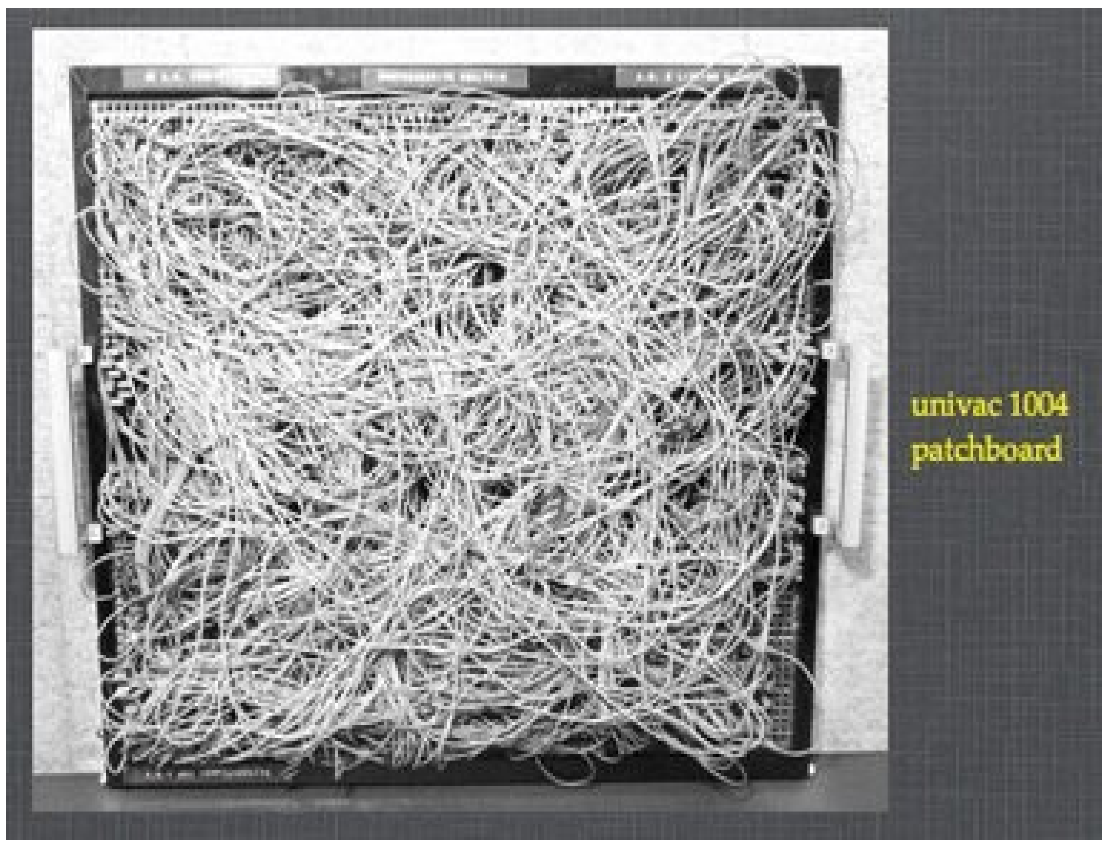
# NUCLEAR REACTION NETWORKS



# Nuclear reaction

Notation: A + b → c + D := A(b,c)D

target nucleus + projectile → ejectile + produce nucleus $^{12}C + ^{4}He \rightarrow ^{16}O + \gamma = C12(\alpha,\gamma)O16$


Reaction rate?

particle energy ("T“),

particle flux ("ρ“),

reaction cross section.


# Nuclear reacti...

All reactions pr...

via reac... ...densi...

Interaction Cross Section: relative velocity of the interacting nuclei

Thermally averaged cross section

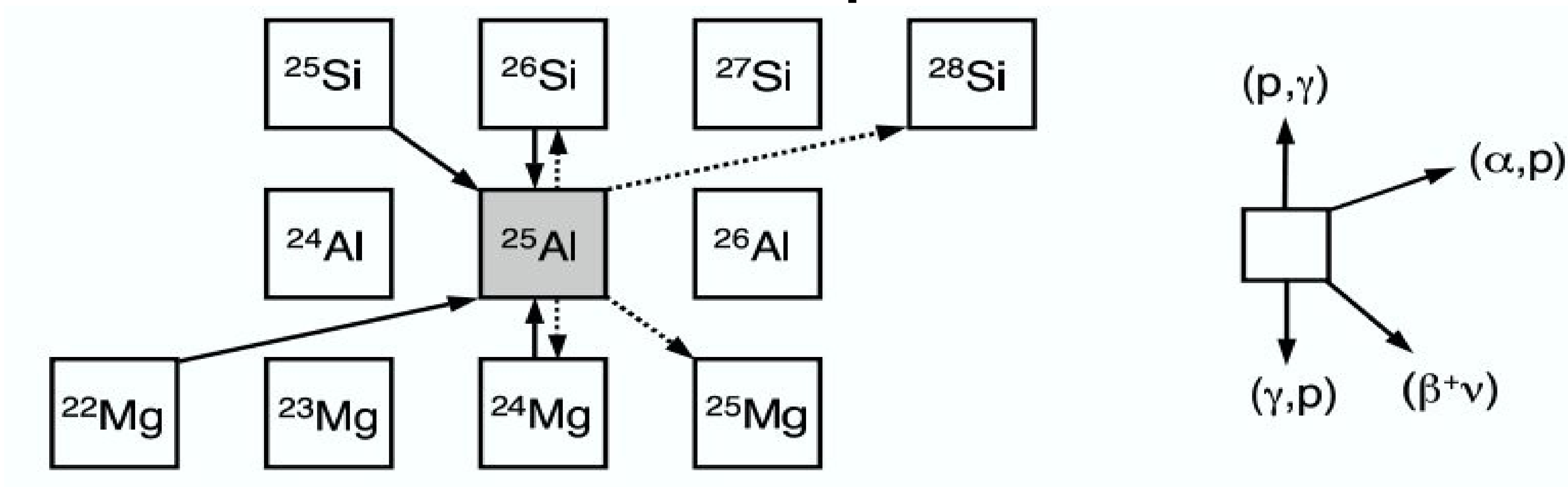
Beta decays of nucle... lead...

Photodisintegration of nucleus i

$$\frac{dN_i}{dt} = \Big[\sum_{j,k} N_j N_k \langle \sigma v \rangle_{jk\rightarrow i} + \sum_{l} \lambda_{\beta,l\rightarrow i} N_l + \sum_{m} \lambda_{\gamma,m\rightarrow i} N_m\Big]$$

$$- \Big[\sum_{n} N_n N_i \langle \sigma v \rangle_{ni} + \sum_{o} \lambda_{\beta,i\rightarrow o} N_i + \sum_{p} \lambda_{\gamma,i\rightarrow p} N_i\Big]$$

# Example




$$\frac{dN_{^{25}Al}}{dt} = N_H N_{^{24}Mg} \langle \sigma v \rangle_{^{24}Mg(p,\gamma)} + N_{^4He} N_{^{22}Mg} \langle \sigma v \rangle_{^{22}Mg(\alpha,p)} + N_{^{25}Si} \lambda_{^{25}Si(\beta^+ \nu)} + N_{^{26}Si} \lambda_{^{26}Si(\gamma,p)} + \dots$$

$$- N_H N_{^{25}Al} \langle \sigma v \rangle_{^{25}Al(p,\gamma)} - N_{^4He} N_{^{25}Al} \langle \sigma v \rangle_{^{25}Al(\alpha,p)} - N_{^{25}Al} \lambda_{^{25}Al(\beta^+ \nu)} - N_{^{25}Al} \lambda_{^{25}Al(\gamma,p)} - \dots$$

Iliadis; Nuclear Physics of Stars

# Simple Solutions

## Steady state

- Some part of the reaction network the time derivatives of all abundances are (nearly) zero
- balanced

## Equilibrium

- Abundances of a group of nuclei are locally balanced
- Because of (almost) equally strong forward and reverse reactions
- Reactionrate is much faster than the hydrodynamical time scale

# Why

Want to explain the origin of the elements

Understand observations

Evolution of stars

Chemical evolution

 of galaxies

Energy release

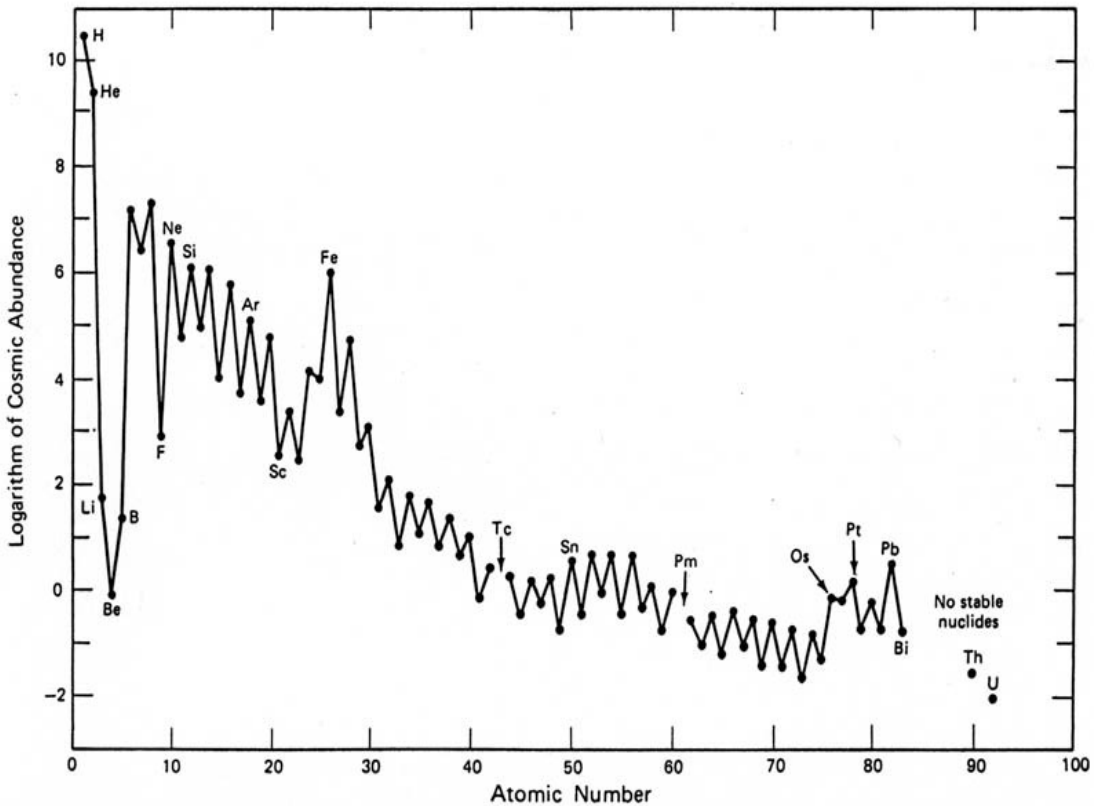



**Figure 2.4** Plot of the abundances of the elements in the solar system versus their atomic number. The abundances are expressed as the logarithm of the number of atoms of each element relative to $10^6$ atoms of silicon. (Data are listed in Table 2.2 after Anders and Ebihara, 1982.)

# requirements

Fast PCs

Hydrodynamic-code

Fast network solver

Partition functions

Masses of the Isotopes

Nuclear level densities

Thermonuclear reaction rates

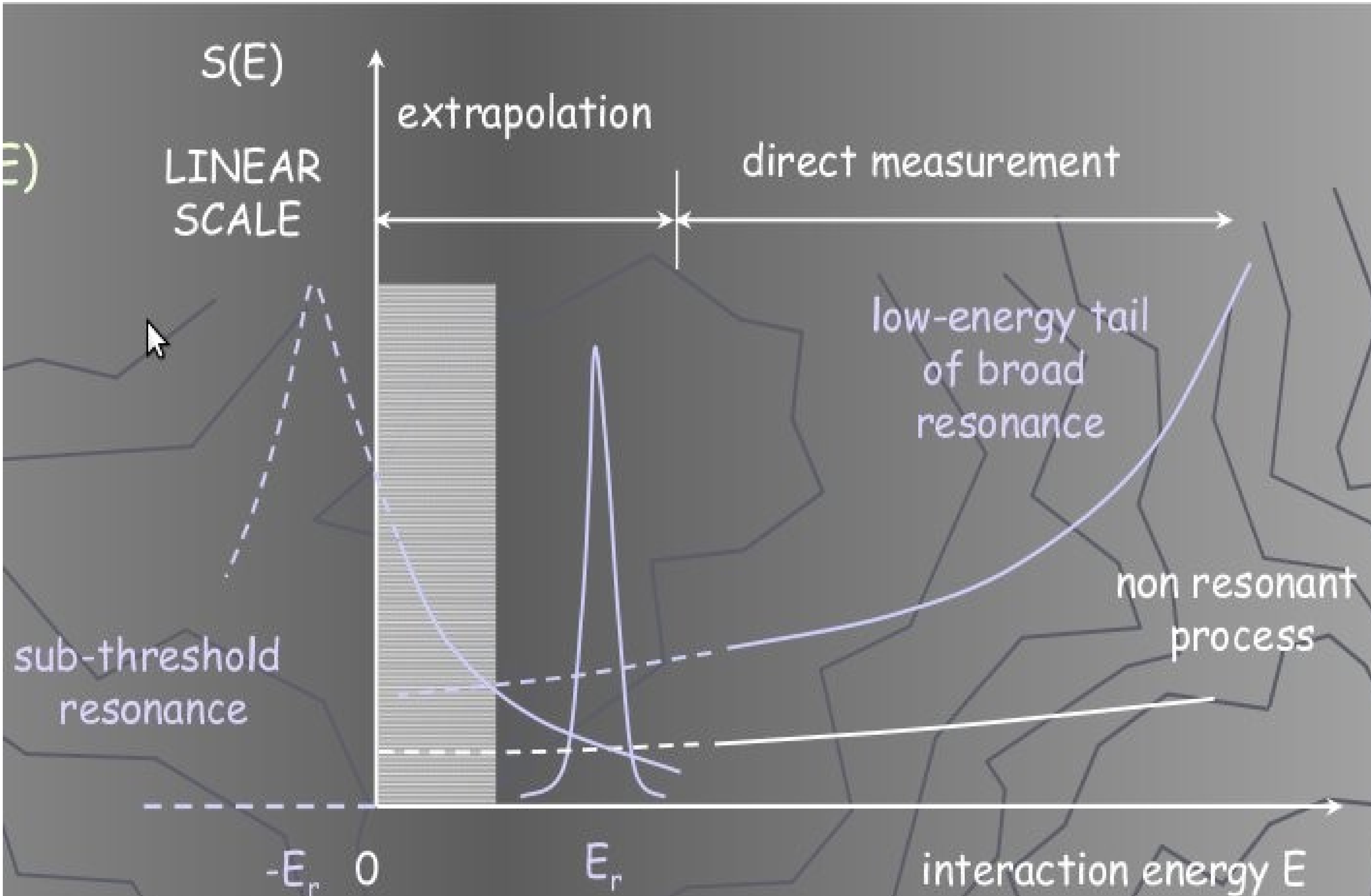
Database

Measurement

Calculations

http://www.nscl.msu.edu/~nero/db/labels.php

http://www.nndc.bnl.gov/masses/mass.mas03



S(E)
E)
LINEAR
SCALE
extrapolation
direct measurement
low-energy tail
of broad
resonance
non resonant
process
sub-threshold
resonance
-E$_r$
0
E$_r$
interaction energy E

# Different approaches for nucleosynthesis

## Follow a massblob

- Tracer particle follows flow lines of simulation
- Record temperature and density as function of time
- Calculate on that trajectory the nucleosynthesis in post-processing step

## Single Zone

- Constant temperature, density

## Multi Zone

- Calculating the energy of every shell and use it for the next one

convection?

# Network Calculations at Constant Temperature and Density

Only reactionrates

Faster

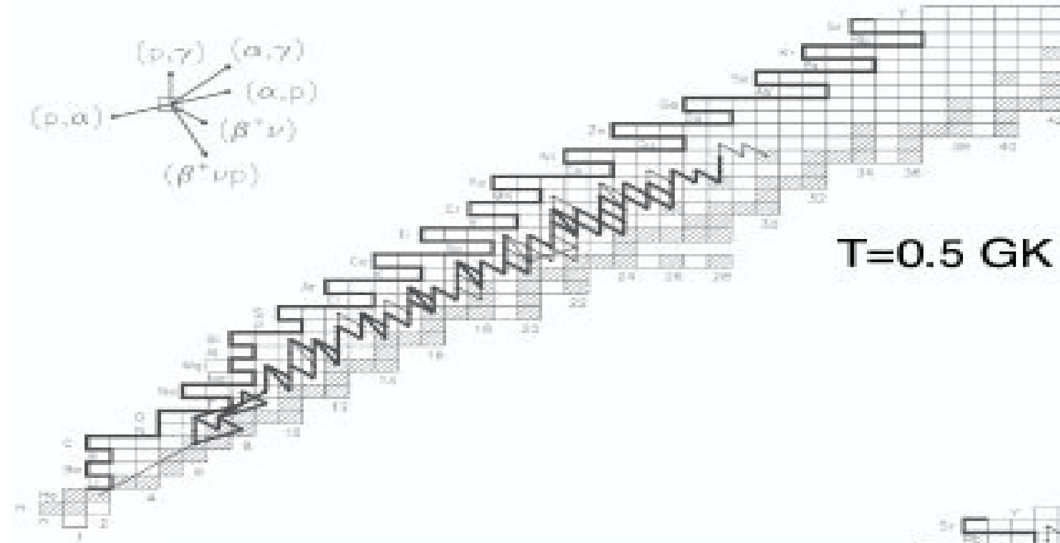
Comparison

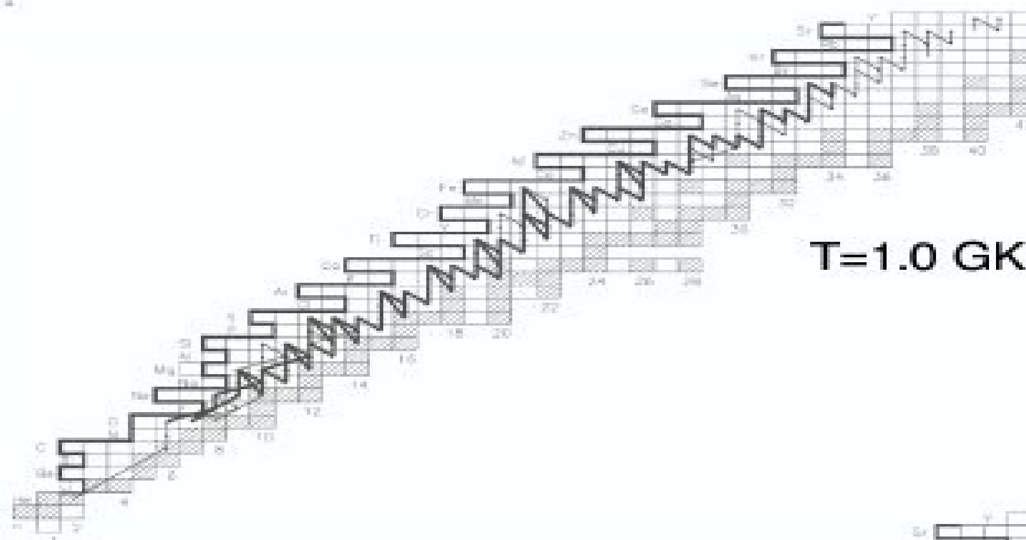
Not necessary to calculate equation of state

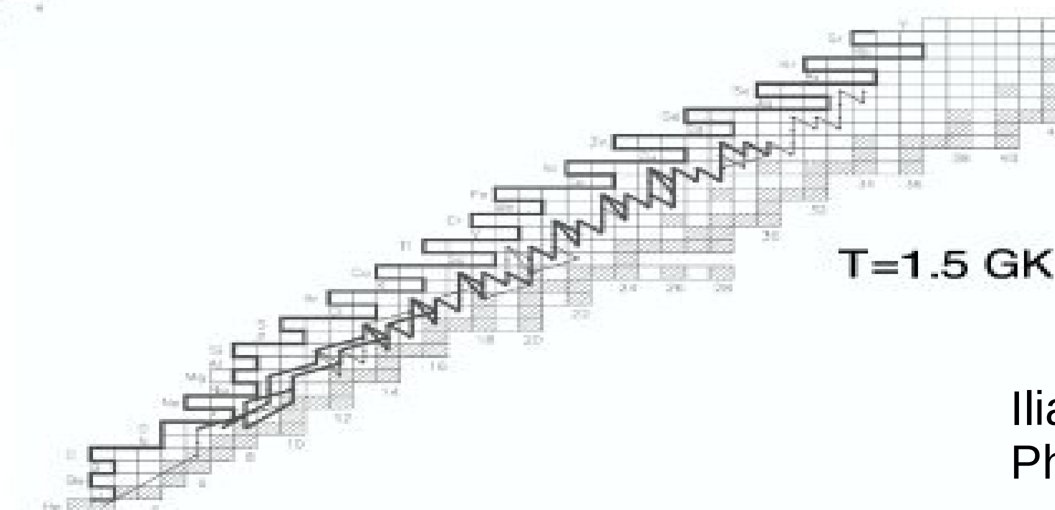
Overview over the nucleosynthesis

Approximation

Not using released energy




Result of numerical reaction network calculations for hydrogen-helium burning

$\rho = 10^4$ g/cm≥

Iliadis; nuclear Physiks of stars

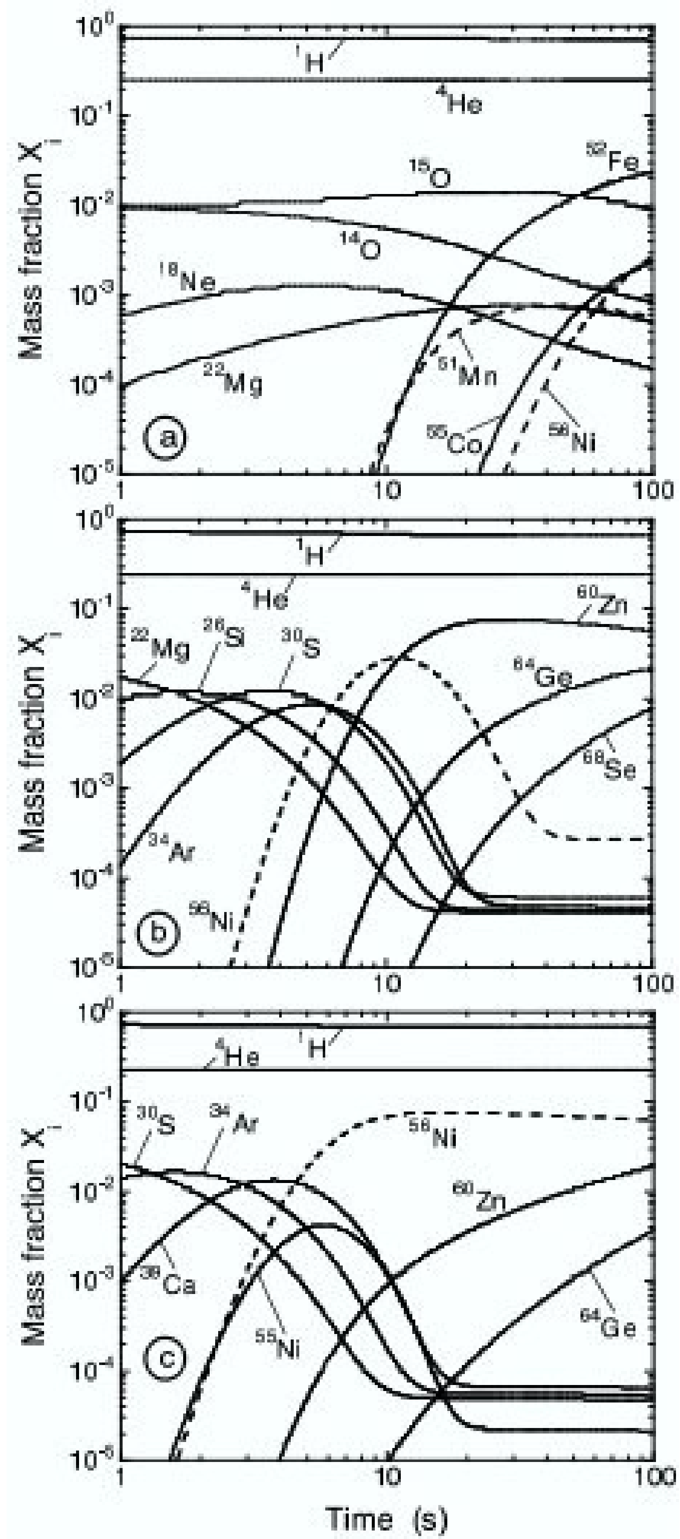
Abundance evolutions during hydrogen-helium burning

a) T=0.5GK

b) T=1.0GK

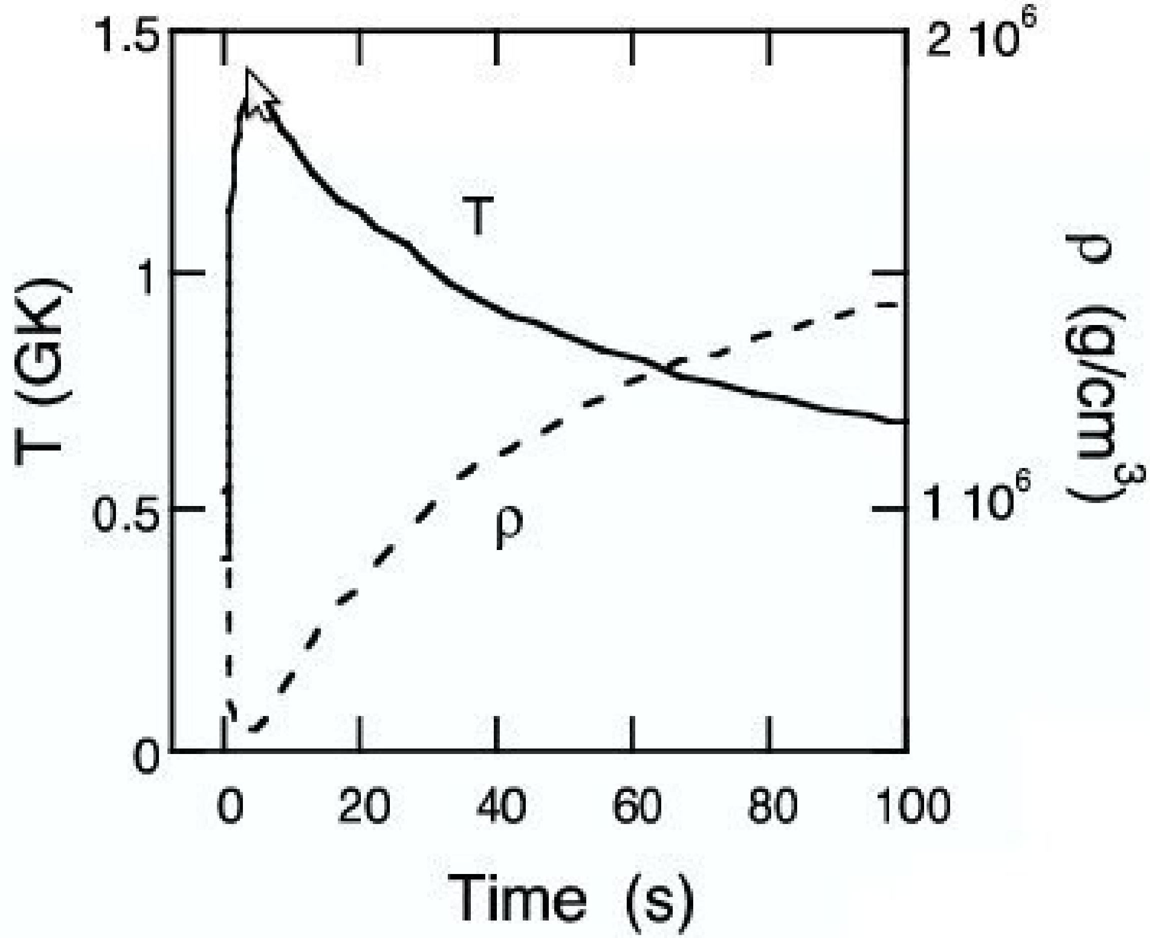
c) T=1.5GK

$\rho = 10^4$ g/cm≥




Iliadis; nuclear Physiks of stars

# Temperature-Density Profiles

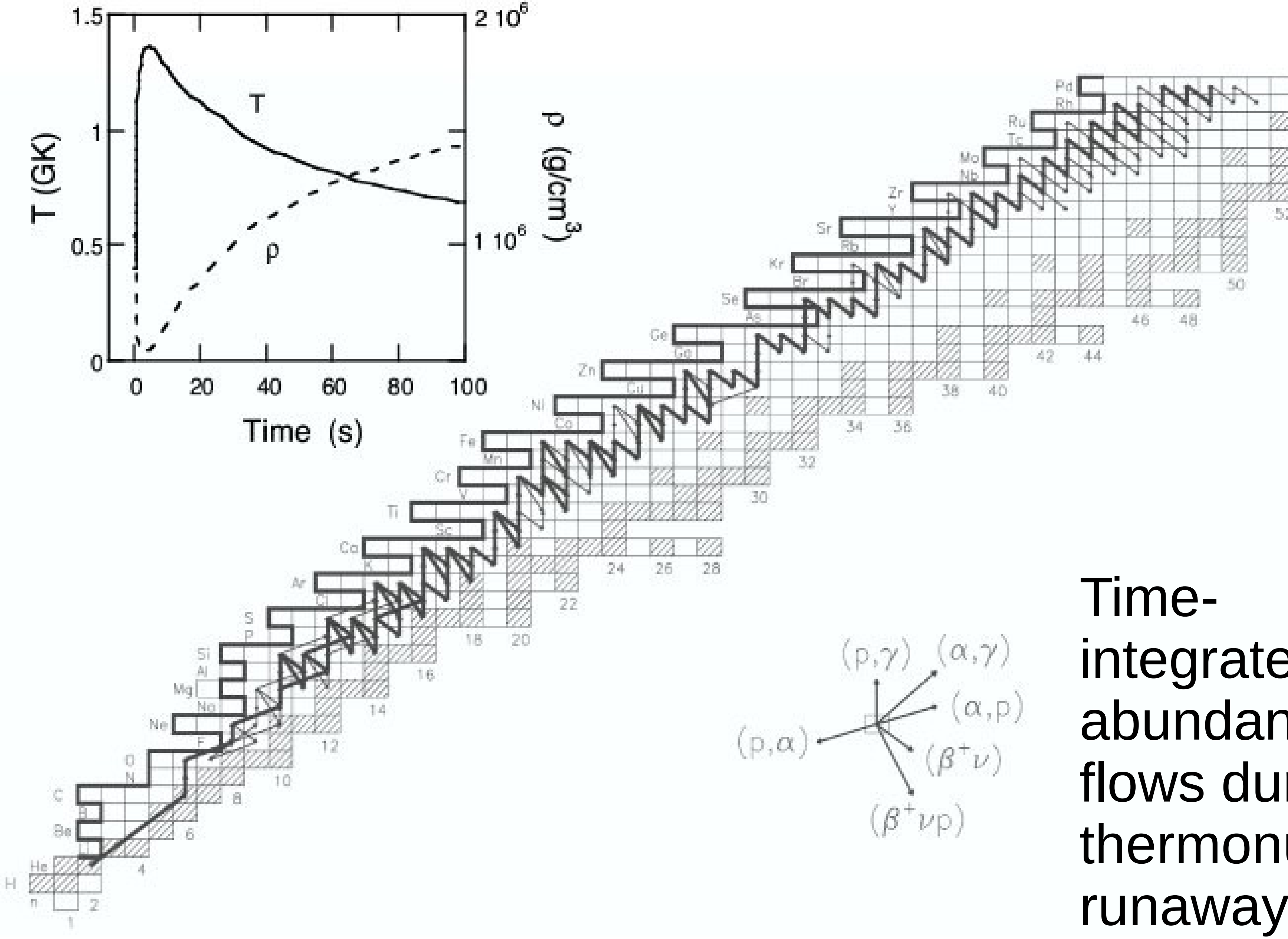


# How to get the profile

When simulating an explosive event, one puts “tracer particles“ in the simulated fluid

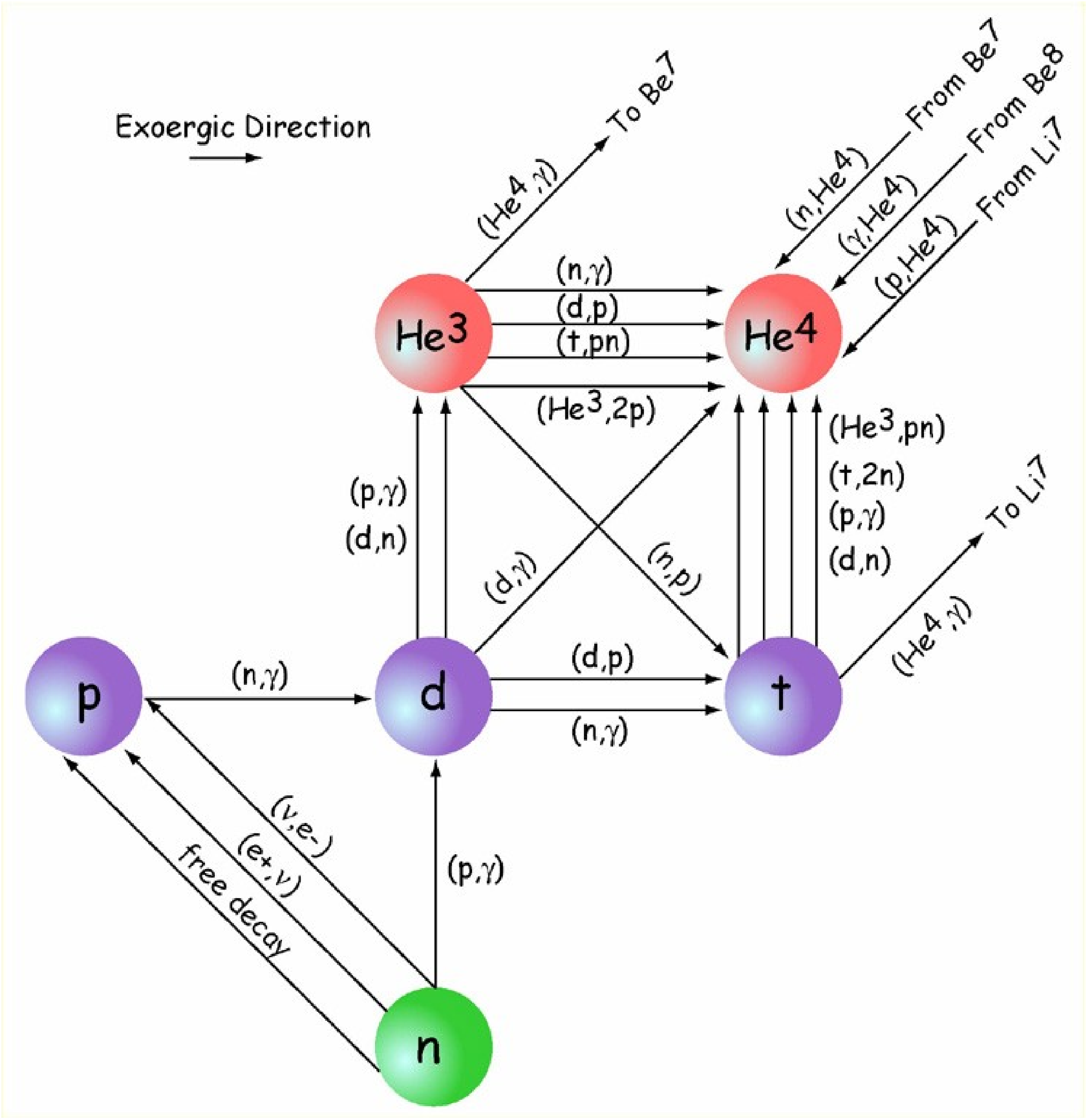
These particles record temperature and density

These information are sufficient to calculate the nucleosynthesis by solving the network equations in a post process




Time-integrated net abundance flows during a thermonuclear runaway

# Big Bang




n neutron

p proton

d deuterium

t tritium

$\rho$ = 0.029e$^4$ g/cm≥, T = 1.5 GK, expansion timescale of 0.83s,
neutron to seed ratio of 92; Nuclear masses are from the FRDM mass model

# rp-Process

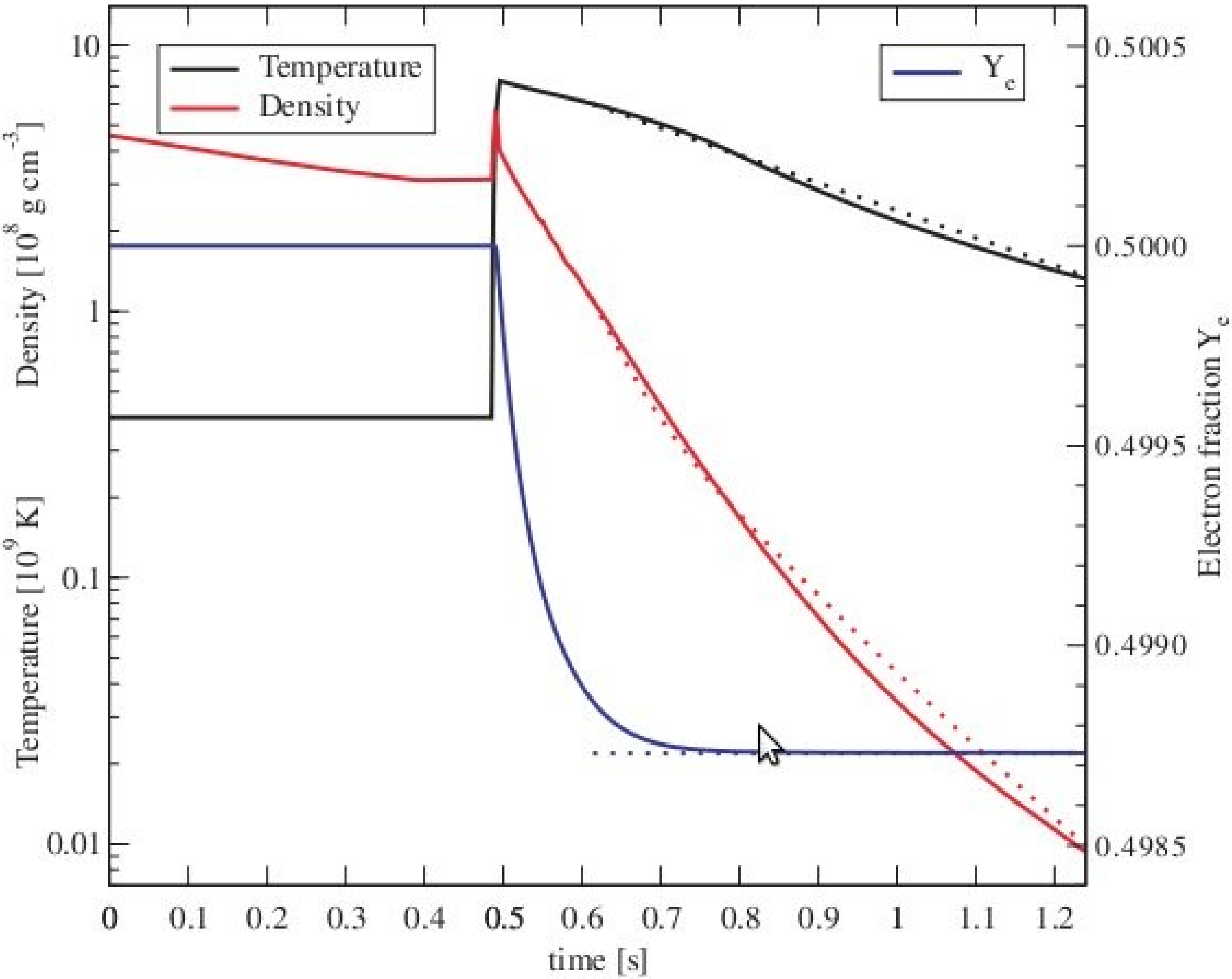
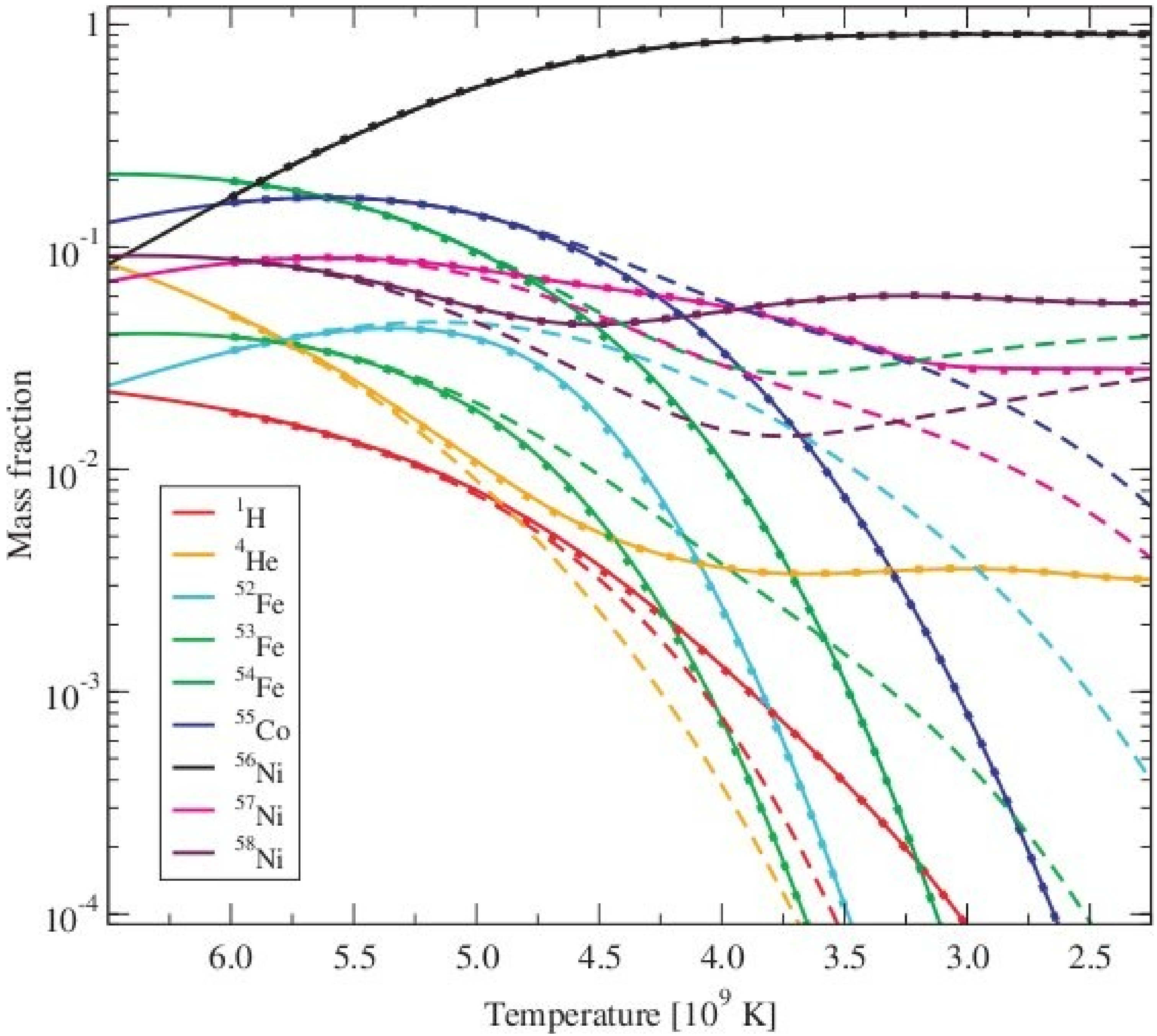



**Figure 13.** Thermodynamic trajectory of a Lagrangian tracer particle having an expansion timescale $\tau = 0.42$ s, final entropy $s = 2.273$ $N_A k$, and final electron mole fraction $Y_e = 0.49873$. The dotted line shows the analytic adiabatic fit to this trajectory, parameterized by $\tau$, $Y_e$, and $s$.

Shows abundance evolution of network (solid lines) along same trajectory starting at T9=6, where NSE has already been established Dashed lines show NSE



# uncertanties

- Rates and nuclear physics
  - Uncertainties in measurements and extrapolation issues
- Numerical scheme for solving the network
  - Choice of scheme limited by computational resources
- Approximations from hydrodynamic input
  - e.g. one zone clearly not exact
  - Tracer particle methods does allow for mixing

# Summary

Nuclear networks are everywhere in theoretical astrophysics

Depending on the situation, the size of the network can vary (different networks needed for r-process or big bang calculations)

The energy generation of the network can (should) be coupled to hydrodynamics, but often post-processing has to be used