Supernova Light Curves

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How does the energy release of supernovae develop in time?

What physics is at the bottom of this temporal evolution?

Is it possible to describe it analytically?

Outline

1. Light Curves
   - Introduction to Light Curves
   - Physics of Type Ia Light Curves

2. Analytic solutions
   - Assumptions
   - Derivation
   - Discussion of solutions

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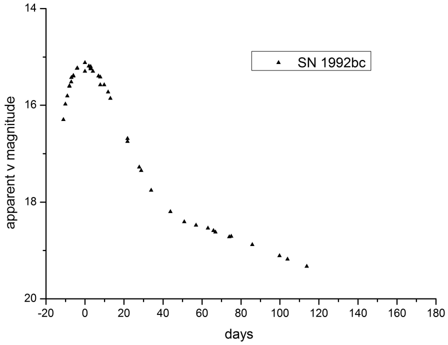
Basics - I

Light curves

Light curves display the temporal evolution of the energy release by supernovae.



Basics - II

- Bolometric light curves:
  - Ideal bolometric light curve: Luminosity integrated over all wavelengths.
  - Practically: Energy due to high energy γ-rays is not included.
  - UVOIR-light curves as a surrogate for bolometric light curves (UV: ultraviolet, O: optical IR: infrared)
  - In the early stages of a Supernova it represents nearly the total energy output.
- This approach is focusing on Type Ia supernovae.
- Starting point:
  - Thermonuclear disruption of a degenerate carbon-oxygen white dwarf
  - Star expands outwards and no compact object is left behind.
  - Thermonuclear burning has stopped: $t = 0$
- Light curves are an important source of information about the physics of supernovae.

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Qualitative Model - I

- First Approach: Qualitative model for the light curve of a Type Ia SN
- The different phases:

1. The ejecta of an SN Ia form an opaque, expanding sphere.
2. Energy is deposited by radioactive decay of $^{56}\mathrm{Ni}$ and $^{56}\mathrm{Co}$ at an exponentially declining rate.
3. At early stages:
   - High opacity $\Rightarrow$ Conversion of energy into kinetic energy of expansion
   - Optically thick ejecta prevents radiation from diffusing out $\Rightarrow$ small Luminosity

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4. Premaximum epoch:
   - Ejecta becomes more dilute.
   - Diffusion time drops below elapsed time.
     $\Rightarrow$ Increasing Luminosity
5. Peak phase:
   - Increasing fraction of emitted energy versus exponentially declining rate of energy input $\Rightarrow$ Luminosity Peak
   - 'Old' photons leak out.
6. Decline phase:
   - Luminosity equals instantanous deposition rate.
   - Shorter diffusion times $\Rightarrow$ More radiation escapes before thermalization.
   - Most of $^{56}\mathrm{Ni}$ has decayed $\Rightarrow$ The light curve tail is powered by $^{56}\mathrm{Co}$ decay.

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Radioactive decay - I

- Radioactive decay is the main source of energy.
- Reactions:

1. Electron capture of $^{56}\mathrm{Ni}$: $^{56}\mathrm{Ni} + \mathrm{e}^- \rightarrow {}^{56}\mathrm{Co} + \nu_\mathrm{e} + \gamma$
   Decay time: $\tau_{Ni} = 6.1\,\mathrm{d}$
   Energies of the most important γ's: 750 keV, 812 keV, 158 keV
2. Electron capture of $^{56}\mathrm{Co}$ (81 %): $^{56}\mathrm{Co} + \mathrm{e}^- \rightarrow {}^{56}\mathrm{Fe} + \nu_\mathrm{e} + \gamma$
   β-decay of $^{56}\mathrm{Co}$ (19 %): $^{56}\mathrm{Co} + \rightarrow {}^{56}\mathrm{Fe} + \mathrm{e}^+ + \nu_\mathrm{e} + \gamma$
   Decay time: $\tau_{Co} = 77.1\,\mathrm{d}$
   Energies of e^+'s: about 600 keV
   Energies of the most important γ's: 847 keV, 1.238 MeV

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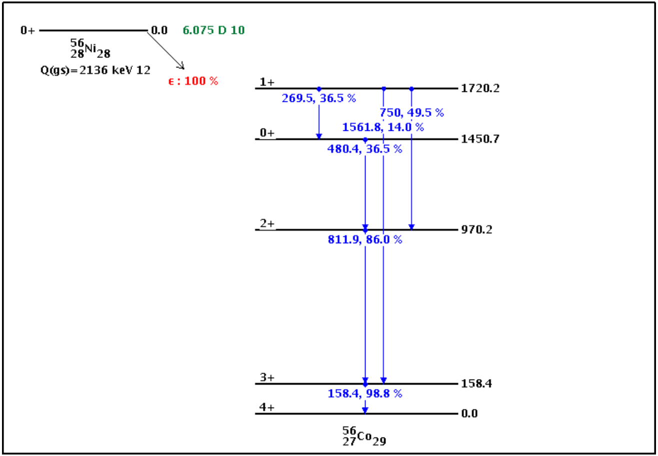

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- Spherical symmetry: The stellar debris is modelled as expanding sphere.
- Homologous expansion: Fractional rate of change of $r(t)$ is constant throughout the expanding sphere:

$$\frac{\dot{r}(t)}{r(t)} = \frac{\dot{R}(t)}{R(t)}$$

- Radiation pressure dominant
- ^{56}Ni present in ejecta. ^{56}Co decay is neglected.
- Constant opacity κ: In general $\kappa = \kappa(T, \rho, composition)$ Assumption that Thomson scattering ($\sigma_T = 6.67 \times 10^{-29}\,\mathrm{m}^2$) dominates opacity $\Rightarrow$ κ independent of T and ρ $\Rightarrow$ strong simplification

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Ansatz - I

- Basic ingredients for our model
- First law of thermodynamics
  - Describes time evolution of the thermal state of the expanding matter:

$$\dot{E} + P\dot{V} = -\frac{\partial L}{\partial m} + \varepsilon \qquad (1)$$

  - E the specific energy
  - V the specific volume: $V = \frac{1}{\rho}$
  - $\frac{\partial L}{\partial m}$ the energy released per unit mass per second
  - L the luminosity: $L = \frac{F_{rad}}{4\pi r^2}$
  - ε the rate of energy injected per unit mass by radioactive decay

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Ansatz - II

- The equations of state for a radiation dominated gas: $E = aT^4V$ and $P = \frac{E}{3V} = \frac{1}{3}aT^4$
- Equation of radiative transfer
  - Diffusion approximation

$$\frac{dP_{rad}}{dr} = -\frac{\kappa\rho}{c}F_{rad} \tag{2}$$

  - The radiation pressure gradient produces a 'photon wind' blowing from high to low P_{rad}.
- $R(t)$ the surface of the expanding sphere
  - For homologous expansion in a coasting phase: $R(t) = R_0 + v_0 t$
  - R_0 the radius of the progenitor
  - v_0 the constant velocity of expansion
  - Constancy of v_0 is a good approximation after the strong cooling of the first days, where most of the thermal energy is converted into kinetic energy.

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- Most simple approach
- Advantage: the crucial concepts are obvious and some simple results are already obtained.

i. The thermodynamics of the trapped radiation is described by a differential equation:

$$4\left(\frac{\dot{T}}{T}+\tau_{exp}^{-1}\right)=\tau_{heat}^{-1}-\tau_{diff}^{-1}$$

with effective timescales:

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$$Heating: \tau_{heat}^{-1}=\frac{\varepsilon}{E}\propto R\exp\frac{-t}{\tau_{Ni}}$$

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$$4\left(\frac{\dot{T}}{T}+\tau_{exp}^{-1}\right)=\tau_{heat}^{-1}-\tau_{diff}^{-1}$$

with effective timescales:

$$Expansion:\ \tau_{exp}^{-1}=\frac{v_0}{R}\propto\frac{1}{R}$$

$$Heating:\ \tau_{heat}^{-1}=\frac{\varepsilon}{E}\propto R\exp\frac{-t}{\tau_{Ni}}$$

$$Diffusion:\ \tau_{diff}^{-1}=\frac{4Rc}{M\kappa}\propto R$$

Toy model - II

ii. Premaximum epoch

- At early times: τ_{exp}^{-1} dominant
- Adiabatic cooling: $T \propto \frac{1}{R}$ and $E \propto \frac{1}{R}$
- At $t = 5\,\mathrm{d}$: $\tau_{exp} = \tau_{heat} \Rightarrow$ reheating of $^{56}\mathrm{Ni}$-decay

iii. Maximum

- $\frac{d(\ln L)}{dt} \stackrel{!}{=} 0 \Rightarrow \tau_{diff} = \tau_{heat}$
- $t_{Max} = 11\,\mathrm{d}$

Arnett's law

The luminosity at peak equals the instantanous energy deposition rate.

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- At $t = 41\,\mathrm{d}$: $\tau_{exp} = \tau_{diff}$
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- Dimensionless radial coordinate: $x=\frac{r}{R(t)}$
- Ansatz for T: $T(x,t)^4=\Psi(x)\Phi(t)T(0,0)^4\frac{R(0)^4}{R(t)^4}$
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- Time differential equation:

$$\dot{\Phi} + \Phi \frac{R(t)}{R_0 \tau_0} = \tilde{\varepsilon} \frac{R(t)}{R_0} e^{\frac{-t}{\tau_{Ni}}}$$

- Solution of differential equation:

$$\Phi(t) = \frac{\varepsilon_{Ni} M_{Ni}^0 \tau_0}{E_{Th}0} \Lambda(y,z) + e^{-z^2}$$

$$\Lambda(y,z) = e^{-z^2} \int_0^z e^{-2k\,y+k^2}\, 2k\, dk$$

$$z = \frac{t}{\tau_m} \quad y = \frac{\tau_m}{2\tau_{Ni}}$$

$\tilde{\varepsilon}$, τ_m and τ_0 are constants depending on $\kappa, \varepsilon_{Ni}, R_0, v_0$

- Luminosity at surface ($x = 1$) of expanding sphere:

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Outline

Mathematical behaviour - I

- The function $\Lambda(y, z(t))$ determines the structure of the light curve.
- It is a one-parameter family of curves. Typical values of the parameter y are between 0.5 and 2
- We want to investigate $\Lambda(y, z)$ in different regions.

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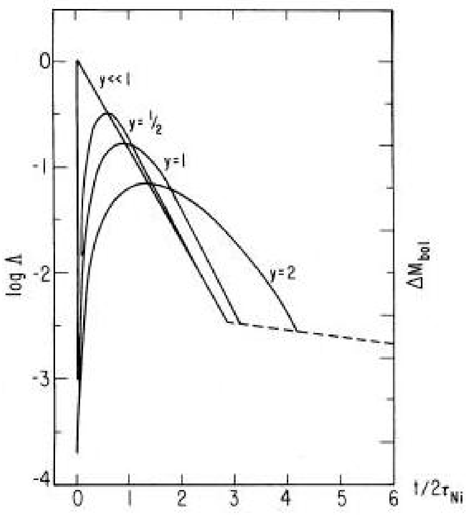

Mathematical behaviour - III

1. Early times: $y \gg z$

$$\Lambda(y,z) \approx \left[1 - e^{-\frac{t}{\tau_{Ni}}}\left(1 + \frac{t}{\tau_{Ni}}\right)\right]$$

   Saturation curve that rises to maximum in a few decay times.
   Solution of (1) without radiative diffusion term ($\frac{\partial L}{\partial m}$).

2. Peak:
   $\frac{\partial \Lambda}{\partial z} \stackrel{!}{=} 0 \Rightarrow \Lambda(y, z_{Max}) = e^{-\frac{t_{Max}}{\tau_{Ni}}} \Rightarrow L_{Max} = M_{Ni}^0 \varepsilon_{Ni} \Lambda = M_{Ni}^0 \varepsilon_{Ni} e^{-\frac{t_{Max}}{\tau_{Ni}}}$
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3. Late times or small y: $y \ll z$

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- The analytic solution is depending on several parameters:

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   - ^{56}Ni decay time: $\tau_{Ni} = 6\,\mathrm{d}$
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2. uncertain parameters:
   - the opacity: κ
   - the total mass of the ejecta: M
   - initial amount of ^{56}Ni: M_{Ni}
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Extending the scope I

- Not all observations are within the scope of our modelling.
- Observational result: “luminosity-width relation” - the brightest supernovae have the broadest light curve peaks.
- This disagrees the behaviour of our one-parameter family solutions $\Lambda(y, z)$.
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ii. Including $^{56}\mathrm{Co}$ decay
- Near maximum light the energy release from Co-decay exceeds that from Ni.
- $^{56}\mathrm{Co}$ decay in optical thin SN environment
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- Processes: Compton scattering, pair-production, free-free transitions, bound-free transitions, bound-bound line transitions, non-radiative processes
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- What is done today?

1. Fully three-dimensional explosion models, including instabilities and turbulence effects $\Rightarrow$ velocites, densities and composition of explosion ejecta
2. Radiative transfer calculations
    - 3D Monte Carlo methods
    - Evolution of energy packets in 3D Cartesian grid
    - Key data:
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    - 100 time steps from 2 to 80 days
    - 50^3-grid
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Summary

- Light curves display the temporal evolution of the energy output of a SN.
- The main energy source is $^{56}\mathrm{Ni}$ and $^{56}\mathrm{Co}$-decay.
- Crucial physical process: Photon diffusion in an expanding SN atmosphere.
- Simple model for bolometric light curves under strong assumptions.
- Complexity of elaborated calculations is due to a non constant opacity.

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