarrow {}^{2}\mathrm{H} + \mathrm{e}^{+} + \nu_{e} + 0.42\mathrm{MeV}$$

$$\mathrm{p} \rightarrow \mathrm{n} + \mathrm{e}^{+} + \nu_{e}$$ (weak transformation from proton to neutron in first reaction)

$$\mathrm{e}^{+} + \mathrm{e}^{-} \rightarrow 2\gamma + 1.022\mathrm{MeV}$$

$$^{2}\mathrm{H} + {}^{1}\mathrm{H} \rightarrow {}^{3}\mathrm{He} + \gamma + 5.49\mathrm{MeV}$$

$$^{3}\mathrm{He} + {}^{3}\mathrm{He} \rightarrow {}^{4}\mathrm{He} + {}^{1}\mathrm{H} + {}^{1}\mathrm{H} + 12.86\mathrm{MeV}$$

Quick historical overview:

1920s: Arthur Stanley Eddington advocated proton-proton reactions as star burning principle, even though tunneling and weak interaction were not yet understood

1936: Weizsäcker suggested the pp reaction as stellar energy source

1938: Bethe & Critchfield derived an actual formalism

1953: pp chains were indentified as the primary energy source of our sun

1967: Bethe wins nobel prize for his work on stellar nucleosynthesis

Quick remark: Why no diproton?

- The two protons in the first reaction overcome the coulumb barrier to fuse to a deuteron

- One of the protons becomes a neutron through weak interaction

- Question: Why is there no direct fusion to ^2He / diproton?

- The diproton has no bound state

- The Pauli principle forbids two identical fermions to be in the same place
  → at least one quantum number has to be different

- For n>0 the excitation energy is higher than the binding energy
  → therefore l,m = 0
  → spin must be antiparallel

- Due to the spin-spin-coupling of strong nuclear force there is no 2-nucleon singulett nucleus, the deuteron also only has a bound state as triplett!

- It has been postulated, that, if the strong force was 2% stronger, the diproton would have a bound state
  → "diproton catastrophe"

Energy output of pp-chain

- The first reaction H + H → D only creates 0.42MeV of energy due to the energy necessary to transform a proton into a neutron. Because of the low energy output, extremely low cross-section ,the necessity of weak interaction and the tunneling process, this step is very slow and limitating the reaction rate of the pp chain.

- In the next step, the positron from proton-proton fusion annihilates with an electron of the plasma, creating two photons (conservation of momentum). This process creates energy equivalent to twice the electron mass:

$$2\text{x } 0.511\text{MeV} = \underline{1.022\text{MeV}}$$

This annihilation process proceeds quickly as the two particles attract each other.

- The third step D + H → ^{3}He sets 5.49MeV of energy free.

The last step is actually a reaction of helium-3, but will be shortly discussed as it provides two protons. $^3\text{He} + {}^3\text{He} \rightarrow {}^4\text{He} + 2\text{ H} + \underline{12.86\text{MeV}}$. As this process needs two helium-3 nuclei, all previous reactions have to be done twice.

→ The total energy output of the pp chain (net reaction $6\text{ H} \rightarrow {}^4\text{He} + 2\text{ H} + 2\ v$) is therefore: $2*(0.42+1.022+5.49)+12.86\ [\text{MeV}] \approx \underline{26.7\ [\text{MeV}]}$

Alternations of the standard pp chain

- Stars may contain lithium. As lithium is not created by stellar fusion processes (it was actually created in the big bang), lithium in stars is only a reactant. Because of this, the lithium concentration of stars can be used to measure their age. This lithium-burning reaction chain is:

$$\begin{aligned}
&{}^{1}\mathrm{H} + {}^{6}\mathrm{Li} \rightarrow {}^{7}\mathrm{Be} \\
&{}^{7}\mathrm{Be} + e^{-} \rightarrow {}^{7}\mathrm{Li} + \nu_e \\
&{}^{1}\mathrm{H} + {}^{7}\mathrm{Li} \rightarrow {}^{8}\mathrm{Be} \\
&{}^{8}\mathrm{Be} \rightarrow 2\ {}^{4}\mathrm{He}
\end{aligned}$$

- There is also beryllium/bor burning reaction chain, responisble for ~0.11% of our sun's energy production. It is interesting for particle physics as it creates high-energy neutrinos (up to 14.06MeV).

$$\begin{aligned}
&{}^{3}\mathrm{He} + {}^{4}\mathrm{He} \rightarrow {}^{7}\mathrm{Be} + \gamma \\
&{}^{7}\mathrm{Be} + {}^{1}\mathrm{H} \rightarrow {}^{8}\mathrm{B} + \gamma \\
&{}^{8}\mathrm{B} \rightarrow {}^{8}\mathrm{Be} + e^{+} + \nu_e + \gamma \\
&{}^{8}\mathrm{Be} \rightarrow 2\ {}^{4}\mathrm{He}
\end{aligned}$$

- Furthermore, there is the pep (proton-electron-proton) reaction. Its frequency ratio to standard pp is roughly 1:400 as it is a three-particle process. As it does not create a positron but capture an electron directly, its neutrino has a much higher energy.

$$^{1}\mathrm{H} + e^{-} + {}^{1}\mathrm{H} \rightarrow {}^{2}\mathrm{H} + \nu_e$$

- A process that has been predicted but never observed is the Hep reaction. It is postulated to be extremely rare (approximately 0.3 ppm). It creates even higher neutrino energies.

$$^{3}\mathrm{He} + {}^{1}\mathrm{H} \rightarrow {}^{4}\mathrm{He} + e^{+} + \nu_e$$

Heat and mass requirements

- The fusion rate of pp reaction is proportional to the temperatur to the power of 6. The basic reaction only requires the matter to be in a plasma state. This is the case at temperatures over 3 million Kelvin. The hydrogen-deuterium reaction requires 12-13 Jupiter masses. This marks the lower limit of a brown dwarf.

- A steady reaction rate of helium-3 fusion requires a temperature of 10-14 million Kelvin.

- The lithium burning is dominant at temperatures 14-23 MK. But it occurs at lower rates even in brown dwarfs of 2.5 MK. It needs at least 60 times the mass of Jupiter.

- Likewise, beryllium/bor burning is dominant when the star exceeds 23 MK.

hydrogen reactions in labs

- So far, only reactions of hydrogen isotopes have been viable for experimentation and possible use as an energy source through nuclear fusion as the coulomb barrier is the lowest at a nuclear charge of 1e.

- Because of its much higher cross-section, deuterium-tritium reactions are most common. It only needs low temperatures. An uncontrolled fusion is prominent in H-bombs. A single DT → He(4) reaction yields 17.6MeV.

- A much lower cross-section but less problems with tritium radioactivity and rarity is present in DD reactions. There are two possible products: tritium + proton (4.032MeV) or helium-3 + neutron (3.268MeV). The possible reaction that might follow are:

$$\begin{aligned}
\mathrm{p}+\mathrm{T} &\rightarrow {}^{4}\mathrm{He}+\gamma+19{,}814\,\mathrm{MeV}\\
\mathrm{D}+\mathrm{T} &\rightarrow \mathrm{n}+{}^{4}\mathrm{He}+17{,}589\,\mathrm{MeV}\\
\mathrm{D}+{}^{3}\mathrm{He} &\rightarrow \mathrm{p}+{}^{4}\mathrm{He}+18{,}353\,\mathrm{MeV}\\
\mathrm{T}+\mathrm{T} &\rightarrow 2\,\mathrm{n}+{}^{4}\mathrm{He}+11{,}332\,\mathrm{MeV}
\end{aligned}$$

Part II: Cold CNO cycles

- Although the reactions go a different path, the net reaction and energy output of a full cycle is the same as with the pp chain. But – there are different distributions of how much energy is radiated in photons (and thus sunlight) and neutrinos

- As catalytic reactions, cycles needs abundances of their catalysts in the stellar material

- The reaction rate of these cycles are proportional to T^15-T^18

- CNO reaction rates are important to understand stars, especially the ones larger than our sun

- Ratio of pp and CNO reactions in low-mass stars are hinting to the age of globular clusters – the derived age of the oldest clusters is about the same as the computed age of the universe in other experiments

a) The Bethe-Weizsäcker cycle (1937-1939)

- C-12 serves as a catalyst of this reaction, going thorugh the cycle of N-13 (p capture), C-13 (beta decay), N-14 (pc), O-15 (pc), N-15 (β) and back to C-12 again through simultaneous proton capture and alpha decay
- Common notation of cycles:

$$^{12}\mathrm{C}(p,\gamma)^{13}\mathrm{N}(\beta^{+}\nu)^{13}\mathrm{C}(p,\gamma)^{14}\mathrm{N}(p,\gamma)^{15}\mathrm{O}(\beta^{+}\nu)^{15}\mathrm{N}(p,\alpha)^{12}\mathrm{C}$$

- The slowest reaction limiting the reaction rate to a full cycle in ~340 million years of this cycle is the proton capture
  N(14) → O(15)
- Starts at T≈14 MK and becomes dominant at T≈30 MK / 1.5 times our suns' mass (larger temperature gradients lead to a convective core in massive stars)

b) The NO cycle (Burbidge et al., 1957)

$$ {}^{16}\mathrm{O}(p,\gamma)^{17}\mathrm{F}(\beta^{+}\nu)^{17}\mathrm{O}(p,\alpha)^{14}\mathrm{N}(p,\gamma)^{15}\mathrm{O}(\beta^{+}\nu)^{15}\mathrm{N}(p,\gamma)^{16}\mathrm{O} $$

- Fueled by the abundance of oxygen in the star

- The strength of the radiative proton capture branch 15N(p,γ)16O is the limiting factor – it competes with the alpha decay of Bethe-Weizsäcker

- 0.04% of our sun's core proton reactions

- This cycle approaches a constant fluor abundance in a star – like the other cycles, it does not accumulate one of its catalytic elements after an equilibrium is reached

c) 3rd CNO cycle (evidence found 1974 by Rolfs & Rodney, outlined by Dearborn & Schramm) and possible 4th cycle

$$^{17}\mathrm{O}(p,\gamma)^{18}\mathrm{F}(\beta^{+}\nu)^{18}\mathrm{O}(p,\alpha)^{15}\mathrm{N}(p,\gamma)^{16}\mathrm{O}(p,\gamma)^{17}\mathrm{F}(\beta^{+}\nu)^{17}\mathrm{O}$$

$$^{18}\mathrm{O}(p,\gamma)^{19}\mathrm{F}(p,\alpha)^{16}\mathrm{O}(p,\gamma)^{17}\mathrm{F}(\beta^{+}\nu)^{17}\mathrm{O}(p,\gamma)^{18}\mathrm{F}(\beta^{+}\nu)^{18}\mathrm{O}$$

- Only significant in massive stars

- Radiative proton captures leading to F(18)/F(19) compete with NO-cycle's/3rd cycle's alpha decays

d) Adding up the cold CNO cycles

As stated, the cycles coexist, their rates depend on mass & temperature of the stars due to the competition of (p,α) and (p,γ) reactions. The lower the temperature, the more the strong interaction alpha decay dominates over the electromagnetic gamma decay. This means that “lower” CNO cycles also dominate.
The alpha decays of 16O and 20Ne are also preferred due to their pronounced alpha cluster structure.

Part III: Hot CNO cycles

- High-temperature conditions provide alternative pathways for radioactive isotopes (eg. 13N, 17F and 18F), as they can perform proton captures before their beta decay.

- While cold CNO processes depend on the proton captures of stable isotopes, hot CNOs are characterized by beta decay rates of 14O, 15O and 18Ne. With sufficient temperatures, reactions can break out of the cycles through alpha capture on these isotopes, triggering a flow to elements of higher masses. This is called αp process.

Again, there are three dominant cycles. These hot CNO cycles are:

12C → 13N → 14O → 14N → 15O → 15N → 12 C (HCNO-I)

15N → 16O → 17F → 18Ne → 18F → 15O → 15N (HCNO-II)

18F → 19Ne → 19F → 16O → 17F → 18Ne → 18F (HCNO-III)

Addendum: Sources

- "Principles of Stellar Evolution and Nucleosynthesis", Donald D. Clayton, Univ Chicago Press 1983

- "The cold and hot CNO cycles", Annu. Rev. Nucl. Part. Sci. 2010. 60:381–404;
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- Wikipedia (for quick overviews)

Thank you for
your attention!