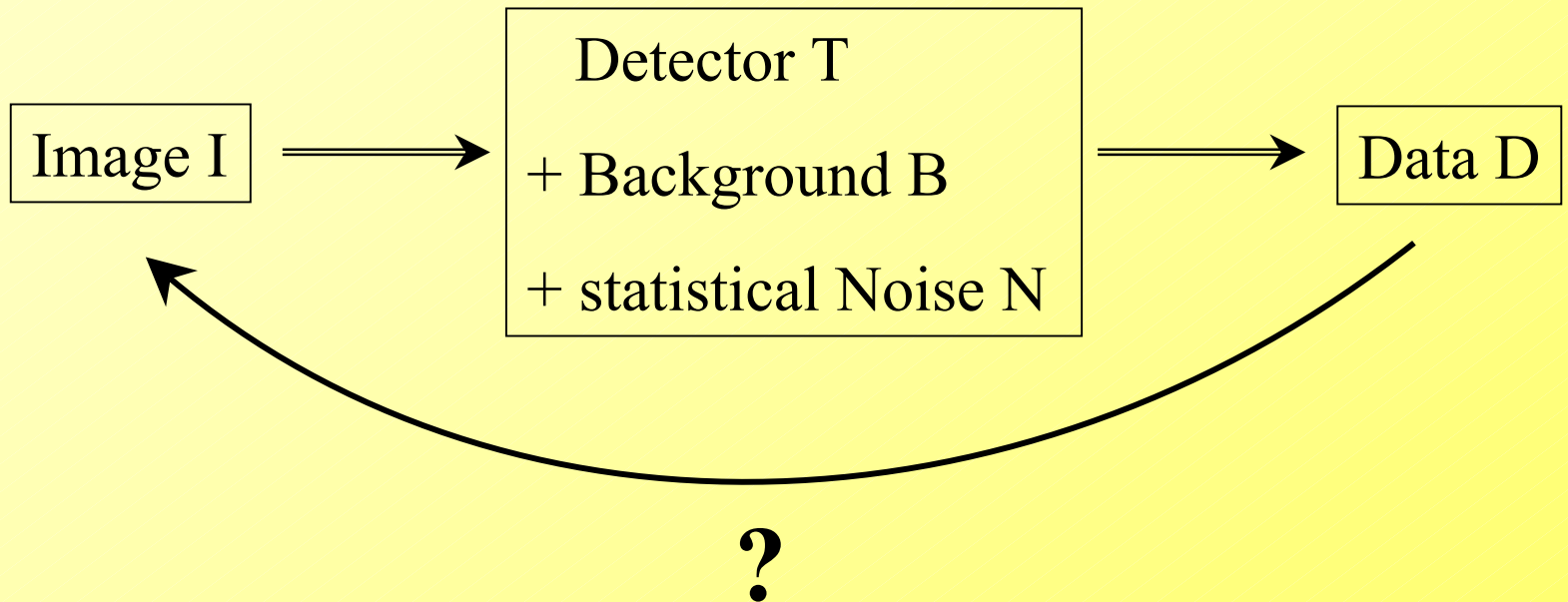


Imaging
in
High-Energy Astronomy

Contents

- Introduction
 - Image Reconstruction
 - Bayes' Theorem
 - Imaging Methods
- Maximum Entropy (ME)
 - Historical ME
 - Classical ME
- Richardson-Lucy
- Multiresolution Regularized Expectation Maximization (MREM)
- Summary

The Problem of Image Reconstruction



$$D = T \cdot I + B + N$$

Bayes' Theorem

$$P(h | D) = \frac{P(h) \cdot P(D | h)}{P(D)}$$

- $P(h | D)$ posterior probability
- $P(h)$ prior
- $P(D)$ evidence
- $P(D | h)$ likelihood

Different Imaging Methods

- Maximum Entropy (ME)
- Maximum Likelihood (ML)
- Richardson-Lucy (RL)
- Multiresolution Regularized Expectation Maximization (MREM)
-

Historical Maximum Entropy

- entropy S of an image:

$$S = - \sum f_i \log f_i$$

where f_i is the intensity in pixel i

➔ S is a measure of structure in the image

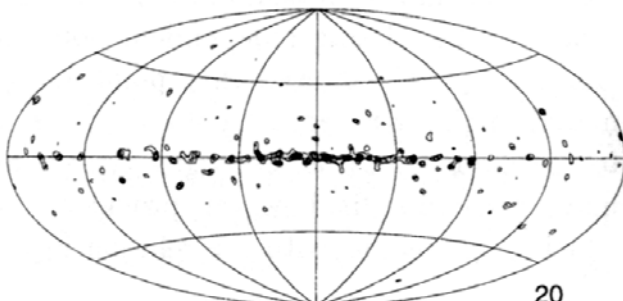
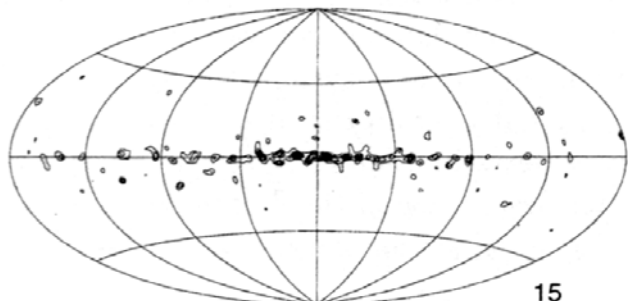
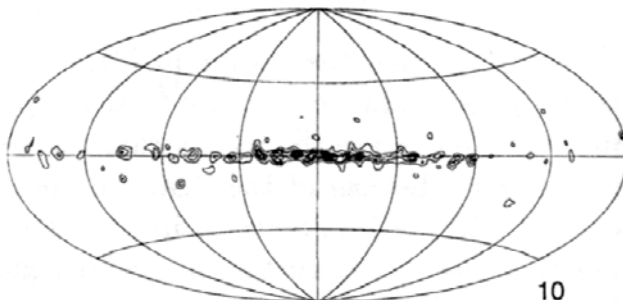
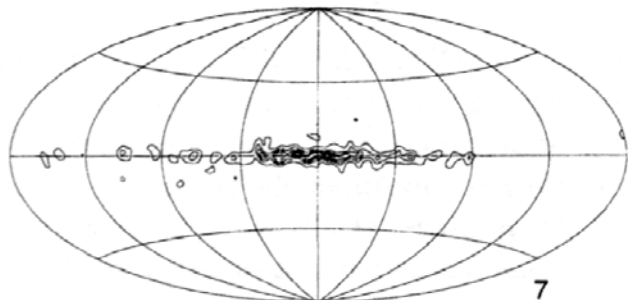
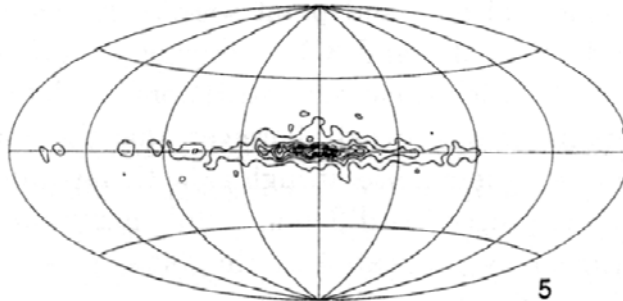
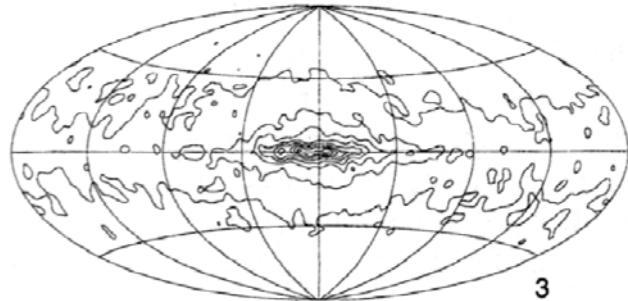
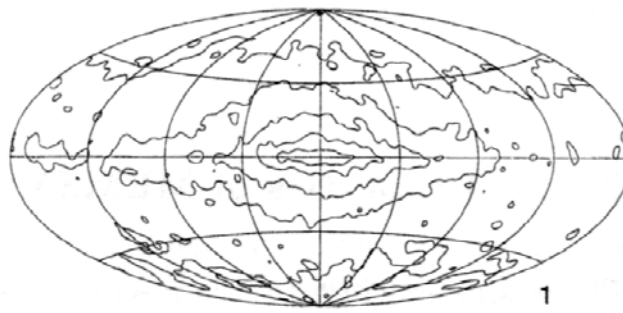
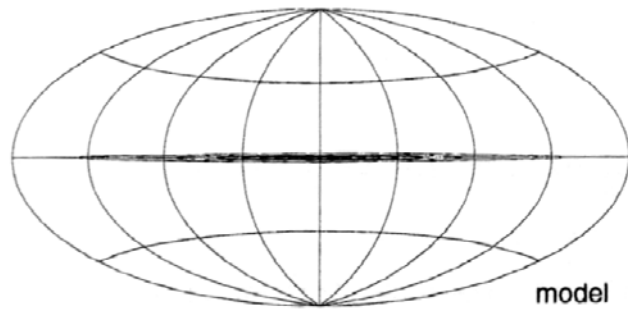
- used statistic: Poisson log-likelihood L

$$L = \log(P(\text{data} | \text{model}))$$

- maximise $C = S - \lambda L$, starting with $\lambda = 0$
progressively increasing λ

Historical Maximum Entropy

- stop process when model first consistent with the data
- ME maximises S over all images which are consistent with the data
 - ➔ i.e. finding the „flattest“ image
- problems:
 - stopping criteria often subjective
 - over-fitting the data
 - no error estimation possible



Example for
data over-
fitting

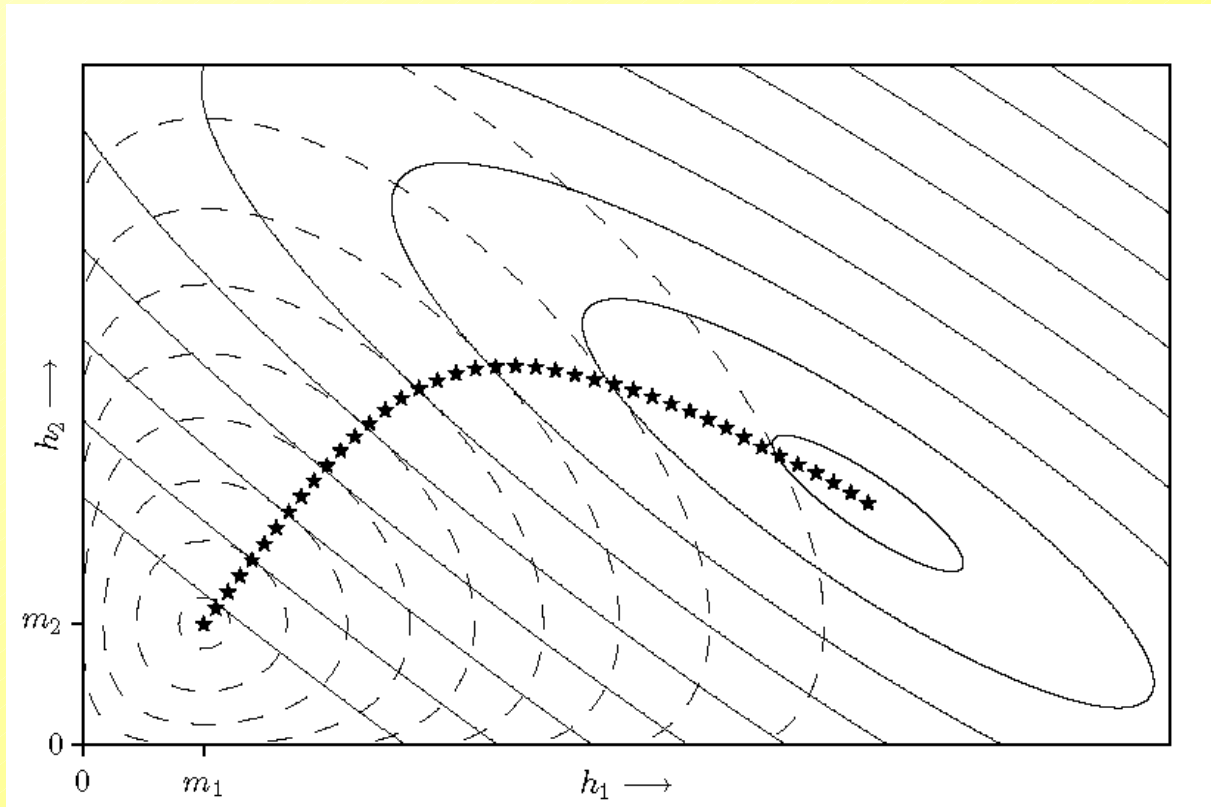
(Knödseder,
PhD Thesis)

Classical Maximum Entropy

- imaging is treated as a Bayesian parameter estimation problem for the pixel intensities
- posterior probability: $P(h | D) \propto e^{\alpha S(h) + L(D|h)}$
- variation of α
 - $\alpha \rightarrow \infty$ entropy dominates (\rightarrow flat image)
 - $\alpha = 0$ data dominates (\rightarrow max. likelihood image)

Classical Maximum Entropy

- find image with maximum evidence



Dis- & Advantages of ME

➤ disadvantages:

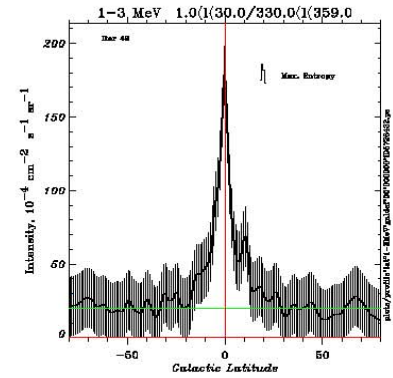
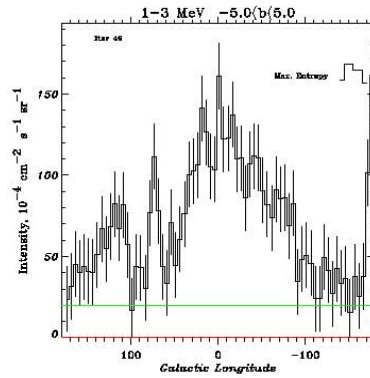
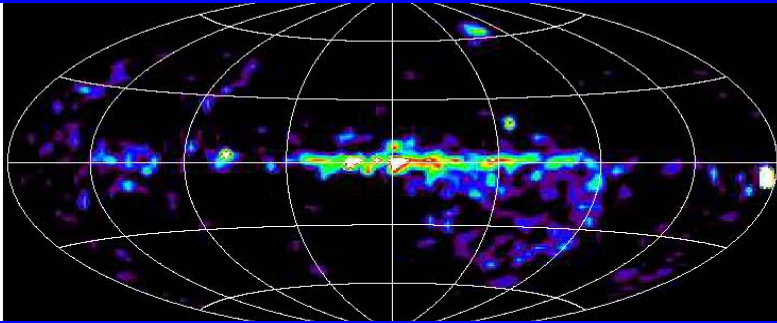
- no correlation between pixels, must/can add it separately
- difficult to handle background

➤ advantages:

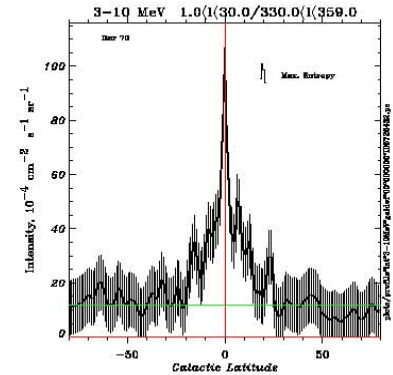
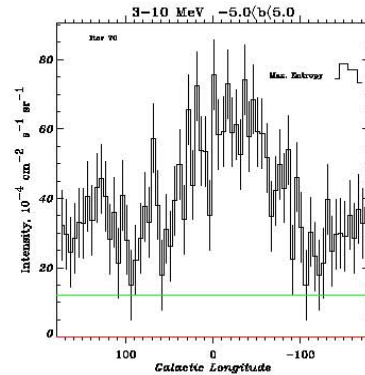
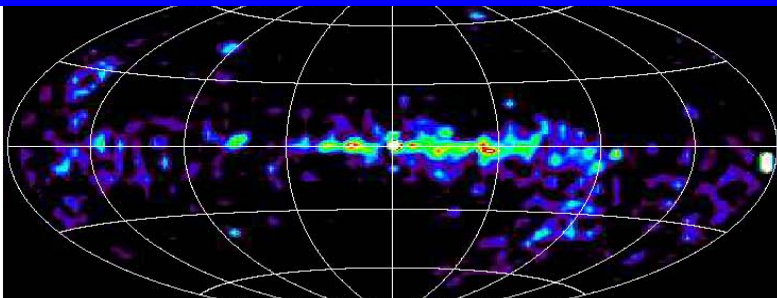
- error bars can be derived from $P(f|D)$
- well-defined criterion for „best“ image

ME on COMPTEL data

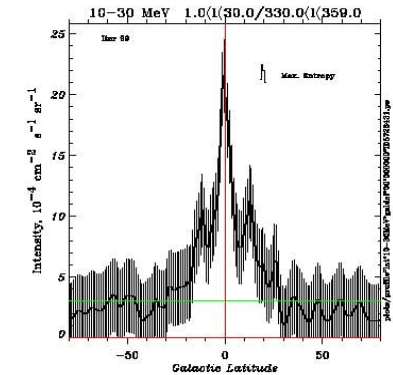
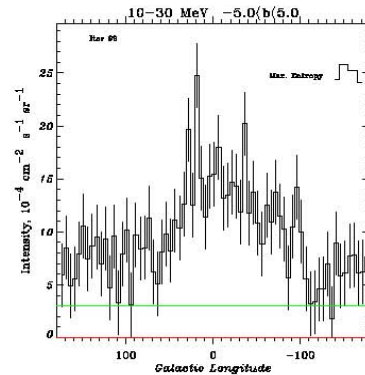
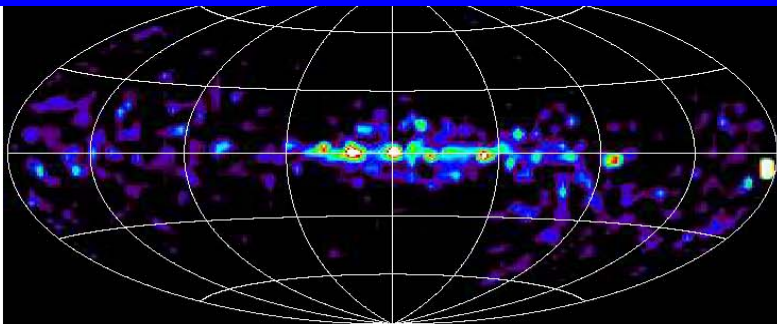
1 - 3 MeV



3 - 10 MeV

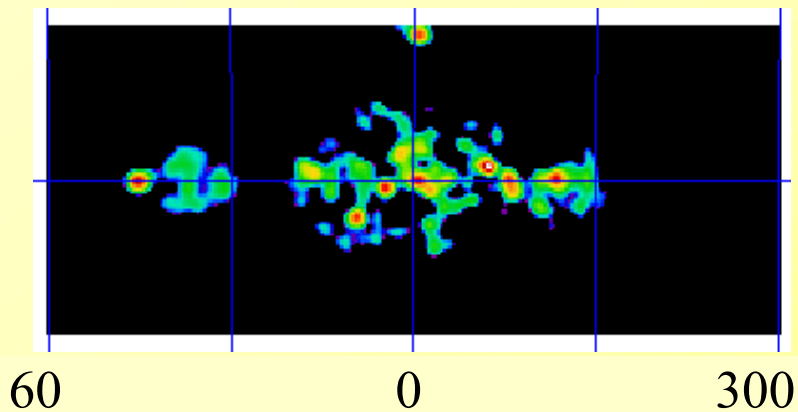


10 - 30 MeV



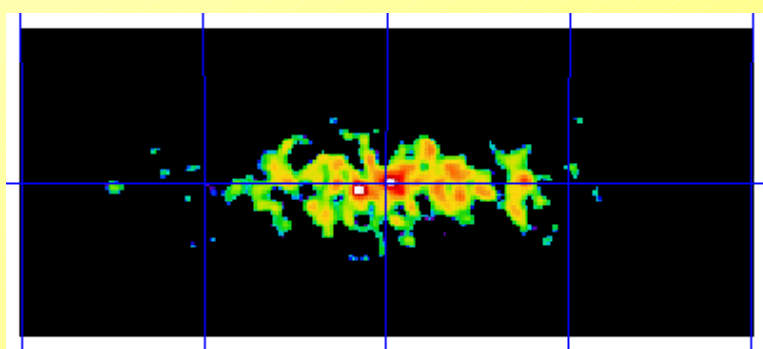
ME on INTEGRAL/SPI data

18-143 keV



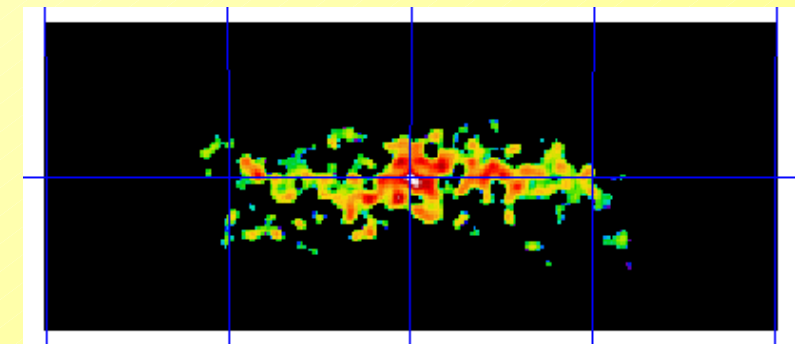
+25

143-268 keV

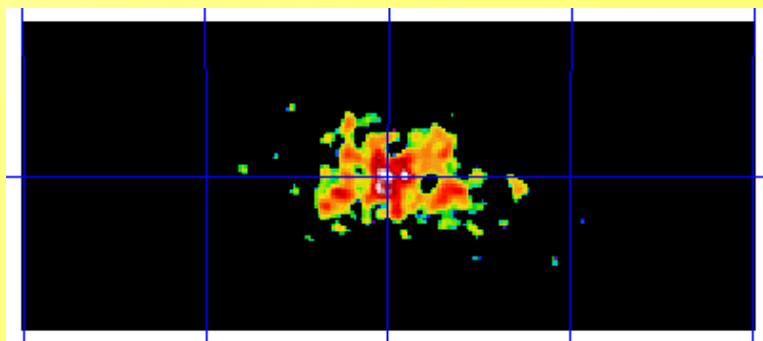


-25

268-393 keV

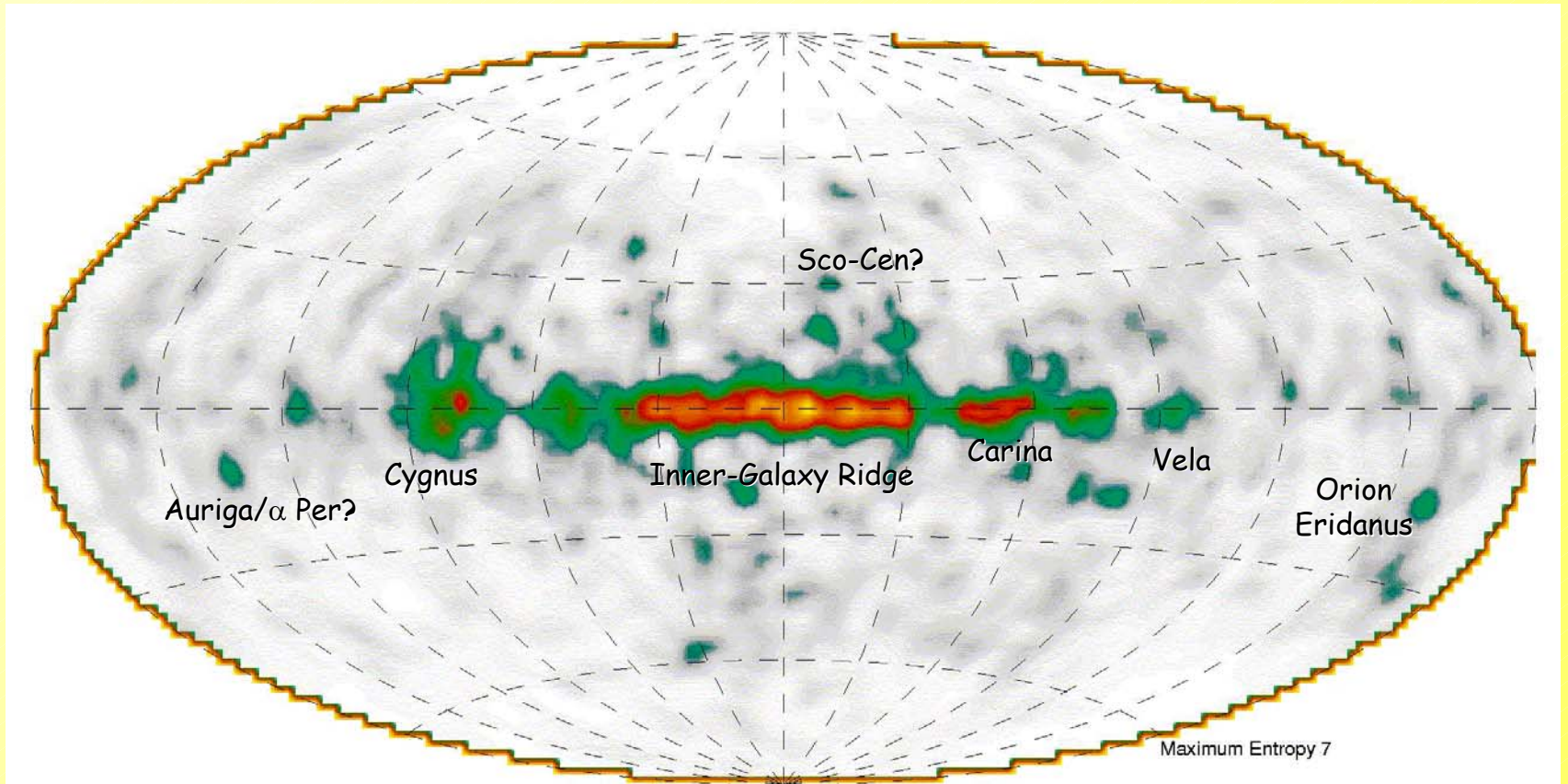


393-518 keV



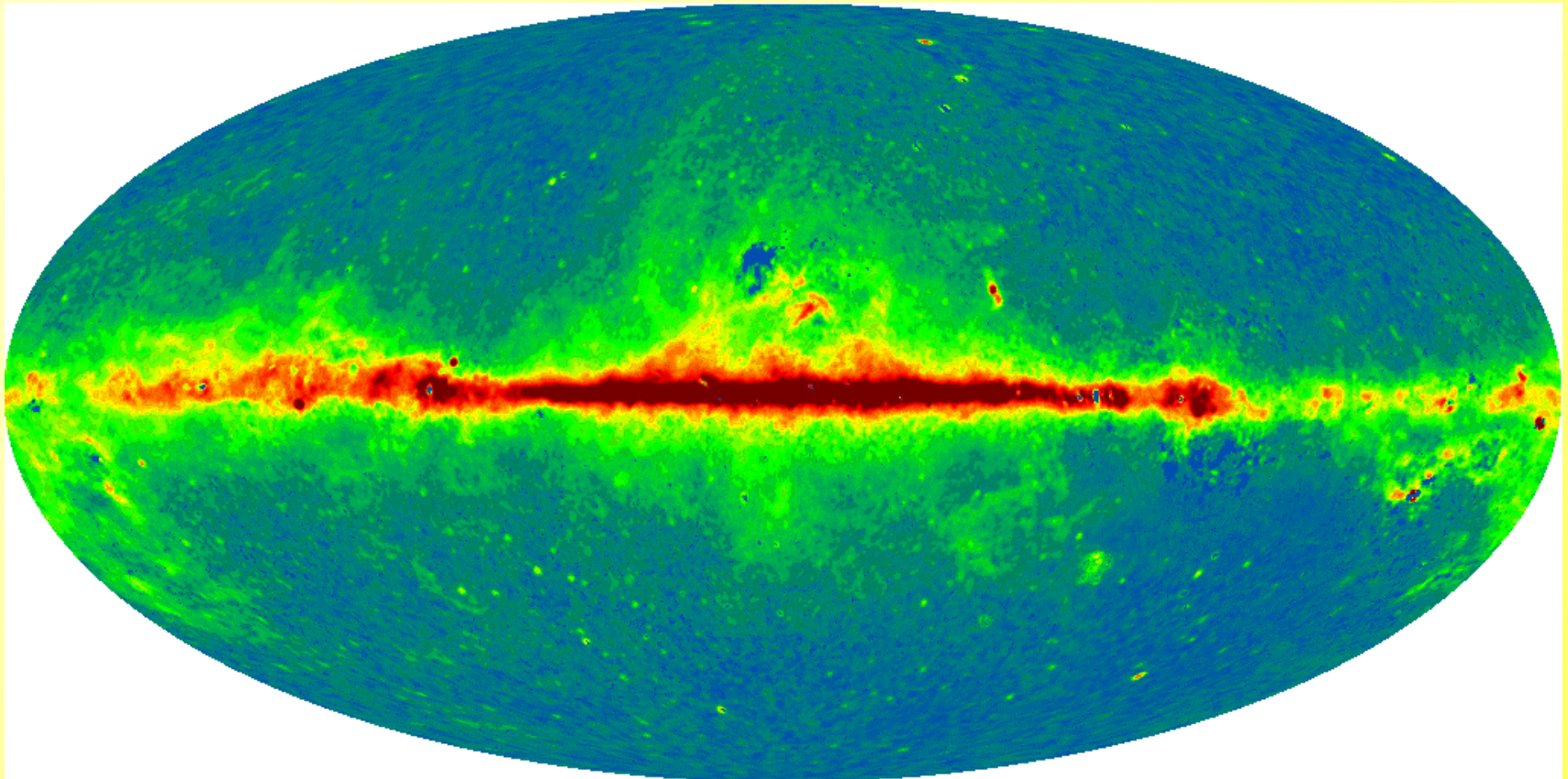
positronium

The Sky at 1809 keV: ^{26}Al



ME on WMAP-data

synchrotron foreground



(WMAP Science Team)

Richardson-Lucy

- iterative improvement of image starting from initial estimate f_j^0

$$f_j^{k+1} = f_j^k \left(\frac{\sum_{i=1}^N \frac{n_i}{e_i^k} R_{ij}}{\sum_{i=1}^N R_{ij}} \right)$$

where $e_i^k = \sum_{j=1}^M R_{ij} f_j^k + b_i$ is the expected number of counts in a data space cell

n_i number of events in data space cell i

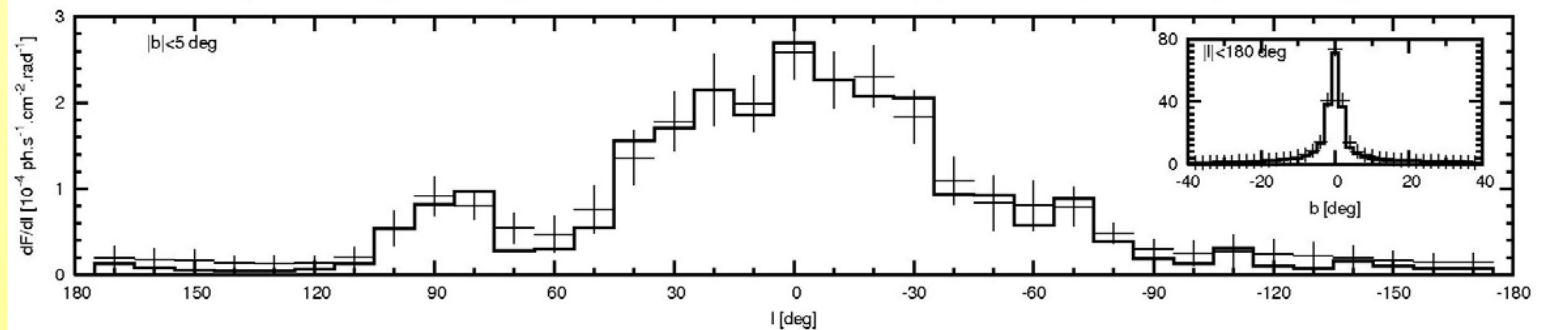
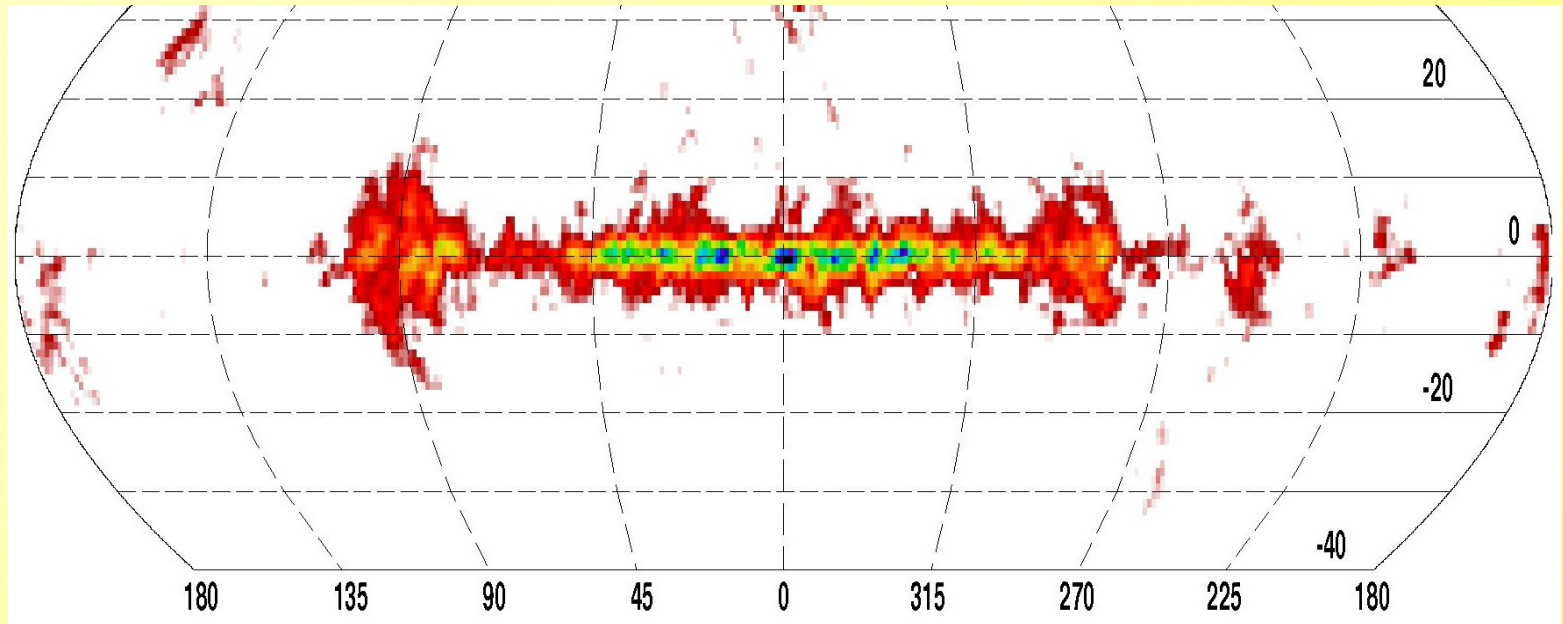
b_i background model

Richardson-Lucy

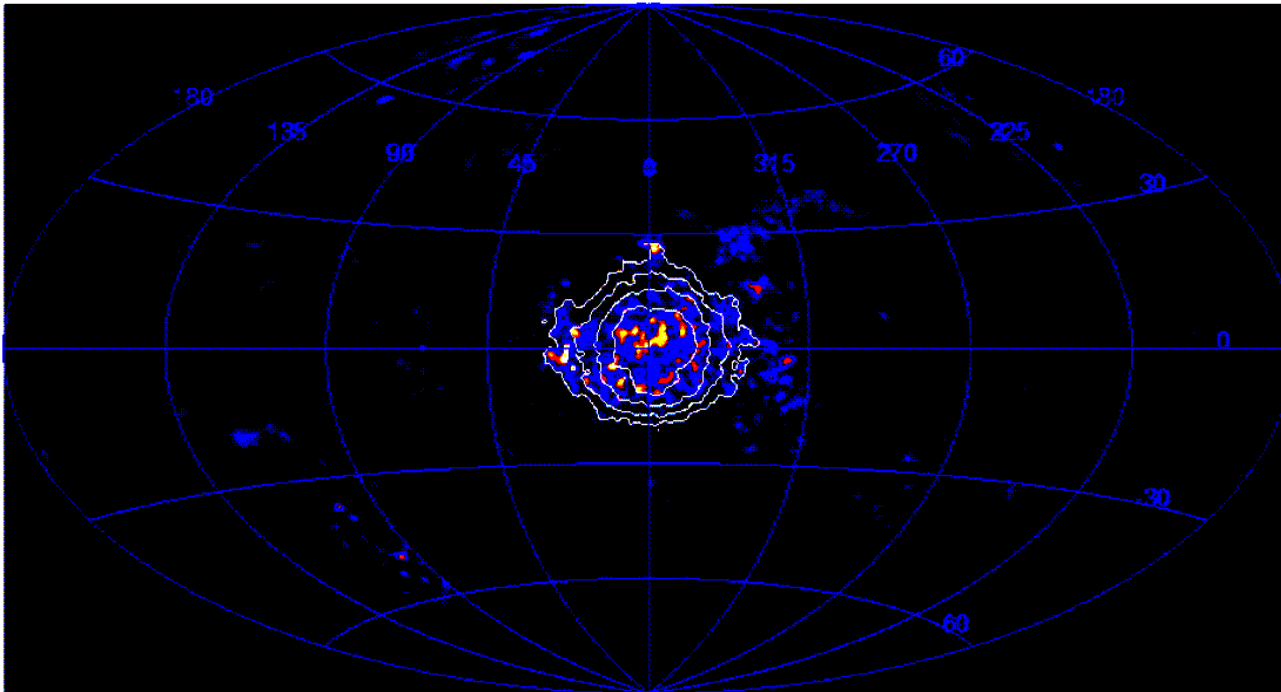
- positivity constraint: $f_j^k > 0 \Rightarrow f_j^{k+1} > 0$
- fast change at the beginning,
slow corrections later on
 - ➔ must stop iterations before
overfitting the data

RL on INTEGRAL-data

^{26}Al -line (1809 keV)

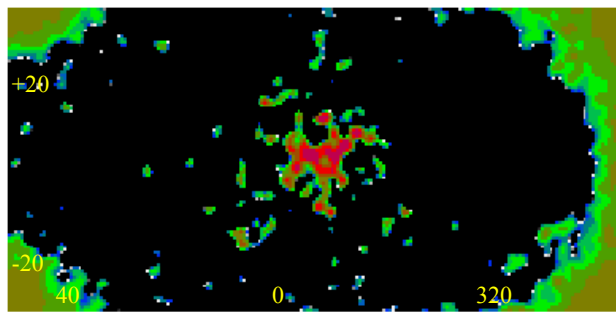


511 keV - line in the Galaxy



Richardson-
Lucy

Knödlseider, 2004



Maximum Entropy

Strong, 2004

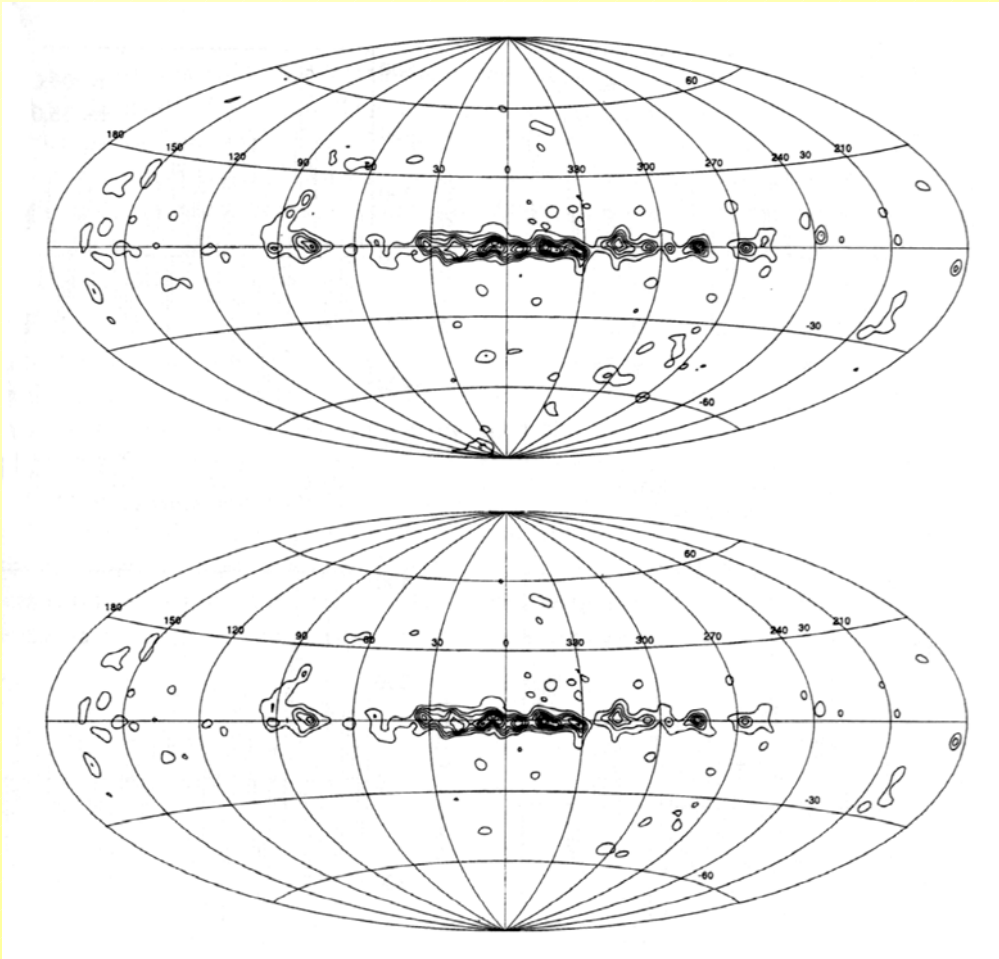
Richardson-Lucy

- disadvantages
 - no clear stopping criterion
 - no error estimation
 - slow convergence speed

- advantages
 - simple to implement

Comparison of ME / RL

COMPTEL 1,8 MeV all-sky maps



Richardson-Lucy
reconstruction
(„optimum“ iteration 6)

Maximum Entropy
reconstruction
(„optimum“ iteration 6)

General Problems

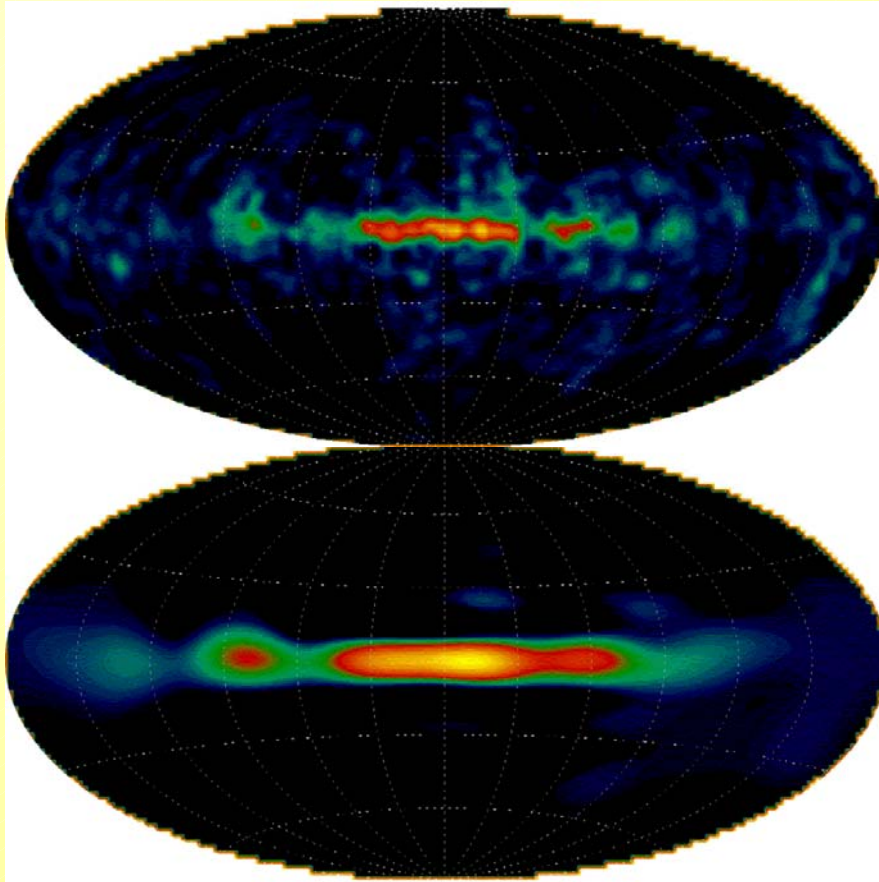
- stopping criterion difficult to find
- grainy restoration of extended features
- clumpy reconstruction of emission, leading to artificial „hot spots“ (indistinguishable from real point-like gamma-ray sources)
- calculations often time and CPU consuming
- What's the real / true image?

MREM

- MREM = **M**ultiresolution **R**egularized **E**xpectation **M**aximization
- developed to reconstruct diffuse gamma-ray emission
- stops when significant structure has been extracted from the data (i.e. not much changing under perturbation of the data)
- significance level of structures a priori chosen, not the angular resolution (by defining pixel size, e.g. ME)

Comparison of MREM / ME

COMPTEL 1.8 MeV Maps



**Maximum-Entropy
Imaging Deconvolution
("ME")**

**Multi-Resolution
Expectation Maximization
Imaging Deconvolution
("MREM")**

Summary

- aim: $D = T \cdot I + B + N \quad \rightarrow \quad I(D) = ?$
- a lot of different methods for imaging:
 - Maximum Entropy
 - Richardson-Lucy
 - MREM
- problems:
 - often difficult to find a stopping criterion
 - estimate the significance of the result

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